Dollarization Dynamics*

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This study explores the consequences of dollarizing an economy with an initial dollar shortage. We show that the resulting transitional dynamics are tantamount to that of a "sudden stop": consumption of tradable goods fall, the real exchange rate depreciates abruptly by a discrete drop in domestic prices and wages followed by a gradual appreciation from positive inflation. With nominal rigidities the economy first falls into a recession. This is true even if all prices and wages are allowed to adjust flexibly on impact. The subsequent recovery in activity always "overshoots" the steady state: the non-tradable sector transitions from the initial recession to a boom, then asymptotes to its steady state.

1 Introduction

Born out of deep frustration with inflation, a country may resign its native currency to adopt a foreign currency like the US dollar as legal tender (from now on "dollarization"). Ecuador famously went down this extreme path more than two decades ago. Recent proposals for dollarization, previously rejected, have resurfaced and gained traction in Argentina in the midst of the presidential race and escalating inflation.[1] Although there are several costs of dollarization, the focus of this paper is on one overlooked cost: we study the transitional dynamics induced by dollarizing an economy that has an initial shortage of dollars.

Unilateral dollarizations have two well-known economic costs which are *not* the subject of this paper: the transfer of seignorage to foreigners and the loss of independent monetary policy, both exchange rate and interest rates, to respond to future shocks.[2] The

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[1] Ecuador dollarized in 2000-01, El Salvador in 2001, Zimbabwe in 2009. Much earlier Panama adopted the dollar in 1904 as its main currency, Goldfajn, Olivares, Frankel and Milesi-Ferretti (2001) provides a historical perspective .

[2] Another costs is the potential loss of a 'lender of last resort' (LOLR) role for banks—a role typically assumed by central banks with the liquidity provided by the potential for money creation. This cost is debatable: what really matters for a LOLR is rapid access to liquidity, so it can potentially done by a well financed Treasury with swift access to liquidity, private or public credit (e.g. via the IMF).

loss and transfer of seignorage is simply a matter of accounting. The foreign country benefits from the increased holdings of their currency. For a currency associated with low inflation and nominal interest rates this cost may be modest, albeit not entirely trivial.[3] The loss of independent monetary policy is essentially equivalent to that of adopting a credible fixed exchange rate or currency board, or to joining a currency union. Ascertaining this cost goes well beyond simple accounting. A large body of work in International Economics (e.g. Optimal Currency Area) suggests that the costs may be quite significant. The lost seignorage is a relatively smooth flow while the cost from lack of monetary independence arrives with some delay and uncertainty. In contrast, the benefits of taming inflation may be immediate. This difference in horizons—longer for costs than benefits—is relevant in explaining the political allure of dollarization.

We will not dwell further on these well-known costs of dollarization.[4] Our contribution is to isolate and study an overlooked short-run potential cost of dollarization, one that must be paid upfront, at inception. We are interested in the following scenario. Suppose that, due to a shortage of foreign reserves or a lack of foreign credit, the conversion of domestic currency to "dollars" is carried out at an unfavorable rate: leaving the initial balance of foreign currency below its long-run steady-state value. How does an economy deal with such a situation?

We build and study a simple model to tackle this question. The model represents an open monetary economy with two sectors: tradable and non-tradable goods. The country is limited in its borrowing capacity, but can save freely abroad.

Now picture this economy using domestic currency (e.g. pesos) but suddenly announcing an immediate dollarization: domestic currency becomes void and, from now on, the dollar must be used as unit of account and medium of exchange. How many dollars does this new dollarization regime start with? The answer depends on various factors. Domestic currency may get exchanged for dollars using some conversion rate in accordance with the availability of Central Bank net reserves; individuals may have existing private holdings of dollar assets; and, finally, the individuals or the government may be able borrow dollars abroad.[5] We imagine these channels are limited but used as much

[3] The flow value is the saved interest payment relative to government debt. At a steady state this amounts to a flow of i/v where i is the nominal interest rate $v = PY/M$ is velocity.

[4] While this paper was partly spurred by dollarization proposals in Argentina, we have not set out to provide a full cost-benefit analysis that tallies all the costs. We hope our focus on an often neglected cost enriches the discussion of dollarization, current and future.

[5] In the case of present-day Argentina, net available reserves are very small, indeed near zero by some estimates. Argentina is currently under an assistance program with the IMF and has likely exhausted its credit in private international markets. Private citizens, however, have savings in dollars. The more affluent private citizens have significant wealth held abroad. Many others have some savings in the form of physical dollars "under the mattress", but estimates vary widely. Of course, a large fraction of the population, less

as possible and take as given the resulting initial dollar balances.

In our model, if initial balances are large enough the economy reaches a steady-state equilibrium immediately, seamlessly switch from domestic to foreign currency. However, this is no longer the case when dollars are initially scarce, due to limited reserves, limited credit and low pre-existing dollar holdings. In this scarcity situation, the economy undergoes a non-trivial transition towards its steady state. Our goal is to study such post-dollarization transitional dynamics.

We consider both flexible prices and nominal rigidities. As our baseline, even under nominal rigidities, we allow prices to be fully flexible at the instant dollarization is declared, based on the idea that this coordinates firms to reset prices in a new unit of account.[6]

It is worth noting that our analysis is post-dollarization. Thus, a country experiencing very high and persistent inflation may have a relatively high frequency of price changes—prices may not be very sticky before dollarization. However, this is not necessarily relevant after dollarization is adopted since inflation if inflation is not expected to remain as high as before. Likewise, desired money balances may be quite low prior to dollarization, due to the high inflation tax. However, what is relevant is the the steady state value for money balances under dollarization, which may be significantly higher.

Our main results highlight how the equilibrium outcome resembles that of a (self-inflicted) "sudden stop": a temporarily positive current account and trade imbalance, brought about by a discrete drop in the consumption of tradable goods that then slowly rises towards its steady state. Indeed, for a benchmark case these dynamics are solved without reference to the non-tradable sector. Intuitively, tradable consumption falls as agents seek to save to build up their stock of foreign currency. Put differently, the low real money balances raise the domestic interest rate (or shadow rate) and this lowers spending.[7]

Our "sudden stop" characterization has immediate implications for non-tradables. Without nominal rigidities, market clearing requires prices and wages in the non-tradable sector to fall discretely, then recover over time in line with the recovery of tradables. Put differently the real exchange rate must drop on impact, then recover over time.

affluent, has no dollar savings. Aggregate dollar holdings is not the correct measure of liquidity.

[6]However, we also explore variants where this initial price is not set optimally. This could capture a number of departures from this flexible and optimal ideal: perhaps only a fraction of firms change their price, or there is some lack of coordination, or lack of perfect foresight with behavioral biases and rules of thumb (e.g. firms selling non-tradable goods bent on charging the same price in dollars they did before dollarization).

[7]This emphasizes the crucial condition that the country be up against a borrowing constraint, so that the domestic interest rate can be higher than the foreign interest rate.

In the presence of nominal rigidities, however, the price of non-tradable goods (or the real exchange rate) may be too high or too low at any point in time, with such price misalignments leading to recessions or a booms in the non-tradable sector. For example, if the price (in dollars) remains at its high original value before dollarization, then there will be a recession and the price will initially tend to fall and later rise back to its steady state. However, because the price rise is slow so prices fall behind and become too low. Thus, the non-tradable sector starts in a recession, but then enter a boom before returning to normal.

In our baseline, all firms are allowed to reset their initial price when the economy becomes dollarized. This leads to a discrete downward jump in the price level. We show that the price jumps to a level that lies above the flexible price level, but below the steady state level. Indeed, the chosen price level is set to make inflation precisely zero in that instant, not too high to merit deflation, nor too low to merit inflation. Intuitively, since all firms have just reset their prices, firms getting a chance to change their price again, immediately after, do not reset price discretely away from other firms, implying zero instantaneous inflation. However, inflation immediately becomes positive and the price of non-tradable goods rises monotonically towards its steady state. The non-tradable sector starts in a recession, but transitions into a boom before returning asymptotically towards neutral.

In addition to our qualitative results, we undertake a quantitative calibration and investigation. Our results suggest that the transitional dynamics may be significant. Our model has the benefit of being quite simple, so the magnitude are transparently driven by a few key parameters. The most important of which is the size of the shortfall in initial dollars relative to its steady state value.

We also explore two relevant extensions. First, we consider situations where the initial price of non tradable goods is not reset, immediately upon dollarization, by all firms. Alternatively, even if the price is reset by all firms these firms may lack the perfect foresight needed to lower their price sufficiently. In our view, this may realistically capture that upon dollarization, private agents may seek to roughly maintain their present prices or wages in dollar terms. In other words, our general-equilibrium macro model may call for a sharp real depreciation, but is it realistic to assume private agents will know this in a flash? We show that, if, for either reason, the initial price is set too high, then the initial recession is deeper and longer lasting. Indeed, the situation is then similar to that of a fixed exchange rate regime, currency peg or currency board.

Second, we consider an extension with heterogenous agents and incomplete markets. In this extension there are two groups of agents: the first group are rentiers specialized in the tradable sector, living off their endowment of the export good (e.g. commodities); the other group of agents is specialized the non-tradable sector, providing labor to produce non-tradable goods. Financial markets are imperfect, so these groups cannot borrow from one another, or insure each other. We show that heterogeneity has non-trivial implications for the aggregate dynamics, but most even more noteworthy is the breakdown across groups. In particular, dollarization may disproportionally hurt the group specialized in the non tradable sector. These agents have no direct access to dollars from exports and the price of the good they sell falls. This negative pecuniary effect makes quickly accumulate dollar balances more difficult for this group.

Related Literature

Our paper relates to work directly dealing with dollarization, as well as a broader literature in international macroeconomics studying monetary and exchange rate policy as well as liquidity crises or "sudden stop" shocks.

Our model has elements from both older and more recent international macro traditions. On the one hand, we draw on ideas of the "Monetary Approach to the Balance of Payments" (e.g., Frenkel and Johnson, eds, 1976) and its focus on money and financial frictions across countries, relevant to our dollarization application. In particular, the backbone of our model is closest to Calvo (1981), but with tradable and non-tradable goods.[8,9] On the other hand, our treatment of nominal rigidities with staggered pricing follows the modern approach in New Keynesian open economy models (e.g., Benigno and Benigno, 2003; Gali and Monacelli, 2005).

Our results show that dollarizing can resemble the effects of a sudden stop. The latter have been extensively studied theoretically, empirically and quantitatively in the literature (e.g. Calvo, 1995, 1998; Mendoza, 2002; Calvo, Izquierdo and Mejía, 2004).

The potential benefits and definite costs of dollarization are discussed extensively in an early paper by Fischer (1982). That paper concludes that the loss of seignorage is significant and that, absent commitment problems, dollarization is dominated by a fixed exchange regime. A large literature addresses the cost from lost monetary policy indepen-

[8]The Bretton Woods international monetary arrangement provided a backdrop motivation for for the lack of free capital mobility in these models. In contrast, most of the subsequent "Intertemporal Approach the Current Account" developed in the 1980s took as a baseline a model a world with free capital markets where interest rates were equalized across countries. The subsequent New Keynesian open-economy literature followed this approach, often adding perfect risk sharing, and essentially dropped money in favor of interest rates.

[9]Calvo and Rodriguez (1977) provided an earlier incarnation where consumption, savings and money demand schedules were imposed, rather than derived. Vegh (2013) provides an excellent overview of this literature and its extensions.

dent. For example, Schmitt-Grohé and Uribe (2001) provides a quantitative investigation and concludes that dollarization minimizes welfare among the regimes they consider. On the benefit side, it has long been argued that the use of foreign currency may impose constraints on policymakers that would otherwise misbehave, helping stabilize inflation. Alesina and Barro (2001) emphasizes this point as well as the potential for lowering trade costs. Calvo (2001; 2002) argues that the net dollarization may be less costly and potentially on net more beneficial for emerging economies which lack credibility, use dollars in transactions,[10] issue debt in dollars, and suffer from volatile nominal exchange rates.

2 A Dollarized Open Economy

We first provide a brief overview of the model and then turn to the details. Time is continuous $t \geq 0$. Dollarization is announced and implemented at $t = 0$. We study the economy post dollarization $t \geq 0$—only briefly discussing the moments before dollarization.

There are two sectors: tradable and non-tradable goods. Agents consume both goods and provide labor towards the production of the non-tradable good. They also obtain utility from the liquidity services provided by money holdings. Our baseline model is based on a representative agent, but we also explore an extension with heterogeneity and financial frictions.

Tradable goods have given international prices.[11] Non-tradable good may face nominal rigidities a la Calvo but in our baseline are completely flexible at $t = 0$ when dollarization is announced. This captures the idea that firms must set prices in dollars.

Individuals have access to international financial markets. To abstract from seignorage, we set the foreign interest and inflation rates to zero. To set aside the issue of loss of monetary independence, we use a deterministic model without fluctuations.[12]

In our baseline analysis there is no government policy. Dollarization abdicates any monetary policy and we do not consider fiscal policy nor trade or tax policy.

[10] Guidotti and Rodriguez (1992) model the slow but persistent and voluntary currency substitution towards the dollar from high inflations.

[11] Results would be similar in a model with tradable goods and home bias or finite trading costs (that induce home bias). However, the non-tradable vs tradable good is simpler and avoid terms of trade manipulation effects which we do not think are essential to the question.

[12] That is, we abstract from fluctuations in demand (e.g. terms of trade, foreign interest rates or domestic discount factors) nor supply (e.g. productivity or work preferences)

2.1 Primitives: Preferences, Technology and Trade

Preferences. Agents have utility

$$\int_0^\infty e^{-\rho t} U(c_{Tt}, c_{Nt}, \ell_t, m_t, p_{Tt}, p_{Nt})\, dt$$

with discount rate $\rho > 0$. We will specialize the utility U below. Here c_T and c_N are consumption of tradable and non-tradable goods, respective, ℓ is labor, m are nominal dollar holdings of money. The prices p_{Tt} and p_{Nt} of tradable and non-tradable goods must be included in utility to adjusts the liquidity services provided nominal dollars m; relatedly, we assume U is homogeneous of degree zero in (m, p_T, p_N), i.e. a doubling of all prices and money balances does not change utility. In what follows we take advantage of the fact that $p_{Tt} = p_T$ is constant to suppress p_{Tt} as an argument and ease the notation.

For our theoretical analysis we adopt an additively separable specification of utility

$$u_T(c_T) + u_N(c_N) - h(\ell) + u_m(m/P(p_N)), \tag{1}$$

for some given increasing concave functions u_T, u_N, u_M, an increasing and convex disutility function h, and a non-decreasing and weakly concave price-index function $P(p_N)$, that deflates nominal balances m into real balances $m/P(p_N)$.[13] Imposing homotheticity over (c_T, c_N) may be desirable and this obtains with $u_T(c)$ and $u_N(c)$ each proportional to some $u(c) = c^{1-1/\epsilon}/(1-1/\epsilon)$. For numerical explorations we also work with $u_{\mathcal{C}}(\mathcal{C}(c_T, c_N)) - h(\ell) + u_m(m/P(p_N))$ where $\mathcal{C}$ is homogeneous of degree one CES aggregator, while $u_{\mathcal{C}}$, u_m and h are power functions.[14]

Technology. Output of the tradable good is given by a constant endowment $y_T > 0$. The non-tradable good is produced from labor. To incorporate price rigidity, we adopt the standard setup where final good producers combine a continuum of input varieties $i \in [0,1]$ using a constant returns Constant Elasticity of Substitution (CES) production function

$$y_{Nt} = \left[\int_0^1 y_{Nt}(i)^{1-\frac{1}{\eta}}\, di\right]^{\frac{1}{1-\frac{1}{\eta}}},$$

[13] Recall that we are omitting p_T without loss of generality because it is being held constant. Making its dependence on p_T explicit the price index function $P(p_T, p_N)$ should be homogeneous of degree one. However, once p_T is held fixed it should be weakly concave.

[14] Note that with $u_c(c) = c^{1-\sigma}/(1-\sigma)$ and $\mathcal{C}(c_T, c_N) = (\alpha c_T^{1-\gamma} + (1-\alpha) c_N^{1-\gamma})^{\frac{1}{1-\gamma}}$ and $\gamma = \sigma$ we obtain the additive case.

with CES elasticity $\eta > 1$. Each variety is produced by a monopolistically competitive firm one-for-one from labor

$$y_{Nt}(i) = \ell_{it}.$$

Aggregate labor equals $\ell_t = \int_0^1 \ell_{it}\, di$. The efficient aggregate production entails $y_{Nt}(i) = y_{Nt}$ and gives $y_{Nt} = \ell_t$, a linear one-for-one aggregate production function between output and labor.[15]

International Trade. The rest of the world offers both intratemporal and intertemporal trade opportunities. Trade in goods is done at given international prices equal for both exports and imports to p_{Tt}. To simplify we also assume that the price of tradables is constant over time, $p_{Tt} = p_T$. Intertemporal trade involves two assets: foreign money m and foreign bonds b. The foreign interest rate on bonds r^* is given and assumed constant. These trade opportunities are summarized by the constraints

$$\begin{aligned} \dot{m}_t + \dot{b}_t &= r^* b_t + p_T(y_T - c_{Tt}), \\ m_t &\geq 0, \\ b_t &\geq -\phi. \end{aligned}$$

Here money holdings must be non-negative, while bond holdings are subject to a borrowing constraint with maximal credit limit $\phi \geq 0$.

