Collusion with Optimal Information Disclosure*

Takuo Sugaya — Stanford GSB
Alexander Wolitzky — MIT

March 12, 2026

Abstract

Motivated by recent concerns surrounding the use of third-party pricing algorithms by competing firms, we study repeated Bertrand competition where market demand or the cost of serving the market is observed by an intermediary (or "algorithm") that selectively discloses demand or cost information to maximize firms' collusive profit. We show that an *upper censorship* disclosure policy is optimal, which leads to *price rigidity* and *supra-monopoly prices* in some states. Improving the algorithm's accuracy reduces expected consumer surplus whenever it does so under monopoly pricing. When the state is positively correlated over time, the algorithm discloses more information when recent demand was lower or costs were higher. The analysis extends to a generalized model that accommodates product differentiation and capacity constraints. We relate our findings to recent antitrust cases.

Keywords: collusion, information disclosure, pricing algorithms, price rigidity, consumer surplus

JEL codes: C73, D43, D82

*For helpful comments, we thank seminar participants at Bonn, ESSET, HKUST, JEA, SAET, SITE, VSET, and the 2025 Northwestern Antitrust Conference and MIT Sloan IO Theory Conference; and Kyle Bagwell, Alessandro Bonatti, Sophie Calder-Wang, Glenn Ellison, Drew Fudenberg, Joe Harrington, Anton Kolotilin, Leslie Marx, Jeanine Miklós-Thal, Stephen Morris, Volker Nocke, Juan Ortner, Jesse Shapiro, Jean Tirole, and Catherine Tucker.

1 Introduction

Firms increasingly use automated algorithms to set prices and other competitive variables, a development that has raised a range of regulatory and antitrust concerns (Mehra, 2015; Ezrachi and Stucke, 2017; Calvano et al., 2020a). A particular focus of some prominent recent cases is third-party algorithms that facilitate information-sharing among competing firms while recommending prices. For example, RealPage, Inc. is a company that markets revenue management software to apartment management firms. RealPage's software gathers detailed, near real-time information on apartment rents and occupancy rates from its clients and other non-public sources and uses this data—including data on market conditions gleaned from competitors—to recommend prices. Following a history of private litigation against RealPage, in August 2024 the US Department of Justice and eight state attorneys general sued RealPage, asserting that, "At bottom, RealPage is an algorithmic intermediary that collects, combines, and exploits landlords' competitively sensitive information," which constitutes an "unlawful scheme to decrease competition among landlords," (USDOJ 2024a,b; see also Calder-Wang and Kim, 2024). Similar algorithmic intermediaries have emerged in a number of other industries, including retail gasoline pricing (A2i Systems and Kalibrate; see Assad et al., 2024) and hotel room pricing (IDeaS and Rainmaker; see Harrington, 2025). In addition, related concerns have also been raised regarding offline cartel facilitators, such as the Swiss consulting firm AC-Treuhand, which was prosecuted by the European Commission for facilitating several European industrial cartels by disclosing competitively sensitive information and recommending prices and market allocations (Harrington, 2006; Marshall and Marx, 2012).

Motivated by this type of setting, this paper develops a simple model of how an intermediary that possesses more detailed aggregate demand or cost information than its client firms can selectively disclose this information to maximize its clients' collusive profit.[1] We work in the canonical setting of repeated Bertrand competition

[1]The model is intended as a benchmark and does not attempt to fully capture the complex

with stochastic demand, introduced by Rotemberg and Saloner (1986).[2] Following Rotemberg and Saloner, our baseline setting assumes homogeneous products and iid demand, although we subsequently relax both of these assumptions. To get a stark and tractable model, we assume that the current demand state is observed *only* by the intermediary (henceforth, the *algorithm*), which then discloses information about the state according to a known policy. We also make a key technical assumption that profit is affine in the unknown state, so that, for any fixed price and any distribution over states, expected profit is determined by the expected state. Under these assumptions, we characterize the disclosure policy and the (pure strategy, subgame perfect) equilibrium that maximizes the firms' profits.

The main result in our baseline model is that the (firm-)optimal policy is *upper censorship* together with *conditional monopoly pricing*: there is a cutoff demand state $\hat{s}$ such that, if the current demand state s is below $\hat{s}$, the algorithm discloses s and recommends the corresponding monopoly price $p^m(s)$ to all firms; and if the current demand state s is above $\hat{s}$, the algorithm discloses only the event $\{s > \hat{s}\}$ and recommends the monopoly price conditional on this information, $p^m(\mathbb{E}[s|s > \hat{s}])$. The optimal equilibrium thus features *rigid prices*: prices are constant unless demand falls below $\hat{s}$. It also involves *supra-monopoly prices* for a range of demand states: for demand states s in the interval $(\hat{s}, \mathbb{E}[\tilde{s}|\tilde{s} > \hat{s}])$, the equilibrium price is $p^m(\mathbb{E}[\tilde{s}|\tilde{s} > \hat{s}])$, which is greater than the monopoly price in state s, $p^m(s)$, whenever the monopoly price $p^m(\cdot)$ is an increasing function of the demand state. Finally, as compared to the observable demand case studied by Rotemberg and Saloner, optimal collusive prices are higher—and consumer surplus is lower—for every demand state.

The logic of these results is as follows. As in Rotemberg and Saloner, firms are

industries mentioned above. For example, in practice the objective of an intermediary like RealPage may or may not be maximizing collusive profit, and the intermediary's information may or may not be a superset of the firms'. We connect our analysis to the RealPage case in detail in Sections 2.2 and 4.3.

[2]Stochastic demand and stochastic cost are equivalent up to a sign change in our model. For concreteness, we mostly discuss the stochastic demand case.

most tempted to undercut the collusive price when demand is high, as this is when the static monopoly profit $\Pi^m(s)$ is largest relative to the equilibrium continuation payoff. In Rotemberg and Saloner—which is identical to the special case of our model where the algorithm fully discloses the demand state—the cartel responds by reducing prices when demand is high, which reduces current-period profit and hence reduces the current-period deviation gain. (This is the logic of Rotemberg and Saloner's "price wars during booms.") However, when an algorithm controls the firms' information, it is more profitable to reduce profit at high demand states by pooling these states with lower demand states and recommending the monopoly price conditional on the disclosed information, rather than cutting prices. In other words, the algorithm reduces firms' temptation to deviate in high demand states *only* by reducing their information, not by reducing the recommended price conditional on their information. Technically, the key observation is that the firms' "capped monopoly profit," $\min\{\Pi^m(s), \Pi^{\max}\}$—where $\Pi^{\max}$ is the maximum industry profit that the firms can obtain in a single period without violating incentive constraints, which is independent of s with homogeneous products and iid demand—is a "convex-then-concave" function of s, and upper censorship is the optimal disclosure policy for a convex-then-concave objective function (Kolotilin, 2018; Dworczak and Martini, 2019; Kolotilin et al., 2022).

The optimal collusive equilibrium displays clean comparative statics. Reducing the number of firms, increasing the discount factor, or improving the algorithm's accuracy makes collusive prices more flexible and increases collusive profit. In addition, if improving the information of a monopoly seller reduces expected consumer surplus, then so does making collusive prices more flexible in our model. This result speaks directly to antitrust concerns regarding algorithmic information-sharing. Specifically, while prior studies have found an ambiguous effect of improved algorithmic demand prediction on consumer surplus (Sugaya and Wolitzky, 2018; Miklós-Thal and Tucker, 2019), our conclusion is more unambiguously negative. The reason is that prior studies assumed that the algorithm fully discloses its information to firms, while we assume that it selectively discloses its information to maximize firm profits, and therefore conceals information that would lead to price cuts if it were disclosed. Thus, while Miklós-Thal and Tucker (2019, p. 1553) find "somewhat reassuring results for antitrust authorities who are worried about the implications for anticompetitive and collusive behavior of the digital environment," we can unfortunately offer no such reassurances for algorithms that selectively disclose information to maximize collusive profit.

We relate our baseline model and results to some recent antitrust cases, focusing on the RealPage case. While the available evidence is limited, it seems consistent with our key mechanism wherein an intermediary with superior demand information selectively discloses this information to maximize its clients' joint profits, resulting in a stylized pricing pattern over the business cycle where prices respond flexibly to demand conditions in industry downturns but are more rigid in upturns.

Our baseline model tractably generalizes Rotemberg and Saloner and delivers sharp and empirically plausible results, but it does assume a rather special market structure. We therefore extend the model in three directions. First, we let the state persist over time, following a Markov process. Here the main results from the iid case go through, and there are also some new results. For example, we show that when demand is positively correlated over time, the algorithm discloses more information when recent demand was lower. (The opposite result holds with negative serial correlation.) The intuition is that with positive serial correlation, firms are more pessimistic about demand—and thus less tempted to deviate—when recent demand was lower, so the algorithm can disclose more information without prompting a deviation. We also show that the optimal collusive price is no longer always equal to the monopoly price for the disclosed mean demand, and that, while price is always monotone in current demand (as in the iid case and in contrast to Rotemberg and Saloner), it can be non-monotone in the previous period's demand, so that the expected price conditional on the last-period demand can display countercyclicality similar to that in Rotemberg and Saloner.

Second, we consider a generalized model that accommodates product differentiation and capacity constraints.[3] The basic mechanism of the baseline model is robust to these extensions. Optimal information disclosure generally entails regions of censorship, price rigidity, and supra-monopoly pricing. However, the form of the optimal disclosure policy can depend on details of the demand system, and in particular on the shape of the firms' incentive-constrained profit function. With a differentiated-goods linear demand system, this profit function is globally convex, implying that full information disclosure is optimal. With homogeneous goods and capacity constraints, by contrast, it is piecewise-convex but can have a concave kink, and the optimal disclosure policy can involve either upper or lower censorship, depending on parameters.

Third, we briefly consider the problem of designing a disclosure policy to maximize a weighted average of producer and consumer surplus, assuming that firms play their optimal equilibrium under the chosen policy. Here, we find that the consumer-optimal disclosure policy under linear demand with an unknown intercept is a binary signal that reveals only whether demand is below or above a cutoff, where the low signal induces the corresponding monopoly price, and the high signal sparks a "price war" (i.e., a sub-monopoly price).

The remainder of the paper is organized as follows. Following a discussion of the literature, Section 2 presents the baseline model, Section 3 solves the model, and Section 4 discusses implications and comparative statics. We connect our model and results to recent antitrust cases, especially the RealPage case, in Sections 2.2 and 4.3. Section 5 contains the extension to a persistent state. Section 6 concludes and discusses further extensions. The generalized model is presented in Appendix A. Finally, Appendix B considers more general objectives, such as maximizing consumer

[3] These features are likely important in the RealPage case, although our discussion of this case in Sections 2.2 and 4.3 focuses on our baseline model for simplicity.

surplus.

Related literature. We contribute to the literatures on pricing algorithms, information-sharing among colluding firms, information design, and repeated games.

Much of the recent literature on pricing algorithms studies how independent algorithms can learn to set supra-competitive prices (Waltman and Kaymak, 2008; Calvano et al., 2020b; Klein, 2021; Asker, Fershtman, and Pakes, 2024; Banchio and Mantegazza, 2024), as well as the commitment value of adopting such algorithms (Cooper et al., 2015; Salcedo, 2015; Hansen, Misra, and Pai, 2021; Brown and MacKay, 2023; Lamba and Zhuk, 2024). We instead ask how a shared algorithm with demand information superior to the firms' optimally discloses information to facilitate collusion. Sugaya and Wolitzky (2018, Example 3) and Miklós-Thal and Tucker (2019) show that the effect of disclosing demand information on collusive profit and consumer surplus is generally non-monotone, as it facilitates more accurate deviations as well as more accurate on-path pricing (a logic similar to Rotemberg and Saloner's). O'Connor and Wilson (2019), Martin and Rasch (2022), and Bonatti, Fiocco, and Piccolo (2024) document similar effects under imperfect monitoring.[4] However, none of these papers characterizes optimal disclosure.

Harrington (2022) notes a reason why our model might *not* be a good fit for a third-party company like RealPage that designs and sells a pricing algorithm to competing firms: if firms independently decide whether to purchase and adopt the algorithm, a profit-maximizing algorithm designer's objective may be to maximize the *difference* in profit between adopters and non-adopters, rather than adopters' profits.[5] This alternative objective could be considered in future research. Harrington (2025) considers a problem closer to ours, where the algorithm maximizes adopter's profit subject to the constraint that adoption is profitable. However, in his model, adopters commit to following the algorithm's price recommendations. Finally, a recent paper

[4] Bonatti, Fiocco, and Piccolo (2024) focus on a comparison between revealing demand information before and after firms set prices.

[5] Bordoli (2025) and Wu (2025) consider a similar designer objective in a static setting.

by Harrington and Ortner (2025) considers a similar problem where adopters do not commit to following the recommendations. The key difference from our model is that Harrington and Ortner consider idiosyncratic demand uncertainty that washes out in aggregate. In that setting (and assuming a linear demand system), they show that full information disclosure is optimal.

The broader literature on information-sharing among colluding firms considers a range of mechanisms, including the impact of improved monitoring (Abreu, Milgrom, and Pearce, 1991; Kandori, 1992; Harrington and Skrzypacz, 2011; Awaya and Krishna, 2016), the benefits of maintaining strategy uncertainty (Bernheim and Madsen, 2017; Sugaya and Wolitzky, 2018; Ortner, Sugaya, and Wolitzky, 2024; Kawai, Nakabayashi, and Ortner, 2026), and the allocative role of communication under incomplete information.[6] These papers find that concealing various types of information can be advantageous for cartels. However, we are not aware of any prior work that studies optimal information disclosure for facilitating collusion.[7]

Optimal information disclosure has been studied extensively in static environments (Rayo and Segal, 2010; Kamenica and Gentzkow, 2011), especially in the affine case we focus on (Gentzkow and Kamenica, 2016; Kolotilin et al., 2017; Kolotilin, 2018; Dworczak and Martini, 2019), as well as in some specific dynamic settings (e.g., Ely, 2017; Renault, Solan, and Vieille, 2017). From a technical perspective, the closest paper is Kolotilin and Li (2021), who study a repeated cheap talk game with voluntary transfers. Like us, they reduce the problem of characterizing the optimal equilibrium to a static information design problem, although the reduction works differently in the two papers.[8] Kolotilin and Li derive conditions under which upper censorship is

[6] The latter literature contains papers on communicating private cost information (McAfee and McMillan, 1992; Athey and Bagwell, 2001; Athey, Bagwell, and Sanchirico, 2004; Skrzypacz and Hopenhayn, 2004), as well as private signals of stochastic market demand (Hanazono and Yang, 2007; Gerlach, 2009; Buehler and Gärtner, 2013; Sahueget and Walckiers, 2017).

[7] Hickok (2024) studies optimal information disclosure by a platform that takes a share of firms' revenue, finding that full disclosure is optimal.

[8] Kolotilin and Li's reduction also imposes a monotonicity constraint, which is absent in our setting. Kuvalekar, Lipnowski, and Ramos (2022) likewise reduce a repeated communication game to a static one.

optimal, which include shape restrictions on utilities beyond affineness. For us, no conditions are required beyond affineness, due to the structure of Bertrand competition. Once the reduction to static information design (Lemma 1) is in place, our proof is essentially the same as Kolotilin and Li's (as well as Kolotilin, 2018; Dworczak and Martini, 2019; and others). However, the structure of Bertrand competition also lets us handle the Markov case in Section 5, and we also characterize more general optimal disclosure policies in Appendix A.

From the viewpoint of repeated game theory, we combine the recursive approach pioneered by Abreu (1986, 1988) with optimal public information disclosure. The approach is to consider static information design in the stage game augmented with continuation payoffs. This is especially tractable in symmetric games where symmetric equilibria can be shown to be optimal. The text of the paper illustrates this approach in the setting of homogeneous-goods Bertrand competition, while the model in Appendix A is a relatively general one where this methodology applies.

2 A Model of Collusion with Information Disclosure

2.1 Model

Prices and profits. We consider homogeneous-goods Bertrand competition among n firms with stochastic demand or a stochastic common production cost. In each period, a non-negative demand or cost state $s \in [\underline{s}, \bar{s}]$ is drawn independently from an atomless distribution F, and each firm i sets a non-negative price p_i, which is publicly observed. The firms' information about s is described below. The lowest-price firm i serves the entire market and makes profit $\Pi(p_i, s)$. The market is shared equally in case of a tie.

We focus on the case where s measures market demand. In this case, we assume that $\Pi(0,s)=0$ for all s (normalizing costs to 0); that $\Pi(p,s)$ is continuous in p with a well-defined monopoly profit $\Pi^m(s)=\max_p \Pi(p,s)$ for each s; and that $\Pi(p,s)$ is *affinely increasing* in s for each p: that is, $\Pi(p,\underline{s}) \le \Pi(p,\bar{s})$ and

$$\Pi(p,s)=\frac{\bar{s}-s}{\bar{s}-\underline{s}}\Pi(p,\underline{s})+\frac{s-\underline{s}}{\bar{s}-\underline{s}}\Pi(p,\bar{s}) \quad \text{for all } p,s.$$

In the alternative case where s measures a common production cost, the assumptions are slightly different. Here, we assume that $\Pi(p,s) \le 0$ for all $p<s$ with equality at $p=s$; that $\Pi(p,s)$ is continuous in p with a well-defined monopoly profit $\Pi^m(s)$ for each s; and that $\Pi(p,s)$ is affinely *decreasing* in s for each p. An interpretation of this case is that the firms are bidders in a procurement auction, where the cost of fulfilling the contract is privately observed by an intermediary who coordinates bid-rigging among the firms.[9]

Affineness in s is our key assumption. It has two important implications. First, expected profit is measurable with respect to mean demand: for any price p and any distribution of demand states $\mu \in \Delta([\underline{s},\bar{s}])$, the expected profit from serving the market at price p is $\mathbb{E}^\mu[\Pi(p,s)]=\Pi(p,\mathbb{E}^\mu[s])$. Second, monopoly profit $\Pi^m(s)=\max_p \Pi(p,s)$ is increasing and convex in s as the maximum of increasing affine functions.[10]

Affineness in s is a strong assumption, but it is satisfied in some important cases, which we return to throughout the paper. First, it holds if there is a binary underlying demand or cost state $\mathbf{s} \in \{\underline{s},\bar{s}\}$, where s is a continuous signal of $\mathbf{s}$ satisfying $\Pr(\mathbf{s}=\bar{s}|s)=(s-\underline{s})/(\bar{s}-\underline{s})$. Second, it holds for *linear demand with an unknown intercept*, where demand equals $D(p,s)=s-p$, and hence $\Pi(p,s)=p(s-p)$.[11]

[9] An example that fits this interpretation is the Kumatori Contractors Cooperative studied by Kawai, Nakabayashi, and Ortner (2026), which we discuss in Section 6.

[10] In the stochastic cost case, $\Pi^m(s)$ is decreasing and convex in s.

