Real Price Rigidity and Investment Irreversibility in Customer Markets: Evidence from Credit Supply Shocks

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Abstract

This paper argues that investment irreversibility leads to greater real price rigidity by strengthening the customer-market incentive. In a simple model, predictable demand from a customer base insures firms against future losses from irreversible capital after bad idiosyncratic demand shocks. Thus, firms with more irreversible capital forgo profits today by selling more output to invest in a larger customer base. But when financial constraints tighten and make current profits more valuable, these firms reduce customer investment more sharply, which puts more upward pressure on prices and markups. This behavior makes markups more countercyclical and thus causes greater real price rigidity. I find evidence consistent with this mechanism using producer price microdata and the bankruptcy of Lehman Brothers as a natural experiment in the credit supplied to firms. After credit contractions, firms in low-irreversibility industries cut prices while firms in high-irreversibility industries do not. Markups also rise temporarily in high-irreversibility industries relative to low-irreversibility industries. A back-of-the-envelope exercise implies that credit contractions lower aggregate prices by almost 40% less than they would if prices in all industries responded like prices in the least irreversible industry.

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1 Introduction

Price rigidity shapes the comovement of inflation with aggregate output. The more rigid prices are, the less inflation falls after a decline in output. But why are prices rigid? One answer is the widespread presence of customer markets, the observation that firms accumulate sticky customer bases by selling more output because customers prefer to buy from the same firm over time.[1] Customer markets create rigidity in *real* prices by making markups more countercyclical, especially during financially driven recessions.[2] But how strong is the effect of the customer-market incentive on real price rigidity, and which firm characteristics strengthen this incentive?

This paper argues that investment irreversibility in tangible capital leads to greater real price rigidity because it strengthens the customer-market incentive. Irreversibility amplifies losses from bad idiosyncratic demand shocks in the future, and predictable demand from a sticky customer base insures firms against these losses. To invest in a larger *future* base, firms with more irreversible capital forgo current profits to sell more output *today*. However, if financial constraints tighten and make current profits more valuable, firms with more irreversible capital scale back today's customer investment more sharply, which creates more upward pressure on today's prices and markups. Consistent with this mechanism, I empirically show that firm-specific cuts in credit supply cause prices to fall by less in industries with higher irreversibility. This result is surprising because, under the standard "ex post" intuition, a firm that avoids selling capital due to irreversibility should absorb a cut in credit

[1] See Phelps and Winter (1970), Bils (1989), Gottfries (1991), Rotemberg and Woodford (1993), Ravn et al. (2006), and Nakamura and Steinsson (2011) for theoretical work on customer markets and real price rigidity. Chevalier and Scharfstein (1996), Foster et al. (2016), Gilchrist et al. (2017), Paciello et al. (2019), Renkin and Züllig (2024), and Bauer et al. (2026) find empirical support for customer markets across many industries. Gourio and Rudanko (2014), Peters and Taylor (2017), Chiavari (2026), and He et al. (2026) show that firms also invest in their customer base through selling expenses.

[2] There is empirical evidence that markups are countercyclical, although recent research is mixed. Bils et al. (2018) argues for countercyclical markups while Nekarda and Ramey (2020) finds evidence in favor of procyclical or acyclical markups. Blampied and Mahadeo (2025) provides one reconciliation by showing markups after monetary policy shocks are countercyclical in recessions and procyclical in expansions.

supply by adjusting its price by more, not by less. The estimates imply a quantitatively meaningful increase in real price rigidity in industries with more irreversible capital.

I formalize the mechanism in a two-period partial equilibrium model. I adapt a standard model of customer markets from Ravn et al. (2006) to a two-period setting with partially irreversible investment and uninsurable idiosyncratic demand risk. Irreversibility penalizes the firm when it sells capital after a bad demand shock in the future (second period). A sticky customer base mitigates this problem because demand from the customer base is predictable and unaffected by the demand shock, so a larger customer base functions like precautionary saving. In other words, a larger base lowers a firm's demand risk, which reduces future capital sales after bad shocks and the related losses due to irreversibility. Therefore, firms with more irreversible capital sell more output today (first period) to accumulate larger customer bases as insurance against bad demand shocks in the future.

This stronger customer-market incentive creates more upward pressure on the price and markup response to tighter financial constraints. In a customer market, firms acquire their marginal customer by forgoing current profits, so firms with more irreversible capital forgo more current profits on the marginal customer. Hence, when tighter financial constraints increase the marginal value of liquidity today, firms with more irreversible capital cut back more sharply on their *customer* investment. This behavior creates more upward pressure on prices and markups for firms with more irreversible capital: when prices rise, they rise by more, and when prices fall, they fall by *less*. The key prediction is about the relative response of prices when irreversibility is higher, not the sign. Motivated by empirical evidence from Kim (2021) and Lenzu et al. (accepted) that firms may deal with cuts in credit supply by cutting prices to fire sell inventories, I extend the baseline model to include inventory accumulation. In this extension, prices can fall after financial constraints tighten, but they fall by less when capital is more irreversible. This partial-equilibrium mechanism should increase real price rigidity in general equilibrium during financially driven recessions.

To test this mechanism, I compare the response of product-level prices to firm-specific credit supply shocks in high- versus low-irreversibility industries. Following Chodorow-Reich (2014) and Kim (2021), I construct a shift-share instrument that uses the bankruptcy of Lehman Brothers as a source of plausibly exogenous variation in the credit supplied to firms. This instrument identifies the causal effect of a credit supply shock on prices, and the comparison across industries shows how the response varies with irreversibility, not the causal effect of irreversibility itself. I combine these credit supply shocks with confidential product-level prices from the Producer Price Index (PPI) of the U.S. Bureau of Labor Statistics (BLS), Census Bureau microdata on export prices and markups, and industry-level estimates of physical irreversibility in tangible capital from Kermani and Ma (2023).

One year after the Lehman bankruptcy, an idiosyncratic cut in credit supply by one standard deviation causes firms in low-irreversibility industries to cut prices by 10% while firms in high-irreversibility industries do not cut prices. This difference in behavior is surprising because the standard "ex post" intuition is that irreversibility amplifies other margins of adjustment after a credit supply shock since firms want to limit capital sales. If a firm with low irreversibility cuts prices, then a firm with high irreversibility should cut prices by *more*, not *less*. However, this behavior is consistent with my model's "ex ante" mechanism. Prices fall by less after cuts in credit supply because tighter financial constraints make firms care more about liquidity today and less about insurance against future losses. As a result, firms with more irreversible capital cut back their customer investment more sharply.

Additional evidence points to a stronger ex ante customer-market incentive in industries with higher irreversibility. I obtain similar results using export prices from the Longitudinal Firm Trade Transactions Database (LFTTD), even when I compare the price responses for products with similar demand elasticities. Thus, the result replicates in a distinct dataset, and irreversibility does not seem to be just a proxy for inelastic demand. Among goods with low demand elasticities, exporters in more irreversible industries even increase prices, so firms

in more irreversible industries are not merely insulated from credit supply shocks. Consistent with a difference in customer-market incentives, firms in high-irreversibility industries also temporarily raise markups relative to firms in low-irreversibility industries. Finally, descriptive evidence is consistent with greater ex ante investment in more predictable demand. In industries with higher irreversibility, firm-to-firm contracts are longer, and selling expenses account for a larger share of sales.

The estimated difference in price responses across industries is large enough to change the amount by which aggregate prices fall after a credit contraction. I consider a back-of-the-envelope partial-equilibrium exercise that compares two counterfactuals. One counterfactual scenario supposes that prices in all industries respond to credit contractions like prices in the least irreversible industry. In this scenario, credit contractions alone would have caused prices to fall by almost 12% one year after the bankruptcy of Lehman Brothers. For comparison, the PPI actually fell by 19% over this period.[3] The other counterfactual instead applies the estimated price response for each industry, so prices in high-irreversibility industries fall by less. In this scenario, credit contractions would have caused the PPI to fall by almost 40% less. Thus, the regression estimates imply a quantitatively meaningful effect from irreversibility on real price rigidity after credit contractions.

To check the role of indirect general equilibrium effects, I build a two-industry DSGE calibrated to the regression estimates. I model a steady-state difference in customer-market incentives across industries in *reduced form* rather than explicitly microfounding it as the result of irreversibility. This choice simplifies the analysis and preserves the interaction between indirect effects and the customer-market incentive, which is the important characteristic for the inflation response to credit supply shocks.[4] In the DSGE, since aggregate

[3]The large magnitude of the PPI decline does not contradict the missing deflation puzzle (e.g. Hall 2011, Ball and Mazumder 2011) because the puzzle is about consumer price inflation.

[4]The key assumption is that an aggregate credit supply shock does not cause firms to substantially re-optimize their ex ante demand for customer investment in a way that eliminates the difference in the strength of the customer-market incentive across industries. This assumption is reasonable because the source of the

inflation falls by more when all prices respond as they do in the least irreversible industry, a central bank following a standard Taylor rule cuts the real interest rate by more, and this behavior attenuates the difference in the inflation response to an aggregate credit supply shock by about one half.

Related Literature. This paper's primary contribution is that investment irreversibility in tangible capital leads to greater real price rigidity because it strengthens the customer-market incentive. To the best of my knowledge, this paper is the first to connect two separate strands of the literature, irreversibility and customer markets, to understand how prices behave, especially during financially driven recessions.

This connection points to irreversible capital—and characteristics of the production technology more broadly—as a novel reason why the literature has found that tighter financial constraints increase prices in some settings and decrease them in others. Gottfries (1991), Chevalier and Scharfstein (1996), Gilchrist et al. (2017), Christiano et al. (2015), and Dou and Ji (2021) show that tighter financial constraints create upward pressure on prices due to customer markets and working capital constraints. Renkin and Züllig (2024), Kim and Park (2025), and Bauducco et al. (2026) use plausibly exogenous variation in credit supply to provide empirical support for these channels. However, Kim (2021) and Lenzu et al. (accepted) also use plausibly exogenous variation in credit supply and find that prices tend to *fall*, not rise, due to fire sales of inventories. To explain the difference in signs, Renkin and Züllig (2024) shows that firms in their sample sell goods with low demand elasticities and do not have significant inventory holdings. My results suggest that investment irreversibility also generates upward pressure on price responses alongside demand-side characteristics and inventory holdings. More broadly, the results relate to research showing that financial frictions make the slope of the Phillips Curve state-dependent, such as Acharya et al. (2026),

ex ante demand is idiosyncratic uncertainty, which is far larger than aggregate uncertainty for most firms.

Drechsler et al. (2026), and Liu (2026).

This paper provides a customer-market explanation for why irreversibility is associated with greater price rigidity. Kermani and Ma (2023) shows that irreversibility is correlated with more rigid nominal prices and explains this empirical result with Woodford (2005)'s and Altig et al. (2011)'s theoretical finding that firm-specific capital makes prices strategic complements.[5] In those papers, firm-specific capital means that firms cannot adjust their stock of capital used in current production and must directly invest to adjust their future stock, in contrast to the standard assumption that firms can rent arbitrary amounts of capital for current production from a perfectly competitive market. This paper interprets Kermani and Ma (2023)'s measure of irreversibility as a measure of costly resale rather than of the inability to rent capital for current production. Under this interpretation, this paper provides evidence that the ex ante interaction between irreversibility and the customer-market incentive also explains the correlation between irreversibility and price rigidity. Section 4.4 discusses the distinction between firm-specific capital and costly resale in more detail.

An ex ante interaction between the customer-market incentive and irreversibility also helps explain why customer capital matters even in industries that are not typically associated with consumer brands or product differentiation. He et al. (2026) finds that many business-facing industries spend substantial amounts on selling expenses like salespeople and marketing. According to Foster et al. (2016), markets for homogeneous products like concrete and industrial gases involve demand accumulation through higher sales. In these industries, demand-side returns to a larger customer base are not as obvious as in, for example, consumer or differentiated markets. This paper points out that customer investment in business-facing or homogeneous-product industries could be explained by production technology characteristics like irreversible capital. Other technological characteristics could include operating

[5]Strategic complementarity is a real price rigidity. With small menu costs, a real price rigidity can lead to less frequent price adjustment, which is Kermani and Ma (2023)'s measure of price rigidity. Aruoba et al. (2024) confirms this intuition in a quantitative model.

leverage (see Carlson et al. 2004 and Novy-Marx 2011) and labor irreversibility (see Ghaly et al. 2017, Donangelo et al. 2019, Jäger et al. 2024, and Meeuwis et al. 2025) since these characteristics would also amplify the cost of bad demand shocks.

Outline. Section 2 explains the theoretical mechanism. Section 3 describes the data, empirical methodology, and interpretation of the irreversibility measure. Section 4 presents the price and markup results as well as checks of alternative explanations. Finally, Section 5 uses a back-of-the-envelope exercise to quantify the effect of the ex ante interaction between investment irreversibility and the customer-market incentive on real price rigidity.

2 Irreversibility and Customer Markets

This section presents a stylized two-period partial equilibrium model in which firms face partially irreversible investment and uninsurable demand risk. Because irreversibility amplifies the cost of bad demand shocks in the future, firms have a stronger *ex ante* customer-market incentive since a larger customer base insures against future low demand. However, when tighter financial constraints cause firms to value current profits more relative to future profits, firms with more irreversible capital scale back their customer investment more sharply. This stronger reaction creates more upward pressure on prices and markups. Critically, this reaction occurs in the first period *before* demand uncertainty resolves. Omitted derivations are in Appendix F.

2.1 Setup

There are two periods $t \in \{0, 1\}$. In $t = 0$, the firm produces, and in $t = 1$, the firm's decision is split into the "morning" and the "afternoon." In the morning, the firm learns what demand will be in the afternoon and adjusts its capital stock. Denote the demand state

by $z \in \{B, G\}$: demand will either be bad or good with equal probability. In the afternoon, the firm produces.

The demand state z represents *idiosyncratic* demand risk, not *aggregate* demand risk. Most firms likely consider idiosyncratic risk, rather than aggregate risk, to be the primary source of concern when it comes to the costs of investment irreversibility because micro uncertainty is far larger than macro uncertainty for most firms. I assume this risk is uninsurable with financial assets, so the customer base will be the only way to insure against low demand.

Production and Investment. The firm produces output using capital and labor according to a Cobb-Douglas production function with constant returns to scale and weight α on capital. Labor is paid an exogenous wage w, and the marginal cost of production is the wage divided by the marginal product of labor. The firm arrives in $t = 0$ with a capital stock of k_0, and capital chosen in the morning of $t = 1$ is given by $k_1^z = (1 - \delta)k_0 + i_1^z$, where the z superscript indicates that i_1^z is chosen conditional on knowledge of the demand state z. Investment is partially irreversible: the firm pays i_1^z when it invests ($i_1^z \geq 0$) but recovers only $(1 - \kappa)i_1^z$ when it disinvests ($i_1^z < 0$), where $\kappa \in (0, 1)$ indexes the degree of irreversibility.

Interpret κ as irreversibility due to the characteristics of a firm's production technology as opposed to the firm's product-market characteristics. For example, κ should arise from mobility costs, not from holdup problems.[6] The wedge κ can be a reduced-form parameter for the remaining amount of irreversibility after a firm adjusts its capital structure, such as holding more cash or arranging for extra credit lines.

[6] Williamson (1989) explains how a firm's capital can become irreversible when it has to be customized or otherwise made specific for a downstream buyer, which creates a holdup problem. Irreversibility is an endogenous outcome of the firm's product market (few potential trading partners) rather than an exogenous characteristic of the production technology.

Customer Demand. In both periods, the firm faces a demand curve with intratemporal elasticity of demand $\eta > 1$. However, the amount of demand at a given price is not the same across time. Specifically, the firm sells output y_t at a price p_t according to

$$y_0 = p_0^{-\eta}, \quad y_1^z = \underbrace{(p_1^z)^{-\eta} z}_{\text{searchers}} + \underbrace{(p_1^z)^{-\eta} \chi y_0^{\theta(1-\eta)}}_{\substack{\text{sticky} \\ \text{customer base}}}, \quad \theta(1-\eta) \in [0, 1). \tag{1}$$

In $t = 1$, the firm gets demand from two types of customers: searchers and a sticky customer base. Demand from searchers is uncertain, but demand from the customer base is predictable.[7] The functional form for demand from the customer base comes from (relative) goods-specific habits as in Ravn et al. (2006), which is a standard way to tractably model customer markets. The parameters $\chi \geq 0$ and $\theta \leq 0$ determine how much an extra sale in $t = 0$ shifts future demand.

Why is there demand uncertainty from searchers? A literal interpretation is that the searchers are customers who do not develop habits, so taste shocks, search frictions, or exogenous churn create uncertainty about demand from searchers. For example, households could derive utility from two baskets of goods. Goods-specific habits apply to one basket, and taste shocks apply to the other basket. The demand curve in (1) could also be a metaphor or simplification of long-term contracts. Searchers represent spot-market buyers whose demand is unknown because they did not sign a contract, and the sticky customer base consists of customers who signed long-term contracts. To acquire customers into long-term contracts, the firm needs to set lower prices in $t = 0$.

[7] According to empirical evidence from Larkin (2013) and Baker et al. (2023), firms with larger or stickier bases have lower future cash flow volatility and are less exposed to negative shocks, consistent with the assumption that demand from a sticky customer base insulates a firm from demand risk.

Objective of the Firm. The firm maximizes the objective

$$\varphi d_0 + \frac{1}{R}\mathbb{E}[d_1^z], \tag{2}$$

where d_t are dividends, φ represents the marginal value of internal liquidity in $t = 0$, and R is the gross risk-free interest rate. It is standard to approximate tighter financial constraints as an increase in φ since tighter constraints in many specifications result in a higher φ. A one percent increase in φ can be interpreted as a one percent increase in the required return on an investment because the firm now values $t = 0$ profits more, and any investment must yield a higher return to be worth the opportunity cost.

Discussion of Model Ingredients. I have designed this model's ingredients to make two points: (i) irreversibility strengthens the customer-market incentive to sell more output today, and (ii) greater irreversibility leads to upward pressure on the price response to tighter financial constraints today. I intentionally put the costs of irreversibility into the future rather than today because this model is about how the *anticipation* of bad future demand shocks leads to an *ex ante* insurance motive to invest in the customer base. Because of this ex ante motive, a firm with greater irreversibility adjusts its price by more when financial constraints tighten in the present, before any idiosyncratic demand uncertainty has resolved. This model is not about how irreversibility affects a firm's decisions *ex post* after a bad shock occurs.

This model focuses on the ability of the customer base to insure against low demand and purposefully ignores other options. Notably, I do not allow the firm to adjust its capital stock in $t = 0$. Shutting off investment in $t = 0$ allows for a sharper statement of the interaction between irreversibility and the customer-market incentive. If I did allow investment in $t = 0$, then an increase in the irreversibility wedge κ would not only increase the cost of selling capital in the future but also likely cause the firm to scale down the entire operation in $t = 0$

because the user cost of capital increases with κ. In reality, however, a higher κ would likely reflect a difference in the type of capital used, and the price of this capital could then adjust so that the expected return on the firm's capital offsets the higher user cost. We could also imagine that the initial period $t = 0$ represents normal times during which a firm does not want to make any adjustment to its capital stock due to large fixed costs. For these reasons, it is a reasonable assumption to only allow the capital stock to adjust in $t = 1$.

Another key assumption is the additive form of the $t = 1$ demand curve (1). An alternative choice would be a multiplicative form, where demand from *all customers* is uncertain. The customer base then just shifts the expected volume of demand. I do not take this alternative approach because it does not capture the common notion that the customer base leads to predictable demand.[8] Under multiplicative demand shocks, while the expected return on capital still rises with a larger customer base, exposure to the idiosyncratic demand shock also rises so that a bad demand shock is larger in magnitude. In contrast, when I use the additive form, exposure to the demand shock falls with a larger customer base, which better fits the idea that customer bases provide predictable demand.

Connection to Price Rigidity. This model has a *real* price rigidity, not a nominal rigidity. At the firm level, prices are not literally rigid. However, there is upward pressure on the price and markup responses when tighter financial constraints increase φ, the marginal value of liquidity. This incentive makes markups more countercyclical in general equilibrium because adverse shocks usually tighten financial constraints and push markups higher. This upward pressure on markups offsets the usual downward pressure on marginal costs that causes inflation to fall after declines in output. See Ravn et al. (2006) and Gilchrist et al. (2017) for how customer markets make aggregate prices more rigid in general equilibrium

[8] Dou et al. (2021) models the customer base as deterministic in an environment with liquidity shocks. Using household transactions data, Baker et al. (2023) finds that firms with less customer churn experience a smaller decline in demand in downturns, consistent with the idea that a sticky customer base insulates against demand risk.

through countercyclical markups, especially during financially driven recessions.

2.2 The Price Response to Tighter Financial Constraints

This section characterizes the elasticity of prices in $t = 0$ to φ, the marginal value of liquidity. First, I study the price response when customer markets are shut off. Second, I study the price response when customer markets are turned on. Third, I study why irreversibility affects the price response by strengthening the customer-market incentive. Fourth, I explain that the model's key prediction is about the relative magnitude of the price response to a shift in φ and not the sign.

No Customer Markets. Without customer markets, the firm sets the price in each period equal to a constant markup over the marginal cost. Because an increase in φ has no effect on the marginal cost in $t = 0$,[9] the firm actually does not respond at all. The firm's choice of production is purely static, so it already maximizes profits from labor in $t = 0$, and the firm has no other way to increase dividends. Further, the size of the irreversibility wedge κ has no effect on the firm's price choice in $t = 0$.

Adding Customer Markets. With customer markets, the firm chooses the price

$$p_0 = \frac{\eta}{\eta - 1}\left(\underbrace{mc_0}_{\text{marginal cost}} - \underbrace{\frac{\lambda}{\varphi}}_{\substack{\text{customer-market} \\ \text{incentive}}} \right), \quad \lambda = \frac{1}{R}\mathbb{E}\left[\frac{\partial d_1}{\partial y_0}\right]. \tag{3}$$

The firm's price in $t = 0$ equals the standard monopolist's markup over the difference of two terms: the marginal cost and the customer-market incentive. Because a sale in $t = 0$ not only generates revenue but also brings future demand in the afternoon of $t = 1$, a firm

[9] The marginal cost comes from hiring more labor. An earnings-based or collateral-based constraint would generally imply no effect on the cost of another hour of labor input, but a working capital constraint on the wage bill would.

has an incentive to cut its price, even though a lower price causes the marginal sale to strictly reduce the firm's profits in $t = 0$. The strength of this customer-market incentive to cut the price depends on how much the marginal sale improves future expected dividends after discounting by the marginal value of liquidity φ and the gross interest rate R. In this model, the firm adjusts its price after a shift in φ as follows:

Proposition 2.1. *The firm increases its log price and log markup in* $t = 0$ *with* $\log \varphi$*:*

$$\frac{\partial \log p_0}{\partial \log \varphi} > 0 \text{ and } \frac{\partial \log \mu_0}{\partial \log \varphi} > 0. \tag{4}$$

This proposition is in line with models of customer markets and financial constraints in previous research (see Gottfries 1991, Chevalier and Scharfstein 1996, and Gilchrist et al. 2017), and it is not the primary contribution of this paper. The primary contribution is a novel mechanism that links irreversibility to price rigidity through the customer-market incentive.

Another intuition for why prices rise is that the customer-market incentive makes the firm behave as though it faces a more elastic demand curve in the long run than in the short run. In this model, the short-run elasticity is $-\partial \log y_0 / \partial \log p_0 = \eta > 1$, so lower prices lead to more output sold.[10] However, the firm's price p_0 affects not only current output sold y_0 but also future output sold y_1, so the firm sets p_0 based on the present value of output sold, not just the current period's value. Thus, the firm behaves as if p_0 has a bigger effect on y_0 than it actually does. In other words, the long-run demand elasticity is higher than the short-run elasticity. An increase in the marginal value of liquidity φ reduces the present value of future output sold and makes the firm behave as if its long-run demand elasticity has fallen, which pushes p_0 up, just like a markup shock to η.

[10] This result holds because I assume customers have *relative* deep habits as opposed to *additive* deep habits. In the latter specification, the short-run elasticity becomes endogenous with the level of output sold. See Ravn et al. (2006) and Binsbergen (2016).

The optimality condition for the firm's price in the afternoon of $t = 1$ remains the same as the model without customer markets because the firm has no incentive to accumulate a customer base in $t = 1$. The level of the firm's price may not be the same because the total amount of demand in $t = 1$ influences the price through the marginal cost, but the firm's markup over marginal cost is just $\frac{\eta}{\eta-1}$.

Higher Irreversibility Strengthens the Customer-Market Incentive. Proposition 2.2 states this model's key result:

Proposition 2.2. *Assume the demand states B and G are sufficiently far from each other, the irreversibility wedge κ is sufficiently small, and χ is not too large relative to k_0, so that the firm disinvests in the morning of the $t = 1$ bad state ($i_1^B < 0$) and invests in the morning of the $t = 1$ good state ($i_1^G \geq 0$). Then a firm with higher irreversibility*

(i) invests more in the customer base, $\dfrac{\partial \log y_0}{\partial \kappa} > 0$, and

(ii) has a higher elasticity of prices and markups to shifts in φ:

$$\frac{\partial^2 \log p_0}{\partial \kappa \, \partial \log \varphi} > 0 \text{ and } \frac{\partial^2 \log \mu_0}{\partial \kappa \, \partial \log \varphi} > 0. \tag{5}$$

The assumptions of this proposition ensure the analysis is not moot. If B is not sufficiently low, then the firm may not sell capital, in which case κ is irrelevant. If G is not high enough, then the good state is not really a good state. If κ is too high, then the firm may simply not sell any capital when $z = B$, in which case a marginal increase in κ has no effect on outcomes. Finally, χ parameterizes the impact of y_0 on the future customer base in the afternoon of $t = 1$. If it is too large relative to the firm's initial capital stock k_0, then the firm may not sell any capital when $z = B$. The mathematically precise statements of these assumptions are provided in Appendix F.

The intuition for Proposition 2.2(i) is that higher irreversibility raises the cost of each capital sale when demand is low, and a larger customer base reduces how much capital is sold by dampening the fall in demand. The following two derivatives illustrate why the firm considers a larger irreversibility wedge κ to be painful. Holding y_0 constant,

$$\frac{\partial d_1^B}{\partial \kappa} = i_1^B < 0, \quad \frac{\partial d_1^G}{\partial \kappa} = 0.$$

These two derivatives show how κ affects future dividends if the firm cannot adjust its sales in $t = 0$. A small increase in κ reduces the firm's dividends in the bad state in proportion to the units of capital already being sold but has no effect on d_1^G because the firm does not sell capital in that state. If we treat y_0 as a parameter instead of an optimal choice and differentiate with respect to it, then we obtain

$$\frac{\partial^2 d_1^B}{\partial y_0 \, \partial \kappa} = \frac{\partial i_1^B}{\partial y_0} \propto \frac{\partial (B + \chi y_0^{\theta(1-\eta)})}{\partial y_0} > 0, \quad \frac{\partial^2 d_1^G}{\partial y_0 \, \partial \kappa} = 0.$$

The first mixed derivative shows that an increase in κ reduces d_1^B by less because higher output sales y_0 cut how much capital the firm sells in the morning of $t = 1$. The reason the firm sells less capital is that a higher y_0 leads to a higher level of demand in the afternoon of $t = 1$. The marginal unit of capital is now worth keeping instead of selling. Since the customer-market incentive λ/φ is just the present discounted expected effect of y_0 on future dividends, the customer-market incentive λ increases with κ and leads to higher output sales in $t = 0$.[11]

Proposition 2.2 shows that higher irreversibility strengthens the customer-market incentive, and as a result, the sensitivity of prices to shifts in liquidity demand increases. When

[11] Higher sales in $t = 0$ mean the firm sets a lower price and thus a lower markup in $t = 0$. Empirically, markups seem to increase with the size of a customer base (see Baker et al. 2023). An alternative demand curve like in Binsbergen (2016) would allow the demand elasticity to fall with the size of the customer base, which would generate higher markups. The results in this model should still go through as long as a firm still has an incentive to make more sales in $t = 0$ to accumulate the customer base.

the irreversibility wedge κ increases, the firm forgoes profits in $t = 0$ to obtain a larger customer base in $t = 1$ if demand from searchers is low. But if the firm needs liquidity in $t = 0$ because φ rises, then the firm cuts back on sales and hikes prices much more aggressively than a firm with lower irreversibility. The intuition is that, for small κ, the firm only loses a little money on the marginal customer because marginal revenue is still close to marginal cost, while, for high κ, the firm accepts a very large loss to make one more sale. Thus, a one percent cut in output sold reduces the firm's marginal loss by a smaller percentage when κ is high. When the marginal value of liquidity in $t = 0$ rises, a firm with more irreversible capital needs to cut back on output sold more sharply to reduce its marginal loss by one percent than a firm with less irreversible capital.

Relative Magnitudes, Not the Sign. The key prediction of this model is that higher irreversibility increases the elasticity of prices to the marginal value of liquidity φ, not that prices rise. In the baseline model with only customer markets, the elasticity is always positive. The elasticity can be negative, however, once the firm has a price-cutting lever to raise liquidity. Motivated by Kim (2021), Lenzu et al. (accepted), and my own empirical evidence, I introduce inventories into the model as a way for the firm to raise liquidity while cutting prices. Appendix G shows that even if the elasticity of prices to φ is negative, a higher irreversibility wedge κ still increases the elasticity. In other words, a firm optimally cuts its price after a rise in $t = 0$ liquidity demand, and a firm with higher irreversibility cuts by less.[12]

The intuition behind this result is that the customer base and inventory stock are two assets from which the firm disinvests after a rise in φ, and the price response depends on whether the firm ultimately cuts or hikes the amount of output sold. With inventories, there is a distinction between output *produced* and output *sold*. A firm can simultaneously cut

[12] I also show that the markup result remains true under an additional *sufficient* condition that the marginal benefit to customer acquisition is sufficiently strong. See Appendix G.3.

output produced and increase output sold, which pushes prices down, if the firm accumulates fewer inventories. However, since inventories do not reduce demand risk the way that a sticky customer base does, greater irreversibility still strengthens the customer-market incentive. As a result, a firm with greater irreversibility will cut back relatively more on its production, which can lead to an elasticity of price p_0 to φ that is negative and increases with the irreversibility wedge κ.

3 Data and Empirical Methodology

This section describes how I will provide empirical support for the theory in Section 2. I first describe my datasets followed by the construction of the credit supply shock, the main regression specification, and the identification assumptions.

3.1 Data Sources

Description. I construct a novel dataset, which I call the PPI-Census-Dealscan dataset, using (i) confidential item-level price data underlying the Producer Price Index (PPI) published by the Bureau of Labor Statistics (BLS),[13] (ii) bank lending information from Dealscan, (iii) estimates of investment irreversibility by industry from Kermani and Ma (2023), (iv) balance-sheet information from Mergent FISD, Compustat, and the Quarterly Financial Report in Economic Census Years (QFRCEN), and (v) other confidential microdata from the Census Bureau. See Appendix E.1 for details on how I clean and construct this dataset.

The BLS constructs the PPI through monthly surveys of about 100,000 items from approximately 25,000 establishments across many industries, including services. Price quotes are based on transactions and adjusted for quality changes or product substitutions so that

[13] My access to the PPI microdata was indefinitely suspended as of September 2025 due to limited resources at the BLS to support external research projects.

true price changes are measured. Items are uniquely identified by price-determining characteristics, such as the type of buyer, color, and size. The BLS samples establishments to be representative of the entire U.S. production sector, and items are randomly sampled from all the items produced by a given establishment. See U.S. Bureau of Labor Statistics (2014a) and U.S. Bureau of Labor Statistics (2014b) for more details on the survey methodology and quality adjustment procedures, respectively.

I use Dealscan to measure bank lending to firms. This database provides comprehensive historical information on large syndicated loans at issuance and after amendments. In this market, a group of banks comes together as a syndicate to issue a loan. One or more banks work as the "lead arranger," which drafts the loan agreement, performs due diligence, and recruits participants. Typically, the lead arranger retains a large share of the loan.[14] While Dealscan does not cover all bank loans, Chodorow-Reich (2014) shows that it comprises nearly half of all commercial and industrial bank loans and two-thirds of lending with a maturity greater than 1 year.

My analysis proxies investment irreversibility at the industry level as one minus the predicted industry-average recovery rate of PPE based on physical attributes.[15] This measure comes from Kermani and Ma (2023), which estimates the recovery rate on property, plant, and equipment (PPE) by 2-digit SIC industry using data from Chapter 11 bankruptcy filings. The authors show that Chapter 11 filings require an orderly liquidation analysis (OLA). An

[14]Using confidential microdata from the Shared National Credit Registry (SNC), Blickle et al. (2022) shows that the lead arranger does not always retain a large share because the arranger often sells a portion of its stake after loan issuance. As a result, the raw Dealscan data may overstate the loans issued by lead arrangers. My analysis, however, uses code provided by Blickle et al. (2022) that predicts the actual stake retained by each participant in a syndicated loan.

[15]This paper focuses exclusively on the irreversibility of *tangible* capital and does not consider *intangible* capital. The primary reason is measurement: I argue Kermani and Ma (2023) provides a credible measure of irreversibility for tangible capital, but there is no similar measure for the irreversibility of intangible capital. Further, such a measure would also need to exclude the role of *customer capital* since investment in future demand is itself a form of intangible capital. Nevertheless, in principle, firms with more irreversible intangible capital, such as intellectual property or organizational capital, should also want stickier customers to deal with demand uncertainty.

orderly liquidation differs from a forced liquidation or fire sale because the firm would get 6–12 months to liquidate, which mitigates the size of fire sale losses. The key object from an OLA is the expected recovery rate for PPE as a fraction of the PPE's book value. This rate can be interpreted as a measure of the irreversibility wedge κ from Section 2 because one minus the recovery rate is precisely the fraction of value that would be lost when the firm sells tangible capital. In Section 3.4, I discuss in more detail why I use the *predicted* recovery rates to measure irreversibility. A 10pp increase in my measure of irreversibility means that a firm expects to get 10pp less as a share of the book value of its tangible capital.

Several of the tests in Kermani and Ma (2023) support this interpretation of the recovery rate as a measure of investment irreversibility for tangible capital. Differences in the average recovery rate across industries can be explained by industry-level differences in the physical attributes of the capital used.[16] Firms in industries with lower recovery rates sell capital less frequently and cut investment rates by more after an increase in uncertainty, which is consistent with a stronger real options channel due to higher irreversibility. These two results also assuage the concern that recovery rates only predict behavior during bankruptcy and not during normal times.

I supplement the previous data sources with firm financial information and additional microdata from the Census Bureau. Mergent FISD provides information about previous bond issuance. For some regressions, I add balance-sheet controls like book leverage using data from Compustat and the QFRCEN, and I control for firm age and employment size using the Longitudinal Business Database (LBD). I also study the behavior of export prices, markups, inventories, and capital stocks with data from the Longitudinal Firm Trade Transactions

[16] They consider four attributes: mobility, customizability, durability, and the share of structures in capital. Mobility is measured as transportation costs of capital equipment from producers to users divided by production costs incurred by the industries which produced the equipment. Customizability is measured as design costs incurred by equipment producers divided by production costs. Durability is measured using an industry's depreciation rates from Compustat. Structure shares are measured from the BEA detailed fixed asset tables.

Database (LFTTD) and Annual Survey/Census of Manufactures (ASM/CMF).

Although the PPI-Census-Dealscan dataset has shortcomings, it has unique advantages that make my empirical design feasible. Critically, I need price data from firms that face idiosyncratic credit supply shocks and heterogeneity in irreversibility. Because my irreversibility measure is at the industry level, ACNielsen scanner price data in Kim (2021) is not usable since it primarily covers the food and beverage industry whereas the PPI covers many more industries. Samples from other countries may be larger in size due to better collection of price and quantity data, but to the best of my knowledge, other countries do not have cross-industry data on irreversibility.

Sample Characteristics. Table B.1 summarizes key characteristics of the main sample. The main sample consists of approximately 350 firms that appear in both the PPI and Dealscan. Most of the firms are public, over half are manufacturers, and over half have issued bonds. Firms in my sample are mostly large employers and old. The mean value of irreversibility is 63% and from 57% at the 10th percentile to 71% at the 90th percentile. The table also summarizes the credit supply shock and Lehman exposure IV, which are defined in Section 3.2. The distributions of these two variables are similar to Kim (2021)'s sample, which only includes producers of consumer packaged goods, so these measures of credit supply are not driven by the industries of the borrowers. Finally, I find that the correlation in monthly inflation rates over 2007:Q3–2009:Q3 between the aggregate price index implied by my sample and the aggregate price index implied by the entire PPI microdata is about 0.59.[17] This moderately high correlation suggests that the pricing behavior in my sample is fairly similar to the broader economy's.

[17]The small sample size limits the number of estimates that I could disclose, so I just report the correlation instead of plotting the time series for both indices.

3.2 The Credit Supply Shock and Regression Specification

This section explains why I choose credit supply shocks to provide empirical support for the theory in Section 2, how I measure credit supply shocks, the regression specification, and the identification assumptions.

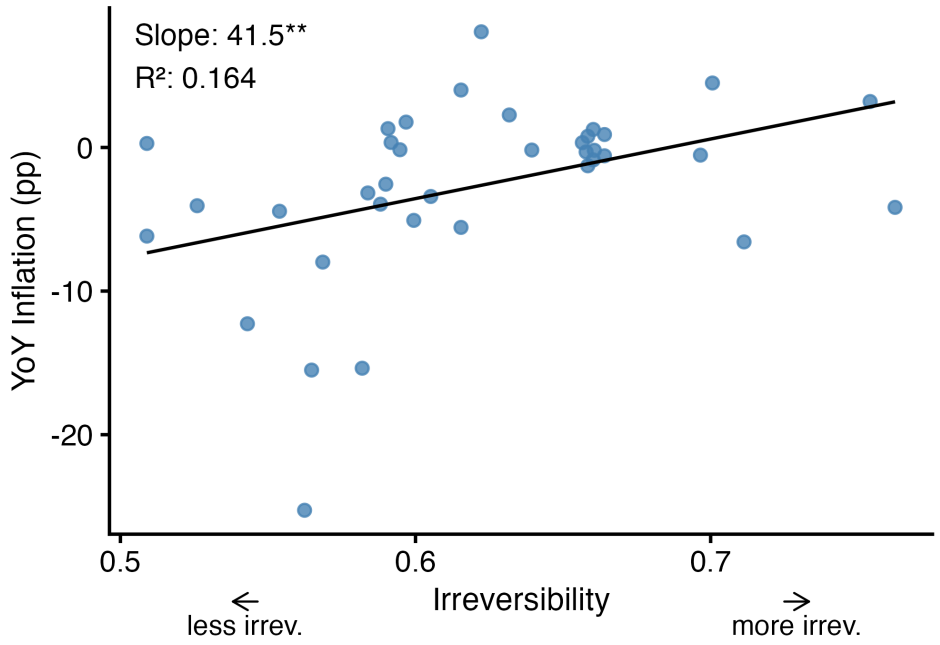
Why Credit Supply Shocks for Empirical Variation? My empirical analysis uses firm-specific credit supply shocks to proxy for shifts in the marginal value of liquidity φ and compares the impact on prices in industries with high irreversibility to the impact in industries with low irreversibility. In principle, other empirical shocks could also test the prediction that higher irreversibility strengthens the customer-market incentive. In my model, an increase in φ is equivalent to an increase in the gross interest rate R. An alternative empirical strategy could be a regression of prices on monetary policy shocks interacted with irreversibility.

This paper uses credit supply shocks over other discount rate shocks like monetary policy shocks because the statistical power of credit supply shocks should be much larger. It is well established (see Ramey 2016, Bauer and Swanson 2022) that many monetary policy shock series involve small shocks. In contrast, credit supply shocks during the Global Financial Crisis were likely both large for a given firm and heterogeneous across firms because some banks were much more exposed to the subprime mortgage crisis.

It is also important to use variation in credit supply across firms within the same industry because a naive comparison of price changes by industry does not provide causal estimates of credit supply shocks on prices. For example, because the bankruptcy of Lehman Brothers reduced credit supply to non-financial firms, we might be tempted to test the theory in Section 2 by plotting industry-level inflation rates after the bankruptcy of Lehman Brothers against irreversibility. Figure 1 shows that this approach is consistent with the theory: industries with high irreversibility tend to have less negative year-over-year inflation

rates across industries at the 3-digit NAICS level. However, industries may experience other shocks like productivity shocks, which confound an interpretation of this figure's price changes as the result of a shift in the marginal value of liquidity φ. For this reason, I turn attention to plausibly exogenous variation in credit supplied to individual firms.

Figure 1: Prices Fall by Less in High-Irreversibility Industries After the Lehman Bankruptcy



Notes: This figure plots the year-over-year inflation in 3-digit NAICS PPI indices over September 2008–2009 against irreversibility from Kermani and Ma (2023). I exclude NAICS 211 (oil & gas extraction) as an outlier since its inflation rate is driven by the collapse in oil prices over 2008–2009.

Definition of the Credit Supply Shock. I measure credit supply shocks caused by the bankruptcy of Lehman Brothers in September 2008 following Chodorow-Reich (2014). The change in bank b's credit supply to firm f from the pre-Lehman period to the post-Lehman period is

$$\Delta L_{f,b} = \frac{\#\text{ loans to all firms except } f,\ \text{Oct. 2008–Jun. 2009}}{\frac{1}{2}(\#\text{ loans Oct. 2005–Jun. 2006} + \#\text{ loans Oct. 2006–Jun. 2007})}. \tag{6}$$

The numerator counts the number of loans lent in the post-Lehman period while the denominator counts the average number of loans over two pre-Lehman periods.[18] A decrease in $\Delta L_{f,b}$ means that bank b has issued relatively fewer loans to firms other than f during the post-Lehman period compared to the pre-Lehman period. If bank b has issued fewer loans to all other firms, then its likelihood to supply loans to firm f has probably fallen, too.

To obtain a firm-level credit supply shock, I aggregate $\Delta L_{f,b}$ across banks in the firm's last loan syndicate. Specifically, let S_f denote the set of banks which comprises the last syndicate for firm f and $s_{f,b,\text{last}}$ the share lent by bank b.[19] Then

$$\Delta L_f = \sum_{b \in S_f} s_{f,b,\text{last}} \, \Delta L_{f,b}. \tag{7}$$

For interpretability of coefficients, I use $\Delta \widetilde{L}_f$, which rescales ΔL_f to unit variance.