To simplify we assume that[16]

$$\begin{aligned} r^* &= 0, \\ \phi &= 0, \\ b_0 &= 0. \end{aligned}$$

The assumption that $r^* = 0$ is not crucial to our analysis, it helps assume away seignorage: bonds and money yield the same return, even if both were held. The assumption that ϕ and b_0 are zero is just a convenient normalization; the crucial condition is that $b_0 = -\phi$. This sets the country up against its borrowing constraint. Naturally, in equilibrium, the borrowing constraint will continue to bind, so that $b_t = 0$ for all $t \geq 0$. By implication, we will be able to ignore foreign bonds and focus exclusively on money m_t.

[15]This obtains under flexible prices. However, due to nominal rigidities, price dispersion can lead to inefficient production, $y_N < \ell$. Starting from zero inflation, however, this inefficiency is second order in the size of the shocks, so it will not be important in our analysis.

[16]Our results hold more generally with $r^* < \rho$. We focus on $r^* = 0$ for presentation purposes. One can also think of $r^* = 0$ as an approximation for low r^*, that is as a limit of $r^* \downarrow 0$.

Finally, we assume that initial foreign money holdings are given and strictly positive

$$m_0 > 0.$$

The level of m_0 turns out to be crucial in our analysis, as it captures the relative scarcity of dollars in the newly dollarized economy. We are interested in cases where $m_0 < m^*$ where m^* is the steady state level, which shall be discussed further below.

2.2 Markets: Household and Firm Optimization

Household Optimization. The representative agent transacts in local markets and faces the budget constraint

$$p_T c_{Tt} + p_{Nt} c_{Nt} + \dot{m}_t + \dot{b}_t = r^* b_t + p_T y_T + w_t \ell_t + \Pi_t$$

as well as $m_t \geq 0$ and $b_t \geq -\phi$. Here w_t is the nominal wage and Π_t are firm profits. Recall that we assume $r^* = \phi = b_0 = 0$ and $m_0 > 0$ given. To ease the notation we now normalize $p_T = 1$.

Agents maximize utility subject to these constraints. This gives the optimality conditions[17]

$$\frac{u_{Tt}}{p_T} = \frac{u_{Nt}}{p_{Nt}} = -\frac{u_{\ell t}}{w_t} = \mu_t \tag{2}$$

so that an extra dollar spent on each good (including the reduction in labor) must be equated to the marginal utility of wealth, μ_t. and the optimality of m_t which gives

$$\dot{\mu}_t = \rho \mu_t - u_{mt} \tag{3}$$

with $e^{-\rho t} \mu_t \to 0$.[18] One can solve $\mu_t = \int_0^\infty e^{-\rho s} u_{mt+s}\, ds$ and express the optimality condition as

$$\frac{u_{Tt}}{p_T} = \int_0^\infty e^{-\rho s} u_{mt+s}\, ds,$$

equating the marginal utility from spending an extra dollar on tradables versus increasing money balances permanently.

Finally, the optimality condition for b_t is $\dot{\mu}_t \leq (\rho - r^*)\mu_t$ with complementary slackness: if $\dot{\mu}_t < (\rho - r^*)\mu_t$ then $b_t = -\phi = 0$. Since $r^* = 0$ and $\dot{\mu}_t = \rho \mu_t - u_{mt} < \rho \mu_t$ given

[17] These conditions can be obtained by applying the Maximum principle.

[18] It is useful to reinterpret the condition that $\dot{\mu}_t = \rho \mu_t - u_{mt}$. Suppose we added a domestic bond and allowed both borrowing and saving in this bond. Then its optimality condition is $\dot{\mu}_t = (\rho - i_t)\mu_t$. This implies that $i_t = u_{mt}/\mu_t = p_T u_{mt}/u_{Tt} > 0$ which can be interpreted as a shadow interest rate. Since $i_t \neq r^*$ this emphasizes that the borrowing constraint prevents interest rate equalization.

$u_{mt} > 0$ we conclude that $b_t = 0$.

Firm Optimality. Final good producers behave competitively and have constant returns technology, implying that in the relevant cases (positive production and zero profits) the price is given by

$$p_{Nt} = \left(\int_0^1 p_{Nt}(i)^{1-\eta}\, di \right)^{\frac{1}{1-\eta}}.$$

where $p_{Nt}(i)$ is the price of input variety i. Optimal demand for variety i is then

$$y(i) = c_{Nt} \left(\frac{p_{Nt}(i)}{P_{Nt}} \right)^{-\eta}.$$

Thus, monopolist producers of varieties have market power. These firms have pricing policies and meet all demand by hiring labor as needed to do so.

We consider two cases: flexible prices and Calvo price rigidity. When prices are flexible,

$$p_{Nt} = \frac{1}{1 - 1/\eta} \cdot w_t \tag{4}$$

at all $t \geq 0$.

With nominal rigidities, firms set prices at random Poisson arrival dates with arrival rate $\lambda > 0$. Thus, a firm's price remains constant over a random, exponentially distributed, time interval. Any firm resetting its price at t is symmetric and solves

$$\max_{p_t^*} \int_t^\infty e^{-\lambda(s-t)} \mu_s (p_{Nt}^R - w_s) c_{Ns} \, (p_t^*)^{-\eta} \, p_{Ns}^\eta \, ds,$$

The first-order condition delivers the reset price as a constant markup over a weighted average of future nominal marginal costs

$$p_{Nt}^R = \frac{1}{1 - 1/\eta} \frac{\int_t^\infty \omega_s \, w_s \, ds}{\int_t^\infty \omega_s \, ds}, \tag{5}$$

where $\omega_s = e^{-\lambda s} \mu_s c_{Nt} p_{Ns}^\eta$.

In our baseline, we assume that $t = 0$ is special due to dollarization: all firms are able to reset prices at $t = 0$. After all, firms now quote prices in dollars and if they were not already doing so this requires a price reset.[19] This implies that $p_{N0} = p_{N0}^R$.

[19] Results are similar if there is a short period of time with an extraordinary high value for λ.

In period t, producers who had their last price reset at $s \leq t$ produce

$$y_t(s) = c_{Nt} \left(\frac{p_s^R}{P_{Nt}} \right)^{-\eta}$$

and demand labor $\ell_t(s) = y_t(s)$. Aggregate labor is then

$$\ell_t = \int_{-\infty}^{t} e^{-\lambda(t-s)} \ell_t(s) \, ds.$$

The price of non-tradable goods satisfies

$$p_{Nt} = \left(\lambda \int_{-\infty}^{t} e^{-\lambda(t-s)} \left(p_{Ns}^R \right)^{1-\eta} ds \right)^{\frac{1}{1-\eta}} \tag{6}$$

where we take $p_{Ns}^R = p_{N0}^R$ for $s \leq 0$ to capture the extraordinary price reset at $t = 0$. Differentiating (6) yields

$$\frac{\dot{p}_{Nt}}{p_{Nt}} = \lambda g \left(\frac{p_{Nt}^R}{p_{Nt}} \right), \tag{7}$$

with $g(x) = \frac{1}{1-\eta}(x^{1-\eta} - 1)$. Note that $\pi_{Nt} = 0$ if and only if $p_{Nt}^R = p_{Nt}$. This implies that at $t = 0$ we have $\pi_{N0} = 0$ since all firms reset prices at $t = 0$ and so $p_{N0}^R = p_{N0}$.

2.3 Equilibrium

Summing up, an equilibrium is a sequence of aggregates $\{c_{Tt}, c_{Nt}, \ell_t, m_t, b_t, p_{Nt}, w_t\}_{t \geq 0}$ satisfying household maximization, firm maximization and

$$\dot{m}_t = p_T(y_T - c_{Tt}),$$

which incorporates the observation that $b_t = 0$ in all equilibria.

Steady State. A steady state is an equilibrium with values for each variable that are constant over time. One can verify that under standard conditions on utility, there exists a steady state and it is unique. We denote the steady state values by a star: $(c_T^*, c_N^*, \ell^*, m^*, p_N^*)$. Note that $c^* = \ell^*$ given aggregate technology. In addition, (c_T^*, c_N^*, ℓ^*) is independent of p_T, whereas m^* and p_N^* are proportional to p_T.

3 Dollarization as a Sudden Stop

We now study the equilibrium immediately after dollarization. We are interested in situations where, when dollarization takes place, dollars are scarce in the precise sense that money balances are below their steady state,

$$m_0 < m^*.$$

Why would this be the case? Suppose that just prior to dollarization private agents had some foreign assets (m_-, b_-) and available credit ϕ_-; similarly, the government may have had holdings (m^g_-, b^g_-) and borrowing capacity ϕ^g_-.[20] Then

$$m_0 \leq \bar{m}_0 \equiv m_- + b_- + \phi_- + m^g_- + b^g_- + \phi^g_- > 0.$$

Setting $m_0 = \bar{m}_0$ requires all economic actors—both private agents and the government—to convert all their assets into money and borrow to their limit. In addition, the government must then distribute all its dollars to private agents—via the conversion of pre-existing domestic currency or via outright transfers. Otherwise, $m_0 < \bar{m}_0$.

Our analysis thus focuses on situations where either $\bar{m}_0 < m^*$ or $m_0 < m^* \leq \bar{m}_0$. That is, situations where dollars are scarce because private holdings, reserves or credit are low or because these were not used to their maximal capacity.

3.1 Decoupling Tradables and Sudden Stops

Our main results will show that, upon dollarization, tradable consumption falls discretely and then gradually recovers towards its steady state—a pattern reminiscent of a "sudden stop". We shall see that the particular form this sudden stop takes has very sharp implications for the non-tradable sector.

To drive this point in the clearest manner, we first discuss a baseline case where the equilibrium in the tradable sector can be solved without reference to the non-tradable sector. This "block diagonal" property is convenient and interesting, although it is not crucial to our results, as we show later.

Additive separability of utility already ensures there are no direct interactions between the tradable and non-tradable sector. However, there is still a potential interaction via the real value of money, given by $m_t/P(p_{Nt})$ for some given price index function P. Thus, if when m_t is low the price of non-tradables p^f_{Nt} is also low, then this mitigates the fall

[20] Depending on whether we interpret the government as the Central Bank, the Treasury, or a consolidation of the two a^g_- may be positive or negative.

in real money balances $m_t/P(p_{Nt})$. This, in turn, may affect the desire to accumulate money, affecting consumption and savings choices. Formally, this interaction is captured in condition (3)

$$\dot{\mu}_t = \rho\mu_t - U_m'(m_t/P(p_{Nt}))/P(p_{Nt}), \tag{8}$$

by the presence of p_{Nt}. However, it turns out that the effects of $P(p_{Nt})$ are, in general, ambiguous because $P(p_{Nt})$ enters twice in this expression. A dichotomy obtains when these two effects cancel out: when utility over real money balances is logarithmic then

$$U_m(m/P) = \log(m/P) = \log m - \log P,$$

$$U_m'(m/P)P = U_m'(m) = 1/m.$$

The first expression shows that utility will be additively separable in nominal dollars m and the price level P. This implies that the price index P does not affect preferences over (c_T, c_N, ℓ, m) and, thus, drops out of the equilibrium determination.[21] The second expression shows that the terms involving P in (8) cancel out. Using $\mu_t = U_T'(c_{Tt})/p_T$ to rewrite (8), we can rewrite the equilibrium conditions for c_{Tt} and m_t as

$$\dot{c}_t = \frac{c_t}{\sigma_T(c_t)}\left(p_T\frac{U_m'(m_t)}{U_T'(c_{Tt})} - \rho\right) \tag{9}$$

$$\dot{m}_t = p_T(y_T - c_{Tt}), \tag{10}$$

and $e^{-\rho t}U_T'(c_{Tt}) \to 0$; here $\sigma_T(c) = -U_T''(c)c/U_T'(c)$ is the local elasticity of marginal utility for tradables.

These conditions pin down the path $\{c_{Tt}, m_t\}$ for any m_0 and make no reference to the non-tradable sector. Thus, we can solve for an equilibrium by first solving for $\{c_{Tt}, m_t\}$ and then using this path to solve for $\{c_{Nt}, \ell_t, p_{Nt}\}$. Since the equilibrium outcome for c_{Tt} and m_t is independent of the non-tradable sector it is *as if* there were no non-tradable sector, just as in Calvo (1981). The dynamics are also easily characterized. There is a unique steady-state and the dynamical system (9)–(10) is saddle-path stable with m_t acting as a state variable and c_{Tt} as a jump variable. These observations are summarized in the next proposition.

Proposition 1 (Dollarization→Sudden Stop)**.** *Suppose*

$$U_m(m) = \log m$$

and $m_0 - m^* < 0$, *then*

[21] By implication, all of our results in this case are robust to the price index function that is employed.

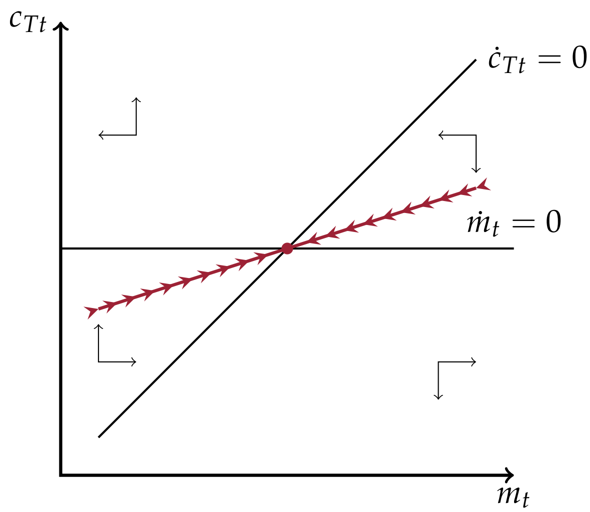

Figure 1: Phase diagram of saddle path dynamics for (c_t, m_t) with additive utility and $U_m = \log$.

1. the path $\{c_{Tt}, m_t\}$ does not depend on non-tradable sector; neither the path (c_{Nt}, ℓ_t, p_{Nt}) nor on any specification for utility or price stickiness in the non-tradable sector;
2. in equilibrium $c_{Tt} \leq c^*$, with c_{Tt} increasing and $c_{Tt} \to c^*$;
3. to a first-order approximation in $\Delta = \log(m_0/m^*) < 0$,

$$\log(c_{Tt}/c_T^*) = \phi \Delta e^{-\delta t}$$
$$\log(m_t/m^*) = \Delta e^{-\delta t}$$

with $\phi > 0$.

Proposition 1 shows that dollarizing when dollars are scarce effectively induces a "sudden stop" that lowers consumption of tradables throughout, creating a surplus in the current account to accumulate dollars. Over time, consumption and dollar balances gradually recover. Figure 1 illustrates the saddle-path dynamics in a phase diagram.

Intuitively, individuals cut their consumption to build up their money balances. Put differently, the shortage of money increases the shadow interest rate, inducing individuals to save. Collectively, these efforts create a current account and trade surplus that increases dollar money balances.

3.2 Non Tradables with Flexible Prices

Next we turn to the the non tradable sector, starting with the case of flexible prices. Let us take as given a path for tradable consumption with $c_{Tt} \leq c^*$, c_{Tt} increasing and $c_{Tt} \to c^*$ as in the conclusion of Proposition 1. The equilibrium conditions (2) are

$$\frac{U'_\ell(\ell^f_t)}{w_t} = \frac{U'_N(c^f_{Nt})}{p_{Nt}} = \frac{U'_T(c^f_{Tt})}{p_T}. \tag{11}$$

Combining the first equality with the firm pricing condition (4) and $c^f_{Nt} = \ell^f_t$ gives

$$\frac{U'_\ell(c^f_{Nt})}{U'_N(c^f_{Nt})} = 1 - \frac{1}{\eta} \qquad \Longrightarrow \qquad c^f_{Nt} = c^*_N,$$

so that non-tradable consumption is constant, at its steady state value. The second equality in (11) gives the equilibrium price

$$p^f_{Nt} = \frac{U'_N(c^*_N)}{U'_T(c_{Tt})} p_T \tag{12}$$

as an increasing function of c_{Tt}.

Proposition 2 (Flexible Prices). *In a flexible price equilibrium $c^f_{Nt} = c^*$ and p_{Nt} is an increasing (time-dependent) function of c_{Tt}. Thus, if $c_{Tt} \leq c^*$, with c_{Tt} increasing and $c_{Tt} \to c^*$ then $p^f_{Nt} \leq p^*_N$, with p^f_{Nt} increasing and $p^f_{Nt} \to p^*_N$.*

These observations are relatively standard and intuitive: an economy experiencing a drop in tradables (without a change in relative preferences of tradables vs non-tradables) will generally require a change in relative prices. With additive utility, non-tradable consumption should remain unchanged so the relative prices of non tradables must fall.

Suppose, as in Proposition 1, that after dollarization c_{Tt} lies below its steady state and increases over time. Then it follows that p_{Nt} starts below the steady state and rises over time, converging to it. In other words, dollarization produces a discrete fall in dollar prices followed by positive inflation. That is, the real exchange rate is initially depreciated and gradually appreciates over time towards its steady state.