[11] These two cases both nest Example 3 of Sugaya and Wolitzky (2018), which assumes a binary demand state and linear demand. The first case also nests the model of Miklós-Thal and Tucker (2019), which assumes a binary demand state and unit demand. Our analysis also applies for linear demand subject to a non-negativity constraint, $D(p,s)=\max\{s-p,0\}$, so long as $\underline{s} \ge \bar{s}/2$, so that demand $D(p^m(s),s')$ is non-negative for any monopoly price $p^m(s)$ and demand state s'.

Third, it holds for *general demand with an unknown constant marginal cost*, where demand equals $D(p)$ and marginal cost equals s, so that $\Pi(p,s) = (p-s)D(p)$.

Information. We assume that the firms do not directly observe the state s. Instead, s is observed by an intermediary—which we refer to as the *algorithm*—which maps s to a (possibly random) signal according to a known rule. We assume that the signal is publicly observed by all firms. Importantly, this assumption restricts the scope of our analysis to public information disclosure and rules out more general private communication.[12] Since expected profit is measurable with respect to mean demand, it is without loss to view the algorithm as choosing a distribution G of the firms' posterior expectations of s, which we denote by x. By Blackwell (1953) (see also Strassen, 1965; Kolotilin, 2018), such a distribution is consistent with Bayesian updating of the prior F if and only if $G \in MPC(F)$, the set of mean-preserving contractions of F. We refer to such a distribution G as a *disclosure policy.*

Repeated game equilibrium. The above game is repeated in discrete time with a common discount factor δ. In principle, the algorithm can choose a different disclosure policy G each period, but we will see that there is no benefit from doing so in the current model with an iid state.[13]

Our solution concept is pure strategy, subgame perfect equilibrium (henceforth, "equilibrium"). Here, pure strategies mean that, in each period, each firm i sets a deterministic price $p_i(x)$ as a function of the disclosed mean demand state x and the history of past mean demand states and all firms' past prices.[14]

[12] With private signals, in each period the problem would become one of characterizing the optimal Bayes correlated equilibrium in a game with a continuum of states and actions and discontinuous payoffs. This problem is generally intractable. For example, see Smolin and Yamashita (2025) for results with concave payoffs, as well as a recent literature review. Private signals do improve on public ones—for example, Ortner, Sugaya, and Wolitzky (2024) find the optimal equilibrium with private signals in a similar model without demand uncertainty. Besides tractability, public communication and pure strategies ensure that the equilibrium will be robust to firms' investigating their competitors' prices before setting their own.

[13] In Section 5, the state follows a Markov process, and the optimal disclosure policy depends on the previous period's state. In Appendix A, the optimal disclosure policy is time-invariant along the equilibrium path but discloses no information off path.

[14] Restricting to pure strategies is standard but may not be without loss of generality, as random-

2.2 Discussion: Connection to Recent Antitrust Cases

The model is motivated by third-party algorithms that facilitate information-sharing among competitors and recommend prices, like RealPage in apartment rentals, IDeaS and Rainmaker in hotel rooms, and A2i Systems and Kalibrate in retail gasoline. Of these cases, the most information is available for RealPage. We therefore focus primarily on this case, drawing on the 2024 US DOJ complaint and the subsequent 2025 proposed settlement, and on Calder-Wang and Kim's (2024) empirical study of RealPage.[15]

We argue that RealPage fits most key assumptions of our model setup reasonably well. (The most important exception in our view is that algorithmic pricing adoption is far from universal among apartment management companies, and our model abstracts from competition with non-adopters as well as firms' decision to adopt algorithmic pricing.[16]) First, RealPage has a near-monopoly in providing algorithmic pricing for multifamily apartment buildings: after acquiring a competing product in 2017, RealPage has over 80% of the algorithmic pricing market in this industry (US-DOJ 2024a, p. 5; Calder-Wang and Kim, p. 2). In addition, while the apartment rental industry is relatively fragmented and algorithmic pricing penetration is nowhere near 100%, in many markets it is reasonable to view RealPage clients as large players with market power. The DOJ complaint reports that five or fewer landlords manage a majority of all multifamily apartment units in 445 US ZIP codes; and Calder-Wang and Kim report that the 20 largest US apartment management companies—each of

ization could deter deviations by making firms unsure of the winning price. This effect is studied in complete-information models by Bernheim and Madsen (2017) and Kawai, Nakabayashi, and Ortner (2026). Combining randomization and incomplete information is a possible direction for future research.

[15] These are the main publicly available information sources on RealPage. In turn, to our knowledge, the RealPage case is the algorithmic collusion facilitation case where the most information is currently available. Information on such cases is relatively scarce because algorithmic collusion facilitation is a new issue and because several cases are currently in litigation.

[16] These features are a key focus of the complementary work of Harrington (2025) and Harrington and Ortner (2025). In turn, these papers abstract from firms' incentives to implement their recommended prices and from aggregate demand uncertainty, respectively, which are key aspects of our model.

which manages over 32,000 units—all use RealPage. While RealPage clients account for only 10.4% of the overall US apartment rental market, they account for 21% of multifamily rentals, over 30% of buildings with over 20 units, and 34% of total units (Calder-Wang and Kim, p. 8). In some markets, RealPage clients' market share is considerably higher: Calder-Wang and Kim's Table 3 reports that in 18% of market segments (consisting of a geographical submarket and a building class), a majority of multifamily apartment buildings use algorithmic pricing.[17] The DOJ complaint also contains anecdotal evidence that RealPage clients perceive themselves to have market power and the ability to impact competitors' behavior. For example, one landlord wrote, "Our very first goal we came out with immediately out of the gate is that we will not be the reason any particular sub-market takes a rate dive. So for us the strategy was to hold steady and to keep an eye on the communities around us and our competitors," (pp. 4-5).

Second, RealPage has better demand information than its clients. RealPage's business is based on its superior, non-public data: e.g., it claims that it "does not have any true competitors, mainly because our data is based on real lease transaction data," and that "we have [the] most data and the best data" (USDOJ 2024a, p. 5, 14). It obtains this data by collecting and combining data from its many clients (the conduct at the heart of its business model and the many antitrust complaints it has attracted), but also by direct private market research. As the DOJ reports (p. 11), "RealPage has an additional, complementary product called Market Analytics. Market Analytics compiles data from over 50,000 monthly phone calls that RealPage makes to landlords across the country. On these calls RealPage collects nonpublic, competitively sensitive information by floor plan on occupancy rates, effective rents, and concessions… These market surveys cover over 11 million units and

[17] Calder-Wang and Kim also report that penetration exceeds 66% in many submarkets (e.g., 68% in Arapahoe County, Denver; 74% in Far Northwest, Austin; 92% in Irvine, Orange County). These statistics indicate considerable heterogeneity across markets in the market share and concentration of RealPage clients. Our model is a better fit for markets where RealPage clients have higher market share and concentration.

approximately 52,000 properties. Landlords, including but not limited to those that use AIRM, YieldStar, or other RealPage products knowingly share this nonpublic information with RealPage." RealPage uses this data to estimate a demand curve for each apartment, which it then uses to determine recommended prices (p. 19). For it to make sense for RealPage to use competitors' data and market surveys to estimate demand, these sources must be providing information on demand variables that are correlated across firms, as in our model.[18] One indication of the importance of this data is that DOJ's proposed settlement bans RealPage from using competitors' data in the last 12 months as well as non-public market surveys to determine recommended prices.

Finally, the DOJ complaint and Calder-Wang and Kim contain evidence that RealPage's price recommendations are oriented more toward joint profit-maximization than individual profit-maximization. For example, the complaint reports that "RealPage frequently tells prospective and current clients that a 'rising tide raises all ships.' A RealPage revenue management vice president explained that this phrase means that 'there is greater good in everybody succeeding versus essentially trying to compete against one another in a way that actually keeps the industry down'," (p. 12). Consistent with these statements, Calder-Wang and Kim estimate that RealPage adopters' pricing fits a model of joint profit-maximization better than individual profit-maximization.[19] So, with the important caveat that we do not model competition with non-adopters or firms' decision to adopt algorithmic pricing, our model seems to fit this market fairly well.

[18] Our baseline model takes this correlation to the extreme by assuming that demand is perfectly correlated across firms. A richer model would allow differentiated products (as in Appendix A) and imperfectly correlated demand.

[19] We take no position on whether the evidence that RealPage maximizes joint rather than individual profit is fully compelling from an economic standpoint, let alone a legal one. We only claim that conceptualizing RealPage as maximizing joint profit is well-motivated.

3 Optimal Information Disclosure and Pricing

We characterize the joint information disclosure and pricing policy that maximizes collusive profits (the sum of the firms' payoffs). We reduce this problem to a static information design problem in Section 3.1 and solve it in Section 3.2.

3.1 Reduction to Static Information Design

For any number $V \geq 0$, define

$$\Pi^{\max}(\delta, n, V) = \frac{\delta V}{(1-\delta)(n-1)}. \tag{1}$$

Intuitively, this is the maximum profit Π such that a firm prefers to receive Π/n today and V/n in every future period rather than Π today and zero in the future. Thus, if the firms expect a future per-period collusive profit of V and the cartel tries to make profit $\Pi > \Pi^{\max}(\delta, n, V)$ in any period, a firm will deviate.

Next, define V^* as the greatest fixed point of the equation

$$V = \max_{G \in MPC(F)} \mathbb{E}^G \left[\min\left\{\Pi^m(x), \Pi^{\max}(\delta, n, V)\right\}\right]. \tag{2}$$

Intuitively, this is the maximum expected profit attainable by a joint information disclosure and pricing policy that never makes profit greater than $\Pi^m(x)$ (the maximum feasible profit at mean demand state x) or $\Pi^{\max}(\delta, n, V^*)$ (the maximum incentive compatible profit at mean demand state x, when future per-period collusive profit equals V^*). Note that the right-hand side of (2) is bounded by $\mathbb{E}^F[\Pi^m(s)]$, so V^* is well-defined.

We show that optimal collusive profit equals V^* and that this profit level is attained by an equilibrium that is *symmetric* ($p_i(x)$ is always identical across firms i), *stationary* (the disclosure policy G is the same in every period and, on path, $p_i(x)$ is independent of the history of past demand realizations), and of a *grim trigger* form (play permanently reverts to the static Nash equilibrium following any deviation).

Lemma 1 *Optimal collusive profit equals V^* and is attained by a symmetric, stationary, grim trigger equilibrium. Moreover, a disclosure policy G is optimal if and only if it solves the maximization problem in (2) with $V = V^*$.*

Lemma 1 reduces the problem of finding an optimal equilibrium to the static information design problem on the right-hand side of equation (2), with $V = V^*$.

Proof. We first show that there exists a symmetric, stationary, grim trigger equilibrium that attains collusive profit V^*. For each x, let $p^m(x) \in \operatorname{argmax}_p \Pi(p,x)$ be a monopoly price in state x, and let

$$p(x) = \begin{cases} p^m(x) & \text{if } \Pi^m(x) \leq \Pi^{\max}(\delta, n, V^*), \\ \min\{p : \Pi(p,x) = \Pi^{\max}(\delta, n, V^*)\} & \text{if } \Pi^m(x) > \Pi^{\max}(\delta, n, V^*). \end{cases}$$

Note that $p(x)$ is well-defined by the intermediate value theorem, as $\Pi(0,x) = 0$ and $\Pi(p,x)$ is continuous in p.[20] Let $G^* \in \operatorname{argmax}_{G \in MPC(F)} \mathbb{E}^G[\min\{\Pi^m(x), \Pi^{\max}(\delta, n, V^*)\}]$. Consider disclosure policy G^*, together with the strategy profile where all firms price at $p(x)$ whenever mean demand x realizes on path, and all firms price at zero off path. This is a symmetric, stationary, grim trigger strategy profile, which yields collusive profit V^* by construction. To see that it is an equilibrium, note that a firm's best deviation when realized mean demand is x is to price just below $p(x)$: this is immediate if $p(x) = p^m(x)$ and otherwise follows because $p(x)$ is the smallest price p satisfying $\Pi(p,x) = \Pi^{\max}(\delta, n, V^*)$, so that $\Pi(p',x) < \Pi^{\max}(\delta, n, V^*)$ for all $p' < p(x)$. This deviation wins the entire market in the current period, but forfeits an expected profit of V^*/n in every future period. Thus, the strategy profile is an equilibrium if and

[20]Here and throughout, we write proofs for the case where s is a demand state. The proofs for the case where s is a cost state are nearly identical.

only if, for all x, we have

$$\begin{aligned}(1-\delta)\Pi(p(x),x)+\delta(0) &\leq \frac{1}{n}((1-\delta)\Pi(p(x),x)+\delta V^{*}) \qquad \Longleftrightarrow \\ \Pi(p(x),x) &\leq \frac{\delta V^{*}}{(1-\delta)(n-1)}=\Pi^{\max}(\delta,n,V^{*}).\end{aligned}$$

Since this inequality holds by construction, the strategy profile is an equilibrium.

We now show that no equilibrium yields higher profit. Fix any equilibrium, and let $\bar{V}$ be the supremum over periods t and histories of play up to and including period t of the expected per-period collusive profit from period $t+1$ onward. Now fix an arbitrary period t and a history of play up to period t, and suppose that when the realized mean demand in period t at this history is x, the prescribed winning price is $p(x)$, and each firm i wins with probability α_i and obtains equilibrium continuation value v_i. (So, $\alpha_i = 1/|j : p_j(x) = p(x)|$ if $p_i(x) = p(x)$, and $\alpha_i = 0$ otherwise. Note that each $p_i(x)$—and thus the winning price $p(x)$—is deterministic by our restriction to pure strategy equilibria.) Since a possible deviation for firm i is to price just below $p(x)$ and a firm's minimax payoff is zero, firm i's incentive constraint implies

$$(1-\delta)\Pi(p(x),x)+\delta(0) \leq \alpha_i(1-\delta)\Pi(p(x),x)+\delta v_i.$$

Averaging this inequality over the n firms, we have

$$(1-\delta)\Pi(p(x),x) \leq \frac{1}{n}\left((1-\delta)\Pi(p(x),x)+\delta\sum_i v_i\right) \leq \frac{1}{n}\left((1-\delta)\Pi(p(x),x)+\delta\bar{V}\right),$$

where the second inequality is by definition of $\bar{V}$. This inequality is equivalent to $\Pi(p(x),x) \leq \Pi^{\max}\left(\delta,n,\bar{V}\right)$. Since we also have $\Pi(p(x),x) \leq \Pi^{m}(x)$ by definition, and these inequalities hold for any x, expected collusive profit in period t is at most $\max_{G\in MPC(F)}\mathbb{E}^{G}\left[\min\left\{\Pi^{m}(x),\Pi^{\max}\left(\delta,n,\bar{V}\right)\right\}\right]$. Since this holds for any period t, we have $\bar{V} \leq \max_{G\in MPC(F)}\mathbb{E}^{G}\left[\min\left\{\Pi^{m}(x),\Pi^{\max}\left(\delta,n,\bar{V}\right)\right\}\right]$. But this implies that equation (2) has a fixed point weakly above $\bar{V}$, and hence $\bar{V} \leq V^{*}$, by definition of

V^*. ∎

A direct implication of Lemma 1 is that collusion is impossible if $\delta < (n-1)/n$. (The same condition implies that collusion is impossible under full information disclosure, as in Rotemberg and Saloner.) Conversely, if $\delta \geq (n-1)/n$ then monopoly profit under no information disclosure, $\Pi^m\left(\mathbb{E}^F[s]\right)$, is attainable.

Lemma 2 *If $\delta < (n-1)/n$ then $V^* = 0$. Conversely, if $\delta \geq (n-1)/n$ then $V^* \geq \Pi^m\left(\mathbb{E}^F[s]\right)$.*

Proof. If $\delta < (n-1)/n$ then $\Pi^{\max}(\delta, n, V) < V$ for all $V > 0$, so the only solution to (2) is $V = 0$. Conversely, if $\delta \geq (n-1)/n$ then no information disclosure together with a constant on-path price of $p^m\left(\mathbb{E}^F[s]\right)$ and zero prices off path is an equilibrium, with expected profit $\Pi^m\left(\mathbb{E}^F[s]\right)$. ∎

Given Lemma 2, we henceforth assume that $\delta \geq (n-1)/n$.

3.2 Optimality of Upper Censorship

The information design problem in (2) is easily solved using recent results from the static information design literature.

First, let $s^* \in [\underline{s}, \bar{s}]$ solve

$$\Pi^m(s^*) = \Pi^{\max}(\delta, n, V^*) \tag{3}$$

if such a demand state exists, and let $s^* = \bar{s}$ otherwise. Note that, by Lemma 2 and our assumption that $\delta \geq (n-1)/n$, we have $V^* \geq \Pi^m\left(\mathbb{E}^F[s]\right)$, and hence $s^* \geq \mathbb{E}^F[s]$.[21] Thus, since F is atomless, there exists $\hat{s} \in [\underline{s}, s^*]$ such that

$$\mathbb{E}^F[s|s \geq \hat{s}] = s^*.$$

[21] This follows because if $s^* < \mathbb{E}^F[s]$ then $\Pi^m\left(\mathbb{E}^F[s]\right) > \Pi^{\max}(\delta, n, V^*)$ by (3) and monotonicity of $\Pi^m(s)$, but then we would have $V^* > \Pi^{\max}(\delta, n, V^*)$ by Lemma 2, contradicting the definition of V^*.

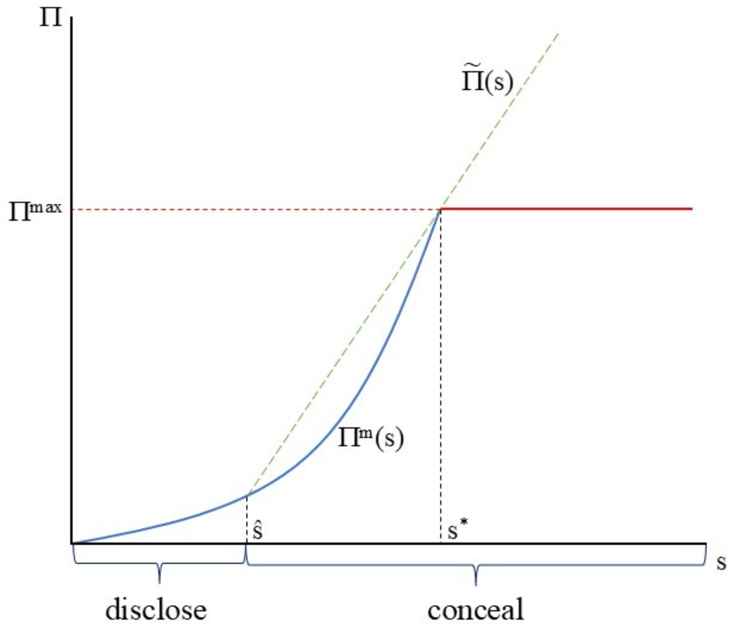

Figure 1: The optimal disclosure policy with undifferentiated products. First, s^* is determined as the solution to $\Pi^m(s^*) = \Pi^{\max}(\delta, n, V^*)$. Then, $\hat{s}$ is determined as the solution to $E^F[s|s \geq \hat{s}] = s^*$. The optimal policy discloses demand states $s < \hat{s}$ and recommends the corresponding monopoly price, $p^m(s)$; and conceals demand states $s \geq \hat{s}$ and recommends the monopoly price conditional on this information, $p^m(s^*)$. The function $\tilde{\Pi}(s)$ is defined in the proof of Theorem 1.