My primary analysis instruments the credit supply shock $\Delta \widetilde{L}_f$ because $\Delta \widetilde{L}_f$ may not be plausibly exogenous for reasons that I discuss below in Section 3.3. My instrument is the Lehman co-syndication IV, which is the fraction of a bank's syndication portfolio where Lehman Brothers had a lead role, following Ivashina and Scharfstein (2010) and Chodorow-Reich (2014).[20] An advantage of using an instrument instead of reporting the reduced form is that the credit supply variable $\Delta \widetilde{L}_f$ has more interpretable units than the co-syndication IV. The IV is a shift-share instrument. There is a *single aggregate* shift (the Lehman bankruptcy in September 2008) and firm-specific exposure weights because a firm's lenders may have co-syndicated loans with Lehman. The intuition for why this IV provides exogenous variation in credit supply is that borrowers with credit lines from Lehman could not draw credit from Lehman anymore, so these borrowers precautionarily drew down their credit lines with the

[18] The number of loans issued by a bank b over some time period T is $\sum_{t \in T} s_{f,b,t}$, where $s_{f,b,t}$ is the share of a loan issued to firm f by bank b at time t.

[19] Many loans in Dealscan do not have lender shares. I impute these shares in a two-step procedure. First, I use Blickle et al. (2022)'s method to predict lender shares. Second, if a lender share is still missing, then I impute the lender share based on the syndicate structure following Chodorow-Reich (2014) and Kim (2021).

[20] I thank Gabriel Chodorow-Reich for providing these instruments at the parent bank level on his website.

other banks in their last syndicate. Focusing on syndicated loans in which Lehman was a lead arranger ensures that these drawdowns were significant in size. Therefore, banks that co-syndicated with Lehman more frequently had less capacity to issue loans to non-financial firms because of disruptions in financial markets, not because of shocks to their borrowers.[21] To get a firm-level version of this IV, I compute the weighted average of the co-syndication fractions based on the loan shares from a firm's last pre-crisis loan. Section 3.3 discusses identification assumptions for this instrument in more detail.

Regression Specification. My main regression specification is the unweighted triple difference-in-differences (DiD) regression

$$\begin{aligned}\pi_{g,f,t} = &\sum_{k \in T} \mathbb{1}(t=k)\left[\beta_{0,k}\left(-\Delta\widetilde{L}_f\right) + \beta_{1,k}\left(-\Delta\widetilde{L}_f \times H(\text{Irrev}_{\text{ind}(f)})\right)\right] \\ &+ \sum_{k \in T} \mathbb{1}(t=k)\left[\gamma_{0,k} H(\text{Irrev}_{\text{ind}(f)}) + \gamma_k^\top X_f\right] + \Gamma_{g,t} + \Gamma_{\text{ind}(f),t} + \varepsilon_{g,f,t}\end{aligned} \tag{8}$$

where f is a firm, g is a product group, t is a quarter, $T = \{\text{2007:Q3}, \ldots, \text{2009:Q3}\} \setminus \{\text{2008:Q3}\}$, $H(x)$ is some function, and X_f is a set of firm characteristics measured before the Lehman bankruptcy.[22] This specification estimates the dynamic effect of exposure to the credit supply shock on product-level inflation while allowing the effect to vary across industries with different levels of irreversibility. I include product group by time fixed effects to absorb common shocks to a product group and 3-digit NAICS industry by time fixed effects so that the coefficient $\beta_{0,k}$ is identified using variation among firms within the same 3-digit NAICS industry that experience different credit supply shocks. The coefficient $\beta_{1,k}$ is identified using variation across industries in how a firm responds to the *same* credit supply

[21] In Appendix C, I check that the two other instruments used by Chodorow-Reich (2014) yield similar results. I focus on the Lehman co-syndication IV because it has the clearest story for why the exclusion restriction should hold and because disclosure avoidance requirements by the Census Bureau limit the number of estimates I can report.

[22] I define a quarter as the price observation in the first month of the quarter, so the year-over-year change in 2009:Q3 is given by the log change in prices over July 2008–2009.

shock. In my analysis, I use either $H(x) = x$ (linear continuous interaction) or $H(x) = \mathbb{1}(x \geq \text{median})$ (binary indicator for whether a firm has above-median irreversibility). I use the negative of the credit supply shock on the right-hand side so that $\beta_{0,k}$ and $\beta_{1,k}$ describe the impact of a credit *tightening* on price changes. To address potential endogeneity of the credit supply shock, I instrument $-\Delta\widetilde{L}_f$ and $-\Delta\widetilde{L}_f \times H(\text{Irrev}_{\text{ind}(f)})$ with the Lehman co-syndication IV and its interaction with $H(\text{Irrev}_{\text{ind}(f)})$. I justify this strategy in Section 3.3. Because I omit 2008:Q3, $\beta_{0,k}$ and $\beta_{1,k}$ describe the impact *relative* to the price change a firm would have chosen in 2008:Q3. I cluster standard errors by a firm's primary lender since firms which primarily borrow from the same lender likely experience similar treatment effects. Chodorow-Reich (2014) and Lenzu et al. (accepted) also make this choice.[23]

The outcome variable $\pi_{g,f,t}$ denotes the year-over-year inflation rate in quarter t on the price index for a product group g sold by firm f. I choose year-over-year rates to mitigate seasonality and to follow Kim (2021)'s event-study specification. A product group g is a 4-digit NAICS code by buyer type pair. I study pricing dynamics at the product-firm level because the BLS only collects prices for a random sample of the products sold by a firm. To reduce measurement error from random sampling, I do not aggregate product-level prices to a firm-specific price. Items in the PPI microdata are also assigned more granular product codes than 4-digit NAICS, but because of the sample size I need to aggregate. I include the buyer type, which can be "Final User," "Intermediate," or "Same Price to All," since the type of buyer may influence how prices are set.[24] I give details on the construction of $\pi_{g,f,t}$ from item-level prices in Appendix E.1.

The controls X_f account for the dynamic effects of ex ante characteristics on price changes that could otherwise threaten a causal interpretation of the estimated responses

[23] Lenzu et al. (accepted) finds that clustering by primary lender leads to more conservative standard errors than the method proposed by Adão et al. (2019).

[24] Products sold as intermediates may have their prices set according to long-term contracts rather than spot prices. The ability of a firm to adjust its price may be significantly different since these contracts may ex ante constrain how prices can move.

to credit supply shocks. My choice of controls follows Chodorow-Reich (2014) and Kim (2021). The first set of controls accounts for differences in access to external financing. I include fixed effects for previous bond market issuance in Mergent FISD, whether the firm was publicly listed, whether the firm ever had multiple lead arrangers on a loan issued before Lehman's bankruptcy, and the number of loans (partitioned into bins) due over 2008:Q4–2009:Q2. Since lending relationships are sticky, I also use characteristics of the last syndicated loan issued before the Lehman bankruptcy as proxies for a borrower's financing access. Specifically, I include continuous covariates for the interest spread, log size, and maturity of the last syndicated loan and fixed effects for whether the last loan was a credit line or other type (typically a term loan), the year of the last loan, and whether the last loan had multiple lead arrangers. The second set of controls comprises log age and log employment since the firm dynamics literature has found firm age and size to be important predictors of behavior. See Appendix E.1 for more details on these control variables and their construction.

3.3 Identification Assumptions for the Credit Supply Shock

The credit supply shock $\Delta\widetilde{L}_f$ (7) and the Lehman co-syndication IV are relevant under the assumption that borrowers and lenders form sticky relationships. In other words, if firm f's previous lenders reduce the supply of credit to f, then f is not able to easily replace the lost credit with loans from other lenders, so f actually faces a decline in its sources of financing. Lending relationships can be sticky because of asymmetric information or search costs. There is empirical support for this assumption in syndicated loan markets (see Sufi 2007, Chodorow-Reich 2014) and for all commercial and industrial loans issued by the largest U.S. banks (see Amiti et al. 2026).

The coefficients $\beta_{0,k}$ and $\beta_{1,k}$ in (8) can be interpreted as the causal impact of a credit supply shock on price changes relative to the changes chosen in 2008:Q3 for firms in high- or low-irreversibility industries if the exposure weights implied by the Lehman co-syndication

IV are exogenous. Recall that the Lehman IV is a shift-share instrument, and Goldsmith-Pinkham et al. (2020) shows that independence of the weights is sufficient (but not necessary) for shift-share instruments to satisfy exclusion restrictions. I argue this assumption is reasonable for the Lehman IV. The decision by a bank to co-syndicate a loan with Lehman seems entirely unrelated to its decision about the type of borrowers to whom it lends, especially since the Lehman bankruptcy was unexpected. Further, it seems implausible that some firm types which happen to change prices differently than other types also systematically prefer to borrow from banks that co-syndicate frequently with Lehman. In other words, this instrument satisfies the exclusion restriction as long as, conditional on observables, banks which co-syndicated with Lehman more frequently did not systematically lend to different types of firms than other banks. The narrative evidence also supports this assumption. The Lehman bankruptcy was widely viewed as unexpected, so concern about exposure to Lehman likely did not affect which bank-firm pairs signed loans.[25] Lehman went bankrupt due to the mortgage market rather than the commercial and industrial lending market. Thus, it is more likely that loan issuance to firms fell because banks had to cut lending to all non-financial firms than because firms which were adversely affected by housing price declines drew down their existing credit lines so much that banks had to limit lending to most firms.

The credit supply shock $\Delta\widetilde{L}_f$ also functions like a shift-share instrument in that a firm's exposure weights are its last syndicate's loan shares across different banks adjusted by how much lending fell to other firms, but this measure requires stronger assumptions for exogeneity. In particular, when firm f's banks decrease lending to other firms, the cause must be that these banks experienced a credit supply shock. However, this assumption is violated if firm f experiences a shock unrelated to credit supply and heavily draws down its credit lines because then firm f's banks have not, in fact, reduced credit to firm f. Instead, firm f's banks have lent less to other firms because they extended substantial credit to firm

[25] See Chodorow-Reich (2014) for a detailed narrative account of the Lehman bankruptcy.

f. Using the Lehman co-syndication IV as an instrument assuages this concern since it is a plausible credit supply shock.

Finally, I run two more checks that my credit supply shock measures are plausibly exogenous. First, the credit supply shock $\Delta\widetilde{L}_f$ passes the test introduced by Khwaja and Mian (2008). This test checks for selection on unobserved firm characteristics by testing whether a bank that cuts its lending portfolio also cuts lending to firms to whom the bank has previously lent credit. The results are reported in Appendix B.2. Second, I regress observable firm characteristics on the Lehman co-syndication instrument.[26] The results are reported in Appendix B.3. The instrument is not correlated with most characteristics, so it likely reflects changes in bank health rather than firm characteristics.

3.4 Assumptions About the Irreversibility Measure

I measure irreversibility as one minus the *predicted* time-averaged recovery rate of PPE based on physical attributes measured in 1997. This section discusses my assumptions about this measure so that I can interpret the empirical analysis as supporting evidence for this paper's key theoretical argument: irreversibility leads to greater real price rigidity by strengthening the customer-market incentive. First, I argue that my measure of irreversibility can be interpreted as a production technology characteristic. Second, I discuss why my regression (8) does not cleanly estimate the causal effect of irreversibility on the price response to credit supply shocks and how I address this concern.

Irreversibility as a Production Technology Characteristic. This paper interprets the predicted recovery rate of PPE based on its physical attributes from Kermani and Ma (2023) as a *production technology characteristic*. This assumption is necessary because the

[26] I focus on sample balance with respect to the Lehman co-syndication instrument rather than the credit supply shock because I instrument most regressions. What matters then for a causal interpretation is whether the Lehman instrument systematically affects certain types of borrowers.

model's key comparative static is the change in the price response to a shift in the marginal value of liquidity when a firm's production technology is more irreversible, holding constant other features of the firm's problem like the characteristics of the demand curve.[27]

A natural concern is that irreversible industries also differ in their demand-side characteristics, in which case the measure would capture product-market differences rather than production technology. To assuage this concern, Appendix A.1 shows that (one minus) the predicted recovery rate is not strongly associated with several demand-side characteristics that might shape the ability to accumulate customers. Notably, irreversibility is not strongly related to the share of domestic customer demand from households, the share of commodities classified as exchange-traded according to Rauch (1999), or the share of firms with major customers in Compustat Segment.[28] The first fact assuages the concern that irreversible industries invest more in their customer base because they sell more to households, who may have stickier demand due to stronger behavioral frictions. The second fact reduces the possibility that irreversible industries have more gains from investments in buyer relationships because they sell more goods which are not traded on exchanges or because their products are more differentiated. Consistent with this evidence, Foster et al. (2016) finds that the customer-market incentive applies even in industries with relatively homogeneous products like chemicals and ready-mix concrete. The third fact mitigates the issue that irreversible industries may tend to have higher buyer concentration, which would strengthen incentives to invest in buyer relationships and could make capital more irreversible due to holdup problems. In the regression analysis, I also check that price responses differ by industry irreversibility for products with similar demand elasticities.

The physical attributes selected by Kermani and Ma (2023) are also plausibly unrelated

[27] In Section 2's model, the disinvestment wedge κ must shift, not the customer-market parameters χ and θ or the demand elasticity η.

[28] If a firm has a customer which accounts for more than 10% of the firm's revenue, then Compustat Segment records the firm-customer relationship.

to the degree of relationship specificity across industries. Capital can be irreversible when it is physically difficult to move. Kermani and Ma (2023) measures this property by an industry's mobility costs using the ratio of equipment transportation costs (from producers to users) to production costs and the share of structures in the capital stock. For example, high mobility costs help explain why the chemicals industry is relatively irreversible among goods producers, whereas the textiles industry is relatively reversible. Equipment in the chemicals industry can be costly to transport because of their size, such as reactors weighing hundreds of metric tons, while looms for textiles reach only a few metric tons.[29] Capital can also be difficult to resell when depreciation makes it less durable because redeploying capital takes time, so quickly depreciating capital will be much less valuable when it is actually redeployed. This reason also helps explain why the chemicals industry is more irreversible than the textiles industry. Equipment for chemicals production is highly susceptible to corrosion from, for example, use of acidic fluids and interaction with moisture in the environment, which continue to depreciate the capital even when idle, unlike looms and knitting machines in the textiles industry.[30] Finally, when equipment requires higher design costs, it becomes less useful to other firms because it likely is customized to a specific production process, so it is either less productive in other uses or costs more to redeploy. For example, Ramey and Shapiro (2001) finds that equipment used in aerospace plants sells for a higher price when purchased by other aerospace firms.

While my measure of irreversibility needs to be physical, irreversibility in the *model* does not need to be. It can also arise from other sources of irreversibility, such as external financing costs and operating leverage, as long as they reduce to features of a firm's production technology. For example, demand-curve characteristics would not be admissible sources of irreversibility because then the primitives are the preferences of a firm's customers, not

[29]For example, see Short (2004) for an example of heavy chemical reactors. Secondary-market listings typically report the weight of looms at a few metric tons.

[30]See Jenkins (2018), Buecker and Dixon (2016).

the production technology.

Do I Identify the Causal Effects of Irreversibility? While the primary regression (8) provides causal estimates of the price response to credit supply shocks for high- and low-irreversibility industries, the interaction coefficient $\beta_{1,k}$ between the credit supply shock and irreversibility *cannot* be cleanly interpreted as the *causal effect of irreversibility* on the price response to credit supply shocks. Because I do not have an exogenous shifter of irreversibility in my analysis,[31] any industry characteristic correlated with irreversibility could be part of the reason why prices adjust differently. Caron (2025) shows that the estimates of $\beta_{1,k}$ have a causal interpretation only if I make the strong assumption that, conditional on covariates, the treatment effect of credit supply shocks on price changes is independent of unobserved characteristics which are correlated with investment irreversibility.[32] In the empirical analysis of Section 4.4, I perform robustness checks to show that it is reasonable to interpret my estimates of $\beta_{1,k}$ as evidence that physical sources of irreversibility change the response of prices to credit supply shocks, although the exact estimates may not be the true causal effects.

4 Evidence from Credit Supply Shocks

The regression results in this section provide evidence in support of Section 2's stylized model. Specifically, in response to the same credit supply shock, firms in industries with low investment irreversibility cut prices relative to their unaffected counterparts while firms in industries with high irreversibility do not cut their prices. This behavior is consistent with the stylized model's mechanism in which irreversibility leads to greater upward pressure on

[31] Chen et al. (2023) estimates the causal effect of a change in partial irreversibility on investment using a VAT reform in China.

[32] Caron (2025) proposes a method to estimate the causal difference for binary treatments, so I would need to extend the method to a continuous setting, which is likely a difficult task and outside the scope of the paper.

the response of prices and markups to shifts in the marginal value of liquidity.

This section first shows estimates from the main regression (8), in which I regress the year-over-year inflation rate on credit supply shocks and the interaction with investment irreversibility. I supplement these results with a similar regression run on export prices instead of the PPI sample. Then I provide supporting evidence that a difference in the customer-market incentive explains the difference in price responses to credit supply shocks. Finally, I explain why it is reasonable to interpret the difference in price responses as the result of differences in investment irreversibility and not some other characteristic correlated with irreversibility.

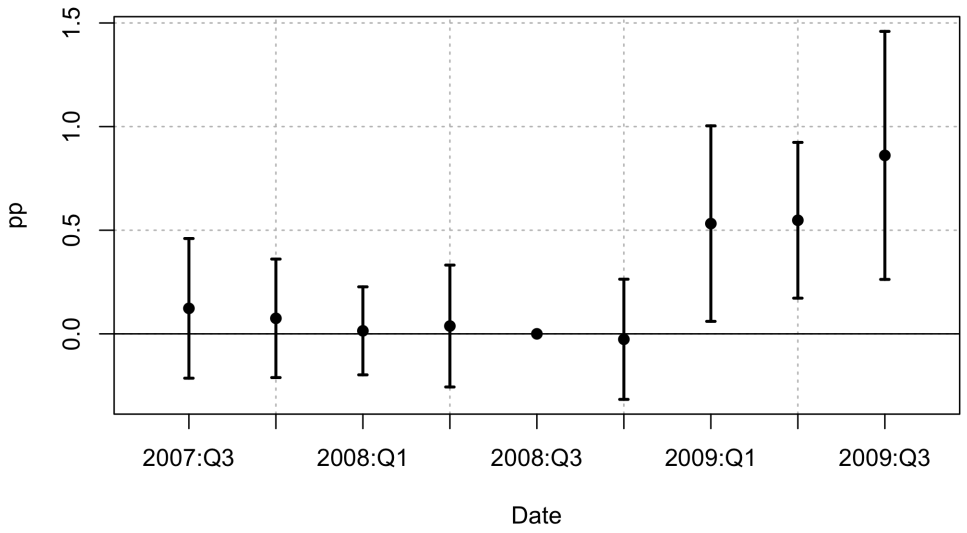
4.1 Irreversibility and the Price Responses to Credit Supply

In response to the same one standard deviation cut in credit supply, firms in industries with 1pp higher irreversibility choose a 0.86pp higher year-over-year price change in 2009:Q3. This estimate is $\beta_{1,2009:Q3}$ in the main difference-in-differences regression (8) when the interaction is linear ($H(x) = x$). Figure 2 shows the corresponding event study. The vertical axis is scaled so that each coefficient represents the effect of a 1 percentage point increase in irreversibility on the causal impact of a one standard deviation credit supply shock on a firm's price change relative to the price change the firm would have chosen in 2008:Q3. Note that this effect of irreversibility is *not* causal unless strong assumptions are invoked. There is no obvious pretrend in the four periods before 2008:Q3, but starting in 2009:Q1, firms with higher irreversibility choose higher year-over-year price changes in response to the same credit supply shock.[33]

This relative difference in pricing behavior still appears when I interact the credit supply shock with whether a firm has above- or below-median irreversibility (i.e. $H(x) = \mathbb{1}(x \geq$

[33] A zero effect in 2008:Q4 is not surprising given that $\pi_{g,f,t}$ in 2008:Q4 would be the year-over-year price change in October 2008, which is only a few weeks after the Lehman bankruptcy. Firms may need more time to adjust prices.

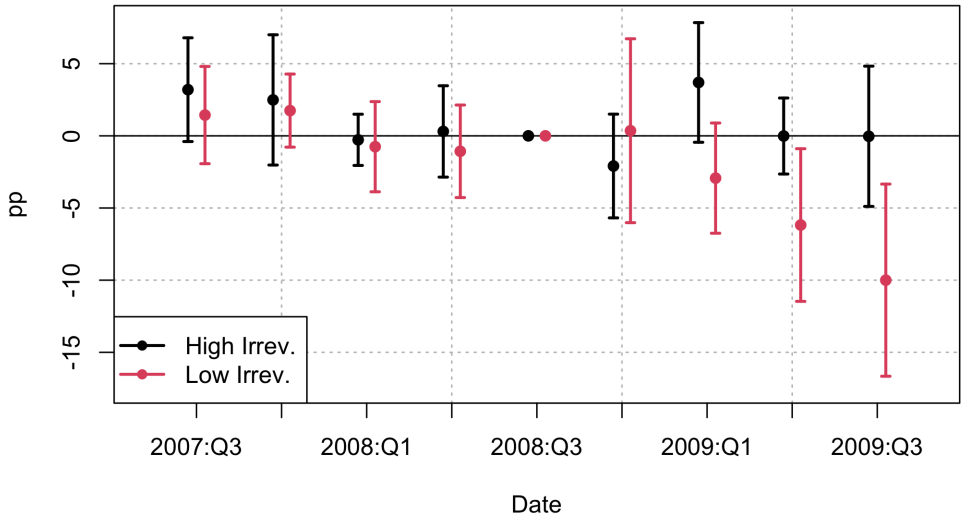
Figure 2: More Upward Pressure on the Price Response When Irreversibility Is Higher



Notes: This figure shows the estimated percentage-point difference in the year-over-year log price change after a one standard deviation cut in credit supply when irreversibility increases by 1pp. The coefficients on the credit supply shock's (linear) interaction with irreversibility are instrumented with the Lehman co-syndication IV linearly interacted with irreversibility. Standard errors are clustered by primary lender, and bars are 95% confidence intervals. Any views expressed are those of the author(s) and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

median)) instead of a linear interaction. Figure 3 plots the impact of the credit supply shock on the price changes for each group instead of just the relative difference. While firms in industries with low irreversibility cut prices after exposure to a credit supply shock, firms in industries with high irreversibility do not cut prices. The differences between the coefficients are also statistically significant at the 5% level over 2009:Q1–2009:Q3. The zero price change can still signify ex ante higher customer-base investment because if firms have inventories, then a firm can simultaneously cut production and inventory accumulation. The first effect decreases output sold while the second increases output sold, and it is entirely possible for the two effects to cancel out.

Figure 3: Low-Irreversibility Firms Cut Prices While High-Irreversibility Firms Do Not



Notes: This figure shows the estimated year-over-year percentage-point log price change relative to 2008:Q3 for firms with high versus low irreversibility after a one standard deviation cut in credit supply. This behavior is surprising because the standard "ex post" intuition is that irreversibility amplifies other margins of adjustment after a cut in credit supply since firms want to limit capital sales. We would expect the red coefficients to be from high-irreversibility industries, not the black ones. However, this behavior is consistent with the "ex ante" story that tighter financial constraints make firms care more about liquidity today and less about insurance against future losses, so these firms cut back their customer investment more sharply. The coefficients on the credit supply shock and its interaction with $H(\text{Irrev})$ are instrumented with the Lehman co-syndication IV and its interaction with $H(\text{Irrev})$. Bars are 95% confidence intervals. I cluster standard errors by primary lender. Any views expressed are those of the author(s) and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

The difference in year-over-year price changes across industries is quantitatively large but also not implausible. After a one standard deviation cut in credit supply, a firm in an industry with above-median irreversibility chooses a year-over-year price change that is 10pp higher than a firm in an industry with below-median irreversibility. For comparison, in Table B.1, the standard deviation of year-over-year price changes in 2009:Q3 is 25pp.

I check the robustness of these results in the appendices. Appendix B.4 checks first-stage diagnostics. Appendix C.1 checks alternative choices of controls, such as the liquid share of assets, and other measures of year-over-year inflation rates. I check that other characteristics correlated with irreversibility do not explain the differences in price responses using export price data that I discuss further below.[34]

While the levels of the price responses in Figure 3 can be rationalized by the theory in Section 2, they are a surprising result because they are not consistent with the simpler intuition that greater irreversibility restricts the options available to a firm when it needs liquidity. Given previous research on price responses to credit supply shocks (e.g. Kim 2021, Renkin and Züllig 2024, Lenzu et al. accepted), a reasonable prior for the results is that firms in industries with higher irreversibility either cut prices by more or raise prices by more. Irreversibility disincentivizes capital sales, so these firms should rely more on other margins to deal with liquidity demand shocks. In other words, irreversibility constrains firms' *ex post* responses to shocks. Further, high-irreversibility firms should lean *harder* on the same margin used by low-irreversibility firms if greater irreversibility constrains a firm's ex-post response options. Thus, if the inventory incentive dominates, as in Kim (2021)'s sample, then all firms should cut prices, and firms with greater irreversibility should cut prices by more. If the customer-market incentive dominates, as in Renkin and Züllig (2024)'s sample, then all firms should hike prices, and firms in high-irreversibility industries should hike prices by more. The fact that firms with low irreversibility cut prices and that firms with high irreversibility do not in the PPI sample is difficult to explain from the ex post perspective. An ex ante story in which irreversible firms precautionarily accumulate inventory to deal with future demand shocks also does not work because then high-irreversibility firms should have cut prices by more, not less.

[34]My access to the PPI microdata was suspended before I could check for omitted variables by including additional interactions of the credit supply shock with other characteristics.

Robustness to Export Prices. A reasonable concern is that these results may be specific to this sample of firms found in the PPI or to my choice to run the regression in growth rates instead of levels. To address these concerns, I re-run the regression analysis using export prices from the Longitudinal Firm Trade Transactions Database (LFTTD), where the left-hand-side variable is now the log price level instead of the log year-over-year price change.[35] I also use the LFTTD export price data to check two other concerns. First, the difference in price responses could result from a difference in demand elasticities across industries with high versus low irreversibility.[36] Second, the previous regressions did not saturate the covariates, so differences in pre-crisis characteristics across industries with high versus low irreversibility may explain the difference in price responses. I address these concerns with the following regression. Let $I_f = H(\text{Irrev}_{\text{ind}(f)})$, $E_g = H(\text{Elast}_g)$, and $Z_{g,f} = (I_f, E_g, I_f \times E_g)^\top$. Then I estimate

$$\begin{aligned}\log p_{g,f,t} = &\sum_{k \in T} \mathbb{1}(t=k) \left[\beta_{0,k}\left(-\Delta \widetilde{L}_f\right) + \boldsymbol{\beta}_{1,k}^\top \left(-\Delta \widetilde{L}_f \times Z_{g,f}\right)\right] \\ &+ \sum_{k \in T} \mathbb{1}(t=k) \left[\gamma_{0,k} I_f + \gamma_{1,k}^\top X_f + \gamma_{2,k}^\top (I_f \times X_f) + \gamma_{3,k}(I_f \times E_g)\right] \\ &+ \Gamma_{g,t} + \Gamma_{\text{ind}(f),t} + \varepsilon_{g,f,t}\end{aligned} \tag{9}$$

where H indicates an above-median value, $\boldsymbol{\beta}_{1,k}$ is the coefficient vector corresponding to $Z_{g,f}$, and the granularity of the product group g is HS6. The main effect of E_g in each period is absorbed by $\Gamma_{g,t}$. By additionally interacting the credit supply shock $-\Delta \widetilde{L}_f$ with the demand elasticity Elast_g, I can compare price responses across industries with different levels of irreversibility within each demand-elasticity group. I measure demand elasticities using estimates from Soderbery (2015), which improves upon estimates from Broda and

[35] I cannot run the regression in levels on the PPI sample because my access to PPI microdata was indefinitely suspended as of September 2025 due to limited resources at the BLS to support external research projects.

[36] Kim (2021) and Renkin and Züllig (2024) both find that demand elasticity helps explain heterogeneity in how prices respond to credit supply shocks.

Weinstein (2006).[37] I find that there are no significant differences in the mean demand elasticity for industries with high versus low irreversibility.[38] I interact the covariates X_f with the irreversibility indicator to better control for pre-treatment differences, although I do not also interact with the elasticity indicator to avoid too many thin cells.[39]

There is greater upward pressure on prices for exporters in industries with high irreversibility than for exporters in industries with low irreversibility after a cut in credit supply, holding constant the product-level demand elasticity group. Further, exporters of goods with low demand elasticities in high-irreversibility industries actually raise prices after a cut in credit supply, not just keep them constant. The relative difference in coefficients in 2009 is also about 10%, like the corresponding regression in Figure 3 using PPI data.[40] For goods with high demand elasticities, prices fall after a cut in credit supply, and they fall by less in industries with high irreversibility. Figure C.4 in the Appendix reports these results. As in the PPI sample, this difference is consistent with different ex ante customer-market incentives and inconsistent with greater ex post sensitivity by firms to credit supply shocks in high-irreversibility industries. When I do not include the demand elasticity as a third interaction with the credit supply shock, the results mirror Figure 3. I find price cuts after cuts in credit supply in low-irreversibility industries and no statistically significant price response in high-irreversibility industries. Table C.3 reports these results.

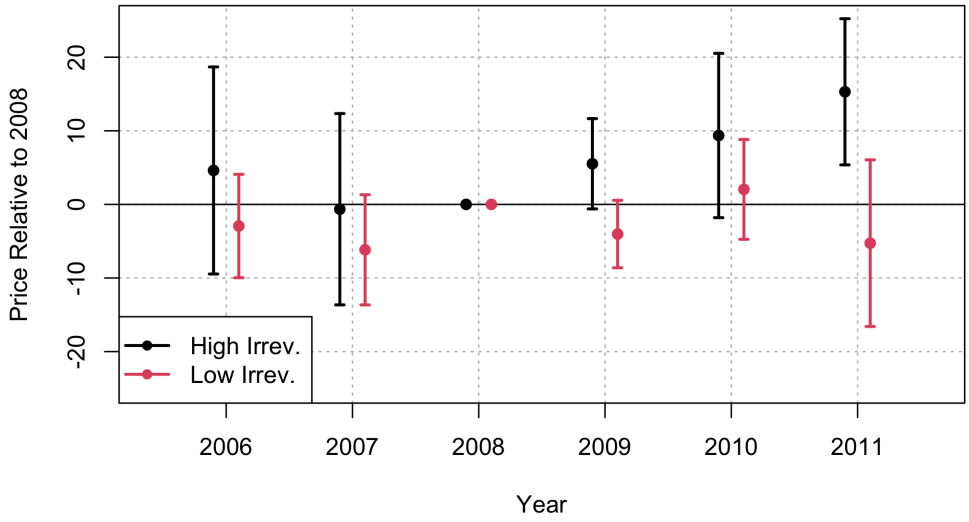
[37] An important difference is that Soderbery (2015) finds upward bias in Broda and Weinstein (2006)'s estimates. Removing this upward bias substantially lowers the median demand elasticity in less irreversible industries, notably the food and beverages industry.

[38] In the export price sample, the mean demand elasticity is also not different between high- and low-irreversibility industries with high demand elasticities or between high- and low-irreversibility industries with low demand elasticities.

[39] Because unit value prices can be noisy, I follow Renkin and Züllig (2024) and apply a two-step estimator that weights firm-product pairs inversely to the variance of their residuals from an unweighted first-step regression.

[40] I construct prices as annual unit values, and since LFTTD records data at a monthly frequency like the PPI, I use the twelve months ending in July to line up with the PPI sample. So 2009 in Figure 4 is constructed from transactions over August 2008-July 2009.

Figure 4: Exporters with High Irreversibility Raise Prices for Low Demand Elasticity Goods



Notes: This figure shows the estimated impact of a one standard deviation cut in credit supply on the log unit value price (relative to 2008) for firms with above-median irreversibility versus below-median irreversibility selling products with below-median demand elasticity. Both coefficients in 2009 are significant at the 10% level. The upward pressure on the price response in high-irreversibility industries still appears when the credit supply shock also varies with the demand elasticity. This result assuages the concern that irreversibility is a proxy for more product-market characteristics that lead to inelastic demand, such as specialized products. This figure is *not* evidence that firms in high-irreversibility industries are more ex post sensitive to credit supply shocks because I find smaller price cuts in industries with high irreversibility for high demand elasticity goods. Also, when I do not split goods into high and low demand elasticities, I get the same result that firms in high-irreversibility industries do not cut prices while firms in low-irreversibility industries cut prices after a cut in credit supply. The coefficients on the credit supply shock and its interaction with $H(\text{Irrev})$ are instrumented with the Lehman co-syndication IV and its interaction with $H(\text{Irrev})$. This is Panel (b) of Figure C.4 in Appendix C.1, which also reports the analogous above-median-demand-elasticity panel. The product-level demand elasticities are import demand elasticities from Soderbery (2015). Standard errors are clustered by primary lender, and bars are 95% confidence intervals. Any views expressed are those of the author(s) and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3432. (CBDRB-FY26-P3432-R13333). This is a part of clearance request #13333.

4.2 Evidence for Differences in Customer-Market Incentives

While a difference in price responses after credit supply shocks is consistent with a difference in customer-market incentives, the reason why the prices behave differently should be a difference in the behavior of markups according to the theory in Section 2, not a difference in marginal costs. This section tests this idea by measuring markups according to the production approach in De Loecker et al. (2020) and Foster et al. (2022) using ASM/CMF data. This data is at the establishment level, so I compute a sales-weighted average markup across establishments with the same 4-digit NAICS code owned by the same firm. I do not fully aggregate to the firm level because markups may behave differently across product categories.[41] I run the LFTTD price regression (9) but use the log markup on the left-hand side and interpret 4-digit NAICS codes as product group codes.[42]

After an idiosyncratic cut in credit supply, firms in industries with high irreversibility set a higher log markup relative to firms in industries with low irreversibility and temporarily increase the log markup in absolute terms. I am in the process of disclosing the numerical estimates, but I can report their sign and significance, as shown in Table 1. Over 2009–2011, there is a marginally significant increase in the log markup in 2009 for firms with high irreversibility, and afterward the significance vanishes, consistent with a temporary rise in markups. The difference in markup responses is also statistically significant at the 5% level only in 2009. The temporary nature of the absolute increase and relative difference in markups is consistent with a rise in markups by firms in high-irreversibility industries to boost short-run liquidity.

[41] When De Loecker et al. (2020) computes markups using Census data, the authors define the firm-level markup as the sales-weighted average markup across establishments with the same 4-digit NAICS code owned by the same firm.

[42] I run the regression using data over 2005–2011 to match the dates of the LFTTD sample. Recall that the LFTTD sample's data is timed so that, for example, the price in 2006 is constructed from data over August 2005 to July 2006. Because the ASM/CMF data is annual, I choose 2007 to be the reference year since production in the fourth quarter of 2008 is treated.

Table 1: Log Markups Rise in High-Irreversibility Industries

	2008	2009	2010	2011
High Irreversibility	+	$+^{*}$	+	+
Low Irreversibility	+	−	−	−
Sig. Diff. ($p < 0.05$)	N	Y	N	N

Notes: This table reports the sign and significance of the credit supply shock coefficient by above-/below-median irreversibility. A plus sign means that a cut in credit supply increases the log sales-weighted average markup across establishments with the same 4-digit NAICS code owned by the same firm. This table shows that a cut in credit supply causes a firm in high-irreversibility industries to raise its log markup in 2009 above the value in 2007 at the 10% level, and it is significantly higher than the log markup response by a firm in the low-irreversibility industry. I compute the establishment markup following De Loecker et al. (2020) and Foster et al. (2022). The coefficients on the credit supply shock and its interaction with $H(\text{Irrev})$ are instrumented with the Lehman co-syndication IV and its interaction with $H(\text{Irrev})$. Standard errors are clustered by primary lender. The bottom row of this table reports the significance at the 5% level of the interaction coefficient. Any views expressed are those of the author(s) and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3432. (CBDRB-FY26-P3432-R13333). This is a part of clearance request #13333. $^{*}\, p < 0.10$, $^{**}\, p < 0.05$, $^{***}\, p < 0.01$.

4.3 Supporting Descriptive Evidence

If irreversibility strengthens the customer-market incentive, then irreversibility should be strongly associated with other measures of investment in a sticky customer base. This prediction holds for two complementary notions of customer investment: contract duration and selling expenses. While this evidence is descriptive, it is favorable for this paper's argument for two reasons. First, the export price evidence shows a difference in price responses even when I control for demand elasticities, so I can address concerns about confounding demand-side characteristics that show up through product-level demand elasticities. Second, Section 3.4 shows that my measure of irreversibility is not closely related to demand-side characteristics that would shift the returns to a customer base.

Contract Duration. Longer contract durations mitigate demand uncertainty by pinning down demand further into the future. Based on firm-to-firm contracts from goods-selling manufacturers collected by Chen et al. (2026), a 10pp increase in irreversibility is associated with a one-year increase in contract duration. While longer contracts have often been interpreted as an optimal response to relationship-specific investments,[43] the fact that my measure of irreversibility is not associated with buyer concentration in Compustat Segment suggests that differences in durations at the industry level may not be the result of relationship specificity.

Selling Expenses. Selling expenses have long been interpreted as another way to invest in a customer base. Gourio and Rudanko (2014) develops a model of customer capital due to product market frictions and supports the model with Compustat data on selling, general, and administrative expenses. Peters and Taylor (2017) and Chiavari (2026) refine this Compustat measure by stripping out unrelated components, such as R&D and rental expenses, while He et al. (2026) uses Capital IQ to directly measure selling expenses and uses SEC filing data to explain why firms use selling expenses.

These papers have typically interpreted selling expenses as a proxy for the strength of product market frictions, but this paper points out that high selling expenses could also be the result of differences in production technology. Consistent with this idea, I find that irreversibility is robustly and strongly associated with higher selling expenses in Appendix A.2, Figure A.3 and Table A.2. The results hold when I control for demand-side characteristics that could shape the return to selling expenses, such as the share of sales to households. Differences in production technology may help explain why He et al. (2026) finds that some business-to-business industries spend a lot on selling expenses, despite the usual explanation

[43] Examples include energy markets (Joskow 1987, Von Hirschhausen and Neumann 2008, and Zahur 2026) and trucking (Hubbard 2001). On the other hand, Darmouni et al. (2026) finds that quantity contracts are used in the pulp and paper industry to mitigate demand uncertainty.

that customer markets result from consumer brands or good-specific habits.

4.4 Alternative Explanations

This section considers alternative explanations for my results and argues why they are less plausible than this paper's argument that irreversibility strengthens the customer-market incentive. Table 2 summarizes this section's arguments.

Customer Markets or Firm-Specific Capital? Kermani and Ma (2023) also shows that irreversibility has positive associations with markup countercyclicality and the frequency of price adjustments, and the authors suggest that this association arises from a difference in the magnitude of desired price changes when firms have firm-specific capital instead of rented capital. The intuition for this mechanism comes from Altig et al. (2011). If capital is "firm-specific," meaning that the capital stock for current production cannot be adjusted, unlike rented capital, then firms will cut prices by less after a decline in aggregate output because firms cannot increase their capital stock to meet the extra demand due to a lower relative price.[44] If unmodeled menu costs deter firms from making small price changes, then we should expect small desired price changes to result in more inaction. Under this interpretation, greater irreversibility in an industry should also mean a lower share of rented capital, which then makes capital stocks harder to adjust in the short run.

However, two reasons lead me to prefer my explanation, which goes through customer markets. First, I find that irreversibility is associated with *more* rented capital, not less, in Figure A.6d, presumably because firms lease capital to hedge against the costs of irreversibility (Fairhurst et al. 2026, Chowdhury et al. 2025) or low borrowing capacity (Eisfeldt and Rampini 2009, Kermani and Ma 2023). The mechanism in Altig et al. (2011) should

[44]The model in Altig et al. (2011) does allow capacity utilization to vary contemporaneously. The key assumption is that the stock of capital itself for current production cannot be adjusted, whereas standard DSGE models assume capital for current production can be freely adjusted through rental markets.

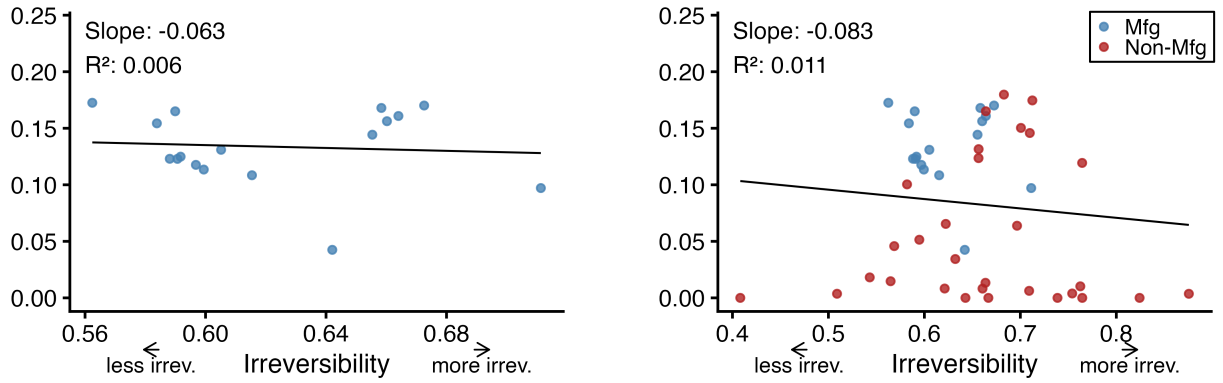
Table 2: Addressing Alternative Explanations for Differences in Price Responses

Alternative	Intuition	Evidence Against This Concern
Firm-specific capital	When capital in production cannot be adjusted immediately because capital is firm-owned, firms desire smaller price changes.	Higher irreversibility is associated with *more* rented capital (Figure A.6d). Less frequent capital sales also point to irreversibility as a proxy for resale costs, not the inability to adjust capital in current production.
Differences in inventories	Inventory stocks could be smaller or harder to liquidate in high-irreversibility industries	Irreversibility is not significantly negatively associated with median inventories/sales (Figure 5) or inventory recovery rates (Figure A.1d).
Other correlated firm characteristics	Interest spread on the last loan, liquidity ratio, book leverage, PPE share of assets, or selling expenses drive the result.	The export-price result for low-elasticity goods survives additional shock interactions with each characteristic (Table C.4).
Insulation from credit supply shocks	Precautionary cash holdings reduce the need to adjust prices at all after a credit supply shock.	High-irreversibility exporters raise prices for low-elasticity goods (Figure 4), and this result is robust to adding an interaction of the liquidity ratio with the credit supply shock. High-irreversibility firms also cut employment growth more (Table C.5).
Survivorship bias	High-irreversibility firms are the firms that did not go bankrupt because they were more insulated.	Credit supply shocks do not seem to cause more deaths among Compustat firms, and death rates do not rise more in high-irreversibility industries (Appendix D).
Different demand elasticities	Product-market characteristics that result in more inelastic demand change the incentive to cut prices.	Export-price responses differ by irreversibility within demand-elasticity groups (Figure 4 and Appendix Figure C.4).
Capital stocks and marginal costs	Firms with high-irreversibility cut more capital, so their marginal costs are higher.	Establishment-level capital stocks significantly fall for low-irreversibility firms but not for high-irreversibility firms (Table C.6).

therefore make prices less rigid in more irreversible industries since an ex ante larger share of leases makes it easier to adjust current production capacity. Second, Kermani and Ma (2023) shows that capital sales are less frequent on average in industries with higher irreversibility, consistent with larger costs to capital sales. There is no cost to capital sales in Altig et al. (2011)'s model, so there should not be a strong incentive to sell capital less frequently. In contrast, I interpret irreversibility as costly resale, and it is precisely this cost that motivates insurance through the customer base.

