

Coordination and Incomplete Information  
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Presidential Address

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## Topic 1: Coordination

Many economic and social environments are "coordination problems" with "strategic complementarities"...

- ▶ when a fixed exchange rate is under attack, speculators have a greater incentive to short the currency if they think other speculators will sell...
- ▶ when a bank has taken a loss, depositors have a greater incentive to withdraw their deposits if they think other investors will withdraw their deposits...
- ▶ when an economy is in recession, firms have an incentive to postpone investment if they think others will postpone investment....
- ▶ and so on....

## Coordination and Multiple Equilibria

When these strategic situations are modelled as a game such as....

	Invest	Not Invest
Invest	2, 2	-1, 0
Not Invest	0, -1	0, 0

these strategic complementarities give rise to multiple equilibria...

- ▶ Two strict Nash equilibria: (Invest, Invest) and (Not Invest, Not Invest)
- ▶ One generic response of game theorists:
  - ▶ look for an all purpose theory of equilibrium selection...

# Coordination and Multiple Equilibria

- ▶ More nuanced response:
  - ▶ The multiplicity is an artifact of a convenient and pervasive - but counterfactual - modelling choice: common knowledge of payoffs
- ▶ What happens if you relax common knowledge of payoffs assumptions?
- ▶ An intuition: "strategic uncertainty" favors "invest, invest" equilibrium in the example, since a player who attaches a 50% chance to the other player investing will choose to invest
  - ▶ Game theory terminology: "Invest, invest" is the "risk dominant" equilibrium
- ▶ This is consistent with experimental evidence
- ▶ But how should we think about relaxing common knowledge assumptions? Equivalently, how should we think about taking incomplete information seriously?

## Topic 2: Taking Incomplete Information Seriously

How should we think about taking incomplete information seriously in applied economic modelling? This talk:

1. A historical perspective on modelling incomplete information and review of how we use "type spaces" to model incomplete information
2. An abstract plea to take incomplete information more seriously in applied modelling
3. An application to coordination games ("global games")

# The Tripartite Distinction in Game Theory

von Neumann and Morgenstern "Theory of Games and Economic Behavior" 1944

## 1. Perfect Information Games

- ▶ There is common knowledge of the structure of a game being played: players, the order in which they move, previous moves, payoffs, etc...
- ▶ LEADING EXAMPLE: Chess

## 2. Complete but Imperfect Information

- ▶ There is common knowledge of the structure of the game being played: players, rules of the game, feasible strategies, payoffs, etc....; but may not know past or current actions of other players or exogenous uncertainty
- ▶ LEADING EXAMPLE: Poker

## 3. Incomplete Information

- ▶ There is not common knowledge of the structure of the game being played
- ▶ LEADING EXAMPLE: Almost all economic environments of interest?

## Some Pessimistic Assessments of Game Theory in Economics

von Neumann and Morgenstern "Theory of Games and Economic Behavior" 1944

*...we cannot avoid the assumption that all subjects under consideration are completely informed about the physical characteristics of the situation in which they operate*

- ▶ Luce and Raiffa "Game and Decisions" 1957

*..the theory assumes... complete knowledge on the part of the player in a very complex situation, where experience indicates that a human being would be far more restricted in his perceptions. The immediate reaction of the empiricist tends to be that, since such assumptions are so at variance with known fact, there is little point to the theory except as a mathematical exercise (p5).*

## John Harsanyi 1967/68

- ▶ incomplete information is not a problem: we can incorporate any incomplete information without loss of generality!
- ▶ got a piece of the first game theory Nobel prize for this observation
- ▶ precursor to game theory takeover of economic theory (at least according to the Nobel prize citation)



## John Harsanyi's Contribution: Type Spaces

- ▶ suppose there is a set of states  $\Theta$  that we care about
- ▶ suppose that are two players, Ann and Bob (generalize straightforwardly to many players)
- ▶ each player has a space of possible "types":  $T_A, T_B$
- ▶ write  $\pi_A(t_B, \theta | t_A)$  for the probability that type  $t_A$  of Ann assigns to both Bob being type  $t_B$  and the state being  $\theta$ ; so we have

$$\pi_A : T_A \rightarrow \Delta(T_B \times \Theta)$$

and analogously

$$\pi_B : T_B \rightarrow \Delta(T_A \times \Theta)$$

# John Harsanyi Contribution: Type Spaces

- ▶ "types" relate to hand in poker
  - ▶ "like" your hand in poker: private information to the players
  - ▶ "unlike" your hand in poker: no physical counterpart in the world; no ex ante stage
- ▶ Type spaces can be used to model arbitrary beliefs and higher order beliefs because....
  - ▶ The state space  $\Theta$  can embed a lot of stuff...e.g., it can encompass payoffs but also the rules of the game....
  - ▶ The type spaces  $T_A$  and  $T_B$  can be as big as you want (e.g., "universal type space") to incorporate as rich higher order beliefs as you want