3.3 Non Tradables with Nominal Rigidities

We now turn to the case with nominal rigidities $\lambda < \infty$. Once again let us solve for an equilibrium taking as given the equilibrium path for $\{c_{Tt}\}$ where $c_{Tt} \leq c^*$ and $c_{Tt} \to c^*$. This is immediately warranted under Proposition 1 and will be further justified below.

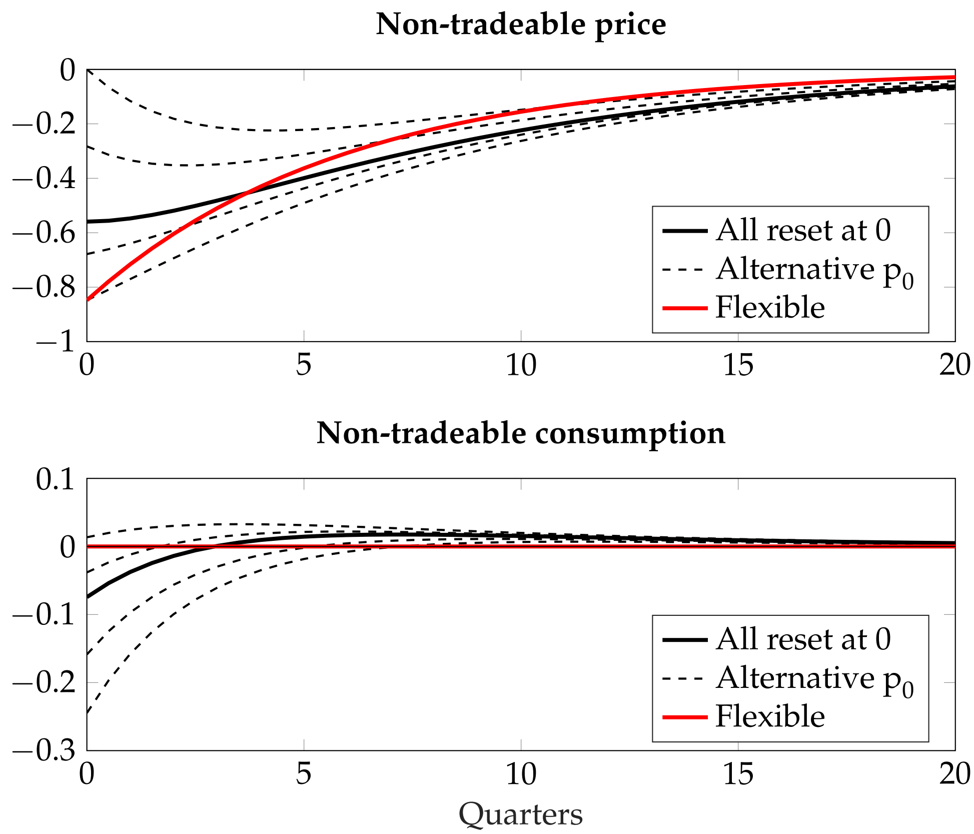

Figure 2: Time paths for $\{p_{Nt}\}$ and $\{c_{Nt}\}$. Dashed lines represent paths for different initial prices p_{N0}; black line represents equilibrium where everyone resets at $t = 0$ and $\pi_{N0} = 0$; red line represents flexible price baseline.

At one extreme, if prices are completely rigid ($\lambda = 0$) and, say, fixed at the steady state value $p_{Nt} = p_N^*$ then quantities are determined by the optimality condition $U_N'(c_{Nt}) = (p_N^*/p_T)U_T'(c_{Tt})$ and $\ell_t = c_{Nt}$. Thus, both consumption of non-tradables and employment are an increasing function of c_{Tt}. This is intuitive: if prices cannot adjust then quantities do the adjustment. In particular, since non-tradable prices needed to fall, when they do not then tradable consumption and, hence, employment fall.

When prices are not flexible and not completely rigid, so that $\lambda \in (0, \infty)$, there are adjustments in both prices and quantities. To solve the equilibrium and we must employ the firm optimality conditions (5) and (7) combined with the agent optimization $w_t = p_T U_\ell'(\ell_t)/U_T'(c_{Tt})$. We work with the log-linearized version of these conditions. Abusing notation we now write variables in log deviations from the steady state. This gives the

Phillips curve relation

$$\dot{\pi}_{Nt} = \rho\pi_{Nt} - \kappa(p_{Nt} - p^f_{Nt}), \tag{13}$$
$$\dot{p}_{Nt} = \pi_{Nt}, \tag{14}$$

for some $\kappa > 0$ and where p^f_{Nt} is the log linear deviation of the flexible price for non-tradable goods, obtained from the path for c_{Tt} as in the previous subsection. Log-linearizing the relation (12) gives $p^f_{Nt} = \sigma_T c_{Tt}$. Finally, our baseline assumes that all prices are reset at $t = 0$ which implies that

$$\pi_{N0} = 0.$$

This initial condition pins down p_{N0} uniquely since the two-dimensional system (13)–(14) is saddle-path stable. The next result sharply characterizes the dynamics: inflation is positive, the initial price is too high to avoid a recession and eventually is too low, generating a boom in non-tradables.

Proposition 3. *Suppose all firms can adjust prices at $t = 0$ and that $c_{Tt} \leq c^*$ with c_{Tt} increasing and $c_{Tt} \to c^*$. Then $\pi_{N0} = 0$ and for all $\Delta = m_0 - m^* < 0$ and $|\Delta|$ small enough*

1. *$\pi_{Nt} > 0$ for $t > 0$ and $p_{Nt} \to p^*_N$;*
2. *$p_{N0} \in (p^f_{Nt}, p^*_N)$ implying a recession in non-tradables $c_{N0} < c^*$;*
3. *$p_{Nt} < p^f_{Nt}$ for some $t > 0$, implying a boom in non-tradables $c_{N0} > c^*$.*

Figure 2 illustrates these dynamics for prices and quantities in the non-tradable sector. The economic intuition is as follows. First, recall that we allow the initial price of non-tradables to adjust freely upon dollarization at $t = 0$. Intuitively, firms then set a price discretely below the steady state p^*. However, since prices are sticky thereafter, for $t > 0$, and because spending is expected to improve over time firms set the price above the flexible one, so that $p_{N0} > p^f_{N0}$. As a result, the economy starts in a recession in the non-tradable sector, $c_{N0} < c^*$.

Because all firms reset their price optimally at $t = 0$ in a forward looking manner, those firms that also gets to reset their prices shortly afterwards will not make drastic discrete revision to their $t = 0$ price. This means there are no immediate inflationary pressures, so that $\pi_{N0} = 0$. However, over time inflation becomes positive and stays positive throughout for $t > 0$ because spending and the flexible price are rising, which provides an upwards trend in the prices set by firms, so that $\pi_{Nt} \geq 0$ for $t > 0$. However, since prices are sticky, the price p_{Nt} rises sluggishly and eventually fall behind p^f_{Nt}, so that $p_{Nt} < p^f_{Nt}$. At this moment, the non-tradable sector experiences a boom, $c_{Nt} > c^*$. In a nutshell, we expect the price to start below but return to its steady state which requires positive inflation. However, inflation, in turn, requires a current or anticipated boom in the non tradable sector. Indeed, $\pi_{N0} = 0$ holds despite a concurrent recession because firms expect a future transition to a boom. Figure 2 shows that all the paths for prices, for different arbitrary p_{N0} reach a minimum (where inflation is zero) when they are above p^f_{Nt}, then inflation rises and the equilibrium price path falls below the its flexible price counterpart.

It is important to note that these results *do not hold under a fixed exchange regime*. In particular, we adopted the assumption that all firms can adjust prices at $t = 0$ as our baseline under dollarization. The motivation is that firms must reset prices in a new unit of account, in the foreign currency (e.g. dollars), whereas their past prices were quoted in the domestic currency (e.g. pesos). In this way, dollarization coordinates an instant of price flexibility. In contrast, under a fixed exchange regime prices will continue to be quoted in the domestic currency so a natural benchmark is to maintain price rigidities unchanged. In continuous time, this implies $p_{Nt} = p^*_N$, which worsens the recession and creates initial deflationary pressures $\pi_{N0} < 0$. We explore this case in Section 5.1.

3.4 Sudden Stops without Decoupling

If the equilibrium $\{c_{Tt}\}$ resembles a "Sudden Stop" of the form

$$c_{Tt} \leq c^* \text{ with } c_{Tt} \text{ increasing and } c_{Tt} \to c^*, \tag{15}$$

then we have been able derive sharp implications for non-tradables, both quantities and prices. We have already justified this particular sudden stop property (15) when $U_m(m) = \log m$—that is when $\sigma_m(m) = 1$ where $\sigma_m(m) = U''_m(m)m/U'_m(m)$ is the local elasticity of marginal utility U'_m.

We now show that as long as $\sigma_m(m^*) \geq 1$ the same conclusion holds, at least to a first-order approximation in the size of $\Delta = m_0 - m^* < 0$. The restriction to $\sigma_m \geq 1$ is the empirically relevant case, since $1/\sigma_m$ is associated with the interest elasticity of demand which is found to be below 1, near 1/2 (Benati et al., 2021). The next proposition shows that we can guarantee (15) and all its implications for small enough Δ with this restriction.

Proposition 4 (Dollarization→Sudden Stop II)**.** *Suppose $\sigma_m(m^*) \geq 1$. Then for $\Delta = m_0 - m^* < 0$ and small enough the equilibrium satisfies* (15)*.*

Intuitively, this result relies on the fact that when $\sigma_m(m^*) \geq 1$ then a fall in prices p_{Nt}

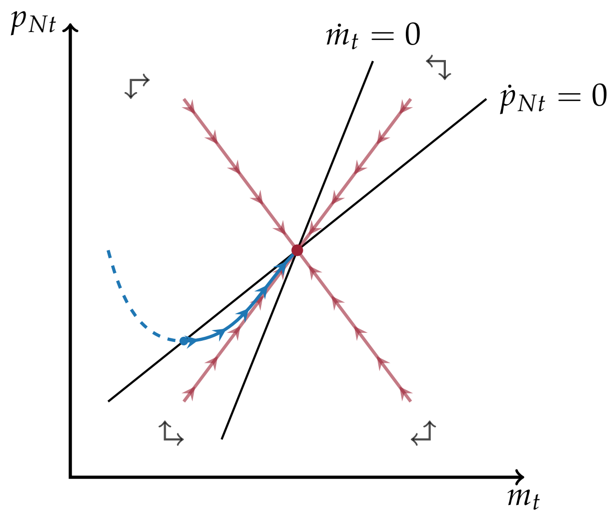

Figure 3: Phase diagram for stable equilibrium dynamics for state variable (m_t, P_t).

makes agents less eager to accumulate dollar balances. Thus, the dynamics are similar but mitigated.

The proof of this result is relatively non trivial and its details are included in an Appendix. Here we provide a graphical sketch of the main argument. We have a differential system in (m, p_N, c_T, π_N). This system is saddle-path stable. Thus, the stable solution satisfies

$$(c_{Tt}, \pi_{Nt})' = \Psi(m_t, p_{Nt})' \tag{16}$$

for some 2x2 matrix Ψ relating the "jump" variable (c_{Tt}, π_{Nt}) to the state variables (m_t, p_{Nt}). If we start with 16 holding at $t = 0$ then it holds for all $t \geq 0$ and we obtain a first order differential system for the state (m_t, p_{Nt}) that is globally stable: $(\dot{m}, \dot{p}_N)' = \Phi(m, p_N)'$ for some 2x2 matrix Φ. The appendix characterizes Ψ and Φ and shows that the implied dynamical system looks like the phase diagram in Figure 3. In particular, the loci for $\dot{p}_N = 0$ and $\dot{m} = 0$ are both upward sloping and the one for $\dot{p}_N = 0$ is flatter. This implies that any path in the (cone) region below the steady state, to the right of $\dot{p} = 0$, and to the left of $\dot{m} = 0$, stays in this region and the path converges to the steady state. Moreover, there exists a linear path in between the loci $\dot{p} = 0$ and $\dot{m} = 0$, that is, a stable eigenvector. For any given $m_0 < m^*$ we set p_{N0} on the $\dot{p}_N = 0$ locus so that $\pi_{N0} = 0$. This implies that (m_t, p_{Nt}) travels along a path towards the steady state to the left of the eigenvector with slope in the (m, p_N) space flatter than the eigenvector and, thus, flatter than the locus for $\dot{m} = 0$.

Recall that c_T is a linear function of (m, p_N). Indeed, along the locus $\dot{m} = 0$ we have c_T constant at the steady state level. Any parallel translation of this locus are iso-c_T lines and those to the left of $\dot{m} = 0$ have c_T below the steady state. This implies that the equilibrium paths, which as argued earlier, are flatter than the $\dot{m} = 0$ locus, must have c_{Tt} increasing over time towards its steady state value.

4 Quantitative Explorations

We now take a numerical approach and provide a more quantitative exploration of dollarization in our model.

Preference Specification. We allow utility to be non-separable between tradables and non-tradables, but will focus on the additive case. The appendix discusses the non-separable case. Utility is

$$\int_0^\infty e^{-\rho t} \left[u_{\mathcal{C}}(\mathcal{C}(c_{Tt}, c_{Nt})) - h(\ell_t) + u_m(m_t/P(p_{Nt})) \right] dt$$

where $u_{\mathcal{C}}(x) = (x^{1-\sigma} - 1)/(1-\sigma)$ and

$$\mathcal{C}(c_T, c_N) = \left((1-\alpha)^{1/\theta} c_{T,t}^{1-\frac{1}{\theta}} + \alpha^{1/\theta} c_{NT,t}^{1-\frac{1}{\theta}} \right)^{\frac{1}{1-\frac{1}{\theta}}}$$

is a CES aggregator. This implies that preferences are homothetic over consumption goods. The parameter α affects the steady state share of non-tradables consumption. As is standard, $\epsilon_C = \sigma^{-1}$ denotes the elasticity of intertemporal substitution and θ is the elasticity of substitution. Low values of θ imply less expenditure switching. When $\sigma^{-1} = \theta$ utility is additively separable; if $\theta > \sigma^{-1}$ then consumption goods are Hicks substitutes ($u_{TN} < 0$); if if $\theta < \sigma^{-1}$ then consumption goods are Hicks complements ($u_{TN} > 0$).

We set $h(x) = v_\ell \frac{x^{1+1/\epsilon_\ell}}{1+1/\epsilon_\ell}$ and $u_m(x) = v_m \frac{x^{1-1/\epsilon_m - 1}}{1-1/\epsilon_m}$ where ϵ_ℓ is the Frisch elasticity of labor and ϵ_m is the interest rate elasticity of money demand. The rest of the model is identical to the previous section.

Calibration. For our calibration we set EIS and inverse Frisch elasticities are chosen to be 2, as it is standard in the small open economy RBC literature.[22] The discount rate is set to 0.01 quarterly, consistent with a 4% annual discount. The value of θ is set to 0.5, very close to empirical estimates (González Rozada et al., 2004). The value is rounded to 0.5 so that the benchmark case features exact separability. Appendix C shows robustness to this choice. The share of non-tradable consumption is chosen to be $\alpha = 0.8$, which is broadly

[22] For a recent example see Schmitt-Grohe and Uribe (2023).

consistent with the values in the literature once we account for the fact that a large part of the cost of "tradable" goods are actually distribution and retailer costs, all non-tradable in nature (Burstein et al., 2005). The parameter γ is also chosen to be 2, consistent with an elasticity of 1 between money and consumption, and a semi-elasticity of money with respect to interest rates of one half, which is in line with the empirical estimates in Benati et al. (2021).

The remaining parameters require a more subtle calibration given the context. The key observation is that the relevant steady state is the one *after* the dollarization occurs. In particular, although the economy might experience high rates of inflation in the situation before the dollarization, it must be the case that after it, the steady-state inflation rate converges to the world inflation rate. In this context, it is natural to assume that the frequencies of price change will correspond to those of the new steady state. The fact that frequencies of price change are responsive to the average inflation rate, even for countries with chronic inflation, is well documented in the literature (Alvarez et al., 2018). Given those considerations, we set $\lambda = 1/3$, which corresponds to an average duration of price spells of 9 months. This is consistent with the median duration at the zero inflation level in Argentina estimated in (Alvarez et al., 2018).

Finally, we need to calibrate the steady state value of money, m^{ss}, and the size of the shock, $\tilde{m}_0$. For m^{ss}, note that the relevant value is money over tradable output. For our exercise, we interpret m as M_0. This is because of two reasons. First, since M_0 is much smaller than other money aggregates, the size of the sudden stop induced by a fall in M is lower. Picking M_0 is thus a conservative choice. Secondly, it makes intuitive sense that, once the economy acquires the necessary high-powered dollars, banks can intermediate them and provide money-like assets. Thus, in a model without banks or several money like assets, M_0 is a clear candidate. For the year 1998 (during Argentina's exchange rate peg), GDP was around 290 billion dollars, M0 was 15 billion. Thus, M_0 over annual GDP was around 5 percent. Because of our calibration, tradable output over GDP is 20 percent. Thus, M_0 over annual GDP is $1/4$, so $m^{ss}/y = 1$ since in our model, a period is a quarter. It is important to note that the legalization of the US dollar for transactional purposes may have depressed the demand for pesos during this period, so the calibration on the size of money is admittedly conservative.