Why Not a Difference in Inventories? This paper argues that inventory accumulation does not explain differences in price responses due to irreversibility. In addition to the model-based discussion in Section 2, irreversibility is not significantly correlated with the median inventories/sales ratio, as Figure 5 shows. Figure A.3 in the appendix reports the corresponding inventories/assets ratios. Thus, it is unlikely that firms in industries with high irreversibility do not cut prices after declines in credit supply because they have fewer inventories on hand. Irreversibility also seems unrelated to the ability of a firm to easily obtain cash from inventories according to inventory recovery rates by industry from Kermani and Ma (2023) in Appendix Figure A.1d.

Figure 5: Median Inventories/Sales



(a) Manufacturing Only

(b) All Industries in Kermani and Ma (2023)

Checking Correlated Characteristics. Firms increase prices by more in high-irreversibility industries for low-demand-elasticity goods even after controlling for another characteristic that could be correlated with irreversibility.[45] Specifically, I add an interaction between the credit supply shock and an additional characteristic to the export price regression (9) to check whether irreversibility is just a proxy for some other channel. The key result from Figure 4 that the price change is higher in industries with greater irreversibility for goods with low demand elasticities is robust across these characteristics: spread on the last loan, liquidity ratio, book leverage, PPE asset share, and SG&A to sales ratio. Riskier borrowers, more cash on hand, lower leverage, lower capital intensity as a share of assets, and more selling expenses as a share of sales do not explain why there is greater upward pressure on prices after credit supply shocks in industries with higher irreversibility. Table C.4 reports the results.

Are Firms with Greater Irreversibility Insulated from Credit Shocks? Another interpretation of Figure 3 is that firms with high irreversibility are insulated from credit supply shocks and therefore do not adjust their prices. One reason why firms with high irreversibility may be insulated is that they precautionarily hold more cash to deal with irreversibility in normal times. With more cash, these firms would not need to liquidate inventory as a way to raise liquidity and thus could keep their prices the same.

However, this explanation is inconsistent with the evidence from export prices in LFTTD and employment growth in the PPI sample. Figure 4 shows a significantly positive response for export prices of goods with low demand elasticities sold in high-irreversibility industries after cuts in credit supply. Firms that are more insulated from shocks should not hike prices. When I regress employment growth on the credit supply shock, firms in industries with high irreversibility also cut employment growth more than firms in industries with

[45] To limit the number of disclosure estimates, I only report sign and significance.

low irreversibility (see Table C.5 in Appendix C.2). While firms likely hold more cash and adjust their capital structure to help with the costs of irreversibility, this paper's evidence indicates that these adjustments in capital structure do not fully remove the incentive to invest in larger customer bases as insurance against idiosyncratic demand risk.

Firms with greater irreversibility may also have been "insulated" because of survivorship bias. My regressions involve balanced panels. If firms in high-irreversibility industries exited more, then a balanced panel may select only for firms that did not exit because they were unaffected by cuts in credit supply. Appendix D mitigates this concern by showing that credit supply shocks did not cause more firm deaths among Compustat firms and that firm death rates did not rise more in industries with high irreversibility.

5 Quantification

This section considers a back-of-the-envelope exercise to show that the estimated differences in the price response to credit supply shocks between high- and low-irreversibility industries imply quantitatively meaningful effects on aggregate price rigidity. Specifically, I use my regression estimates to first predict the amount of deflation caused by credit supply shocks during the Global Financial Crisis and then compute how much more flexible prices would have been if prices in all industries responded like prices in the least irreversible industry. These estimates also point to differences in production technology as another reason, besides demand elasticities, for why Kim (2021) estimates price cuts while Renkin and Züllig (2024) estimates price increases after credit contractions.

This exercise abstracts from general equilibrium effects. To check whether indirect effects substantially change the predictions of the back-of-the-envelope exercise, I build a two-industry DSGE in which I model the difference in customer-market incentives across industries in reduced form rather than explicitly microfounding it with a difference in partial

irreversibility.

5.1 A Back-of-the-Envelope Calculation

This subsection describes the two counterfactuals behind the back-of-the-envelope calculation and reports the implied increase in aggregate price rigidity.

Procedure. I consider two counterfactuals, C1 and C2. In C1, I suppose that firms with tighter credit supply shocks than the 10th percentile have their credit counterfactually relaxed to the 10th percentile. For example, a firm whose credit supply tightened by a 90th percentile amount now counterfactually borrows from a bank that only experienced a 10th percentile credit supply shock. I then use the empirical estimates from the primary regression (8), in which I regress the year-over-year log price change using PPI data on the credit supply shock, its linear interaction with irreversibility (i.e. the specification for Figure 2), and controls to predict the price change for all goods produced by firms in a given industry. In particular, I use log price changes over July 2008–2009 from publicly available goods-level price indices from the BLS PPI, concord these indices to 2-digit SIC codes, compute the average counterfactual credit supply shock for each 2-digit SIC code, and use the coefficient estimates from the regression (8) to compute the counterfactual price change implied by the average credit supply shock.[46] Note that I apply the same adjustment to the price of every good mapped to the same 2-digit SIC industry, and this adjustment takes into account the level of irreversibility in a firm's industry. I do not apply a price adjustment to goods indices which do not link to an SIC code in Kermani and Ma (2023)'s dataset. I use credit supply shocks from firms in Dealscan for which I can compute credit supply shocks, not just firms

[46] I use public goods-level price indices because I computed this back-of-the-envelope calculation after my access to the microdata was indefinitely suspended. I use price changes over July 2008–2009 to align with the timing used by the regressions on the PPI microdata. Specifically, I define the year-over-year price change for a quarter based on the year-over-year change in the first month of the quarter. Thus, the year-over-year price change underlying the 2009:Q3 coefficient is the change over July 2008–2009.

in the PPI or LFTTD samples.[47] Finally, I aggregate the goods-level counterfactual price changes using the PPI's importance weights on each goods index.[48]

In counterfactual C2, I perform the same procedure but I adjust price responses using the estimates for the least irreversible industry among the 2-digit SIC codes that are linked to the goods-level price indices. Note that this counterfactual C2 does not capture the *causal* effect of a change in irreversibility. Instead, it shows how aggregate prices would behave if we could change each industry's characteristics that are correlated with irreversibility to the characteristics of the least irreversible industry. Nevertheless, it is still a useful way to gauge the magnitude of the estimated effects, even if a causal interpretation is difficult to credibly make.

Results. In both counterfactuals, credit supply shocks cause additional deflation, and the magnitudes are large. The actual PPI fell by about 19% over July 2008–2009.[49] The first counterfactual suggests that credit supply shocks cause over 7% more deflation. If we could hypothetically change the irreversibility (and correlated characteristics) of the industries in the PPI sample to the irreversibility of the least irreversible industry, then the additional deflation increases to nearly 12%. This additional deflation implies that prices would fall by 39.1% less because of differences in irreversibility (and correlated characteristics) across industries.

5.2 What About General Equilibrium?

Since the back-of-the-envelope exercise abstracts from general equilibrium effects, I cannot rule out that the predicted additional rigidity due to irreversibility would vanish if I

[47] This sample is the same as the one I use in Appendix C.4 to study the extensive margin effects of the credit supply shock, so I also require these borrowers to have non-missing loan and borrower characteristics in Dealscan.

[48] I have checked that an aggregation of the goods-level price indices using weights from the BLS nearly replicates the official PPIACO series in FRED.

[49] This PPI is the aggregation of the goods-level indices using 2008 importance weights.

Table 3: Back-of-the-Envelope Counterfactuals

	Actual	C1	C2
Percent Price Change	-19.0	-11.8	-7.1
Deflation Due to Credit Supply Shocks		-7.3	-11.9
% More Rigidity in C1 vs. C2			39.1

Notes: This table shows that aggregate prices fall by less and are thus "more rigid" after credit supply shocks compared to the counterfactual scenario in which prices in all industries responded like prices in the least irreversible industry. Percent changes in this table are log point changes. The Actual column reports the percent change in the aggregate PPI index computed by aggregating goods-level price indices using the 2008 importance weights over July 2008–2009. The other two columns report the predicted percent change in the aggregate PPI index in the two counterfactuals and use the Actual column (Column 1) to back out the effect of credit supply shocks on aggregate prices. C1 relaxes firm-specific credit supply shocks larger (and tighter) than the 10th percentile to the 10th percentile, holding each industry's actual level of irreversibility fixed. Counterfactual C2 applies the same credit easing as C1, but additionally assumes prices in all industries respond like prices in the least irreversible industry in the sample. The final row computes how much less prices fell in C1 versus C2 and normalizes by the predicted deflation due to credit supply shocks in C2. Any views expressed are those of the author(s) and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

accounted for general equilibrium forces. To assuage this concern, Appendix I builds a medium-scale two-industry DSGE that accounts for general equilibrium effects on inflation through the nominal wage, aggregate demand, and the real interest rate. I mirror counterfactuals C1 and C2 by comparing the inflation response to credit supply shocks when there is a stronger customer-market incentive in one industry (C1) and when both industries have weak customer-market incentives (C2).

Since the point of this model is just to check whether general equilibrium effects nullify the back-of-the-envelope estimate, I choose a reduced-form modeling device that generates a difference in customer-market incentives across industries to simplify the DSGE and its calibration. The modeling device adds a non-pecuniary preference to the firm objective for

making sales to the same customer over time, mirroring how goods-specific habits create a preference for purchases from the same firm. This paper's core mechanism is that irreversibility affects real price rigidity because it strengthens the customer-market incentive, so what actually matters for aggregate price rigidity is the difference in the customer-market incentive across industries. The DSGE would be much more complicated if I microfounded the difference in customer-market incentives by extending the model in Section 2 into an infinite-horizon setting with heterogeneous firms. I leave this extension for ongoing work in Chen (2026). Further, since idiosyncratic uncertainty is far larger for most firms than aggregate uncertainty (see Bloom et al. 2018), aggregate shocks probably do not have substantial effects on the interaction between irreversibility and the customer-market incentive. Thus, a microfoundation is probably not quantitatively crucial for the first-order effects of MIT shocks to aggregate credit supply.

I find that general equilibrium forces attenuate the back-of-the-envelope's predicted increase in price rigidity by 49.2%. The primary reason is the behavior of real interest rates. Firms rely more on cutting their inventory stocks to deal with the cut in credit supply in counterfactual C2 compared to C1. Inflation falls by more as a result. Since monetary policy follows a Taylor rule which cuts the nominal rate when inflation deviates from target, real interest rates endogenously fall by more, too. Because lower real interest rates incentivize firms to hold onto their inventory stocks, the endogenously larger cut in real interest rates causes firms to cut inventories less intensively, which dampens the fall in inflation and brings the total inflation response in C2 closer to ther esponse in C1. If we neglect the real interest rate's indirect effect on inflation, then the remaining indirect effects attenuate the back-of-the-envelope's prediction about the increase in price rigidity by only 21.5%.

6 Conclusion

This paper shows that investment irreversibility leads to greater real price rigidity because irreversibility strengthens the customer-market incentive. The reason is that irreversibility makes a sticky customer base valuable as insurance against bad idiosyncratic demand shocks in the future. Using producer and export price microdata as well as markups from manufacturing microdata, I find empirical evidence in support of this ex ante mechanism.

These results can help central banks when they make forecasts during financially driven recessions. A key concern during any recession is the behavior of inflation because central banks use inflation as a gauge to determine the appropriate size of their interest rate response. Forecasting inflation during financially driven recessions may be especially difficult given the literature's conflicting evidence on whether financial shocks tend to push prices up or down. While inventory stocks are observable, the customer-market incentive is not easily measured. Financially driven recessions may also be concentrated in specific industries. This paper's results offer the following guidance. Suppose a large financial shock occurs. A central bank can combine proxies for physical irreversibility in its own economy with predictions of which industries will be hit harder by the shock. If the shock tends to affect very irreversible industries, then the central bank can predict that inflation will likely fall by less because the shock will affect industries with stronger customer-market incentives.

I leave several avenues for future research. In ongoing work in Chen (2026), I am developing a heterogeneous-firm model with partially irreversible investment, customer markets, and endogenous demand uncertainty. That paper aims to show that differences in irreversibility can generate quantitatively significant differences in the customer-market incentive in general equilibrium. Investment irreversibility can also be seen as one specific example of the broader problem of operating leverage in the presence of uninsurable demand risk. Bad demand shocks are especially painful when a firm has costs it must pay because

it cannot scale down its operations. Other input adjustment frictions generate operating leverage and could therefore shape a firm's incentive to acquire a sticky customer base. For example, human capital is a significant and partially irreversible investment by firms because workers acquire human capital over time and are imperfectly substitutable. Jäger et al. (2024) estimates that workers are very costly to replace due to imperfect substitutability, and Meeuwis et al. (2025) shows that firms respond to risk-premium shocks by firing workers whose human capital is easier to replace. When firms find it harder to replace their workers, they may invest more in sticky customer bases to ensure that they do not need to frequently adjust their employment.

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Appendices

A Descriptive Evidence about Irreversibility

A.1 Irreversibility and Demand-Side Characteristics

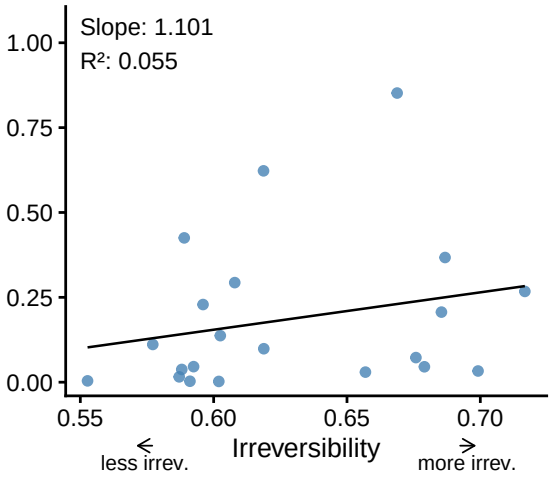
This appendix provides descriptive evidence that my measure of physical irreversibility does not appear to be a proxy for specific demand-side characteristics that might influence the incentive to accumulate a customer base or otherwise shape price responses to credit supply shocks. Figures A.1 and A.2 plot these relationships for manufacturing and mining industries. I specifically consider these characteristics, in order: (i) share of private domestic sales to households (PCE), (ii) share of output classified as exchange-traded according to the conservative classification of Rauch (1999), using domestic-shipment weights, (iii) share of Compustat firms reporting a major customer in Compustat Segment, (iv) inventory liquidation recovery rate from Kermani and Ma (2023), (v) share of output classified as a durable good, and (vi) purchase frequency by households from the Consumer Expenditure survey.

A.2 Irreversibility and Customer Markets

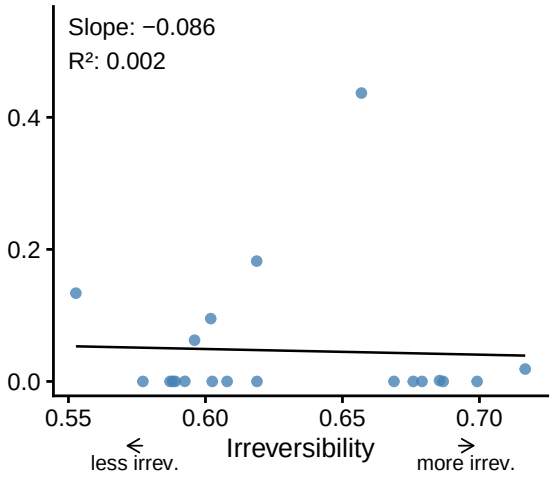
This appendix provides descriptive evidence that irreversibility and customer markets interact. In particular, industries with more irreversible investment also exhibit behaviors consistent with greater investment in future demand. While this evidence does not directly show that firms adjust prices differently in industries with higher irreversibility, as the theory predicts in Section 2, this descriptive evidence provides further support for the idea that irreversibility interacts with investment in a sticky customer base.

Contract Durations. Manufacturing firms in more irreversible industries sign longer contracts on average. This behavior is consistent with the idea that irreversibility amplifies the

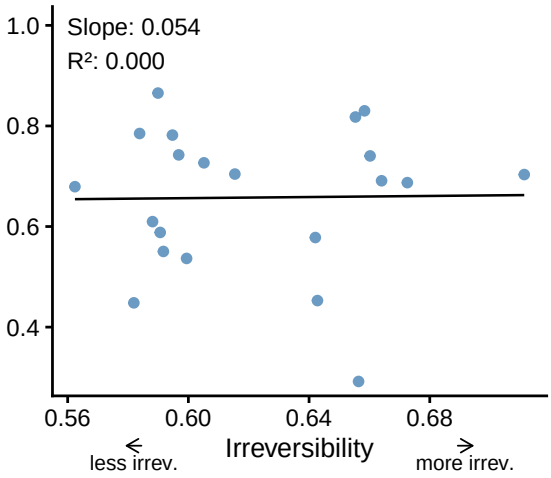
Figure A.1: Investment Irreversibility and Demand-Side Characteristics



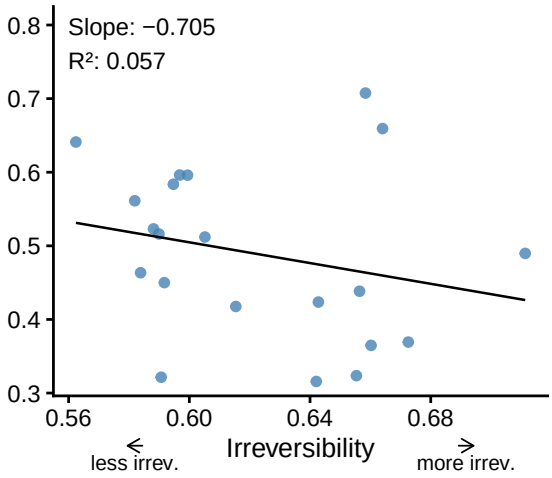
(a) Share of sales to households (PCE)



(b) Exchange-traded share



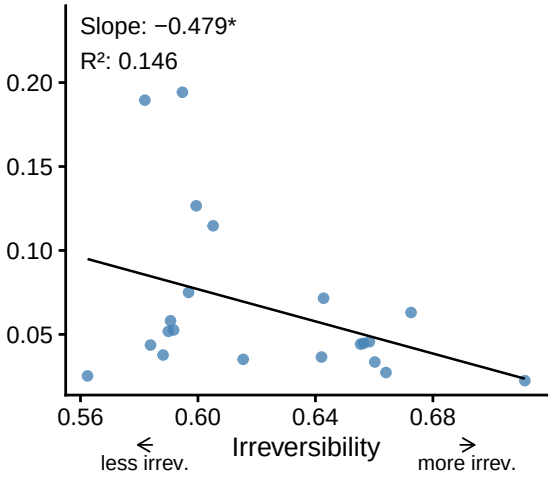
(c) Share of firms reporting a major customer



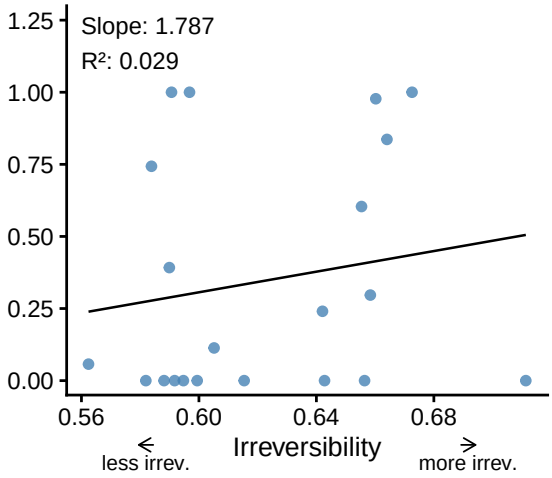
(d) Inventory liquidation recovery rate

Notes: Manufacturing and mining industries. Share of sales to households is the 1997 PCE share of private domestic sales. The exchange-traded share uses the conservative classification of Rauch (1999) and product weights from the 1997 Economic Census (Census of Manufactures and Census of Mineral Industries). Major-customer reporting share uses Compustat Segment data over 1997–2019. Inventory liquidation recovery rates are from Kermani and Ma (2023). The OLS estimates use robust standard errors. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

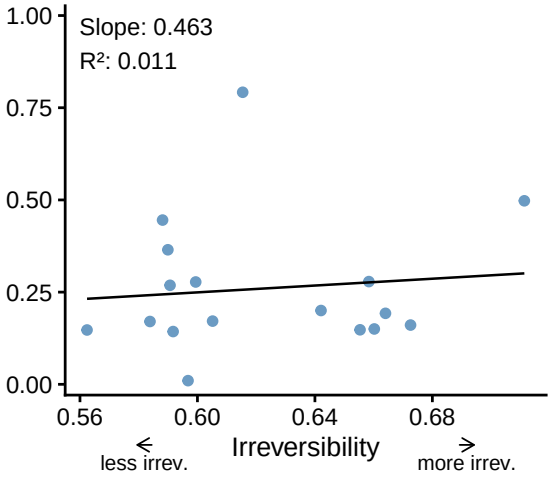
Figure A.2: Investment Irreversibility and Demand-Side Characteristics



(a) Producer concentration (HHI of sales)



(b) Share of output that is durable



(c) Household purchase frequency

Notes: Manufacturing and mining industries. Purchase frequency uses Consumer Expenditure survey diary data for 2004–2007. Mining industries lack purchase-frequency data. The OLS estimates use robust standard errors. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

cost of bad demand shocks, so firms in a more irreversible industry want to lock in demand for a longer period of time because longer contract durations give firms more certainty about their future sales. Table A.1 shows this result by regressing the duration of a contract's initial term on my measure of irreversibility. Some contracts have fixed term lengths, but others have an initial term length and then automatically renew unless one side wants to terminate or renegotiate. Column (2) restricts attention to contracts with no auto-renewal clause. The sample is limited to manufacturing firms since coverage is more complete in the contracts data from Chen et al. (2026).[50] The positive coefficients in Table A.1 indicate that firms in more irreversible industries lock up their demand further into the future by signing longer contracts. The magnitude is also quantitatively meaningful. A difference of 10 percentage points in the degree of irreversibility is associated with an increase in the average contract duration by about a year.

Table A.1: Contract Duration and Investment Irreversibility

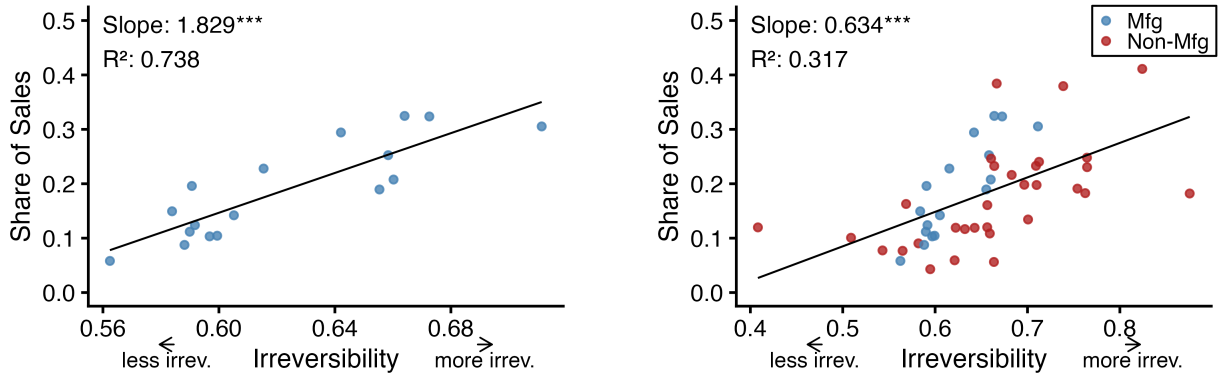
	Duration	
	(1)	(2)
Irreversibility (pp)	1.20**	1.30**
	(0.51)	(0.49)
Include Auto-Renewals	Y	N
R^2	0.234	0.251
Industries	17	17
Contracts	908	589

Notes: Relationship between contract duration and irreversibility of tangible capital at the 2-digit SIC industry level for manufacturers. Contracts in the sample are goods contracts from U.S. manufacturing sellers filed over 2001-2026. Column (2) restricts to the subset of contracts without auto-renewal clauses. The outcome variable is the average duration in months. The units of irreversibility is measured in percent so that the coefficient represents the association with an increase in irreversibility by 1pp. Robust standard errors are reported. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

[50] A contract is included in the sample if the *seller*'s industry is manufacturing, the seller is domiciled in the U.S., the seller provides a good to the buyer, and the contract is not a distribution contract.

Selling Expenses. Industries with higher irreversibility also devote a larger share of sales to selling expenses, which are another way that firms can accumulate a larger customer base. Following Chiavari (2026), I compute selling expenses from Compustat as Selling Expenses = xsga − xrent − xpr − recd − xrd − rdip. This measure takes SG&A expenses and subtracts out costs unrelated to customer acquisition.[51] Figure A.3 plots median selling expenses divided by sales over 1997-2019 against irreversibility by 2-digit SIC industry, separately for manufacturing and again for all industries. The relationship is strong. Among manufacturers, the R^2 is very high at 0.73. Across all industries, the R^2 weakens, but the relationship is still positive and significant. The weaker relationship suggests that the relationship between demand uncertainty and investment irreversibility of tangible capital matters less outside of manufacturing. The mean version involves larger coefficients because of outliers that increase the mean selling expense to sales ratio.

Figure A.3: Selling Expenses / Sales Positively Correlate with Investment Irreversibility



(a) Manufacturing Only

(b) All Industries in Kermani and Ma (2023)

This relationship is robust across aggregation choices and definitions of the selling expenses ratio. The same results apply when I use the mean instead of the median or

[51] xrent = rental expenses (e.g. structures, operating leases), xpr = pension and retirement expenses, recd = doubtful or uncollectible receivables, xrd = R&D expenses, and rdip = in-progress R&D expenses. I follow Peters and Taylor (2017) and do not remove xrd and rdip when xrd > xsga and xrd < cogs since R&D expenses are sometimes put under COGS.

the ratio of industry-total selling expenses to industry-total sales instead of a firm-weighted statistic. I also check robustness to Peters and Taylor (2017)'s measure of selling expenses, which only strips out R&D costs, and SG&A expenses, as in Gourio and Rudanko (2014). Results are available upon request.

The relationship between investment irreversibility and selling expenses is robust and stable to the inclusion of demand-side controls. predicts Table A.2 reports the relationship between irreversibility and the He et al. (2026) and Chiavari (2026) measures of selling expenses, with and without controls for an industry's share of sales to households, inventory recovery rates from Kermani and Ma (2023) and the share of firms reporting a major customer. The point estimates do not change much when I add controls. I choose these specific measures because they are available across most industries. The Rauch (1999) commodity classification, for example, only applies to goods.

Table A.2: Selling Expenses and Investment Irreversibility with Demand-Side Controls

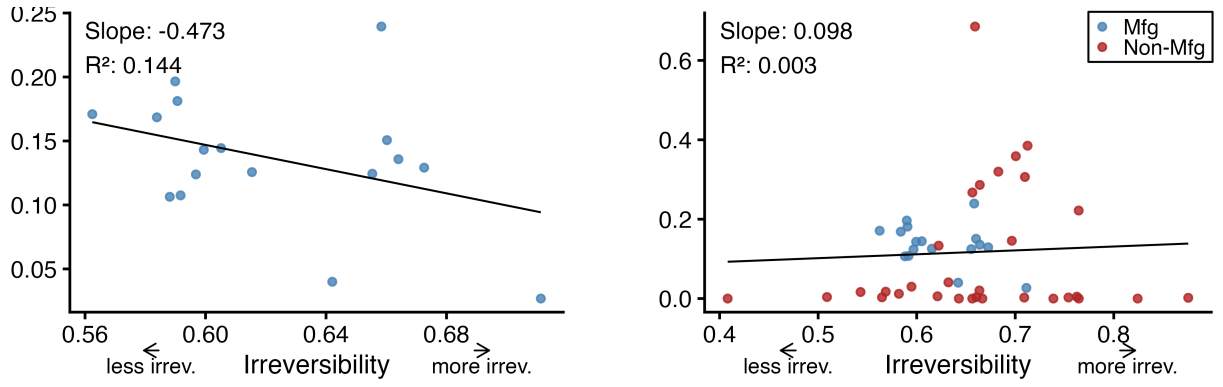
	HMS median S&M / revenue		Chiavari median selling / sales	
	(1)	(2)	(3)	(4)
Irreversibility	0.1464**	0.1420*	0.4905***	0.4860***
	(0.0674)	(0.0741)	(0.1079)	(0.1285)
Demand-Side Controls	N	Y	N	Y
N	42	42	43	43
R^2	0.146	0.189	0.369	0.501

Notes: This table regresses two different measures of selling expenses on the irreversibility measure and checks that their point estimates are stable when I control for four demand-side characteristics: share of sales to households, inventory recovery rates from Kermani and Ma (2023), share of firms reporting a major customer, and HHI concentration. I choose this subset of characteristics because the others are not available for a broad range of industries. Each reported coefficient is scaled so that it is the percentage point response to a one percentage point increase in irreversibility. Robust standard errors are reported. * $p < .10$, ** $p < .05$, *** $p < .01$.

A.3 Investment Irreversibility and Inventory Accumulation

Figure A.4 reports median inventories/assets ratios from Compustat.

Figure A.4: Median Inventories/Assets



(a) Manufacturing Only

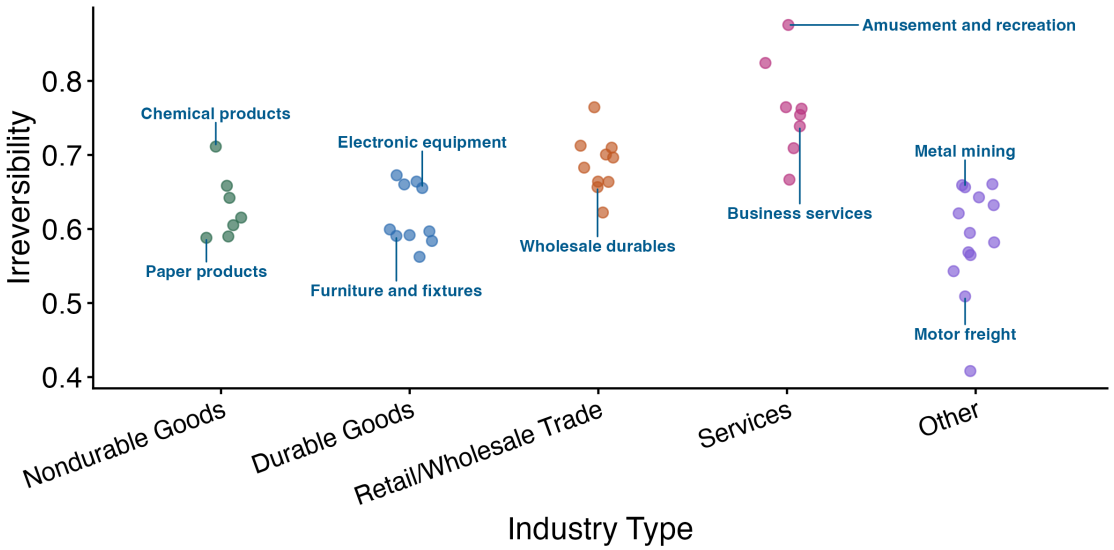
(b) All Industries in Kermani and Ma (2023)

A.4 Correlations of Investment Irreversibility with Other Industry Characteristics

Investment irreversibility varies widely within different types of industries, as shown in Figure A.5. For instance, in non-durable and durable goods manufacturing, the difference in irreversibility exceeds 10pp between the most and least irreversible industries. Retail/wholesale trade and services industries generally have more irreversible tangible capital. In this paper, we should conceptualize irreversibility as the difficulty of reducing a firm's scale rather than large upfront costs which discourage the initial investment. For example, one might suspect that, because durables manufacturing can be very capital intensive, irreversibility is higher in these industries. However, Figure A.5 shows the opposite is true.

Table A.3 restricts to manufacturing industries and groups them by quartiles of irreversibility. Within each quartile, the industries are listed in ascending order of irreversibility,

Figure A.5: Irreversibility by Broad Industry Types



Notes: Services includes information, business/professional services, health care, education. Other combines mining, agriculture, utilities, construction, transportation/pipeline services.

so the top row of Quartile 1 (primary metal) is the manufacturing industry with the lowest irreversibility and the bottom row of Quartile 4 (chemical products) is the single most irreversible industry.

Table A.3: Mfg. Industries by Irreversibility Quartile

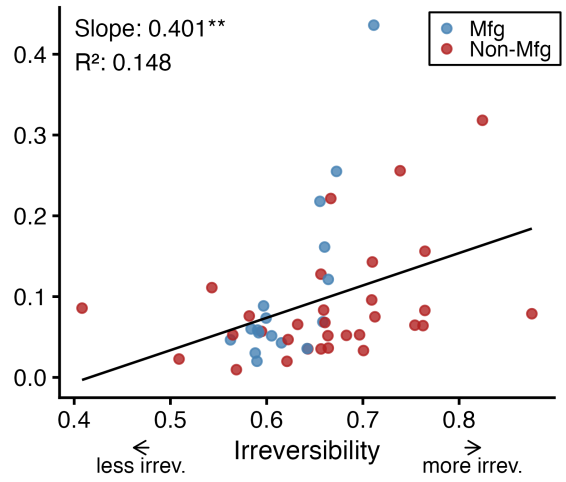
Quartile 1 (lowest irrev.)	Quartile 2	Quartile 3	Quartile 4 (highest irrev.)
Primary metal	Stone, clay, glass, and concrete	Food products	Machinery
Fabricated metal	Transportation equipment	Printing and publishing	Misc. manufacturing
Paper products	Wood products	Electronic equipment	Analytical instruments
Textile products	Rubber and plastics products	Apparel products	Chemical products
Furniture and fixtures			

In this appendix section, I also investigate other industry characteristics that correlate with irreversibility and discuss why these other characteristics are either supportive of my interpretation or unlikely to be the explanation behind the results from the price difference-in-differences in Section 4. At the 5% significance level, higher irreversibility is associated with higher cash/assets, a higher operating lease share, a higher perceived discount rate, and

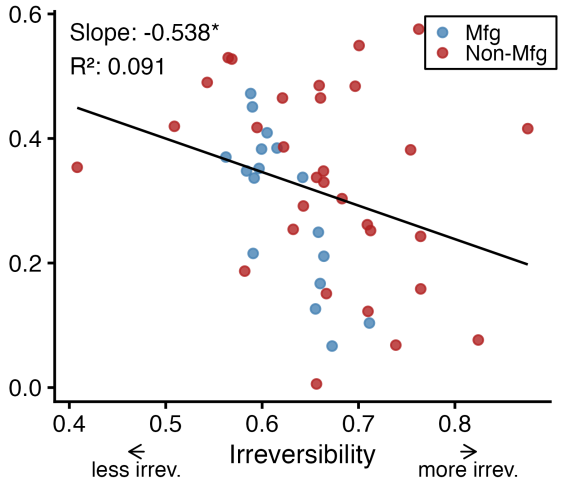
a lower frequency of price adjustment, and, at the 10% level, with lower book leverage.[52] I discuss most of these characteristics and their association with irreversibility in the main text. The association with discount rates is consistent with the idea that irreversibility increases the hurdle rate for investments.

[52]The industry characteristics are measured using Compustat data over the years 1997-2019, with the exception of the operating lease capital share, for which I use data over 2019-2025, and capital intensity, which is measured using BLS industry data. For any firm-year-level characteristic, I aggregate by taking the median within an industry. The frequency of price adjustment is computed by Pasten et al. (2020) using PPI data over 2005-2011 concorded to the BEA summary level, and the perceived discount rates are from Gormsen and Huber (2022).

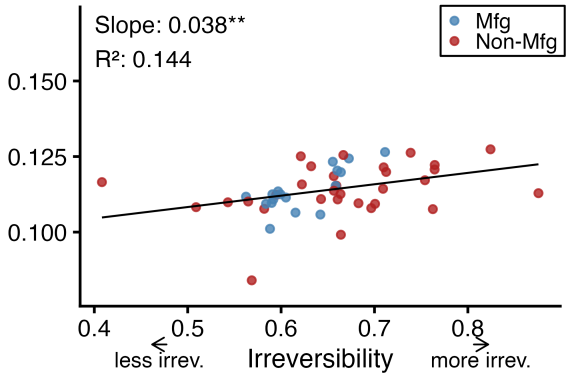
Figure A.6: Characteristics of Irreversible Industries



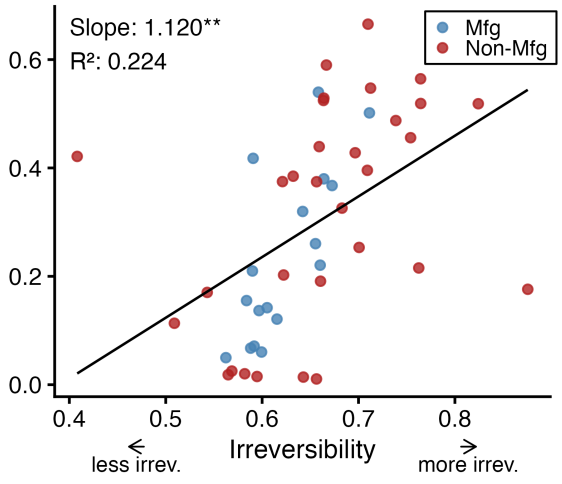
(a) Cash / Assets



(b) Book Leverage

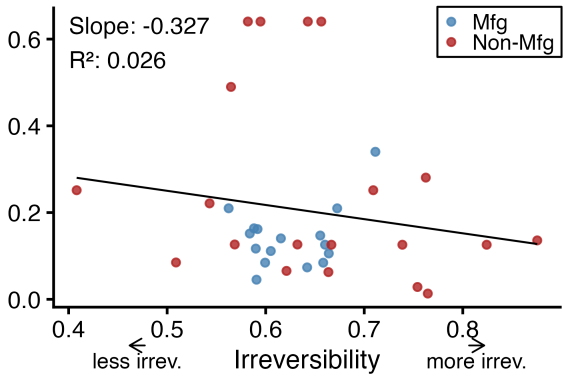


(c) Perceived Discount Rate

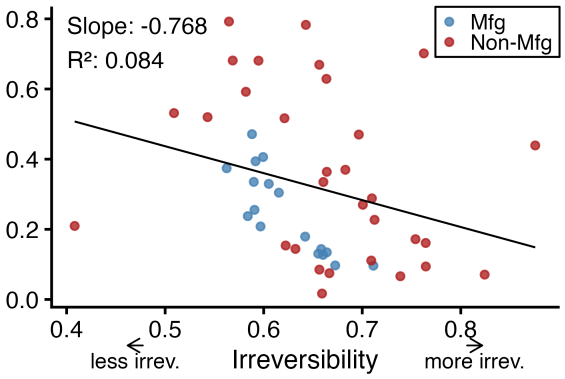


(d) Operating Lease Capital Share

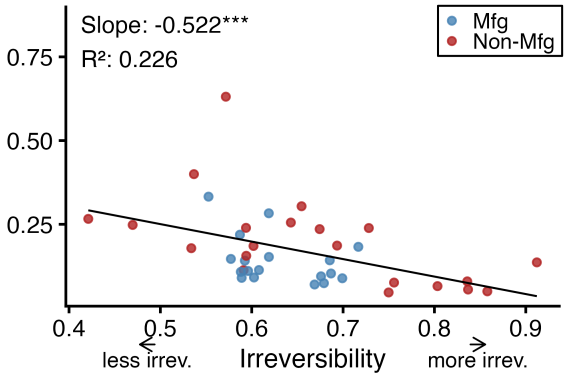
Figure A.7: Characteristics of Irreversible Industries



(a) Capital Intensity



(b) PPENT / Assets



(c) Freq. of Price Adjustment

B Diagnostic Checks for Empirical Analysis

B.1 Summary Statistics

Table B.1: Summary Statistics of the PPI-Census-Dealscan Dataset

Variable	N	Mean	SD	P10	P50	P90
Public	350	0.86				
Manufacturer	350	0.57				
Bond Access	350	0.57				
Credit Supply Shock (Dealscan)	350	0.42	0.12	0.28	0.41	0.56
Lehman Co-Syndication IV (%)	350	0.89	0.42	0.56	0.79	1.3
Employment (Mar. 2007)	350	17900	42800	724	5520	35700
Last Loan Amount (mil.)	350	711	1270	45	300	1600
Age	350	38.5	11.2	32	32	58
Irreversibility (%)	350	63	5	57	62	71
Last Loan All-In Spread (bp)	350	114	91.5	25.5	75.5	250
Last Loan Maturity (months)	350	53.6	14.5	36	60	62
Loan Due	350	0.23				
YoY % Price Chg. 2009:Q3	350	-6.65	25.1	-38	0	8.2

Notes: Manufacturers form a majority of the sample, but the PPI also surveys firms in many other industries, including mining, agriculture, retail, and wholesale trade. A firm's industry code comes from Census Bureau data, but the PPI may collect prices on products unrelated to a firm's primary industry. Any views expressed are those of the author(s) and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

B.2 Khwaja and Mian (2008) Test

This appendix reports the Khwaja and Mian (2008) test discussed in Section 3.3. In short, this test considers firms which signed a new loan over 2008:Q4-2009:Q2 and computes the growth in lending for each lender-firm pair across lenders in the firm's last syndicate. Note that many lenders will extend zero dollars to the firm in the post-Lehman period, so my

measure of the lending change must allow for zeros. I consider two versions. First, the lending change is $\log(1+\text{loan}_{\text{post}}) - \log(\text{loan}_{\text{last}})$. Second, I use the Davis et al. (1996) (DHS) growth rate.

