Information Design

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Peking University Guanghua School June 2017

Mechanism Design and Information Design

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   - ...and this lecture

This Lecture

- a general framework in two slides
- leading examples at length
- applications in brief
- various elaborations if time

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- Basic Game $G : (A_i, u_i)_{i=1..,I}$ where $u_i : A \times \Theta \to \mathbb{R}$
- Information Structure S: $(T_i)_{i=1..,I}$ and $\pi : \Theta \to \Delta(T)$

Information Designer's Problem

- *Decision rule* $\sigma : T \times \Theta \to \Delta(A)$ is *obedient* for (G, S) if, for all i, t_i, a_i and a_i',

$$\begin{aligned} & \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a_i, a_{-i}\right), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi\left(\theta\right) \\ \geq & \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a_i', a_{-i}\right), \theta\right) \sigma\left(a \middle| t, \theta\right) \pi\left(t \middle| \theta\right) \psi\left(\theta\right); \end{aligned}$$

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  Obedient decision rule σ is a *Bayes correlated equilibrium* (BCE). Characterizes implementability.

- Information designer with payoff $v : A \times \Theta \rightarrow \mathbb{R}$ picks a Bayes correlated equilibrium $\sigma \in BCE(G, S)$ to maximize

$$V_S(\sigma) \equiv \sum_{a, t, \theta} \psi(\theta) \pi\left(t \middle| \theta\right) \sigma\left(a \middle| t, \theta\right) v(a, \theta).$$

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2. Metaphorical: e.g., adversarial / worst case
3. Informational robustness: family of objectives characterize set of attainable outcomes

One Uninformed Player: Benchmark Investment Example

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- leading example of Kamenica-Gentzkow 11

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- interpretation: firm observes signal = "action recommendation," drawn according to (p_B, p_G)

Obedience Constraints

- if "advised" to invest, invest has to be a best response:

$$\begin{aligned} -\frac{1}{2}p_B + \frac{1}{2}p_G x &\geq 0 \Leftrightarrow \\ p_G &\geq \frac{p_B}{x} \end{aligned}$$

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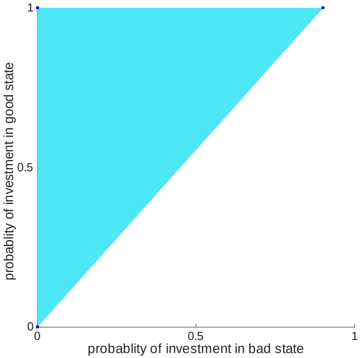
Bayes Correlated Equilibria

equilibrium outcomes (p_B, p_G) for $x = 0.9$



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- Optimal for government to obfuscate, partially pooling good state and bad state

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- Firm receives a signal which is "correct" with probability $q > 1/2$.
- Formally, the firm observes a signal g or b, with signals g and b being observed with conditionally independent probability q when the true state is G or B respectively:

	bad state B	good state G
bad signal b	q	$1 - q$
good signal g	$1 - q$	q

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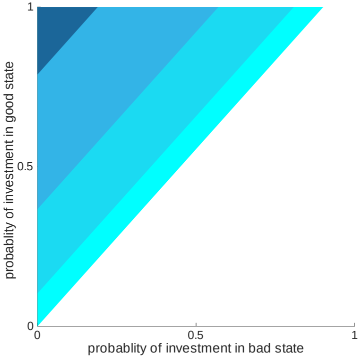
- the interesting question: what if we project $\left(p_B^b, p_G^b, p_B^g, p_G^g\right)$ back into ex ante behavior $\left(p_B^b, p_G^b, p_B^g, p_G^g\right)$? e.g.

$$p_G = q p_G^g + (1-q)\, p_G^b$$

One Informed Player: Bayes Correlated Equilibrium

equilibrium set (for $x = 0.9$ and $q = 0.5$, 0.575, 0.7 and 0.875)



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- will now have profile of action recommendations depending on the state

Two Firms: Strategic Complementarities

- If $\varepsilon > 0$, optimal rule is

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- Classic Question: are firms better off if they share their information?
- Consider quantity competition when firms uncertain about level of demand (intercept of linear demand curve) with symmetry, normality and linear best response; two effects in conflict:
  1. Individual Choice Effect: Firms would like to be as informed as possible about the state of demand
  2. Strategic Effect: Firms would like to be as uncorrelated with each other as possible

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  - For low enough price sensitivity (and thus strategic substitutability), individual choice effect wins and full information is optimal
  - For intermediate price sensitivity, there is a non-trivial trade-off and it is optimal to have firms observe noisy signals of demand, but with uncorrelated noise and thus conditionally independent signals, and thus signals which are as uncorrelated as possible conditional on their accuracy

Application 2 - Aggregate Volatility: Wacky Designer Objective

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- Which information structure maximizes variance of average action?