Finally, we must calibrate the size of the initial imbalance. There are two different reasons why, initially, money starts below steady state. On the one hand, the new steady state after dollarizing features zero inflation, in contrast with the situation before dollarizing. Therefore, even if all pesos were converted to dollars at the current exchange rate, the stock of dollars would be below steady state. Secondly, the fact that international reserves

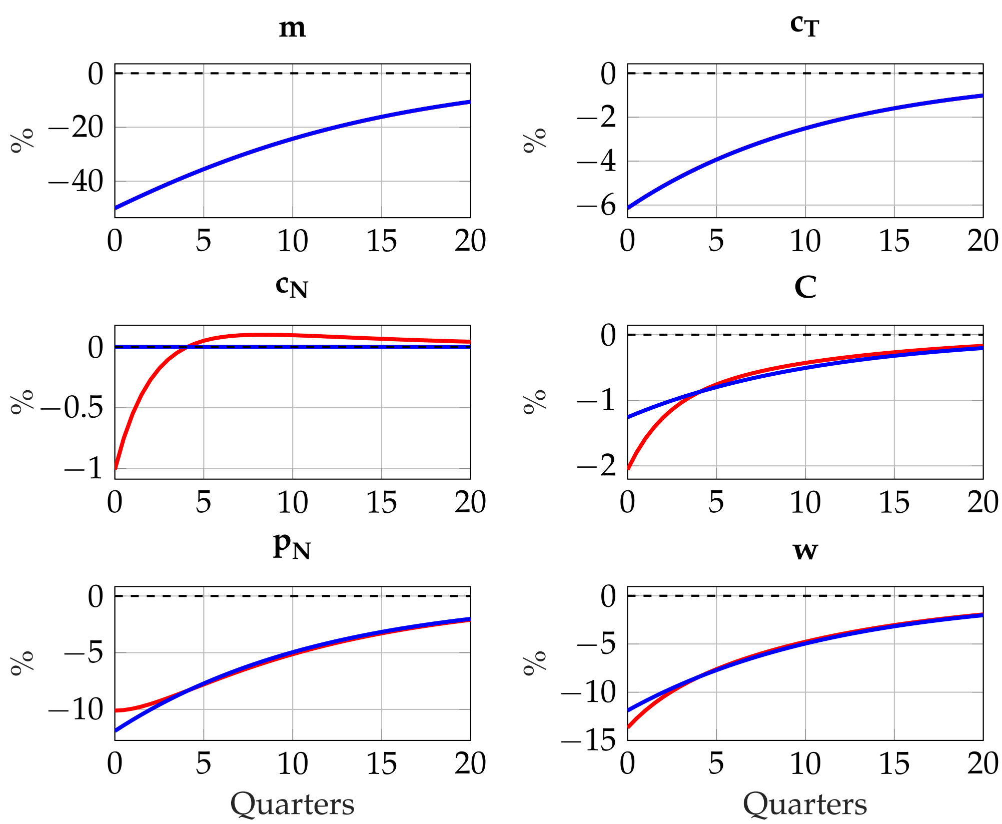

Figure 4: IRFs under the baseline calibration. All firms reset their price at $t = 0$. Units on the vertical axis are percentage point deviations from the final steady state.

are not enough to fully convert the stock of pesos to dollars implies that, absent any other source of liquidity, the exchange rate would have to depreciate if a dollarization were to occur. Put it simply, at the pre-dollarization exchange rate, there are not enough dollars to convert all existing pesos. Because of the large uncertainty regarding the implementation of a dollarization policy, any calibration is admittedly speculative. Note that, as of April 2023, $M0$ in pesos as fraction of annual GDP is around 4%. Thus, because of the first motive alone, dollarization would imply that the stock of money would start 20% below the new steady state. Any extra liquidation via devaluation would add to this number. Furthermore, even if pesos were exchanged by dollars at some rate and the private sector wouldn't suffer from this, the public sector will: it has to pay the private sector dollars in exchange for pesos that are useless. Thus, the economy-wide cost of dollarizing is the total stock of pesos, as argued by Fischer (1982) among others. In our calibration, due to the initial amount of money starting 50 percent below steady state is a reasonable starting point to gauge the magnitudes of this.

Results. Figure 4 shows the response of the economy to dollarization. Given that quantity of money starts below steady state and since the economy does not have access to capital markets, a drop in tradable consumption is required to rebuild money balances. Furthermore, demand of non-tradables is depressed. In the flexible price economy, $\tilde{p}_N$ drops in order to exactly offset this fall in demand. As a result, consumption of non-tradables stays at the steady state level.[23] The price of non-traded goods falls and gradually recovers back to steady state, as money balances are gradually rebuilt. Note however that convergence takes a long time: even after 16 quarters (4 years), the stock of money is around 15 percentage points below steady state.

Focusing on the response when prices are sticky, the fact that we allow all firms to reset at time $t = 0$ gets the response close to the flexible price allocation, but as we argued $p_{N0} > p^f_{N0}$. This implies that, initially, there is also a recession in the non-traded sector: as prices are higher than the flexible price allocation, quantities are lower. In line with the results of the previous section, non-tradable consumption overshoots the steady state value. Aggregate consumption initially falls below the flexible price case, and then rises above it, mirroring the overshooting in non-tradables.

5 Two Extensions

We now consider two relevant extensions. First, we discuss the dynamics if the initial dollar price is not fully flexible at the instant of dollarization, the recession may then be larger, as in a fixed exchange regime. Second, we consider an extension with heterogeneous groups of agents, with one group specialized in the tradable sector and the other in the non-tradable sectors. We assume that financial frictions and incomplete markets prevent these groups from borrowing from each other or offering insurance arrangements. Our main finding is that heterogeneity matters for the aggregate response and that the the cost of dollarization may be very asymmetrically felt, falling largely on the non-tradable agents, who are not endowed with the dollar providing exports.

5.1 Non-Flexible Initial Price

In order to gain perspective on how important the assumption of full reset at $t = 0$, Figure 5 shows the response of the economy if prices at time $t = 0$ start at the post-dollarization steady state. The purpose of the Figure is mainly illustrative: the economy is not at the

[23]This stark prediction hinges on the assumption that $\theta = \sigma^{-1}$, departures from that would imply that c_N goes above or below steady state depending on whether $\theta > \sigma^{-1}$ or $\theta < \sigma^{-1}$ respectively.

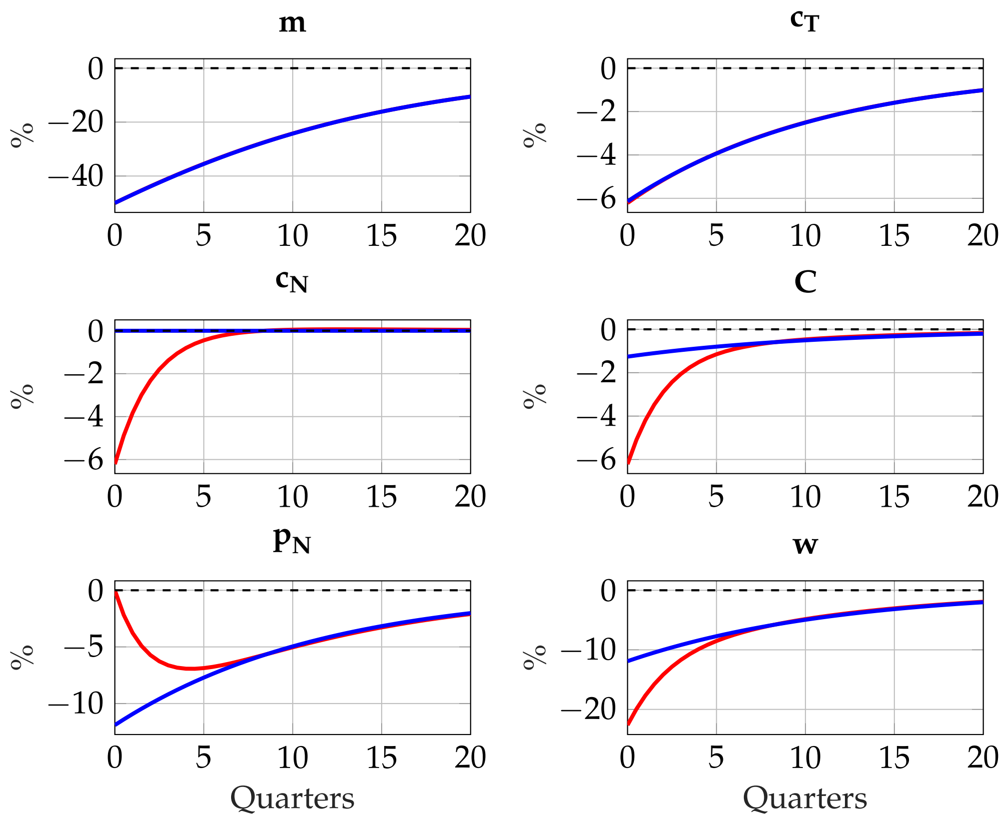

Figure 5: IRFs under the baseline calibration. The initial price level is set at the steady state level. Units on the vertical axis are percentage point deviations from the final steady state.

terminal steady state before $t = 0$. However, because of behavioral frictions or inertia, it can be the case the price at $t = 0$ is not exactly fully flexible. Any friction at time $t = 0$ may induce a p_{N0} that is above the one in the sticky price case in Figure 4. The purpose of Figure 5 is to quantify the implications of stickiness at time 0. As we observe, the recession is now much more abrupt initially. Since the price of non-tradables adjusts by far less initially, quantities adjust much more: aggregate consumption initially falls by around 6 percent. The overshooting is still present, but is quantitatively very small compared to the initial recession.

5.2 Household Heterogeneity and Incomplete Markets

The second extension incorporates heterogeneous agents. The motivation is two-fold. First, advocates for dollarization state that agents already have a stock of foreign currency that could be used as money should a dollarization occur. This in principle can mitigate the negative effects of a dollarization. However, the distribution of foreign currency holdings is heterogeneous across agents. Thus, dollarization will have a different impact depending on the who holds existing stock of dollars. Second, households have different sources of income. If a household gets income from the tradable sector, their income is likely co-move with output in this sector. For example, if an individual owns a farm that produces exportable crops and gets her income from selling those crops, her income co-moves one to one with what happens in the tradable sector. On the other hand, agents that are tied to the non-tradable sector (for example, workers in the non-tradable sector, which is the largest by far in terms of employment in Argentina) will be much more exposed to fluctuations in the real exchange rate. This heterogeneity changes the dynamics of the accumulation of foreign currency when a dollarization occurs. On the one hand, households that get their income from the tradable sector need only to reduce consumption in order to accumulate dollars; whereas on the other hand households working in the non-tradable sector need to purchase dollars from someone else.

We extend our baseline model from Section 4 to incorporate this heterogeneity. We consider two types of households. One type of households get their income exclusively from the tradable sector. In particular, they get the total endowment of tradable goods. The other type consists in agents that work and own the firms producing non-tradables. Let $i \in \{1,2\}$ index the type of households. Type-1 households work in the tradable sector, and have a constant endowment y. Type-2 households work in the non-tradable sector for a wage w, and get the profits of non-tradable firms if there were any. To isolate this channel, we assume homogeneous preferences across both sectors: all that changes is their source of income. We further assume that households do not trade foreign assets. This can be motivated by assuming that agents type 2 do not hold any foreign bonds.

The optimality conditions are identical to the ones without heterogeneity, but now there is one for each agent. The difference lies in the budget constraint. Each agent's budget constraint is (Π denotes the profits of the non-tradable producing firms).

$$\begin{aligned}\dot{m}^1 &= p_T\left(y - c_T^1\right) - p_N c_N^1 \\ \dot{m}^2 &= w\ell^2 + \Pi - p_T c_T^2 - p_N c_N^2\end{aligned}$$

In equilibrium, using market clearing we can rewrite the Type-2 budget constraint as

$$\dot{m} = p_N c_N^1 - p_T c_T^2 \tag{17}$$

This equation is the formalization of the intuition explained earlier: if households from the non-tradable sector want to increase their holdings of dollars, in equilibrium their income comes from the expenditure in non-tradables of the agents type 1. Recall that, in

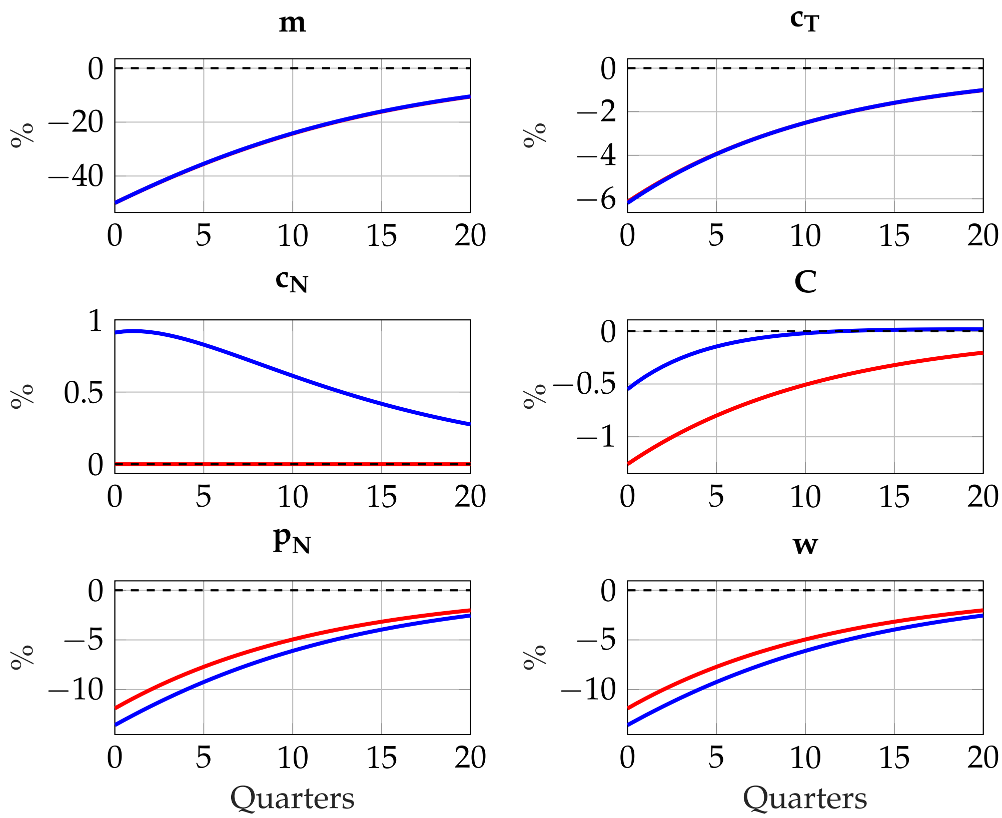

Figure 6: Impulse response to dollarization, both agents start below steady state. Blue line indicates two agent model. Red line indicates a representative agent model with the same calibration and flexible prices. Response of aggregate variables.

the benchmark representative agent case, c_N was constant and p_N dropped. If the same applies for agent 1, that means that the total expenditure on non-tradables falls. This makes accumulating dollars harder for agents type 2, potentially amplifying the recession and delaying the transition back to steady state.