With this data, I regress the change in lending on the firm-bank credit supply shock $\Delta L_{f,b}$ and either observable firm characteristics or a borrower fixed effect. Under certain assumptions, the bias introduced when firm-bank matching is not as good as random equals the difference in the point estimates for the coefficient on $\Delta L_{f,b}$ across these two specifications. Because my sample size falls substantially, I adjust the covariates from the baseline sample to make the regression estimable. Specifically, I use 2-digit NAICS fixed effects instead of 3-digit, an indicator for whether a loan was due instead of the bins for the number of loans due, an indicator for bond issuance or not, group together firms whose last loan was signed in 2004 or earlier, and drop altogether the indicator for whether a firm had multiple leads in the pre-period. As Table B.2 shows, the point estimates are quite close to each other. The differences have similar magnitudes to what Chodorow-Reich (2014) and Kim (2021) find.

Table B.2: Khwaja and Mian (2008) Test

	(1)	(2)	(3)	(4)
-$\Delta L_{f,b}$	-2.49^{***}	-2.43^{***}	-0.33^{***}	-0.30^{***}
	(0.40)	(0.43)	(0.05)	(0.05)
Borrower FE	Y	N	Y	N
Controls	N	Y	N	Y
R-Squared	0.31	0.22	0.40	0.26
Observations	350	350	350	350

Note: Estimates of the coefficient on the firm-bank credit supply shock $-\Delta L_{f,b}$. The outcome variable in columns (1)-(2) is the log growth and in columns (3)-(4) is the DHS growth rate. The columns differ in the covariates used (borrower FE or observables). Standard errors are two-way clustered by firm and bank. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

B.3 Balance of the Lehman Instrument

Table B.3 reports the association between the Lehman co-syndication instrument and observed firm characteristics. The Lehman co-syndication IV is generally not associated with observable covariates. The only variables that are statistically significant at the 5% level are the interest rate spread, bond access, and book leverage. I control for the first two covariates in all regressions, and when I add book leverage as well as other balance sheet controls in Appendix C.1, the results still hold.

Table B.3: Balance Test of Observables with Lehman Co-Syndication IV

Variable	Association	Variable	Association
Last Loan All-In Spread (bp)	$+^{****}$	PPENT/Assets	+
Age	+	Liq. Assets/Assets	−
Last Loan Maturity (months)	+	Book Leverage	$+^{**}$
Bond Access	$+^{***}$	Interest/Debt	+
Public	+	Inventory/Sales	−
Loan Due	+	Inventory/Assets	−
Loan Issued After 2005	+	PPENT	+
Multiple Leads on Last Loan	+	Assets	+
Multiple Leads on A Loan in 2006Q4-7Q2	+	Sales	+
Employment (Mar. 2007)	+	Log(Inventory)	+
Irreversibility (pp)	+	Log(Assets)	+
Last Loan Amount (mil.)	+	Log(Liq. Assets)	+
Recoverable PPENT/Assets	+		

Sign and significance when I regress firm characteristics on the Lehman co-syndication IV to check whether these characteristics are balanced. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

B.4 First-Stage Diagnostics

Table B.4 reports first-stage diagnostics for the instrumental-variables specification. Note that the credit supply shock is the same in every period for the main diff-in-diff, so I run the first stage for a single period. When using the Lehman co-syndication IV as a single instrument for the credit supply shock (no interaction with irreversibility), Olea and Pflueger (2013)'s robust test for a weak IV rejects the null hypothesis when using the standard critical value of 10%. In the case of a single endogenous regressor that is just identified, Angrist and Kolesár (2024) recommends that researchers screen by the sign of the first stage predicted by economic theory. I find that the first stage for (8) is positive and statistically significant at the 5% level. This sign is precisely what I expect a priori: higher exposure due to co-syndication with Lehman predicts a higher value of $-\Delta\widetilde{L}_f$, i.e. tighter credit supply.

Table B.4: Olea and Pflueger (2013) Weak IV Test

Statistic	Estimate
Eff. F-Stat	69.4
Crit. Val. for 10%	23.1

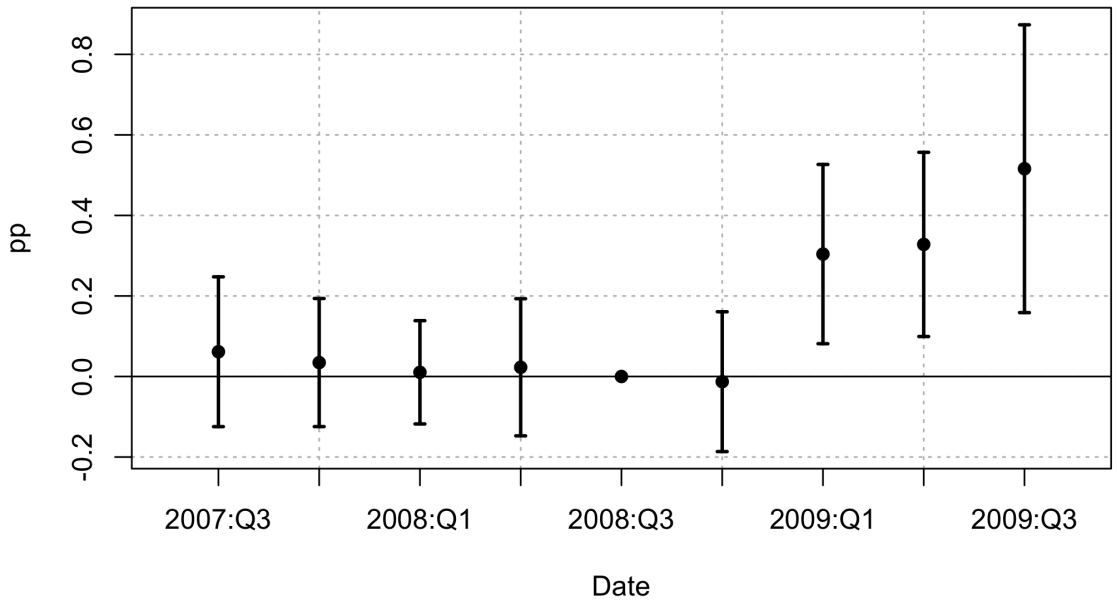
Note: The effective F-stat from Olea and Pflueger (2013)'s robust test for weak IVs is large and exceeds the critical value for a maximum relative bias of 10%. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

C Additional Results Using Credit Supply Shocks

C.1 Robustness Checks for Price Regressions

Results Using Lehman Co-Syndication IV. Figure C.1 shows the reduced-form estimate of $\beta_{1,k}$ when using the Lehman co-syndication IV. Figure C.2 shows the estimate of $\beta_{1,k}$ when book leverage, average interest rate, cash-to-assets ratio, and inventory-to-sales ratio are added as additional covariates.

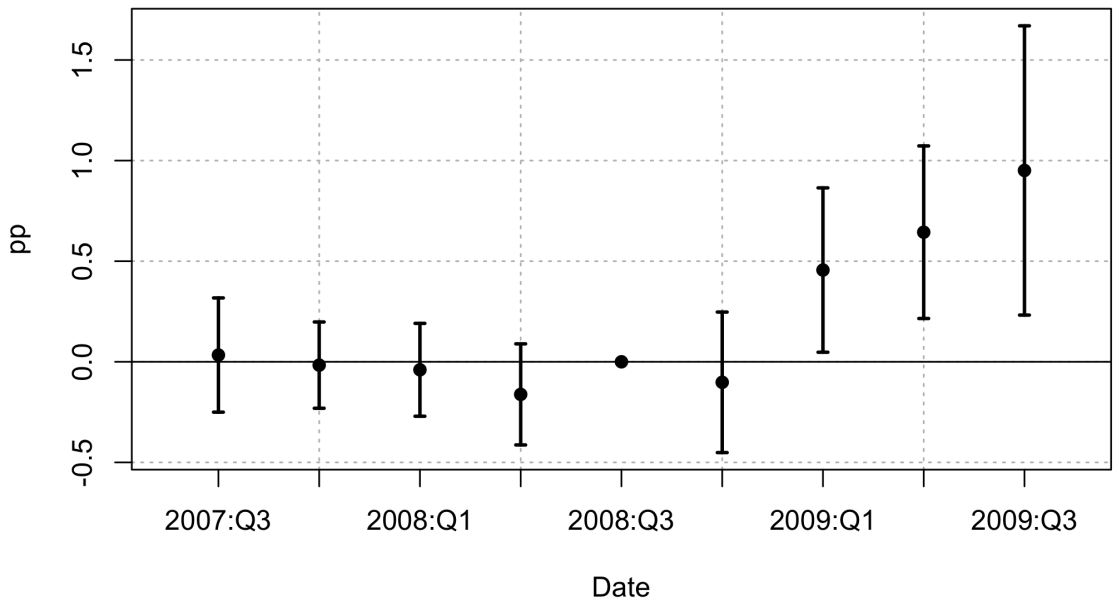
Figure C.1: Reduced Form of Regression with Linear Interaction in Main Text



Note: The figure displays the estimated coefficients and confidence intervals for the primary specification. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

In addition to these diff-in-diffs, I verify that the sign and significance of $\beta_{1,2009:Q3}$ remains robust across a variety of other specifications. The sign and significance at the 5%

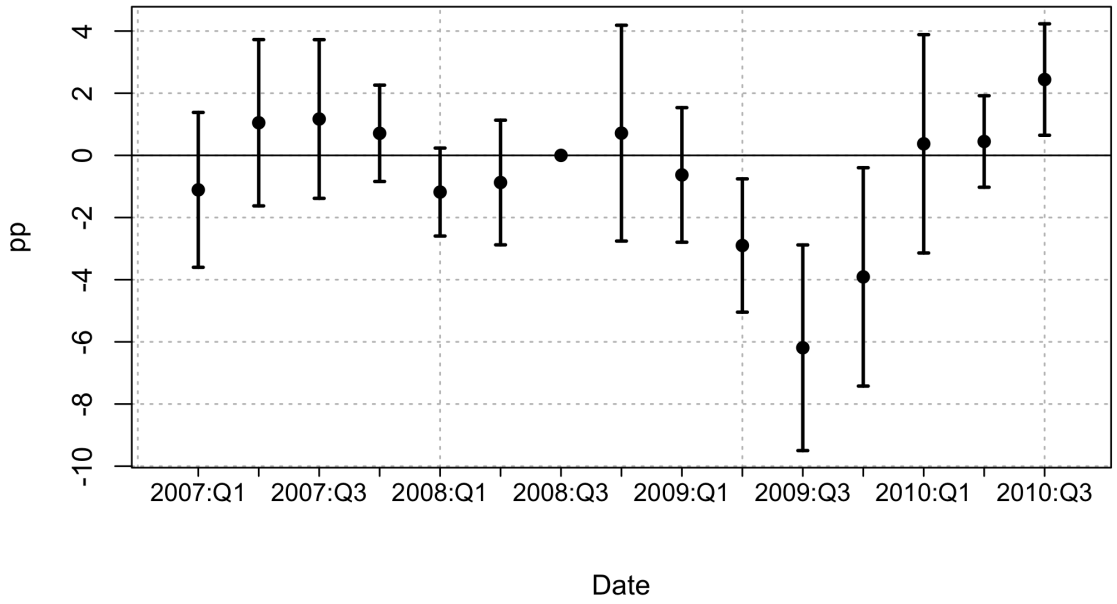
Figure C.2: Adding Balance Sheet Controls to Baseline Regression in Main Text



Note: This figure presents estimated effects when I also include as covariates the liquid assets ratio, book leverage, average interest rate on debt, and inventory to sales ratio measured in 2007. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

level is unchanged when I use 2-digit NAICS instead of 3-digit (with and without balance sheet controls) or proxy irreversibility with one minus the actual time-averaged recovery rate *without projecting onto physical attributes*. If I construct the outcome variable by first aggregating to a single price within a year-quarter as an average or geometric average of all items within a 4-digit NAICS by buyer type product group and then computing year-over-year log price changes from the aggregated index, the sign on the coefficient in 2009:Q3 remains the same, but it is no longer significant. However, the sign and significance of the interaction coefficient in 2009:Q1 remains positive at the 5% level. This result could stem from measurement error since a product group at the 4-digit NAICS level does combine many

Figure C.3: Effects on Prices Over a Longer Horizon



Note: This figure regresses the year-over-year price change on the instrumented credit supply shock over 2007:Q1-2010:Q3. Due to sample size concerns, this regression does not check for heterogeneous effects across high/low irreversibility industries. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

different types of products. My baseline measure avoids this measurement error because I compute goods-level inflation rate, which compares the price over time for the exact same good, and then average across goods to the product group level to get an inflation rate.

Potential Pre-Trend in the Levels of the Price Response. One reasonable concern about the estimates in Figure 3, which interacts the credit supply shock with an indicator for above or below median irreversibility, is that there may be evidence of pretrends. Specifically, the coefficient in 2007:Q3 for the above-median group appears quite close to being statistically significant at the 5% level. I argue that this issue is not too big of a concern

for two reasons. First, the key novel contribution of my paper is the relative difference in pricing behavior between the two groups. For a triple difference-in-differences, even if there are pretrends, as long as the pretrends are similar across the groups of the third difference (i.e. high versus low irreversibility), we can still claim that a decline in credit supply causes firms in industries with higher investment irreversibility to set relatively higher prices than firms in industries with lower irreversibility.[53] In my case, the coefficients in Figure 3 are very close to each other in every quarter of the pre-period. Second, in Appendix C.1, I run the reduced-form regression without heterogeneity by irreversibility for the subsample of firms with panel data over 2007:Q1-2010:Q3. With the longer preperiod, this regression does not have evidence of a concerning pretrend, which assuages the concern that exposure to the Lehman co-syndication IV is not independent of shocks or unobserved borrower characteristics that affect price changes. In addition, the shape of the response in the post-period resembles what Kim (2021) finds with data on consumer packaged goods.

Results Using Other Instruments. My results are robust to using two other intruments from Chodorow-Reich (2014), namely exposure to the ABX index and bank deposit-to-assets ratio in 2007. The evidence in Table C.1 is weaker, but the signs are consistent with what I find using the Lehman co-syndication instrument. These regressions also include the balance sheet controls of Figure C.2.

Export Prices. My results are robust to export prices, as discussed in the main text. A key advantage of the export price data is that I can check the role of demand elasticities. Kim (2021) and Renkin and Züllig (2024) find that the demand elasticity in a product market explains heterogeneity in how prices responds to credit supply shock. Figure C.4 reports the event-study figures for both high and low demand elasticity groups. The difference in point estimates are larger for low demand elasticities while the difference for goods with

[53] See Olden and Møen (2022) and Caron (2025)

Table C.1: Sign and Significance of When Using ABX and Deposits/Assets Instruments.

Coefficient	ABX	Deposits/Assets
$\beta_{1,2009:Q1}$	$+$	$+^{*}$
$\beta_{1,2009:Q2}$	$+$	$+^{*}$
$\beta_{1,2009:Q3}$	$+^{*}$	$+$

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high demand elasticities are marginally significant at the 10% level. However, alternative specifications in which vary how I construct the unit-value price lead to significance at the 5% level. Table C.2 reports those results. Notably, while the baseline results show only marginally significant differences in the relative price response for goods with high demand elasticities, alternative constructions of the unit-value price show stronger significance. I also use the export price data to check for omitted variables that could be correlated with irreversibility and are alternative explanations for my results in Table C.4 .

Table C.2: Sign and Significance of Price Responses Using Alternative Unit-Value Price Measures

Price measure	2009	2010	2011
A. Price response within HL			
Geometric Weighted Average, No Constrained Elasticity Estimates	$+^{*}$	$+^{*}$	$+^{***}$
Geometric Weighted Average	$+$	$+^{*}$	$+^{**}$
Weighted Average, No Constrained Elasticity Estimates	$+$	$+$	$+^{***}$
Weighted Average	$+$	$+^{*}$	$+^{**}$
B. Price response within LL			
Geometric Weighted Average, No Constrained Elasticity Estimates	$-^{*}$	$+$	$-$
Geometric Weighted Average	$-$	$-$	$-$
Weighted Average, No Constrained Elasticity Estimates	$-^{**}$	$+$	$-$
Weighted Average	$-$	$-$	$-$
C. Price response within HH			
Geometric Weighted Average, No Constrained Elasticity Estimates	$-$	$-$	$-^{***}$
Geometric Weighted Average	$+$	$+$	$-$
Weighted Average, No Constrained Elasticity Estimates	$-$	$+$	$-$
Weighted Average	$+$	$+$	$-$
D. Price response within LH			
Geometric Weighted Average, No Constrained Elasticity Estimates	$-^{***}$	$-^{***}$	$-^{***}$
Geometric Weighted Average	$-^{***}$	$-^{***}$	$-^{***}$
Weighted Average, No Constrained Elasticity Estimates	$-^{***}$	$-^{***}$	$-^{***}$
Weighted Average	$-^{***}$	$-^{***}$	$-^{***}$
E. Relative price response: HL − LL			
Geometric Weighted Average, No Constrained Elasticity Estimates	$+^{**}$	$+$	$+^{***}$
Geometric Weighted Average	$+$	$+^{**}$	$+^{**}$
Weighted Average, No Constrained Elasticity Estimates	$+^{***}$	$+$	$+^{***}$
Weighted Average	$+^{*}$	$+^{*}$	$+^{**}$
F. Relative price response: HH − LH			
Geometric Weighted Average, No Constrained Elasticity Estimates	$+$	$+^{*}$	$+^{*}$
Geometric Weighted Average	$+^{**}$	$+^{**}$	$+^{**}$
Weighted Average, No Constrained Elasticity Estimates	$+^{**}$	$+^{***}$	$+^{***}$
Weighted Average	$+^{***}$	$+^{***}$	$+^{***}$

Note: The first letter of each group denotes high (H) or low (L) irreversibility; the second denotes high or low demand elasticity. Panels A–D report the sign and significance of each group's credit-supply-shock coefficient; panels E and F report differences between high- and low-irreversibility groups within each demand-elasticity group. Rows compare geometric and arithmetic weighted averages of unit values, with and without constrained elasticity estimates. The baseline results in the main text correspond to "Geometric Weighted Average, No Constrained Elasticity Estimates." Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3432. (CBDRB-FY26-P3432-R13333). This is a part of clearance request #13333. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

Table C.3: Sign and Significance of Export Price Responses by Irreversibility Only

Price measure	2009	2010	2011
A. High irreversibility			
Geometric Weighted Average, No Constrained Elasticity Estimates	$+$	$+$	$+$
Geometric Weighted Average	$+$	$+^{*}$	$+$
B. Low irreversibility			
Geometric Weighted Average, No Constrained Elasticity Estimates	$-^{***}$	$-^{*}$	$-^{**}$
Geometric Weighted Average	$-^{**}$	$-^{*}$	$-^{**}$
C. Relative price response: High − Low			
Geometric Weighted Average, No Constrained Elasticity Estimates	$+^{***}$	$+^{**}$	$+^{***}$
Geometric Weighted Average	$+^{**}$	$+^{***}$	$+^{**}$

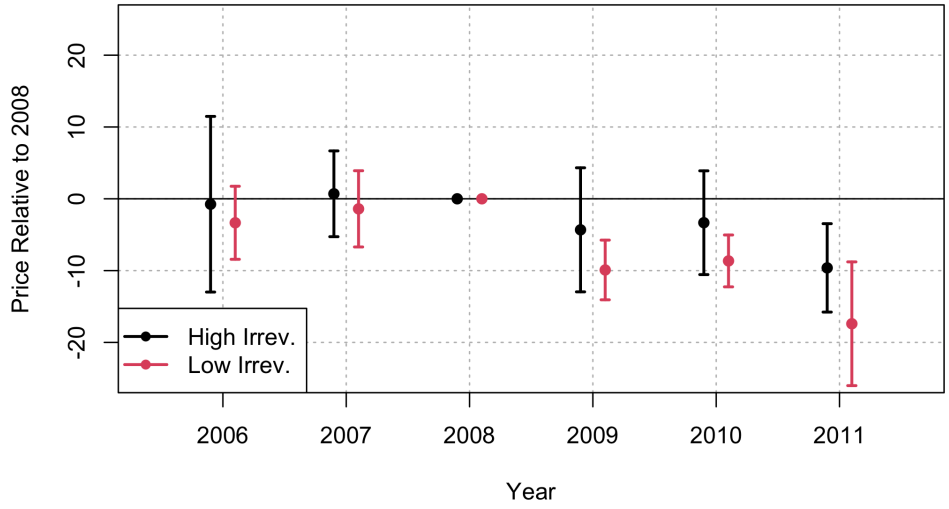
Note: This table shows that the responses of export prices to credit supply shocks in high- versus low-irreversibility industries resemble the responses in the PPI data. Thus, the split by demand elasticity simply uncovers the fact that, for sufficiently inelastic demand, the customer-market incentive becomes strong enough that firms in high-irreversibility industries hike their prices instead of merely neglecting to cut them. Panels A and B report the sign and significance of the credit-supply-shock coefficient for firms in high- and low-irreversibility industries, where the credit supply shock is not interacted with the demand elasticity. Panel C reports the difference between the high- and low-irreversibility coefficients. Rows are alternative constructions of the log unit-value price; the first row, "Geometric Weighted Average, No Constrained Elasticity Estimates," is the baseline. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3432. (CBDRB-FY26-P3432-R13333). This is a part of clearance request #13333. $^{*}\,p < 0.1$, $^{**}\,p < 0.05$, $^{***}\,p < 0.01$.

Table C.4: Checks for Omitted Variable Bias in Treatment Effects

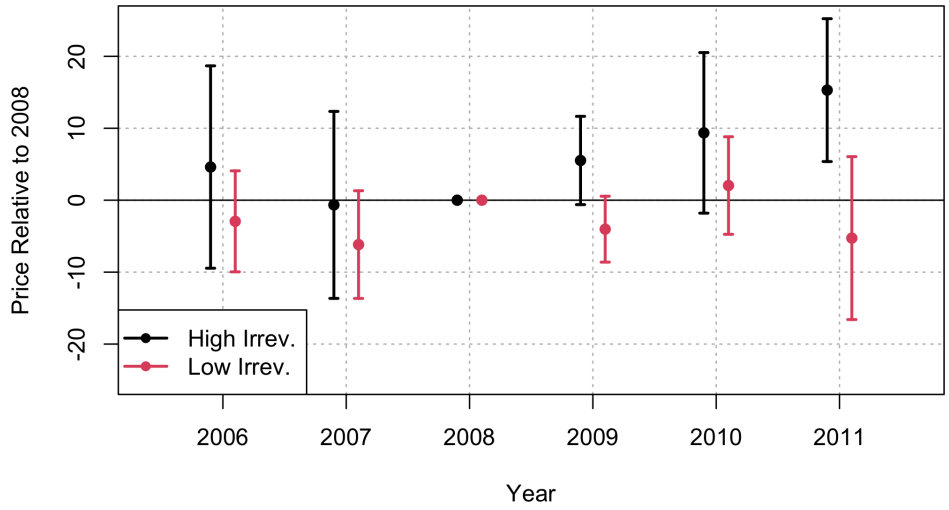
Additional control	2009	2010	2011
A. Relative price response: HL − LL			
Baseline	$+^{**}$	$+$	$+^{***}$
Spread on last loan	$+^{**}$	$+$	$+^{***}$
Liquid assets / assets	$+^{***}$	$+$	$+$
Book leverage	$+^{***}$	$+$	$+^{**}$
PPE / assets	$+^{***}$	$+$	$+^{***}$
SG&A / sales	$+^{**}$	$-$	$+$
B. Relative price response: HH − LH			
Baseline	$+$	$+^{*}$	$+^{*}$
Spread on last loan	$+$	$+$	$+^{*}$
Liquid assets / assets	$+$	$+$	$+$
Book leverage	$+$	$+$	$-$
PPE / assets	$+$	$+$	$+$
SG&A / sales	$+^{*}$	$+$	$+$
C. Price response within HL			
Baseline	$+^{*}$	$+^{*}$	$+^{***}$
Spread on last loan	$+$	$+^{*}$	$+^{***}$
Liquid assets / assets	$+^{*}$	$+$	$-$
Book leverage	$+$	$+$	$+$
PPE / assets	$+^{**}$	$+^{**}$	$+^{**}$
SG&A / sales	$+^{**}$	$+$	$-$

Note: HL, LL, HH, and LH refer to the irreversibility by demand elasticity bin, so HL refers to high-irreversibility and low demand elasticity. Panels A and B report the significance of the difference in price responses to the credit supply shock across industries with high versus low irreversibility for products with similar demand elasticities. Baseline reports the regression from the main text. The remaining rows check robustness to adding additional characteristics as interactions with the credit supply shock. All characteristics are predetermined variables measured before the Lehman bankruptcy. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3432. (CBDRB-FY27-P3432-R13463). This is a part of clearance request #13463. $^{*}\,p < 0.1$, $^{**}\,p < 0.05$, $^{***}\,p < 0.01$.

Figure C.4: Prices Also Respond Differently to Credit Supply Shocks for Exporters



Panel (a): High Demand Elasticity



Panel (b): Low Demand Elasticity

Notes: Each panel plots the estimated impact of a credit supply shock on the log unit value price (relative to 2008) for firms with above-median irreversibility (black) versus below-median irreversibility (red) selling products with above-median demand elasticity (panel (a)) versus below-median demand elasicity (panel (b)). The product-level demand elasticities are import demand elasticities from Soderbery (2015). Standard errors are clustered by primary lender, and bars are 95% confidence intervals. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3432. (CBDRB-FY26-P3432-R13333). This is a part of clearance request #13333.

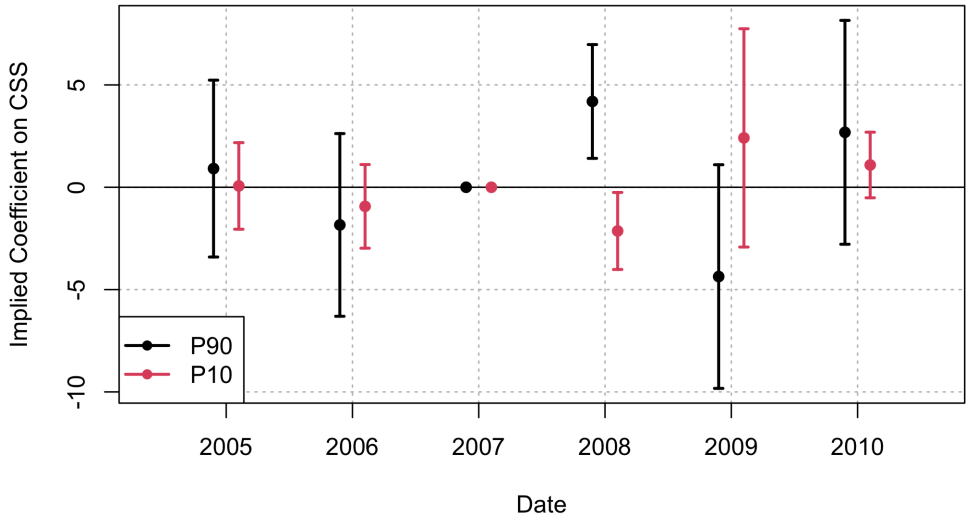
TFPR as a Proxy for Prices. The response of log TFPR growth to the credit supply shock provides additional evidence about the behavior of firms' prices. TFPR is "revenue" TFP and is distinct from TFP because TFPR measures productivity in terms of revenue rather than physical output, meaning a firm's TFPR equals physical productivity *times the firm's price*. An increase in TFPR can reflect either an increase in physical productivity or any shift that raises prices (see Foster et al. 2008). Under the assumption that an adverse credit supply shock does not increase productivity, evidence of positive TFPR growth can be interpreted as an increase in prices.

I measure TFPR using estimates from Blackwood et al. (2023)'s Collaborative Micro-Productivity Project (CMP). The CMP estimates TFPR using a production function accounting approach with the Annual Survey of Manufactures (ASM) and Census of Manufactures (CMF). These estimates deflate revenue with industry deflators, so any price changes should reflect within-industry changes, which bolsters the interpretation of positive TFPR growth as an increase in a firm's price to generate liquidity relative to other firms in the same industry. I compute the sales-weighted average TFPR across establishments with the same 4-digit NAICS code and parent firm. Then I use year-over-year growth in firm by 4-digit NAICS TFPR as the outcome variable in a difference-in-differences regression similar to (8). The product group by time fixed effects are replaced with 4-digit NAICS by time fixed effects since many firms operate establishments in multiple industries at the 4-digit NAICS level.[54]

After a decline in credit supply, firms in industries with high investment irreversibility temporarily increase their prices while firms in industries with low investment irreversibility temporarily cut their prices. Figure C.5 shows this result in an event study by reporting the estimated impact of a credit supply shock on TFPR growth for firms with irreversibility at the

[54]It is not necessarily the case that the 4-digit NAICS codes of a parent firm's establishments are strict subsets of the parent firm's 3-digit NAICS code. In the LBD, an establishment's industry code is determined by its production independent of the parent firm. For example, the 2-digit NAICS code for the headquarter establishment of a multi-unit firm may be 55 because the establishment primarily provides management services, even if the firm is a manufacturer.

Figure C.5: High (Low) Irreversibility Firms Temporarily Raise (Cut) TFPR



Notes: This figure shows the predicted effect of a one standard deviation credit supply shock on TFPR growth for firms with irreversibility at the 90th percentile of the distribution versus firms with irreversibility at the 10th percentile. Bars are 95% confidence intervals. I cluster standard errors by primary lender. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

90th percentile of the distribution versus firms with irreversibility at the 10th percentile.[55] There is a lack of a pre-trend in 2005 and 2006, but firms with high irreversibility have positive TFPR growth in 2008 followed by negative TFPR growth of similar magnitude in 2009. Thus, the level of TFPR rises only temporarily. The opposite occurs for firms with low irreversibility. Note that annual TFPR is based off the entire year's production, so the data behind the TFPR estimate in 2008 includes 2008:Q4, the quarter after the Lehman bankruptcy. This spike in TFPR growth is consistent with the nearly significant response of year-over-year price changes in 2009:Q1 in Figure 3, as that would have been the year-over-

[55]Due to the smaller sample size for this regression, I report OLS estimates and do not instrument with the Lehman co-syndication IV.

year price change in January of 2009.

C.2 Employment Growth

I compute the firm-level annual growth rate in employment in March using LBD data.[56] Table C.5 reports the sign on the interaction between the credit supply shock and irreversibility as well as the predicted impact on employment growth for firms with irreversibility at the 90th percentile. The growth in employment from Mar. 2008 to Mar. 2009 is not statistically different across industries with high or low irreversibility, but the growth in employment from Mar. 2009 to Mar. 2010 is lower for firms in industries with higher irreversibility. For a 1pp increase in irreversibility, the impact on employment growth is about -0.6pp lower. The second row in the table confirms that firms with irreversibility at the 90th percentile cut employment growth by almost 9pp over Mar. 2009-2010 after a cut in credit supply.[57]

[56] I also remove the product group by time fixed effect and log of employment from the controls to avoid any spurious correlation. Then I add the firm's lagged employment growth (as a time-varying control) to account for the persistence of employment growth and the weighted employment growth across counties in which a borrower has establishments to control for local demand shocks that might be driving a borrower's employment response. The regression uses employment data over 2006-2010. Coefficients are relative to 2008, which is the impact of the credit supply shock on employment growth from Mar. 2007 to Mar. 2008. Note that the time-varying lagged employment growth may be a bad control, but that fact, if anything, should attenuate the estimated effects.

[57] No standard error is reported for Mar. 2008-2009 because, to limit the number of disclosed estimates, I did not disclose the covariance term needed to correctly compute the standard error of the predicted impact.

Table C.5: Firms with High Irrev. Cut Employment More After Credit Supply Shock

Year	2009	2010
	(1)	(2)
CSS $\times$Irrev$_{\text{ind}(f)}$	0.060	-0.613^{**}
	(0.289)	(0.291)
Predicted Effect, P90 Irreversibility	1.731	-8.623^{**}
		(3.254)

Notes: This table report the coefficient on a linear interaction between the credit supply shock and the irreversibility as well as the predicted effect of the credit supply shock on employment growth when irreversibility is at the 90th percentile. Column (1) is Mar. 2008-2009 and (2) is Mar. 2009-2010. No standard error is reported for the predicted effect in Column (1) to limit the number of disclosed estimates. I cluster standard errors by primary lender. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601. $^{*}p < 0.1$, $^{**}p < 0.05$, $^{***}p < 0.01$.

C.3 Establishment Capital and Inventory Responses

The following table uses establishment-level data on capital equipment and inventory stocks. I find evidence that firms in high-irreversibility industries do not cut their equipment stocks after a cut in credit supply, consistent with an unwillingness to downscale capital, while firms in low-irreversibility industries do seem to cut their equipment stock. These cuts are large enough that the log equipment and structures capital stock also falls at the 10% level in 2009 for low-irreversibilty firms. Consistent with the cut in prices by low-irreversibility firms, their log inventory stocks significantly decline over 2008-2009. Note that this data is annual and based on the calendar. year, so the data in 2008 refers to equipment and inventory stocks at the end of 2008. Thus, the statistically negative effect in 2008 for inventories does not raise an identification concern.

Table C.6: Capital Stock and Inventory Responses from the LFTTD Sample

	2008	2009	2010
Equipment			
High Irreversibility	−	−	−
Low Irreversibility	−0.029	−0.049*	−
	(0.022)	(0.025)	
Inventory			
High Irreversibility	+	+	−
Low Irreversibility	−0.11**	−0.13**	−
	(0.05)	(0.06)	

Notes: This table reports the effect of the credit supply shock on establishment-level log equipment and log inventory stocks relative to 2007 for firms in high- versus low-irreversibility industries. To limit the number of disclosed estimates, I only report the magnitudes of coefficients for firms in low-irreversibility industries over 2008-2009. For other years and for high-irreversibility firms, I report only sign and significance. Consistent with irreversibility inducing more inaction, firms with high irreversibility do not significantly cut their capital stocks while firms with low irreversibility do. The point estimates for firms with low irreversibility are also lower than for firms with high irreversibility. Consistent with a fall in prices, firms in low-irreversibility industries cut their inventory stocks whereas firms in high-irreversibility industries do not. Standard errors are clustered by a firm's primary lender. Any views expressed are those of the author(s) and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3432. (CBDRB-FY26-P3432-R13333). This is a part of clearance request #13333. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

C.4 Credit Market Outcomes and Characteristics

This section checks the impact of credit supply shocks on credit market outcomes following Chodorow-Reich (2014) and extends the analysis to study whether investment irreversibility is associated with differential credit-market responses. Chodorow-Reich (2014) estimates both extensive and intensive margins of credit-market outcomes. On the extensive margin, he estimates the effect of the Dealscan credit supply shock on the probability that a borrower obtains a new loan or a favorable modification of an existing loan over 2008:Q4-2009:Q2. On the intensive margin, he estimates the change in the all-in drawn spread between the last syndicated loan before the Lehman bankruptcy and a newly signed loan over 2008:Q4-2009:Q2. I replicate these regressions and then study heterogeneity by irreversibility for the extensive margin. The sample size for the intensive margin is too small. As in Chodorow-Reich (2014), I run the regressions on all available Dealscan borrowers, not just those in the PPI-Census-Dealscan dataset.

There are four differences in my data and how I run the regression compared to Chodorow-Reich (2014). First, I correct the lender shares that construct the credit supply shock using Blickle et al. (2022)'s code, which accounts for the fact that lead arrangers are likely to sell their stake for certain types of loans. Second, I use Dealscan data downloaded in 2024. Since Dealscan updates its data over time, company characteristics like the primary SIC code may not match what Chodorow-Reich (2014) used. Third, I classify borrowers as public or private based on Chava and Roberts (2008)'s file instead of Dealscan's own classification. Fourth, I do not interact all-in drawn spread with the loan type, and I do not interact the public firm indicator with the bond access indicator for the same reason. The reason is that I run a specification in which I fully saturate covariates with an indicator for above/below median irreversibility. I do not interact public with bond access to avoid splitting small cells.

Results. Tighter credit supply reduces loan supply along both the extensive and intensive margins. Table C.7 reports the results. Columns (1) and (2) are replications of Chodorow-Reich (2014)'s analysis, while Column (3) extends the extensive margin regression by interacting the credit supply shock and all covariates except the state fixed effect with an indicator for whether the borrower's industry has above- or below-median irreversibility. Columns (1) and (2) show that a one-standard-deviation credit tightening lowers the probability of obtaining a new loan or a favorable modification by about 4 percentage points and raises the all-in-drawn spread by 0.36 percentage points. The third column shows that the extensive margin result applies to firms with high and low irreversibility. Although the probability of a new loan or favorable modifications falls by less for firms with high irreversibility, the difference is not statistically significant at the 5% level. Nonetheless, the small attenuation is consistent with Section A.4, which shows that firms in irreversible industries have more cash on hand and lower leverage, so they may be less sensitive to credit supply shocks.

The small size of the intensive margin effects makes the working-capital channel seem like an unlikely explanation for the large difference in pricing behaviors that I estimate. If the intensive margin effects of the credit supply shock for across high and low irreversibility industries resemble the extensive margin effects, then we should expect a similar or a smaller increase in the cost of working capital for firms in industries with high irreversibility. Also, even if all variable inputs in a period required working capital loans to finance, we would need a difference in the intensive margin effect by almost 10pp. Since a 1 standard deviation credit supply shock only increases the interest rate on a new loan by less than 1pp and since working capital does not seem to explain the effect of tighter financial constraints on prices in from Scandinanvian economies (see Renkin and Züllig 2024, Bauer et al. 2026), the customer-market incentive seems far more plausible than the working-capital channel.[58]

[58] Bauducco et al. (2026) do find that the working capital constraint predominantly drives the response of prices to credit supply shocks in Chile, so this channel can matter.

Table C.7: Impact of Credit Supply Shocks on Credit Market Outcomes

	(1)	(2)	(3)
CSS	-4.37***	0.36*	
	(1.33)	(0.19)	
CSS (Low Irrev)			-6.54***
			(2.28)
CSS (High Irrev)			-3.40**
			(1.63)
High-Low Sig. Diff.			N
Observations	4,365	222	4,100

Notes: Column (1) reports the effect of a credit tightening on the probability of obtaining a new loan or a favorable modification. Column (2) estimates the effect of tighter credit supply on the change in the all-in-drawn loan spread (percentage points) between a borrower's last pre-Lehman loan and a loan signed over 2008:Q4-2009:Q2. Column (3) repeats (1) but splits the sample by whether the borrower's industry has above- or below-median irreversibility measured by irreversibility. Controls and fixed effects follow Chodorow-Reich (2014). Column (3) saturates controls and fixed effects by the indicator for above/below median irreversibility. Standard errors clustered by primary lender in parentheses. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

D Firm Death by Irreversibility

The regression results impose balanced panels, which raises the concern that the results are driven by survivorship bias. To assuage these concerns, I consider two tests. First, I show that adverse credit supply shocks to Compustat firms did not lead to more acquisitions or bankruptcies in industries with high irreversibility. Second, I show that firm death rates in the Census Bureau's Business Dynamics Statistics (BDS) data were not higher in industries with higher irreversibility.

Table D.1 reports the effect of tighter credit supply on firm death. It reports three regressions, where the outcome variables are the cumulative shares of firms that went bankruptcy, entered liquidation, or were acquired over 2008-2010. So the first column is just for 2008 while the last column includes all three years. The specifications otherwise mirror the credit market outcome regressions in Appendix C.4.

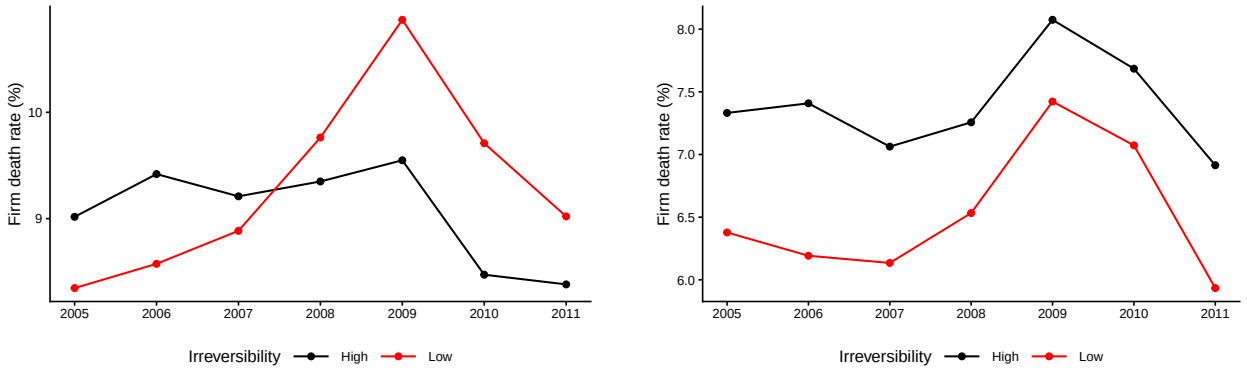
Table D.1: Credit Supply Shocks and Cumulative Firm Exit by Irreversibility

	2008	2009	2010
High Irreversibility	−0.016	0.006	0.049
	(0.020)	(0.028)	(0.031)
Low Irreversibility	0.028	0.014	0.022
	(0.026)	(0.027)	(0.034)
Sig. Diff. ($p < 0.05$)	N	N	N

Notes: Each column reports a separate regression, where the outcome variables equal one if the firm exited due to bankruptcy, liquidation, or acquisition in 2008, 2008-2009, and 2008-2010, respectively. The reported coefficients are the effects of tighter credit supply on firm death. Standard errors clustered by primary lender are in parentheses. The bottom row reports whether the high-minus-low difference is significant at the 5% level. $^{*}p < 0.10$, $^{**}p < 0.05$, $^{***}p < 0.01$.

The Census Bureau's BDS data shows that firms did not exit more in industries with high irreversibility during the Global Financial Crisis. Figure D.1 plots the firm death rate for high- and low-irreversibility industries across all industries and just within manufacturing. If anything, the evidence suggests that exit rates increased more for low-irreversibility industries.

Figure D.1: Firm Death Rates by Industry Irreversibility



(a) All Industries

(b) Manufacturing

Notes: Firm death rates for all firm sizes across matched industries (left) and in manufacturing (right). Black and red denote industries with above-median and below-median irreversibility, respectively. In manufacturing, the split uses the manufacturing-specific median.

E Data

This appendix section describes the construction and cleaning of all datasets used in the empirical analysis.

E.1 Cleaning by Dataset

Definition of a Parent Firm. In several of the datasets I use, there is a distinction between an establishment and the parent firm. An establishment refers to a place of business at a single physical location. Many firms operate multiple establishments. In the LBD, establishments owned by the same parent firm are linked within a year using the identifier firmid. When a firm is privately owned, I use firmid as the parent identifier. When a firm can be linked to Compustat using the Census Bureau's internal bridge with the LBD, I use gvkey as the parent identifier. This choice matters because a single gvkey can be linked to multiple firmids.