# The Misunderstanding of John Harsanyi

- ▶ the good news:
  - ▶ by working with the universal type space, we could in principle dispense with common knowledge assumptions
- ▶ the bad news:
  - ▶ the economics profession went straight back to making the very strong common knowledge assumptions that seemed so problematic to von Neumann-Morgenstern and Luce-Raiffa
  - ▶ the logical possibility of relaxing common knowledge assumptions legitimized making common knowledge assumptions that once seemed unreasonable?
  - ▶ very strong common knowledge assumptions are buried

# The "Asymmetric Information" Approach

- ▶ The typical modelling approach:
  - ▶ Ann's payoffs depend on her "payoff type"  $\theta_A \in \Theta_A$ , Bob's payoffs depend on his "payoff type"  $\theta_B \in \Theta_B$ ,  $\Theta = \Theta_A \times \Theta_B$
  - ▶ Common knowledge of common prior on payoffs  $\pi^* \in \Delta(\Theta)$
  - ▶ Beliefs derived from common prior by Bayes' rule:  $\pi_A(\theta_B|\theta_A)$  derived by Bayes rule from  $\pi^*$
- ▶ Implicit common knowledge assumptions:
  1. known own payoffs
  2. each payoff uniquely associated with belief about others payoffs (i.e.,  $T_A = \Theta_A$ )
    - ▶ common knowledge of second order beliefs (independence) or "beliefs determines preferences" (Neeman 04)
    - ▶ assume away higher order beliefs
  3. common prior assumption (= "no trade")

# A Research Agenda: Taking Incomplete Information Seriously

- ▶ re-visit incomplete information recognizing that implicit common knowledge assumptions are a real issue
  - ▶ theorists: make those *implicit* common knowledge assumptions *explicit* and (where possible) relax them
  - ▶ applied economists: take higher-order beliefs seriously in applications...
- ▶ many different directions one can go, e.g.,
  - ▶ relaxing common prior assumption, close relationship to relaxing solution concepts, e.g., to dominant strategies, iterated dominance, etc...
  - ▶ maintain common prior assumption, allowing for possibility of analyst/econometrician does not know the information structure....
  - ▶ this talk: back to coordination games.....
    - ▶ coordination games and incomplete information, an interpretation of "global games" (work with Hyun Song Shin and Muhamet Yildiz)

## Relaxing Common Knowledge of Payoffs

	Invest	Not Invest
Invest	$\theta - 1, \theta - 1$	$-1, 0$
Not Invest	$0, -1$	$0$

- ▶ Players have cost of investment of 1 (always) and a return  $\theta$  only if the other firm invests
- ▶ If common knowledge that  $\theta = 3$ , then we are back to the previous game
  - ▶ Invest is "risk dominant" - i.e., best response to 50/50 conjecture - if  $\theta \geq 2$
- ▶ We want to relax common knowledge assumption

## Sensitivity to Higher Order Beliefs

- ▶ By now large theoretic game theory literature establishes that even if we are intuitively close to common knowledge (i.e., close in the product topology) that  $\theta = 3$ , we can obtain either invest OR not invest as unique equilibrium prediction (Rubinstein (1989), Weinstein and Yildiz (2007))
- ▶ So von Neumann - Morgenstern, Luce-Raiffa and their contemporaries were right: we must make *some* common knowledge assumptions to get anywhere .....
- ▶ But let's see if there are interesting intermediate explicit assumptions

## Alternative Common Knowledge Assumptions: Rank Beliefs

- ▶ A player's "rank belief" is the probability that she assigns to being more optimistic about  $\theta$  than the other player's
- ▶ Thus

$$r(t_A) = \Pr(\mathbb{E}_A(\theta|t_A) < \mathbb{E}_A(\mathbb{E}_B(\theta)) | t_A)$$

- ▶ Uniform rank belief:  $r(t_A) = \frac{1}{2}$
- ▶ Common knowledge of uniform rank beliefs:  $r(t_A) = \frac{1}{2}$  for all  $t_A$
- ▶ Similarly for Bob.....
- ▶ Defined on universal type space where  $\Theta = \mathbb{R}$
- ▶ This is a major but explicit common knowledge assumption different from perfect information, or independent types, or usual assumptions we make....



## Alternative Common Knowledge Assumptions: Rank Beliefs

**Proposition.** If there is common knowledge of uniform rank beliefs, then players choose *risk dominant* actions, i.e., Ann invests if  $\mathbb{E}_A(\theta|t_A) > 2$  and not invest if  $\mathbb{E}_A(\theta|t_A) < 2$ .

