Competition with Imperfectly Observed Offers

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Abstract

This paper studies competition in product quality when prices are fixed and quality is imperfectly observed. In an efficient equilibrium, the buyer purchases the product with the highest signal of quality that exceeds a threshold. Equilibrium is in pure strategies, so signals do not carry quality information on path, but the buyer's decision rule provides constrained efficient incentives. Competition creates trust: the threshold signal for acceptance and the probability that trade fails decrease with the number of sellers, even as quality remains constant. Implications for the make-or-buy decision are discussed.

1 Introduction

This paper studies markets where sellers compete by offering "deals" that buyers observe only imperfectly. A leading class of applications is competition in product quality with fixed prices, such as in procurement with a fixed budget, or markets where insured consumers have fixed deductibles (e.g., healthcare, home or auto repair). One can also consider a market with flexible prices, but where consumers choose based on an overall assessment of "quality-adjusted price."

In such markets, consumers' trust is critical. If consumers believe that a seller produces goods of unacceptably low quality in equilibrium, they will never buy from her, regardless of their signals of her products' quality (a fact familiar from Bagwell, 1995, assuming full support signals). If there is just one seller in the market, consumers cannot purchase with probability 1 in any equilibrium, as the seller would exploit their trust by shirking quality (as noted in Wolitzky, 2023). The current paper investigates how seller competition increases consumer trust, product quality, and market efficiency.

I focus on the simplest possible model of a market with imperfectly observed deals. There are n identical sellers and a population of identical buyers with unit demand. Each seller simultaneously chooses a quality q_i of goods to produce, with a per-unit production cost that is increasing and convex in quality but constant in quantity.[1] Each buyer observes a conditionally independent signal s_i of each good's quality and decides which good, if any, to purchase at a fixed price.

The buyer's signal distribution $F(s_i|q_i)$ satisfies standard assumptions: smoothness, full support, and the monotone likelihood ratio property (MLRP). I also make one less standard assumption, which is that $F(s_i|q_i)$ is convex in q_i. This ensures that each seller's quality-choice problem has a unique solution, so equilibria are in pure strategies.

I characterize the Pareto efficient equilibrium that maximizes consumer surplus.[2] It falls in one of three regimes.

First, if competition is fierce (n is large or quality signals are accurate), all sellers choose the same quality, which gives the buyer strictly positive surplus, and the buyer purchases

[1] Fixed costs are discussed in Section 7.

[2] Foundations for this equilibrium selection are discussed in Section 6.

the product with the highest signal. Although the signal carries no information in this (symmetric) equilibrium, the buyer's purchasing rule is pinned down by efficiency, as any other rule weakens sellers' incentives to provide high quality.

Next, if competitive pressure is moderate, all sellers choose the quality that makes the buyer indifferent to purchasing, and the buyer purchases the product with the highest signal if it exceeds a cutoff s^*, and otherwise does not purchase. This regime features the following *trust effect*, which is one of this paper's main insights: increasing the number of sellers keeps quality constant (within the moderate-trust regime), but leads to a reduction in s^*, and hence an increase in the probability of purchase and market efficiency. The logic is that, as competition increases, each seller has less to gain by shirking quality, because a lower quality product is less likely to yield the highest signal when there are more competitors. The buyer can therefore use a more permissive cutoff signal s^* without tempting sellers to shirk. Notably, the trust effect does not come from competition raising equilibrium quality: rather, it comes from competition reducing the gain from deviating to lower quality, under the efficient buyer purchasing rule.

Finally, if quality signals are too noisy, there is no trade in equilibrium, regardless of the number of sellers.

These results have implications for a range of economic questions. Most directly, the model provides a simple framework for studying quality competition in markets with fixed prices and imperfectly observed quality. Such markets are not unimportant (e.g., healthcare, procurement), and prior modeling attempts have not provided a full equilibrium analysis.[3] More abstractly, it also links the degree of market competition and the efficiency of bargaining. Such a linkage is often informally discussed in the literature on the theory of firm boundaries (the "make-or-buy" decision), following Williamson (1975) and Klein, Crawford, and Alchian (1978). However, there are very few formal models where market thickness reduces bargaining failures.

[3] Gravelle and Sivey (2010) assume consumers purchase from the firm that generates the highest signal, even if equilibrium quality choices are asymmetric. Shelegia (2012) notes that this consumer behavior is suboptimal and finds an equilibrium with symmetric firms. Maggi (1999) states that Bagwell's (1995) irrelevance result implies that firms provide low quality; however, this conclusion does not hold when multiple firms choose the same equilibrium quality, as then consumers have multiple optimal choices, in contrast to Bagwell's assumption of a unique best response.

Related literature. The baseline model in Wolitzky (2023) is essentially the one-seller version of the present paper. With a single seller, buyers cannot accept with probability 1, so only the moderate trust regime or the trivial, no-trade regime can arise. The current paper introduces seller competition and explores the trust effect. Ravid (2020) and Denti, Marinacci, and Rustichini (2022) consider related bilateral bargaining models, with rational inattention or information acquisition on the buyer side, rather than exogenous signals.

The closest paper, which was an inspiration for this work, is Cusumano, Fabbri, and Pieroth (2024, henceforth CFP). They study a model similar to present one, but with rational attention rather than exogenous signals. (So, the relation between CFP and Ravid (2020) is similar to that between the present paper and Wolitzky (2023).) CFP's main finding is an *attention effect*, wherein greater seller competition changes the buyer's optimal endogenous signal so that she purchases more often. The attention effect is similar to the trust effect I emphasize, albeit in my model the trust effect refers to a change in the buyer's equilibrium strategy for a fixed information structure, while in CFP the buyer's equilibrium strategy and information structure are intertwined, so competition affects them both. Other differences include that, in CFP, quality is exogenous and stochastic, and sellers choose prices; CFP's equilibrium refinement is akin to trembling-hand perfection rather than efficiency; and CFP assume an entropy-based information cost that generates multinomial logit demand. Still, perhaps the more significant contribution of the current paper is showing that a market-expansion effect similar to CFP's attention effect also arises in a completely standard, fully rational model.

In another inspiring contribution, Ellingsen and Miettinen (2026) study a competitive bargaining model with stochastically revocable offers. Their model is very different from mine, but it shares the feature that seller competition reduces disagreement risk by reducing the gains from seller rent-seeking. They develop a theory of firm boundaries based on this observation.