We can now characterize the optimal information disclosure and pricing policy. Figure 1 illustrates the optimal disclosure policy, as well as the construction of s^* and $\hat{s}$.

Theorem 1 *With stochastic demand, the unique optimal disclosure policy is the* upper censorship *policy that discloses demand states below $\hat{s}$ and conceals demand states above $\hat{s}$. The unique optimal collusive price $p(s)$ in state s is given by*

$$p(s) = \begin{cases} p^m(s) & \text{if } s < \hat{s}, \\ p^m(s^*) & \text{if } s \geq \hat{s}. \end{cases} \tag{4}$$

With stochastic costs, the unique optimal disclosure policy is the analogous lower censorship *policy that discloses cost states above $\hat{s}$ satisfying $\mathbb{E}^F[s|s \leq \hat{s}] = s^*$ and* *conceals cost states below* $\hat{s}$.[22] *The unique optimal collusive price* $p(s)$ *in state* s *is given by (4) with the reversed inequalities.*

With stochastic demand, note that $\hat{s} = \underline{s}$—so no disclosure is optimal—iff $s^* = \mathbb{E}^F[s]$, which holds iff $\delta = (n-1)/n$. Conversely, $\hat{s} = \bar{s}$—so full disclosure is optimal—iff $s^* = \bar{s}$, which holds iff $\Pi^m(\bar{s}) \leq \Pi^{\max}(\delta, n, V^*)$. Otherwise, we have $\Pi^m\left(\mathbb{E}^F[s]\right) < \Pi^{\max}(\delta, n, V^*) < \Pi^m(\bar{s})$, and partial disclosure is optimal.

To understand Theorem 1, note that disclosing demand information increases expected monopoly profits—as $\Pi^m(s)$ is convex—but revealing that expected demand is too high requires cutting price to deter a deviation (as in Rotemberg and Saloner). The theorem says that it is optimal to disclose low demand states and conceal high ones, such that the mean concealed state s^* is the highest state s that does not require a price cut from the corresponding monopoly price $p^m(s)$ to deter a deviation.

The intuition is that it cannot be optimal to disclose a mean demand state $s > s^*$, as pooling s with lower demand states would increase expected profit. In particular, full disclosure together with price cuts during booms—Rotemberg and Saloner's equilibrium—is suboptimal in our model. In addition, it cannot be optimal to pool two demand states below s^*, as separating these states increases expected profit since $\Pi^m(s)$ is convex. Finally, it is more profitable to pool demand states above s^* with intermediate states $s \in [\hat{s}, s^*]$ rather than low states $s < \hat{s}$, as this spreads out the distribution of disclosed mean demand states s on the interval $[\underline{s}, s^*]$, where $\Pi^m(s)$ is convex.

A more technical explanation is that the objective function $\min\{\Pi^m(s), \Pi^{\max}(\delta, n, V^*)\}$ is increasing and convex in s for $s \leq s^*$ and is constant in s for $s > s^*$. Thus, (2) describes a mean-measurable information design problem with an objective function that is "S-shaped": first convex, then concave. It is well-known that the solution to such a problem is upper censorship (e.g., Kolotilin, 2018; Dworczak and Martini, 2019; Kolotilin and Li, 2021; Kolotilin et al., 2022). Moreover, adapting the standard proofs

[22] The definitions of $\Pi^{\max}$ and s^* remain as in (1) and (3).

to the current setting where the objective function is not only convex-then-concave but convex-then-constant implies that the solution must take the prescribed form, where the mean censored state s^* lies at the kink of the objective function.[23] In particular, the disclosed mean demand state s always satisfies $\Pi^m(s) \leq \Pi^{\max}(\delta, n, V^*)$, which implies that conditional monopoly pricing is optimal.

Proof. Define an auxiliary objective function

$$\tilde{\Pi}(s) = \begin{cases} \Pi^m(s) & \text{if } s < \hat{s}, \\ \frac{s^*-s}{s^*-\hat{s}}\Pi^m(\hat{s}) + \frac{s-\hat{s}}{s^*-\hat{s}}\Pi^m(s^*) & \text{if } s \geq \hat{s}. \end{cases}$$

Note that $\tilde{\Pi}(s)$ is convex and $\tilde{\Pi}(s) \geq \min\{\Pi^m(s), \Pi^{\max}(\delta, n, V^*)\}$ for all s. (See Figure 1.) Consider the auxiliary problem, $\max_{G \in MPC(F)} \mathbb{E}^G\left[\tilde{\Pi}(s)\right]$. Since $\tilde{\Pi}(s)$ is convex, the solution is full disclosure ($G = F$), and the resulting value is

$$\begin{aligned} \mathbb{E}^F\left[\tilde{\Pi}(s)\right] &= F(\hat{s})\,\mathbb{E}^F\left[\Pi^m(s)\,|s < \hat{s}\right] \\ &\quad + (1 - F(\hat{s}))\,\mathbb{E}^F\left[\frac{s^* - s}{s^* - \hat{s}}\Pi^m(\hat{s}) + \frac{s - \hat{s}}{s^* - \hat{s}}\Pi^m(s^*)\,|s \geq \hat{s}\right] \\ &= F(\hat{s})\,\mathbb{E}^F\left[\Pi^m(s)\,|s < \hat{s}\right] + (1 - F(\hat{s}))\,\Pi^m(s^*)\,. \end{aligned}$$

Since $\tilde{\Pi}(s) \geq \min\{\Pi^m(s), \Pi^{\max}(\delta, n, V^*)\}$ for all s, this value is an upper bound for $\max_{G \in MPC(F)} \mathbb{E}^G\left[\min\{\Pi^m(s), \Pi^{\max}(\delta, n, V^*)\}\right]$. But it is attained by upper censorship with cutoff $\hat{s}$, so this policy is optimal. Moreover, this policy is the unique one that induces only posteriors s where $\tilde{\Pi}(s) = \min\{\Pi^m(s), \Pi^{\max}(\delta, n, V^*)\}$, so it is the unique optimal policy. Finally, this disclosure policy is optimal only in conjunction with the prescribed prices. ■

[23] Specifically, we adapt the proofs of Proposition 3 of Dworczak and Martini (2019) and Proposition 4 of Kolotilin and Li (2021). Theorem 1 is also a special case of Theorem 3 in Appendix A.

4 Implications and Comparative Statics

We now discuss the baseline model's predictions, comparative statics, and consumer welfare implications. At the end of this section, we relate the model's mechanics and predictions to the RealPage case.

4.1 Model Predictions and Impact of Selective Disclosure

The optimal disclosure and pricing policy characterized in Theorem 1 has the following features. In the subsequent discussion, we assume that the monopoly price $p^m(s)$ is unique and increasing in s.

Conditional monopoly pricing. Along the equilibrium path of play, the cartel prevents deviations solely by reducing its members' information, not by reducing collusive prices below the monopoly price conditional on their information. Consequently, collusive prices for a cartel aided by an algorithm that observes a state with distribution F is identical to monopoly pricing for a single firm that observes a state with a less informative distribution G^*. This observation will be useful for the comparative statics and consumer welfare results in Section 4.2, as it implies that comparative statics for collusive prices are equivalent to comparative statics for monopoly prices with respect to the monopoly's information.

Price rigidity—not price wars—during booms. The collusive price $p(s)$ is increasing for $s < \hat{s}$ and is constant (at a higher level) for $s \geq \hat{s}$. Collusive prices thus display rigidity for demand states $s \geq \hat{s}$, rather than "price wars" as in Rotemberg and Saloner. This result gives a novel rationale for oligopoly price rigidity in intermediated collusion settings: prices are rigid because colluding firms optimally limit their own information about market conditions to deter deviations.[24]

[24]While not focused on intermediated cartels per se, Carlton (1986) and others find that prices are more rigid in concentrated industries, and Harrington (2008) and others suggest price rigidity as a collusive marker. Existing theories of rigid collusive prices include Athey, Bagwell, and Sanchirico (2004) and Hanazono and Yang (2007) (based on incentive costs of inducing firms to reveal private

In addition, this result shows that algorithmic pricing does not unambiguously increase price flexibility under collusion. This result somewhat goes against the received view following Chen, Mislove, and Wilson's (2016) study of retailer pricing on Amazon, which finds that very flexible prices are a hallmark of algorithmic pricing under competition.

Supra-monopoly pricing. Collusive prices are *above monopoly* at intermediate demand states: for $s \in (\hat{s}, s^*)$, the optimal collusive price is $p(s) = p^m(s^*) > p^m(s)$. Moreover, these demand states satisfy $\Pi^m(s) < \Pi^{\max}(\delta, n, V^*)$, so monopoly profit can be attained at any one of these states s by disclosing s and recommending price $p^m(s)$ (holding the rest of the equilibrium fixed). Thus, for a range of demand states where monopoly profit is attainable, the algorithm instead implements supra-monopoly prices that deliver lower profits. The reason why is that recommending the supra-monopoly price $p^m(s^*) > p^m(s)$ in states $s \in (\hat{s}, s^*)$ lets the algorithm recommend the same price in states $s > s^*$, where this price would be too high to be incentive compatible if the state were disclosed. In other words, price rigidity for demand states above $\hat{s}$ results in an inefficiently high price for demand states in $(\hat{s}, s^*)$, but thereby supports a higher price for demand states above s^* than would be attainable under full information.[25]

Impact of selective information disclosure. The algorithm's ability to conceal information leads to higher prices and lower consumer surplus in every demand state and thus unambiguously harms consumers. To see this, let V^{FD} be optimal collusive profit under full disclosure, which is given by the greatest fixed point of the equation

$$V^{FD} = \mathbb{E}^F\left[\min\left\{\Pi^m(s), \Pi^{\max}\left(\delta, n, V^{FD}\right)\right\}\right],$$

cost or demand information) and Maskin and Tirole (2001) (who model "kinked demand curves" as a result of Markov perfect equilibria with staggered price setting).

[25] Supra-monopoly pricing at intermediate demand states is analogous to "over-pooling"—where first-best actions are not taken even in some states where they are implementable—in Kolotilin and Li (2021).

and let s^{FD} solve

$$\Pi^m\left(s^{FD}\right) = \Pi^{\max}\left(\delta, n, V^{FD}\right).$$

As in Rotemberg and Saloner, optimal collusive prices under full disclosure are given by

$$p^{FD}(s) = \begin{cases} p^m(s) & \text{if } s \leq s^{FD}, \\ \min\left\{p : \Pi(p,s) = \Pi^{\max}\left(\delta, n, V^{FD}\right)\right\} & \text{if } s > s^{FD}. \end{cases}$$

Since $p^m(s)$ is increasing and $\Pi(p,s)$ is increasing in s, it follows that $p^{FD}(s)$ is increasing for $s < s^{FD}$ and decreasing for $s \geq s^{FD}$. The latter "price wars during booms" result is Rotemberg and Saloner's key message.

Proposition 1 *As compared to collusive prices under full disclosure, collusive prices under the optimal disclosure policy are higher at each demand state. Consequently, selective information disclosure reduces consumer surplus.*

Proof. Note that $V^{FD} \leq V^*$, and hence $s^{FD} \leq s^*$. Therefore, letting $p(s)$ be the optimal collusive price in (4), for $s < s^*$ we have $p(s) \geq p^m(s) \geq p^{FD}(s)$, and for $s > s^*$ we have $p(s) = p^m(s^*) \geq p^m\left(s^{FD}\right) \geq p^{FD}(s)$, where the first inequality holds because $s^{FD} \leq s^*$ and p^m is increasing, and the second holds because $s^{FD} \leq s$ and p^{FD} is decreasing for $s \geq s^{FD}$. ■

Empirical predictions, collusive markers, and the interpretation of price wars. The baseline model has three main empirical predictions:

1. The support of the distribution of equilibrium prices consists of an interval $[p^m(\underline{s}), p^m(\hat{s})]$ and a single higher price $p^m(s^*)$.

2. Prices are rigidly fixed at $p^m(s^*)$ for all demand states except the lowest ones. For low demand states, prices are discretely lower than $p^m(s^*)$ but vary flexibly in the interval $[p^m(\underline{s}), p^m(\hat{s})]$. Overall, prices are pro-cyclical: $p(s)$ is non-decreasing.

3. While prices are pro-cyclical, the gap between monopoly and collusive prices, $p^m(s) - p(s)$, is non-monotone: first zero, then negative, then positive.

The predicted form of price rigidity—a rigid, high price together with an interval of flexible lower prices—is distinctive to our model and is thus a possible collusive marker.

The pro-cyclical relationship between prices and demand in our model gives an alternative interpretation of the "price wars" predicted by Green and Porter (1984) and other models of collusion under imperfect monitoring. In Green and Porter, prices are pro-cyclical: prices drop following demand shortfalls as part of an optimal repeated game equilibrium under imperfect monitoring. In contrast, in Rotemberg and Saloner, prices are counter-cyclical in high demand states. While our model is much closer to Rotemberg and Saloner's, our prediction of pro-cyclical prices coincides with Green and Porter's, albeit by a different mechanism: perfect monitoring and selectively disclosed demand information, rather than imperfect monitoring. This observation is relevant for a line of papers that have tested the competing predictions of Green and Porter and Rotemberg and Saloner (e.g., Porter, 1983; Ellison, 1994) and have typically found results more favorable to Green and Porter's prediction of pro-cyclical prices. Relative to this literature, our analysis shows that perfect monitoring and selective information disclosure is an alternative explanation for pro-cyclical prices.

One way to distinguish our theory from Green and Porter's would be to estimate the gap between monopoly and collusive prices, $p^m(s) - p(s)$, over the cycle. In Green and Porter, the gap is larger following low demand states: collusive and monopoly prices are closer when recent demand was high. In our model, the gap is non-monotone: collusive and monopoly prices are closest (in the model, identical) when current demand is low or equal to s^*.

4.2 Comparative Statics and Consumer Welfare

We now turn to comparative statics. In what follows, we say that collusive prices are *more flexible* if the optimal disclosure policy G^* spreads out in the mean-preserving spread sense. By Blackwell (1953), this is equivalent to the algorithm's output becoming more informative. Holding fixed the algorithm's information F, this is also equivalent to increasing the censorship cutoff $\hat{s}$, so a wider range of demand states are disclosed.[26] We also continue to assume that $\delta \geq (n-1)/n$, as otherwise collusion is impossible by Lemma 2.

Proposition 2 *Collusive profit V^* is higher and collusive prices are more flexible when*

1. *there are fewer firms (n decreases),*
2. *the firms are more patient (δ increases), or*
3. *the algorithm is more accurate (F increases in the mean-preserving spread sense).*

The intuition for the first two results is that decreasing n or increasing δ relaxes the firms' incentive constraints, which lets the algorithm disclose a wider range of states without prompting a deviation. In addition, a more accurate algorithm generates greater collusive profits, which again relaxes incentive constraints and allows greater disclosure.[27]

Proof. For the first two results, note that n and δ affect V^* and p only through the function $\Pi^{\max}$, which is decreasing in n and increasing in δ. Thus, decreasing n or increasing δ shifts up the right-hand side of (2) as a function of V, which increases the

[26] If F changes (as in Proposition 2.3, the comparative static with respect to the algorithm's information), then G^* can spread out even as $\hat{s}$ decreases.

[27] A similar effect is documented in Theorem 5 of Harrington (2025). Improving the algorithm's information corresponds to taking a mean-preserving spread of F by Blackwell (1953)—the interpretation is that there is a distribution H of an underlying state $\mathbf{s}$, and the distribution of the algorithm's expectation of the underlying state is F, a mean-preserving contraction of H.

greatest fixed point V^*. In turn, an increase in V^* increases s^* and $\hat{s}$, which makes prices more flexible.

For the third result, we prove in the appendix that a more informative prior implies a more informative optimal disclosure policy in static information design problems with a convex-constant objective: for any distributions (F_1, F_2, G_1, G_2) where F_2 is a mean-preserving spread of F_1, G_1 is the distribution of x under an optimal disclosure policy for prior F_1, and G_2 is the distribution of x under an optimal disclosure policy for prior F_2, we have that G_2 is a mean-preserving spread of G_1. Given this result, spreading out F implies that the optimal disclosure policy on the right-hand side of (2) is more informative for any fixed value for V. In addition, since $MPC(F_1) \subseteq MPC(F_2)$, spreading out F clearly shifts up the right-hand side of (2) as a function of V, which increases the greatest fixed point V^*. Since increasing V also implies a more informative optimal disclosure policy for any fixed prior (as is obvious from Theorem 1), the result follows. ■

Proposition 2 gives sufficient conditions for collusive prices to become more flexible. In general, the effect of price flexibility on expected consumer surplus is ambiguous. However, since collusive prices equal monopoly prices for a monopoly facing state distribution G^*, price flexibility benefits consumers if and only if improving the information of a monopoly does the same.

To state this result, we must assume that expected consumer surplus is measurable with respect to the distribution of posterior mean states x. To this end, let $CS(p, s)$ denote consumer surplus at price p and state s. We say that consumer surplus is *quasi-linear* in s if there exist functions f, g, and h such that

$$CS(p, s) = f(s) + g(p)s + h(p) \qquad \text{for all } p, s.$$

For example, under linear demand with an unknown intercept, we have $CS(p, s) = (s - p)^2/2 = f(s) + g(p)s + h(p)$, where $f(s) = s^2/2$, $g(p) = -p$, and $h(p) = p^2/2$; and under linear demand with an unknown constant marginal cost, we have

$CS(p,s) = (1-p)^2/2 = f(s) + g(p)s + h(p)$, where $f(s) = 0$, $g(p) = 0$, and $h(p) = (1-p)^2/2$.

Quasi-linearity implies that expected consumer surplus is measurable with respect to the distribution of posterior mean states x, whenever prices $p(x)$ are measurable with respect to x. To see this, take any distribution $\tau \in \Delta(\Delta([\underline{s}, \bar{s}]))$ of distributions $\mu \in \Delta([\underline{s}, \bar{s}])$ of the state $\tilde{s}$ such that $\mathbb{E}^{\tau}[\mu] = F$. Letting $x_{\mu} = \mathbb{E}^{\mu}[s]$ be the mean state under distribution μ and letting G be the distribution of mean state x, we have

$$\begin{aligned} \mathbb{E}^{\tau}\left[\mathbb{E}^{\mu}\left[CS\left(p\left(x_{\mu}\right), s\right)\right]\right] &= \mathbb{E}^{\tau}\left[\mathbb{E}^{\mu}\left[f(s) + g\left(p\left(x_{\mu}\right)\right)s + h\left(p\left(x_{\mu}\right)\right)\right]\right] \\ &= \mathbb{E}^{F}\left[f(s)\right] + \mathbb{E}^{\tau}\left[g\left(p\left(x_{\mu}\right)\right)x_{\mu} + h\left(p\left(x_{\mu}\right)\right)\right] \\ &= \mathbb{E}^{F}\left[f(s)\right] + \mathbb{E}^{G}\left[g\left(p(x)\right)x + h\left(p(x)\right)\right], \end{aligned}$$

where the second line uses the law of iterated expectation. Thus, the expected consumer surplus under any disclosure policy and (mean-measurable) pricing policy equals $\mathbb{E}^{G}[g(p(x))x + h(p(x))]$, where G is the distribution of the disclosed mean state x, plus a constant $\mathbb{E}^{F}[f(s)]$.