## Many Player Symmetric Payoff Generalization (in words)

- ▶ Many (finite or continuum) players
- ▶ Supermodular payoffs: players' payoff gain to investing increasing in number of other players investing
- ▶ Rank belief is belief about number of players with lower expectation of the gain to investment
- ▶ Uniform distribution over the proportion of players with lower expectation

**Proposition.** If there is common knowledge of uniform rank beliefs, then players choose *Laplacian* actions, i.e., player  $i$  invests if and only if it is a best response to a uniform belief over the number of other players investing

# Many Player Symmetric Payoff Generalization (in formulas)

- ▶ Players  $1, \dots, N$
- ▶ Player  $i$ 's net payoff to investing if  $n$  other players invest is  $\pi(n, \theta)$ ; payoff to not investing is 0
- ▶ Rank belief is belief about number of players with lower return to investment
- ▶ Rank belief:

$$r_i(n|t_i) = \Pr(\#\{\mathbb{E}_i(\theta|t_i) < \mathbb{E}_i(\mathbb{E}_j(\theta))\} | t_i)$$

- ▶ Uniform rank belief:  $r_i(n|t_i) = \frac{1}{N}$  for all  $n$ .
- ▶ Claim: if there is common knowledge of uniform rank beliefs, then players choose *Laplacian* actions, i.e., player  $i$  invests if and only if

$$\sum_{n=0}^{N-1} \pi(n, \theta) > 0$$

## "Global Games" in two slides

- ▶ We just passed the 25th anniversary of remarkable paper Carlsson and van Damme (1993) on "Global Games and Equilibrium Selection"
- ▶ Classical common prior "asymmetric information" analysis
  - ▶ Suppose that a state  $\theta$  has a smooth commonly known prior distribution
  - ▶  $x_i = \theta + \sigma\varepsilon_i$ , where the  $\varepsilon_i$  are i.i.d. noise
  - ▶ Ann observes  $x_A$  and forms conjecture about  $\theta$  and  $x_B$  by Bayes updating...
  - ▶ Bob observes  $x_B$  and forms conjecture about  $\theta$  and  $x_A$  by Bayes updating...

## "Global Games" in two slides

- ▶ If prior is uniform, there is common knowledge of uniform rank beliefs
- ▶ If prior is smooth and  $\sigma$  is small, then there is (approximate) common knowledge of uniform rank beliefs
- ▶ Significantly generalized and applied in wide variety of economic settings
- ▶ Many applications fit the binary action, symmetric payoff, Laplacian analysis earlier (Morris and Shin (2003))

# Asymmetric Information versus Incomplete Information

- ▶ Classical Asymmetric Interpretation:
  - ▶ "Types" or signals are drawn according to a common prior distribution
  - ▶ There is common knowledge of the structure of signals and the common prior: a strong assumption
  - ▶ Surely a metaphor?
  - ▶ A stale metaphor?
- ▶ Incomplete Information (rank beliefs) Interpretation:
  - ▶ argument above gives incomplete information interpretation
  - ▶ captures what is really important for results
  - ▶ this was well known, but was formalized in Morris, Shin and Yildiz (2015)

# Laplacian Heuristic and "As If" Theories of Rationality

- ▶ Relaxing common knowledge assumptions equivalent to allowing for richer type spaces
- ▶ Does it require more "sophistication" on the part of players?
- ▶ Not necessarily? Our analysis could be "as if"
  - ▶ **behavioral foundation**: each player naively assumes a uniform conjecture on opponents' player (independent of her  $\theta_i$ ). This conjecture will not turn out to be correct but play will correspond to the unique equilibrium.
  - ▶ **learning foundation**: Steiner and Stewart (2008)

## Concluding Comments on Coordination and Rank Beliefs

- ▶ Should *not* view this as an all purpose equilibrium selection device: rather, offers predictions if when common knowledge of rank beliefs is a reasonable assumption
  - ▶ strong public signals will reduce the likelihood of common knowledge of rank beliefs
- ▶ Should directly evaluate assumptions about rank beliefs directly in the lab or in the world (rather than try and test asymmetric information story directly)
- ▶ Can examine the consequences of different assumptions about rank beliefs: Morris and Yildiz (AER current issue): signal story predicts uniform rank beliefs only after large shocks....



## Comments on Incomplete Information

- ▶ Standard modelling of asymmetric information should sometimes/often be understood as a stale metaphor and this should have implications....
- ▶ Interesting and insightful to consider relaxing implicit common knowledge assumptions
- ▶ Discussed coordination games today, recent work on "informationally robust" analysis in games and mechanisms goes in the same direction