A literature in industrial organization studies competition with experience goods (Wolinsky, 1983; Riordan, 1986; Judd and Riordan, 1994). The closest paper is Shelegia (2011). In his model, each firm i chooses mean quality q_i and price p_i; a consumer's utility from firm i's product equals q_i plus a Gaussian error term; and consumers observe Gaussian signals of their utilities. Shelegia's model is thus richer than mine, but the resulting linear equilibrium is not directly comparable—for example, his stochastic quality model implies a direct efficiency gain from increasing the number of sellers, which I rule out to focus on the trust effect.

Finally, my result that purchasing the product with the highest acceptable signal is the equilibrium policy that maximizes incentives for quality relates to the result that a winner-take-all contest can maximize contestant effort (Krishna and Morgan, 1998; Drugov and Ryvkin, 2020). One difference is that all contestants pay effort costs, while I assume that only the selected seller pays the production cost; another difference is that my buyer lacks commitment, which affects the equilibrium cutoff signal.

2 Model

There are $n \geq 1$ identical sellers and a population of identical buyers—or, without loss, a single buyer. Price is administratively fixed at $p > 0$. Each seller i simultaneously chooses a quality $q_i \in Q = [0, 1]$ of goods to produce. Buyers have unit demand, and a buyer's utility from consuming a good of quality q is $v(q)$, which is strictly increasing and continuous, with $v(0) < p < v(1)$. The cost of supplying a good of quality q is $c(q)$, with $c(0) < p < c(1), c' > 0, c'' > 0$. Thus, if a buyer purchases from a seller with quality q, this generates a payoff of $v(q) - p$ for the buyer and $\pi(q) = p - c(q)$ for the seller. Assume that there exists q such that $c(q) < p < v(q)$, so there are gains from trade at the administered price

Define the competitive quality q^C by $q^C = c^{-1}(p)$ and the monopoly quality q^M by $q^M = v^{-1}(p)$. Note that $c\left(q^C\right) = p < v\left(q^C\right)$, $c\left(q^M\right) < p = v\left(q^M\right)$, and $0 < q^M < q^C < 1$.

Buyers observe noisy signals of quality, drawn from a closed interval $S = [\underline{s}, \bar{s}]$, with $\underline{s} \in \mathbb{R} \cup \{-\infty\}$, $\bar{s} \in \mathbb{R} \cup \{+\infty\}$, and $\underline{s} < \bar{s}$. A buyer's signal s_i of seller i's quality q_i is distributed $F(s_i|q_i)$, i.i.d. across sellers. I maintain the following assumption.

Assumption 1 The signal distributions $F(s_i|q_i)$ satisfy:

1. *Smoothness:* $F(\cdot|q_i)$ admits a continuously differentiable density $f(\cdot|q_i)$, for all q_i.
2. *Full support:* $\operatorname{supp} F(\cdot|q_i) = S$, for all q_i.

3. *Strict MLRP:* $f(s_i|q_i)/f(s_i|q_i')$ is strictly increasing in s_i, for all $q_i > q_i'$.

4. *Convexity in q:* $F(s_i|q_i)$ is convex in q_i, for all s_i.

This assumption is essentially the same as in Wolitzky (2023), except that paper only needed log-concavity of $1 - F(s_i|q_i)$ in q_i, rather than concavity (i.e., convexity of $F(s_i|q_i)$). With a single seller, log-concavity ensures that the seller's quality-choice problem has a unique solution; with multiple sellers, this requires concavity.

A strategy for a seller is a probability distribution over Q. A strategy for the buyer specifies, for each signal profile $s \in S^n$, a probability $\sigma_i(s)$ of purchasing from seller i, such that $\sum_i \sigma_i(s) \leq 1$. Since all signals are on path as F has full support, Nash equilibrium is a suitable solution concept.

Call an equilibrium *non-trivial* if the buyer purchases with positive probability. In a non-trivial equilibrium, call a seller *active* if she sells with positive probability. Since F has full support, there is always a trivial equilibrium where the buyer never purchases (Bagwell, 1995)—more generally, for any set of sellers, there is always an equilibrium where these sellers are inactive.

A first observation is that, in any non-trivial equilibrium, all active sellers use the same pure strategy.

Lemma 1 *In any non-trivial equilibrium:*

1. *Any active seller uses a pure strategy.*

2. *All active sellers choose the same quality q, which lies in $\left[q^M, q^C\right]$.*

3. *If the active quality is strictly greater than q^M, the buyer purchases with probability* 1.

Proof. 1. Suppose for contradiction that there is an equilibrium where a seller i is active but mixes. Then, by an implication of full support and MLRP (Milgrom, 1981, Proposition 1), $\mathbb{E}[v(q_i)|s_i]$ is strictly increasing in s_i. Therefore, for each profile of the other sellers' signals s_{-i}, the buyer purchases from seller i if and only if s_i exceeds the cutoff $s_i^*(s_{-i})$ where $\mathbb{E}[v(q_i)|s_i]$ reaches $\max\{\max_{j\neq i} \mathbb{E}[v(q_j)|s_j], p\}$. Let P_i be the equilibrium distribution of

$s_i^*(s_{-i})$. Then seller i's problem is

$$\max_{q_i} \int_{s_i^*} \pi(q_i)(1 - F(s_i^*|q_i))\, dP_i(s_i^*).$$

Observe that: (i) any optimal q_i is at most q^C, as otherwise i's expected profit is strictly negative (by full support); (ii) $\pi(q_i)$ is non-negative, strictly decreasing, and strictly concave on $[0, q^C]$ (as $c', c'' > 0$); and (iii) $1 - F(s_i^*|q_i)$ is strictly positive, strictly increasing, and concave on $[0, q^C]$ (as $F(s_i^*|q_i)$ is strictly less than 1 by full support, strictly decreasing in q_i by MLRP, and convex in q_i by assumption). Thus, on $[0, q^C]$, the product $\pi(q_i)(1 - F(s_i^*|q_i))$ is the product of a non-negative, strictly decreasing, and strictly concave function, and a strictly positive, strictly increasing, and concave function, and hence is strictly concave. The seller's objective is therefore a convex combination of strictly concave functions, and hence is strictly concave.[4] But then it must have a unique maximum, contradicting that seller i mixes.

2. If two active sellers chose different qualities, the buyer would never buy the lower one. As well, the active quality is at least q^M (or the buyer would rather not buy), and at most q^C (or an active seller would rather not sell).

3. If $q > q^M$, the buyer strictly prefers to buy. ■

By Lemma 1, any non-trivial equilibrium is of one of two types:

High Quality: The active quality q strictly exceeds q^M, and the buyer always purchases.