In particular, if $g(p^m(x))x + h(p^m(x))$ is concave (resp., convex) in x, then expected consumer surplus under monopoly pricing is higher when the monopoly has less (resp., more) information. Thus, since collusive prices when the algorithm observes a state with distribution F equal monopoly prices when the monopoly observes a state with distribution G^*, the concavity or convexity of $g(p^m(x))x + h(p^m(x))$ determines the implications of the comparative statics in Proposition 2 for expected consumer surplus (with the caveat that increasing n or decreasing δ always benefits consumers if it causes δ to fall below $(n-1)/n$, rendering collusion impossible).

Proposition 3 *Assume that consumer surplus is quasi-linear in x. If $g(p^m(x))x + h(p^m(x))$ is concave (resp., convex) in x, then expected consumer surplus is lower (resp., higher) when there are fewer firms, the firms are more patient, or the algorithm is more accurate (resp., so long as $\delta \geq (n-1)/n$).*

Proof. Immediate from Proposition 2 and Jensen's inequality. ■

As a corollary, we obtain the corresponding consumer surplus implications under linear demand with an unknown intercept or unknown constant marginal cost.

Corollary 1 *Under linear demand with an unknown intercept, expected consumer surplus is lower when there are fewer firms, the firms are more patient, or the algorithm is more accurate.*

Under linear demand with an unknown constant marginal cost, expected consumer surplus is higher when there are fewer firms, the firms are more patient, or the algorithm is more accurate (so long as $\delta \geq (n-1)/n$).

Proof. Under linear demand with an unknown intercept, we have $p^m(x) = x/2$ and $CS(p,s) = f(s) + g(p)s + h(p)$, where $f(s) = s^2/2$, $g(p) = -p$, and $h(p) = p^2/2$. Hence,

$$g(p^m(x))x + h(p^m(x)) = -\frac{x^2}{2} + \frac{x^2}{8} = -\frac{3x^2}{8},$$

a concave function of x.

Under linear demand with an unknown constant marginal cost, we have $p^m(x) = (1+x)/2$ and $CS(p,s) = f(s) + g(p)s + h(p)$, where $f(s) = 0$, $g(p) = 0$, and $h(p) = (1-p)^2/2$. Hence,

$$g(p^m(x))x + h(p^m(x)) = \frac{(1-x)^2}{8},$$

a convex function of x. ■

Proposition 3 shows that improving the algorithm's accuracy reduces consumer surplus whenever improving a monopoly's information does so, which Corollary 1 shows holds under linear demand with an unknown intercept.[28] This finding contrasts with results of Sugaya and Wolitzky (2018, Example 3) and Miklós-Thal and

[28] Versions of the result that improving a monopoly's information about the intercept of a linear demand curve decreases expected consumer surplus were shown by Pigou (1920), Vives (2001), and Farboodi, Haghpanah, and Shourideh (2025).

Tucker (2019), who find that a more accurate demand prediction algorithm increases consumer surplus when the firms' discount factor δ lies in an intermediate range. The reason for the difference is that those papers assume that the algorithm fully discloses its information, which sparks price wars during booms. In contrast, with optimal information disclosure, a more accurate algorithm makes prices more flexible without triggering price wars, which reduces expected consumer surplus whenever improving a monopoly's information does so. Our assessment of the likely impact of improved algorithmic demand prediction on consumer surplus is thus considerably more pessimistic.

However, improving the algorithm's accuracy also *increases* consumer surplus whenever improving a monopoly's information does so, which Corollary 1 shows holds under linear demand with an unknown constant marginal cost.[29] In this case, expected consumer surplus also increases when there are fewer or more patient firms—so long as $\delta \geq (n-1)/n$, so that collusion on the no-disclosure monopoly price $p^m\left(\mathbb{E}^F[s]\right)$ is an equilibrium. The explanation is that, once $\delta \geq (n-1)/n$, collusive prices equal monopoly prices for some information structure, so when consumers benefit from improving a monopoly's information, they also benefit from the more flexible collusive prices that result from reducing n or increasing δ.

4.3 Discussion: Connection to the RealPage Case

We discuss how our model mechanisms and results relate to the RealPage case. First, we argue that RealPage intentionally withholds some information from its clients, and that this is plausibly done to make it easier to control its clients' pricing (and specifically to prevent them from undercutting RealPage's recommended prices). Second—and more speculatively—we argue that the resulting price dynamics over the business cycle appear consistent with our prediction of price cuts during downturns and price

[29] A version of the result that improving a monopoly's information about its constant marginal cost increases expected consumer surplus was shown by Vives (2001).

rigidity during booms.

RealPage uses its superior data to recommend prices to its clients (indeed, this is the core business that drew regulatory scrutiny), but it does not make the data itself available to its clients. Furthermore, RealPage does not provide more detailed data even if clients explicitly request it. The stated reason for this practice appears to be to reduce antitrust risk, rather than preventing price deviations as in our model. As the DOJ reports (p. 48), "When a property owner asked for information on specific competitors, Landlord 3's director of revenue management replied that the requested tool, RealPage's Performance Analytics with Benchmarking, did not provide information on specific competitors. The reason? Performance Analytics with Benchmarking 'tracks transactional information therefore due [to] the potential pricing collusion, it's anonymize[d] by RealPage,'."

However, RealPage takes a number of steps to prevent price deviations by its clients, and it is very plausible that withholding detailed demand information is helpful in this regard. RealPage's efforts to control its clients' pricing are well-documented and are a centerpiece of the DOJ complaint and subsequent settlement. As the complaint puts it, "[RealPage]'s recommendations are more than just 'recommendations'," (p. 3), because "RealPage uses multiple mechanisms to increase compliance with price recommendations," (p. 23). These include several mechanisms that make accepting recommendations easier than declining them, including encouraging the use of an "Auto Accept" feature; allowing managers to accept recommendations in bulk but requiring them to decline recommendations one by one; and elevating attempted price overrides to a RealPage "pricing advisor," who may further escalate the disagreement to a regional manager. According to one RealPage client, RealPage's design is "trying to persuade [clients] to take the recommendations (almost like we made it hard to do anything but)," (p. 24).[30]

[30]It would be straightforward to incorporate these mechanisms in our model by assuming that a firm must pay an "override cost" to set a price other than the recommended one. This would just relax firms' incentive constraints by an amount equal to the override cost, leaving our qualitative results unchanged. It would also give natural comparative statics: e.g., a higher override cost leads

Withholding information from its clients is another means through which RealPage can prevent them from overriding its recommendations. The logic of the revelation principle (Myerson, 1982) implies that communicating any information to its clients beyond their recommended prices can only tempt the clients to override RealPage's recommendations. We lack direct evidence that RealPage consciously withholds information from its clients *in order to* reduce their temptation to override. However, such evidence is available in other settings where a cartel facilitator manages firms' information to support collusion.[31] Overall, it is clear that RealPage takes multiple measures to prevent its clients from overriding its recommendations; that RealPage has detailed demand information that it withholds from its clients; and that withholding this information reduces its clients' potential gains from overriding its recommendations and thus facilitates their acceptance.

We can also ask whether the resulting pricing patterns over the business cycle fit the predictions of our model. We focus on our central prediction that algorithmic intermediaries disclose more precise information in economic downturns—leading to lower and more flexible prices—and coarser information in booms—leading to higher and more rigid prices. The evidence in the DOJ complaint and Calder-Wang and Kim on the role of algorithmic pricing over the business cycle is broadly consistent with this pattern. First, Calder-Wang and Kim find that the calendar-year treatment effect of RealPage adoption on price is strongly negative during the financial crisis and modestly positive in other years (see their Figure 7). This is certainly not a direct test of our theory, but it is consistent with viewing RealPage as providing information in accordance with the theory, where, relative to non-adopters, adopters receive precise bad news about demand in down markets (leading to sharply lower

to higher collusive prices and lower consumer surplus.

[31] For example, Harrington (2006) reports that at meetings of the European isostatic graphic cartel, the firms' representatives passed around a calculator where each representative secretly entered their own sales volume, so that ultimately only the sum of the reported sales was observable, which let each firm calculate their own market share but not their competitors'. For more on this and other related examples, see Sugaya and Wolitzky (2018).

prices) and receive coarse good news about demand in normal and strong markets (leading to modestly higher prices).[32]

Furthermore, the DOJ complaint provides evidence that RealPage's data and analytics play an especially important role in price-setting during downturns. The complaint asserts that, "in down markets… [RealPage] instills pricing discipline in landlords, curbing normal fully independent competitive reactions by substituting them with interdependent decision-making," (p. 47). Indeed, RealPage itself advertised that the "AI and the robust data in the RealPage ecosystem" helps its clients "avoid the race to the bottom in down markets," (p. 46) and "curbs [clients'] instincts to respond to down-market conditions by either dramatically lowering price or by holding price when they are losing velocity and/or occupancy," (p. 47)—moreover, given that Calder-Wang and Kim find a strong negative treatment effect of RealPage adoption on price in down markets, these quotations should be interpreted as emphasizing RealPage clients' high prices in down markets relative to competitive pricing, not relative to collusion without RealPage's demand information. RealPage also emphasizes the particular value of its price recommendations in down markets, arguing that, without its recommendations, "you'll be pricing your renewals in the dark without insight into actual lease transaction data that [YieldStar] uses to help you make pricing decision. This is critical to price renewals right[,] especially in a downturn," (p. 14). These quotations are all consistent with RealPage providing more precise information in down markets. In sum, while the available evidence is admittedly limited, the evidence in both the DOJ complaint and Calder-Wang and Kim is broadly consistent with our prediction that RealPage provides more precise demand information and recommends more flexible prices in downturns than in booms.

[32] This comparison assumes that both RealPage adopters and non-adopters sustain markups relative to the static Nash benchmark, differing primarily in how markups respond to demand conditions. This viewpoint is suggested by Calder-Wang and Kim's reduced-form results, which find strongly time-varying RealPage adoption effects but little to no time-averaged adoption effect.

5 Persistent Demand or Cost

We now consider the case where the state follows a Markov process: we assume that the current state s' is drawn from a distribution F_s, where s is the previous period's state. This realistic extension of the baseline iid model illustrates how our results generalize and also yields some new insights. The analysis of this section is inspired by Haltiwanger and Harrington (1991), Kandori (1992), and Bagwell and Staiger (1997), who extended Rotemberg and Saloner's (1986) iid model to various Markov processes.

To accommodate the Markov case, we need to preserve the property that expected current and future profit is measurable with respect to current mean demand (or cost, but we continue to focus on the demand case). This require two assumptions. First, we assume that the current demand state is revealed at the end of each period, so the algorithm does not carry private information across periods. This is realistic if firms observe their sales at the end of the each period.[33] Second, we assume that the Markov transition rule F_s is affine in s, so the distribution over tomorrow's state depends only on today's mean state:

$$F_s\left(s'\right) = \frac{\bar{s}-s}{\bar{s}-\underline{s}} F_{\underline{s}}\left(s'\right) + \frac{s-\underline{s}}{\bar{s}-\underline{s}} F_{\bar{s}}\left(s'\right) \qquad \text{for all } s, s'.$$

For example, F_s is affine in s when there is a binary underlying demand state $\mathbf{s}$ and s is a continuous signal of $\mathbf{s}$ satisfying $\Pr\left(\mathbf{s}=\bar{s}|s\right) = \left(s-\underline{s}\right)/\left(\bar{s}-\underline{s}\right)$. We also assume that the distribution of s in period 1 equals F_{s_0} for some $s_0 \in [\underline{s}, \bar{s}]$.

Affineness allows both *positive persistence*—where $F_{\bar{s}}$ first-order stochastically dominates $F_{\underline{s}}$—and *negative persistence*—where $F_{\underline{s}}$ first-order stochastically dominates $F_{\bar{s}}$. Both cases are of interest: positive persistence is arguably more natural, while negative persistence has been used to capture cyclical demand movements

[33]The assumption that the state is revealed at the end of each period distinguishes our model from dynamic information design, which can involve a tradeoff between optimal disclosure in the current period and in future periods (e.g., Ely, 2017; Renault, Solan, and Vieille, 2017).

(Haltiwanger and Harrington, 1991).

The characterization of the optimal disclosure policy and collusive prices are the same as in the iid case, except that now the expected value of collusive profit $V(s)$ depends on the previous period's state s. The optimal collusive profit for each last-period demand state s must now be calculated simultaneously as the component-wise greatest fixed point $(V^*(s))_{s\in[\underline{s},\bar{s}]}$ of the following system of equations in s:

$$V(s) = (1-\delta) \max_{G\in MPC(F_s)} \mathbb{E}^{x\sim G}\left[\min\left\{\Pi^m(x), \Pi^{\max}\left(\delta, n, \mathbb{E}^{F_x}[V(s')]\right)\right\}\right] + \delta\mathbb{E}^{F_s}[V(s')]. \tag{5}$$

The right-hand side of (5) is bounded and increasing in $V(s')$ for all s, s', so the greatest fixed point is well-defined by Tarski's theorem. We also define $W^*(s) = \mathbb{E}^{F_s}[V^*(s')]$, so we have

$$V^*(s) = (1-\delta) \max_{G\in MPC(F_s)} \mathbb{E}^{G}[\min\{\Pi^m(x), \Pi^{\max}(\delta, n, W^*(x))\}] + \delta W^*(s) \quad \text{for all } s. \tag{6}$$

Note that, since F_s is affine in s, so is $W^*(s)$.

With persistent demand, the appropriate notion of a (symmetric) stationary strategy is that the disclosure policy G depends only the previous period's demand state, while the on-path price $p(s)$ at realized mean demand state s remains independent of the history of past demand realizations (and, in particular, is independent of the current-period disclosure policy). With this definition, Lemma 1 generalizes as follows.

Lemma 3 *The expected present value of optimal collusive profit in each state s equals $V^*(s)$ and is attained by a symmetric, stationary, grim trigger equilibrium. Moreover, a collection of disclosure policies $(G_s)_{s\in[\underline{s},\bar{s}]}$, one for each last-period demand state s, is optimal if and only if, for each s, G_s solves the maximization problem in (5) with $V(\cdot) = V^*(\cdot)$.*

Lemma 3 reduces the problem of finding an optimal equilibrium to the family of static information design problems on the right-hand side of (5), where the function $V^*(\cdot)$ satisfies the fixed point condition.[34]

As in the iid case, collusion is impossible if $\delta < (n-1)/n$. Conversely, if $\delta \geq (n-1)/n$ then monopoly profit under no disclosure given the least-favorable previous period demand state (e.g., $\underline{s}$ in the positively persistent case; $\bar{s}$ in the negatively persistent case), $\Pi^m\left(\min\left\{\mathbb{E}^{F_{\underline{s}}}[s], \mathbb{E}^{F_{\bar{s}}}[s]\right\}\right)$, is attainable for any initial state.

Lemma 4 *If $\delta < (n-1)/n$ then $V^*(s) = 0$ for all s. Conversely, if $\delta \geq (n-1)/n$ then $V^*(s) \geq \Pi^m\left(\min\left\{\mathbb{E}^{F_{\underline{s}}}[s], \mathbb{E}^{F_{\bar{s}}}[s]\right\}\right)$ for all s.*

Proof. If $\delta < (n-1)/n$ then $\Pi^{\max}(\delta, n, V) < V$ for all $V > 0$. Let $s_0 = \operatorname{argmax}_s V^*(s)$, which is well-defined because $\Pi^m(s)$ is continuous and $W^*(s)$ is affine. Suppose for contradiction that $V^*(s_0) > 0$. Then, since $W^*(s) \leq V^*(s_0)$ for all s (as $W^*(s) = \mathbb{E}^{F_s}[V^*(s')]$), the right-hand side of (6) at $s = s_0$ is strictly less than $V^*(s_0)$, a contradiction. Hence, $V^*(s_0) = 0$, and therefore $V^*(s) = 0$ for all s.

Conversely, if $\delta \geq (n-1)/n$ then under no information disclosure it is an equilibrium to set on-path price $\min\left\{p : \Pi\left(p, \mathbb{E}^{F_s}[\tilde{s}]\right) = \Pi^m\left(\min\left\{\mathbb{E}^{F_{\underline{s}}}[\tilde{s}], \mathbb{E}^{F_{\bar{s}}}[\tilde{s}]\right\}\right)\right\}$ when the previous period demand state is s (noting that this price is well-defined by the intermediate value theorem, as $\Pi\left(p, \mathbb{E}^{F_s}[\tilde{s}]\right)$ is continuous in p, $\Pi\left(0, \mathbb{E}^{F_s}[\tilde{s}]\right) = 0$, and $\Pi\left(p^m\left(\mathbb{E}^{F_s}[\tilde{s}]\right), \mathbb{E}^{F_s}[\tilde{s}]\right) = \Pi^m\left(\mathbb{E}^{F_s}[\tilde{s}]\right) \geq \Pi^m\left(\min\left\{\mathbb{E}^{F_{\underline{s}}}[\tilde{s}], \mathbb{E}^{F_{\bar{s}}}[\tilde{s}]\right\}\right)$) and off-path price zero. ■

We now characterize the optimal disclosure policy as a function of the last-period state s in the non-trivial case where $\delta \geq (n-1)/n$. First, let s^* solve

$$\Pi^m(s^*) = \Pi^{\max}(\delta, n, W^*(s^*)) \tag{7}$$

if such a demand state exists, and let $s^* = \bar{s}$ otherwise.[35] Next, for each last-period

[34] The proof is a straightforward generalization of the proof of Lemma 1: the only difference is that the present value of equilibrium profits, the probability distribution over next-period demand states, and the values $\bar{V}$ and v_i defined in the second part of the proof are all now functions of the current expected state s.