PPI. The initial cleaning of the PPI uses Gilchrist et al. (2017)'s script from the replication kit, with the addition of some fields like the buyer type. The cleaning steps are as follows. I remove matrix-weighted and imputed prices. The microdata assigns each item a unique code, where an item includes many price-determining characteristics, including type of buyer, color, and size. I retain an item if it has at least three non-missing price observations, and if reported shipment values are strictly positive. Shipment values are deflated to millions of dollars. I aggregate buyer types into one of three categories ("Final User", "Intermediate", and "Same Price to All") from a much larger number of codes. I exclude exports and sales to government entities.

The BLS records both a transaction price and a base price for each item; the base price is updated to reflect changes in product quality or specifications. The quality-adjusted

price is computed as the ratio of the transaction price to the base price, and all price-change regressions use this quality-adjusted measure. Specifically, if $\widetilde{p}_{i,t}$ denotes the transaction price on item i at time t and $p^b_{i,t}$ denotes the base price, then the quality-adjusted price is

$$p_{i,t} \equiv \frac{\widetilde{p}_{i,t}}{p^b_{i,t}}. \tag{E.1}$$

Relative shipment weights in the microdata are reported at the establishment level rather than at the firm-parent level. Formally, the firm-level weight on good i for establishment e in firm f at time t is

$$w_{i,e,f,t} = w^*_{i,e,f,t} \frac{\sum_i w^*_{i,e,f,t}}{\sum_e \sum_i w^*_{i,e,f,t}}, \tag{E.2}$$

where $w^*_{i,e,f,t}$ denotes the relative weight of good i in the production structure of firm f at establishment e as recorded by the BLS, and the second term adjusts for the fact that PPI weights on goods are defined by establishment rather than parent firm.

Monthly log price changes are winsorized at ± 100 log points at the item level before aggregation. I compute quarterly log price changes from the monthly series. Four-quarter log price changes are computed as the sum of quarterly log changes over a rolling window. I retain product groups with at least five distinct firms so that the product group fixed effects are not too noisily estimated.

LBD. The LBD provides establishment-level employment, industry classification, and geographic identifiers for 2004–2011. I harmonize NAICS to 2002 using the vintage-consistent version from Fort and Klimek (2016). I assign a firm a 3-digit NAICS code using the employment-weighted mode across its establishments, excluding management establishments with 3-digit NAICS code 551.

I use the LBD to create longitudinal panels. The LBD's longitudinal identifier for

parent firms, lbdfid, works reasonably well but has several deficiencies.[59] I augment this identifier in two ways. First, I use gvkey as a longitudinal identifier when possible. Second, if the current year's lbdfid/gvkey does not exist in the following year, then I identify the headquarters establishment in the current year and use its lbdfid/gvkey in the next year as the successor firm-level identifier. The headquarters is defined as the largest management establishment by employees if one exists and otherwise the largest establishment across any industry code by employees. Ties are broken by payroll. A longitudinal panel created in this manner will vary with the year in which the panel is initialized. I construct two linkages using 2004 and 2005 as starting cohorts.

I also use the LBD to measure firm employment size and age for covariates. Firm employment in March 2007 is aggregated from the LBD to the firm level using the firm parent identifier. When I construct the PPI-Census-Dealscan dataset, I drop any firms with fewer than 10 employees. Firm birth year is determined by the earliest of three sources: the year the LBD first records positive employment, the year the LBD first records positive payroll, or the Compustat–CRSP link date for public firms (based on the permco). Firm age is then 2007 minus the birth year plus one.

Finally, I measure employment growth over time using the LBD. First, over 2006-2010, I compute the Davis et al. (1996) (DHS) growth rate at the establishment level. Second, for each year from 2006 to 2010, firm-level employment and DHS employment growth are constructed by aggregating LBD establishment-level data to the firm level. See Chodorow-Reich (2014) for details on how the establishment-level growth rates can be used to create the firm-level growth rates.

Dealscan. The loan sample is constructed following Chodorow-Reich (2014). Dealscan loan facilities are restricted to borrowers that satisfy all of the following criteria: (i) U.S.-

[59] See Chow et al. (2021) for more details.

domiciled, (ii) not in the finance, insurance, or real estate sectors (SIC codes 60–69), (iii) primary loan purpose is either "Corporate purposes" or "Working capital", and (iv) either the facility start date falls between January 2004 and August 2008, or the facility ends after October 2007. When constructing the credit supply shocks, I consolidate banks to their parent bank.[60] I impute lender shares using the predicted values from regression estimates provided by Blickle et al. (2022) when available and otherwise the average lender share for a lead/participant in syndicates with the same structure. Finally, multiple borrowers can be linked to the same parent firm in the Census Bureau. For example, Dealscan may identify subsidiaries as separate borrowers. I construct the credit supply shock, instruments, and controls initially at the Dealscan borrower level and then aggregate to the parent firm level using the size of the last loans for each Dealscan borrower as weights. For categorical controls, I aggregate using the economically meaningful choice. For example, the indicator for a credit line equals one if any of the last loans was a credit line.

FISD. I exclude firms in the finance, insurance, and real estate industries and retain bonds with offering dates before September 1, 2008. I classify a firm's access to bond markets into three categories. Firms which have no matched FISD issuer id are grouped together as having zero bond issuance. Firms which have issued at least one investment grade bond in their history are classified as "investment grade". All other firms are grouped together into a third category.

Export Prices I construct unit value price indices at the HS6 level to correspond roughly the 4-digit NAICS granularity of the PPI data. In LFTTD, export values and quantities are recorded at the firm by HS10 level on a monthly frequency. I compute unit value prices using values and quantities over August in year $t-1$ to July in year t. Thus, the 2008 unit value corresponds to exports over August 2007-July 2008. This choice lines up with the

[60] I thank Gabriel Chodorow-Reich for generously providing me a file with the consolidations he used.

timing of prices in the PPI sample. I follow Heise (2024) and restrict attention to exports between firms that have arms-length relationships (to avoid intra-firm transfers, which are not actual transactions), trim products with price changes in excess of 400 log points at an annual frequency or have price levels below the 1st or above the 99th percentile of the price distribution for that product code, and aggregate to a price index at the HS6 level as a geometric weighted average, where weights are the average of the good's value across years $t-1$ and t.

The sample in Figures 4 and C.4 restrict attention to goods whose demand elasticities do not require the constrained estimator from Soderbery (2015) out of concern that the data for those goods is too thin to precisely estimate the demand elasticity. Specifically, one way that Soderbery (2015) improves upon the Broda and Weinstein (2006) estimates of demand elasticities is that some products imply elasticities which fail Soderbery (2015)'s model assumptions. To ensure these elasticities are economically reasonable, Soderbery (2015) implements a second step in which nonlinear constrained estimator does not allow the elasticity estimate to reach infeasible regions. However, goods with few observations or thin volumes tend to require the constrained estimator, so estimates of these demand elasticities may be quite noisy, motivating me to exclude them from the main results. Nonetheless, the results are robust to keeping all goods for which Soderbery (2015) has estimates of demand elasticities. Note that I use Soderbery (2015)'s estimates assigned at the HS8 level. When I determine the demand elasticity of a firm's HS6 price index, I aggregate using the median across the underlying HS8 goods.

Markups from the ASM/CMF. I construct markups using data from Blackwood et al. (2023)'s Collaborative Micro-Productivity Project (CMP), which assembles a cleaned panel of establishments from the ASM/CMF. I compute the markup of a manufacturing establish-

ment e owned by firm f in industry s at time t as

$$\mu_{e,f,s,t} = (\alpha^L_{s,t} + \alpha^M_{s,t}) \frac{\text{Nominal Output}_{e,f,s,t}}{\text{LaborCost}_{e,s,f,t} + \text{MaterialsCost}_{e,s,f,t}}. \tag{E.3}$$

The output elasticities $\alpha^L_{s,t}$ and $\alpha^M_{s,t}$ are measured by the cost shares of labor and materials at the 6-digit NAICS level. Raval (2023) finds that choosing only materials or only labor as the flexible input can produce vastly different markup patterns whereas the composite markup exhibits less dispersion across firms and less extreme time-series movements. Further, in Monte Carlo experiments, the markup implied by the composite better tracked the true markup. Following Foster et al. (2022), I define nominal output as the total value of shipments plus inventory changes since I cannot distinguish input costs due to sold goods versus inventoried goods. I restrict the sample to establishments owned by firms observed in the LFTTD sample. To mirror the granularity of the product groups in the LFTTD regression (9) for Figure 4, I construct the sales-weighted average markup across establishments with the same 4-digit NAICS code owned by the same firm, as 4-digit NAICS is approximately HS6.[61]

Compustat. I construct balance sheet controls from Compustat within each quarter and then average over 2006:Q4-2007:Q2. Variables constructed include book leverage, cash-to-assets ratio, average interest rate (interest payments divided by total debt), and the inventory-to-sales ratio. Total debt equals short-term plus long-term debt. Book leverage is total debt divided by the sum of total debt and common/ordinary equity. These controls are used in a robustness check of the main diff-in-diff.

[61] When De Loecker et al. (2020) compute firm-level markups using the ASM/CMF data, they also define a firm as a group of establishments with the same 4-digit NAICS code and the same Census parent firm identifier.

QFRCEN. The 2007 QFRCEN is used to provide balance sheet information for firms not observed in Compustat. QFRCEN firms are mapped to the LBD by EIN. When there are duplicate matches by EIN to a parent firm in a given year-quarter, I identify the best matches using fuzzy name matching and then manually review the suggested candidates.

E.2 Firm Matching Algorithm

I match firms from Dealscan, FISD, and the PPI to Census Bureau records using their name, address, and industry information. To perform matching, I use tools from the fedmatch package created by Cohen et al. (2021).

I create candidate matches in multiple stages. In each stage, the candidate match is proposed based on a name and exact state match.

1. Exact name match of the long name
2. Match of the long name based on weighted Jaccard distance. This distance penalizes differences involving rare words more heavily than differences involving common words, like "corporation". See Cohen et al. (2021) for more details.
3. Fuzzy matching using the Jaro–Winkler distance
4. Repeat 1-3 using the short name.

Exact name matches from stages 1 and 4 as well as matches from stage 2 which would become exact name matches after dropping common words are automatically accepted if there exists an establishment under the same firmid in the LBD with an exact city/zipcode match or matching industry information. All other candidate matches are manually reviewed.

My matching strategy also accounts for differences in how the datasets handle names. The PPI records historical names, so I can just match PPI respondents which have data in

the year 2007 to the 2007 CBPBR. I choose this specific year as the anchor year for the PPI-CBPBR bridge because the Census Bureau's company organization data is most accurate in an Economic Census year, and this year occurs right before the Lehman bankruptcy. Public firms in Dealscan and FISD can be indirectly linked via Compustat. Chava and Roberts (2008) created and updates a link from Dealscan firms to Compustat, which allows me to use the Census Bureau's Compustat-SSEL bridge for these firms. A subset of FISD can be linked to Compustat using historical issuer CUSIPs in the Compustat-CRSP linking file. Other firms in Dealscan and FISD are linked by name. Note that Dealscan and FISD are "header" datasets, so their company data reflects current information rather than historical information. To deal with this issue, I match firms to the 2020 CBPBR first and then check if there are matches in 2007 at the establishment level by lbdnum, which is an accurate longitudinal identifier according to Chow et al. (2021). This approach deals with the possibility that a firm changed names or headquarter locations between 2007 and the present. Firms in Dealscan or FISD with no matches at this point are then matched directly to the 2007 CBPBR. For example, firms that shut down before 2020 will not appear in the 2020 database. Finally, to maximize coverage, I also link unmatched PPI respondents directly to Dealscan and use the Dealscan-CBPBR link to assign a firmid to more PPI respondents, since the address information in Dealscan is higher quality than the PPI's.

F Omitted Proofs for the PE Model

This appendix solves the two-period model of Section 2 backward, from the afternoon of $t = 1$ to $t = 0$, and proves Propositions 2.1 and 2.2. I will occasionally define new non-primitive terms to keep the math compact. I will use time subscripts or explicit parenthetical arguments for new terms that are functions of endogenous choices. Non-primitive terms are purely functions of parameter primitives.

F.1 Setup

Demand. Let $\widetilde{\theta} \equiv \theta(1-\eta)$. The demand curves (1) are

$$\begin{aligned} y_0 &= p_0^{-\eta}, \\ y_1^z &= (p_1^z)^{-\eta}\, \mathcal{H}(y_0 \mid z), \\ \mathcal{H}(y_0 \mid z) &\equiv z + \chi y_0^{\widetilde{\theta}}, \end{aligned} \tag{F.1}$$

where $\mathcal{H}(y_0 \mid z)$ is the level of $t = 1$ demand in state $z \in \{B, G\}$, where $B < G$. The probability of each state is $1/2$, for simplicity. The marginal increase in demand from higher sales in $t = 0$ is given by

$$\mathcal{H}'(y_0) \equiv \chi \widetilde{\theta}\, y_0^{\widetilde{\theta}-1} \geq 0. \tag{F.2}$$

Because the increase is the same in either state, I omit the dependence on z.

Production, Investment, and Dividends. Output is $y_0 = k_0^{\alpha} \ell_0^{1-\alpha}$ at $t = 0$ and $y_1^z = (k_1^z)^{\alpha}(\ell_1^z)^{1-\alpha}$ in the afternoon of $t = 1$, where $k_1^z = (1-\delta)k_0 + i_1^z$. Investment expenses in the

morning of $t = 1$ is $\Phi_\kappa(i) \equiv \max\{i, (1-\kappa)i\}$. Dividends in each period are given by

$$d_0 = p_0 y_0 - w\ell_0, \quad d_1^z = p_1^z y_1^z - w\ell_1^z - \Phi_\kappa(i_1^z),$$

and the firm maximizes the present value of dividends in (2), subject to a multiplier φ on d_0 representing the marginal value of liquidity. The model ends after the afternoon of $t = 1$, and capital cannot be sold for an additional return.

Parameter Restrictions. These are technical regularity restrictions rather than economically meaningfully assumptions: $\eta > 1$, $\alpha, \delta \in (0,1)$, $\kappa \in (0,1)$, $\chi \geq 0$, and $\widetilde{\theta} \in [0,1)$.

F.2 The Firm's Problem at $t = 1$

I solve $t = 1$ taking y_0 as given, so the demand level $H \equiv \mathcal{H}(y_0 \mid z)$ is known in both the morning and the afternoon. Write $\bar{k} \equiv (1-\delta)k_0$ for the depreciated capital stock the firm brings into the morning.

Afternoon. Because y_1^z does not affect demand in any later period, the afternoon problem is static. Producing y with capital k costs

$$C(y,k) \equiv w\, y^{\frac{1}{1-\alpha}} k^{-\frac{\alpha}{1-\alpha}},$$

and the marginal cost is

$$mc(y,k) = \frac{w}{1-\alpha}\, y^{\frac{\alpha}{1-\alpha}} k^{-\frac{\alpha}{1-\alpha}}. \tag{F.3}$$

I define

$$\widetilde{\alpha}_k \equiv \frac{\alpha(\eta-1)}{1+\alpha(\eta-1)} \in (0,1),$$

which satisfies

$$1-\widetilde{\alpha}_k = \frac{1}{1+\alpha(\eta-1)}, \quad \widetilde{\alpha}_k = \alpha(\eta-1)(1-\widetilde{\alpha}_k).$$

Given demand $H>0$ and capital $k>0$, the firm sets $p_1 = \frac{\eta}{\eta-1}\, mc(y_1,k)$, and its profit is

$$\pi_1(H,k) \equiv \max_{y>0} \left\{ H^{1/\eta} y^{1-1/\eta} - C(y,k) \right\} = \bar{A}\, H^{1-\widetilde{\alpha}_k} k^{\widetilde{\alpha}_k}, \tag{F.4}$$

where

$$\bar{A} \equiv \frac{1+\alpha(\eta-1)}{\eta} \left[\frac{(\eta-1)(1-\alpha)}{\eta w} \right]^{(\eta-1)(1-\alpha)(1-\widetilde{\alpha}_k)} > 0.$$

It can be shown that the profit function π_1 is jointly concave in (H,k).

Morning. After observing z, the firm chooses k_1 to maximize afternoon profit minus investment expenses. By (F.4), its optimized dividend is $d_1^z = T(\mathcal{H}(y_0 \mid z);\kappa)$, where

$$T(H;\kappa) \equiv \max_{k_1>0} \left\{ \bar{A}\, H^{1-\widetilde{\alpha}_k} k_1^{\widetilde{\alpha}_k} - \Phi_\kappa(k_1 - \bar{k}) \right\}. \tag{F.5}$$

For a price of capital $p_k \in (0,1]$, let

$$\widehat{k}(p_k) \equiv \left(\frac{\widetilde{\alpha}_k \bar{A}}{p_k} \right)^{\frac{1}{1-\widetilde{\alpha}_k}}, \tag{F.6}$$

$$R_H(p_k) \equiv \max_{k>0} \left\{ \bar{A}\, k^{\widetilde{\alpha}_k} - p_k\, k \right\} = (1-\widetilde{\alpha}_k) \bar{A}\, \widehat{k}(p_k)^{\widetilde{\alpha}_k}. \tag{F.7}$$

The first is the capital per unit of demand that the firm wants when capital trades at p_k. The second is afternoon profit per unit of demand minus investment expenses.

Lemma F.1. *(a) The function R_H satisfies*

$$R'_H(p_k) = -\widehat{k}(p_k),$$
$$\frac{\partial R_H(1-\kappa)}{\partial \kappa} = \widehat{k}(1-\kappa) > 0.$$

(b) The optimal capital choice k_1^* in (F.5) is

condition	k_1^*	$T(H;\kappa)$
(sell) $\widehat{k}(1-\kappa)H < \bar{k}$	$\widehat{k}(1-\kappa)H$	$R_H(1-\kappa)H + (1-\kappa)\bar{k}$
(inaction) $\widehat{k}(1)H \le \bar{k} \le \widehat{k}(1-\kappa)H$	$\bar{k}$	$\bar{A}\,H^{1-\widetilde{\alpha}_k}\bar{k}^{\widetilde{\alpha}_k}$
(buy) $\widehat{k}(1)H > \bar{k}$	$\widehat{k}(1)H$	$R_H(1)H + \bar{k}$

(F.8)

(c) $T(\cdot;\kappa)$ is concave, with

$$T_H = (1-\widetilde{\alpha}_k)\bar{A}\left(\frac{k_1^*}{H}\right)^{\widetilde{\alpha}_k}.$$

In particular, $T_H = R_H(1-\kappa)$ in the sell region and $T_H = R_H(1)$ in the buy region.

(d) In the sell region,

$$\frac{\partial T}{\partial \kappa} = k_1^* - \bar{k} < 0, \quad \frac{\partial^2 T}{\partial \kappa\, \partial H} = \widehat{k}(1-\kappa) = \frac{k_1^*}{H} > 0.$$

In the interior of the inaction and buy regions, T and T_H do not depend on κ.

Proof. (a) The envelope theorem applied to (F.7) gives $R'_H(p_k) = -\widehat{k}(p_k)$.

(b) The firm buys capital at price 1 and sells it at price $1-\kappa$. So it buys up to $\widehat{k}(1)H$

if it holds less than that, sells down to $\widehat{k}(1-\kappa)H$ if it holds more than that, and otherwise keeps $\bar{k}$. When the firm actively adjusts capital at price p_k, substituting $k_1^* = \widehat{k}(p_k)H$ into (F.5) gives

$$T = R_H(p_k)H + p_k\bar{k}.$$

(c) It is straightforward to show that T is concave. The formula for T_H follows from the envelope theorem, and the special cases from $k_1^*/H = \widehat{k}(p_k)$.

(d) In the sell region, $T = R_H(1-\kappa)H + (1-\kappa)\bar{k}$. Differentiating with part (a) gives

$$\frac{\partial T}{\partial \kappa} = \widehat{k}(1-\kappa)H - \bar{k}, \quad \frac{\partial^2 T}{\partial \kappa\, \partial H} = \widehat{k}(1-\kappa).$$

The inaction and buy rows of (F.8) do not contain κ. □

Intuition for Proposition 2.2. Since $d_1^z = T(\mathcal{H}(y_0 \mid z); \kappa)$, the customer-market incentive in (3) is

$$\lambda = \frac{1}{R}\,\mathbb{E}\left[\frac{\partial d_1^z}{\partial y_0}\right] = \frac{1}{2R}\sum_{z\in\{B,G\}} T_H\big(\mathcal{H}(y_0 \mid z); \kappa\big)\,\mathcal{H}'(y_0), \tag{F.9}$$

which is positive when $\chi\widetilde{\theta} > 0$ because $T_H > 0$ by Lemma F.1(c). When the firm strictly sells capital in the bad state and does not sell in the good state, Lemma F.1(d) gives the derivatives in Section 2:

$$\begin{aligned}
\frac{\partial d_1^B}{\partial \kappa} &= i_1^B < 0, \\
\frac{\partial^2 d_1^B}{\partial y_0\, \partial \kappa} &= \widehat{k}(1-\kappa)\,\mathcal{H}'(y_0) = \frac{\partial i_1^B}{\partial y_0} > 0, \\
\frac{\partial d_1^G}{\partial \kappa} &= \frac{\partial^2 d_1^G}{\partial y_0\, \partial \kappa} = 0.
\end{aligned}$$

F.3 The Firm's Problem at $t = 0$

Let $\nu \equiv \alpha/(1-\alpha)$ and $\varsigma \equiv 1/\eta + \nu$. The firm's profit, marginal revenue, and marginal cost in $t = 0$ are

$$\pi_0(y) \equiv y^{1-1/\eta} - C(y, k_0), \quad mr_0(y) \equiv \left(1 - \frac{1}{\eta}\right) y^{-1/\eta}, \quad mc_0(y) \equiv mc(y, k_0).$$

The marginal loss on a sale at $t = 0$ is

$$L(y) \equiv mc_0(y) - mr_0(y),$$

the amount by which the marginal cost of the sale exceeds its marginal revenue. By (F.5), the firm chooses y_0 to maximize

$$J(y) \equiv \varphi\, \pi_0(y) + \frac{1}{2R} \sum_{z \in \{B,G\}} T\big(\mathcal{H}(y \mid z); \kappa\big).$$

The firm's FOC is

$$\varphi\, L(y_0) = \lambda(y_0), \tag{F.10}$$

where $\lambda(y)$ is (F.9) evaluated at $y_0 = y$. Since $mr_0(y_0) = \frac{\eta-1}{\eta} p_0$, dividing (F.10) by φ gives the main-text price equation (3).

Proof of Proposition 2.1. Assume $\chi\widetilde{\theta} > 0$, so that $\lambda > 0$. First, I prove that $y_0(\varphi)$ is a strictly decreasing function. Fix $\varphi' > \varphi$ and let $y = y_0(\varphi)$. By (F.10), $L(y) = \lambda(y)/\varphi > 0$, so

$$J'(y; \varphi') = J'(y; \varphi) - (\varphi' - \varphi)L(y) = -(\varphi' - \varphi)L(y) < 0.$$

Since $J(\cdot;\varphi')$ is strictly concave, $J'(\cdot;\varphi')$ is strictly decreasing. Because $J'(y;\varphi') < 0$ and because J is strictly concave, the root $y_0(\varphi')$ must increase J' up, hence $y_0(\varphi') < y = y_0(\varphi)$.

The price $p_0 = y_0^{-1/\eta}$ therefore strictly increases in φ, and so does the markup

$$\mu_0 \equiv \frac{p_0}{mc_0(y_0)} = \frac{1-\alpha}{w}\, k_0^{\nu}\, y_0^{-\varsigma}.$$

Assume that, for each z, $\mathcal{H}(y_0 \mid z)$ lies in the interior of a region of Lemma F.1(b), so that λ is differentiable at y_0. Define the elasticities

$$\epsilon_L(y) \equiv \frac{d\log L(y)}{d\log y}, \tag{F.11}$$

$$\epsilon_\lambda(y) \equiv -\frac{d\log \lambda(y)}{d\log y}, \tag{F.12}$$

Since $L'(y) = \nu\, mc_0(y)/y + mr_0(y)/(\eta y) > 0$ and $L(y_0) > 0$, $\epsilon_L(y_0) > 0$. By (F.9) and (F.2),

$$\lambda(y) = \frac{\chi\widetilde{\theta}}{2R}\, y^{\widetilde{\theta}-1} \sum_{z\in\{B,G\}} T_H\big(\mathcal{H}(y \mid z);\kappa\big).$$

Since $\mathcal{H}(y_0 \mid z)$ lies in the interior of a region, T_H is differentiable there. Taking logarithms and differentiating in $\log y$ gives

$$\epsilon_\lambda(y) = 1 - \widetilde{\theta} - \frac{y\,\mathcal{H}'(y)\sum_{z\in\{B,G\}} T_{HH}\big(\mathcal{H}(y \mid z);\kappa\big)}{\sum_{z\in\{B,G\}} T_H\big(\mathcal{H}(y \mid z);\kappa\big)}.$$

The fraction is weakly negative because T is concave, so $T_{HH} \le 0$, while $\mathcal{H}'(y) \ge 0$ and $T_H > 0$ by Lemma F.1(c). Hence $\epsilon_\lambda(y_0) \ge 1 - \widetilde{\theta} > 0$. Differentiating the logarithm of (F.10) in $\log\varphi$ gives

$$\frac{\partial \log y_0}{\partial \log \varphi} = -\frac{1}{\epsilon_L(y_0) + \epsilon_\lambda(y_0)} < 0,$$

$$\frac{\partial \log p_0}{\partial \log \varphi} = \frac{1}{\eta(\epsilon_L(y_0) + \epsilon_\lambda(y_0))} > 0,$$
$$\frac{\partial \log \mu_0}{\partial \log \varphi} = \frac{\varsigma}{\epsilon_L(y_0) + \epsilon_\lambda(y_0)} > 0.$$

□

Proof of Proposition 2.2. The proof follows these steps. First, I compute a number of expressions that will be used to perform the main proof. Second, I show that irreversibility wedge κ increases the elasticity by showing the FOC $J'(y_0) = 0$ takes a log-linear form in $\log y_0$ with slope S_0. The wedge κ only shifts the level of the FOC (changes y_0), *but* the elasticity of y_0 with respect to φ does not directly depend on κ. Instead, it depends solely on ω_0. However, shifts in κ shift y_0, which then shifts ω_0.

Useful Expressions: Suppose that at y_0 the firm strictly sells capital in the bad state and strictly buys capital in the good state. Appendix F.4 gives a condition on parameters, Assumption F.1, under which this behavior is optimal. In these regimes, Lemma F.1(c) gives $T_H = R_H(1-\kappa)$ in the bad state and $T_H = R_H(1)$ in the good state. Then (F.9) becomes $\lambda(y) = A(\kappa)\, y^{\widetilde{\theta}-1}$, where

$$A(\kappa) \equiv \frac{\chi^{\widetilde{\theta}} \left[R_H(1-\kappa) + R_H(1)\right]}{2R}. \tag{F.13}$$

Substituting into (F.10) gives

$$\varphi\, L(y_0) = A(\kappa)\, y_0^{\widetilde{\theta}-1}.$$

Taking logarithms gives

$$\log \varphi + \log L(y_0) = \log A(\kappa) + (\widetilde{\theta} - 1) \log y_0.$$

Let $\omega_0 \equiv L(y_0)/mc_0(y_0)$ be the share of marginal cost at $t = 0$ that marginal revenue does not cover. Since $L(y_0) > 0$ and $mr_0(y_0) > 0$, $\omega_0 \in (0, 1)$. By (F.3), $mc_0(y)$ is proportional to y^ν, and $mr_0(y)$ is proportional to $y^{-1/\eta}$. So $L'(y) = \nu\, mc_0(y)/y + mr_0(y)/(\eta y)$, and

$$\epsilon_L(y) = \frac{y\, L'(y)}{L(y)} = \frac{\nu\, mc_0(y) + mr_0(y)/\eta}{mc_0(y) - mr_0(y)}.$$

At $y = y_0$, dividing the numerator and the denominator by $mc_0(y_0)$ and using $mr_0(y_0)/mc_0(y_0) = 1 - \omega_0$ gives

$$\epsilon_L(y_0) = \frac{\nu + (1 - \omega_0)/\eta}{\omega_0} = \frac{\nu + 1/\eta}{\omega_0} - \frac{1}{\eta} = \frac{\varsigma}{\omega_0} - \frac{1}{\eta}.$$

In the regimes of Assumption F.1, $\epsilon_\lambda(y_0) = 1 - \widetilde{\theta}$. The elasticity of the FOC with respect to y is

$$S_0 \equiv \epsilon_L(y_0) + \epsilon_\lambda(y_0) = \frac{\varsigma}{\omega_0} - \frac{1}{\eta} + 1 - \widetilde{\theta},$$

which is positive because $\varsigma/\omega_0 - 1/\eta > \varsigma - 1/\eta = \nu > 0$.

Proof of Part (i): higher irreversibility raises sales at $t = 0$, $\partial \log y_0/\partial\kappa > 0$. Recall the first-order condition (F.10) of the objective J, which under this proposition's assumption becomes

$$\log \varphi + \log L(y_0) = \log A(\kappa) + (\widetilde{\theta} - 1) \log y_0.$$

Differentiating it in κ gives

$$\epsilon_L(y_0)\, \frac{\partial \log y_0}{\partial \kappa} = \epsilon_{\lambda,\kappa} + (\widetilde{\theta} - 1) \frac{\partial \log y_0}{\partial \kappa},$$

where, by (F.13) and Lemma F.1(a),

$$\epsilon_{\lambda,\kappa} \equiv \frac{A'(\kappa)}{A(\kappa)} = \frac{\widehat{k}(1-\kappa)}{R_H(1-\kappa)+R_H(1)} > 0.$$

Hence

$$\frac{\partial \log y_0}{\partial \kappa} = \frac{\epsilon_{\lambda,\kappa}}{S_0} > 0.$$

Proof of Part (ii) and (iii): higher irreversibility raises the elasticities of the price and the markup to φ. By the proof of Proposition 2.1, with $\epsilon_\lambda(y_0) = 1 - \widetilde{\theta}$,

$$\frac{\partial \log p_0}{\partial \log \varphi} = \frac{1}{\eta S_0},$$
$$\frac{\partial \log \mu_0}{\partial \log \varphi} = \frac{\varsigma}{S_0}.$$

The parameter κ enters S_0 only through y_0, via ω_0. Since $mr_0(y)/mc_0(y) \propto y^{-\varsigma}$,

$$\frac{d\omega_0}{d \log y_0} = \varsigma(1-\omega_0),$$
$$\frac{dS_0}{d\omega_0} = -\frac{\varsigma}{\omega_0^2}.$$

By part (i),

$$\frac{\partial S_0}{\partial \kappa} = \frac{dS_0}{d\omega_0}\frac{d\omega_0}{d \log y_0}\frac{\partial \log y_0}{\partial \kappa} = -\frac{\varsigma^2(1-\omega_0)\epsilon_{\lambda,\kappa}}{\omega_0^2 S_0} < 0.$$

Therefore

$$\frac{\partial^2 \log p_0}{\partial \kappa \, \partial \log \varphi} = -\frac{1}{\eta S_0^2}\frac{\partial S_0}{\partial \kappa} = \frac{\varsigma^2(1-\omega_0)\epsilon_{\lambda,\kappa}}{\eta \, \omega_0^2 S_0^3} > 0,$$

and

$$\frac{\partial^2 \log \mu_0}{\partial \kappa \, \partial \log \varphi} = \eta\varsigma \, \frac{\partial^2 \log p_0}{\partial \kappa \, \partial \log \varphi} > 0.$$

□

F.4 The Capital-Regime Assumption

This subsection gives a condition on parameters under which the capital regimes used in the proof of Proposition 2.2 hold. Let $y^*(\varphi, \kappa)$ be the positive root of

$$\varphi \, L(y) = A(\kappa) \, y^{\widetilde{\theta}-1}. \tag{F.14}$$

The root exists and is unique because L is strictly increasing over the real line while the right-hand side of (F.14) is positive and strictly decreasing. The root depends only on parameters and is the firm's optimal level of production in $t = 0$. This level of production is affected by B and G to the extent that the demand states determine the firm's capital regime in the morning of $t = 1$. Otherwise, the level of production does not depend on B or G. The following assumption is the precise version of the hypothesis of Proposition 2.2.

Assumption F.1. At $y^* = y^*(\varphi, \kappa)$, the firm strictly sells capital in the bad state and strictly buys capital in the good state:

$$B + \chi(y^*)^{\widetilde{\theta}} < \frac{\bar{k}}{\widehat{k}(1-\kappa)},$$
$$G + \chi(y^*)^{\widetilde{\theta}} > \frac{\bar{k}}{\widehat{k}(1)}.$$

A sufficient condition for Assumption F.1 to hold is for B and G to satisfy

$$0 < B < \frac{\bar{k}}{\widehat{k}(1-\kappa)} - \chi(y^*)^{\widetilde{\theta}},$$
$$G > \frac{\bar{k}}{\widehat{k}(1)} - \chi(y^*)^{\widetilde{\theta}}.$$

Note that y^* and $\widehat{k}$ do not depend on B or G, so it is possible to freely choose B and G to ensure these conditions hold. These are the conditions stated in words in Proposition 2.2. Since $\widehat{k}(1-\kappa)$ increases with κ, the bound on B tightens as κ rises. The bound also requires the demand from the customer base, $\chi(y^*)^{\widetilde{\theta}}$, to be small relative to the depreciated capital stock $\bar{k} = (1-\delta)k_0$.

G Adding Inventories to the Model

This appendix adds inventories to the model of Section 2. I show three results. First, under some parameter conditions, the firm holds a positive stock of inventory into $t = 1$. Otherwise, the analysis would be moot. Second, a rise in the marginal value of liquidity φ can make the firm sell more at $t = 0$ and thus cut its price. Third, a higher irreversibility wedge κ raises the elasticity of the price to φ, even if the elasticity itself is negative.

This appendix sets up the model, establishes some properties of the production technology, and proves the main results. I relegate regularity conditions and precise statements of parameter restrictions to Appendix G.4.

G.1 Setup

In $t = 0$, the firm sells its output as a continuum of parts whose demand is subject to taste shocks, as in Wen (2011). Unsold output becomes inventory that is then sold in the morning of $t = 1$ at an exogenous price p^m. Production in $t = 1$ still only occurs in the

afternoon. To simplify, I assume the firm does not start with any inventories on hand in $t = 0$. Thus, if the $t = 0$ price p_0 falls when φ increases, then it results from the firm accumulating fewer inventories and not from liquidation of existing inventories on hand. Nonetheless, including inventories on hand at the start of $t = 0$ should not qualitatively change this model's implications. It should only weaken the conditions under which a firm prefers cutting prices instead of hiking to deal with an increase in φ.

Inventories. The final good sold by a firm is a CES aggregate of different parts, each of which gets an i.i.d. taste shock. For example, the final good may come in different colors, and in every period, demand for specific colors will be higher. A firm now uses capital and labor to produce each part using an identical Cobb-Douglas production technology. I make the timing assumption that firms choose their production plans before they observe the taste shocks. As Wen (2011) proves, the firm optimally chooses their price and a cutoff value for the taste shock such that all parts with demand higher than this cutoff will get stocked out, and parts with lower demand will have goods remaining on the shelves that are stored as inventory. Using this cutoff, it is possible to write output sold and inventory accumulated in "aggregate" terms without tracking the identities of the stocked-out and inventoried parts. Appendix H provides more detail.

Define q as output *produced* by the firm. Since the firm does not start with any inventories, the stock of goods on the shelf in $t = 0$ comes entirely from production. Let x_n^* be the cutoff value for the i.i.d. taste shocks, y the amount of output sold, and n the stock of inventories accumulated into the morning of $t = 1$. Appendix H shows that there exist functions $\widetilde{\mathcal{Y}}(x_n^*)$ and $\widetilde{\mathcal{N}}(x_n^*)$ such that $y = \widetilde{\mathcal{Y}}(x_n^*)q$ and $n = \widetilde{\mathcal{N}}(x_n^*)q$. Further, for a given y, there is a one-to-one mapping between n and x_n^*. The chosen cutoff uniquely determines the stock of accumulated inventories. The cost of production in $t = 0$ now depends on n as well. I suppress the dependence on the initial capital stock with the

notation $\widetilde{C}(y,n) \equiv C\big(q(y,n),k_0\big)$.

A fraction $\delta_n \in [0,1)$ of the stock left unsold at $t=0$ depreciates. The firm can sell the remainder in the morning of $t=1$ at the price $p^m > 0$. The different parts become perfect substitutes in the morning market. Morning sales also do not affect the customer base. These sales happen because there exists a continuum of perfectly competitive markets for each part. Because the parts are all produced with identical technologies, they sell for the same price across markets. No customer base is built because the firm is selling the individual inventories, not its final good. The firm still behaves as a monopolist for its own good because the aggregation of the different parts into the final good differentiates the parts.

This modeling approach to inventories makes three stylized assumptions. First, the firm does not get to sell inventories in the afternoon of $t=1$. Second, inventory sales in the morning do not affect the customer base. Third, the firm gets to treat inventories as substitutes and sell at an exogenous price p^m. I make the first two assumptions because the point of inventories is primarily to give firms a reason to cut prices after an increase in the marginal value of liquidity φ. I do not introduce inventories to study interactions between the customer-market motive and inventory choices. By requiring inventories to be sold in the morning instead of the afternoon and assuming away an effect of morning sales on the size of the customer base, I prevent excessive interactions between customer markets and inventories. Otherwise, it becomes difficult to obtain analytical results with this microfoundation of inventories.[62] The third assumption simplifies the characterization of the incentive to hold inventories. Because my model has a finite horizon, the firm optimally dumps all inventories in the last period of sales. However, that means equal amounts of on-the-shelf goods may not be optimal under some parameters since some parts will have

[62] I could probably write down reduced-form functions for how inventories affect output sold to get analytical results. However, the current approach is simpler and clearer about the role of inventories since they have limited interactions with customer markets by assumption.

stocked out, and it may not be optimal to fully replenish those parts. It is not economically interesting to establish the precise parameters under which equal amounts of on-the-shelf goods is optimal or to account for the effects on the inventory incentive when the firm cannot offer equal amounts of on-the-shelf parts in the morning of $t = 1$. The infinite-horizon version in Wen (2011) avoids this issue because the problem is recursive there.

The Objective. The firm's objective can be written as

$$\begin{aligned} J(y,n) &\equiv \varphi\big[y^{1-1/\eta} - \widetilde{C}(y,n)\big] + bn + \mathcal{V}(y), \\ b &\equiv \frac{(1-\delta_n)p^m}{R}, \\ \mathcal{V}(y) &\equiv \frac{1}{2R}\sum_{z\in\{B,G\}} T\big(\mathcal{H}(y \mid z);\kappa\big). \end{aligned} \tag{G.1}$$

As in Appendix F, I can define the marginal loss and write it normalized by the marginal cost as

$$\begin{aligned} L(y,n) &\equiv \widetilde{C}_y(y,n) - mr_0(y), \\ \omega(y,n) &\equiv \frac{L(y,n)}{\widetilde{C}_y(y,n)}. \end{aligned}$$

At an optimum with $n_0 > 0$, the first-order conditions are

$$\varphi\, L(y_0,n_0) = \lambda(y_0), \tag{G.2}$$

$$\varphi\, \widetilde{C}_n(y_0,n_0) = b. \tag{G.3}$$

Since $mr_0(y_0) = \frac{\eta-1}{\eta}p_0$, (G.2) is the price equation (3) with $\widetilde{C}_y$ in place of mc_0. To ensure the analysis is not moot, I assume parameters are such that $n_0 > 0$ is optimal. In Appendix G.4, I establish these conditions.

G.2 Properties of the Production Technology

This section establishes some key properties of the firm's production costs, now that inventories are part of the problem. The curvature of these costs will be important in later proofs. Until Subsection G.4, $y > 0$ and $n > 0$, so that $x_n^* > 1$.

Define

$$\psi_n(x_n^*) \equiv \widetilde{\mathcal{N}}(x_n^*) - \frac{\widetilde{\mathcal{N}}'(x_n^*)}{\widetilde{\mathcal{Y}}'(x_n^*)}\widetilde{\mathcal{Y}}(x_n^*), \quad \psi_y(x_n^*) \equiv \widetilde{\mathcal{Y}}(x_n^*) - \frac{\widetilde{\mathcal{Y}}'(x_n^*)}{\widetilde{\mathcal{N}}'(x_n^*)}\widetilde{\mathcal{N}}(x_n^*), \tag{G.4}$$

$$s_y(x_n^*) \equiv \frac{\widetilde{\mathcal{Y}}(x_n^*)}{\psi_y(x_n^*)}, \quad s_n(x_n^*) \equiv \frac{\widetilde{\mathcal{N}}(x_n^*)}{\psi_n(x_n^*)}. \tag{G.5}$$

The function ψ_n represents the marginal increase in the stock of accumulated inventories from an increase in production when a firm holds sales constant while the second function ψ_y represents the marginal increase in output sold from an increase in production when a firm holds the stock of accumulated inventories constant. The next two terms decompose how production gets allocated to sales and to inventories. Using (G.4) and (G.5), I also define the inverse elasticity of transformation between sales and inventories, holding fixed production q, as the expression

$$\bar{\varepsilon}(x_n^*) \equiv \frac{d\log\left(\psi_y(x_n^*)/\psi_n(x_n^*)\right)}{d\log(n/y)}.$$

Interpret the ratio ψ_y/ψ_n as the marginal cost of inventory relative to sales. A high ratio means that it is expensive to accumulate more inventories instead of sales without an increase in production q.

It will also be useful to define these elasticities of the cost function $\widetilde{C}$:

$$\varepsilon_{yy}(y,n) \equiv \frac{y\widetilde{C}_{yy}(y,n)}{\widetilde{C}_y(y,n)}, \quad \varepsilon_{ny}(y,n) \equiv \frac{y\widetilde{C}_{yn}(y,n)}{\widetilde{C}_n(y,n)}, \quad \varepsilon_{nn}(y,n) \equiv \frac{n\widetilde{C}_{nn}(y,n)}{\widetilde{C}_n(y,n)}.$$

Finally, define

$$\widehat{\rho}(x_n^*) \equiv \frac{s_n(x_n^*)\big(\nu - \bar{\varepsilon}(x_n^*)\big)}{\nu s_n(x_n^*) + \bar{\varepsilon}(x_n^*)s_y(x_n^*)}. \tag{G.6}$$

This expression represents the strength of the inventory incentive when adjusting prices.