Steady state and Calibration. The rest of the model is as before, with the caveat that we consider the flexible price case. Appendix D contains the details and Appendix D.2 shows the results with sticky prices. We look for a steady state in which $w = p_N = 1$, to be compared with the other case. Importantly, agent 1 gets a share $1 - \alpha$ of total output and agent 2 gets a share α. The fact that income shares equal consumption shares arises from our assumption about income sources. We assume that the steady state level of money is shared between both agents according to those shares as well. Thus, $m^1_{ss} = 1 - \alpha$, $m^2 = \alpha$. We consider two shocks: in one, we start both agents 50% below their steady state money holdings. In the second one, the aggregate quantity of money still falls by 50%, but all the

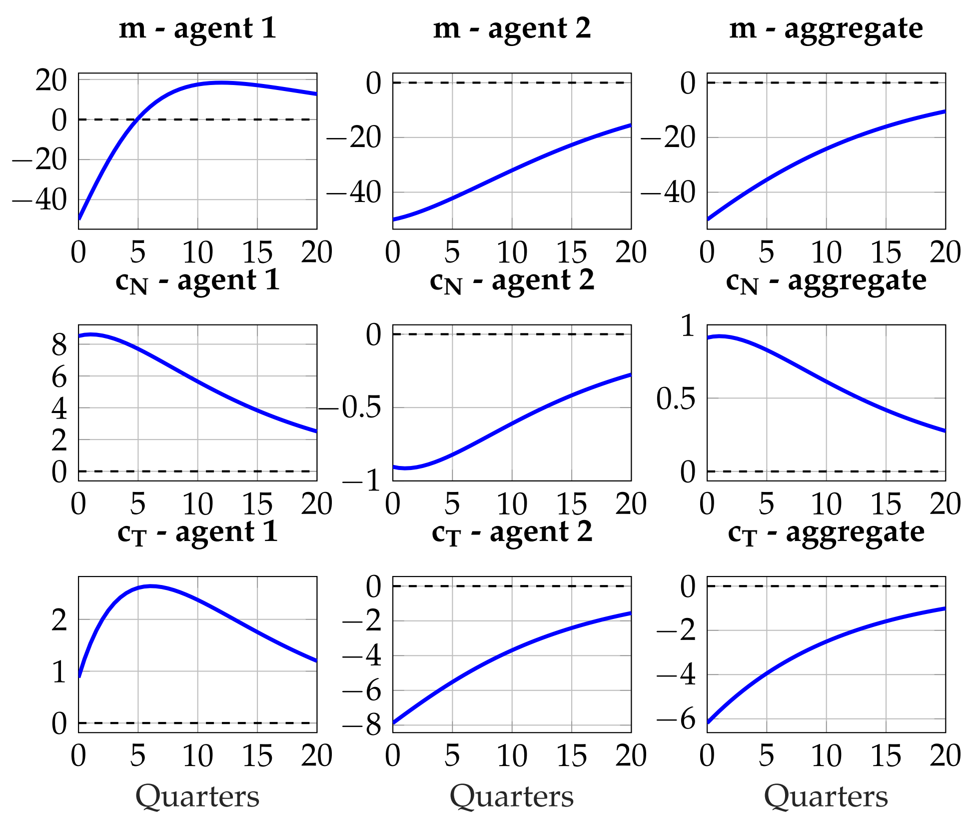

Figure 7: Impulse response to dollarization, both agents start below steady state. Blue line indicates two agent model. Red line indicates a representative agent model with the same calibration and flexible prices. Response of consumption and money for each agent.

burden falls on agent 2. That is, we set $\tilde{m}_0^1 = 0$ and $\tilde{m}_0^2 = -\log(2)/\alpha$. he intuition here is that richer agents (that get their income from tradables) already have dollars, so they do not have to accumulate assets for transactional purposes.

Results. Figures 6 and 7 show the response when the shock is to both agents. Figures and 8 depict the case where the shock only hits agent 2. As we can see, the aggregate effects of both shocks are similar. In particular, the paths for money and tradable consumption are similar as in the representative agent model. On the other hand, even though preferences are separable, the consumption of non-traded goods does change. The intuition is that the decline in m^2 generates a large negative wealth effect on agent 2. The drop in p_N exacerbates this. Given that prices are flexible, the wage in terms of non-traded goods does not change. Therefore, because of the negative income effect, this agent wants to consume less and work more. Given that demand for non-tradables does not fall as much as in the representative agent case, the equilibrium output of non-tradables goes

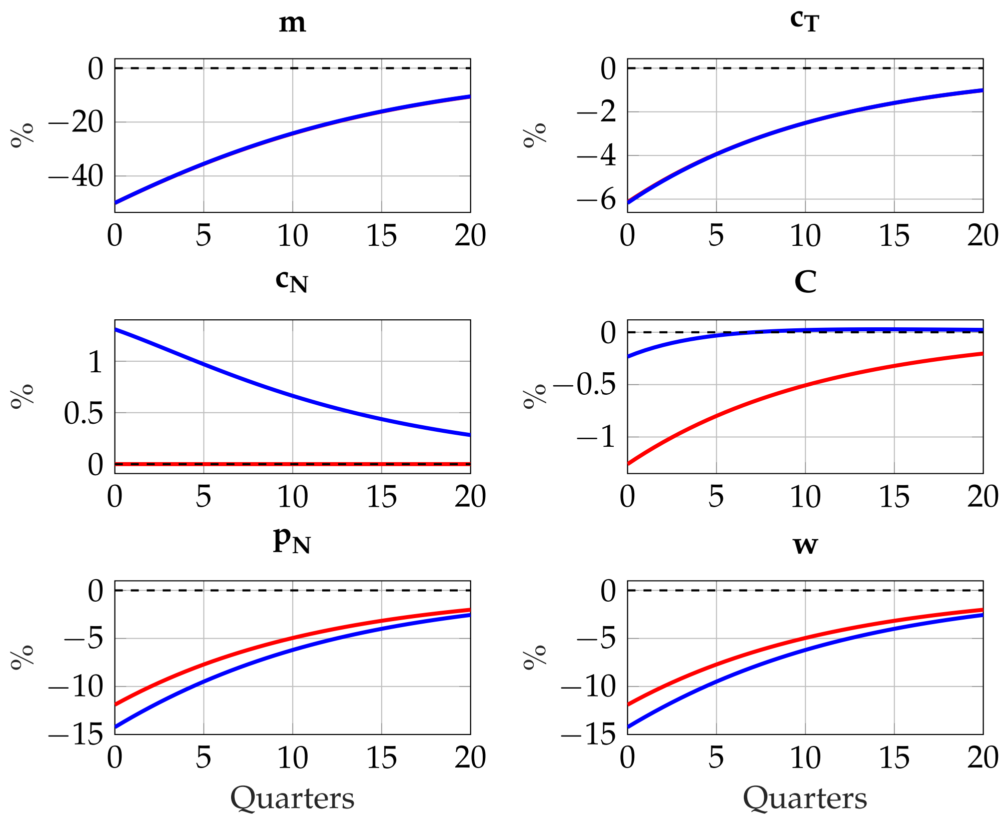

Figure 8: Impulse response to dollarization, only agent 2 starts below steady state. Blue line indicates two agent model. Red line indicates a representative agent model with the same calibration and flexible prices. Response of aggregate variables.

up.[24]

This exercises shows several important takeaways. First, heterogeneity matters for aggregate responses: under the calibration presented, a representative agent model would predict no response in non-tradables. However, the present two agent model feature a boom in the non-traded sector. Second and most importantly, the aggregates do not accurately reflect what happens for most agents in the economy: agent 2, which in steady state represents a share α of the economy, has both consumption of tradables and non-tradables dropping. Thus, even though aggregate consumption of non-tradables goes up, most agents suffer a drop in consumption in both types goods.

[24]Appendix D.1 shows that, given this specification of preferences, aggregate non-traded output goes up in the case in which we have logarithmic utility for money and both types of consumption.

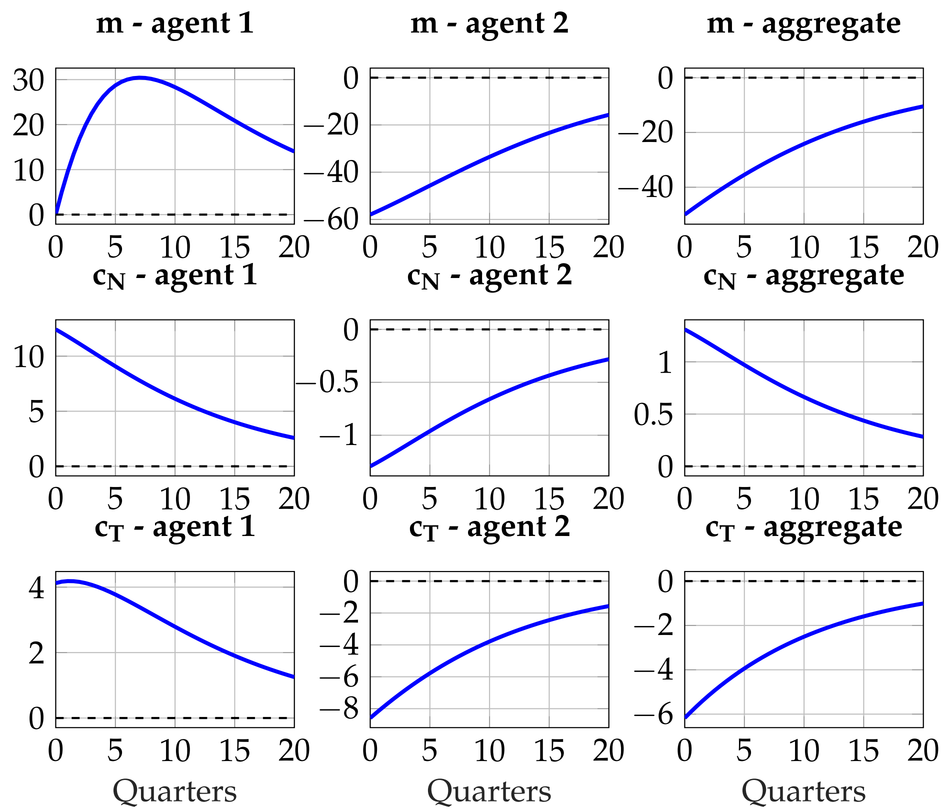

Figure 9: Impulse response to dollarization, only agent 2 starts below steady state. Blue line indicates two agent model. Red line indicates a representative agent model with the same calibration and flexible prices. Response of consumption and money for each agent.

6 Conclusions

This paper studied the transitional dynamics of "dollarization" for a country that starts with a shortage of foreign currency. Our main result is that these dynamics resemble those of a "sudden stop": a temporary drop in the consumption of tradable goods to accumulate of foreign currency. In general this requires a temporary drop in the real exchange rate, followed a gradual real appreciation via positive inflation. When prices are not fully flexible a recession initially ensues—even when prices are fully adjusted upon announcement of dollarization—because the exchange rate is overvalued, relative to the flexible price equilibrium. This recession eventually gives way to to a boom as the economy accumulates dollars and the exchange rate falls behind and becomes undervalued, relative to the flexible price equilibrium.

From a welfare perspective these dynamics are immediately costly, in the short-run, for two distinct reasons. First, we have the temporary drop in tradable consumption. Second, we have the inefficient fluctuations—recession followed by boom—in non-tradable consumption and employment. Whether or not a country undergoes these dire dynamics in to significant degree depends crucially on the initial dollar balances upon dollarization, which undoubtedly depends on the particular circumstances faced by each country. This paper has spelled out why this is an important consideration, not just at the individual level, but also for its macroeconomic implications.

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A Proof of Proposition 3

We first show that $\pi_{Nt} > 0$ for all $t > 0$. Let $\lambda_1 < 0 < \lambda_2$ be the two eigenvalues of the characteristic polynomial satisfying $\lambda_1 + \lambda_2 = \rho$ and $\lambda_1 \lambda_2 = -\kappa < 0$. Then

$$\pi_{Nt} = -|\lambda_1| \, p_{Nt} + z_t \tag{18}$$

with

$$z_t = \kappa \int e^{-|\lambda_2| s} p^f_{Nt+s} \, ds.$$

Thus, $\pi_{N0} = 0$ pins down $p_{Nt} = z_t / |\lambda_1|$.

Since p^f_{Nt} is increasing and $p^f_{Nt} \to p^*_N$ then

$$\dot{z}_t = \kappa \int e^{-|\lambda_2| s} \pi^f_{Nt+s} \, ds > 0$$

for all $t \geq 0$ with $p^f_{Nt} < p^*_N$, which must be the case in a neighborhood of $t = 0$. Differentiating (18) gives

$$\dot{\pi}_{Nt} = -|\lambda_1| \, \pi_{Nt} + \dot{z}_t.$$

Solving this linear differential equation for π_{Nt} then gives

$$\pi_{Nt} = \int_0^t e^{-|\lambda_1| s} \dot{z}_{t-s} \, ds > 0$$

as desired.

To show that $p_{Nt} \in (p^f_{Nt}, p^*_N)$ note that

$$\pi_{Nt} = \kappa \int_0^\infty e^{-\rho s} (p^f_{Nt+s} - p_{Nt+s}) \, ds.$$

For $t > 0$ we have that $\pi_{Nt} > 0$ which implies that $p^f_{Nt+s} - p_{Nt+s} > 0$ for some positive measure of s.

Finally, $\pi_{N0} = 0$ implies that $p^f_{Ns} - p_{Ns} < 0$ for positive measure of s. Indeed, $p^f_{N0} - p_{N0} < 0$ since

$$\dot{\pi}_{N0} = \rho \pi_{N0} - \kappa (p^f_{N0} - p_{N0}) = \dot{\pi}_{N0} = -\kappa (p^f_{N0} - p_{N0}) > 0.$$

This completes the proof.

B Proof of Proposition 3

The linearized system is

$$\begin{aligned}
\dot{M} &= \sigma \frac{y}{M_{ss}} \mu \\
\dot{p}_N &= \pi \\
\dot{\mu} &= \gamma\rho M - \alpha(\gamma-1)\rho p_N + \rho\mu \\
\dot{\pi} &= -\kappa(-\mu/\sigma - \theta p_N) + \rho\pi
\end{aligned}$$

written in matrix form:

$$\begin{pmatrix} \dot{M} \\ \dot{p}_N \\ \dot{\mu} \\ \dot{\pi} \end{pmatrix} = \begin{pmatrix} 0 & 0 & \sigma^{-1}\frac{y}{M_{ss}} & 0 \\ 0 & 0 & 0 & 1 \\ \gamma\rho & -\alpha(\gamma-1)\rho & \rho & 0 \\ 0 & \theta\kappa & \kappa\theta & \rho \end{pmatrix} \begin{pmatrix} M \\ p_N \\ \mu \\ \pi \end{pmatrix} \tag{19}$$

Two negative real roots In order to get the eigenvalues λ, we compute:

$$det \begin{pmatrix} 0-\lambda & 0 & \sigma^{-1}\frac{y}{M_{ss}} & 0 \\ 0 & 0-\lambda & 0 & 1 \\ \gamma\rho & -\alpha(\gamma-1)\rho & \rho-\lambda & 0 \\ 0 & \theta\kappa & \kappa\theta & \rho-\lambda \end{pmatrix}$$

$$= -\lambda[-\lambda(\rho-\lambda)^2 - \alpha(\gamma-1)\rho\kappa\theta - \kappa\theta(\rho-\lambda)] + \sigma^{-1}\frac{y}{M_{ss}}[\gamma\rho\theta\kappa + \gamma\rho\lambda(\rho-\lambda)]$$

operating:

$$0 = \lambda^2(\lambda-\rho)^2 + \lambda\alpha(\gamma-1)\rho\kappa\theta + (\kappa\theta + \sigma^{-1}\frac{y}{M_{ss}}\gamma\rho)\lambda(\rho-\lambda) + \frac{y}{M_{ss}} \tag{20}$$

$$\underbrace{\lambda^2(\lambda-\rho)^2 - \theta(\kappa + \frac{y}{M_{ss}}\gamma\rho)\lambda(\lambda-\rho)}_{LHS(\lambda)} = \underbrace{-\lambda\theta\alpha(\gamma-1)\rho\kappa - \frac{y}{M_{ss}}\gamma\rho\kappa\theta^2}_{RHS(\lambda)} \tag{21}$$

We can write the LHS as $f(x) = x^2 - \theta(\kappa + \frac{y}{M_{ss}}\gamma\rho)x$, where $x = \lambda(\lambda - \rho)$. This is useful to find the roots of the LHS. Since it is a quadratic in x, the roots are λ such that:

$$x = \begin{cases} 0 \\ \theta(\kappa + \frac{y}{M_{ss}}\gamma\rho) \end{cases}$$

for each x, there are two λ that achieve that x, giving us the four roots:

$$\lambda = \begin{cases} 0, \rho & \text{for } x = 0 \\ \dfrac{\rho \pm \sqrt{\rho^2 + 4\rho\theta(\kappa + \frac{y}{M_{ss}}\gamma\rho)}}{2} & \text{for } x = \theta(\kappa + \frac{y}{M_{ss}}\gamma\rho) \end{cases}$$

The roots ordered in increasing order are $(\frac{\rho - \sqrt{\rho^2 + 4\rho\theta(\kappa + \frac{y}{M_{ss}}\gamma\rho)}}{2}, 0, \rho, \frac{\rho + \sqrt{\rho^2 + 4\rho\theta(\kappa + \frac{y}{M_{ss}}\gamma\rho)}}{2})$. The RHS is a line.