Low Quality: The active quality q equals q^M, and the buyer purchases with positive probability.

In a high quality equilibrium, the buyer's payoff is increasing is q, and the active sellers' payoff is decreasing in q. In a low quality equilibrium, the buyer's payoff is 0, and the active sellers' payoff is increasing in the probability of trade. Thus, the *buyer-optimal efficient equilibrium* is the highest quality equilibrium, if a high quality equilibrium exists, and otherwise is the equilibrium with the highest probability of trade. I focus on this equilibrium, providing a foundation for this selection in Section 6.

[4] If instead $1 - F(s_i|q_i)$ is only log-concave in q_i, then $\pi(q_i)(1 - F(s_i^*|q_i))$ is log-concave in q_i for each s_i^*, but its expectation over s_i^* need not be log-concave.

3 The Efficient Equilibrium and the Trust Effect

To characterize the buyer-optimal efficient equilibrium, I use the following terminology (ignoring ties).

Definition 1 *In a* best signal equilibrium*, the buyer always purchases from the seller with the highest signal s_i.*

In a best acceptable signal equilibrium*, there is a cutoff signal $s^* < \bar{s}$ such that the buyer purchases from the seller with the highest signal s_i that exceeds s^* and does not purchase if $\max_i s_i < s^*$.*

Note that, since signals have full support, all sellers are active in either a best signal equilibrium or a best acceptable signal equilibrium.

The main result of this paper is as follows.

Proposition 1 *Whenever a non-trivial equilibrium exists, the buyer-optimal efficient equilibrium is:*

1. *A best signal equilibrium, if a high quality equilibrium exists.*

2. *A best acceptable signal equilibrium, otherwise.*

   Moreover, in the high quality regime, quality strictly increases with the number of sellers; and, in the low quality regime, the probability of trade strictly increases with the number of sellers.

To sketch the argument, in a symmetric equilibrium with buyer strategy σ and active quality q, a seller i's expected payoff from choosing quality q_i is

$$\pi(q_i) \int_s \sigma_i(s) f(s_i|q_i) \prod_{j \neq i} f(s_j|q) \, ds.$$

The derivative with respect to q_i (i.e., the seller's return to quality), evaluated at $q_i = q$, is

$$\int_s \sigma_i(s) \left(\pi(q) \ell(s_i|q) - c'(q)\right) \prod_j f(s_j|q) \, ds,$$

where $\ell(s_i|q)$ is the *score* of the signal distribution, defined by

$$\ell(s_i|q) = \frac{f_q(s_i|q)}{f(s_i|q)}.$$

Similarly to standard moral hazard models (Mirrlees, 1975; Holmström, 1979), the score measures the sensitivity of the signal to quality, or, equivalently, the informativeness of the signal about marginal variation in quality. Since the score $\ell(s_i|q)$ is increasing in the signal s_i, by MLRP, the return to quality is maximized when the buyer purchases from the seller with the highest signal s_i that exceeds a cutoff s^*. In a low quality equilibrium, the cutoff is determined by the condition that q^M is the optimal quality for each seller; in a high quality equilibrium, the buyer purchases with probability 1, so the cutoff must equal $\underline{s}$.

The *trust effect* is that, in an efficient low quality equilibrium, the cutoff signal s^* decreases and the probability of trade increases with the number of sellers.[5] To see why this holds, define

$$L^n_{s^*}(q) = \frac{\int_{s>s^*} f_q(s|q) F(s|q)^{n-1} ds}{\int_{s>s^*} f(s|q) F(s|q)^{n-1} ds} = \mathbb{E}\left[\ell\left(s_{(n)}, q\right) | s_{(n)} > s^*\right], \tag{1}$$

where $s_{(n)}$ is the maximum of n i.i.d. draws from $F(\cdot|q)$. That is, $L^n_{s^*}(q)$ is the expectation of the score of the maximum of n i.i.d. draws from $F(\cdot|q)$, conditional on the maximum draw exceeding s^*. Note that $L^n_{s^*}(q)$ is strictly increasing in s^* and n, by MLRP. In a low quality, best acceptable signal equilibrium with n sellers, a seller's problem is

$$\max_{q_i} \pi(q_i) \int_{s>s^*} f(s_i|q_i) F\left(s|q^M\right)^{n-1} ds.$$

Taking log's, the FOC is

$$L^n_{s^*}\left(q^M\right) = \chi\left(q^M\right),$$

where

$$\chi(q) := \frac{c'(q)}{\pi(q)}$$

[5] A similar effect implies that, in an efficient high quality equilibrium, quality increases with the number of sellers.

denotes *marginal cost over profit*. Since $L^n_{s^*}(q)$ is strictly increasing in s^* and n, the value for s^* that satisfies the FOC is strictly decreasing in n. Intuitively, as n increases, the probability of generating the highest of n signals (conditional on $s_{(n)} > s^*$) becomes more sensitive to quality, which increases the return to quality; so, for the return to quality at q^M to remain constant as n increases, the buyer must choose a lower minimum acceptable signal, s^*. Finally, the probability of trade $1 - F(s^*(n)|q)^n$ strictly increases with n, for two reasons: the cutoff signal $s^*(n)$ decreases, and the probability $1 - F(s^*|q)^n$ that the highest signal exceeds any fixed cutoff s^* increases.

The trust effect has several implications. Notably, it predicts that the probability that trade fails to materialize in markets with imperfect quality observations decreases with the number of sellers. A consequence is that integration between the buyer and a seller is likely more profitable in thinner markets. This is reminiscent of Williamson (1975) and Klein, Crawford, and Alchian (1978), who view integration as an efficient response to bargaining frictions in thin markets, with the important difference that these authors (especially Williamson) emphasize ex post market thinness following relationship-specific investment, rather than simply a thin spot market. Relative to these authors, my model suggests that in the presence of moral hazard in product quality (or other forms of seller opportunism in designing "deals"), market thinness is a factor favoring integration, even in spot markets with no relationship-specific investments or potential hold-up.

4 Existence of Low and High Quality Equilibria

Recall the definition of $L_{s^*}(q)$ in (1) (suppressing the number of sellers n), and define

$$\begin{aligned} L_-(q) &= \lim_{s^* \to \underline{s}} L_{s^*}(q) = \mathbb{E}\left[\ell\left(s_{(n)}, q\right)\right], \\ L_+(q) &= \lim_{s^* \to \bar{s}} L_{s^*}(q) = \lim_{s \to \bar{s}} \ell(s, q), \end{aligned} \tag{2}$$

where the last equality is by L'Hôpital's rule. Note that, for each q, $\lim_{n\to\infty} L^n_-(q) = L_+(q)$.