[35] There is at most one solution to (7). If $W^*(s)$ is decreasing, this is immediate, as the

state s, let $\hat{s}(s)$ satisfy

$$\mathbb{E}^{F_s}[\tilde{s}|\tilde{s} \geq \hat{s}(s)] = s^* \tag{8}$$

if such a state exists, and let $\hat{s}(s) = \underline{s}$ otherwise. Note that, by Lemma 4 and our assumption that $\delta \geq (n-1)/n$, we have $V^*(\underline{s}) \geq \Pi^m\left(\min\left\{\mathbb{E}^{F_{\underline{s}}}[s], \mathbb{E}^{F_{\bar{s}}}[s]\right\}\right)$, and hence $s^* \geq \min\left\{\mathbb{E}^{F_{\underline{s}}}[s], \mathbb{E}^{F_{\bar{s}}}[s]\right\}$, so (8) admits a solution $\hat{s}(s) \in [\underline{s}, \bar{s}]$ for $s = \text{argmin}_{s \in \{\underline{s}, \bar{s}\}} \mathbb{E}^{F_s}[s']$. However, in contrast to the iid case, (8) does not always admit a solution $\hat{s}(s)$ for all last-period demand states s: in this case, the distribution F_s is so high that $\mathbb{E}^{F_s}[s'] > s^*$, in which case no disclosure of the current demand state is optimal, and the optimal price is $\min\left\{p : \Pi\left(p, \mathbb{E}^{F_s}[s']\right) = \Pi^{\max}\left(\delta, n, W^*\left(\mathbb{E}^{F_s}[s']\right)\right)\right\}$, which is less than the corresponding monopoly price $p^m\left(\mathbb{E}^{F_s}[s']\right)$. Thus, in the Markov case, following last-period states that make firms sufficiently optimistic about the current state, the optimal collusive policy can entail no information disclosure and a price below the corresponding monopoly price.

We can now characterize the optimal disclosure and pricing policy in the Markov case.

Theorem 2 *With stochastic demand, the unique optimal disclosure policy as a function of the last-period demand state s is the* upper censorship *policy that discloses demand states below $\hat{s}(s)$ and conceals demand states above $\hat{s}(s)$. The optimal collusive price $p(\tilde{s}; s)$ (which is unique except when $\hat{s}(s) = \underline{s}$) when the current realized mean demand state is $\tilde{s}$ and the last-period demand state is s is given by*

$$p(\tilde{s}; s) = \begin{cases} p^m(\tilde{s}) & \text{if } \tilde{s} < \hat{s}(s), \\ p^m(s^*) & \text{if } \tilde{s} \geq \hat{s}(s) > \underline{s}, \\ \min\left\{p : \Pi\left(p, \mathbb{E}^{F_s}[s']\right) = \Pi^{\max}\left(\delta, n, W^*\left(\mathbb{E}^{F_s}[s']\right)\right)\right\} & \text{if } \hat{s}(s) = \underline{s}. \end{cases} \tag{9}$$

left-hand side of (7) is increasing and the right-hand side is decreasing. If $W^*(s)$ is increasing, this follows because, since $\delta \geq (n-1)/n$, we have $\Pi^m(\underline{s}) \leq \Pi^{\max}(\delta, n, \Pi^m(\underline{s})) < \Pi^{\max}\left(\delta, n, \Pi^m(\min\left\{\mathbb{E}^{F_{\underline{s}}}[s], \mathbb{E}^{F_{\bar{s}}}[s]\right\})\right) \leq \Pi^{\max}(\delta, n, W^*(\underline{s}))$, and the left-hand side of (7) is convex while the right-hand side is linear.

Moreover, under positive persistence, $\hat{s}(s)$ is decreasing, so the optimal policy discloses less information when last-period demand is higher; conversely, under negative persistence, $\hat{s}(s)$ is increasing, so the optimal policy discloses more information when last-period demand is higher.

With stochastic costs, the unique optimal disclosure policy is the analogous lower censorship *policy that discloses cost states above $\hat{s}(s)$ satisfying $\mathbb{E}^{F_s}[\tilde{s}|\tilde{s} \leq \hat{s}(s)] = s^*$ and conceals cost states below $\hat{s}(s)$. The unique optimal collusive price $p(s)$ in state s is given by (9) with the reversed inequalities and $\bar{s}$ in place of $\underline{s}$. Moreover, under positive persistence, $\hat{s}(s)$ is decreasing, so the optimal policy discloses more information when last-period cost is higher; conversely, under negative persistence, $\hat{s}(s)$ is increasing, so the optimal policy discloses less information when last-period cost is higher.*

Proof. The proof is a straightforward generalization of the proof of Theorem 1. The main difference is that, since $W^*(s)$ is affine, the function $\min\{\Pi^m(s), \Pi^{\max}(\delta, n, W^*(s))\}$ is now "convex-then-linear" in s, rather than "convex-then-constant" as in the iid case. The same argument as in the proof of Theorem 1 implies that, when $\hat{s}(s) > \underline{s}$, upper censorship is optimal, with mean demand among concealed states equal to the point s^* where $\Pi^m(s^*) = \Pi^{\max}(\delta, n, W^*(s^*))$. A similar argument shows that, when $\hat{s}(s) = \underline{s}$, no disclosure is optimal, with a price p being the smallest price to satisfy $\Pi\left(p, \mathbb{E}^{F_s}[s']\right) = \Pi^{\max}\left(\delta, n, W^*\left(\mathbb{E}^{F_s}[s']\right)\right)$. Finally, it is immediate from (8) that $\hat{s}(s)$ is decreasing under positive persistence and increasing under negative persistence. ■

The new insights of Theorem 2 concern how optimal disclosure depends on last-period demand. When the optimal disclosure policy is non-trivial (i.e., $\hat{s}(s) \in (\underline{s}, \bar{s})$, so some states are disclosed and others are censored), the mean censored state is fixed at s^*, regardless of the last-period demand state s. With positive persistence, this requires greater censoring (lower $\hat{s}$) when the last-period demand state is higher: intuitively, the algorithm discloses less information following good periods, when firms are optimistic about current demand and are thus more tempted to deviate. Conversely, with negative persistence, the algorithm discloses more information following good periods, when firms are pessimistic and are thus less tempted to deviate.

In contrast to the iid case, for certain last-period states it can be optimal for the algorithm to disclose no information and recommend prices below the corresponding monopoly price.[36] For example, with positive persistence, it can be optimal to fully reveal current demand when last-period demand was low (so $\hat{s}(s) = \bar{s}$), partially reveal current demand when last-period demand was intermediate (so $\hat{s}(s) \in (\underline{s}, \bar{s})$), and reveal nothing about current demand when last-period demand was high (so $\hat{s}(s) = \underline{s}$)—moreover, in the last case, the optimal price $p\left(\mathbb{E}^{F_s}[s']; s\right)$ satisfies $\Pi\left(p, \mathbb{E}^{F_s}[s']\right) = \Pi^{\max}\left(\delta, n, W^*\left(\mathbb{E}^{F_s}[s']\right)\right)$, and so is less than the monopoly price $p^m\left(\mathbb{E}^{F_s}[s']\right)$ and can even be decreasing in s. Thus, while collusive prices are always monotone in current demand (as in the iid case and in contrast to Rotemberg Saloner), they may be non-monotone in last-period demand. Notably, the expected price conditional on last-period demand can be single-peaked, a result that recovers some of Rotemberg and Saloner's intuition.[37]

In addition to these novel points, Theorem 2 shows that the main results from the iid case generalize to the Markov case. In particular, for any last-period demand state s, optimal collusion entails price rigidity at high current demand states, supra-monopoly prices over an intermediate range of states, and more flexible prices when n is lower, δ is higher, or F_s is more informative.

Finally, in Appendix D, we provide a numerical example showing that the effect of greater demand persistence on collusive profit, consumer surplus, and the amount of information disclosure can all be non-monotone.

[36] Recall that in the iid case, no disclosure is only optimal in the knife-edge case where $\delta = (n-1)/n$.

[37] Whether prices actually display this pattern depends on whether $\Pi(p, \tilde{s})$ or $\Pi^{\max}(\delta, n, W^*(\tilde{s}))$ increases faster in $\tilde{s} = \mathbb{E}^{F_s}[s']$ over the range $\{s : \hat{s}(s) = \underline{s}\}$.

6 Conclusion

This paper has developed a tractable model of an intermediary that possesses information on market demand or the cost of serving the market that is superior to that of the firms competing for the market and that selectively discloses this information to maximize the firms' profit in the best collusive equilibrium. Our main motivation is the rise of third-party pricing algorithm providers such as RealPage in apartment rentals, A2i Systems and Kalibrate in retail gasoline, and IDeaS and Rainmaker in hotel rooms. We adapt the canonical Rotemberg Saloner (1986) model of repeated Bertrand competition with stochastic demand by letting an intermediary selectively disclose demand or cost information. Assuming that expected profit is determined by the expected state, we show that with homogeneous goods, optimal information disclosure takes an *upper censorship* form: demand states s below a cutoff $\hat{s}$ are disclosed and result in the corresponding monopoly price $p^m(s)$, while demand states above $\hat{s}$ are concealed and result in the monopoly price for the mean concealed state, $p^m(\mathbb{E}[s|s > \hat{s}])$. The resulting pricing policy entails *price rigidity*, as well as *supra-monopoly prices* for a range of intermediate demand states. Prices are more flexible when the market is more concentrated, the firms are more patient, or the algorithm is more accurate. In turn, price flexibility reduces expected consumer surplus whenever improving a monopoly's information does so. This result suggests that improved algorithmic demand prediction is likely to reduce expected consumer surplus, in contrast to prior studies that find more optimistic results when the algorithm always discloses its predictions (Sugaya and Wolitzky, 2018; Miklós-Thal and Tucker, 2019). Finally, most results survive in more general specifications with demand persistence, product differentiation, or capacity constraints, although the specific form of the optimal censorship policy depends on the demand system, and collusive prices can sometimes fall short of the corresponding monopoly prices.

As for applications, we have focused on the recent RealPage case, but the theory can also potentially speak to a number of other cases. Cases concerning algorithmic pricing of hotel rooms (e.g., IDeaS and Rainmaker) raise similar issues to RealPage. Algorithmic pricing of retail gasoline also raises similar issues, although the empirical literature on this topic to date (e.g., Assad et al., 2024) has not emphasized business cycle fluctuations. Moreover, while our main motivation is algorithmic collusion facilitation, the theory applies equally to any cartel facilitator that controls the participating firms' information. An important example is information-sharing in bidding rings in procurement auctions, such as the bidding ring organized by the Kumatori Contractors Cooperative, studied by Kawai, Nakabayashi, and Ortner (2026). This ring took drastic steps to limit bidders' information about the cost of completing (only) the largest construction project they bid on: "[The director of the Cooperative] told the members that he would be collecting, from each of the invited bidders, the detailed project plan that the town distributes at the on-site briefing. This was understood by the members of the Cooperative as a preventative measure to make defection more difficult by making it harder for other firms to estimate costs," (ibid.). This behavior matches our result that cartels censor information in those states of the world where deviating is most tempting.[38]

Many of our assumptions can be further relaxed at the cost of a more intricate analysis. First, if expected profit depended on the entire distribution of the unknown state rather than only its mean, we would have a non-linear information design problem, where disclosure policies that pool intervals of states together (like upper censorship) are typically sub-optimal (Kolotilin, Corrao, and Wolitzky, 2025). However, upper censorship is approximately optimal if the information design problem is close to linear.[39] Second, if the intermediary's objective differs from maximizing industry profit or if some firms do not use the intermediary, as in Harrington (2022) and

[38] A subtlety in mapping this example to our model is that the "tempting state" is a large procurement project, which has both a higher (observed) reserve price and a higher (unobserved) completion cost than a typical project. To capture this example, one could extend the model to allow both an observable stochastic component (the reserve price) and an unobserved one (the cost).

[39] It is also likely that, if the problem is close to linear, every optimal disclosure policy approximates upper censorship. A result along these lines in a related class of information design problems is Theorem 3 of Kolotilin and Wolitzky (2026).

Harrington and Ortner (2025), the model must be extended to incorporate the intermediary's incentives and firms' incentives to use the intermediary. Third, if the colluding firms are asymmetric, there is no longer a unique cartel-optimal equilibrium, and one must instead analyze the Pareto frontier of the equilibrium payoff set and consider non-stationary equilibria as in Abreu (1986). Fourth, in practice, algorithmic intermediaries can also facilitate collusion by systematizing monitoring of firms' prices. This could be incorporated by considering an imperfect monitoring version of our model.[40] Fifth, as discussed in Section 4.3, algorithmic intermediaries like RealPage sometimes take steps to make it difficult for firms to override their recommendations. This could be addressed in an extended model where the intermediary partially delegates price-setting to its clients. Finally, allowing private price recommendations from the algorithm—or private information for the firms, which could possibly be elicited by the algorithm—appears challenging but potentially realistic and insightful.[41] These are all interesting directions for future research.

A Generalized Model

In this appendix, we derive a version of our main result for a general class of payoff functions that includes the homogeneous-goods model analyzed in the text, as well as a differentiated-goods model and a model of homogeneous goods with capacity constraints.

[40] Recent algorithmic cartel facilitation cases where the central issue appears to be facilitating monitoring of competitors' actions rather than demand conditions involve the data analytics companies AgriStats and Circana, which provide detailed production and price information for the poultry and pork markets (AgriStats) and the frozen potato market (Circana).

[41] The former question would combine the forces in the present paper with those in Ortner, Sugaya, and Wolitzky (2024). The latter would concern optimal Bayes correlated equilibria (as in, e.g., Smolin and Yamashita, 2025) or communication equilibria (as in, e.g., Goltsman and Pavlov, 2014).

A.1 General Payoff Structure

Let $\pi_i(\mathbf{p}, s)$ denote firm i's profit at price vector $\mathbf{p} = (p_1, \ldots, p_n)$ and demand state s.[42] We define firm i's maximum deviation profit at price vector $\mathbf{p}$ by

$$\pi_i^d(\mathbf{p}, s) = \max_{p_i} \pi_i(p_i, \mathbf{p}_{-i}, s).$$

We impose four assumptions on payoffs. First, we assume that payoffs are symmetric, continuous, and quasi-concave in a firm's own price.[43]

Assumption 1 For any price vector $(p_1, \ldots, p_n)$, state s, firm i, and permutation ϕ on $\{1, \ldots, n\}$, we have

$$\pi_i\left(p_{\phi(1)}, \ldots, p_{\phi(n)}, s\right) = \pi_{\phi(i)}(p_1, \ldots, p_n, s).$$

In addition, $\pi_i(\mathbf{p}, s)$ is continuous in $\mathbf{p}$ and quasi-concave in p_i for all $\mathbf{p}_{-i}$.

Second, we assume that $\pi_i(\mathbf{p}, s)$ is affinely increasing in s.

Assumption 2 For any price vector $\mathbf{p}$ and firm i, we have $\pi_i(\mathbf{p}, \underline{s}) \leq \pi_i(\mathbf{p}, \bar{s})$ and

$$\pi_i(\mathbf{p}, s) = \frac{\bar{s} - s}{\bar{s} - \underline{s}} \pi_i(\mathbf{p}, \underline{s}) + \frac{s - \underline{s}}{\bar{s} - \underline{s}} \pi_i(\mathbf{p}, \bar{s}) \qquad \text{for all } s.$$

Assumptions 1 and 2 imply that, for any mean public belief s, there exists a symmetric, pure-strategy, static Nash equilibrium $\mathbf{p}$. Let $\underline{\pi}$ denote the lowest payoff from such a Nash equilibrium at mean public belief $\mathbb{E}^F[s]$. We characterize the

[42] This appendix focuses on stochastic demand. The stochastic cost case is analogous.

[43] The symmetry notion in Assumption 1 is known as *total symmetry*. The results in this appendix also hold under the weaker notion of *weak symmetry*: for any pair of firms i and j, there exists a permutation ϕ on $\{1, \ldots, n\}$ such that $\phi(i) = j$ and, for any price vector $(p_1, \ldots, p_n)$, state s, and firm k, we have

$$\pi_k\left(p_{\phi(1)}, \ldots, p_{\phi(n)}, s\right) = \pi_{\phi(k)}(p_1, \ldots, p_n, s).$$

The examples in the current paper are totally symmetric, but some oligopoly models (e.g., Salop's circle model) are only weakly symmetric. See Plan (2023).

pure-strategy, subgame perfect equilibrium that maximizes collusive profit among all equilibria where off-path expected profit equals $\underline{\pi}$. In contrast to the homogeneous goods case analyzed in the text, where the static Nash profit equals the minimax payoff of 0, here $\underline{\pi}$ can be strictly greater than the minimax payoff. We are thus focusing on equilibria sustained by the threat of *Nash reversion.* This entails a loss of optimality in the class of all pure-strategy, subgame perfect equilibria: to find the optimal equilibrium in this larger class, one would simultaneously find the worst equilibrium for each firm as a fixed point, following Abreu (1988). However, except for giving a different value for the off-path payoff $\underline{\pi}$, this procedure would yield the same characterization of on-path prices and information disclosure. Our qualitative results are thus insensitive to the specification of off-path payoffs.

Next, define per-firm profit when all firms set the same price p in state s by

$$\pi(p,s) = \pi_i(p,\ldots,p,s),$$

and define the corresponding maximum deviation profit by

$$\pi^d(p,s) = \pi_i^d(p,\ldots,p,s).$$

Our third assumption is that, for any price vector, there exists a common price that weakly increases profit without increasing the firms' average deviation gain.

Assumption 3 For any price vector $\mathbf{p}$ and state s, there exists a price $p \geq 0$ such that

$$\pi(p,s) \geq \frac{1}{n}\sum_i \pi_i(\mathbf{p},s) \qquad \text{and} \qquad \pi^d(p,s) - \pi(p,s) \leq \frac{1}{n}\sum_i \left(\pi_i^d(\mathbf{p},s) - \pi_i(\mathbf{p},s)\right).$$

Finally, we assume that $\pi(p,s)$ is strictly quasi-concave in p and that the deviation gain at the monopoly price is increasing in s.

Assumption 4 $\pi(p,s)$ is strictly quasi-concave in p with a well-defined monopoly price $p^m(s)$ and monopoly profit

$$\pi^m(s) = \pi(p^m(s), s) = \max_p \pi(p,s)$$

for each s. Moreover, the deviation gain

$$\pi^d(p^m(s), s) - \pi(p^m(s), s)$$

is strictly increasing in s.

Assumptions 1–4 are satisfied in the baseline homogeneous-goods model, as well as in two additional examples described below: differentiated goods with a linear demand system, and homogeneous goods with capacity constraints.

We now characterize the optimal collusive profit, prices, and information disclosure policy in this general model. For each v, let $s^*(v)$ solve

$$\pi^d(p^m(s), s) - \pi(p^m(s), s) = \frac{\delta}{1-\delta}(v - \underline{\pi}),$$

if such an s exists, and let $s^*(v) = \bar{s}$ otherwise. By Assumption 4, $s^*(v)$ is unique. For $s \le s^*(v)$, let $p^{\max}(s,v) = p^m(s)$, and for $s > s^*(v)$, let $p^{\max}(s,v) \in \arg\max_{p\ge 0} \pi(p,s)$ subject to

$$\pi^d(p,s) - \pi(p,s) \le \frac{\delta}{1-\delta}(v - \underline{\pi}). \tag{10}$$

Next, let

$$\pi^{\max}(s,v) = \pi(p^{\max}(s,v), s).$$

Note that $\pi^{\max}(s,v) = \pi^m(s)$ if $s \le s^*(v)$, while $\pi^{\max}(s,v) < \pi^m(s)$ if $s > s^*(v)$. In addition, $\pi^{\max}(s,v)$ is non-decreasing in v for all s, since increasing v relaxes (10).