In order to see there is at least two negative roots note that when $\lambda \to -\infty$, $LHS > RHS$ because it is a polinomial of higher order with positive principal coefficient. Note also that at $\lambda = 0$, $LHS(0) = 0 > RHS(0)$ since th constant on the RHS is negative. If we can find a $\lambda < 0$ such that $LHS(\lambda) < RHS(\lambda)$, this implies that there are two negative roots (draw it). Let's show that the RHS is always above the local minimum on the LHS. The local minimum of the LHS satisfies:

$$\begin{aligned} LHS(\lambda)' &= 0 \\ 0 &= f'(\lambda(\lambda - \rho))(2\lambda - \rho) \end{aligned}$$

the minimum we are interested is the one which is negative, thus $\lambda = \rho/2$ doesn't work. Solving $f'(\lambda(\lambda - \rho))$ we get that the minimum we are interested is given byL

$$\lambda = \frac{\rho \pm \sqrt{\rho^2 + \frac{\theta}{2}(\kappa + \frac{y}{M_s}\gamma\rho)}}{2} < 0$$

for notational simplicity denote $b = \frac{y}{M_{ss}}\gamma\rho$. Then, evaluating the LHS at that point yields:

$$LHS = f(\theta(\kappa + b)/2) = -\frac{1}{4}(\theta(\kappa + b))^2$$

since the λ we are interested in solves $f'(x) = 0$, which is achieved at $\theta(\kappa + b)/2$. On the other hand, the line evaluated at that λ gives:

$$-(\frac{\rho \pm \sqrt{\rho^2 + \frac{\theta}{2}(\kappa + \frac{y}{M_s}\gamma\rho)}}{2})\theta\alpha(\gamma - 1)\rho\kappa - \frac{y}{M_{ss}}\gamma\rho\kappa\theta^2$$

for the line to be above the LHS we need:

$$(\frac{\rho \pm \sqrt{\rho^2 + \frac{\theta}{2}(\kappa + \frac{y}{M_s}\gamma\rho)}}{2})\theta\alpha(\gamma - 1)\rho\kappa - b\kappa\theta^2 < \frac{1}{4}(\theta(\kappa + b))^2$$

(i.e the line is "less negative" than the LHS). Note that the first term, $-\lambda\theta\alpha(\gamma - 1)\rho\kappa$, is negative since λ is negative. Notice that this requires the assumption that $\gamma > 1$. Operating:

$$4(\frac{\rho \pm \sqrt{\rho^2 + \frac{\theta}{2}(\kappa + \frac{y}{M_s}\gamma\rho)}}{2})\theta\alpha(\gamma - 1)\rho\kappa - 2b\kappa\theta^2 < (\theta(\kappa + b))^2$$

$$4(\frac{\rho \pm \sqrt{\rho^2 + \frac{\theta}{2}(\kappa + \frac{y}{M_s}\gamma\rho)}}{2})\theta\alpha(\gamma - 1)\rho\kappa < (\theta(\kappa - b))^2$$

where we solved the square on the RHS and put it toghether with the term on the LHS. Since

$$4(\frac{\rho \pm \sqrt{\rho^2 + \frac{\theta}{2}(\kappa + \frac{y}{M_s}\gamma\rho)}}{2})\theta\alpha(\gamma - 1)\rho\kappa < 0$$

and the RHS is a square, the equation always holds. Thus, there as long as $\gamma > 1$ there are two real negative roots.

B.1 Phase diagram

In this subsection, we characterize the shape of the phase diagram for the separable case ($\theta = \sigma^{-1}$). In order to do so, we start from the following observation: since there are exactly two negative roots, we know that the solution for money is given by:

$$\tilde{M}(t) = a_1 e^{\lambda_1 t} + a_2 e^{\lambda_2 t}$$

for some constants a_1, a_2 that depend on the initial conditions $(M(0), p_N(0))$. Given that, we can obtain the evolution for $\hat{\mu}$ as:

$$\dot{M} = \lambda_1 a_1 e^{\lambda_1 t} + \lambda_2 a_2 e^{\lambda_2 t}$$
$$= \sigma^1 \frac{y}{M_{ss}} \hat{\mu}$$

from the equation for $\dot{M}$, and thus:

$$\begin{aligned}\hat{\mu}(t) &= \sigma \frac{M_{ss}}{y} [\lambda_1 a_1 e^{\lambda_1 t} + \lambda_2 a_2 e^{\lambda_2 t}] \\ &= \bar{m}[\lambda_1 a_1 e^{\lambda_1 t} + \lambda_2 a_2 e^{\lambda_2 t}]\end{aligned}$$

where $\bar{m} = \sigma \frac{M_{ss}}{y}$ for notational simplicity. Given that $\hat{\mu}(t)$, we can use what we derived above to obtain p_N. Recall that, given $\hat{\mu}$, p_N satisfies a second order differential equation:

$$\ddot{p}_N - \rho \dot{p}_N - \theta \kappa p_N = \theta \kappa \hat{\mu} \tag{22}$$

denote the roots for the characteristic polynomial as θ_1, θ_2 with $\theta_1 < 0$ and $\theta_2 > 0$. Solving θ_2 forwards and then θ_1 backwards, we obtain the following expression for p_N

$$p_N(t) = e^{\theta_1 t} \left[p_N(0) - \frac{\theta\kappa\bar{m}\lambda_1}{P(\lambda_1)} a_1 - \frac{\theta\kappa\bar{m}\lambda_2}{P(\lambda_2)} \right] + \theta\kappa\bar{m}[\frac{\lambda_1}{P(\lambda_1)} a_1 e^{\lambda_1 t} + \frac{\lambda_2}{P(\lambda_2)} a_2 e^{\lambda_2 t}]$$

where $P(X) = x^2 - \rho x - \kappa\theta$ is the characteristic polynomial from the differential equation (22). For the solution that have the correct functional form (i.e a linear combination of exactly two roots) it must be the case that $p_N(0) - \frac{\theta\kappa\lambda_1}{P(\lambda_1)} a_1 - \frac{\theta\kappa\lambda_2}{P(\lambda_2)} = 0$, we check that later on. Thus, the final expression for prices is given by:

$$p_N(t) = \theta\kappa\bar{m}[\frac{\lambda_1}{P(\lambda_1)} a_1 e^{\lambda_1 t} + \frac{\lambda_2}{P(\lambda_2)} a_2 e^{\lambda_2 t}]$$

the crucial link between the coefficients for money and the ones for prices are given by the terms $\frac{\lambda_1}{P(\lambda_1)}$ and $\frac{\lambda_2}{P(\lambda_2)}$. In what follows, we show that we can sign those two.

Lemma 1. *Let λ_1, λ_2 be the two real, negative eigenvalues of the matrix in (19), with $|\lambda_2| > |\lambda_1|$. If $\gamma > 1$, then $P(\lambda_2) > 0 > P(\lambda_1)$.*

Proof. Since λ_1, λ_2 are the solutions to the characteristic polynomial of the matrix in (19), we know that those solve equation (21). Furthermore, because of our earlier proof on the existence of two negative real eigenvalues, we know that

$$\frac{\rho - \sqrt{\rho^2 + 4\rho\theta(\kappa + \frac{y}{M_{ss}}\gamma\rho)}}{2} < \lambda_2 < \lambda_1 < 0.$$

On the other hand, we know that, since $P(X)$ is a quadratic with positive principal coefficient, $P(x) < 0$ for all $x < \theta_1$ or $x > \theta_2$ and $P(x) < 0 for x \in (\theta_1, \theta_2)$. Thus, if we can somehow compare θ_1 with λ_1 and λ_2, then we can get the signs of $P(\lambda_1), P(\lambda_2)$.
Our strategy is as follows. Recall that we can express λ as the solution to $LHS(\lambda) = RHS(\lambda)$ as per equation (21). If we can show that $LHS(\theta_1) < RHS(\theta_1)$, since $\theta_1 < 0$, that automatically implies that $\lambda_2 < \theta_1 < \lambda_1$. We know this from the shape of RHS and LHS, the fact that $\theta_1 < 0$ tells us that we are in the left side of the curve.
We proceed by simply evaluating the terms:

$$\begin{aligned} LHS(\theta_1) &= (\theta_1(\theta_1 - \rho))^2 - \theta(\kappa + \frac{y}{M_s s}\gamma\rho)(\theta_1(\theta_1 - \rho))^2 \\ &= \kappa\theta\left[-\theta\frac{y}{M_s s}\gamma\rho\right] = -\frac{y}{M_{ss}}\gamma\rho\kappa\theta^2 \end{aligned}$$

since $\theta_1(\theta_1 - \rho) = \kappa\theta$, given that θ_1 is a root of $P(x)$. Similarly:

$$RHS(\theta_1) = -\theta_1\theta\alpha(\gamma - 1)\rho\kappa - \frac{y}{M_{ss}}\gamma\rho\kappa\theta^2$$

Thus:

$$RHS(\theta_1) - LHS(\theta_1) = -\theta_1\theta\alpha(\gamma - 1)\rho\kappa > 0$$

since $\gamma > 1$ and $\theta_1 < 0$. Therefore, $\lambda_2 < \theta_1 < \lambda_1$ and thus $P(\lambda_2) > P(\lambda_1)$. ■

Equipped with Lemma 1, we now proceed to derive the system that we can actually plot on the phase diagram. The objective is to get $\dot{y} = \mathbf{\Phi} y$ where $y = [M; p_N]$ and $\mathbf{\Phi}$ is a matrix with known coefficients. We know proceed to derive $\mathbf{\Phi}$. First, note that from our solutions, we can map the exponentials to y as:

$$\begin{pmatrix} M(t) \\ p_N(t) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \theta\kappa\bar{m}\frac{\lambda_1}{P(\lambda_1)} & \theta\kappa\bar{m}\frac{\lambda_2}{P(\lambda_2)} \end{pmatrix} \begin{pmatrix} a_1 e^{\lambda_1 t} \\ a_2 e^{\lambda_2 t} \end{pmatrix}$$

$$\frac{1}{det(\cdot)} \begin{pmatrix} \theta\kappa\bar{m}\frac{\lambda_2}{P(\lambda_2)} & -1 \\ -\theta\kappa\bar{m}\frac{\lambda_1}{P(\lambda_1)} & 1 \end{pmatrix} \begin{pmatrix} M(t) \\ p_N(t) \end{pmatrix} = \begin{pmatrix} a_1 e^{\lambda_1 t} \\ a_2 e^{\lambda_2 t} \end{pmatrix}$$

where $det(\cdot) = \theta\kappa\bar{m}\frac{\lambda_2}{P(\lambda_2)} - \theta\kappa\bar{m}\frac{\lambda_1}{P(\lambda_1)} < 0$ is the determinant of the matrix. Using that, we can write the time derivatives of the states as a function of those states:

$$\dot{M} = \lambda_1 a_1 e^{\lambda_1 t} + \lambda_2 a_2 e^{\lambda_2 t}$$
$$\dot{M} = \underbrace{\theta\kappa\bar{m}\frac{\lambda_1\lambda_2}{det(\cdot)}\left[\frac{1}{P(\lambda_2)} - \frac{1}{P(\lambda_1)}\right]}_{\mathbf{\Phi}_{11}} M(t) + \underbrace{\frac{\lambda_2 - \lambda_1}{det(\cdot)}}_{\mathbf{\Phi}_{12}} p_N(t)$$

for money and:

$$\dot{p}_N = \theta\kappa\bar{m}[\frac{\lambda_1^2}{P(\lambda_1)} a_1 e^{\lambda_1 t} + \frac{\lambda_2^2}{P(\lambda_2)} a_2 e^{\lambda_2 t}]$$
$$= \underbrace{\frac{(\theta\kappa\bar{m})^2}{det(\cdot)}\frac{\lambda_1\lambda_2}{P(\lambda_1)P(\lambda_2)}[\lambda_1 - \lambda_2]}_{\mathbf{\Phi}_{21}} M(t) + \underbrace{\frac{\theta\kappa\bar{m}}{det(\cdot)}\left[\frac{\lambda_2^2}{P(\lambda_2)} - \frac{\lambda_1^2}{P(\lambda_1)}\right]}_{\mathbf{\Phi}_{22}} p_N(t)$$

from those formulas, we can obtain the following sequence of lemmas, which imply that the phase diagram has the form we depicted. Importantly, these formulas hold when $\gamma > 1$. When $\gamma = 1$, $P(\lambda_2) = 0$. But in that case, we get the monotonicity results already derived above.

Lemma 2. $\mathbf{\Phi}_{11}, \mathbf{\Phi}_{22} < 0$ *and* $\mathbf{\Phi}_{12}, \mathbf{\Phi}_{21} > 0$

Proof. Direct from inspecting the formulas for coefficients. ■

Lemma 3. *Let* $s_M = -\frac{\mathbf{\Phi}_{11}}{\mathbf{\Phi}_{12}}$ *and* $s_p = -\frac{\mathbf{\Phi}_{21}}{\mathbf{\Phi}_{22}}$ *be the slopes of the loci where* $\dot{M} = 0$ *and* $\dot{p}_N = 0$ *respectively. If* $\gamma > 1$ *then* $s_M > s_p > 0$

Proof. The fact that are both positive follow from the previous lemma. In order to see the magnitudes, using the formulas we get:

$$s_M = (\theta\kappa\bar{m})\frac{\lambda_1\lambda_2}{\lambda_2 - \lambda_1}[1/P(\lambda_2) - 1/P(\lambda_1)]$$
$$s_p = (\theta\kappa\bar{m})\frac{\lambda_1\lambda_2}{\lambda_1^2 P(\lambda_2) - \lambda_1^2 P(\lambda_1)}(\lambda_2 - \lambda_1)$$

the desired inequality follows from:

$$-\lambda_2^2\frac{P(\lambda_1)}{P(\lambda_2)} - \lambda_1^2\frac{P(\lambda_2)}{P(\lambda_1)} > -2\lambda_1\lambda_2 \tag{23}$$

which holds because the LHS is positive and the RHS is negative. In order to see why, operating:

$$\begin{aligned}
\lambda_1^2 - \lambda_2^2 \frac{P(\lambda_1)}{P(\lambda_2)} - \lambda_1^2 \frac{P(\lambda_2)}{P(\lambda_1)} + \lambda_2^2 &> \lambda_1^2 - 2\lambda_1\lambda_2 + \lambda_2^2 \\
(1/P(\lambda_2) - 1/P(\lambda_1))(\lambda_1^2 P(\lambda_2) - \lambda_2^2 P(\lambda_1)) &> (\lambda_2 - \lambda_1)^2 \\
(1/P(\lambda_2) - 1/P(\lambda_1))(\lambda_1^2 P(\lambda_2) - \lambda_2^2 P(\lambda_1)) &> (\lambda_2 - \lambda_1)^2 \\
\frac{1}{\lambda_2 - \lambda_1}(1/P(\lambda_2) - 1/P(\lambda_1)) &< \frac{\lambda_2 - \lambda_1}{\lambda_1^2 P(\lambda_2) - \lambda_2^2 P(\lambda_1)} \\
\kappa\theta\bar{m}\frac{1}{\lambda_2 - \lambda_1}(1/P(\lambda_2) - 1/P(\lambda_1)) &< \kappa\theta\bar{m}\frac{\lambda_2 - \lambda_1}{\lambda_1^2 P(\lambda_2) - \lambda_2^2 P(\lambda_1)}
\end{aligned}$$

■

thus, the phase diagram has the form depicted in the main text.

Consumption is Montone Using the above results, we can prove very directly that $\tilde{c}_T$ is monotone.

Lemma 4 (Montoninicy)**.** *Under the above assumptions. Assume $\hat{p}_N(0) \in [\hat{p}^*(0), 0]$, where $\hat{p}_N(0) = \hat{p}^*(0)$ implies $\pi_N = 0$. Then $\tilde{c}_T$ is monotone.*

Proof. Decompose $\mathbf{\Phi}$ in its the eigenvalues and eigenvectors as $\mathbf{\Phi} = VDV^{-1}$. The columns in matrix V contain the eigvenvectors. First, we show that the eigenvector corresponding to λ_1 has two positive entries, and the one corresponding to λ_2 has one negative and one positive. Consider first the eigenvector for λ_1. It comes from the solution to:

$$\begin{pmatrix} \mathbf{\Phi}_{11} - \lambda_1 & \mathbf{\Phi}_{12} \\ \mathbf{\Phi}_{21} & \mathbf{\Phi}_{22} - \lambda_1 \end{pmatrix} v = 0$$

since $\mathbf{\Phi}_{21} > 0$, it sufficies to show that $\mathbf{\Phi}_{22} - \lambda_1 < 0$. This follows from:

$$\begin{aligned}
\mathbf{\Phi}_{22} - \lambda_1 &= \frac{\lambda_2^2 P(\lambda_2) - \lambda_1^2 P(\lambda_1)}{\lambda_2 P(\lambda_2) - \lambda_1 P(\lambda_1)} - \lambda_1 \\
&= \frac{\lambda_2(\lambda_2 - \lambda_1)P(\lambda_1)}{\lambda_2 P(\lambda_2) - \lambda_1 P(\lambda_1)} < 0
\end{aligned}$$

the sign follows from our earlier lemmas. For λ_2 we have:

$$\begin{aligned}
\mathbf{\Phi}_{22} - \lambda_2 &= \frac{\lambda_2^2 P(\lambda_2) - \lambda_1^2 P(\lambda_1)}{\lambda_2 P(\lambda_2) - \lambda_2 P(\lambda_1)} - \lambda_1 \\
&= \frac{\lambda_1(\lambda_2 - \lambda_1)P(\lambda_2)}{\lambda_2 P(\lambda_2) - \lambda_1 P(\lambda_1)} > 0
\end{aligned}$$

Thus, the matrix of eigenvectors can be written as:

$$V = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix}$$

where wlog $v_{11}, v_{12}, v_{21} > 0$ and $v_{22} < 0$. Thus, the solution for M and p_N satisfies:

$$\begin{pmatrix} M(t) \\ p_N(t) \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{pmatrix} \begin{pmatrix} ae^{\lambda_1 t} \\ be^{\lambda_1 t} \end{pmatrix}$$

using the intial conditions that $\dot{p}_N(0) = 0$ and $\tilde{M}_0 < 0$ we have:

$$\begin{pmatrix} M(0) \\ 0 \end{pmatrix} = \begin{pmatrix} v_{11} & v_{12} \\ v_{21}\lambda_1 & v_{22}\lambda_2 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix}$$

$$\begin{pmatrix} a \\ b \end{pmatrix} = \frac{1}{v_{22}v_{11}\lambda_2 - v_{12}v_{21}\lambda_1} \begin{pmatrix} v_{22}\lambda_2 & -v_{12} \\ -v_{21}\lambda_1 & v_{11} \end{pmatrix} \begin{pmatrix} M(0) \\ 0 \end{pmatrix}$$

which implies:

$$a = M(0) \frac{v_{22}\lambda_2}{v_{22}v_{11}\lambda_2 - v_{12}v_{21}\lambda_1} < 0$$
$$b = -M(0) \frac{v_{21}\lambda_1}{v_{22}v_{11}\lambda_2 - v_{12}v_{21}\lambda_1} < 0$$