The following conditions will determine the existence of low and high quality equilibria.

Tail Informativeness (TI):

$$L_{+}\left(q^{M}\right) > \chi\left(q^{M}\right).$$

Leader Informativeness (LI):

$$L_{-}\left(q^{M}\right) > \chi\left(q^{M}\right).$$

Monotone Leader Informativeness (MLI):

$$L_{-}(q) \text{ is weakly decreasing on } \left(q^{M}, q^{C}\right).$$

Tail Informativeness says that, at $q = q^M$, the highest possible score exceeds marginal cost over profit. Leader Informativeness says the same is true of the expectation of the highest of n scores (the "leading" seller score). By MLRP, LI implies TI. Conversely, the two conditions coincide in the $n \to \infty$ limit.

Monotone Leader Informativeness says that the sensitivity to quality of the probability of generating the highest of n signals is decreasing in quality over the relevant range, $\left(q^M, q^C\right)$. This is a decreasing returns condition, which will simplify several results. It holds, together with Assumption 1, in the following examples (as follows from straightforward calculations):

Linear mixture family: There exist distributions F_0, F_1 such that $F(s_i|q) = (1-q)\,F_0(s_i) + qF_1(s_i)$ for all s_i, q, and $f_1(s_i)/f_0(s_i)$ is increasing in s. An interpretation (under which u is also linear in q) is that the true quality of a product is binary, and q is the probability that a seller successfully produces a high quality product.

Power-CDF family: There exists a smooth, full-support CDF H and an increasing and concave function α such that $F(s_i|q) = H(s_i)^{\alpha(q)}$ for all s_i, q.

The following result characterizes equilibrium existence.

Proposition 2 *The following hold:*

1. *A high quality equilibrium exists if Leader Informativeness holds. Conversely, under Monotone Leader Informativeness, a high quality equilibrium exists only if Leader Informativeness holds*

2. *A low quality equilibrium exists if and only if Leader Informativeness fails and Tail Informativeness holds. Conversely, a non-trivial equilibrium exists only if Tail Informativeness holds.*

In particular, under Monotone Leader Informativeness, only trivial equilibria exist iff $\chi\left(q^M\right) \geq L_+\left(q^M\right)$*; a low quality equilibrium exists iff* $L_+\left(q^M\right) > \chi\left(q^M\right) \geq L_-\left(q^M\right)$*; and a high quality equilibrium exists iff* $L_-\left(q^M\right) > \chi\left(q^M\right)$*.*

Thus, under MLI, only trivial equilibria exist iff competitive pressure for supra-monopoly quality is low, in that marginal cost over profit, $\chi\left(q^M\right)$, exceeds tail informativeness, $L_+\left(q^M\right)$; a low quality equilibrium exists iff competitive pressure for supra-monopoly quality is moderate, in that $\chi\left(q^M\right)$ is less than tail informativeness, but exceeds leader informativeness, $L_-\left(q^M\right)$; and a high quality equilibrium exists iff competitive pressure for supra-monopoly quality is high, in that $\chi\left(q^M\right)$ is less than leader informativeness.

Proposition 2 holds because, by Proposition 1, a high quality equilibrium exists iff there exists $q \in \left(q^M, q^C\right)$ such that the FOC,

$$L_-\left(q\right) = \chi\left(q\right),$$

holds; a low quality equilibrium exists iff there exists $s^* \in \{-\infty\} \cup \mathbb{R}$ such that the FOC,

$$L_{s^*}\left(q^M\right) = \chi\left(q^M\right),$$

holds; and L_-, L_{s^*}, and χ are monotone (by MLRP and $c'' > 0$).

A corollary is that Tail Informativeness determines whether there is no trade for any number of sellers (if TI fails), or whether a high quality equilibrium exists with sufficiently many sellers (if TI holds).

Corollary 1 *If Tail Informativeness holds, then, for sufficiently large* n*, a high quality equilibrium exists. In this case, under Monotone Leader Informativeness, if* $L_+\left(q\right)$ *is bounded then, as* $n \to \infty$*, quality converges to the unique value* $q^* \in \left(q^M, q^C\right)$ *such that*

$$L_+\left(q^*\right) = \chi\left(q^*\right).$$

Conversely, if $L_+(q) = \infty$ for all $q \in \left(q^M, q^C\right)$, then quality converges to q^C as $n \to \infty$.

Proof. Follows from the FOC, $L_-(q) = \chi(q)$, because, for each q, $L_-(q) \to L_+(q)$ as $n \to \infty$. ■

5 Payoff Comparative Statics

This section treats the payoff comparative statics of the buyer-optimal efficient equilibrium.

First, consider comparative statics in the number of sellers, n. Let Π^n denote a seller's payoff in the buyer-optimal efficient equilibrium with n sellers. Since this equilibrium is symmetric, total seller surplus equals $n\Pi^n$.

Proposition 3 *Fix any $n < n'$. The following hold:*

1. *If $L_+\left(q^M\right) > \chi\left(q^M\right) \geq L_-^{n'}\left(q^M\right)$, then $n\Pi^n < n'\Pi^{n'}$.*
2. *If $L_-^n\left(q^M\right) > \chi\left(q^M\right)$, then $n\Pi^n > n'\Pi^{n'}$.*
3. *If MLI holds, then $\Pi^n \geq \Pi^{n'}$.*

Part 1 says that seller surplus increases with n in the low quality regime.[6] This holds because, by the trust effect, the probability of trade increases with n, while quality is fixed at q^M. Buyer surplus is independent of n in this regime, so total surplus increases with n.

Part 2 says that seller surplus decreases with n in the high quality regime. This holds because quality increases with n, while the probability of trade is fixed at 1. Buyer surplus increases with n in this regime, with ambiguous consequences for total surplus.

Part 3 says that, under MLI, each seller's payoff always decreases with n. In particular, in the low quality regime, the trust effect cannot overturn the direct effect of competition on each seller's sale probability: that is,

$$\frac{1 - F\left(s_n^* | q^M\right)^n}{n} \geq \frac{1 - F\left(s_{n+1}^* | q^M\right)^{n+1}}{n+1}, \qquad \text{for all } n,$$

where s_n^* is the cutoff signal in the buyer-optimal efficient equilibrium with n sellers.

[6] This result echoes CFP's Theorem 2 and Proposition 4.(iv).