Application 2 - Aggregate Volatility

What information structure maximizes variance of average action?

- critical information structure has a confounding (c.f., Lucas 72) signal $s_i = \lambda x_i + (1-\lambda)\,y$ *without noise...*
- variance of average action is maximized when

$$\lambda = \frac{\sigma}{2\sigma + \sqrt{\sigma^2 + \tau^2}}$$

and maximum variance of average action is

$$\left(\frac{\sigma + \sqrt{\sigma^2 + \tau^2}}{2}\right)^2$$

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What information structure maximizes variance of average action?

- "optimal" information structure has a confounding (c.f., Lucas 72) signal $s_i = \lambda x_i + (1 - \lambda) y$ *without noise...*

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- this was actually a non-strategic problem: logic can be extended to strategic setting
- can then be embedded in a richer setting (Angeletos La'O 13)

Application 3 - First Price Auction: Information Shrinking BCE, Adversarial Information Designer, Robust Predictions and Partial Identification

- Example: Two bidders and valuations independently and uniformly distributed on the interval $[0, 1]$
- Plot: (expected bidders' surplus, expected revenue) pairs
- green = feasible pairs, blue = unknown value pairs, red = known value pairs

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3. Partial Identification: Winning bid distribution $\Longrightarrow$ Lower bound on Value Distribution (w/o identifying private vs. common values)

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  3. Non-Communicating Designer: the designer can tell players about the state but without conditioning on players' information

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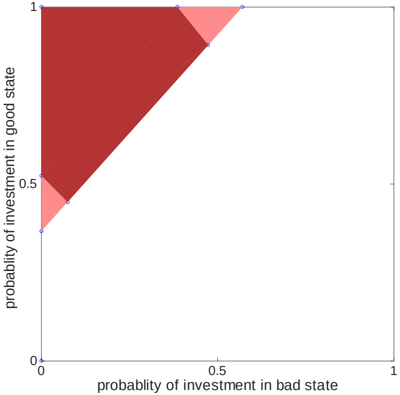
- incentive constraint: add truth-telling to obedience
- to insure truth-telling, differences in recommendations must be bounded across states

Communicating Designer

- adding truth-telling constraints...($x = 0.9,\ q = 0.7$)



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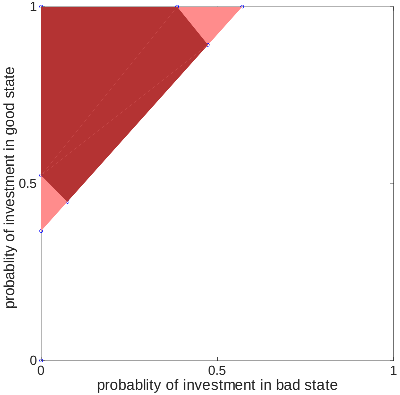
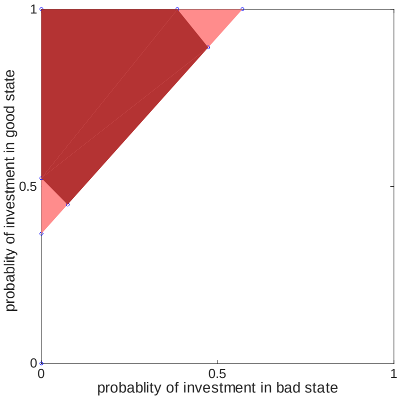


- communicating (red), omniscient (pink)