Given those signs, we have that:

$$\ddot{M} = (\lambda_1)^2 v_{11} ae^{\lambda_1 t} + (\lambda_2)^2 v_{12} be^{\lambda_1 t} < 0 \quad \forall t$$

but since $\dot{M} = -\frac{y}{M_s s}\tilde{c}_T$ then :

$$\dot{\tilde{c}}_T = -\frac{M_s s}{y}\ddot{M} > 0$$

showing that consumption is monotone. Notice that if $p_N(0)$ was such that $\dot{\pi}(0) < 0$ we would have:

$$a = M(0) \frac{v_{22}\lambda_2}{v_{22}v_{11}\lambda_2 - v_{12}v_{21}\lambda_1} - v_{12}\pi_N(0)$$
$$b = -M(0) \frac{v_{21}\lambda_1}{v_{22}v_{11}\lambda_2 - v_{12}v_{21}\lambda_1} + v_{11}\pi_N(0) < 0$$

so for $\pi_N(0) < 0$ but not too large, the coefficient a remains negative. In particular, if

$p_N = 0$, then we have:

$$a = M(0)\frac{v_{22}}{v_{22}v_{11} - v_{12}v_{21}} < 0 \tag{24}$$

$$b = M(0)\frac{-v_{21}}{v_{22}v_{11} - v_{12}v_{21}} < 0 \tag{25}$$

so by continuity, for all $\hat{p}_N(0)$ such that $p_N(0) \in [\hat{p}^*(0), 0]$ we have that $\dot{\hat{c}}_T > 0$ ∎

C Equilibrium

Additional results for calibrated version

Equilibrium conditions. Omitting time subscripts, the equations characterizing the optimum are given by:

$$\frac{\dot{\mu}}{\mu} = \rho - \frac{1}{P}u'_m(m/P)/\mu \tag{26}$$

$$\frac{\dot{m}}{m} = \frac{y - c_T}{m} \tag{27}$$

$$\frac{v_0 \ell^{\varphi}}{C^{-\sigma}\alpha^{1/\theta}\left(c_N/C\right)^{-1/\theta}} = \frac{w}{p_N} \tag{28}$$

$$(\frac{\alpha}{1-\alpha})\frac{c_T}{c_N} = (p_N)^{\theta} \tag{29}$$

$$\mu = C^{1/\theta-\sigma}c_T^{-1/\theta} \tag{30}$$

$$C = \mathcal{C}(c_N, c_T) \tag{31}$$

And the two equations that characterize the pricing block.

Steady state. We focus on the steady state after dollarizing. In that case, given that we assume that p_T is constant, the new steady state features zero inflation. We normalize $w = p_N = 1$. The steady state features $c_T = y$, $c_N = \frac{\alpha}{1-\alpha}y$, $C = \frac{y}{1-\alpha}$, $\frac{m}{Y} = m^{ss}$. We pick scale parameters v_0, a_0 to ensure this is the case.

First order approximation. We take a first order approximation of the system around the steady state. Appendix C shows that we can reduce the system to four variables:

$(\tilde{m}, \tilde{\mu}, \tilde{p}^*, \tilde{p}_N)$. The log-linearized system is given by:

$$\dot{\tilde{m}} = -\frac{y}{\bar{M}}\left(-\alpha(1/\sigma - \theta)\tilde{p}_N - \tilde{\mu}/\sigma\right) \tag{32}$$

$$\dot{\tilde{\mu}} = \gamma\rho\tilde{m} - \alpha(\gamma - 1)\rho\tilde{p}_N + \rho\tilde{\mu} \tag{33}$$

$$\dot{\tilde{p}}^* = -(\rho + \lambda)\left(-\varphi(\alpha(\frac{1}{\sigma} - \theta) + \theta)\tilde{p}_N - (1 + \frac{\varphi}{\sigma})\tilde{\mu}\right) + (\rho + \lambda)\tilde{p}^* \tag{34}$$

$$\dot{\tilde{p}}_N = \lambda(\tilde{p}^* - \tilde{p}_N) \tag{35}$$

The state variables are $\tilde{m}$ and $\tilde{p}_N$, whereas the other two are jump variables. The initial value $\tilde{m}_0 < 0$ is given. As discussed, we let all firms reset at $t = 0$, so the initial value for p_N is chosen so that $p_N(0) = p^*$.

Derivation of the log-linear system. Log-linearizing the system given by (26)-(31) plus the two pricing equations around the steady state we obtain:

$$\dot{\tilde{\mu}} = \gamma\rho(\tilde{m} - \alpha\tilde{p}_N) + \rho\alpha p_N + \rho\tilde{\mu} \tag{36}$$

$$\tilde{\mu} = (\frac{1}{\theta} - \sigma)\tilde{C} - \frac{1}{\theta}\tilde{c}_T \tag{37}$$

$$\dot{\tilde{m}} = -\frac{y}{\bar{M}}\tilde{c}_T \tag{38}$$

$$\tilde{c}_T - \tilde{c}_N = \theta\tilde{p}_N \tag{39}$$

$$(\varphi + 1/\theta)\tilde{c}_N + (\sigma - 1/\theta)\tilde{C} = \tilde{w} - \tilde{p}_N \tag{40}$$

$$\tilde{p}^* = -(\rho + \lambda)(\tilde{w}) + (\rho + \lambda)p^* \tag{41}$$

$$\dot{\tilde{p}}_N = \lambda(\tilde{p}^* - \tilde{p}_N) \tag{42}$$

$$\tilde{C} = \alpha\tilde{c}_N + (1 - \alpha)\tilde{c}_T \tag{43}$$

we can reduce the system to a first order system that only depends on μ, M, p_N, p^*. Using the static equations, is easy to show that:[25]

$$\tilde{c}_T = -\alpha(\frac{1}{\sigma} - \theta)\tilde{p}_N - \tilde{\mu}/\sigma$$
$$\tilde{c}_N = -(\alpha(\frac{1}{\sigma} - \theta) + \theta)\tilde{p}_N - \tilde{\mu}/\sigma$$

using that, we can write the deviations of nominal wage as:

$$\begin{aligned}
(\varphi + 1/\theta)\tilde{c}_N - (\tilde{\mu} + \frac{1}{\theta}\tilde{c}_T) &= \tilde{w} - \tilde{p}_N \\
\varphi\tilde{c}_N - \tilde{\mu} - \frac{1}{\theta}(\tilde{c}_T - \tilde{c}_N) &= \tilde{w} - \tilde{p}_N \\
\varphi\left(-(\alpha(\frac{1}{\sigma} - \theta) + \theta)\tilde{p}_N - \tilde{\mu}/\sigma\right) - \tilde{\mu} &= \tilde{w} \\
-\varphi(\alpha(\frac{1}{\sigma} - \theta) + \theta)\tilde{p}_N - \tilde{\mu}(1 + \frac{\varphi}{\sigma}) &= \tilde{w}
\end{aligned}$$

Robustness to θ. We maintain the baseline calibration, but vary $\theta \in [1/4, 1/2, 1]$. Notice that the first case features $\sigma^{-1} > \theta$ and the last case $\theta < \sigma^{-1}$. For the sticky price case, we allow all firms to reset its price at $t = 0$. As we can see, under flexible prices, whether $\theta > \sigma^{-1}$ or $\theta < \sigma^{-1}$ determines whether c_N goes above or below the steady state value under the flexible price response. As expected, the magnitude of the response in prices is affected by the choice of this parameter: lower θ means prices react more for the same change in quantities. However, the qualitative conclusions are not altered by the exact choice of θ.

D Additional results for two agents version

Steady state We look for a steady state in which $w = p_N = 1$. Since $p_N = 1$, we have that consumption is given by $c_T^i = a_i(1-\alpha)$, $c_N^i = a_i\alpha$ for each agent i. Thus, $C^i = a^i$.

[25] Using the intratemporal condition between T and NT and the aggregate consumption equation into the expression for μ:

$$\begin{aligned}
\tilde{\mu} &= (\frac{1}{\theta} - \sigma)(\alpha\tilde{c}_N + (1-\alpha)\tilde{c}_T) - \frac{1}{\theta}\tilde{c}_T \\
\tilde{\mu} &= (\frac{1}{\theta} - \sigma)(\tilde{c}_T - \alpha\theta\tilde{p}_N) - \frac{1}{\theta}\tilde{c}_T \\
\tilde{\mu} &= -\sigma\tilde{c}_T - \alpha(1 - \sigma\theta)\tilde{p}_N \\
\tilde{c}_T &= -\tilde{\mu}/\sigma - \alpha(1/\sigma - \theta)\tilde{p}_N
\end{aligned}$$

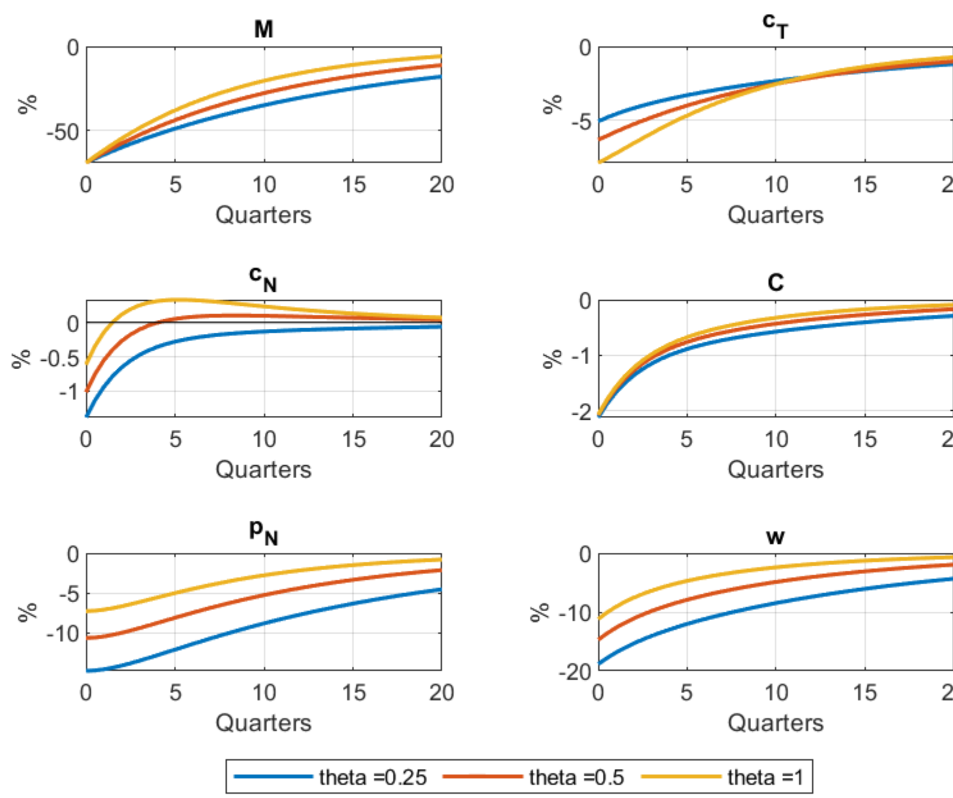

Figure 10: IRFs for different θ, sticky prices.

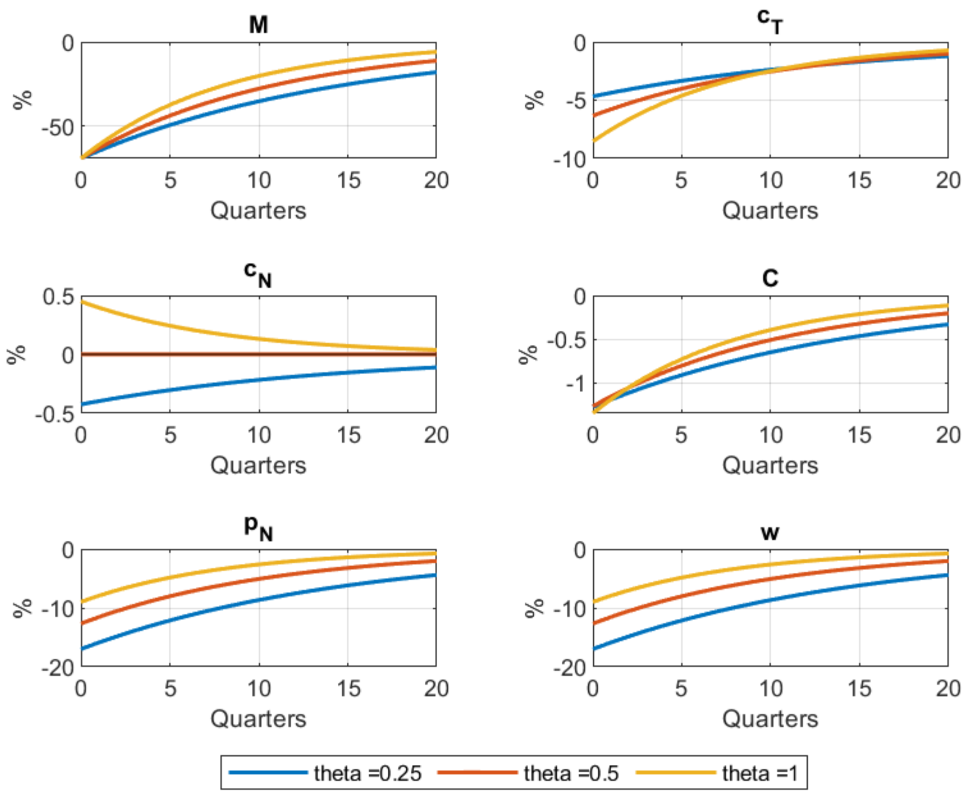

Figure 11: IRFs for different θ, flexible prices

Using that $p_N = 1$, then the price index is 1. Using that into the budget constraint of agent 1, $y = PC^1$ so $a^1 = C_1$. Using that into the budget constraint of agent 2. in equilibrium, $p_N c_N^1 = c_T^2$, which yields:

$$Y\alpha = (1-\alpha)a^2 \to a_2 = \frac{\alpha}{1-\alpha}y$$

from which we can get consumption of tradables and non-tradables for agent 2. Using that we have the consumption of non-tradables for both agents, we can get L_N. Finally, the value of v_0 is picked so that in steady state $W/p_N = 1$ and

$$\begin{aligned}
C_1 &= y \\
C_2 &= \frac{\alpha}{1-\alpha}y \\
c_T^1 &= (1-\alpha)y \\
c_N^1 &= \alpha y \\
c_T^2 &= \alpha y \\
c_N^2 &= \frac{\alpha^2}{1-\alpha}y \\
p_N &= 1 \\
L_N &= \frac{\alpha}{1-\alpha}y
\end{aligned}$$

and parameters on U_m make sure that a given level of M_{ss} is optimal for each agent, and v_0 makes sure that L_N is supplied in steady state.