Lemma G.1. *(a) The first derivatives of* $\widetilde{C}$ *are*

$$\widetilde{C}_y(y,n) = \frac{mc_0(q)}{\psi_y(x_n^*)}, \quad \widetilde{C}_n(y,n) = \frac{mc_0(q)}{\psi_n(x_n^*)}.$$

(b) Suppressing arguments,

$$\begin{aligned}
\frac{\partial \log q}{\partial \log y} &= s_y = \frac{y\widetilde{C}_y(y,n)}{q\, mc_0(q)}, \\
\frac{\partial \log q}{\partial \log n} &= s_n = \frac{n\widetilde{C}_n(y,n)}{q\, mc_0(q)}, \\
\varepsilon_{yy} &= \nu s_y + \bar{\varepsilon} s_n, \\
\varepsilon_{ny} &= (\nu - \bar{\varepsilon}) s_y, \\
\varepsilon_{nn} &= \nu s_n + \bar{\varepsilon} s_y,
\end{aligned}$$

so that $s_y\varepsilon_{yy} + s_n\varepsilon_{ny} = \nu s_y$ *and* $\varepsilon_{ny} + \varepsilon_{nn} = \nu$.

(c) $\psi_y > 0$, $\psi_n > 0$, $s_y \in (0,1)$, $s_n \in (0,1)$, $s_y + s_n = 1$ *and* $\bar{\varepsilon} > 0$.

Proof. I reduce notation by suppressing arguments of various terms. By Lemma G.2(a), $\widetilde{\mathcal{Y}}' < 0 < \widetilde{\mathcal{N}}'$, so $\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}' - \widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}' > 0$, and the definitions of ψ_n, ψ_y, s_y and s_n give

$$\begin{aligned}
\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}' - \widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}' &= \widetilde{\mathcal{N}}'\psi_y, \\
\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}' - \widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}' &= -\widetilde{\mathcal{Y}}'\psi_n, \\
s_y &= \frac{\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}'}{\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}' - \widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}'},
\end{aligned}$$

$$s_n = -\frac{\widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}'}{\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}' - \widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}'}.$$

So $\psi_y > 0$, $\psi_n > 0$, $s_y + s_n = 1$, $s_y \in (0,1)$ and $s_n \in (0,1)$. Lemma G.2 also gives $\bar{\varepsilon} > 0$. This proves (c).

(a) By Subsection G.4, q and x_n^* are continuously differentiable in (y,n). Differentiating $y = \widetilde{\mathcal{Y}}q$ and $n = \widetilde{\mathcal{N}}q$ with respect to q and x_n^* gives $dy = \widetilde{\mathcal{Y}}\,dq + q\widetilde{\mathcal{Y}}'dx_n^*$ and $dn = \widetilde{\mathcal{N}}\,dq + q\widetilde{\mathcal{N}}'dx_n^*$, so $\widetilde{\mathcal{N}}'dy - \widetilde{\mathcal{Y}}'dn = (\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}' - \widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}')\,dq$. By the first two identities, the partial derivatives of q with respect to y and n are $q_y = 1/\psi_y$ and $q_n = 1/\psi_n$. These partials describe how much production needs to change to increase either output sales or inventory accumulation. Since $\widetilde{C}(y,n) = C\big(q(y,n), k_0\big)$, $\widetilde{C}_y = mc_0(q)q_y$ and $\widetilde{C}_n = mc_0(q)q_n$.

(b) By the proof of (a), $yq_y/q = \widetilde{\mathcal{Y}}/\psi_y = s_y$ and $nq_n/q = \widetilde{\mathcal{N}}/\psi_n = s_n$, so

$$d\log q = s_y\,d\log y + s_n\,d\log n,$$

and $y\widetilde{C}_y/(q\,mc_0(q)) = s_y$ and $n\widetilde{C}_n/(q\,mc_0(q)) = s_n$. The first two identities give $\widetilde{\mathcal{Y}}'/\psi_y + \widetilde{\mathcal{N}}'/\psi_n = 0$, so differentiating $\widetilde{\mathcal{Y}}/\psi_y + \widetilde{\mathcal{N}}/\psi_n = 1$ in x_n^* gives $s_y\psi_y'/\psi_y + s_n\psi_n'/\psi_n = 0$. With the definition of $\bar{\varepsilon}$ and $s_y + s_n = 1$,

$$\frac{d\log\psi_y}{d\log x_n^*} = \bar{\varepsilon}s_n\frac{d\log(\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}})}{d\log x_n^*}, \quad \frac{d\log\psi_n}{d\log x_n^*} = -\bar{\varepsilon}s_y\frac{d\log(\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}})}{d\log x_n^*}.$$

Since $n/y = \widetilde{\mathcal{N}}(x_n^*)/\widetilde{\mathcal{Y}}(x_n^*)$,

$$d\log\psi_n = -\bar{\varepsilon}s_y\,(d\log n - d\log y), \quad d\log\psi_y = \bar{\varepsilon}s_n\,(d\log n - d\log y).$$

By (F.3), $mc_0(q) = mc_0(1)q^\nu$. Differentiating $\log\widetilde{C}_y = \log mc_0(1) + \nu\log q - \log\psi_y$ in $\log y$, and $\log\widetilde{C}_n = \log mc_0(1) + \nu\log q - \log\psi_n$ in $\log y$ and in $\log n$, gives ε_{yy}, ε_{ny} and ε_{nn}. The two identities follow from the formulas and $s_y + s_n = 1$. □

G.3 The Price Response

Proposition G.1 states the main result of this section. Its precise assumptions on parameters are stated in Appendix G.4.

Proposition G.1. *Assume that the demand states B and G are far apart, κ is small, and χ is small relative to k_0, so that the firm sells capital in the morning of the $t = 1$ bad state and buys capital in the morning of the $t = 1$ good state. Also assume that the value of morning sales from inventories is sufficiently high, the customer-market incentive is not too strong, and the cost of production is sufficiently convex. Then the following claims are true.*

(i) If the inventory incentive is stronger than the customer-market incentive, then its log price and inventory accumulation fall: $\dfrac{\partial \log p_0}{\partial \log \varphi} < 0$ *and* $\dfrac{\partial \log n_0}{\partial \log \varphi} < 0$.

(ii) A firm with higher irreversibility invests more in the customer base and has a higher elasticity of its price to φ:

$$\frac{\partial \log y_0}{\partial \kappa} > 0, \quad \frac{\partial^2 \log p_0}{\partial \kappa \, \partial \log \varphi} > 0.$$

(iii) For a sufficiently strong effect of y_0 on the customer base, $\theta < -1/\eta$, the elasticity of the markup of the price over output sold to φ also increases with κ.

Proof. I first characterize the price elasticity now that inventories are in the model and then prove the proposition part by part.

Under the assumptions, the customer-market incentive is given by $\lambda(y) = A(\kappa) y^{\widetilde{\theta}-1}$, where

$$A(\kappa) \equiv \frac{\chi^{\widetilde{\theta}} \left[R_H(1-\kappa) + R_H(1) \right]}{2R}.$$

For reference, this expression comes from (F.9), (F.13) and Lemma F.1(c).

Price Elasticity. In this part, I suppress the dependence of various functions on their arguments to economize on notation. Differentiate the FOCs of (y, n) (G.2) and (G.3) with respect to φ to get the linear system

$$\begin{pmatrix} J_{yy} & J_{yn} \\ J_{yn} & J_{nn} \end{pmatrix} \begin{pmatrix} \partial y_0/\partial\varphi \\ \partial n_0/\partial\varphi \end{pmatrix} = \begin{pmatrix} \omega_0 \widetilde{C}_y \\ \widetilde{C}_n \end{pmatrix}. \tag{G.7}$$

The terms on the right-hand side emerge because φ appears directly in the FOCs. The J refers to the firm's objective (G.1). Since $J_{nn} < 0$ (shown below), the second row gives $\partial n_0/\partial\varphi = (\widetilde{C}_n - J_{yn}\,\partial y_0/\partial\varphi)/J_{nn}$. Substitute into the first row

$$\left(J_{yy} - \frac{J_{yn}^2}{J_{nn}}\right)\frac{\partial y_0}{\partial\varphi} = \omega_0 \widetilde{C}_y - \frac{\widetilde{C}_n J_{yn}}{J_{nn}}. \tag{G.8}$$

I now characterize the unknown terms on each side of (G.8).

Starting with the right side, differentiate J to get

$$\begin{aligned} -J_{yn} &= \varphi \widetilde{C}_{yn} = \varphi\, \widetilde{C}_n\, \varepsilon_{ny}/y_0, \\ -J_{nn} &= \varphi \widetilde{C}_{nn} = \varphi\, \widetilde{C}_n\, \varepsilon_{nn}/n_0. \end{aligned}$$

By Lemma G.1(a)–(c), $\widetilde{C}_n > 0$ and $\varepsilon_{nn} = \nu s_n + \bar{\varepsilon} s_y > 0$, so $J_{nn} < 0$. By Lemma G.1(b), $s_y n_0 \widetilde{C}_n = s_n y_0 \widetilde{C}_y$, so

$$\frac{\widetilde{C}_n J_{yn}}{J_{nn}} = \widetilde{C}_n \frac{n_0}{y_0}\frac{\varepsilon_{ny}}{\varepsilon_{nn}} = \widetilde{C}_y \frac{s_n \varepsilon_{ny}}{s_y \varepsilon_{nn}}.$$

Recall $\widehat{\rho}$ from (G.6), which describes the strength of the inventory incentive. By Lemma

G.1(b), $\widehat{\rho} = s_n\varepsilon_{ny}/(s_y\varepsilon_{nn})$, so $\widetilde{C}_n J_{yn}/J_{nn} = \widehat{\rho}\,\widetilde{C}_y$, so (G.8) becomes

$$\Big(J_{yy} - \frac{J_{yn}^2}{J_{nn}}\Big)\frac{\partial y_0}{\partial \varphi} = \widetilde{C}_y\big(\omega_0 - \widehat{\rho}\big). \tag{G.9}$$

Now consider the coefficient on the left-hand side. Differentiate J to get

$$-J_{yy} = \varphi\Big(\widetilde{C}_{yy} + \frac{mr_0(y_0)}{\eta y_0}\Big) - \lambda'(y_0).$$

Since $\lambda(y_0) = \varphi\omega_0\widetilde{C}_y$ and $\lambda(y)$ is proportional to $y^{\widetilde{\theta}-1}$, $-y_0\lambda'(y_0)/\widetilde{C}_y = \varphi(1-\widetilde{\theta})\omega_0$. With $\varepsilon_{yy} = y_0\widetilde{C}_{yy}/\widetilde{C}_y$ and $mr_0(y_0)/\widetilde{C}_y = 1-\omega_0$,

$$-J_{yy} = \frac{\varphi\widetilde{C}_y}{y_0}\Big(\varepsilon_{yy} + \frac{1-\omega_0}{\eta} + (1-\widetilde{\theta})\,\omega_0\Big).$$

By the expression for J_{yn} and $\widetilde{C}_n J_{yn}/J_{nn} = \widehat{\rho}\,\widetilde{C}_y$,

$$\frac{J_{yn}^2}{J_{nn}} = -\frac{\varphi\widetilde{C}_y}{y_0}\,\widehat{\rho}\,\varepsilon_{ny}.$$

Plugging both into the coefficient gives

$$J_{yy} - \frac{J_{yn}^2}{J_{nn}} = -\frac{\varphi\widetilde{C}_y}{y_0}\Big(\varepsilon_{yy} - \widehat{\rho}\,\varepsilon_{ny} + \frac{1-\omega_0}{\eta} + (1-\widetilde{\theta})\,\omega_0\Big). \tag{G.10}$$

By Lemma G.1(b),

$$\varepsilon_{yy} - \widehat{\rho}\varepsilon_{ny} = \nu - \varepsilon_{ny}(s_n/s_y + \widehat{\rho}) = \nu - \nu s_n\varepsilon_{ny}/(s_y\varepsilon_{nn}) = \nu(1-\widehat{\rho}).$$

So the term in parentheses in (G.10) can be written as the term $\Sigma(\omega_0, \widehat{\rho})$, where

$$\Sigma(\omega, \tilde{\rho}) \equiv \nu(1-\tilde{\rho}) + \frac{1-\omega}{\eta} + (1-\widetilde{\theta})\,\omega. \tag{G.11}$$

The function Σ is the elasticity of the FOC with respect to y when I restrict inventory accumulation n to its optimal value for each y. I will frequently use the abbreviation $\Sigma_0 \equiv \Sigma(\omega_0, \widehat{\rho})$.

I will now solve for the output and price elasticity to φ. First, I show $\Sigma_0 > 0$. By the definition of $\widehat{\rho}$,

$$1 - \widehat{\rho} = \frac{\bar{\varepsilon}}{\nu s_n + \bar{\varepsilon} s_y} > 0,$$

and $\omega_0 \in (0,1)$ because, by (G.2), $\omega_0 = \lambda(y_0)/(\varphi \widetilde{C}_y) > 0$, and $1 - \omega_0 = m r_0 / \widetilde{C}_y > 0$. So $\Sigma_0 > 0$. Now substitute (G.10) into (G.9) and solve to get

$$\frac{\partial \log y_0}{\partial \log \varphi} = \frac{\varphi}{y_0} \frac{\partial y_0}{\partial \varphi} = \frac{\widehat{\rho} - \omega_0}{\Sigma_0},$$

and $p_0 = y_0^{-1/\eta}$ gives

$$\frac{\partial \log p_0}{\partial \log \varphi} = \frac{\omega_0 - \widehat{\rho}}{\eta \, \Sigma_0}. \tag{G.12}$$

Proof of Part (i): an increase in φ can lead to lower prices and inventory accumulation.

Assume $\omega_0 < \widehat{\rho}(x^*_{n,0})$, which is the condition that the inventory incentive is stronger than the customer-market incentive. By (G.12) and $\Sigma_0 > 0$, $\partial \log p_0 / \partial \log \varphi < 0$. The second row of the linear system (G.7) gives $J_{nn} \, \partial n_0 / \partial \varphi = \widetilde{C}_n - J_{yn} \, \partial y_0 / \partial \varphi$. By the formula for $\partial \log y_0 / \partial \log \varphi$, $J_{yn} \, \partial y_0 / \partial \varphi = \widetilde{C}_n \varepsilon_{ny} (\omega_0 - \widehat{\rho}) / \Sigma_0$, so

$$J_{nn} \frac{\partial n_0}{\partial \varphi} = \widetilde{C}_n \Big[1 + \frac{\varepsilon_{ny} (\widehat{\rho} - \omega_0)}{\Sigma_0} \Big].$$

By Lemma G.3(a), $\varepsilon_{ny} > 0$. Since $\widehat{\rho} - \omega_0 > 0$ and $\Sigma_0 > 0$, the bracket exceeds 1. Since $J_{nn} < 0$, $\partial n_0 / \partial \varphi < 0$, and since $n_0 > 0$, $\partial \log n_0 / \partial \log \varphi < 0$.

Proof of Parts (ii) and (iii): higher irreversibility raises sales, the price elasticity to φ, and the markup elasticity to φ.

The proof has five steps. The first step, Output, differentiates the FOCs in κ and shows $\partial \log y_0/\partial\kappa > 0$, which is the first claim. The second step, Cutoff, combines this with the FOC for n to show that the cutoff $x^*_{n,0}$ falls with κ. The third step, Marginal Loss to Cost Ratio, uses the FOC for n to write ω_0 as a function ω^N of the cutoff alone, so that the price elasticity (G.12) depends on κ only through $x^*_{n,0}$. The fourth step, Change in Price Elasticity, signs the derivative of this elasticity in the cutoff and combines it with $\partial x^*_{n,0}/\partial\kappa < 0$. The fifth step, Markup, shows that the markup response to φ is also higher.

Output. By Subsection G.4, the steps above apply at every point of the neighborhood of (φ, κ). Differentiating (G.2) and (G.3) in κ, with $\lambda(y) = A(\kappa)y^{\widetilde{\theta}-1}$, gives

$$\begin{pmatrix} J_{yy} & J_{yn} \\ J_{yn} & J_{nn} \end{pmatrix} \begin{pmatrix} \partial y_0/\partial\kappa \\ \partial n_0/\partial\kappa \end{pmatrix} = - \begin{pmatrix} A'(\kappa) y_0^{\widetilde{\theta}-1} \\ 0 \end{pmatrix}. \tag{G.13}$$

Eliminating $\partial n_0/\partial\kappa$ and using (G.10) gives

$$\frac{\varphi \widetilde{C}_y \Sigma_0}{y_0} \frac{\partial y_0}{\partial\kappa} = A'(\kappa) y_0^{\widetilde{\theta}-1}.$$

Since $A(\kappa)y_0^{\widetilde{\theta}-1} = \lambda(y_0) = \varphi\omega_0\widetilde{C}_y$,

$$\frac{\partial \log y_0}{\partial\kappa} = \frac{A'(\kappa)}{A(\kappa)} \frac{\omega_0}{\Sigma_0} > 0,$$

because $A'(\kappa) = \chi\widetilde{\theta}\widehat{k}(1-\kappa)/(2R) > 0$ by the definition of $A(\kappa)$ and Lemma F.1(a).

Cutoff. The second row of the linear system (G.13) and $J_{yn}/J_{nn} = n_0\varepsilon_{ny}/(y_0\varepsilon_{nn})$ give

$$\frac{\partial \log n_0}{\partial\kappa} = -\frac{\varepsilon_{ny}}{\varepsilon_{nn}} \frac{\partial \log y_0}{\partial\kappa}.$$

Also $n_0/y_0 = \widetilde{\mathcal{N}}(x^*_{n,0})/\widetilde{\mathcal{Y}}(x^*_{n,0})$, and $\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}}$ has a positive derivative by Lemma G.2(a), so $x^*_{n,0}$ is differentiable in κ, and these two equations and $\varepsilon_{ny} + \varepsilon_{nn} = \nu$ give

$$\frac{d\log(\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}})}{d\log x^*_n}\frac{\partial \log x^*_{n,0}}{\partial \kappa} = \frac{\partial \log n_0}{\partial \kappa} - \frac{\partial \log y_0}{\partial \kappa} = -\frac{\nu}{\varepsilon_{nn}}\frac{\partial \log y_0}{\partial \kappa}.$$

Since $d\log(\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}})/d\log x^*_n > 0$ and $\varepsilon_{nn} > 0$, $\partial x^*_{n,0}/\partial\kappa < 0$.

Marginal Loss to Cost Ratio. This part establishes that the ratio of the marginal loss to marginal cost reduces to a function of the cutoff after using the firm's FOC with respect to n (G.3).

Define the level of output optimally produced by a static monopolist by

$$q^{\text{sm}} \equiv \left(\frac{(1-1/\eta)\widetilde{\mathcal{Y}}(1)^{1-1/\eta}}{mc_0(1)}\right)^{1/\varsigma}. \tag{G.14}$$

The evaluation of $\widetilde{Y}$ and mc_0 at 1 means that the cutoff is set to 1, in which case the firm accumulates zero inventories. Define the morning value of a unit of inventory relative to the marginal cost implied by q^{sm} as

$$b^{\text{sm}} \equiv \frac{b}{\varphi\, mc_0(q^{\text{sm}})}. \tag{G.15}$$

By Lemma G.1(a), which establishes the first derivatives of $\widetilde{C}$ with respect to sales and inventory stock, and the firm's marginal cost of production (F.3), $\widetilde{C}_n(y,n) = mc_0(q^{\text{sm}})(q/q^{\text{sm}})^\nu/\psi_n$, so the firm's FOC for n (G.3) holds if and only if

$$q = q^{\text{sm}}\left(b^{\text{sm}}\,\psi_n(x^*_n)\right)^{1/\nu}. \tag{G.16}$$

Consider a level of production q that satisfies (G.16). By the definition of q^{sm},

$$mc_0(q^{\mathrm{sm}}) = mc_0(1)(q^{\mathrm{sm}})^{\nu} = (1-1/\eta)\widetilde{\mathcal{Y}}(1)^{1-1/\eta}(q^{\mathrm{sm}})^{-1/\eta}. \tag{G.17}$$

Further, $\widetilde{C}_y(y,n) = mc_0(q^{\mathrm{sm}})(q/q^{\mathrm{sm}})^{\nu}/\psi_y$ and $mr_0(y) = (1-1/\eta)\big(\widetilde{\mathcal{Y}}q\big)^{-1/\eta}$. Using these expressions, I can take $\omega(y,n)$, change it to a function of (q, x_n^*), and pin down q as a function of x_n^*, which results in a function $\omega^N(x_n^*)$. In particular,

$$1-\omega(y,n) = \frac{mr_0(y)}{\widetilde{C}_y(y,n)} = \frac{\psi_y(x_n^*)\widetilde{\mathcal{Y}}(x_n^*)^{-1/\eta}}{\widetilde{\mathcal{Y}}(1)^{1-1/\eta}\big(b^{\mathrm{sm}}\,\psi_n(x_n^*)\big)^{\varsigma/\nu}} \equiv 1-\omega^N(x_n^*). \tag{G.18}$$

Change in Price Elasticity. Define

$$\mathcal{E}(\omega,\tilde{\rho}) \equiv \frac{\omega-\tilde{\rho}}{\Sigma(\omega,\tilde{\rho})}$$

so that, at the optimal choice of (y,n),

$$\eta\,\frac{\partial \log p_0}{\partial \log \varphi} = \mathcal{E}\big(\omega^N(x_{n,0}^*), \widehat{\rho}(x_{n,0}^*)\big).$$

Neither ω^N nor $\widehat{\rho}$ depends on κ, so κ affects the elasticity only through the cutoff $x_{n,0}^*$.

Define $\Gamma(x) \equiv \Sigma(x,x)$. If $\omega, \tilde{\rho} \in [0,1]$, then $\Sigma(\omega,\tilde{\rho}) > 0$ by (G.11), and direct differentiation gives

$$\begin{aligned}
\mathcal{E}_{\omega}(\omega,\tilde{\rho}) &= \frac{\Gamma(\tilde{\rho})}{\Sigma(\omega,\tilde{\rho})^2},\\
\mathcal{E}_{\tilde{\rho}}(\omega,\tilde{\rho}) &= -\frac{\Gamma(\omega)}{\Sigma(\omega,\tilde{\rho})^2}.
\end{aligned}$$

The function Γ is affine with $\Gamma(0) = \varsigma > 0$ and $\Gamma(1) = 1-\widetilde{\theta} > 0$, so it is positive on $[0,1]$. Here $\omega^N(x_{n,0}^*) = \omega_0$ and $\widehat{\rho}(x_{n,0}^*)$ lie in $(0,1)$, by Lemmas G.5 and G.3(a).

By Lemma G.3(a), $\widehat{\rho}'(x^*_{n,0}) > 0$ and $\omega^{N\prime}(x^*_{n,0}) \leq 0$ by Lemma G.4. Write $\Sigma_0 = \Sigma\big(\omega^N(x^*_{n,0}), \widehat{\rho}(x^*_{n,0})\big)$. By the chain rule,

$$\eta \frac{\partial^2 \log p_0}{\partial \kappa \, \partial \log \varphi} = \frac{\Gamma\big(\widehat{\rho}(x^*_{n,0})\big)\, \omega^{N\prime}(x^*_{n,0}) - \Gamma\big(\omega^N(x^*_{n,0})\big)\, \widehat{\rho}'(x^*_{n,0})}{\Sigma_0^2} \frac{\partial x^*_{n,0}}{\partial \kappa} > 0,$$

because the numerator of the fraction is negative and $\partial x^*_{n,0}/\partial \kappa < 0$.

Markup. This part proves the elasticity of the markup of price over output sold to φ increases with κ. Define the markup over the marginal cost of output sold as $\mu_0 \equiv p_0/\widetilde{C}_y(y_0, n_0)$. The elasticity of $\widetilde{C}_y$ with respect to n is

$$\frac{n_0 \widetilde{C}_{yn}}{\widetilde{C}_y} = \varepsilon_{ny} \frac{n_0 \widetilde{C}_n}{y_0 \widetilde{C}_y} = \frac{s_n \varepsilon_{ny}}{s_y},$$

where the first equality uses the definition of ε_{ny} and the second uses $s_y n_0 \widetilde{C}_n = s_n y_0 \widetilde{C}_y$ from Lemma G.1(b). With the definition of ε_{yy}, this gives

$$\frac{\partial \log \widetilde{C}_y(y_0, n_0)}{\partial \log \varphi} = \varepsilon_{yy} \frac{\partial \log y_0}{\partial \log \varphi} + \frac{s_n \varepsilon_{ny}}{s_y} \frac{\partial \log n_0}{\partial \log \varphi}.$$

Taking logs of the inventory n FOC (G.3), differentiating in $\log \varphi$ holding fixed b (the value of inventory if stored into $t = 1$), and using the definitions of ε_{ny} and ε_{nn} gives

$$\varepsilon_{ny} \frac{\partial \log y_0}{\partial \log \varphi} + \varepsilon_{nn} \frac{\partial \log n_0}{\partial \log \varphi} = -1.$$

Re-arrange to get

$$\frac{\partial \log n_0}{\partial \log \varphi} = -\frac{1}{\varepsilon_{nn}} \Big(1 + \varepsilon_{ny} \frac{\partial \log y_0}{\partial \log \varphi}\Big).$$

By Lemma G.1(b) and the definition of $\widehat{\rho}$ (G.6),

$$\frac{s_n \varepsilon_{ny}}{s_y \varepsilon_{nn}} = \frac{s_n(\nu - \bar{\varepsilon})}{\nu s_n + \bar{\varepsilon} s_y} = \widehat{\rho}.$$

Substitute the inventory response.

$$\frac{\partial \log \widetilde{C}_y(y_0, n_0)}{\partial \log \varphi} = \big(\varepsilon_{yy} - \widehat{\rho}\,\varepsilon_{ny}\big)\frac{\partial \log y_0}{\partial \log \varphi} - \widehat{\rho} = \nu(1-\widehat{\rho})\,\frac{\partial \log y_0}{\partial \log \varphi} - \widehat{\rho}.$$

It follows that

$$\begin{aligned}\frac{\partial \log \mu_0}{\partial \log \varphi} &= \widehat{\rho} - \Big(\nu(1-\widehat{\rho}) + \frac{1}{\eta}\Big)\frac{\widehat{\rho} - \omega_0}{\Sigma_0} \\ &= \omega_0\Big(1 + \big(1 - \widetilde{\theta} - 1/\eta\big)\frac{\widehat{\rho} - \omega_0}{\Sigma_0}\Big) \\ &= \frac{\omega_0\,\Gamma(\widehat{\rho})}{\Sigma_0}. \end{aligned} \tag{G.19}$$

Since $\omega_0 > 0$, $\Gamma(\widehat{\rho}) > 0$ (as $\widehat{\rho} \in (0,1)$ by Lemma G.3(a)) and $\Sigma_0 > 0$, $\partial \log \mu_0 / \partial \log \varphi > 0$, whatever the sign of $\omega_0 - \widehat{\rho}$.

By (G.12), the second line of (G.19) is

$$\frac{\partial \log \mu_0}{\partial \log \varphi} = \omega^N(x^*_{n,0})\Big(1 - \big(\eta(1-\widetilde{\theta}) - 1\big)\frac{\partial \log p_0}{\partial \log \varphi}\Big).$$

Differentiating in κ,

$$\frac{\partial^2 \log \mu_0}{\partial \kappa\, \partial \log \varphi} = \frac{\Gamma\big(\widehat{\rho}(x^*_{n,0})\big)}{\Sigma_0}\,\omega^{N\prime}(x^*_{n,0})\,\frac{\partial x^*_{n,0}}{\partial \kappa} - \omega_0\big(\eta(1-\widetilde{\theta}) - 1\big)\frac{\partial^2 \log p_0}{\partial \kappa\, \partial \log \varphi} > 0.$$

Positivity follows because $\omega^{N\prime}(x^*_{n,0}) \le 0$ and $\partial x^*_{n,0}/\partial \kappa < 0$ makes the first term non-negative, and since the assumption that $\theta < -1/\eta$ is the same as $\eta(1-\widetilde{\theta}) < 1$, then $\omega_0 > 0$ and $\partial^2 \log p_0 / (\partial \kappa\, \partial \log \varphi) > 0$ make the second term positive.

□

Remark on Prop. G.1(iii). There is upward pressure on the markup response to an increase in φ if $\theta < -1/\eta$. This is a sufficient condition, not a necessary one, and it behaves more like a regularity condition in the following sense. If a firm accumulates fewer inventories in $t = 0$, then the firm accumulates more customers, all else equal. If $\theta \geq -1/\eta$, then the additional sale reduces the marginal benefit of an extra customer by more than it reduces marginal revenue. As a result, a firm finds it optimal to raise its markup a little because the firm now acquires customers by accumulating fewer inventories rather than cutting the markup. Requiring $\theta < -1/\eta$ ensures that the returns to customers are large enough that the firm does not stop using its markup to invest in the customer base when the firm accumulates fewer inventories.

G.4 Regularity

Properties of the Production Technology. Define $\mathcal{I}$ as the amount sold of on-the-shelf parts that do not stock out, as in Appendix H. Also note that $\psi_y(1) \equiv \widetilde{\mathcal{Y}}(1)$ and $\psi_n(1) \equiv \zeta/(\zeta - 1)$. The following lemma provides closed-form solutions for ψ_n, ψ_y, s_y, and s_n as well as characterizes some of their properties.

Lemma G.2. *(a) On* $(1, \infty)$, $\widetilde{\mathcal{Y}} > 0$, $\widetilde{\mathcal{N}} > 0$, $\widetilde{\mathcal{Y}}' < 0 < \widetilde{\mathcal{N}}'$ *and* $(\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}})' > 0$.

(b) For $x_n^* > 1$,

$$\begin{aligned}
\psi_n(x_n^*) &= 1 + \frac{(x_n^*)^{-\zeta}}{\zeta - 1}, \\
\psi_y(x_n^*) &= \psi_n(x_n^*)\mathcal{Y}(x_n^*)^{\frac{1-\varrho_n}{\varrho_n}}, \\
s_y(x_n^*) &= \frac{\mathcal{Y}(x_n^*)}{\mathcal{Y}(x_n^*) + \mathcal{N}(x_n^*)}, \\
s_n(x_n^*) &= \frac{\mathcal{N}(x_n^*)}{\mathcal{Y}(x_n^*) + \mathcal{N}(x_n^*)}.
\end{aligned}$$

ψ_y is strictly increasing and ψ_n is strictly decreasing on $[1,\infty)$, and $\psi_n(x_n^*) \to 1$ as $x_n^* \to \infty$.

(c) The inverse elasticity of transformation between sales and inventory at fixed production q is positive:

$$\bar{\varepsilon}(x_n^*) = \frac{\zeta(1-\varrho_n)\mathcal{N}(x_n^*)(x_n^*)^{-\zeta}}{(\zeta-1)\psi_n(x_n^*)\mathcal{I}(x_n^*)} > 0.$$

(d) $s_n' > 0$ and $d\log\bar{\varepsilon}/d\log s_n < 1$.

Proof. Primes denote derivatives in x_n^*, and for $x_n^* > 1$ every function of the cutoff is evaluated at x_n^*.

(a) By (H.9) and the closed forms of $\mathcal{Y}$ and $\mathcal{N}$ in Appendix H, $\widetilde{\mathcal{Y}} > 0$ because $\mathcal{Y} > 0$ by (H.4). By Lemma H.1, $\widetilde{\mathcal{N}}(1) = 0$ and $\widetilde{\mathcal{Y}}' < 0 < \widetilde{\mathcal{N}}'$ on $(1,\infty)$, so $\widetilde{\mathcal{N}} > 0$ there, and $(\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}})' = (\widetilde{\mathcal{Y}}\widetilde{\mathcal{N}}' - \widetilde{\mathcal{N}}\widetilde{\mathcal{Y}}')/\widetilde{\mathcal{Y}}^2 > 0$.

(b) Lemma H.1 gives

$$\frac{d\log\widetilde{\mathcal{Y}}}{d\log x_n^*} = -\frac{\mathcal{I}}{(1-\varrho_n)\mathcal{Y}},$$
$$\frac{d\log\widetilde{\mathcal{N}}}{d\log x_n^*} = \frac{\mathcal{I}}{(1-\varrho_n)\mathcal{N}}.$$

So $\widetilde{\mathcal{N}}'\widetilde{\mathcal{Y}}/\widetilde{\mathcal{Y}}' = -\widetilde{\mathcal{N}}\mathcal{Y}/\mathcal{N}$ and $\widetilde{\mathcal{Y}}'\widetilde{\mathcal{N}}/\widetilde{\mathcal{N}}' = -\widetilde{\mathcal{Y}}\mathcal{N}/\mathcal{Y}$, and the definitions of ψ_n and ψ_y and (H.9) give

$$\psi_n = (x_n^*)^{-\frac{1}{1-\varrho_n}}\big(\mathcal{Y}+\mathcal{N}\big), \quad \psi_y = \frac{\widetilde{\mathcal{Y}}\,(\mathcal{Y}+\mathcal{N})}{\mathcal{Y}}.$$

The closed forms of $\mathcal{Y}$ and $\mathcal{N}$ give $\mathcal{Y}+\mathcal{N} = (x_n^*)^{\frac{1}{1-\varrho_n}}\big(1+(x_n^*)^{-\zeta}/(\zeta-1)\big)$, which gives the formula for ψ_n. With $\widetilde{\mathcal{Y}}\,(x_n^*)^{\frac{1}{1-\varrho_n}} = \mathcal{Y}^{1/\varrho_n}$ and $\widetilde{\mathcal{N}}\,(x_n^*)^{\frac{1}{1-\varrho_n}} = \mathcal{N}$, by (H.9), this gives the formulas for ψ_y, s_y and s_n. Also, $x_n^*\psi_n' = -\zeta(x_n^*)^{-\zeta}/(\zeta-1) < 0$, and $\psi_n(x_n^*) \to 1$ as

$x_n^* \to \infty$. Differentiating the closed form of $\mathcal{Y}$ gives $x_n^*\mathcal{Y}' = \frac{\varrho_n}{1-\varrho_n}\zeta(x_n^*)^{\frac{1}{1-\varrho_n}-\zeta}/(\zeta-1)$. With $\mathcal{Y}+\mathcal{N} = (x_n^*)^{\frac{1}{1-\varrho_n}}\psi_n$,

$$\begin{aligned}\frac{d\log\psi_y}{d\log x_n^*} &= \frac{x_n^*\psi_n'}{\psi_n} + \frac{1-\varrho_n}{\varrho_n}\frac{x_n^*\mathcal{Y}'}{\mathcal{Y}} \\ &= \frac{\zeta(x_n^*)^{-\zeta}\left((x_n^*)^{\frac{1}{1-\varrho_n}}\psi_n - \mathcal{Y}\right)}{(\zeta-1)\psi_n\mathcal{Y}} \\ &= \frac{\zeta(x_n^*)^{-\zeta}\mathcal{N}}{(\zeta-1)\psi_n\mathcal{Y}},\end{aligned}$$

which is positive. So ψ_y is strictly increasing on $[1,\infty)$, and so is $-\psi_n$.

(c) By the proof of (b),

$$\begin{aligned}\frac{d\log(\psi_y/\psi_n)}{d\log x_n^*} &= \frac{\zeta(x_n^*)^{-\zeta}(\mathcal{Y}+\mathcal{N})}{(\zeta-1)\psi_n\mathcal{Y}}, \\ \frac{d\log(\widetilde{\mathcal{N}}/\widetilde{\mathcal{Y}})}{d\log x_n^*} &= \frac{\mathcal{I}\,(\mathcal{Y}+\mathcal{N})}{(1-\varrho_n)\mathcal{N}\mathcal{Y}}.\end{aligned}$$

Their ratio is the formula for $\bar{\varepsilon}$ in the statement. It is positive because $\mathcal{I} > 0$ by definition, $\mathcal{N} = (x_n^*)^{\frac{1}{1-\varrho_n}}\widetilde{\mathcal{N}} > 0$ by (a), and $\psi_n > 0$.

(d) I use the identities (H.8) for $\mathcal{I}$, $\mathcal{N}$ and $\mathcal{S}$. Define $a \equiv x_n^* s_n'/s_n$ and $c \equiv x_n^*\bar{\varepsilon}'/\bar{\varepsilon}$. By (b), $s_n = (x_n^*)^{-\frac{1}{1-\varrho_n}}\mathcal{N}/\psi_n$, so

$$a = \frac{x_n^*\mathcal{N}'}{\mathcal{N}} - \frac{1}{1-\varrho_n} - \frac{x_n^*\psi_n'}{\psi_n} = \frac{\mathcal{I}}{(1-\varrho_n)\mathcal{N}} + \frac{\zeta(x_n^*)^{-\zeta}}{(\zeta-1)\psi_n} = \frac{(1+\bar{\varepsilon})\mathcal{I}}{(1-\varrho_n)\mathcal{N}},$$

where the second line uses the identities for $\mathcal{N}$ and $x_n^*\mathcal{N}'$ and the formula for ψ_n, and the third uses the closed form in (c). So $a > 0$ and $s_n' > 0$. By the closed form in (c), $c = x_n^*\mathcal{N}'/\mathcal{N} - \zeta - x_n^*\psi_n'/\psi_n - x_n^*\mathcal{I}'/\mathcal{I}$, so

$$a - c = \zeta - \frac{1}{1-\varrho_n} + \frac{\zeta\mathcal{S}}{\mathcal{I}} = \frac{\zeta}{\mathcal{I}},$$

where the first equality uses $x_n^* \mathcal{I}' = \zeta \mathcal{S}$, and the second uses $(\zeta - \frac{1}{1-\varrho_n})\mathcal{I} = \zeta(1-\mathcal{S})$. Hence $d\log\bar{\varepsilon}/d\log s_n = c/a = 1 - \zeta/(a\mathcal{I}) < 1$. □

The Technology Restriction. Drawing inventory into sales lowers the marginal cost of output sold only if it is cheaper to move inventory from storage onto the shelves by adjusting the cutoff x_n^* than to increase production. The inverse elasticity of transformation between sales and inventory $\bar{\varepsilon}$ pins down this trade-off. This section provides a sufficient condition to ensure that higher inventory accumulation raises the gains from moving inventories onto shelves for sale.

Define

$$\widetilde{\nu}(x_n^*) \equiv \frac{\bar{\varepsilon}(x_n^*)}{1 - d\log\bar{\varepsilon}(x_n^*)/d\log s_n(x_n^*)},$$
$$\nu_\rho \equiv \sup_{x_n^* > 1} \widetilde{\nu}(x_n^*).$$

The object $\widetilde{\nu}(x_n^*)$ is the threshold on $\nu = \frac{\alpha}{1-\alpha}$, which captures curvature in the firm's marginal cost function, above which $\widehat{\rho}$ increases at the cutoff x_n^*.

Lemma G.3. *(a) If $\nu > \nu_\rho$, then $\varepsilon_{ny} > 0$, $\widehat{\rho} \in (0,1)$ and $\widehat{\rho}' > 0$.*

(b) $(\zeta - 1)\nu_\rho < \frac{1}{4} + \frac{1}{e}$, hence $(\zeta - 1)\nu \geq \frac{1}{4} + \frac{1}{e}$ is sufficient to ensure $\nu > \nu_\rho$.

Proof. Primes denote derivatives in x_n^*, and for $x_n^* > 1$ every function of the cutoff is evaluated at x_n^*. Let $a \equiv x_n^* s_n'/s_n$ and $c \equiv x_n^* \bar{\varepsilon}'/\bar{\varepsilon}$. By the proof of Lemma G.2(d),

$$a = \frac{(1+\bar{\varepsilon})\mathcal{I}}{(1-\varrho_n)\mathcal{N}} > 0, \tag{G.20}$$
$$a - c = \frac{\zeta}{\mathcal{I}} > 0. \tag{G.21}$$

Since $c/a = d\log\bar{\varepsilon}/d\log s_n$, the definition of $\widetilde{\nu}$ gives $\widetilde{\nu} = \bar{\varepsilon}a/(a-c) > 0$.

(a) The quotient rule applied to the definition of $\widehat{\rho}$ in (G.6), with $s_y' = -s_n'$, gives

$$\begin{aligned}(\nu s_n + \bar{\varepsilon} s_y)^2 \widehat{\rho}' &= \bar{\varepsilon}(\nu - \bar{\varepsilon}) s_n' - \nu s_n \bar{\varepsilon}' \\ &= \frac{s_n \bar{\varepsilon}}{x_n^*}\big[\nu(a-c) - \bar{\varepsilon} a\big] \\ &= \frac{s_n \bar{\varepsilon}(a-c)}{x_n^*}\big(\nu - \widetilde{\nu}\big). \end{aligned} \tag{G.22}$$

Since $\nu > \nu_\rho \geq \widetilde{\nu}$, $\widehat{\rho}' > 0$ on $(1, \infty)$. Since $\bar{\varepsilon} > 0$ and $\nu s_n + \bar{\varepsilon} s_y \geq \bar{\varepsilon} s_y$,

$$-\frac{s_n}{s_y} \leq \widehat{\rho} \leq \frac{\nu s_n}{\bar{\varepsilon} s_y}.$$

By Lemma G.2(b) and (c),

$$\frac{s_n}{s_y} = \frac{\mathcal{N}}{\mathcal{Y}}, \quad \frac{s_n}{\bar{\varepsilon} s_y} = \frac{(\zeta - 1)\psi_n \mathcal{I}\,(x_n^*)^\zeta}{\zeta(1 - \varrho_n)\mathcal{Y}}.$$

Both tend to 0 as $x_n^* \downarrow 1$, because $\mathcal{N}(1) = \mathcal{I}(1) = 0$ and $\mathcal{Y}(1) = \psi_n(1) = \zeta/(\zeta - 1)$. So $\widehat{\rho}(x_n^*) \to 0$ as $x_n^* \downarrow 1$, and $\widehat{\rho} > 0$ on $(1, \infty)$ because $\widehat{\rho}$ is strictly increasing. Since $s_n > 0$ and $\nu s_n + \bar{\varepsilon} s_y > 0$, $\widehat{\rho} > 0$ gives $\nu > \bar{\varepsilon}$, so $\varepsilon_{ny} = (\nu - \bar{\varepsilon}) s_y > 0$ by Lemma G.1(b) and (c). Finally, $1 - \widehat{\rho} = \bar{\varepsilon}/(\nu s_n + \bar{\varepsilon} s_y) > 0$.