Comparative statics in the buyer's signal precision can also be considered. For any number of sellers $n \geq 2$, if quality is perfectly observed then $q = q^C$, so all surplus accrues to the buyer. With imperfect observability, seller surplus is positive in any non-trivial equilibrium. So, sellers benefit from imperfect information, so long as signal precision is not so low as to entirely preclude trade. This contrasts with the monopoly seller case, where the seller obtains the first-best surplus with perfect information and becomes worse off as buyer signal precision decreases (Wolitzky, 2023). Similar effects arise in various models of competition with imperfect buyer information, where a monopoly seller is harmed by imperfect information, but competing sellers benefit because imperfect information softens competition (e.g., Moscarini and Ottaviani, 2001; Johnson and Myatt, 2006; Armstrong and Zhou, 2022).

6 Payoff-Relevant Signals and Equilibrium Selection

In the model analyzed so far, a buyer has no hard information regarding her value from consuming each product. It is probably more realistic to suppose that a buyer's signal reveals at least a small amount of stochastic, payoff-relevant product information. In this section, I model this by assuming that the buyer's utility from purchasing from seller i equals $\beta v(q_i) + (1-\beta) s_i - p$, where $\beta \in (0,1]$ is a known parameter (and $\beta = 1$ is the baseline model). For instance, if q_i is the average or intended quality of seller i's goods, which determines the bulk of the buyer's long-run consumption utility, s_i may reflect the buyer's enjoyment of a trial or sample of the good, which determines her short-run consumption utility immediately after purchasing. I also assume that $\bar{s} = \infty$, so that, for any $\beta < 1$, the buyer purchases from each seller with positive probability. Besides realism, this extension makes the equilibrium strict for the buyer. More importantly, it uniquely selects the buyer-optimal efficient equilibrium in the baseline model as $\beta \to 1$, under log-concavity of $F(s_i|q)$ and a strengthening of MLI.

First, while the baseline model always has asymmetric equilibria where some sellers are inactive and ignored by the buyer, all equilibria are symmetric when $\beta < 1$, under log-concavity.

Proposition 4 *If $\bar{s} = \infty$, $\beta < 1$, and $F(s_i|q)$ is log-concave in s_i for every q, then every equilibrium is symmetric.*

The logic is that, under log-concavity, a seller with lower anticipated quality has higher marginal return to quality, precluding equilibrium asymmetry.

Next, there is a unique symmetric equilibrium under the following strengthening of MLI.

Monotone Cutoff Informativeness (MCI):

$$L_{s^*}(q) \text{ is weakly decreasing on } \left(0, q^C\right), \text{ for all } s^* \in \mathbb{R}.$$

MCI strengthens MLI by requiring that $L_{s^*}(q)$ is decreasing for any cutoff s^*, not just $s^* = -\infty$, and is decreasing on $\left(0, q^C\right)$, not just $\left(q^M, q^C\right)$. It is satisfied in the above examples where MLI holds (linear mixture family; power-CDF family).

Proposition 5 *Under $\bar{s} = \infty$ and Monotone Cutoff Informativeness, for any $\beta < 1$, there is a unique symmetric equilibrium. Moreover, as $\beta \to 1$, this equilibrium converges to the buyer-optimal efficient equilibrium of the baseline model.*

In the baseline model where $\beta = 1$, under MCI, the buyer-optimal efficient equilibrium is the unique symmetric equilibrium where the buyer uses a cutoff strategy (i.e., it is the unique best signal or best acceptable signal equilibrium). The baseline model can also have low quality equilibria where the buyer uses a non-cutoff strategy that nonetheless rationalizes quality choice q^M. However, such equilibria are never limits of equilibria in the $\beta < 1$ game, where the buyer uses a cutoff strategy in any symmetric equilibrium.

Combining Propositions 4 and 5, we have:

Corollary 2 *Under $\bar{s} = \infty$, log-concavity, and MCI, for any $\beta < 1$, there is a unique equilibrium, which converges to the buyer-optimal efficient equilibrium as $\beta \to 1$.*

7 Discussion

I mention a couple possible extensions.

The model has only variable costs of quality provision, not fixed costs. Fixed costs would be a countervailing force to the trust effect I highlight. The trust effect arises because, with many sellers, each one must provide higher quality to generate the highest signal. However, with fixed costs, the amortized cost of providing high quality increases with the number of competitors. In a symmetric equilibrium with strictly increasing fixed costs, quality must converge to 0 as the number of active sellers goes to ∞. It can also be inefficient for all sellers to be active, because all but one investment in quality will be wasted ex post, whereas in my model all sellers are active in the buyer-optimal efficient equilibrium. For a related model with both fixed and variable costs, see Shelegia (2011).

Another natural extension would make quality itself stochastic. The payoff-relevant signal model of Section 6 is a simple version of this. The model in Shelegia (2011) is another version, where both realized quality and the buyer's signal of realized quality are Gaussian.

One could also introduce heterogeneous buyers. With a continuum of buyer types, only a measure 0 set of buyers are indifferent to purchasing in a pure equilibrium. The trust effect then takes the form of greater competition simultaneously increasing both quality and the probability of trade, in contrast to the current model, where only quality increases in the high quality regime, and only the probability of trade increases in the low quality regime.

Finally, assuming an administratively fixed price makes the model one where sellers offer imperfectly observed, one-dimensional "deals." If instead prices are flexible and perfectly observed, while quality remains unobserved, the model becomes a signaling-like game, where buyer beliefs about quality following off-path prices are indeterminate. Despite this complication, that model is also worth analyzing.

A Appendix

A.1 Proof of Proposition 1

The proposition asserts that, if a high quality equilibrium exists, quality is maximized in a high quality, best signal equilibrium; and, if only low-quality and trivial equilibria exist, the probability of trade is maximized in a low quality, best acceptable signal equilibrium.

First, suppose that there is no high quality equilibrium, and consider the problem of maximizing the probability of trade over all low quality equilibria. Let I^* be the set of active sellers. The problem is

$$\max_{\sigma(s)} \int_s \sum_{i\in I^*} \sigma_i\left(s\right) \prod_{j\in I^*} f\left(s_j|q^M\right) ds \qquad \text{s.t.}$$

$$q^M \in \operatorname*{argmax}_{q_i} \left(p - c\left(q_i\right)\right) \int_s \sigma_i\left(s\right) f\left(s_i|q_i\right) \prod_{j\neq i\in I^*} f\left(s_j|q^M\right) ds \qquad \forall i \in I^*,$$

$$\sum_{i\in I^*} \sigma_i\left(s\right) \leq 1 \qquad \forall s.$$

A relaxed problem replaces seller optimality (the first constraint) with the FOC

$$\int_s \sigma_i\left(s\right)\left(\ell\left(s_i|q^M\right) - \chi\left(q^M\right)\right) \prod_{j\in I^*} f\left(s_j|q^M\right) ds = 0 \qquad \forall i \in I^*.$$

I show that the solution to the relaxed problem is a best acceptable signal equilibrium (and hence satisfies seller optimality).