Linearizing Linearizing and using equilibrium conditions we get the following set of equations:

$$\dot{\tilde{M}}^1 = -\frac{y}{M_{ss}^1}((1-\alpha)\tilde{c}_T^1 + \alpha(\tilde{c}_N^1 + \tilde{p}_N)) \tag{44}$$

$$\dot{\tilde{M}}^2 = \frac{y}{M_{ss}^2}(\alpha(\tilde{c}_N^1 + \tilde{p}_N) - \alpha\tilde{c}_T^2) \tag{45}$$

$$\dot{\hat{\mu}}^1 = \gamma\rho(\tilde{M}^1 - \alpha\tilde{p}_N) + \rho\alpha\tilde{p}_N + \rho\hat{\mu}^1 \tag{46}$$

$$\dot{\hat{\mu}}^2 = \gamma\rho(\tilde{M}^2 - \alpha\tilde{p}_N) + \rho\alpha\tilde{p}_N + \rho\hat{\mu}^2 \tag{47}$$

$$\tilde{\mu}^i = (\frac{1}{\theta} - \sigma)\tilde{C}^i - \frac{1}{\theta}\tilde{c}_T \tag{48}$$

$$\tilde{c}_T^i - \tilde{c}_N^i = \theta\tilde{p}_N \tag{49}$$

$$\tilde{C}^i = \alpha\tilde{c}_N^i + (1-\alpha)\tilde{c}_T^i \tag{50}$$

$$\varphi\tilde{L}_N + (\sigma - 1/\theta)\tilde{C}^2 + \frac{1}{\theta}\tilde{c}_N^2 = \tilde{W} - p_N \tag{51}$$

$$\tilde{L}_N = (1-\alpha)\tilde{c}_N^1 + \alpha\tilde{c}_N^2 \tag{52}$$

$$\tilde{W} - p_N = 0 \tag{53}$$

We can collapse all the static conditions as a function of $\hat{\mu}^1, \hat{\mu}^2$ as follows. From the labor and NT, using labor market clearing and the definition of μ for agent 2:

$$\varphi((1-\alpha)\tilde{c}_N^1 + \alpha\tilde{c}_N^2) - \hat{\mu}^2 - \hat{p}_N = \tilde{W} - p_N \tag{54}$$

using that prices are flexible, and the solution of the intratemporal problem for both agents as:

$$\varphi((1-\alpha)\tilde{c}_N^1 + \alpha\tilde{c}_N^2) - \hat{\mu}^2 = \hat{p}_N \tag{55}$$

$$\varphi((1-\alpha)(-(\alpha(1/\sigma - \theta) + \theta)\tilde{p}_N - \hat{\mu}^1/\sigma) + \alpha(-(\alpha(1/\sigma - \theta) + \theta)\tilde{p}_N - \hat{\mu}^2/\sigma)) - \hat{\mu}^2 = \hat{p}_N \tag{56}$$

$$-\varphi(\alpha(1/\sigma - \theta) + \theta)\tilde{p}_N - (1-\alpha)(\frac{\varphi}{\sigma})\hat{\mu}^1 - (1 + \alpha\frac{\varphi}{\sigma})\hat{\mu}^2 = \hat{p}_N \tag{57}$$

so we can solve for p_N as a function of $\hat{\mu}^1, \hat{\mu}^2$:

$$p_N = -\frac{1}{1 + \varphi(\alpha(1/\sigma - \theta) + \theta)}\Big((1-\alpha)(\frac{\varphi}{\sigma})\hat{\mu}^1 + (1 + \alpha\frac{\varphi}{\sigma})\hat{\mu}^2\Big) \tag{58}$$

$$p_N = \psi_1\hat{\mu}^1 + \psi_2\hat{\mu}^2 \tag{59}$$

replacing that into the solution for c_T^i, c_N^i, we can reduce the system to a 4 variable linear differential equation system in $(\tilde{M}^1, \tilde{M}^2, \hat{\mu}^1, \hat{\mu}^2)$. For $\dot{\tilde{M}}^1$:

$$
\begin{aligned}
\dot{\tilde{M}}^1 &= -\frac{y}{M_{ss}^1}(\tilde{c}_T^1 - \alpha(\tilde{c}_T^1 - \tilde{c}_N^1) + \alpha\tilde{p}_N)) \\
&= -\frac{y}{M_{ss}^1}(\tilde{c}_T^1 - \alpha(\theta - 1)\hat{p}_N) \\
&= -\frac{y}{M_{ss}^1}(-\hat{\mu}^1/\sigma - \alpha(1/\sigma - \theta)\hat{p}_N - \alpha(\theta - 1)\hat{p}_N) \\
&= -\frac{y}{M_{ss}^1}(-\hat{\mu}^1/\sigma - \alpha/\sigma\hat{p}_N + \alpha\hat{p}_N) \\
&= -\frac{y}{M_{ss}^1}(-\hat{\mu}^1/\sigma - \alpha(1/\sigma - 1)\hat{p}_N \\
&= -\frac{y}{M_{ss}^1}(-\hat{\mu}^1/\sigma - \alpha(1/\sigma - 1)(\psi_1\hat{\mu}^1 + \psi_2\hat{\mu}^2)) \\
&= -\frac{y}{M_{ss}^1}(\hat{\mu}^1(-1/\sigma - \alpha(1/\sigma - 1)\psi_1) - \alpha(1/\sigma - 1)\psi_2\hat{\mu}^2))
\end{aligned}
$$

analogously for the second one:

$$
\begin{aligned}
\dot{\tilde{M}}^2 &= \frac{y}{M_{ss}^2}(\alpha(\tilde{c}_N^1 + \tilde{p}_N) - \alpha\tilde{c}_T^2) \\
&= \frac{y}{M_{ss}^2}(\alpha\hat{p}_N - \alpha(\tilde{c}_T^2 - \tilde{c}_N^1)) \\
&= \frac{y}{M_{ss}^2}(\alpha\hat{p}_N - \alpha(-\hat{\mu}^2/\sigma + \theta\hat{p}_N + \hat{\mu}^1/\sigma)) \\
&= \frac{y}{M_{ss}^2}(\alpha\hat{p}_N(1 - \theta) - \alpha(-\hat{\mu}^2/\sigma + \hat{\mu}^1/\sigma)) \\
&= \frac{y\alpha}{M_{ss}^2}\left((-\frac{1}{\sigma} + (1 - \theta)\psi_1)\hat{\mu}^1 + (\frac{1}{\sigma} + (1 - \theta)\psi_2)\hat{\mu}^2\right)
\end{aligned}
$$

regarding the equations for $\tilde{\mu}$, we have:

$$
\begin{aligned}
\dot{\hat{\mu}}^1 &= \gamma\rho(\tilde{M}^1 - \alpha\tilde{p}_N) + \rho\alpha\tilde{p}_N + \rho\hat{\mu}^1 \\
&= \tilde{M}^1 - \alpha\rho(\gamma - 1)\tilde{p}_N + \rho\hat{\mu}^1 \\
&= \tilde{M}^1 - \alpha\rho(\gamma - 1)(\psi_1\hat{\mu}_1 + \psi_2\hat{\mu}_2) + \rho\hat{\mu}^1
\end{aligned}
$$

and

$$
\begin{aligned}
\dot{\hat{\mu}}^2 &= \gamma\rho(\tilde{M}^2 - \alpha\tilde{p}_N) + \rho\alpha\tilde{p}_N + \rho\hat{\mu}^2 \\
&= \tilde{M}^2 - \alpha\rho(\gamma-1)\tilde{p}_N + \rho\hat{\mu}^2 \\
&= \tilde{M}^2 - \alpha\rho(\gamma-1)(\psi_1\hat{\mu}_1 + \psi_2\hat{\mu}_2) + \rho\hat{\mu}^2
\end{aligned}
$$

this yields:

$$\dot{\tilde{M}}^1 = -\frac{y}{M_{ss}^1}(\hat{\mu}^1(-1/\sigma - \alpha(1/\sigma-1)\psi_1) - \alpha(1/\sigma-1)\psi_2\hat{\mu}^2)) \tag{60}$$

$$\dot{\tilde{M}}^2 = \frac{y\alpha}{M_{ss}^2}\left((-\frac{1}{\sigma} + (1-\theta)\psi_1)\hat{\mu}^1 + (\frac{1}{\sigma} + (1-\theta)\psi_2)\hat{\mu}^2\right) \tag{61}$$

$$\dot{\hat{\mu}}^1 = \gamma\rho\tilde{M}^1 + \rho(1-\alpha(\gamma-1)\psi_1)\hat{\mu}^1 - \rho\alpha(\gamma-1)\psi_2\hat{\mu}^2 \tag{62}$$

$$\dot{\hat{\mu}}^2 = \gamma\rho\tilde{M}^2 + \rho(1-\alpha(\gamma-1)\psi_2)\hat{\mu}^2 - \rho\alpha(\gamma-1)\psi_1\hat{\mu}^1 \tag{63}$$

D.1 Separable case

What happens with non-tradable consumption under flexible prices when we have multiple agents? Consider first a parametrization in which $\sigma = \theta^{-1}$. Then from the intratemporal condition for labor and non-tradables:

$$\varphi\hat{L}_N + \frac{1}{\theta}\hat{c}_N^2 = \hat{W} - \hat{p}_N \tag{64}$$

$$\varphi((1-\alpha)\tilde{c}_N^1 + \alpha\tilde{c}_N^2) + \frac{1}{\theta}\hat{c}_N^2 = \hat{W} - \hat{p}_N \tag{65}$$

$$\varphi(1-\alpha)\tilde{c}_N^1 + (\varphi\alpha + \frac{1}{\theta})\hat{c}_N^2 = \hat{W} - \hat{p}_N \tag{66}$$

under flexible prices $\hat{W} - \hat{p}_N = 0$ so

$$\hat{c}_N^2 = -\frac{1-\alpha}{\alpha + \frac{\sigma}{\varphi}}\tilde{c}_N^1 \tag{67}$$

this equation states that in equilibrium, under flexible prices the consumption of both agents *must* move in opposite directions. The reason is simple: given market clearing, for $\tilde{c}_N^1$ to go up, we need either L_N to go up or $\hat{c}^2$ to go down. Under flexible prices $\hat{W} - \hat{p}_N = 0$, so optimality of agent 2 implies that, in equilibrium, both things will happen: labor will go up somewhat and $\hat{c}_N^2$ will go down. In order to see this more clearly, note that aggregate consumption of non-tradables equals:

$$\hat{c}_N = \alpha\hat{c}_N^2 + (1-\alpha)\hat{c}_N^1 \tag{68}$$

$$- \alpha\frac{1-\alpha}{\alpha+\frac{\sigma}{\varphi}}\hat{c}_N^1 + (1-\alpha)\hat{c}_N^1 \tag{69}$$

$$= (1-\alpha)\left(1-\frac{\alpha}{\alpha+\frac{\sigma}{\varphi}}\right)\hat{c}_N^1 \tag{70}$$

$$= \frac{(1-\alpha)}{\alpha+\frac{\sigma}{\varphi}}\left(\frac{\sigma}{\varphi}\right)\hat{c}_N^1 \tag{71}$$

so the sign of aggregate consumption of non-tradables coincides with the one of $\hat{c}_N^1$. The reason is simple: if $\tilde{c}_N^1$ goes up, either L_N goes up or $\hat{c}_N^2$ goes down. Because of optimality, given that the wage in non-tradables doesn't change, it is optimal to do a little bit of both: labor goes up and $\hat{c}_N^2$ goes down. But because labor goes up, we know that total consumption of non-tradables goes up. Thus, as long as consumption of non-tradables goes up for agents type 1 (owners of the tradables good), aggregate consumption of non-tradables will go up. Note that in the types of shocks we consider, the price of non-tradables will drop strongly, whereas the income of agents type 1 does not. Therefore, at least intuitively it is plausible that consumption of non-traded goods goes up for these sector. This is consistent with anecdotal evidence: when real depreciations occur, instead of going for holidays to Brazil or Europe, richer agents may substitute towards a destination in Argentina. Of course, the magnitude of this effect depends on the amount of expenditure switching. Note however that, under our baseline case with a representative agent and $\sigma = \frac{1}{\theta}$, consumption of non-traded goods didn't change after the shock.

Log case In order to build intuition, consider the log case: $\gamma = \sigma = \theta = 1$ and assume that $\varphi = 1$ for simplicity. Impose also that aggregate SS money is equal to y, so $\frac{y}{M_{ss}^1} = (1-\alpha)^{-1}$ and $\frac{y}{M_{ss}^2} = (\alpha)^{-1}$. The above system then simplifies to:

$$\dot{\tilde{M}}^1 = (1-\alpha)^{-1}\hat{\mu}^1 \tag{72}$$

$$\dot{\tilde{M}}^2 = \left(-\hat{\mu}^1 + \hat{\mu}^2\right) \tag{73}$$

$$\dot{\hat{\mu}}^1 = \rho\tilde{M}^1 + \rho\hat{\mu}^1 \tag{74}$$

$$\dot{\hat{\mu}}^2 = \rho\tilde{M}^2 + \rho\hat{\mu}^2 \tag{75}$$

Let's consider a shock in which only the money of agents type 2 gets liquidated. We can motivate by thinking that agent 1 has already enough dollars for transactional purposes, so the liquation does not affect her. In that case, we have $\hat{M}_0^1 = 0$ and $\hat{M}_0^2 < 0$. How does this shock get transmitted?

First, note that the equations for $\hat{\mu}^1, \hat{M}^1$ can be solved autonomously. Since $\hat{M}_0^1 = 0$, that part of the system is at steady state, so $\hat{M}_t^1 = \hat{\mu}_t^1 = 0$ for all t. Intuitively, under this calibration $p_N c_N$ is constant in equilibrium. Thus, regardless of what happens with the price of non-tradables, expenditure does not change. In the equation for $\dot{\hat{\mu}}^2$, the price would show up because of revaluation effects of money. However, since preferences are log for money $\gamma = 1$, this effect is zero. This is what gives us separability.

Let's turn now to what happens with agent 2. Since $\hat{\mu}_t^1 = 0$ for all t, the system for $\hat{\mu}^2, \hat{M}^2$ can be easily solved. The solution features monotone, exponential convergence back to steady state.

Given $\hat{\mu}_t^2$, what happens to prices? The solution for prices in this context is given by:

$$\hat{p}_N = \psi_1 \hat{\mu}_t^2 = -\frac{1+\alpha}{2}\hat{\mu}_t^2 \tag{76}$$

Replacing into the equation for aggregate consumption of non-tradables, and using that under this calibration $c_N^i = -\hat{p}_N - \hat{\mu}^i$ for both agents, we get:

$$\hat{c}_N = \alpha c_N^2 + (1-\alpha)c_N^1 \tag{77}$$

$$= -\hat{p}_N - \alpha\hat{\mu}^2 \tag{78}$$

$$= \frac{1-\alpha}{2}\hat{\mu}^2 \tag{79}$$

so *aggregate* consumption of non-tradables goes *up*. The intuition is as follows. First, tradable consumption for agent 1 doesn't change, because of the separability assumptions. If the price of non-tradables drops, she consumes more non-tradables, leaving her tradable consumption unchanged. On the other hand, agent 2 has to reduce her tradable consumption to accumulate money (her income does not change given that $p_N c_1^N$ is constant).

D.2 Quantitative results with sticky prices

Figures D.2 and D.2 show the response of the two agent economy when prices are sticky

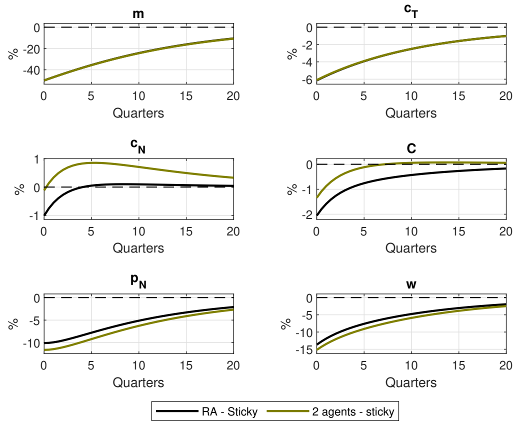

Figure 12: Aggregates, shock to both agents. Orange line indicates two agent model. Black line indicates a representative agent model with the same calibration. Both solutions are computed with sticky prices and full reset at time $t = 0$.

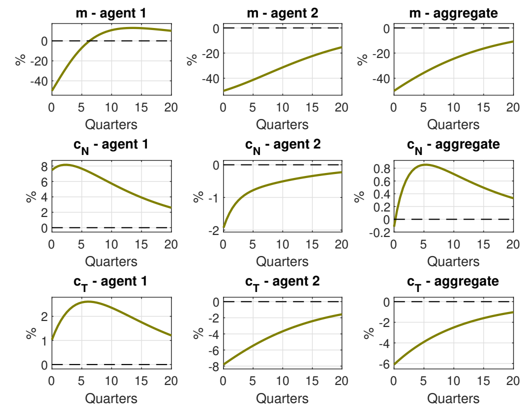

Figure 13: Distribution, shock to both agents. Orange line indicates two agent model. Black line indicates a representative agent model with the same calibration. Both solutions are computed with sticky prices and full reset at time $t = 0$.

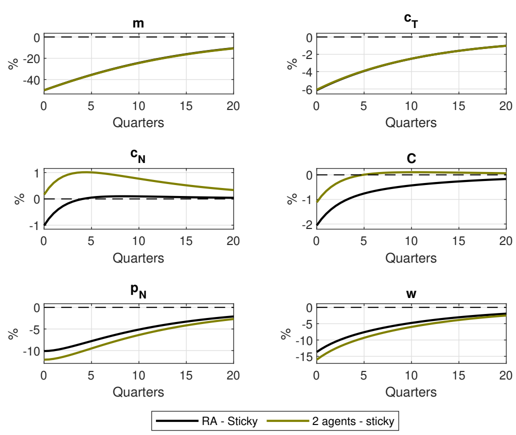

Figure 14: Aggregates, shock only to agent 2. Orange line indicates two agent model. Black line indicates a representative agent model with the same calibration. Both solutions are computed with sticky prices and full reset at time $t = 0$.

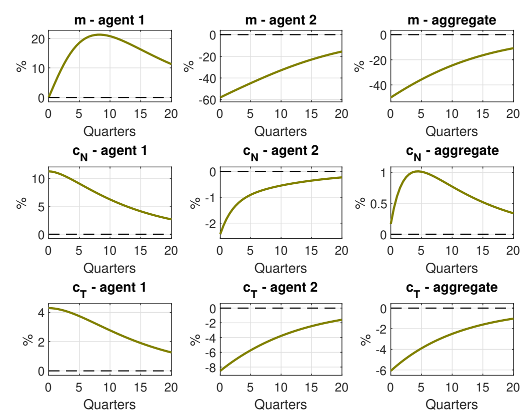

Figure 15: Distribution, shock only to agent 2. Orange line indicates two agent model. Black line indicates a representative agent model with the same calibration. Both solutions are computed with sticky prices and full reset at time $t = 0$.