Finally, let v^* be the greatest fixed point of the equation

$$v = \max_{G \in MPC(F)} \mathbb{E}^G \left[\pi^{\max}(s, v) \right], \tag{11}$$

which is well-defined by Tarski's theorem, since $\pi^{\max}(s, v)$ is non-decreasing in v and bounded, and let

$$s^* = s^*(v^*).$$[44]

Lemma 1 extends as follows.

Lemma 5 *Among equilibria with off-path payoffs $\underline{\pi}$, optimal per-firm collusive profit equals v^* and is attained by a symmetric, stationary, grim trigger equilibrium. Moreover, a disclosure policy G is optimal if and only if it solves the maximization problem in (11) with $v = v^*$.*

Proof. The construction of a symmetric, stationary, grim trigger equilibrium that attains per-firm collusive profit v^* is the same as in the proof of Lemma 1.

We show that no equilibrium can attain higher profit. Fix any equilibrium, and define $\bar{v}$ as in the proof of Lemma 1. Fix an arbitrary period t and a history of play up to period t, and suppose that when the realized mean demand in period t at this history is s, the equilibrium price vector is $\mathbf{p}(s)$ and firm i's equilibrium continuation payoff is v_i. The resulting incentive constraint for firm i is

$$(1-\delta)\pi_i^d(\mathbf{p}(s), s) + \delta\underline{\pi} \leq (1-\delta)\pi_i(\mathbf{p}(s), s) + \delta v_i.$$

Averaging this inequality over the n firms, we obtain

$$\begin{aligned}(1-\delta)\frac{1}{n}\sum_i \pi_i^d(\mathbf{p}(s), s) &\leq (1-\delta)\frac{1}{n}\sum_i \pi_i(\mathbf{p}(s), s) + \delta\frac{1}{n}\sum_i (v_i - \underline{\pi}) \\ &\leq (1-\delta)\frac{1}{n}\sum_i \pi_i(\mathbf{p}(s), s) + \delta(\bar{v} - \underline{\pi}),\end{aligned}$$

[44] Here, we use lower-case letters for per-firm payoffs, whereas in the baseline model we used capital letters for the corresponding industry payoffs.

where the second inequality is by definition of $\bar{v}$. Therefore,

$$\frac{1}{n}\sum_{i}\left(\pi_{i}^{d}\left(\mathbf{p}\left(s\right),s\right)-\pi_{i}\left(\mathbf{p}\left(s\right),s\right)\right)\leq\frac{\delta}{1-\delta}\left(\bar{v}-\underline{\pi}\right).$$

By Assumption 3, there exists $p\left(s\right)\geq 0$ such that

$$\begin{aligned}\pi\left(p\left(s\right),s\right) &\geq \frac{1}{n}\sum_{i}\pi_{i}\left(\mathbf{p}\left(s\right),s\right) \qquad \text{and}\\ \pi^{d}\left(p\left(s\right),s\right)-\pi\left(p\left(s\right),s\right) &\leq \frac{\delta}{1-\delta}\left(\bar{v}-\underline{\pi}\right).\end{aligned}$$

Since $p^{\max}\left(s,\bar{v}\right)$ maximizes $\pi\left(p,s\right)$ subject to $\pi^{d}\left(p,s\right)-\pi\left(p,s\right)\leq\left(\delta/\left(1-\delta\right)\right)\left(\bar{v}-\underline{\pi}\right)$, expected collusive profit in period t is at most

$$\max_{G\in MPC(F)}\mathbb{E}^{G}\left[\pi^{\max}\left(s,\bar{v}\right)\right].$$

Since this holds for every period t and history, we have

$$\bar{v}\leq\max_{G\in MPC(F)}\mathbb{E}^{G}\left[\pi^{\max}\left(s,\bar{v}\right)\right].$$

Hence $\bar{v}\leq v^{*}$, by definition of v^{*} as the greatest fixed point of (11). ■

It remains to solve the information design problem in (11). In general, this problem can be solved using techniques from the static information design literature (e.g., Dworczak and Martini, 2019), and the solution depends on the shape of $\pi^{\max}\left(s,v\right)$ as a function of s. However, an explicit solution is available under the following condition, which holds in our examples.[45]

Condition 1 $\pi^{\max}\left(s,v\right)$ *is convex on* $s\in\left[s^{*}\left(v\right),\bar{s}\right]$, *for any* v.

Since $\pi^{m}\left(s\right)$ is convex in s and $\pi^{\max}\left(s,v^{*}\right)=\pi^{m}\left(s\right)$ if $s\leq s^{*}$, Condition 1 implies that either $\pi^{\max}\left(s,v^{*}\right)$ is globally convex in s, or it is piecewise-convex with a concave

[45]It suffices that $\pi^{\max}\left(s,v^{*}\right)$ is convex in s for $s\geq s^{*}$, but this weaker condition depends on the endogenous object v^{*}.

kink at $s = s^*$. The linear demand system and homogeneous goods with capacity constraints examples below illustrate these cases, respectively.

To state the generalized version of Theorem 1, define a pair of states $(\hat{s}_L, \hat{s}_H)$ as follows. If $\pi^{\max}(s, v^*)$ is globally convex, define $\hat{s}_L = \hat{s}_H = s^*$. Otherwise, define $(\hat{s}_L, \hat{s}_H)$ so that

$$\begin{aligned} \hat{s}_L &< \hat{s}_H, \\ \mathbb{E}^F[s|s \in [\hat{s}_L, \hat{s}_H]] &= s^*, \qquad \text{and} \\ \frac{\hat{s}_H - s^*}{\hat{s}_H - \hat{s}_L}\pi^m(\hat{s}_L) + \frac{s^* - \hat{s}_L}{\hat{s}_H - \hat{s}_L}\pi^{\max}(\hat{s}_H, v^*) &= \pi^m(s^*), \end{aligned}$$

if such a pair exists. Otherwise, if $s^* \leq \mathbb{E}^F[s]$, define $\hat{s}_L = \underline{s}$ and define $\hat{s}_H$ so that $s^* = \mathbb{E}^F[s|s \leq \hat{s}_H]$; and if $s^* \geq \mathbb{E}^F[s]$, define $\hat{s}_L$ so that $s^* = \mathbb{E}^F[s|s \geq \hat{s}_L]$ and define $\hat{s}_H = \bar{s}$.

Theorem 3 *Assume that Assumptions 1–4 and Condition 1 hold. Then the unique optimal disclosure policy discloses demand states below $\hat{s}_L$ and above $\hat{s}_H$ and conceals demand states in the interval $[\hat{s}_L, \hat{s}_H]$. The unique optimal collusive price $p(s)$ in state s is given by*

$$p(s) = \begin{cases} p^m(s) & \text{if } s < \hat{s}_L, \\ p^m(s^*) & \text{if } s \in [\hat{s}_L, \hat{s}_H], \\ p^{\max}(s, v^*) & \text{if } s > \hat{s}_H. \end{cases}$$

Proof. Analogous to the proof of Theorem 1. When $\hat{s}_L = \hat{s}_H = s^*$, interpret the middle branch below as the constant $\pi^m(s^*)$.[46] Define an auxiliary objective function

$$\tilde{\pi}(s) = \begin{cases} \pi^m(s) & \text{if } s < \hat{s}_L, \\ \frac{\hat{s}_H - s}{\hat{s}_H - \hat{s}_L}\pi^m(\hat{s}_L) + \frac{s - \hat{s}_L}{\hat{s}_H - \hat{s}_L}\pi^{\max}(\hat{s}_H, v^*) & \text{if } s \in [\hat{s}_L, \hat{s}_H], \\ \pi^{\max}(s, v^*) & \text{if } s > \hat{s}_H. \end{cases}$$

[46]Later, we follow the convention $\frac{F(\hat{s}_H) - F(\hat{s}_L)}{\hat{s}_H - \hat{s}_L} = 0$ for $\hat{s}_H = \hat{s}_L$.

Note that $\tilde{\pi}(s)$ is convex and $\tilde{\pi}(s) \geq \pi^{\max}(s, v^*)$ for all s. Consider the auxiliary problem

$$\max_{G \in MPC(F)} \mathbb{E}^G [\tilde{\pi}(s)].$$

Since $\tilde{\pi}(s)$ is convex, the solution is full disclosure ($G = F$), and the resulting value is

$$\begin{aligned} & \mathbb{E}^F [\tilde{\pi}(s)] \\ = \; & F(\hat{s}_L) \mathbb{E}[\pi^m(s) | s < \hat{s}_L] \\ & + (F(\hat{s}_H) - F(\hat{s}_L)) \mathbb{E}\left[\frac{\hat{s}_H - s}{\hat{s}_H - \hat{s}_L} \pi^m(\hat{s}_L) + \frac{s - \hat{s}_L}{\hat{s}_H - \hat{s}_L} \pi^{\max}(\hat{s}_H, v^*) | s \in [\hat{s}_L, \hat{s}_H]\right] \\ & + (1 - F(\hat{s}_H)) \mathbb{E}[\pi^{\max}(s, v^*) | s > \hat{s}_H] \\ = \; & F(\hat{s}_L) \mathbb{E}[\pi^m(s) | s < \hat{s}_L] + (F(\hat{s}_H) - F(\hat{s}_L)) \pi^m(s^*) \\ & + (1 - F(\hat{s}_H)) \mathbb{E}[\pi^{\max}(s, v^*) | s > \hat{s}_H]. \end{aligned}$$

Since $\tilde{\pi}(s) \geq \pi^{\max}(s, v^*)$ for all s, this value is an upper bound for $\max_{G \in MPC(F)} \mathbb{E}^G [\pi^{\max}(s, v^*)]$. However, since $\pi^{\max}(s, v^*) = \pi^m(s)$ for all $s \leq s^*$, this value is attained by disclosing demand states below $\hat{s}_L$ and above $\hat{s}_H$ and concealing demand states in the interval $[\hat{s}_L, \hat{s}_H]$, so this policy is optimal. Moreover, this policy is the unique one that induces only posteriors s where $\tilde{\pi}(s) = \pi^{\max}(s, v^*)$, so it is the unique optimal policy. Finally, this disclosure policy is optimal only in conjunction with the prescribed prices. ■

A.2 Linear Demand System

The symmetric linear demand system

$$\pi_i(\mathbf{p}, s) = p_i \left(s + b \sum_{j \neq i} p_j - p_i \right),$$

with $b \in [0, 1/(n-1))$, is a workhorse model for oligopoly pricing under uncertainty (e.g., Vives, 2001, Chapter 8) and has recently been used by Harrington (2022, 2025) to study oligopoly pricing with a third-party pricing algorithm.[47] Theorem 3 applies as follows.

Proposition 4 *The symmetric linear demand system satisfies Assumptions 1–4 and Condition 1. The function* $\pi^{\max}(s, v^*)$ *is globally convex, so full disclosure is optimal. The unique optimal collusive price in state* s *is given by*

$$p(s) = \begin{cases} p^m(s) & \text{if } s < s^*, \\ \frac{s + 2\sqrt{\frac{\delta}{1-\delta}(v^* - \underline{\pi})}}{2-(n-1)b} & \text{if } s > s^*. \end{cases}$$

Proof. Assumptions 1 and 2 obviously hold. Assumption 3 holds because, for any price vector $\mathbf{p} = (p_1, \ldots, p_n)$, we have

$$\pi\left(\frac{1}{n}\sum_i p_i, s\right) \geq \frac{1}{n}\sum_i \pi_i(\mathbf{p}, s) \qquad \text{and} \qquad \pi^d\left(\frac{1}{n}\sum_i p_i, s\right) \leq \frac{1}{n}\sum_i \pi_i^d(\mathbf{p}, s),$$

(by straightforward calculation), so replacing any asymmetric price vector $\mathbf{p}$ with the symmetric vector $(\sum_i p_i/n, \ldots, \sum_i p_i/n)$ increases profits without violating incentive constraints. For Assumption 4, we have

$$\begin{aligned} \pi(p, s) &= p(s + (n-1)bp) - p^2, \qquad \pi^d(p, s) = \left(\frac{s + (n-1)bp}{2}\right)^2, \\ p^m(s) &= \frac{s}{2(1-(n-1)b)}, \qquad \text{and} \qquad \pi^m(s) = \frac{s^2}{4(1-(n-1)b)}. \end{aligned}$$

Straightforward calculation shows that $\pi^d(p^m(s), s) - \pi(p^m(s), s)$ is strictly increasing in s, so Assumption 4 holds.

The unique symmetric Nash equilibrium price and per-firm Nash profit are

$$p^N(s) = \frac{s}{2-(n-1)b} \qquad \text{and} \qquad \pi^N(s) = \left(\frac{s}{2-(n-1)b}\right)^2.$$

[47] The condition $b \geq 0$ implies that the firms' products are substitutes. The condition $b \leq 1/(n-1)$ implies that profits are bounded and is satisfied whenever the demand system results from utility maximization by a representative consumer (Amir, Erickson, and Jin, 2017).

For any $v \geq 0$, we have $p^{\max}(s,v) = p^m(s)$ if $p^m(s)$ satisfies (10), and otherwise $p^{\max}(s,v)$ is the larger value of p where (10) holds with equality, namely

$$p^{\max}(s,v) = \frac{s + 2\sqrt{\frac{\delta}{1-\delta}(v - \underline{\pi})}}{2 - (n-1)b}.$$

Thus,

$$\begin{aligned}
\pi^{\max}(s,v) &= \min\left\{\pi^m(s), \frac{s^2 + 2(n-1)bs\sqrt{\frac{\delta}{1-\delta}(v-\underline{\pi})} - 4(1-(n-1)b)\frac{\delta}{1-\delta}(v-\underline{\pi})}{(2-(n-1)b)^2}\right\}, \\
s^*(v) &= \frac{4(1-(n-1)b)\sqrt{\frac{\delta}{1-\delta}(v-\underline{\pi})}}{(n-1)b}.
\end{aligned}$$

Since $\pi^{\max}(s,v)$ is convex on $s \in [s^*(v), \bar{s}]$, Condition 1 holds. Moreover, the left and right derivatives of $\pi^{\max}(s,v)$ coincide at $s = s^*(v)$, so $\pi^{\max}(s,v)$ is globally convex.[48] Theorem 3 therefore implies that full disclosure is optimal. ■

A.3 Homogeneous Goods with Capacity Constraints

Consider homogeneous-goods Bertrand competition with a capacity constraint C for each firm and linear demand with an unknown intercept $\mathbf{s} \in \{\underline{s}, \bar{s}\}$, where $s = \mathbb{E}[\mathbf{s}|s]$. Then,

$$\begin{aligned}
\pi_i(\mathbf{p}, s) &= \mathbf{1}\left\{p_i = \min_j p_j\right\} \\
&\quad \times p_i\left(\frac{\bar{s}-s}{\bar{s}-\underline{s}}\min\left\{\frac{\underline{s}-p_i}{|\{j : p_j = p_i\}|}, C\right\} + \frac{s-\underline{s}}{\bar{s}-\underline{s}}\min\left\{\frac{\bar{s}-p_i}{|\{j : p_j = p_i\}|}, C\right\}\right).
\end{aligned}$$

[48]This can be seen from an envelope theorem argument: If $\frac{\partial}{\partial p}\left(\pi^d(p,s) - \pi(p,s)\right)\Big|_{(p,s)=(p^m(s^*),s^*)} \neq 0$, then the implicit function theorem implies that $p^{\max}(s,v)$ is differentiable at $s = s^*$. Since $p^m(s^*) = p^{\max}(s^*,v)$ maximizes $\pi(p,s^*)$, the envelope theorem then implies that $\frac{d}{ds}\pi(p^{\max}(s,v),s) = \frac{d}{ds}\pi^m(s)$, so $\pi^{\max}(s,v)$ is differentiable at $s = s^*$, and hence is globally convex.

This specification clearly satisfies Assumptions 1–3, and it also satisfies Assumption 4 if $C > \bar{s}/n$, which we assume holds.

If $C > \bar{s} - s^*/2$ then the capacity constraint is slack at $s = s^*$, so $\pi^{\max}(s, v^*)$ has a concave kink at $s = s^*$, as in the baseline model. In particular, $\pi^{\max}(s, v^*)$ is locally constant for s just above s^*. As s increases beyond s^*, $p^{\max}(s, v^*)$ decreases until $\bar{s} - p^{\max}(s, v^*)$ reaches C, so the capacity constraint binds in state $\bar{s}$ for a deviating firm. At this point, $\pi^{\max}(s, v^*)$ becomes strictly increasing in s. For some parameters, $\pi^{\max}(s, v^*)$ is convex on $s \in [s^*, \bar{s}]$ and is strictly convex on $[\tilde{s}, \bar{s}]$ for some $\tilde{s} \in (s^*, \bar{s})$.[49] In this case, $\pi^{\max}(s, v^*)$ is piecewise-convex with a concave kink, and the optimal disclosure policy is either lower censorship or upper censorship, as follows from Theorem 3.

B Consumer or Total Surplus Objective

In this appendix, we find the optimal disclosure policy for maximizing a weighted average of producer and consumer surplus. We assume as in Section 4.2 that consumer surplus is quasi-linear in s, so that $CS(p, s) = f(s) + g(p)s + h(p)$ for functions f, g, h. Since $\mathbb{E}[f(s)]$ is independent of the disclosure policy, we consider the problem of maximizing a weighted average of $\mathbb{E}[g(p)s + h(p)]$ and $\mathbb{E}[\Pi(p, s)]$, with weights $1 - \alpha$ and α, for $\alpha \in [0, 1)$ (the $\alpha = 1$ case already having been considered in the text). We assume that, for any disclosure policy G, the firm-optimal subgame perfect equilibrium is played, resulting in an expected profit V given by the greatest solution to

$$V = \mathbb{E}^G[\min\{\Pi^m(x), \Pi^{\max}(n, \delta, V)\}],$$

and a price $p(x, V)$ at disclosed mean demand state x given by $p(x, V) = p^m(x)$ if $p^m(x) \leq \Pi^{\max}(n, \delta, V)$, and $p(x, V) = \min\{p : \Pi(x, s) = \Pi^{\max}(n, \delta, V)\}$ otherwise. Given this function $p(x, V)$, the designer's problem is to first solve the "inner

[49] For example, this holds if $\underline{s} = 14$, $\bar{s} = 18$, $C = 12$, $\delta = 0.79$, $n = 5$, and $s \sim U[14, 18]$.

problem,"

$$\max_{G\in MPC(F)} \mathbb{E}^G\left[(1-\alpha)\left(g\left(p\left(x,V\right)\right)x+h\left(p\left(x,V\right)\right)\right)+\alpha\min\left\{\Pi^m\left(x\right),\Pi^{\max}\left(n,\delta,V\right)\right\}\right]$$

$$\text{s.t.} \qquad V \text{ is the greatest solution to } V=\mathbb{E}^G\left[\min\left\{\Pi^m\left(x\right),\Pi^{\max}\left(n,\delta,V\right)\right\}\right],$$

for each V that is the greatest solution to $V=\mathbb{E}^G\left[\min\left\{\Pi^m\left(x\right),\Pi^{\max}\left(n,\delta,V\right)\right\}\right]$ for some $G\in MPC\left(F\right)$, and then to maximize over V.