It is without loss to take σ symmetric among active sellers. Otherwise, for any permutation $\rho \in \mathcal{P}\left(I^*\right)$, let $\rho\left(s\right) = \left(s_{\rho(1)}, \ldots, s_{\rho(n)}\right)$, and let

$$\hat{\sigma}_i\left(s\right) = \frac{1}{|I^*|!} \sum_{\rho\in\mathcal{P}(I^*)} \sigma_{\rho^{-1}(i)}\left(\rho\left(s\right)\right),$$

the average over all permutations of the active sellers' signals of the sales probability of an active seller with signal s_i. Then $\hat{\sigma}$ is symmetric among active sellers; it gives the same acceptance probability as σ by construction; and it satisfies the FOC for each seller i, because

$$\begin{aligned} &\int_s \hat{\sigma}_i(s)\left(\ell\left(s_i|q^M\right)-\chi\left(q^M\right)\right)\prod_{j\in I^*} f\left(s_j|q^M\right)ds \\ =\ & \frac{1}{|I^*|}\sum_{i\in I^*}\int_s \sigma_i(s)\left(\ell\left(s_i|q^M\right)-\chi\left(q^M\right)\right)\prod_{j\in I^*} f\left(s_j|q^M\right)ds = 0, \end{aligned}$$

where the first equation is by symmetry, and the second is because the FOC holds at σ for all $i \in I^*$.

Let λ_i be the multiplier on the i^{th} FOC, and form the Lagrangian

$$\min_{(\lambda_i)_{i\in I^*}} \max_{\sigma(s):\sum_{i\in I^*}\sigma_i(s)\le 1} \int_s \sum_{i\in I^*} \sigma_i(s)\left(1+\lambda_i\left(\ell\left(s_i|q^M\right)-\chi\left(q^M\right)\right)\right)\prod_{j\in I^*} f\left(s_j|q^M\right)ds.$$

Note that $\lambda_i \ge 0$ for all i, as if $\lambda_i < 0$ for some i then $\sigma_i(s) = 1$ for all s gives a value of $1 - \lambda_i \chi\left(q^M\right) > 1$, which is greater than the maximum value of 1 when $\lambda_i = 0$ for all i, so λ cannot be optimal. Since there is a solution where σ is symmetric, there must also be a solution where λ is symmetric: $\lambda_i = \lambda$ for all i. Imposing this and maximizing the objective pointwise (ignoring tied signals), the solution is

$$\sigma_i(s) = \mathbf{1}\left\{\lambda\left(\ell\left(s_i|q^M\right)-\chi\left(q^M\right)\right) \ge \max\left\{-1, \max_{j\ne i\in I^*}\lambda\left(\ell\left(s_j|q^M\right)-\chi\left(q^M\right)\right)\right\}\right\}.$$

Since $\lambda \ge 0$ and $\ell\left(s_i|q^M\right)$ is increasing in s_i, this equals

$$\sigma_i(s) = \mathbf{1}\left\{\ell\left(s_i|q^M\right) \ge \max\left\{\chi\left(q^M\right)-\frac{1}{\lambda}, \max_{j\ne i\in I^*}\ell\left(s_j|q^M\right)\right\}\right\},$$

with convention $1/0 = \infty$. Letting s^* satisfy $\ell\left(s^*|q^M\right) = \chi\left(q^M\right) - 1/\lambda$ if such $s^* \in S$ exists, and $s^* = -\infty$ otherwise, this equals

$$\sigma_i(s) = \mathbf{1}\left\{s_i \ge \max\left\{s^*, \max_{j\ne i\in I^*} s_j\right\}\right\}.$$

Hence, the probability of trade is maximized when the buyer purchases from the active seller with the highest acceptable signal. With this buyer strategy, an active seller's problem is strictly concave as established in the proof of Lemma 1, so the FOC implies seller optimality. Finally, the argument in the text shows that the probability of trade is maximized when all sellers are active.

Next, suppose that a high quality equilibrium exists, and consider the problem of maximizing the active quality. The problem is

$$\max_{q,\sigma(s)} q \qquad \text{s.t.}$$

$$\int_s \sigma_i(s)\left(\ell(s_i|q)-\chi(q)\right)\prod_{j\in I^*} f(s_j|q)\,ds = 0 \qquad \forall i\in I^*,$$

$$\sum_i \sigma_i(s) = 1 \qquad \forall s,$$

where the first constraint is a seller's (necessary and sufficient) FOC, and the second constraint holds because the buyer always purchases in a high quality equilibrium.

Let λ_i be the multiplier on the i^{th} FOC, and form the Lagrangian

$$\min_{(\lambda_i)_{i\in I^*}} \max_{q,\sigma(s):\sum_i \sigma_i(s)=1\ \forall s} q + \int_s \sum_{i\in I^*} \sigma_i(s)\left[1+\lambda_i\left(\ell(s_i|q)-\chi(q)\right)\right]\prod_{j\in I^*} f(s_j|q)\,ds.$$

As in the low quality case, there is a solution where $\lambda_i = \lambda \geq 0\ \forall i$. Imposing this and maximizing the objective pointwise subject to the constraint $\sum_i \sigma_i(s) = 1\ \forall s$, and using MLRP as in the low quality case, the solution is

$$\sigma_i(s) = \mathbf{1}\left\{s_i \geq \max_{j\neq i\in I^*} s_j\right\},$$

with quality given by the largest solution to the FOC

$$L_-^n(q) = \chi(q),$$

where $L_-^n(q)$ is defined in (2). Finally, quality is maximized when all sellers are active, because $L_-^n(q)$ is increasing in n and $\chi(q)$ is increasing in q, so the largest solution to the FOC is increasing in n.

A.2 Proof of Proposition 2

First, by Proposition 1, a high quality equilibrium exists iff a best signal equilibrium exists. Since a seller's problem is concave in a symmetric equilibrium, this holds iff there exists $q \in \left(q^M, q^C\right)$ such that

$$L_{-}\left(q\right) = \chi\left(q\right).$$

Suppose that LI holds. If $q = q^M$, the LHS strictly exceeds the RHS, by LI. As $q \to q^C$, the RHS converges to ∞, as $\pi\left(q\right) \to 0$, while the LHS is bounded. Both sides are continuous, so they intersect by the intermediate value theorem. Conversely, if LI fails, then at $q = q^M$, the RHS exceeds the LHS; so, under MLI, the RHS strictly exceeds the LHS for all $q \in \left(q^M, q^C\right)$ (as the LHS is decreasing under MLI, and the RHS is increasing by $c'' > 0$).