(b) Dividing (G.20) by (G.21) and multiplying by the closed form of $\bar{\varepsilon}$ in Lemma G.2(c),

$$\bar{\varepsilon} = \frac{\zeta(1 - \varrho_n)\mathcal{N}(x_n^*)^{-\zeta}}{(\zeta - 1)\psi_n \mathcal{I}},$$

gives

$$(\zeta - 1)\widetilde{\nu} = \frac{\mathcal{I}(x_n^*)^{-\zeta}}{\psi_n} + \bar{\varepsilon}\,\frac{\mathcal{I}(x_n^*)^{-\zeta}}{\psi_n}.$$

I bound the two terms separately. Both bounds use $\frac{1}{1-\varrho_n} < \zeta$, by the regularity condition

$\zeta(1-\varrho_n)>1$, and $(\zeta-1)\psi_n(x_n^*)^\zeta=(\zeta-1)\big((x_n^*)^\zeta-1\big)+\zeta$, by Lemma G.2(b), which also gives $\psi_n>1$.

First term. Since $\tilde{x}^{\frac{1}{1-\varrho_n}-\zeta}<1$ for $\tilde{x}>1$, the definition of $\mathcal{I}$ gives

$$\mathcal{I}=\zeta\int_1^{x_n^*}\tilde{x}^{\frac{1}{1-\varrho_n}-\zeta-1}\,d\tilde{x}<\zeta\int_1^{x_n^*}\tilde{x}^{-1}\,d\tilde{x}=\zeta\log x_n^*.$$

With $u\equiv\zeta\log x_n^*$, so that $(x_n^*)^{-\zeta}=e^{-u}$, and $\psi_n>1$, the first term is below ue^{-u}, which is at most $1/e$.

Second term. Since the integrand in the definition of $\mathcal{N}$ is below $(x_n^*)^{\frac{1}{1-\varrho_n}}$,

$$\mathcal{N}<(x_n^*)^{\frac{1}{1-\varrho_n}}F(x_n^*)=(x_n^*)^{\frac{1}{1-\varrho_n}-\zeta}\big((x_n^*)^\zeta-1\big)<(x_n^*)^\zeta-1.$$

The closed form of $\bar{\varepsilon}$ gives

$$\bar{\varepsilon}\,\frac{\mathcal{I}(x_n^*)^{-\zeta}}{\psi_n}=\frac{\zeta(\zeta-1)(1-\varrho_n)\mathcal{N}}{\big((\zeta-1)\psi_n(x_n^*)^\zeta\big)^2}=\frac{\zeta(\zeta-1)(1-\varrho_n)\mathcal{N}}{\big((\zeta-1)((x_n^*)^\zeta-1)+\zeta\big)^2}.$$

Since $(A+B)^2\geq 4AB$ for $A,B\geq 0$, with $A=(\zeta-1)\big((x_n^*)^\zeta-1\big)$ and $B=\zeta$, the denominator is at least $4\zeta(\zeta-1)\big((x_n^*)^\zeta-1\big)$. This lower bound on the denominator and the upper bound $\mathcal{N}<(x_n^*)^\zeta-1$ on the numerator give

$$\bar{\varepsilon}\,\frac{\mathcal{I}(x_n^*)^{-\zeta}}{\psi_n}<\frac{\zeta(\zeta-1)(1-\varrho_n)\big((x_n^*)^\zeta-1\big)}{4\zeta(\zeta-1)\big((x_n^*)^\zeta-1\big)}=\frac{1-\varrho_n}{4}<\frac{1}{4}.$$

Hence $(\zeta-1)\widetilde{\nu}(x_n^*)<\frac{1}{e}+\frac{1-\varrho_n}{4}$ at every $x_n^*>1$, and $(\zeta-1)\nu_\rho\leq\frac{1}{e}+\frac{1-\varrho_n}{4}<\frac{1}{4}+\frac{1}{e}$. The final statement is immediate from the fact that we want ν to exceed ν_ρ. □

Reducing the Firm's Problem to the Optimal Cutoff. Define the sales of q^{sm} (G.14) when no inventory stock is accumulated by $y^{\text{sm}}\equiv\widetilde{\mathcal{Y}}(1)q^{\text{sm}}$ and the marginal value of the

customer base relative to marginal revenue at y^{sm} by

$$\Lambda^{\text{sm}} \equiv \frac{A(\kappa)(y^{\text{sm}})^{\widetilde{\theta}-1}}{\varphi\, mr_0(y^{\text{sm}})},$$

where $\lambda(y) = A(\kappa)y^{\widetilde{\theta}-1}$ and $A(\kappa) \equiv \chi\widetilde{\theta}\left[R_H(1-\kappa) + R_H(1)\right]/(2R)$ as in (F.13). Also, define the value of Λ^{sm} at which cutoff x_n^* satisfies both first-order conditions, where b^{sm} is from (G.15) and ω^N from (G.18), by

$$\Lambda^N(x_n^*) \equiv \omega^N(x_n^*)\left(\frac{\widetilde{\mathcal{Y}}(x_n^*)\left(b^{\text{sm}}\,\psi_n(x_n^*)\right)^{1/\nu}}{\widetilde{\mathcal{Y}}(1)}\right)^{1-\widetilde{\theta}}\frac{\widetilde{\mathcal{Y}}(1)\,b^{\text{sm}}\,\psi_n(x_n^*)}{\psi_y(x_n^*)}.$$

The function Λ^N is constructed so that the first-order condition for y reduces to an equation in the cutoff alone. Consider (y,n) with $\varphi\widetilde{C}_n(y,n) = b$, let x_n^* be the cutoff at (y,n), and evaluate every function of the cutoff at x_n^*. Since $\omega = L/\widetilde{C}_y$, $\varphi L = A(\kappa)y^{\widetilde{\theta}-1}$ if and only if $\omega(y,n) = A(\kappa)y^{\widetilde{\theta}-1}/\left(\varphi\widetilde{C}_y(y,n)\right)$. By the proof of Proposition G.1, the output at (y,n) satisfies (G.16), which is a re-arrangement of the inventory FOC, and $\omega(y,n) = \omega^N(x_n^*)$. By Lemma G.1(a), $\widetilde{C}_y/\widetilde{C}_n = \psi_n/\psi_y$, so $\varphi\widetilde{C}_y = b\,\psi_n/\psi_y = \varphi\, mc_0(q^{\text{sm}})\,b^{\text{sm}}\,\psi_n/\psi_y$. Since $y = \widetilde{\mathcal{Y}}(x_n^*)q$ and $y^{\text{sm}} = \widetilde{\mathcal{Y}}(1)q^{\text{sm}}$, sales relative to the static-monopoly level are $y/y^{\text{sm}} = \widetilde{\mathcal{Y}}(b^{\text{sm}}\psi_n)^{1/\nu}/\widetilde{\mathcal{Y}}(1)$. With $mc_0(q^{\text{sm}}) = \widetilde{\mathcal{Y}}(1)\,mr_0(y^{\text{sm}})$ by (G.17) and the definition of Λ^{sm},

$$\omega(y,n) = \frac{A(\kappa)y^{\widetilde{\theta}-1}}{\varphi\widetilde{C}_y(y,n)} = \Lambda^{\text{sm}}\left(\frac{y}{y^{\text{sm}}}\right)^{\widetilde{\theta}-1}\frac{\psi_y(x_n^*)}{\widetilde{\mathcal{Y}}(1)\,b^{\text{sm}}\,\psi_n(x_n^*)}.$$

So $\varphi L = A(\kappa)y^{\widetilde{\theta}-1}$ if and only if $\omega^N(x_n^*)$ equals the right side, i.e. if and only if $\Lambda^N(x_n^*) = \Lambda^{\text{sm}}$.

Lemma G.4. *ω^N is strictly decreasing on $[1,\infty)$.*

Proof. By Lemma G.2(b), ψ_y is strictly increasing on $[1,\infty)$ and ψ_n is strictly decreasing. Also $\widetilde{\mathcal{Y}}$ is strictly decreasing on $[1,\infty)$ by (H.9) and Lemma H.1. Recall that ω^N is defined

by (G.18) (where b^{sm} is from (G.15)) as

$$\omega^N(x_n^*) = 1 - \frac{\psi_y(x_n^*)\widetilde{\mathcal{Y}}(x_n^*)^{-1/\eta}}{\widetilde{\mathcal{Y}}(1)^{1-1/\eta}\big(b^{\text{sm}}\,\psi_n(x_n^*)\big)^{\varsigma/\nu}}.$$

So the fraction in the definition of ω^N is positive and strictly increasing on $[1,\infty)$. Hence ω^N is strictly decreasing on $[1,\infty)$. □

For $b^{\text{sm}} > (\zeta-1)/\zeta$ (where b^{sm} is from (G.15)), define

$$v_1 \equiv \Big(\frac{b^{\text{sm}}\zeta}{\zeta-1}\Big)^{1/\nu} > 1, \quad \bar{\Lambda}(b^{\text{sm}}) \equiv \big(v_1^{\varsigma}-1\big)v_1^{1-\widetilde{\theta}-1/\eta} > 0.$$

The first term v_1 is production relative to q^{sm} when the firm accumulates no inventory, and the inequality just means the first unit of inventory is worth more than it costs. The second term $\bar{\Lambda}$ is the value of Λ^{sm} at which the zero-inventory corner is optimal. When $\Lambda^{\text{sm}} < \bar{\Lambda}$, a firm optimally accumulates some inventories.

Lemma G.5. *Let $\widetilde{\theta} \in (0,1)$ and $b^{\text{sm}} > (\zeta-1)/\zeta$. Then ω^N has a unique zero $x_n^{*,0}$ on $(1,\infty)$, $\omega^N \in (0,1)$ on $[1,x_n^{*,0})$, and $\Lambda^N(1) = \bar{\Lambda}(b^{\text{sm}})$. The function Λ^N is strictly decreasing on $[1,x_n^{*,0}]$, and it maps $(1,x_n^{*,0})$ onto $(0,\bar{\Lambda}(b^{\text{sm}}))$.*

Proof. This proof first establishes the endpoint values of Λ^N on $[1,x_n^{*,0}]$ and then proves monotonicity over this interval.

The fraction in the definition of ω^N (G.18) is positive, so $\omega^N < 1$. At $x_n^* = 1$, $\psi_y(1) = \widetilde{\mathcal{Y}}(1)$ and $b^{\text{sm}}\psi_n(1) = v_1^{\nu}$, so $1-\omega^N(1) = v_1^{-\varsigma}$, and $\omega^N(1) > 0$ because $v_1 > 1$. As $x_n^* \to \infty$, $\widetilde{\mathcal{Y}}(x_n^*) \to 0$ by Lemma H.1, and $\psi_n(x_n^*) \to 1$ and $\psi_y(x_n^*) \geq \psi_y(1) = \widetilde{\mathcal{Y}}(1)$ by Lemma G.2(b), so $\omega^N \to -\infty$. By Lemma G.4, ω^N has a unique zero $x_n^{*,0} > 1$ and lies in $(0,1)$ on $[1,x_n^{*,0})$. At $x_n^* = 1$, $y/y^{\text{sm}} = v_1$ and $\widetilde{\mathcal{Y}}(1)b^{\text{sm}}\psi_n(1)/\psi_y(1) = v_1^{\nu}$, so $\Lambda^N(1) = (1-v_1^{-\varsigma})v_1^{1-\widetilde{\theta}+\nu} = \bar{\Lambda}(b^{\text{sm}})$, because $\nu-\varsigma = -1/\eta$. Also $\Lambda^N > 0$ on $[1,x_n^{*,0})$ and $\Lambda^N(x_n^{*,0}) = 0$.

On $[1, x_n^{*,0})$, $\omega^N \in (0,1)$ is strictly decreasing by Lemma G.4. $\widetilde{\mathcal{Y}}$ and ψ_n are positive and strictly decreasing, and $\nu > 0$ and $1 - \widetilde{\theta} > 0$, so the second factor of Λ^N is positive and strictly decreasing. By Lemma G.2(b), ψ_n/ψ_y is positive and strictly decreasing. Thus, Λ^N is strictly decreasing on $[1, x_n^{*,0})$. Since $\Lambda^N(x_n^{*,0}) = 0$, it strictly decreases on $[1, x_n^{*,0}]$, and by the intermediate value theorem it maps $(1, x_n^{*,0})$ onto $(0, \bar{\Lambda}(b^{\mathrm{sm}}))$. □

Precise Statement of Assumptions for Proposition G.1. Refer to (G.14) and (G.15) for q^{sm} and b^{sm}, respectively.

Assumption G.1. $\chi > 0$, $\widetilde{\theta} \in (0,1)$, $\nu > \nu_\rho$, $b^{\mathrm{sm}} > (\zeta - 1)/\zeta$, $\Lambda^{\mathrm{sm}} < \bar{\Lambda}(b^{\mathrm{sm}})$, and

$$B + \chi(y^{\mathrm{sm}} v_1)^{\widetilde{\theta}} < \frac{\bar{k}}{\widehat{k}(1-\kappa)}, \quad G > \frac{\bar{k}}{\widehat{k}(1)}.$$

Sufficient Conditions. Here are a set of sufficient conditions for Assumption G.1. By Lemma G.3(b), $\nu(\zeta - 1) \geq \frac{1}{2} + \frac{1}{e}$ implies $\nu > \nu_\rho$. Let $\chi\widetilde{\theta} > 0$, so that $\Lambda^{\mathrm{sm}} > 0$. If $b^{\mathrm{sm}} \geq (1 + \Lambda^{\mathrm{sm}})(\zeta - 1)/\zeta$, then $v_1^\nu \geq 1 + \Lambda^{\mathrm{sm}} > 1$, so $b^{\mathrm{sm}} > (\zeta - 1)/\zeta$, and $v_1^{-1/\eta} < 1$ and $v_1^{\widetilde{\theta}-1} < 1$ give $v_1^\nu - v_1^{-1/\eta} - \Lambda^{\mathrm{sm}} v_1^{\widetilde{\theta}-1} > 0$. Multiplying by $v_1^{1-\widetilde{\theta}}$ and using $\nu + 1 - \widetilde{\theta} = \varsigma + 1 - \widetilde{\theta} - 1/\eta$ gives $\Lambda^{\mathrm{sm}} < (v_1^\varsigma - 1) v_1^{1-\widetilde{\theta}-1/\eta} = \bar{\Lambda}(b^{\mathrm{sm}})$.

Existence. The desired capital regimes hold at $y > 0$ if the two inequalities of Assumption F.1 hold with y in place of y^*. Under Assumption G.1, $0 < \Lambda^{\mathrm{sm}} < \bar{\Lambda}(b^{\mathrm{sm}})$, so Lemma G.5 gives a unique $x_n^{*,\dagger} \in (1, x_n^{*,0})$ with $\Lambda^N(x_n^{*,\dagger}) = \Lambda^{\mathrm{sm}}$, and, by (G.16) and the construction of Λ^N, the point $(y^\dagger, n^\dagger)$ with cutoff $x_n^{*,\dagger}$ and production $q^{\mathrm{sm}}\big(b^{\mathrm{sm}}\psi_n(x_n^{*,\dagger})\big)^{1/\nu}$ satisfies (G.3) with $n^\dagger > 0$ and (G.2) with $\lambda(y) = A(\kappa) y^{\widetilde{\theta}-1}$. Since $\widetilde{\mathcal{Y}}$ and ψ_n are strictly decreasing, $y^\dagger < y^{\mathrm{sm}} v_1$, so the capital regimes hold at $y^\dagger$ and, as strict inequalities continuous in y, on an open interval of y containing $y^\dagger$, on which $\lambda(y) = A(\kappa) y^{\widetilde{\theta}-1}$ by Lemma F.1(b), (F.1) and (F.13). So $(y^\dagger, n^\dagger)$ satisfies (G.2) and (G.3), with $x_{n,0}^* = x_n^{*,\dagger}$ and $\lambda(y) = A(\kappa) y^{\widetilde{\theta}-1}$ near $y^\dagger$.

The Sign of the Price Response. Define

$$x_n^{*,p} \equiv \inf\left\{x_n^* > 1 : \omega^N(x_n^*) < \widehat{\rho}(x_n^*)\right\},$$
$$\Lambda^p \equiv \Lambda^N(x_n^{*,p}).$$

Lemma G.6. *(a) Let $\widetilde{\theta} \in (0,1)$, $\nu > \nu_\rho$ and $b^{\mathrm{sm}} > (\zeta-1)/\zeta$. Then $\{x_n^* > 1 : \omega^N(x_n^*) < \widehat{\rho}(x_n^*)\} = (x_n^{*,p}, \infty)$, $x_n^{*,p} \in (1, x_n^{*,0})$, and $\Lambda^p \in (0, \bar{\Lambda}(b^{\mathrm{sm}}))$.*

(b) Under Assumption G.1, $\omega^N(x_{n,0}^*) < \widehat{\rho}(x_{n,0}^*)$ if and only if $\Lambda^{\mathrm{sm}} < \Lambda^p$.

(c) Let $\nu > \nu_\rho$, $\widetilde{\theta} \in (0,1)$, $\hat{b} > (\zeta-1)/\zeta$ and $\hat{\Lambda} \in \left(0, \bar{\Lambda}(\hat{b})\right)$. Then Assumption G.1 holds at some values of $(w, k_0, R, \varphi, \kappa, \delta, \delta_n, p^m, \chi, B, G)$ with $b^{\mathrm{sm}} = \hat{b}$ and $\Lambda^{\mathrm{sm}} = \hat{\Lambda}$. In particular, Assumption G.1 holds at some parameters with $\Lambda^{\mathrm{sm}} < \Lambda^p$ and at some parameters with $\Lambda^{\mathrm{sm}} > \Lambda^p$.

Proof. (a) By Lemma G.4 and Lemma G.3(a), $\omega^N - \widehat{\rho}$ is strictly decreasing on $(1,\infty)$. By the definition of $\widehat{\rho}$ and Lemma G.2(b) and (c),

$$0 < \widehat{\rho}(x_n^*) \leq \frac{\nu s_n(x_n^*)}{\bar{\varepsilon}(x_n^*) s_y(x_n^*)} = \frac{\nu(\zeta-1)\psi_n(x_n^*)\mathcal{I}(x_n^*)(x_n^*)^\zeta}{\zeta(1-\varrho_n)\mathcal{Y}(x_n^*)},$$

which tends to 0 as $x_n^* \downarrow 1$, because $\mathcal{I}(1) = 0$ and $\mathcal{Y}(1) = \zeta/(\zeta-1)$. Since $\omega^N(1) > 0$ by Lemma G.5, $\omega^N - \widehat{\rho}$ is positive near 1, and it is negative at $x_n^{*,0}$, where $\omega^N = 0 < \widehat{\rho}$. So $\omega^N - \widehat{\rho}$ has a unique zero on $(1,\infty)$, which lies in $(1, x_n^{*,0})$ and equals $x_n^{*,p}$, and $\omega^N < \widehat{\rho}$ exactly on $(x_n^{*,p}, \infty)$. By Lemma G.5, $\Lambda^p = \Lambda^N(x_n^{*,p}) \in (0, \bar{\Lambda}(b^{\mathrm{sm}}))$.

(b) Under Assumption G.1, the paragraph on existence gives $x_{n,0}^* = x_n^{*,\dagger} \in (1, x_n^{*,0})$ and $\Lambda^N(x_{n,0}^*) = \Lambda^{\mathrm{sm}}$. So $\omega^N(x_{n,0}^*) < \widehat{\rho}(x_{n,0}^*)$ if and only if $x_{n,0}^* > x_n^{*,p}$. Since Λ^N is strictly decreasing on $[1, x_n^{*,0}]$ by Lemma G.5, this holds if and only if $\Lambda^{\mathrm{sm}} = \Lambda^N(x_{n,0}^*) < \Lambda^N(x_n^{*,p}) = \Lambda^p$.

(c) Take any $w > 0$, $R > 0$, $\varphi > 0$, $\kappa \in (0,1)$, $\delta \in (0,1)$ and $\delta_n \in [0,1)$, and let $k_0 > 0$. The objects q^{sm} and y^{sm} do not depend on p^m or χ, b^{sm} is p^m times a positive factor that does not depend on χ, and Λ^{sm} is χ times a positive factor that does not depend on p^m. So $p^m > 0$ and $\chi > 0$ can be chosen with $b^{\text{sm}} = \hat{b}$ and $\Lambda^{\text{sm}} = \hat{\Lambda}$. Then $\nu > \nu_\rho$, $b^{\text{sm}} > (\zeta - 1)/\zeta$ and $\Lambda^{\text{sm}} < \bar{\Lambda}(b^{\text{sm}})$, and the definitions of Λ^{sm} and mr_0 give

$$\chi(y^{\text{sm}} v_1)^{\widetilde{\theta}} = \frac{2R\varphi(1-1/\eta)\,\hat{\Lambda}\,v_1^{\widetilde{\theta}}}{\widetilde{\theta}\,[R_H(1-\kappa)+R_H(1)]}\,(y^{\text{sm}})^{1-1/\eta}.$$

By (F.3), $mc_0(1) = wk_0^{-\nu}/(1-\alpha)$, so y^{sm} is proportional to $k_0^{\nu/\varsigma}$, while v_1, R_H and $\widehat{k}$ do not depend on k_0. Since $\nu(1-1/\eta) < \varsigma$, $\chi(y^{\text{sm}} v_1)^{\widetilde{\theta}}/k_0 \to 0$ as $k_0 \to \infty$, while $\bar{k}/\widehat{k}(1-\kappa) = (1-\delta)k_0/\widehat{k}(1-\kappa)$. So, for large k_0, $\chi(y^{\text{sm}} v_1)^{\widetilde{\theta}} < \bar{k}/\widehat{k}(1-\kappa)$, and Assumption G.1 holds at every $B \in \left(0, \bar{k}/\widehat{k}(1-\kappa) - \chi(y^{\text{sm}} v_1)^{\widetilde{\theta}}\right)$ and $G > \max\{B, \bar{k}/\widehat{k}(1)\}$. Finally, $\bar{\Lambda}(b^{\text{sm}})$ and Λ^p depend only on ν, η, ζ, ϱ_n, $\widetilde{\theta}$ and b^{sm}, by their definitions, and $\Lambda^p \in (0, \bar{\Lambda}(\hat{b}))$ at $b^{\text{sm}} = \hat{b}$ by (a). Taking $\hat{\Lambda} \in (0, \Lambda^p)$ and $\hat{\Lambda} \in \left(\Lambda^p, \bar{\Lambda}(\hat{b})\right)$ gives the last claim. □

H Inventory Microfoundation

In this section, I describe a microfoundation for inventories based on Wen (2011).

H.1 Setup

The firm sells its output as a continuum of parts $i \in [0,1]$, and its customers buy the CES bundle

$$y_t = \left[\int_0^1 \widetilde{x}_{n,i,t}\, y_{i,t}^{\varrho_n}\, di\right]^{1/\varrho_n}, \qquad \varrho_n \in (0,1), \tag{H.1}$$

of the quantities $y_{i,t}$ that it sells of each part. Here $\widetilde{x}_{n,i,t}$ is an i.i.d. taste shock for part i, drawn from a Pareto distribution with CDF $F(\widetilde{x}_n) = 1 - \widetilde{x}_n^{-\zeta}$ on $[1,\infty)$, tail parameter $\zeta > 1$, and the standard regularity condition $\zeta(1-\varrho_n) > 1$, which ensures finite aggregates.

The firm's demand curves apply to the bundle y_t. The firm stocks the shelf of each part *before* the realization of $\widetilde{x}_{n,i,t}$: it places new production $q_{i,t}$ on the shelf of part i, and the units that it does not sell become inventory. This timing makes inventory valuable as a way to avoid costly stock-outs.

Optimal Stock-Out Cutoff. Define $n_{i,t-1}$ as the inventory of part i at the start of period t. A fraction δ_n depreciates before use. The firm stocks every shelf before the taste shocks are drawn, and the shocks are i.i.d. across parts, so it puts the same available stock $\mathcal{Q}_t$ on every shelf. It does so by replenishing each part with new production, so a part that stocked out in the previous period receives more new production. The firm optimally chooses a cutoff $x^*_{n,t} \geq 1$ that partitions parts into two regimes:

- *Stock-out* ($\widetilde{x}_{n,i,t} \geq x^*_{n,t}$): demand for the part exceeds its available stock, so all of the stock is sold, including all inventories on hand.
- *Excess supply* ($\widetilde{x}_{n,i,t} < x^*_{n,t}$): only some of the stock is sold and the rest becomes inventory.

Aggregation. This section derives aggregate products sold and inventory accumulated.

It will be useful to define a few terms first:

$$D(x_n^*) \equiv \int_1^{x_n^*} \widetilde{x}_n^{\frac{1}{1-\varrho_n}}\, dF(\widetilde{x}_n) + (x_n^*)^{\frac{1}{1-\varrho_n}} \left[1 - F(x_n^*)\right], \tag{H.2}$$

$$\mathcal{N}(x_n^*) \equiv \int_1^{x_n^*} \left[(x_n^*)^{\frac{1}{1-\varrho_n}} - \widetilde{x}_n^{\frac{1}{1-\varrho_n}}\right] dF(\widetilde{x}_n), \tag{H.3}$$

$$\mathcal{Y}(x_n^*) \equiv \int_1^{x_n^*} \widetilde{x}_n^{\frac{1}{1-\varrho_n}}\, dF(\widetilde{x}_n) + (x_n^*)^{\frac{\varrho_n}{1-\varrho_n}} \int_{x_n^*}^{\infty} \widetilde{x}_n\, dF(\widetilde{x}_n), \tag{H.4}$$

with $D(x_n^*) + \mathcal{N}(x_n^*) = (x_n^*)^{\frac{1}{1-\varrho_n}}$. When $\widetilde{x}_n$ is Pareto-distributed, Lenzu et al. (accepted)

shows[63]

$$\mathcal{Y}(x_n^*) = \frac{\zeta}{\frac{1}{1-\varrho_n} - \zeta}\left((x_n^*)^{\frac{1}{1-\varrho_n}-\zeta} - 1\right) + \frac{\zeta}{\zeta - 1}(x_n^*)^{\frac{1}{1-\varrho_n}-\zeta}, \tag{H.5}$$

$$D(x_n^*) = \left(1 + \frac{\zeta}{\frac{1}{1-\varrho_n} - \zeta}\right)(x_n^*)^{\frac{1}{1-\varrho_n}-\zeta} - \frac{\zeta}{\frac{1}{1-\varrho_n} - \zeta}, \tag{H.6}$$

$$\mathcal{N}(x_n^*) = (x_n^*)^{\frac{1}{1-\varrho_n}}\left(1 - (x_n^*)^{-\zeta}\right) - \frac{\zeta}{\frac{1}{1-\varrho_n} - \zeta}\left((x_n^*)^{\frac{1}{1-\varrho_n}-\zeta} - 1\right). \tag{H.7}$$

Define also $\mathcal{I}(x_n^*) \equiv \int_1^{x_n^*} \widetilde{x}_n^{\frac{1}{1-\varrho_n}}\, dF(\widetilde{x}_n)$, which is positive for $x_n^* > 1$, $\mathcal{J}(x_n^*) \equiv \int_{x_n^*}^{\infty} \widetilde{x}_n\, dF(\widetilde{x}_n)$, which is finite because $\zeta > 1$, and $\mathcal{S}(x_n^*) \equiv (x_n^*)^{\frac{1}{1-\varrho_n}-\zeta}$. The Pareto distribution gives $\mathcal{I} = \frac{\zeta(1-\varrho_n)}{\zeta(1-\varrho_n)-1}(1-\mathcal{S})$ and $\mathcal{J} = \zeta(x_n^*)^{1-\zeta}/(\zeta-1)$, and these closed forms and those of $\mathcal{N}$ imply

$$\begin{aligned}
(\zeta - \frac{1}{1-\varrho_n})\mathcal{I}(x_n^*) &= \zeta\big(1 - \mathcal{S}(x_n^*)\big), \\
\mathcal{N}(x_n^*) &= (x_n^*)^{\frac{1}{1-\varrho_n}} - \mathcal{I}(x_n^*) - \mathcal{S}(x_n^*), \\
x_n^*\mathcal{I}'(x_n^*) &= \zeta\mathcal{S}(x_n^*), \\
x_n^*\mathcal{N}'(x_n^*) &= \frac{(x_n^*)^{\frac{1}{1-\varrho_n}} - \mathcal{S}(x_n^*)}{1-\varrho_n}.
\end{aligned} \tag{H.8}$$

Using these expressions, define

$$\widetilde{\mathcal{Y}}(x_n^*) \equiv \frac{\mathcal{Y}(x_n^*)^{1/\varrho_n}}{(x_n^*)^{\frac{1}{1-\varrho_n}}}, \qquad \widetilde{\mathcal{N}}(x_n^*) \equiv \frac{\mathcal{N}(x_n^*)}{(x_n^*)^{\frac{1}{1-\varrho_n}}}, \tag{H.9}$$

so that

$$y_t = \widetilde{\mathcal{Y}}(x_{n,t}^*)(q_t + (1-\delta_n)n_{t-1}), \qquad n_t = \widetilde{\mathcal{N}}(x_{n,t}^*)(q_t + (1-\delta_n)n_{t-1}). \tag{H.10}$$

[63] As notation, I use ζ and ϱ_n while Lenzu et al. (accepted) uses ξ and ρ, respectively. Lenzu et al. (accepted) writes G and H for the functions that I write $\mathcal{Y}$ and $\mathcal{N}$. I write $\mathcal{Y}$, $\mathcal{N}$ and $\mathcal{I}$ to avoid a notation clash with the demand states B and G and the demand level H of Appendix F.

H.2 Properties of the Aggregates

Lemma H.1. *With $\widetilde{\mathcal{Y}},\widetilde{\mathcal{N}}$ as in (H.9), for every $x_n^* > 1$, $\widetilde{\mathcal{Y}}'(x_n^*) < 0$ and $\widetilde{\mathcal{N}}'(x_n^*) > 0$. Further, $\widetilde{\mathcal{N}}$ maps $[1,\infty)$ onto $[0,1)$, $\widetilde{\mathcal{Y}}(x_n^*) \to 0$ as $x_n^* \to \infty$, and*

$$\frac{d\log\widetilde{\mathcal{Y}}}{d\log x_n^*} = -\frac{\mathcal{I}}{(1-\varrho_n)\mathcal{Y}},$$
$$\frac{d\log\widetilde{\mathcal{N}}}{d\log x_n^*} = \frac{\mathcal{I}}{(1-\varrho_n)\mathcal{N}}.$$

Proof. For $\widetilde{\mathcal{N}}$, differentiating the definition of $\mathcal{N}$ gives $\mathcal{N}'(x_n^*) = \frac{1}{1-\varrho_n}(x_n^*)^{\frac{\varrho_n}{1-\varrho_n}}F(x_n^*)$, and the definition also gives $(x_n^*)^{\frac{1}{1-\varrho_n}}F(x_n^*) - \mathcal{N}(x_n^*) = \mathcal{I}(x_n^*)$. Hence

$$\widetilde{\mathcal{N}}'(x_n^*) = \frac{\mathcal{N}'(x_n^*)}{(x_n^*)^{\frac{1}{1-\varrho_n}}} - \frac{\mathcal{N}(x_n^*)}{(1-\varrho_n)(x_n^*)^{\frac{1}{1-\varrho_n}+1}} = \frac{\mathcal{I}(x_n^*)}{(1-\varrho_n)(x_n^*)^{\frac{1}{1-\varrho_n}+1}} > 0.$$

Also $\widetilde{\mathcal{N}}(1) = 0$, and $\widetilde{\mathcal{N}}(x_n^*) = F(x_n^*) - (x_n^*)^{-\frac{1}{1-\varrho_n}}\mathcal{I}(x_n^*) \to 1$ as $x_n^* \to \infty$, because $\mathcal{I}$ is bounded by its closed form. So $\widetilde{\mathcal{N}}$ is a strictly increasing map from $[1,\infty)$ onto $[0,1)$. Since $\widetilde{\mathcal{N}} = \mathcal{N}/(x_n^*)^{\frac{1}{1-\varrho_n}}$, the formula for $\widetilde{\mathcal{N}}'$ also gives $d\log\widetilde{\mathcal{N}}/d\log x_n^* = \mathcal{I}/\big((1-\varrho_n)\mathcal{N}\big)$.

For $\widetilde{\mathcal{Y}}$, the Leibniz rule applied to $\mathcal{Y} = \mathcal{I} + (x_n^*)^{\frac{\varrho_n}{1-\varrho_n}}\mathcal{J}$ gives

$$\mathcal{Y}'(x_n^*) = (x_n^*)^{\frac{1}{1-\varrho_n}}\,f(x_n^*) + \frac{\varrho_n}{1-\varrho_n}(x_n^*)^{\frac{\varrho_n}{1-\varrho_n}-1}\mathcal{J}(x_n^*) - (x_n^*)^{\frac{1}{1-\varrho_n}}f(x_n^*) = \frac{\varrho_n}{1-\varrho_n}(x_n^*)^{\frac{\varrho_n}{1-\varrho_n}-1}\mathcal{J}(x_n^*) > 0,$$

where f is the density of F. Since $\log\widetilde{\mathcal{Y}} = \frac{1}{\varrho_n}\log\mathcal{Y} - \frac{1}{1-\varrho_n}\log x_n^*$,

$$\frac{d\log\widetilde{\mathcal{Y}}}{d\log x_n^*} = \frac{1}{\varrho_n}\,\frac{x_n^*\mathcal{Y}'(x_n^*)}{\mathcal{Y}(x_n^*)} - \frac{1}{1-\varrho_n} = \frac{1}{1-\varrho_n}\left[\frac{(x_n^*)^{\frac{\varrho_n}{1-\varrho_n}}\mathcal{J}(x_n^*)}{\mathcal{Y}(x_n^*)} - 1\right] = -\frac{\mathcal{I}(x_n^*)}{(1-\varrho_n)\mathcal{Y}(x_n^*)} < 0,$$

where the third equality uses $\mathcal{Y} = \mathcal{I} + (x_n^*)^{\frac{\varrho_n}{1-\varrho_n}}\mathcal{J}$. Thus,

$$\widetilde{\mathcal{Y}}'(x_n^*) = -\widetilde{\mathcal{Y}}(x_n^*)\mathcal{I}(x_n^*)/\big((1-\varrho_n)x_n^*\mathcal{Y}(x_n^*)\big) < 0.$$

Finally, $\widetilde{\mathcal{Y}} \to 0$ by the closed form of $\mathcal{Y}$. □

I Results from the Two-Industry DSGE

This section shows that general equilibrium attenuates the increase in price rigidity when there is a difference in the customer-market incentive across industries. The reason is that monetary policy responds to adverse credit supply shocks with a lower real interest rate. When the customer-market incentive is stronger in some industries, prices fall by less so the real interest rate endogenously falls by less, which changes firms' incentives to adjust prices.

Below, I give a brief overview of the model, decompose the inflation response into direct and indirect effects, discuss how the real interest rate channel attenuates the additional rigidity from differences in the customer-market incentive, and show how prices also become more rigid after monetary policy shocks, not just credit supply shocks.

I.1 Overview of the Quantitative Model

I briefly summarize the model and report the full details in Appendix J. The model is a two-industry DSGE. The two industries are identical except that firms in industry 1 have non-pecuniary value for the customer base. Firms in industry 2 have no such value. In both industries, firms hold inventories, face an earnings binding constraint, and have quadratic dividend adjustment costs. These ingredients allow me to simulate credit supply shocks in the model and calibrate it to the empirical evidence. To simplify the calibration, the model's time frequency is annual rather than quarterly and matches the year-over-year price response. The difference in responses identifies θ_f, which indexes industry 1's non-pecuniary value for a larger customer base.[64] I also allow for mild deep habits on the household side, in

[64] In reality, there may be differences in other characteristics between high and low irreversibility industries that influence the relative difference in price changes, such as their inventory technology. For parsimony and to focus on the customer-market incentive, I assume θ_f is the sole source of differences.

line with the model in Section 2, where goods-specific habits are needed to make customer acquisition valuable for high-irreversibility firms.

I add nominal rigidities to the model so that we can study the implications for inflation. Prices and wages are both sticky due to Rotemberg costs. I calibrate the costs to match a 9.5% monthly frequency of price changes and an average duration of time between wage changes of 6 quarters, following Nakamura and Steinsson (2008) and Grigsby et al. (2021). The price stickiness is relatively mild because firms operate in customer markets, and I calibrate to the micro-evidence instead of estimates of the aggregate Phillips Curve, such as Hazell et al. (2022). The central bank follows a standard Taylor rule with persistence in the nominal rate and penalties for deviations from the inflation targets.

I.2 Impact of An Aggregate Credit Supply Shocks on Inflation

The Missing Intercept. After a credit supply shock that lowers aggregate debt by 1%, impact inflation is -1.29% in the economy with differences in the customer-market incentive and -1.59% in a counterfactual economy in which both industries have weak customer-market incentives because I shut off the non-pecuniary value for the customer base. Irreversibility therefore dampens the impact response by 19.2%, where I define the dampening as $1 - C_1/C_2$ with C_1 the baseline economy and C_2 the counterfactual response.

The micro difference-in-differences evidence alone implies a much smaller response of inflation to a credit supply shock. Inside the model, the difference-in-differences slopes imply an aggregate effect of roughly -0.40% in the economy with differences in the customer-market incentive and -0.60% in the counterfactual economy per 1% fall in debt, compared with the model responses of -1.29% and -1.59%. These differences indicate a large negative missing intercept, at least within this DSGE.[65]

To investigate the role of different general-equilibrium channels, I decompose the impact

[65] See Wolf (2021) for an extensive discussion of the missing intercept problem.

response of inflation into direct and indirect effects. I compute the direct effect by taking the firm's linearized partial-equilibrium block and solving the implied inflation response to an aggregate credit supply shock, holding fixed all endogenous aggregate variables. I compute the indirect effect of a given channel by feeding in the channel's equilibrium aggregate paths into the firm block and differencing out the direct effect. The channels I consider are the nominal wage channel (Π_w), aggregate demand channel (p_X, X, and I), and the real interest rate channel (R).[66] The wage channel amplifies the inflation response ($-0.53\,\%$ in the economy with differences in the customer-market incentive and $-0.63\,\%$ in the counterfactual economy) because production cuts by firms result in lower equilibrium labor supplied, which pushes wages down. The demand channel mitigates the fall in inflation ($0.26\,\%$ and $0.27\,\%$) because the fire sale of inventories in $t = 0$ incentivizes temporarily higher demand, and that higher demand increases firms' desired prices. The real interest rate channel amplifies the inflation response (-0.09) in the economy with differences in the customer-market incentive but dampens the response (0.06) in the counterfactual economy. Monetary policy endogenously lowers the real interest rate in my model (there is no zero-lower bound). When firms in industry 1 have strong customer-market incentives, a lower real rate incentivizes firms to partially undo their disinvestment from their customer base, which pushes prices down. When the two industries are identical, firms instead respond to a lower real rate by holding onto more of their inventories, which mitigates the decline in prices.

In the DSGE, heterogeneity in the customer-market incentive increases price rigidity by less because monetary policy implements a lower real interest rate. In the model, prices are more rigid by 19.2% compared to the predicted increase in rigidity by 33.2% based on the diff-in-diff estimates within the model. The predicted increase based on the direct effect has

[66] The real interest rate is directly linked to aggregate demand by the Euler equation. The decomposition presented here is about the direct effects of the paths of equilibrium aggregates on the firm block's decisions, not necessarily how those aggregates get determined in equilibrium. The small effect of the interest rate channel here simply signifies that movements in the real rate do not affect the firm's price decision directly by much. Instead, the real rate affects the firm's price indirectly through wages and aggregate demand.

a comparable magnitude to the main text's back of the envelope calculation. The indirect effect, however, makes prices more flexible in the economy with differences in the customer-market incentive and makes prices more rigid in the counterfactual economy, which shrinks how much rigidity occurs because of the customer-market incentive. If we ignore the real interest rate channel in both economies, then prices are more rigid by 27.5%, which is much closer to the rigidity implied by the model's diff-in-diff of 33.2%.

Comparison to the Back-of-the-Envelope. The dampening due to an asymmetry in the customer-market incentive does not map directly to the main text's back-of-the-envelope calculation. The magnitude of the credit supply shock was normalized to one in the previous part, and the average credit supply shock for high-irreversibility industries is slightly smaller in the back-of-the-envelope calculation, so a symmetric aggregate shock is not a perfect match. Since this DSGE has just two industries, I also recompute the back-of-the-envelope using the coefficient estimates from the regression with an above/below median irreversibility indicator for the interaction. The back-of-the-envelope calculation suggests that prices are more rigid by 31.4% when some industries have more irreversible investment. For the DSGE, this number becomes 16.0%. As I discussed previously, general equilibrium attenuates the reduction in rigidity because the indirect effects of the credit supply shock do not shrink by as much as the direct effect. If we turn off the real interest rate channel, then the increase in rigidity becomes 24.7%.

Monetary Policy Shocks. Recall that the partial-equilibrium model in Section 2 implies that shifts in the marginal value of liquidity are equivalent to shifts in the discount rate. The intuition is that a shift in the discount rate modify the marginal value of the customer base. When demand falls persistently because of a decline in the real interest rate, a firm gets fewer customers for the same price. The marginal customer will bring less demand with it in the future, so the firm optimally invests less in the customer base. The same

force that generates countercyclical markups after tighter financial constraints applies to demand shocks more generally. After a monetary policy contraction, the extra rigidity of prices can be seen in two ways. First, on impact, inflation falls by less in the economy with differences in the customer-market incentive. Second, we can compare the inflation-output sensitivity at different horizons. I define the inflation-output sensitivity as the ratio of the cumulative log price change to the cumulative output impulse response. Specifically, the cumulative log change in the price level is $\log P_t - \log P_{ss} = \sum_t \log \Pi_t$. The cumulative output impulse response is the cumulative sum of log deviations in output from steady state, $\sum_t (\log Y_t - \log Y_{ss})$. Table I.1 reports the numbers from these experiments after a monetary policy shock. While the additional rigidity is not as large as in the case of credit supply shocks, there is still some attenuation. After 10 years, the amount of inflation after the same change in output is 10% smaller when there is a difference in the customer-market incentive.

J Details of the Two-Industry DSGE

J.1 Industries

Firms are partitioned into industries $s \in \{1, \ldots, S\}$, and industry s is a continuum of atomistic firms with total mass m_s, where $\sum_s m_s = 1$. The firms of an industry are identical, and in the aggregate experiments every firm of an industry is hit by the same shocks. In equilibrium every firm-level variable is therefore common across the firms of an industry, and I index it by the industry alone, as in $p_{s,t}$, $c_{s,t}$ and $h_{s,t}$. Each firm takes as given the industry indices $p_{X_s,t}$, $p_{s,t}$, $X_{s,t}$ and $I_{s,t}$, the industry demand levels $\mathcal{H}^c_{s,t}$ and $\mathcal{H}^I_{s,t}$, and every aggregate variable, even though its own price equals the industry price index $p_{s,t}$ in equilibrium.