The inner problem can be rewritten as an unconstrained information design problem by letting β be the multiplier on the constraint (which exists by Slater's condition for any $V<V^*$, the solution when $\alpha=1$), normalizing the objective by $1/\left(1-\alpha\right)$, and letting $\lambda=\left(\alpha+\beta\right)/\left(1-\alpha\right)$, to obtain

$$\max_{G\in MPC(F)} \mathbb{E}^G\left[\left(g\left(p\left(x,V\right)\right)x+h\left(p\left(x,V\right)\right)\right)+\lambda\min\left\{\Pi^m\left(x\right),\Pi^{\max}\left(n,\delta,V\right)\right\}\right]. \tag{12}$$

In general, (12) can be solved by standard techniques (e.g., Dworczak and Martini, 2019), given the functions p, g, h, and Π^m and the multiplier λ. Here, we consider the leading case of linear demand with an unknown intercept or an unknown constant marginal cost.

Unknown demand. As shown in Section 4.2, $\Pi\left(p,x\right)=p\left(x-p\right)$, $g\left(p\right)=-p$, and $h\left(p\right)=p^2/2$. We thus have

$$\Pi^m\left(x\right)=\frac{x^2}{4} \qquad \text{and} \qquad g\left(p^m\left(x\right)\right)x+h\left(p^m\left(x\right)\right)=-\frac{3x^2}{8}.$$

Suppressing the argument of $\Pi^{\max}\left(n,\delta,V\right)$ and letting $p^{\max}\left(x,V\right)$ solve $\Pi\left(p^{\max}\left(x,V\right),x\right)=\Pi^{\max}$ (for $x\geq s^*\equiv 2\sqrt{\Pi^{\max}}$), we calculate

$$\begin{aligned} p^{\max}\left(x,V\right) &= \frac{x-\sqrt{x^2-4\Pi^{\max}}}{2} \qquad \text{and} \\ g\left(p^{\max}\left(x,V\right)\right)x+h\left(p^{\max}\left(x,V\right)\right) &= \frac{-x^2-2\Pi^{\max}+x\sqrt{x^2-4\Pi^{\max}}}{4}. \end{aligned}$$

The designer's (inner) problem is thus

$$\max_{G\in MPC(F)} \mathbb{E}^G \left[\begin{array}{c} \mathbf{1}\{x < s^*\}(-3+2\lambda)\frac{x^2}{8} \\ +\mathbf{1}\{x \geq s^*\}\left(\frac{-x^2+x\sqrt{x^2-4\Pi^{\max}}}{4} + \left(-\frac{1}{2}+\lambda\right)\Pi^{\max}\right) \end{array} \right].$$

Note that if $\lambda < 3/2$ then the objective is decreasing and concave in x for $x < s^*$ and is increasing and concave in x for $x \geq s^*$ (as the function $-x^2 + x\sqrt{x^2 - 4\Pi^{\max}}$ is increasing and concave for $x \geq 2\sqrt{\Pi^{\max}}$), with a convex kink at s^*. Thus, if the optimal multiplier λ (i.e., the multiplier in the inner problem with the optimal value for V) is less than 3/2, the optimal policy is a binary signal that discloses only whether demand is above or below a cutoff s^{**}, by an argument similar to the proof of Proposition 4 of Dworczak and Martini (2019). Intuitively, when $\lambda < 3/2$ the objective has the same shape as consumer surplus $g(p(x))x + h(p(x))$; and when $x < s^*$, (monopoly) price is linearly increasing in x, so consumer surplus is decreasing and concave in x; whereas when $x \geq s^*$, price is decreasing and convex in x (by Rotemberg and Saloner's logic), so consumer surplus is increasing and concave in x. The optimal policy is thus a binary signal that discloses only whether demand is "low" (in which case firms set the corresponding monopoly price) or "high" (causing a "price war").

If instead the optimal multiplier λ is greater than 3/2, the objective is increasing and convex for $x < s^*$, so the objective is S-shaped overall. In this case, the solution is upper censorship with a cutoff $\hat{s} < s^*$, as in the problem considered in the text.

The next result summarizes this discussion and also shows that a binary signal maximizes consumer surplus. (Equivalently, the optimal multiplier is less than 3/2 when $\alpha = 0$.) This follows because the optimal signal for any weight α is either a binary signal or is upper censorship with a cutoff $\hat{s} \in (\underline{s}, s^*)$, but given upper censorship with a cutoff $\hat{s}$, pooling all states below $\hat{s}$ in a single signal realization increases expected consumer surplus, so the optimal signal when $\alpha = 0$ must be binary.

Proposition 5 *Under linear demand with an unknown intercept, the optimal disclosure policy is either a binary signal that reveals only whether demand is below or above a cutoff; or it is upper censorship. The former policy is optimal if the weight on consumer surplus is sufficiently high; the latter policy is optimal if the weight on producer surplus is sufficiently high.*

Proof. Given the above discussion, it suffices to show that the optimal multiplier λ^* is at most $3/2$ when $\alpha = 0$. To see this, suppose toward a contradiction that $\lambda^* > 3/2$. Then, as shown above, the optimal disclosure policy is an upper censorship policy G with a cutoff $\hat{s} < s^*$. We can also assume that $\hat{s} > \underline{s}$, as otherwise this policy is no disclosure, which is also a binary signal.

To obtain a contradiction, we show that the binary signal $\hat{G}$ that discloses only whether demand is above or below $\hat{s}$ yields strictly higher expected consumer surplus than G. To see this, let V and $\hat{V}$ denote expected profit under G and $\hat{G}$, respectively. Recall that $p(x, V) = p^m(x)$ for all $x < \hat{s}$ (as $\hat{s} < s^*$) and that $g(p^m(x))x + h(p^m(x))$ is strictly concave in x, so $\hat{G}$ yields strictly higher expected consumer surplus than G if $p\left(\mathbb{E}^F[s|s < \hat{s}], \hat{V}\right) \leq p\left(\mathbb{E}^F[s|s < \hat{s}], V\right)$ and $p\left(\mathbb{E}^F[s|s \geq \hat{s}], \hat{V}\right) \leq p\left(\mathbb{E}^F[s|s \geq \hat{s}], V\right)$. In turn, since $p\left(x, \tilde{V}\right)$ is increasing in $\tilde{V}$, it suffices to show that $V \geq \hat{V}$. But this holds because V and $\hat{V}$ are, respectively, the greatest fixed points of the equations

$$\tilde{V} = \mathbb{E}^G\left[\min\left\{\Pi^m(x), \Pi^{\max}\left(\delta, n, \tilde{V}\right)\right\}\right] \quad \text{and} \quad \tilde{V} = \mathbb{E}^{\hat{G}}\left[\min\left\{\Pi^m(x), \Pi^{\max}\left(\delta, n, \tilde{V}\right)\right\}\right];$$

but, for any $\tilde{V}$, we have

$$\begin{aligned}
\mathbb{E}^{G}\left[\min\left\{\Pi^{m}(x),\Pi^{\max}\left(\delta,n,\tilde{V}\right)\right\}\right] &= \int_{\underline{s}}^{\hat{s}}\Pi^{m}(s)\,dF(s)+(1-F(\hat{s}))\,\Pi^{\max}\left(\delta,n,\tilde{V}\right)\\
&\geq F(\hat{s})\,\Pi^{m}\left(\mathbb{E}^{F}[s|s<\hat{s}]\right)+(1-F(\hat{s}))\,\Pi^{\max}\left(\delta,n,\tilde{V}\right)\\
&\geq F(\hat{s})\min\left\{\Pi^{m}\left(\mathbb{E}^{F}[s|s<\hat{s}]\right),\Pi^{\max}\left(\delta,n,\tilde{V}\right)\right\}\\
&\quad+(1-F(\hat{s}))\min\left\{\Pi^{m}\left(\mathbb{E}^{F}[s|s\geq\hat{s}]\right),\Pi^{\max}\left(\delta,n,\tilde{V}\right)\right\}\\
&= \mathbb{E}^{\hat{G}}\left[\min\left\{\Pi^{m}(x),\Pi^{\max}\left(\delta,n,\tilde{V}\right)\right\}\right],
\end{aligned}$$

where the first inequality is by convexity of $\Pi^{m}(s)$, so the greatest fixed point of the first equation is not lower than that of the second. ■

A natural conjecture is that there exists a weight on producer surplus $\alpha^{*}\in(0,1)$ such that if $\alpha<\alpha^{*}$ then the optimal multiplier λ is less than $3/2$, so a binary signal is optimal; and if $\alpha\geq\alpha^{*}$ then $\lambda\geq 3/2$, so upper censorship is optimal. This holds whenever the optimal multiplier β on the constraint $\mathbb{E}^{G}[\Pi^{m}(x),\Pi^{\max}(n,\delta,V)]=V$ is monotone in α, as then λ is monotone in α, so that $\lambda\geq 3/2$ iff α exceeds a cutoff α^{*}. However, we do not have a proof that β is monotone in α.

Unknown cost. Since $\Pi(p,x)=(p-x)(1-p)$ and $CS(p,x)=h(p)$ for $h(p)=(1/2)(1-p)^{2}$, we have

$$p^{m}(x)=\frac{1+x}{2},\qquad \Pi^{m}(x)=\frac{(1-x)^{2}}{4},\qquad \text{and}\quad h(p^{m}(x))=\frac{(1-x)^{2}}{8}.$$

Suppressing the argument of $\Pi^{\max}(n,\delta,V)$ and letting $p^{\max}(x,V)$ solve $\Pi(p^{\max}(x,V),x)=\Pi^{\max}$ (for $x\leq s^{*}\equiv 1-2\sqrt{\Pi^{\max}}$), we calculate

$$\begin{aligned}
p^{\max}(x,V) &= \frac{1+x-\sqrt{(1-x)^{2}-4\Pi^{\max}}}{2}\qquad\text{and}\\
h(p^{\max}(x,V)) &= \frac{(1-x)^{2}-2\Pi^{\max}+(1-x)\sqrt{(1-x)^{2}-4\Pi^{\max}}}{4}.
\end{aligned}$$

The designer's (inner) problem is thus

$$\max_{G \in MPC(F)} \mathbb{E}^G \left[\begin{array}{c} \mathbf{1}\{x \le s^*\} \left(\frac{(1-x)^2+(1-x)\sqrt{(1-x)^2-4\Pi^{\max}}}{4} + \left(-\frac{1}{2}+\lambda\right)\Pi^{\max} \right) \\ +\mathbf{1}\{x > s^*\}(1+2\lambda)\frac{(1-x)^2}{8} \end{array} \right].$$

Note that the objective is decreasing and convex for $x \le s^*$ (as the function $(1-x)^2 + (1-x)\sqrt{(1-x)^2 - 4\Pi^{\max}}$ is decreasing and convex for $x \le s^*$); whereas for $x > s^*$, the objective is increasing and concave if $\lambda \le -1/2$ and is decreasing and convex if $\lambda > -1/2$. In the former case, the objective consists of a decreasing and convex piece followed by an increasing and concave piece, and it is straightforward to show that the optimal disclosure policy is upper censorship. In the latter case, the objective consists of two decreasing and convex pieces that meet at a kink, and in general the optimal disclosure policy censors an intermediate range of states around the kink and discloses the lowest and highest states. However, unlike in the unknown demand case, we have not been able to determine which of these cases applies for the problem of maximizing consumer surplus ($\alpha = 0$).[50]

C Proof of Proposition 2.3

Fix any convex function $\Pi^m(s)$ and any constant $\Pi^{\max}$. Let (F_1, F_2, G_1, G_2) be such that

$$\begin{aligned} F_1 &\in MPC(F_2), \\ G_1 &\in \underset{G \in MPC(F_1)}{\operatorname{argmax}} \mathbb{E}^G[\min\{\Pi^m(x), \Pi^{\max}\}], \qquad \text{and} \\ G_2 &\in \underset{G \in MPC(F_2)}{\operatorname{argmax}} \mathbb{E}^G[\min\{\Pi^m(x), \Pi^{\max}\}]. \end{aligned}$$

[50] Conversely, as $\lambda \to \infty$ (approaching the $\alpha = 1$ case considered in the text), the objective flattens out for $s \le s^*$, implying that lower censorship is optimal for sufficiently high λ.

We show that $G_1 \in MPC(G_2)$, or equivalently $\int_{\underline{s}}^{s} G_1(s)\,ds \leq \int_{\underline{s}}^{s} G_2(s)\,ds$ for all x (since G_1 and G_2 have the same mean). As shown in the text, this completes the proof of Proposition 2.3.

By Theorem 1, we have

$$G_1(s) = \begin{cases} F_1(s) & \text{if } s \leq \hat{s}_1, \\ F_1(\hat{s}_1) & \text{if } \hat{s}_1 < s < s^*, \\ 1 & \text{if } s \geq s^*, \end{cases}$$

where $s^* = E^{F_1}[s|s > \hat{s}_1]$ satisfies $\Pi^m(s^*) = \Pi^{\max}$, and similarly for G_2. Note that, for any $s \leq \hat{s}_2$, we have

$$\int_{\underline{s}}^{s} (G_1(s) - G_2(s))\,ds = \int_{\underline{s}}^{s} (G_1(s) - F_2(s))\,ds \leq \int_{\underline{s}}^{s} (F_1(s) - F_2(s))\,ds \leq 0,$$

where the equality is by $G_2(s) = F_2(s)$ for all $s \leq \hat{s}_2$, the first inequality is by $G_1(s) \leq F_1(s)$ for all $s < s^*$, and the second inequality is because $F_1 \in MPC(F_2)$. Next, $G_1(s) - G_2(s)$ is non-decreasing on the interval $[\hat{s}_2, s^*)$, as on this interval $G_1(s)$ is non-decreasing and $G_2(s) = F_2(\hat{s}_2)$ is constant. In addition, $G_1(s) - G_2(s) = 0$ for $s \geq s^*$. Thus, $\int_{\underline{s}}^{s} (G_1(s) - G_2(s))\,ds$ is convex on $[\hat{s}_2, s^*]$ and constant on $(s^*, \bar{s}]$. Therefore, since $\int_{\underline{s}}^{s} (G_1(s) - G_2(s))\,ds \leq 0$ for all $s \leq \hat{s}_2$, if $\int_{\underline{s}}^{s} (G_1(s) - G_2(s))\,ds$ is ever strictly positive then for some $s \in (s^*, \bar{s}]$, then it must be strictly positive at $s = s^*$ (since, as a convex function on $[\hat{s}_2, s^*]$, it is bounded above by its linear interpolation over this interval). But, by integration by parts,

$$\begin{aligned} \mathbb{E}^{G_1}[s] &= \bar{s} - \int_{\underline{s}}^{\bar{s}} G_1(s)\,ds = s^* - \int_{\underline{s}}^{s^*} G_1(s)\,ds, \qquad \text{and} \\ \mathbb{E}^{G_2}[s] &= \bar{s} - \int_{\underline{s}}^{\bar{s}} G_2(s)\,ds = s^* - \int_{\underline{s}}^{s^*} G_2(s)\,ds, \end{aligned}$$

so since $\mathbb{E}^{G_1}[s] = \mathbb{E}^{G_2}[s]$ we have $\int_{\underline{s}}^{s^*} (G_1(s) - G_2(s))\,ds = 0$. Thus, $\int_{\underline{s}}^{s} (G_1(s) - G_2(s))\,ds \leq 0$ for all s, completing the proof.

D Example with Persistent Demand

Assume a binary demand state ($s \in \{\underline{s}, \bar{s}\}$) and linear demand ($\Pi(p, s) = p(s - p)$), and consider the parameters $\underline{s} = 1$, $\bar{s} = 2$, $\delta = .55$, $n = 2$, and $\Pr(s_{t+1} = \bar{s} | s_t = \bar{s}) = \Pr(s_{t+1} = \underline{s} | s_t = \underline{s}) = \rho \in (1/2, 1)$. Binary demand violates our assumption that the distribution of states is atomless; however, that assumption is easily relaxed. With a binary state, an upper censorship policy now corresponds to disclosing state $\underline{s}$ with some probability q (conditional on $s = \underline{s}$) and pooling state $\underline{s}$ together with state $\bar{s}$ otherwise. Upper censorship is optimal by essentially the same proof as in the atomless case, so we can parameterize an optimal disclosure policy by $q \in [0, 1]$, with $q = 0$ being no disclosure, $q \in (0, 1)$ being non-trivial upper censorship, and $q = 1$ being full disclosure.

Figures 2–4 display the optimal disclosure policy, firm profit, and consumer surplus as ρ ranges from 1/2 to 1. In Figure 2, the blue curve plots the optimal disclosure policy q at last-period demand state $\underline{s}$, while the orange line plots q at last-period demand state $\bar{s}$. Note that the blue curve is always above the orange curve, as under positive persistence ($\rho > 1/2$) more information is disclosed when the last-period state is lower, as shown in Theorem 2. In addition, Figure 2 displays three distinct equilibrium regimes. In Regime 1, non-trivial upper censorship is optimal for both last-period demand states. For $\rho \in (1/2, 0.727)$, Regime 1 prevails, and increasing ρ leads to more disclosure at last-period state $\underline{s}$ and less disclosure at last-period state $\bar{s}$. Intuitively, increasing ρ makes firms more pessimistic at last-period state $\underline{s}$ and more optimistic at last-period state $\bar{s}$, which increases disclosure at last-period state $\underline{s}$ and decreases disclosure at last-period state $\bar{s}$. Once ρ reaches 0.727, firms are so optimistic at last-period state $\bar{s}$ that no disclosure becomes optimal, while further increases in ρ continue to increase disclosure at last-period state $\underline{s}$. This second regime persists until ρ reaches 0.863. At this point, demand is so persistent that future profits are much higher at last-period state $\bar{s}$ than at last-period state $\underline{s}$, which makes partial disclosure optimal again at last-period state $\bar{s}$, so the equilibrium is

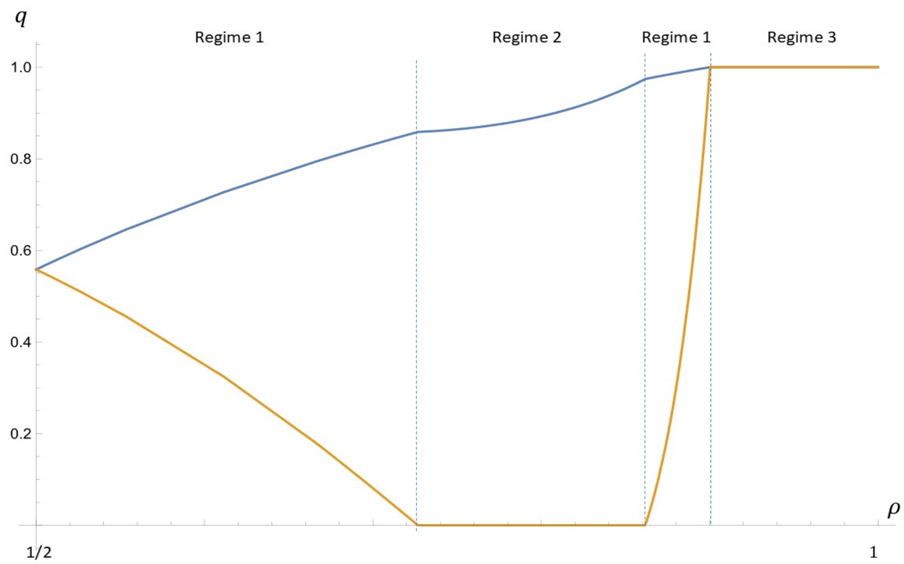

Figure 2: Optimal disclosure policy. The blue curve is the probability of disclosing $\underline{s}$ when the current state is $\underline{s}$ and the last-period state was $\underline{s}$; the orange curve is the probability of disclosing $\underline{s}$ when the current state is $\underline{s}$ and the last-period state was $\bar{s}$.

again in Regime 1. Further increases in ρ then rapidly increase disclosure for both last-period states, until ρ reaches 0.902, at which point full disclosure becomes optimal for both last-period states.