Second, by Proposition 1, a low quality equilibrium exists iff a best acceptable signal equilibrium exists. Since a seller's problem is concave in a symmetric equilibrium, this holds iff there exists $s^* \in S \cup \{-\infty\}$ such that

$$L_{s^*}\left(q^M\right) = \chi\left(q^M\right).$$

If LI fails, the RHS weakly exceeds the LHS at $s^* = \underline{s}$; if TI holds, the LHS strictly exceeds the RHS at $s^* = \bar{s}$. By continuity, an intersection in $S \cup \{-\infty\}$ exists. Conversely, if TI fails, then the RHS exceeds the LHS at $s^* = \bar{s}$; so, the RHS strictly exceeds the LHS for all $s^* < \bar{s}$ (as the LHS is strictly increasing in s^* by MLRP).

A.3 Proof of Proposition 3

Parts 1 and 2 are immediate from Propositions 1 and 2.

For part 3, recall that quality in the (buyer-optimal) efficient equilibrium is non-decreasing in n, and hence $\pi\left(q\right)$ is non-increasing, so it suffices to establish

$$\frac{1 - F\left(s_n^*|q_n^*\right)^n}{n} \geq \frac{1 - F\left(s_{n+1}^*|q_{n+1}^*\right)^{n+1}}{n+1} \qquad \text{for all } n, \tag{3}$$

which shows that each seller's probability of sale is also non-increasing. This is obvious if

$q_n^* > q^M$, as then $F(s_n^*|q_n^*) = F\left(s_{n+1}^*|q_{n+1}^*\right) = 1$. So, assume that $q_n^* = q^M$.

First, suppose that $q_{n+1}^* = q^M$. Transform signals to quantiles by letting $r = F\left(s|q^M\right)$ and $r_n = F\left(s_n^*|q^M\right)$, so (3) becomes

$$\frac{1-r_n^n}{n} \geq \frac{1-r_{n+1}^{n+1}}{n+1}. \tag{4}$$

To establish this inequality, let $\bar{r}$ satisfy

$$\frac{1-\bar{r}^n}{n} = \frac{1-r_{n+1}^{n+1}}{n+1}, \qquad \text{or} \qquad \bar{r}^n = \frac{1+nr_{n+1}^{n+1}}{n+1}.$$

Let

$$K^n(r) = \frac{\int_r^1 r^{n-1}\ell\left(F^{-1}(r)\right)dr}{\int_r^1 r^{n-1}dr},$$

where $F^{-1}(r)$ is the signal s such that $F\left(s|q^M\right) = r$. I show below that $K^n(\bar{r}) \geq K^{n+1}(r_{n+1})$. This completes the proof, because the equilibrium FOC's are

$$K^n(r_n) = \chi\left(q^M\right) = K^{n+1}(r_{n+1}),$$

so, since K^n is increasing by MLRP, it follows that $r_n \leq \bar{r}$, and hence (4) holds.

Now suppose that $q_{n+1}^* > q^M$. By MLI, this implies that $L_-^{n+1}\left(q^M\right) > \chi\left(q^M\right)$, or equivalently $K^{n+1}(0) > \chi\left(q^M\right)$. As above, let $r_n = F\left(s_n^*|q^M\right)$, let $r_{n+1} = 0$, and let $\bar{r}^n = \left(1+nr_{n+1}^{n+1}\right)/(n+1) = 1/(n+1)$. By Lemma 2, $K^n(\bar{r}) \geq K^{n+1}(0)$, and hence

$$K^n(\bar{r}) \geq K^{n+1}(0) > \chi\left(q^M\right) = K^n(r_n).$$

This implies that $r_n < \bar{r}$, and hence (4) holds, completing the proof.

Lemma 2 $K^n(\bar{r}) \geq K^{n+1}(r_{n+1})$.

Proof. Note that $K^n(\bar{r})$ is the expectation of $\ell\left(F^{-1}(r)\right)$, when r has CDF

$$\frac{\int_{\bar{r}}^r x^{n-1}dx}{\int_{\bar{r}}^1 x^{n-1}dx} = \frac{r^n-\bar{r}^n}{1-\bar{r}^n},$$

with quantile function

$$Q^n(t) = (\bar{r}^n + t(1-\bar{r}^n))^{1/n}.$$

Similarly, $K^{n+1}(r_{n+1})$ is the expectation of $\ell(F^{-1}(r))$, when r has CDF

$$\frac{\int_{r_{n+1}}^{r} x^n dx}{\int_{r_{n+1}}^{1} x^n dx} = \frac{r^{n+1} - r_{n+1}^{n+1}}{1 - r_{n+1}^{n+1}},$$

with quantile function

$$Q^{n+1}(t) = \left(r_{n+1}^{n+1} + t\left(1 - r_{n+1}^{n+1}\right)\right)^{1/(n+1)}.$$

Let $y = r_{n+1}^{n+1} + t\left(1 - r_{n+1}^{n+1}\right)$, so by definition of $\bar{r}$,

$$(Q^n(t))^n = \bar{r}^n + t(1-\bar{r}^n) = \frac{1+ny}{n+1}, \qquad \text{and} \qquad \left(Q^{n+1}(t)\right)^{n+1} = y.$$

Hence,

$$Q^n(t) \geq Q^{n+1}(t) \iff \frac{1+ny}{n+1} \geq y^{n/(n+1)},$$

which holds by the weighted AM-GM inequality. Thus, the distribution underlying $K^n(\bar{r})$ first-order stochastically dominates the distribution underlying $K^{n+1}(r_{n+1})$. Since $\ell(F^{-1}(r))$ is increasing by MLRP, it follows that $K^n(\bar{r}) \geq K^{n+1}(r_{n+1})$. ∎

A.4 Proof of Proposition 4

A similar argument as in Lemma 1 implies that every equilibrium is pure. So, consider a pure equilibrium with qualities $(q_1, \ldots, q_n)$. The buyer's perceived payoff from purchasing from seller i after signal s_i is $\beta v(q_i) + (1-\beta)s_i - p$. So, he purchases from seller i iff

$$i \in \text{argmax}_j\, \beta v(q_j) + (1-\beta)s_j, \qquad \text{and} \qquad s_i \geq s_i^* := \frac{p - \beta v(q_i)}{1-\beta}.$$

Hence, if the signal of i's quality equals s_i, i sells with probability

$$w_i\left(s_i\right) := \mathbf{1}\left\{s_i \geq s_i^*\right\} \prod_{k \neq i} F\left(s_i + \frac{\beta}{1-\beta}\left(v\left(q_i\right) - v\left(q_k\right)\right) | q_k\right).$$

Thus, if i chooses quality q, she sells with total probability

$$D_i\left(q\right) = \int w_i\left(s\right) dF\left(s|q\right),$$

and her expected payoff is $\left(p - c\left(q\right)\right) D_i\left(q\right)$.