Table I.1: Customer-Market Incentive Raises Price Rigidity After Monetary Policy Shock

	Diff. in Cust. Mkt. Incentive	Same Weak Cust. Mkt. Incentive	Extra Rigidity (%)
Impact Inflation Response	−0.35	−0.38	7.7
Inflation-Output Sensitivity			
Impact	1.352	1.580	14.5
10 Years	0.604	0.673	10.2
Long Run	0.211	0.233	9.5

Notes: This table reports the impact inflation response after a monetary policy shock as well as the inflation-output sensitivity on impact, after 10 years, and in the long run. The inflation-output sensitivity is the ratio of two terms. The numerator is the cumulative log change in price level, which is $\log P_t - \log P_{ss} = \sum_t \log \Pi_t$. The denominator is the cumulative output impulse response, which is the cumulative sum of log deviations in output from steady state, $\sum_t (\log Y_t - \log Y_{ss})$. Column 1 is the DSGE model in which industry 1 has a stronger customer-market incentive because of the non-pecuniary value for the customer base. Column 2 is the DSGE model in which industry 1 and 2 are the same: firms in either industry do not have a non-pecuniary value for the customer base. The extra rigidity column equals one minus the ratio of column 1 to 2. The monetary policy shock is scaled to 1pp. Any views expressed are those of the authors and not those of the U.S. Census Bureau or Bureau of Labor Statistics (BLS). The Census Bureau and BLS have reviewed this data product to ensure appropriate access, use, and disclosure avoidance protection of the confidential source data used to produce this product. This research was performed at a Federal Statistical Research Data Center under FSRDC Project Number 3211. (CBDRB-FY25-P3211-R12601). This is a part of clearance request #12601.

J.2 Household

Preferences are over the consumption index X_t of (J.14)–(J.16) and labor ℓ_t:

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t U\left(X_t, \ell_t\right), \qquad U\left(X_t, \ell_t\right) = \frac{X_t^{1-\sigma}}{1-\sigma} - \chi \frac{\ell_t^{1+\nu}}{1+\nu}, \qquad \sigma \geq 0,\ \nu > 0. \tag{J.1}$$

A labor packer combines varieties with elasticity η_w, and a union sets the nominal wage $W_{\iota,t}$ of each variety:

$$\ell_{\iota,t} = \left(\frac{W_{\iota,t}}{W_t}\right)^{-\eta_w} \ell_t^W, \quad W_t = \left(\int_0^1 W_{\iota,t}^{1-\eta_w}\, d\iota\right)^{\frac{1}{1-\eta_w}}, \quad \ell_t^W = \left(\int_0^1 \ell_{\iota,t}^{\frac{\eta_w-1}{\eta_w}}\, d\iota\right)^{\frac{\eta_w}{\eta_w-1}}. \tag{J.2}$$

The household pays a labor tax τ_w and the Rotemberg wage-adjustment costs $AC_{w,\iota,t}$, holds real bonds B_t paying the gross real return R_t, and receives firm profits π_t, labor-packer and capital-goods profits π_t^ℓ, π_t^I, and the net transfer $\mathcal{T}_t$. The budget constraint is

$$\begin{aligned}\underbrace{\sum_s m_s\, p_{s,t} c_{s,t}}_{=p_{X,t}X_t} + B_t = (1-\tau_w)\int_0^1 w_{\iota,t}\ell_{\iota,t}\, d\iota - \int_0^1 AC_{w,\iota,t}\, d\iota \\ + \pi_t + \pi_t^\ell + \pi_t^I + R_{t-1}B_{t-1} + \mathcal{T}_t.\end{aligned} \tag{J.3}$$

A fiscal authority runs a balanced budget, so $\mathcal{T}_t$ collects the labor tax, rebates every private adjustment cost lump-sum, and funds the tax advantage on debt in (J.35):

$$\mathcal{T}_t = \tau_w\, w_t \ell_t^W - \tau\,(R_{t-1}-1)\,B_{t-1} + \sum_s m_s\,(PAC_{s,t} + DAC_{s,t} + \mathrm{KAC}_{s,t}) + \int_0^1 AC_{w,\iota,t}\, d\iota. \tag{J.4}$$

Each union takes its own rebate as given, so the wage-adjustment cost is a private cost in (J.3) and a public rebate in (J.4).

The multiplier on (J.3), the real stochastic discount factor, and the Euler equations for nominal bonds paying $R_{n,t}$ and for real bonds paying R_t are

$$\mathcal{E}_t = \frac{U_X\,(X_t, \ell_t)}{p_{X,t}}, \tag{J.5}$$

$$M_{t,t+k} = \beta^k \frac{\mathcal{E}_{t+k}}{\mathcal{E}_t} = \beta^k \frac{U_X\,(X_{t+k}, \ell_{t+k})}{U_X\,(X_t, \ell_t)} \frac{p_{X,t}}{p_{X,t+k}}, \tag{J.6}$$

$$1 = \mathbb{E}_t\left[M_{t,t+1}\frac{R_{n,t}}{\Pi_{t+1}}\right], \tag{J.7}$$

$$1 = \mathbb{E}_t\left[M_{t,t+1}R_t\right], \qquad R_t = \frac{1}{\mathbb{E}_t[M_{t,t+1}]}. \tag{J.8}$$

Wage Phillips Curve. The wage Phillips Curve is standard and given by

$$\begin{aligned} 0 = & -\frac{\eta_w}{\kappa_w}\left(\frac{U_\ell\left(X_t,\ell_t\right)}{U_X\left(X_t,\ell_t\right)}p_{X,t}+\frac{\eta_w-1}{\eta_w}(1-\tau_w)\,w_t\right)\ell_t \\ & -\left(\Pi_t\frac{w_t}{w_{t-1}}-\overline{\Pi}_w\right)\Pi_t\frac{w_t}{w_{t-1}}\,w_t\ell_t \\ & +\mathbb{E}_t\left[M_{t,t+1}\left(\Pi_{t+1}\frac{w_{t+1}}{w_t}-\overline{\Pi}_w\right)\Pi_{t+1}\frac{w_{t+1}}{w_t}\,w_{t+1}\ell_{t+1}\right], \end{aligned} \tag{J.9}$$

with the marginal rate of substitution

$$\frac{U_\ell}{U_X}=-\chi\,\ell_t^\nu X_t^\sigma. \tag{J.10}$$

I set $\tau_w=-1/(\eta_w-1)$ to eliminate the steady-state wage markup. THus, in steady-state,

$$\chi\,\ell^\nu X^\sigma\,p_X=w \qquad \text{(steady state)}. \tag{J.11}$$

Equation (J.11) pins χ to implement the target level of labor supply ℓ_{target} in Table J.1.

The Rotemberg Coefficients. The parameters κ_p and κ_w are defined by the first-order equivalence between the Rotemberg and Calvo price and wage curves. Linearized around $\overline{\Pi}$, the Rotemberg price curve's inflation slope is $(\eta-1)/\kappa_p$. Equate it to the Calvo slope $(1-\theta_p)(1-\beta\theta_p)/\theta_p$ and re-arrange to

$$\kappa_p=\frac{(\eta-1)\,\theta_p}{(1-\theta_p)(1-\beta\theta_p)}. \tag{J.12}$$

For the wage Phillips Curve, the Calvo slope has the extra term $1/(1+\nu\eta_w)$, I have

$$\kappa_w=\frac{\eta_w\,\theta_w\,(1+\nu\eta_w)}{(1-\theta_w)(1-\beta\theta_w)}. \tag{J.13}$$

The parameters θ_p and θ_w are the non-reset probabilities.

J.3 The Consumption Bundle

Industry s's bundle is a habit-adjusted CES aggregator, with elasticity η, over the industry's firms, in which a purchase c from a firm with customer base h counts as $c\,h^{-\theta}$. The firms within an industry are identical in equilibrium, so the bundle is

$$X_{s,t} = c_{s,t} h_{s,t-1}^{-\theta}, \tag{J.14}$$

with the habit-adjusted index

$$p_{X_s,t} = p_{s,t} h_{s,t-1}^{\theta}. \tag{J.15}$$

The industry bundles are aggregated by a Cobb–Douglas function, with consumption and its price index given by

$$X_t = \prod_s X_{s,t}^{m_s}, \qquad p_{X,t} = \prod_s p_{X_s,t}^{m_s}. \tag{J.16}$$

Industry demand and the numéraire normalization are

$$X_{s,t} = \frac{p_{X,t}}{p_{X_s,t}} X_t, \qquad \prod_s p_{s,t}^{m_s} = 1. \tag{J.17}$$

By symmetry, the industry price index equals the common firm price $p_{s,t}$, and industry output equals the common firm sales $y_{s,t}$. Aggregate output is

$$Y_t = \prod_s y_{s,t}^{m_s}. \tag{J.18}$$

Habit and Customer Demand. The customer base accumulates on the household's purchases of the variety:

$$h_{s,t} = \rho_h h_{s,t-1} + (1 - \rho_h)\, c_{s,t}, \qquad 0 \leq \rho_h < 1. \tag{J.19}$$

A firm sells output as final consumption to customers from the representative household.

$$c_{s,t} = p_{s,t}^{-\eta}\, \mathcal{H}_{s,t}^{c}\, h_{s,t-1}^{\theta(1-\eta)}, \qquad \mathcal{H}_{s,t}^{c} = p_{X_s,t}^{\eta-1}\, p_{X,t}\, X_t. \tag{J.20}$$

The firm also sells output as intermediate inputs to the capital-goods producer. Thus, the firm's demand curve is

$$y_{s,t} = p_{s,t}^{-\eta}\, \mathcal{H}_{s,t}, \qquad \mathcal{H}_{s,t} = \mathcal{H}_{s,t}^{I} + \mathcal{H}_{s,t}^{c}\, h_{s,t-1}^{\theta(1-\eta)}, \qquad \eta > 1,\ \theta \leq 0, \tag{J.21}$$

and the habit share of demand is

$$\vartheta_{s,t} \equiv \frac{c_{s,t}}{y_{s,t}} = \frac{\mathcal{H}_{s,t}^{c}\, h_{s,t-1}^{\theta(1-\eta)}}{\mathcal{H}_{s,t}^{I} + \mathcal{H}_{s,t}^{c}\, h_{s,t-1}^{\theta(1-\eta)}} \in (0,1). \tag{J.22}$$

J.4 Capital-Goods Producer and the Monetary Block

The capital-goods producer aggregates varieties with the same two-level structure, without habits. Its industry-s input $I_{s,t}$ is a CES aggregate, with elasticity η, of its purchases from the industry's firms, which in equilibrium equals $y_{s,t}^{I}$ from each firm:

$$I_t = \prod_s I_{s,t}^{m_s}, \qquad I_{s,t} = y_{s,t}^{I}. \tag{J.23}$$

Cost minimization and the numéraire give the investment demand faced by a firm that charges $p_{s,t}$

$$y^I_{s,t} = p^{-\eta}_{s,t}\,\mathcal{H}^I_{s,t}, \qquad \mathcal{H}^I_{s,t} \equiv p^{\eta-1}_{s,t}\,I_t. \tag{J.24}$$

The producer sells the investment good at p^I_t and maximizes $\mathbb{E}_0 \sum_t M_{0,t}\left[p^I_t I_t - I_t\right]$. It follows then that

$$p^I_t = 1. \tag{J.25}$$

Monetary policy follows

$$R_{n,t} = R^{\gamma_R}_{n,t-1}\left[\overline{R}_n \left(\frac{\Pi_t}{\overline{\Pi}}\right)^{\gamma_\Pi}\left(\frac{Y_t}{\overline{Y}}\right)^{\gamma_Y}\epsilon^R_t\right]^{1-\gamma_R}, \qquad \overline{R}_n = \overline{\Pi}/\beta, \tag{J.26}$$

where $R_{n,t}$ is the nominal gross rate and $\overline{Y}$ is the deterministic steady state level of output. The real gross rate is $R_t \equiv 1/\mathbb{E}_t[M_{t,t+1}]$ of (J.8), and the Fisher equation is

$$R_{n,t} = \frac{1}{\mathbb{E}_t\left[M_{t,t+1}/\Pi_{t+1}\right]}. \tag{J.27}$$

J.5 The Firm's Problem

A firm of industry s faces the demand curve (J.21), produces with a Cobb–Douglas technology on predetermined capital, holds inventories, sets its price subject to a Rotemberg cost, smooths dividends, and faces an earnings-based constraint. I microfound inventories as in Appendix H.

Technology and Capital. Output of the firm's good is

$$q_{s,t} = A_t\,\overline{q}\,k^{\alpha_k}_{s,t-1}\,\ell^{\alpha_\ell}_{s,t}, \qquad \alpha_k, \alpha_\ell \in (0,1), \tag{J.28}$$

with labor hired at the market wage w_t and capital predetermined. Aggregate TFP follows a log AR(1) around $A_{ss} = 1$,

$$\log A_t = \rho_A \log A_{t-1} + \varepsilon_{A,t}, \tag{J.29}$$

and the capital stock accumulates with investment $i_{s,t}$ a single unrestricted control purchased at p_t^I:

$$k_{s,t} = (1 - \delta_k)\, k_{s,t-1} + i_{s,t}. \tag{J.30}$$

Inventories, Frictions, Dividend. Production is pooled with inherited inventory into the available stock of parts $\mathcal{Q}_{s,t}$. Those parts become either sales or inventory held into the next period based on the stock-out cutoff $z^*_{n,s,t} \geq 1$ of (H.9):

$$\mathcal{Q}_{s,t} = q_{s,t} + n_{s,t-1}, \quad y_{s,t} = \widetilde{G}\left(z^*_{n,s,t}\right) \mathcal{Q}_{s,t}, \quad n_{s,t} = \widetilde{H}\left(z^*_{n,s,t}\right) \mathcal{Q}_{s,t}, \tag{J.31}$$

with $n_{s,t} \geq 0$.

Rotemberg price stickiness and the quadratic dividend adjustment costs are

$$PAC_{s,t} \equiv \frac{\kappa_p}{2} \left(\Pi_t \frac{p_{s,t}}{p_{s,t-1}} - \overline{\Pi} \right)^2 Y_t \ \geq 0, \tag{J.32}$$

$$DAC_{s,t} \equiv \frac{\kappa_d}{2} \left(d_{s,t} - \bar{d}_s \right)^2 \ \geq 0, \tag{J.33}$$

both rebated lump-sum to the household.

Debt $b_{s,t}$ is chosen at t and carried into $t+1$. The beginning-of-period stock, $b_{s,t-1}$, is tax-advantaged. Following Lian and Ma (2020) and Drechsel (2023), I assume firms face an earning-based constraint.

$$b_{s,t} \leq \exp\left(e_{\varphi,s,t}\right) \varphi \left(p_{s,t} y_{s,t} - w_t \ell_{s,t} \right), \qquad \text{(multiplier } \lambda_{\varphi,s,t} \geq 0\text{)}, \tag{J.34}$$

$$R_{\tau,t} \equiv 1 + (1-\tau)(R_t - 1), \tag{J.35}$$

$$e_{\varphi,s,t} = \rho_\varphi \, e_{\varphi,s,t-1} + \varepsilon_{\varphi,s,t}, \qquad e_{\varphi,s,ss} = 0. \tag{J.36}$$

The parameter φ is the EBITDA multiple.

Capital adjustment is subject to the quadratic cost

$$\mathrm{KAC}_{s,t} \equiv \frac{\kappa_k}{2} \frac{(k_{s,t} - k_{s,t-1})^2}{k_{s,t-1}}, \qquad \kappa_k \geq 0, \tag{J.37}$$

and is rebated lump-sum through (J.4).

Dividends are given by

$$\begin{aligned} d_{s,t} = {} & p_{s,t} y_{s,t} - w_t \ell_{s,t} - p_t^I \, i_{s,t} \\ & - R_{\tau,t-1} b_{s,t-1} + b_{s,t} - PAC_{s,t} - DAC_{s,t} - \mathrm{KAC}_{s,t}. \end{aligned} \tag{J.38}$$

The Problem. The firm takes as given the wage w_t, investment-good price p_t^I, the household stochastic discount factor $M_{t,t+1}$, aggregate inflation Π_t, output Y_t, the demand levels $\left(\mathcal{H}_{s,t}^I, \mathcal{H}_{s,t}^c\right)$, aggregate TFP A_t, the tax-adjusted rate $R_{\tau,t}$, the borrowing-limit shock $e_{\varphi,s,t}$, and the external customer-base benchmark $\overline{h}_{s,t-1}$ from Appendix J.7; together with the initial states $(p_{s,-1}, h_{s,-1}, n_{s,-1}, b_{s,-1}, k_{s,-1})$ with $k_{s,-1} > 0$. It chooses the sequences

$$\left(p_{s,t},\ y_{s,t},\ q_{s,t},\ z^*_{n,s,t},\ n_{s,t},\ b_{s,t},\ h_{s,t},\ d_{s,t},\ \ell_{s,t},\ i_{s,t},\ k_{s,t}\right)_{t\geq 0}$$

subject to the demand curve (J.21) with its components (J.20), the habit law (J.19), the stock-out technology (J.31) with $n_{s,t} \geq 0$ and $z^*_{n,s,t} \geq 1$, the production constraint

$$q_{s,t} \ \leq \ A_t \, \overline{q} \, k_{s,t-1}^{\alpha_k} \ell_{s,t}^{\alpha_\ell} \qquad (\text{multiplier } mc^q_{s,t} \geq 0), \tag{J.39}$$

the capital law of motion (J.30), the borrowing constraint (J.34), and dividends (J.38). The objective is

$$\max \quad \mathbb{E}_0 \sum_{t\geq 0} M_{0,t} \left[d_{s,t} \; + \; v\left(c_{s,t},\, h_{s,t-1},\, \overline{h}_{s,t-1}\right) \right]. \tag{J.40}$$

The Lagrangian. The industry index is suppressed here to economize on notation. Attaching multipliers, each scaled by $M_{0,t}$, the Lagrangian is

$$\begin{aligned}
\mathcal{L} = \mathbb{E}_0 \sum_{t\geq 0} M_{0,t} \Big\{ & \; d_t + v\left(c_t, h_{t-1}, \overline{h}_{t-1}\right) \\
& + \lambda_{d,t} \Big[p_t y_t - w_t \ell_t - p_t^I \, i_t - R_{\tau,t-1} b_{t-1} + b_t \\
& \qquad - PAC_t - DAC_t - \mathrm{KAC}_t - d_t \Big] \\
& + \lambda_{\varphi,t} \left[\exp\left(e_{\varphi,t}\right) \varphi\left(p_t y_t - w_t \ell_t\right) - b_t \right] + m c_t^q \left[A_t \overline{q} \, k_{t-1}^{\alpha_k} \ell_t^{\alpha_\ell} - q_t \right] \\
& + \lambda_{p,t} \left[p_t^{-\eta} \mathcal{H}_t - y_t \right] + \lambda_{h,t} \left[\rho_h h_{t-1} + (1-\rho_h) c_t - h_t \right] \\
& + \lambda_{y,t} \left[\widetilde{G}\left(z_{n,t}^*\right)\left(q_t + n_{t-1}\right) - y_t \right] \\
& + \lambda_{n,t} \left[\widetilde{H}\left(z_{n,t}^*\right)\left(q_t + n_{t-1}\right) - n_t \right] \\
& + \lambda_{n,t}^+ \, n_t + \lambda_{z,t}\left(z_{n,t}^* - 1\right) + \lambda_{d,t} \, q_t^k \left[(1-\delta_k) k_{t-1} + i_t - k_t \right] \Big\}.
\end{aligned} \tag{J.41}$$

The inequality multipliers satisfy complementary slackness $\lambda_{\varphi,t} \geq 0$; $\lambda_{n,t}^+ \geq 0$, $n_t \geq 0$, $\lambda_{n,t}^+ n_t = 0$; and $\lambda_{z,t} \geq 0$, $z_{n,t}^* - 1 \geq 0$, $\lambda_{z,t}(z_{n,t}^* - 1) = 0$.

J.6 First-Order Conditions

The Dividend. The FOC w.r.t. $d_{s,t}$ gives the shadow value of liquidity,

$$\lambda_{d,s,t} = \frac{1}{1 + \kappa_d \left(d_{s,t} - \bar{d}_s \right)}. \tag{J.42}$$

The real marginal cost of a produced unit is

$$\tilde{p}_{s,t} \equiv \frac{mc^q_{s,t}}{\lambda_{d,s,t}}. \tag{J.43}$$

The Capital Adjustment-Cost Wedge.

$$\lambda_{K,t} = \kappa_k \left(\frac{k_{s,t}}{k_{s,t-1}} - 1 \right) + \mathbb{E}_t \left[M_{t,t+1} \frac{\lambda_{d,s,t+1}}{\lambda_{d,s,t}} \frac{\kappa_k}{2} \left[1 - \left(\frac{k_{s,t+1}}{k_{s,t}} \right)^2 \right] \right]. \tag{J.44}$$

The Inventory Layer. The FOC w.r.t. output $y_{s,t}$ gives

$$\lambda_{y,s,t} = \left(\lambda_{d,s,t} + \lambda_{\varphi,s,t} \exp\left(e_{\varphi,s,t} \right) \varphi \right) p_{s,t} - \lambda_{p,s,t}, \tag{J.45}$$

and the FOC w.r.t. q_t, combined with the FOC w.r.t. $z^*_{n,t}$, gives the marginal cost of a produced unit,

$$mc^q_{s,t} = \widetilde{\Phi}\left(z^*_{n,s,t} \right) \left(\left(\lambda_{d,s,t} + \lambda_{\varphi,s,t} \exp\left(e_{\varphi,s,t} \right) \varphi \right) p_{s,t} - \lambda_{p,s,t} \right), \tag{J.46}$$

equivalently $mc^q = \lambda_y \widetilde{\Phi}$ with $\widetilde{\Phi} = \widetilde{G} - (\widetilde{G}'/\widetilde{H}')\widetilde{H}$.

The FOC w.r.t. the cutoff $z^*_{n,s,t}$ gives

$$\lambda_{n,s,t} = -\lambda_{y,s,t} \frac{\widetilde{G}'(z^*_{n,s,t})}{\widetilde{H}'(z^*_{n,s,t})}.$$

The FOC w.r.t. the next-period inventory stock $n_{s,t}$ is then

$$-\lambda_{y,s,t} \frac{\widetilde{G}'\left(z^*_{n,s,t} \right)}{\widetilde{H}'\left(z^*_{n,s,t} \right)} = \lambda^+_{n,s,t} + \mathbb{E}_t\left[M_{t,t+1}\, mc^q_{s,t+1} \right]. \tag{J.47}$$

In the interior case $\lambda_n^+ = 0$, so the steady-state cutoff solves

$$\frac{\widetilde{G}'\left(z_{n,ss}^*\right)}{\widetilde{H}'\left(z_{n,ss}^*\right)} = \frac{-\beta\,\widetilde{G}\left(z_{n,ss}^*\right)}{1-\beta\,\widetilde{H}\left(z_{n,ss}^*\right)}. \tag{J.48}$$

Input Choices. The FOC w.r.t. $i_{s,t}$ sets the shadow value of installed capital equal to the purchase price:

$$q_t^k = p_t^I. \tag{J.49}$$

The labor FOC is

$$mc_{s,t}^q\,\alpha_\ell\,\frac{q_{s,t}}{\ell_{s,t}} = \left(\lambda_{d,s,t} + \lambda_{\varphi,s,t}\exp\left(e_{\varphi,s,t}\right)\varphi\right)\,w_t, \tag{J.50}$$

equivalently, dividing by $\lambda_{d,s,t} > 0$ and using (J.43),

$$\alpha_\ell\,\tilde{p}_{s,t}\,\frac{q_{s,t}}{\ell_{s,t}} = \left(1 + \frac{\lambda_{\varphi,s,t}\exp\left(e_{\varphi,s,t}\right)\varphi}{\lambda_{d,s,t}}\right)w_t.$$

Capital FOC is

$$p_t^I + \lambda_{K,t} = \mathbb{E}_t\left[M_{t,t+1}\frac{\lambda_{d,s,t+1}}{\lambda_{d,s,t}}\left(\alpha_k\,\tilde{p}_{s,t+1}\,\frac{q_{s,t+1}}{k_{s,t}} + (1-\delta_k)\,p_{t+1}^I\right)\right], \tag{J.51}$$

where (J.49) has replaced q^k by p^I at both dates.

Given $(\tilde{p}_t, \lambda_{\varphi,t}, k_{t-1})$, the labor condition and the technology (J.28) solve jointly in closed form:

$$\ell_{s,t} = \left[\frac{\alpha_\ell\,\tilde{p}_{s,t}\,A_t\overline{q}\,k_{s,t-1}^{\alpha_k}}{\left(1 + \frac{\lambda_{\varphi,s,t}\exp\left(e_{\varphi,s,t}\right)\varphi}{\lambda_{d,s,t}}\right)w_t}\right]^{\frac{1}{1-\alpha_\ell}}, \qquad q_{s,t} = A_t\overline{q}\,k_{s,t-1}^{\alpha_k}\ell_{s,t}^{\alpha_\ell}. \tag{J.52}$$

The Price Margin. The current Rotemberg price-adjustment wedge is

$$\mathcal{P}_{s,t}^{\text{now}} \equiv \kappa_p \left(\Pi_t \frac{p_{s,t}}{p_{s,t-1}} - \overline{\Pi} \right) \Pi_t \frac{p_{s,t}}{p_{s,t-1}} Y_t, \tag{J.53}$$

which is zero at the deterministic steady state. It is obtained from (J.32) by $\partial PAC_t/\partial p_t = \mathcal{P}_t^{\text{now}}/p_t$ and $\partial PAC_{t+1}/\partial p_t = -\mathcal{P}_{t+1}^{\text{now}}/p_t$.

The FOC w.r.t. $p_{s,t}$ is, after some algebra,

$$\begin{aligned} \lambda_{p,s,t} = & \frac{(\lambda_{d,s,t} + \lambda_{\varphi,s,t} \exp(e_{\varphi,s,t})\, \varphi)\, p_{s,t}}{\eta} - \lambda_{h,s,t}(1-\rho_h)\vartheta_{s,t} \\ & - \frac{\lambda_{d,s,t}}{\eta\, y_{s,t}} \left(\mathcal{P}_{s,t}^{\text{now}} - \mathbb{E}_t \left[M_{t,t+1} \frac{\lambda_{d,s,t+1}}{\lambda_{d,s,t}} \mathcal{P}_{s,t+1}^{\text{now}} \right] \right). \end{aligned} \tag{J.54}$$

Eliminating $\lambda_{p,s,t}$ between (J.46) and (J.54) and factoring out $\lambda_{d,s,t} p_{s,t}(\eta-1)/\eta$ gives the reduced pricing condition

$$\begin{aligned} mc_{s,t}^q = \lambda_{d,s,t}\, p_{s,t}\, \frac{\eta-1}{\eta}\, \widetilde{\Phi}\left(z_{n,s,t}^*\right) \Bigg[& 1 + \frac{\lambda_{\varphi,s,t} \exp(e_{\varphi,s,t})\, \varphi}{\lambda_{d,s,t}} + \frac{\lambda_{h,s,t}(1-\rho_h)\vartheta_{s,t}}{\lambda_{d,s,t} p_{s,t}(\eta-1)/\eta} \\ & + \frac{1}{(\eta-1) p_{s,t} y_{s,t}} \Bigg(\kappa_p \left(\Pi_t \frac{p_{s,t}}{p_{s,t-1}} - \overline{\Pi} \right) \Pi_t \frac{p_{s,t}}{p_{s,t-1}} Y_t \\ & - \mathbb{E}_t \left[M_{t,t+1} \frac{\lambda_{d,s,t+1}}{\lambda_{d,s,t}} \kappa_p \left(\Pi_{t+1} \frac{p_{s,t+1}}{p_{s,t}} - \overline{\Pi} \right) \Pi_{t+1} \frac{p_{s,t+1}}{p_{s,t}} Y_{t+1} \right] \Bigg) \Bigg]. \end{aligned} \tag{J.55}$$

At $\kappa_p = 0$, we would get

$$\lambda_{p,s,t} = (\lambda_{d,s,t} + \lambda_{\varphi,s,t} \exp(e_{\varphi,s,t})\, \varphi)\; p_{s,t}/\eta - \lambda_{h,s,t}(1-\rho_h)\, \vartheta_{s,t}. \tag{J.56}$$

Debt. Debt FOC is

$$\lambda_{\varphi,s,t} = \lambda_{d,s,t} - R_{\tau,t}\, \mathbb{E}_t[M_{t,t+1} \lambda_{d,s,t+1}]. \tag{J.57}$$

The tax-advantage of debt forces the borrowing constraint to bind in the steady state. In the quantitative analysis, I assume the constraint always binds, as is standard in the literature.

The Customer-Base Margin. The control h_t enters (J.41) at date t only through $-\lambda_{h,t}h_t$, so

$$\lambda_{h,t} = \mathbb{E}_t \left[M_{t,t+1} \frac{\partial \mathcal{L}_{t+1}}{\partial h_t} \right]. \tag{J.58}$$

At fixed controls and multipliers, h_{t-1} enters the date-t Lagrangian at three sites: the demand constraint, through $\mathcal{H}_t \supset \mathcal{H}^c_{s,t} h_{t-1}^{\theta(1-\eta)}$; the habit law $\lambda_{h,t}\left[\rho_h h_{t-1} + (1-\rho_h)c_t - h_t\right]$, both directly and through c_t; and the objective's flow v, both directly and through c_t. No other term of (J.41) is a function of h_{t-1}. With $\partial c_t/\partial h_{t-1} = \theta(1-\eta)c_t/h_{t-1}$, the first two sites give

$$\theta(1-\eta)\frac{c_{s,t}}{h_{s,t-1}}\left[\lambda_{p,t} + \lambda_{h,t}(1-\rho_h)\right] + \lambda_{h,t}\rho_h. \tag{J.59}$$

J.7 Customer-Specific Habits for Firms

Setup. Firms are indexed by f and households by $j \in [0,1]$. The relationship between firm f and household j is measured by the habit stock $h_{f,j,t}$, which follows

$$h_{f,j,t} = \rho_h \, h_{f,j,t-1} + (1-\rho_h)\, c_{f,j,t}, \qquad h_{f,t} \equiv \int_0^1 h_{f,j,t}\, dj. \tag{J.60}$$

Firm f charges a household-specific price $p_{f,j,t}$

$$c_{f,j,t} = p_{f,j,t}^{-\eta}\, \mathcal{H}_h \, h_{f,j,t-1}^{\theta(1-\eta)}, \qquad \eta > 1,\ \theta \leq 0. \tag{J.61}$$

Customer-Specific Habits. To give firm f a value for specific customers, I imitate deep habits from Ravn et al. (2006) and assume that the firm is operated by a manager who maximizes the present discounted value of dividends *plus* an additional preference over the flow of sales to each household j given by

$$v_{f,t} \;=\; \left(\mathcal{V}_{f,t} - \mathcal{V}_{f,t}|_{\theta_f=0}\right). \tag{J.62}$$

where

$$\mathcal{V}_{f,t} = \left[\int_0^1 \left(c_{f,j,t} \left(\frac{h_{f,j,t-1}}{\overline{h}_{j,t-1}} \right)^{-\theta_f} \right)^{\frac{\eta_f - 1}{\eta_f}} dj \right]^{\frac{\eta_f}{\eta_f - 1}}, \quad \theta_f \leq 0, \eta_f > 1 \tag{J.63}$$

$$\mathcal{V}_{f,t}|_{\theta_f = 0} = \left[\int_0^1 c_{f,j,t}^{\frac{\eta_f - 1}{\eta_f}} dj \right]^{\frac{\eta_f}{\eta_f - 1}}, \tag{J.64}$$

$$\overline{h}_{j,t-1} = \int_{f'} h_{f',j,t-1} \, \mathrm{d}f'. \tag{J.65}$$

The firm's non-pecuniary flow payoff $v_{f,t}$ is the difference between an aggregator with habits and that same aggregator with the habit parameter θ_f set to zero. I choose this functional form so that, when $\theta_f = 0$, the firm derives zero additional payoff from its customer relationships. Unlike the household side, the habit adjustment is relative to the average habit stock of household j across all firms, $\overline{h}_{j,t-1}$, which firm f takes as given since it is atomistic. The reason for $\overline{h}_{j,t-1}$ is that I want the functional form to mathematically resemble the effects of the household's deep-habits on the firm's pricing incentives. The household always evaluates the marginal utility of another purchase from a particular firm relative to the marginal utility from other firms. Without $\overline{h}_{j,t-1}$, the firm does not evaluate the "utility" of sales to one customer relative to the sales it makes to other customers. Aside from this mathematical requirement, it is plausible that a firm evaluates the importance of investment in specific customers based on what their competitors are doing. If all their competitors have been investing in sticky bases, then they do not want to deviate from that benchmark.

The firm's objective function becomes

$$\mathbb{E}_0 \sum_{t \geq 0} M_{0,t} \left[d_{f,t} \; + \; v_{f,t} \right]. \tag{J.66}$$

Aggregated Form. In my model, households are identical, so a symmetric equilibrium generically exists in which each firm has the same habit stock with respect to each household, $h_{f,j,t} = h_{f,t}$. Then (J.62) reduces to the functional form

$$v\left(c, h, \overline{h}\right) \equiv c\left[\left(\frac{h}{\overline{h}}\right)^{-\theta_f} - 1\right], \tag{J.67}$$

so that $v_{f,t} = v(c_{f,t}, h_{f,t-1}, \overline{h}_{f,t-1})$. The two key partial derivatives are

$$v_c\left(c, h, \overline{h}\right) = \left(\frac{h}{\overline{h}}\right)^{-\theta_f} - 1, \tag{J.68}$$

$$v_h\left(c, h, \overline{h}\right) = -\,\theta_f \,\frac{c}{h}\left(\frac{h}{\overline{h}}\right)^{-\theta_f}, \tag{J.69}$$

When firms are also symmetric, these derivatives simplify to

$$v_c(c, h, h) = 0, \quad v_h(c, h, h) = -\,\theta_f \,\frac{c}{h}, \tag{J.70}$$

hence the firm derives no additional value directly from another sale but does value a larger habit stock. The fact that $v_c = 0$ mirrors the fact that, with goods-specific habits, a firm does not value sales to households depending on a household's level of consumption, only because the household has a habit stock for that firm. This zero is precisely why I need the external benchmark.

It follows that the shadow value of the customer base becomes

$$\begin{aligned}\lambda_{h,s,t} = \mathbb{E}_t\Bigg[M_{t,t+1}\Bigg(&\theta(1-\eta)\,\frac{c_{s,t+1}}{h_{s,t}}\left(\lambda_{p,s,t+1} + \lambda_{h,s,t+1}(1-\rho_h)\right)\\ &+ \lambda_{h,s,t+1}\,\rho_h - \theta_{f,s}\,\frac{c_{s,t+1}}{h_{s,t}}\Bigg)\Bigg],\end{aligned} \tag{J.71}$$

J.8 Market Clearing and Equilibrium

For every t:

$$\text{(labor)} \quad \sum_s m_s \, \ell_{s,t} = \ell_t^W, \tag{J.72}$$

$$\text{(goods, per industry)} \quad y_{s,t} = c_{s,t} + y_{s,t}^I, \tag{J.73}$$

$$\text{(inventory, per industry)} \quad n_{s,t} = \widetilde{H}\left(z_{n,s,t}^*\right) \mathcal{Q}_{s,t}, \tag{J.74}$$

$$\text{(habit, per industry)} \quad h_{s,t} = \rho_h h_{s,t-1} + (1-\rho_h) c_{s,t}, \tag{J.75}$$

$$\text{(bonds)} \quad B_t = \sum_s m_s \, b_{s,t}, \tag{J.76}$$

$$\text{(capital goods)} \quad I_t = \sum_s m_s \, i_{s,t}. \tag{J.77}$$

Definition J.1. An equilibrium is a collection of sequences, indexed by $t \geq 0$ and by s where relevant:

(i) firm policies $\{p_{s,t}, y_{s,t}, q_{s,t}, c_{s,t}, z_{n,s,t}^*, n_{s,t}, b_{s,t}, h_{s,t}, \ell_{s,t}, i_{s,t}, k_{s,t}, d_{s,t}\}$ with the multipliers and wedges $\{\lambda_d, \lambda_p, \lambda_h, \lambda_\varphi, \lambda_n^+, q^k, \lambda_K\}_{s,t}$;

(ii) household policies $\{X_t, \ell_{\iota,t}, B_t\}$ and nominal wages $\{W_{\iota,t}\}$;

(iii) capital-goods policies $\{I_t, y_{s,t}^I\}$;

(iv) prices $\{p_{s,t}, p_{X_s,t}, p_{X,t}, w_t, \tilde{p}_{s,t}, p_t^I, R_{n,t}, \Pi_t, R_t, R_{\tau,t}\}$;

such that, at every t, households, firms, and capital goods producers are optimizing subject to their constraints, and all markets clear.

J.9 Calibration

I split the parameters into standard ingredients in DSGEs and non-standard ones.

Table J.1: Calibration

Parameter	Value	Source
β	0.96	standard
σ	2.00	standard
ν	0.50	standard
ℓ_{target}	1/3	standard target (set by labor disutility χ)
η_w	4.5	Galí (2015)
τ_w	$-1/(\eta_w - 1)$	eliminate s.s. wage markup
α_k	0.33	capital exponent
α_ℓ	0.67	labor exponent
$\overline{q}$	1.00	normalize
δ_k	0.10	standard
τ	0.35	Hennessy and Whited (2007)
$\bar{d}_s$	s.s. dividend of industry s	Jermann and Quadrini (2012)
φ	4.6	Dealscan, Drechsel (2023)
ρ_φ	0.9^4	standard
ρ_A	0.9^4	standard
θ_w	$(5/6)^4$	Grigsby et al. (2021)
κ_w	25.37	Calvo to Rotemberg via (J.13)
θ_p	$(1 - 0.095)^{12}$	Nakamura and Steinsson (2008)
κ_p	0.609	Calvo to Rotemberg via (J.12)
$\overline{\Pi}$	1.0	standard
$\overline{\Pi}_w$	1.0	standard
γ_R	0.75^4	standard
γ_Π	1.50	standard
γ_Y	0.00	standard

J.9.1 Standard Parameters

I only make a note of sticky wage and price calibration parameters; the remaining parameter values are standard in Table J.1. Grigsby et al. (2021) estimates that the average duration between wage changes for continuously employed workers in the U.S. is 6 quarters. A Calvo wage price non-reset parameter of 5/6 at the quarterly frequency would give you that average duration, which I then annualize by exponentiating to the fourth. Nakamura and Steinsson (2008) provides a range of estimates for the monthly frequency of price changes. A value of 9.5% at the monthly frequency is well within the middle of its reported range. The calibration converts this monthly reset probability to an annual non-reset probability.

J.9.2 Non-Standard Parameters

The model has two industries: industry 1 is the "high-irreversibility" industry, and industry 2 is the "low-irreversibility" industry. I set the sizes of these two industries to match the (relative) commodity weights mapped to the above and below median irreversibility industries in the back-of-the-envelope calculation, which are 0.34 and 0.66, respectively. I pick $\eta = 2$ because Soderbery (2015) estimates a median demand elasticity on imported goods to be 1.9. The estimation method used by Soderbery (2015) improves upon the method used by Broda and Weinstein (2006). Gilchrist et al. (2017) also uses a value of 2. I set the household deep habit parameter $\theta = -0.1$, which is much smaller than the -0.8 used by Gilchrist et al. (2017). I calibrate the persistence of the customer base to be 0.9^4 to generate a persistent base.

To simulate a difference-in-differences regression, I take the linearized partial-equilibrium firm block and study the impulse response to a credit supply shock, holding fixed every endogenous industry and aggregate variable at steady state (Appendix J.1). Lenzu et al. (accepted) also takes this approach. However, I only compute the point estimate and do not simulate a panel of firms to literally reproduce the regression. The endogenous aggregate

variables determined outside the firm block are X, I, p_X, Π_w, and R. To simulate a credit supply shock, I tighten the earnings-based constraint. To determine the appropriate size of the shock, I scale based on the equilibrium change in debt borrowed.

I calibrate the parameters $(\theta_{f,1}, \zeta, \varrho_n, \kappa_d, \kappa_k)$ to $(-0.350, 2.74, 0.190, 0.150, 15.87)$ so that I match the following moments: (i) diff-in-diff slope of 0.0010 in industry 1 to a credit supply shock, (ii) diff-in-diff slope of -0.6 in industry 2 to a credit supply shock, (iii) average inventory/sales ratio of 1.000 based on calculations for the U.S. economy from Wen (2011), (iv) average dividend absorption share of 0.200, and (v) average capital absorption share of 0.050. The averages are averages over the two industries weighted by their size. The calibrated model attains diff-in-diff slopes of 0.0009 in industry 1 and -0.608 in industry 2, an average inventory/sales ratio of 1.006, an average dividend absorption share of 0.180, and an average capital absorption share of 0.033.

The target slope of -0.6 in industry 2 comes from a 1 standard deviation credit supply shock (about 12pp) leading to a 10% cut in year-over-year prices by firms in low-irreversibility industries. It is assumed that this 12% shock corresponds to a 12% decline in a firm's own debt stock. These numbers imply a slope of -0.83 instead of -0.6, but I found that it was difficult to hit this number without requiring a much lower dividend absorption number. Adding labor adjustment costs would help so that firms in industry 2 would need to rely more on inventory cuts than production cuts, but I chose to avoid labor adjustment costs to avoid adding another set of bells and whistles.

The last two moments control how much firms adjust dividends and capital to deal with the reduction in debt after their earnings-based constraints tighten. A tighter constraint implies a reduction in actual borrowing, and a firm has to "pay" for those forgone dollars from debt by changing input expenditures and dividends. A dividend absorption share of 20% means that 20% of the borrowing reduction gets paid with lower dividends. The capital absorption share is defined similarly. I choose 20% as a reasonable target because Dwenger

et al. (2020) estimates that roughly 22% of lost debt financing gets replaced with dividend cuts using a sample of German firms during the GFC.[67] I target 5% for capital absorption because capital needs to be slowly adjust, or else firms would liquidate capital instead of inventories. My chosen value for the capital adjustment cost is close to the value of 20 used by Lenzu et al. (accepted) and 15 used by Kim (2021).

[67]The PPI and LFTTD samples do not have data on dividend issuance except for publicly listed firms in the sample. Also, I do not have direct data on the lost debt financing for individual firms. Despite Dwenger et al. (2020)'s sample being German firms instead of U.S. firms, the estimates are probably better than what I could obtain with my data.