Figures 3 and 4 trace the implications of these effects for firm profit and consumer surplus. In Figure 3, the blue curve plots a firm's continuation value (discounted sum of profits) at last-period demand state $\underline{s}$; the orange curve plots this value at last-period demand state $\bar{s}$; and the green curve is the average of the two, which equals a firm's ex ante expected profit. In Regime 1, increasing ρ decreases the continuation value at last-period state $\underline{s}$ and increases it at last-period state $\bar{s}$. The net effect is to (slightly) increase expected profits, as increasing the continuation value at last-period state $\bar{s}$ relaxes the binding incentive constraint. In contrast, the effect of increasing ρ on profits is non-monotone in Regime 2 and is zero in Regime 3 (where optimal profits are first-best). In Figure 4, the blue curve plots the current-period consumer surplus at last-period demand state $\underline{s}$; the orange curve plots it at last-period demand state $\bar{s}$; and the green curve is the average of the two, which equals ex ante expected

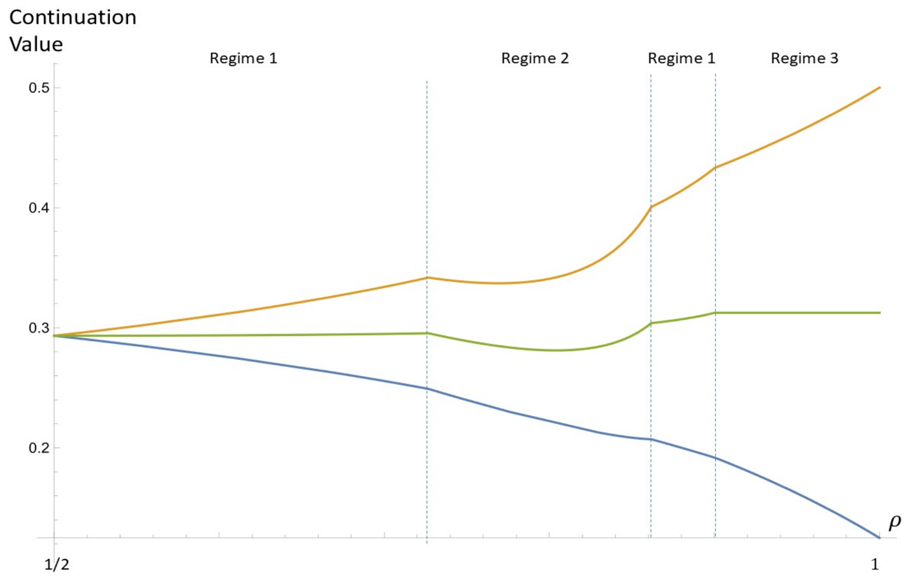

Figure 3: Optimal continuation values. The blue curve is a firm's continuation value at last-period state $\underline{s}$. The orange curve is the corresponding value at last-period state $\bar{s}$. The green curve is the ex ante expected profit.

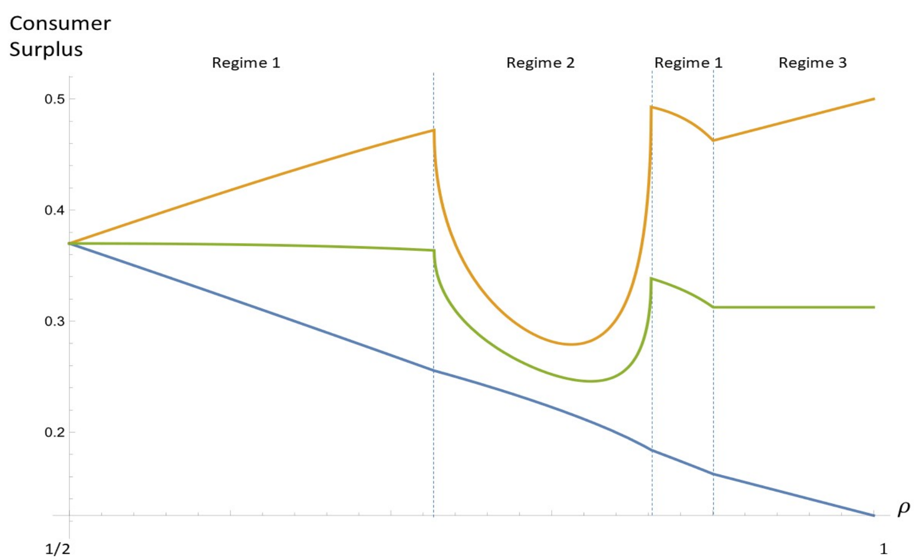

Figure 4: Consumer surplus. The blue curve is the current-period consumer surplus at last-period state $\underline{s}$. The orange line is the corresponding value at last-period state $\bar{s}$. The green curve is ex ante expected consumer surplus.

consumer surplus. Expected consumer surplus is decreasing in ρ in Regime 1 (albeit only slightly when $\rho \in (1/2, 0.727)$), non-monotone in ρ in Regime 2, and constant in ρ in Regime 3.

References

[1] Abreu, Dilip. "Extremal Equilibria of Oligopolistic Supergames." *Journal of Economic Theory* 39.1 (1986): 191-225.

[2] Abreu, Dilip. "On the Theory of Infinitely Repeated Games with Discounting." *Econometrica* 56.2 (1988): 383-396.

[3] Abreu, Dilip, Paul Milgrom, and David Pearce. "Information and Timing in Repeated Partnerships." *Econometrica* 59.6 (1991): 1713-1733.

[4] Amir, Rabah, Philip Erickson, and Jim Jin. "On the Microeconomic Foundations of Linear Demand for Differentiated Products." *Journal of Economic Theory* 168 (2017): 641-665.

[5] Asker, John, Chaim Fershtman, and Ariel Pakes. "The Impact of Artificial Intelligence Design on Pricing." *Journal of Economics & Management Strategy* 33.2 (2024): 276-304.

[6] Assad, Stephanie, et al. "Algorithmic Pricing and Competition: Empirical Evidence from the German Retail Gasoline Market." *Journal of Political Economy* 132.3 (2024): 723-771.

[7] Athey, Susan, and Kyle Bagwell. "Optimal Collusion with Private Information." *RAND Journal of Economics* 32.3 (2001): 428-465.

[8] Athey, Susan, Kyle Bagwell, and Chris Sanchirico. "Collusion and Price Rigidity." *Review of Economic Studies* 71.2 (2004): 317-349.

[9] Awaya, Yu, and Vijay Krishna. "On Communication and Collusion." *American Economic Review* 106.2 (2016): 285-315.

[10] Bagwell, Kyle, and Robert W. Staiger. "Collusion over the Business Cycle." *Rand Journal of Economics* 28.1 (1997): 82-106.

[11] Banchio, Martino, and Giacomo Mantegazza. "Artificial Intelligence and Spontaneous Collusion." *arXiv preprint* arXiv:2202.05946 (2024).

[12] Bernheim, B. Douglas, and Erik Madsen. "Price Cutting and Business Stealing in Imperfect Cartels." *American Economic Review* 107.2 (2017): 387-424.

[13] Blackwell, David. "Equivalent Comparisons of Experiments." *Annals of Mathematical Statistics* (1953): 265-272.

[14] Bonatti, Alessandro, Raffaele Ficco, and Salvatore Piccolo. "Data Analytics for Algorithm Pricing." *Working Paper* (2024).

[15] Bordoli, Davide, "Selling Information with Strategic Externalities." *Working Paper* (2025).

[16] Brown, Zach Y., and Alexander MacKay. "Competition in Pricing Algorithms." *American Economic Journal: Microeconomics* 15.2 (2023): 109-156.

[17] Buehler, Stefan, and Dennis L. Gärtner. "Making Sense of Nonbinding Retail-Price Recommendations." *American Economic Review* 103.1 (2013): 335-359.

[18] Calder-Wang, Sophie, and Gi Heung Kim. "Coordinated vs Efficient Prices: The Impact of Algorithmic Pricing on Multifamily Rental Markets." *Working Paper*, University of Pennsylvania, SSRN 4403058 (2024).

[19] Calvano, Emilio, et al. "Protecting Consumers from Collusive Prices due to AI." *Science* 370.6520 (2020a): 1040-1042.

[20] Calvano, Emilio, et al. "Artificial Intelligence, Algorithmic Pricing, and Collusion." *American Economic Review* 110.10 (2020b): 3267-3297.

[21] Carlton, Dennis. "The Rigidity of Prices." *American Economic Review* 76.4 (1986): 637-58.

[22] Cooper, William L., Tito Homem-de-Mello, and Anton J. Kleywegt. "Learning and Pricing with Models that do not Explicitly Incorporate Competition." *Operations Research* 63.1 (2015): 86-103.

[23] Dworczak, Piotr, and Giorgio Martini. "The Simple Economics of Optimal Persuasion." *Journal of Political Economy* 127.5 (2019): 1993-2048.

[24] Ellison, Glenn. "Theories of Cartel Stability and the Joint Executive Committee." *Rand Journal of Economics* 25.1 (1994): 37-57.

[25] Ely, Jeffrey C. "Beeps." *American Economic Review* 107.1 (2017): 31-53.

[26] Ezrachi, Ariel, and Maurice E. Stucke. "Artificial Intelligence & Collusion: When Computers Inhibit Competition." *University of Illinois Law Review* (2017): 1775.

[27] Farboodi, Maryam, Nima Haghpanah, and Ali Shourideh. "Good Data and Bad Data: The Welfare Effects of Price Discrimination." *arXiv preprint* arXiv:2502.03641 (2025).

[28] Gentzkow, Matthew, and Emir Kamenica. "A Rothschild-Stiglitz Approach to Bayesian Persuasion." *American Economic Review* 106.5 (2016): 597-601.

[29] Gerlach, Heiko. "Stochastic Market Sharing, Partial Communication and Collusion." *International Journal of Industrial Organization* 27.6 (2009): 655-666.

[30] Goltsman, Maria, and Gregory Pavlov. "Communication in Cournot oligopoly." *Journal of Economic Theory* 153 (2014): 152-176.

[31] Green, Edward J., and Robert H. Porter. "Noncooperative Collusion under Imperfect Price Information." *Econometrica* 52.1 (1984): 87-100.

[32] Haltiwanger, John, and Joseph E. Harrington Jr. "The Impact of Cyclical Demand Movements on Collusive Behavior." *RAND Journal of Economics* 22.1 (1991): 89-106.

[33] Hanazono, Makoto, and Huanxing Yang. "Collusion, Fluctuating Demand, and Price Rigidity." *International Economic Review* 48.2 (2007): 483-515.

[34] Hansen, Karsten T., Kanishka Misra, and Mallesh M. Pai. "Frontiers: Algorithmic Collusion: Supra-Competitive Prices via Independent Algorithms." *Marketing Science* 40.1 (2021): 1-12.

[35] Harrington Jr, Joseph E. "How do Cartels Operate?." *Foundations and Trends in Microeconomics* 2.1 (2006): 1-105.

[36] Harrington Jr, Joseph E. "Detecting Cartels." *Handbook of Antitrust Economics*, Buccirossi ed. (2008).

[37] Harrington Jr, Joseph E. "The Effect of Outsourcing Pricing Algorithms on Market Competition." *Management Science* 68.9 (2022): 6889-6906.

[38] Harrington Jr, Joseph E. "Hub-and-Spoke Collusion with a Third-Party Pricing Algorithm." *Working Paper* (2025).

[39] Harrington, Joseph E., and Juan Ortner. "The Equilibrium Design of a Third-Party's Pricing Algorithm." *Working Paper* (2025).

[40] Harrington, Joseph E., and Andrzej Skrzypacz. "Private Monitoring and Communication in Cartels: Explaining Recent Collusive Practices." *American Economic Review* 101.6 (2011): 2425-2449.

[41] Hickok, Nathaniel. "Algorithm Design Meets Information Design: Price Recommendation Algorithms on Online Platforms." *Working Paper*, MIT.

[42] Kamenica, Emir, and Matthew Gentzkow. "Bayesian Persuasion." *American Economic Review* 101.6 (2011): 2590-2615.

[43] Kandori, Michihiro. "The Use of Information in Repeated Games with Imperfect Monitoring." *Review of Economic Studies* 59.3 (1992): 581-593.

[44] Kandori, Michihiro. "Correlated Demand Shocks and Price Wars during Booms." *Review of Economic Studies* 58.1 (1991): 171-180.

[45] Kawai, Kei, Jun Nakabayashi, and Juan M. Ortner. "The Value of Privacy in Cartels: An Analysis of the Inner Workings of a Bidding Ring." *Review of Economic Studies* 93.1 (2026): 600–628.

[46] Kolotilin, Anton. "Optimal Information Disclosure: A Linear Programming Approach." *Theoretical Economics* 13.2 (2018): 607-635.

[47] Kolotilin, Anton, et al. "Persuasion of a Privately Informed Receiver." *Econometrica* 85.6 (2017): 1949-1964.

[48] Kolotilin, Anton, and Hongyi Li. "Relational Communication." *Theoretical Economics* 16.4 (2021): 1391-1430.

[49] Kolotilin, Anton, Timofiy Mylovanov, and Andriy Zapechelnyuk. "Censorship as Optimal Persuasion." *Theoretical Economics* 17.2 (2022): 561-585.

[50] Kolotilin, Anton, Roberto Corrao, and Alexander Wolitzky. "Persuasion and Matching: Optimal Productive Transport." *Journal of Political Economy* 133.4 (2025): 1334-1381.

[51] Kolotilin, Anton, and Alexander Wolitzky. "The Economics of Partisan Gerrymandering." *Econometrica* 94.1 (2026): 71-103.

[52] Klein, Timo. "Autonomous Algorithmic Collusion: Q-Learning under Sequential Pricing." *RAND Journal of Economics* 52.3 (2021): 538-558.

[53] Kuvalekar, Aditya, Elliot Lipnowski, and Joao Ramos. "Goodwill in Communication." *Journal of Economic Theory* 203 (2022): 105467.

[54] Lamba, Rohit, and Sergey Zhuk. "Pricing with Algorithms." *arXiv preprint* arXiv:2205.04661 (2024).

[55] Marshall, Robert C., and Leslie M. Marx. *The Economics of Collusion: Cartels and Bidding Rings*. Mit Press, 2012.

[56] Martin, Simon, and Alexander Rasch. "Demand Forecasting, Signal Precision, and Collusion with Hidden Actions." *International Journal of Industrial Organization* 92 (2024): 103036.

[57] Maskin, Eric, and Jean Tirole. "A Theory of Dynamic Oligopoly, II: Price Competition, Kinked Demand Curves, and Edgeworth Cycles." *Econometrica* 56.3 (1988): 571-599.

[58] McAfee, R. Preston, and John McMillan. "Bidding Rings." *American Economic Review* (1992): 579-599.

[59] Mehra, Salil K. "Antitrust and the robo-seller: Competition in the Time of Algorithms." *Minnesota Law Review* 100 (2015): 1323.

[60] Miklós-Thal, Jeanine, and Catherine Tucker. "Collusion by Algorithm: Does Better Demand Prediction Facilitate Coordination Between Sellers?." *Management Science* 65.4 (2019): 1552-1561.

[61] Myerson, Roger B. "Optimal Coordination Mechanisms in Generalized Principal–Agent Problems." *Journal of Mathematical Economics* 10.1 (1982): 67-81.

[62] O'Connor, Jason, and Nathan E. Wilson. "Reduced Demand Uncertainty and the Sustainability of Collusion: How AI Could Affect Competition." *Information Economics and Policy* 54 (2021): 100882.

[63] Ortner, Juan, Takuo Sugaya, and Alexander Wolitzky. "Mediated Collusion." J*ournal of Political Economy* 132.4 (2024): 1247-1289.

[64] Pigou, A.C. *The Economics of Welfare.* (1920).

[65] Porter, Robert H. "A Study of Cartel Stability: the Joint Executive Committee, 1880-1886." *Bell Journal of Economics* (1983): 301-314.

[66] Plan, Asaf. "Symmetry in n-Player Games." *Journal of Economic Theory* 207 (2023): 105549.

[67] Renault, Jérôme, Eilon Solan, and Nicolas Vieille. "Optimal Dynamic Information Provision." *Games and Economic Behavior* 104 (2017): 329-349.

[68] Rotemberg, Julio J., and Garth Saloner. "A Supergame-Theoretic Model of Price Wars During Booms." *American Economic Review* 76.3 (1986): 390-407.

[69] Sahuguet, Nicolas, and Alexis Walckiers. "A Theory of Hub-and-Spoke Collusion." *International Journal of Industrial Organization* 53 (2017): 353-370.

[70] Salcedo, Bruno. "Pricing Algorithms and Tacit Collusion." *Working Paper*, Pennsylvania State University (2015).

[71] Smolin, Alex and Takuro Yamashita. "Information Design in Smooth Games." *Working Paper*, Toulouse School of Economics (2025).

[72] Skrzypacz, Andrzej, and Hugo Hopenhayn. "Tacit Collusion in Repeated Auctions." *Journal of Economic Theory* 114.1 (2004): 153-169.

[73] Strassen, Volker. "The Existence of Probability Measures with Given Marginals." *Annals of Mathematical Statistics* 36.2 (1965): 423-439.

[74] Sugaya, Takuo, and Alexander Wolitzky. "Maintaining Privacy in Cartels." *Journal of Political Economy* 126.6 (2018): 2569-2607.

[75] US Department of Justice. *United States of America et al. v. RealPage* (2024a).

[76] US Department of Justice. "Justice Department Sues RealPage for Algorithmic Pricing Scheme that Harms Millions of American Renters" (2024b).

[77] US Department of Justice. *United States of America et al. v. RealPage, Proposed Final Judgement* (2025).

[78] Vives, Xavier. *Oligopoly Pricing: Old Ideas and New Tools.* MIT Press, Cambridge, Mass., 2001.

[79] Waltman, Ludo, and Uzay Kaymak. "Q-Learning Agents in a Cournot Oligopoly Model." *Journal of Economic Dynamics and Control* 32.10 (2008): 3275-3293.

[80] Wu, Karen. "Welfare Effects of Market Research." *Working Paper* (2025).