Suppose for contradiction that there are two sellers i, j with $q_i > q_j$. Let

$$\Delta = \frac{\beta}{1-\beta}\left(v\left(q_i\right) - v\left(q_j\right)\right) > 0.$$

Note that

$$s_j^* = s_i^* + \Delta,$$

and, for any $s \geq s_j^*$,

$$\frac{w_j\left(s\right)}{w_i\left(s\right)} = \frac{F\left(s-\Delta|q_i\right)}{F\left(s+\Delta|q_j\right)} \prod_{k \neq i,j} \frac{F\left(s + \frac{\beta}{1-\beta}\left(v\left(q_j\right) - v\left(q_k\right)\right) | q_k\right)}{F\left(s + \frac{\beta}{1-\beta}\left(v\left(q_i\right) - v\left(q_k\right)\right) | q_k\right)}.$$

This ratio is strictly increasing in s. Indeed, each factor in the product for $k \neq i, j$ has the form

$$\frac{F\left(t|q_k\right)}{F\left(t+\Delta|q_k\right)},$$

which is increasing in t by log-concavity. In addition, the first factor,

$$\frac{F\left(s-\Delta|q_i\right)}{F\left(s+\Delta|q_j\right)} = \frac{F\left(s-\Delta|q_i\right)}{F\left(s-\Delta|q_j\right)} \frac{F\left(s-\Delta|q_j\right)}{F\left(s+\Delta|q_j\right)},$$

is strictly increasing in s, as the first term is strictly increasing by MLRP (as $q_i > q_j$) and the second term is increasing by log-concavity.

Therefore,

$$\frac{D_i(q_i)}{D_i(q_j)} = \frac{\int w_i(s)\, dF(s|q_i)}{\int w_i(s)\, dF(s|q_j)} < \frac{\int w_j(s)\, dF(s|q_i)}{\int w_j(s)\, dF(s|q_j)} = \frac{D_j(q_i)}{D_j(q_j)},$$

where the inequality is equivalent to

$$\begin{aligned}
& \left(\int \left(\frac{w_j(s)}{w_i(s)}\right)\left(\frac{f(s|q_i)}{f(s|q_j)}\right) w_i(s)\, dF(s|q_j)\right)\left(\int w_i(s)\, dF(s|q_j)\right) \\
& -\left(\int \left(\frac{w_j(s)}{w_i(s)}\right) w_i(s)\, f(s|q_j)\, ds\right)\left(\int \left(\frac{f(s|q_i)}{f(s|q_j)}\right) w_i(s)\, f(s|q_j)\, ds\right) \\
= & \left(\int w_i(s)\, dF(s|q_j)\right)^2 \mathrm{Cov}_\mu\left(\frac{w_j(s)}{w_i(s)}, \frac{f(s|q_i)}{f(s|q_j)}\right) > 0,
\end{aligned}$$

where μ is the measure proportional to $w_i(s)\, dF(s|q_j)$, which holds because $w_j(s)/w_i(s)$ and $f(s|q_i)/f(s|q_j)$ are both strictly increasing.

Finally, since each seller prefers her own quality,

$$\pi(q_i)\, D_i(q_i) \geq \pi(q_j)\, D_i(q_j) \qquad \text{and} \qquad \pi(q_j)\, D_j(q_j) \geq \pi(q_i)\, D_j(q_i),$$

which implies

$$\frac{D_i(q_i)}{D_i(q_j)} \geq \frac{\pi(q_j)}{\pi(q_i)} \geq \frac{D_j(q_i)}{D_j(q_j)},$$

a contradiction.

A.5 Proof of Proposition 5

Let

$$s^*_\beta(q) = \frac{p - \beta v(q)}{1-\beta},$$

and let

$$L_{s^*_\beta(q)}(q) = \frac{\int_{s^*_\beta(q)}^\infty f_q(s|q)\, F(s|q)^{n-1}\, ds}{\int_{s^*_\beta(q)}^\infty f(s|q)\, F(s|q)^{n-1}\, ds} = \mathbb{E}\left[\ell\left(S_{(n)}, q\right) | S_{(n)} > s^*_\beta(q)\right].$$

By MCI, $L_{s^*_\beta}(q)$ is weakly decreasing on $\left(0, q^C\right)$ for any fixed cutoff s^*_β. Since $s^*_\beta(q)$ is decreasing, MLRP implies that $L_{s^*_\beta(q)}(q)$ is also weakly decreasing on $\left(0, q^C\right)$. So, since $c'' > 0$,

$$\frac{\partial}{\partial q'} \log \Pi_i \left(q'|q\right)\bigg|_{q'=q} = L_{s^*_\beta(q)}(q) - \chi(q)$$

is strictly decreasing in q on $\left(0, q^C\right)$. Since $\frac{\partial}{\partial q'} \log \Pi_i \left(q'|q\right)\Big|_{q'=q} = 0$ is the FOC for a symmetric equilibrium with quality q, there is a unique symmetric equilibrium quality $q^*(\beta)$ (possibly $q^*(\beta) = 0$). Moreover, since the same FOC applies when $\beta = 1$ and the buyer uses a cutoff strategy, there is a unique such equilibrium, which is the buyer-optimal efficient equilibrium. Letting the cutoff $s^*_\beta(q)$ take values in the compact set $[-\infty, +\infty]$, upper hemicontinuity of the Nash correspondence implies that any limit of equilibria $\left(q^*(\beta), s^*_\beta(q^*(\beta))\right)$ as $\beta \uparrow 1$ must be an equilibrium when $\beta = 1$. Since the buyer uses a cutoff strategy in every equilibrium when $\beta < 1$, the same holds at any limit point. Finally, since the $\beta = 1$ game has a unique symmetric cutoff equilibrium, this must be the unique limit.

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