Communicating Designer



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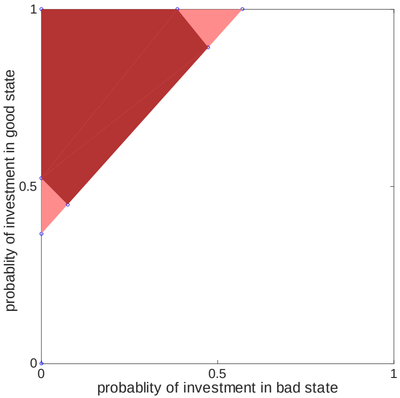

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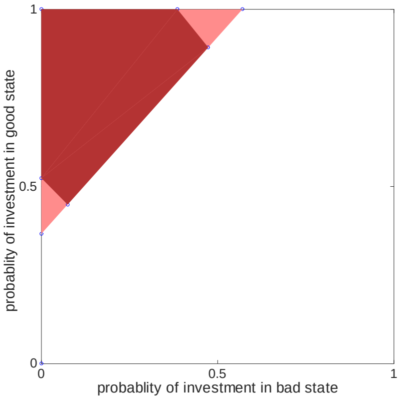

- if there is a large discrepency in recommendations, then firm has an incentive to misreport his signal
- e.g., at maximum investment BCE (top right), firm with good signal is always told to invest;
- might as well mis-report good signal as bad signal to get

Non-communicating designer

- firm observes his signal

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- government offers a recommendation, independent of the signal, depending on the true state

Non-communicating designer

- firm observes his signal
- government offers a recommendation, independent of the signal, depending on the true state
- In our example, communicating and non-communicating designer can attain the same set of outcomes; Kotolin et al show this in a more general - but still restrictive - class of environments

Taxonomy

	Single Agent	Many Agent Uninformed Designer	Many Agent Informed Designer
Omniscient	.	Bayesian Solution	BCE
Communicating	Kolotilin et al	Communication Equilibrium	.
Non Communicating	KG informed receiver	Strategic Form Correlated Equilibrium	.

Elaborations

1. Other Objectives
   - Ely 15, Arieli 15, Taneva 16
2. Comparing Information
   - many player Blackwell order generalization
3. Concavification and its many player generalizations
   - Kamenica-Gentzkow 11 get a lot of action out of "concavification" (Aumann-Maschler 95); many player generalization harder - Mathevet, Perego and Taneva 16
4. Adversarial Information Design
   - Carroll 15, Taneva et al 16, Kajii-Morris 97
5. Incomplete information correlated equilibrium literature
   - Forges 93
6. Relation to Mechanism Design
   - Myerson 82, 87, 91

Literature

- Our methodology papers:
  - "Robust Predictions in Incomplete Information Games," Ecta 13 (includes Cournot results)
  - "Bayes Correlated Equilibrium and The Comparison of Information Structures," TE 16
  - "Information Design, Bayesian Persuasion and Bayes Correlated Equilibrium," AER P&P 2016
  - Full paper on material in lecture in preparation
- Examples
  - Kamenica-Gentzkow 11 and Bergemann Morris 16 (see also Taneva 16)
  - "Information and Volatility" (with Tibor Heumann) JET ; see also "Information and Market Power" (working paper)
  - "First Price Auctions with General Information Structures: Implications for Bidding and Revenue" (with Ben Brooks) Ecta forthcoming; see also "The Limits of Price Discrimination" AER

1. Other Objectives

- Suppose the government was interested in maximizing the probability of at least one firm investing
- (Assuming $x > 1/2$) This can always be achieved with probability 1....

$\theta = B$	I	N
I	0	$\frac{1}{2}$
N	$\frac{1}{2}$	0

$\theta = G$	I	N
I	1	0
N	0	0

This is true for $\varepsilon = 0$ and by continuity for $|\varepsilon|$ independent of the sign...

- Compare Ely 15, Arieli 15, Taneva 16

Other Objectives and a Benevolent Information Designer

- In one firm case, if government had the same objective as the firm, he would always give them full information...
- But in the two firm case, a benevolent government maximizing the (joint) profits of the two firms might still manipulate information in order to correct for externalities and coordinate behavior
- In game

$\theta = B$	I	N
I	$-1+\varepsilon+z$	-1
N	z	0

$\theta = G$	I	N
I	$x+\varepsilon+z$	x
N	z	0

benevolent government will behave as an investment maximizing government if z is large enough

2. Ordering Information

- Intuition: more information for the player imposes more constraints on the information designer and reduces the set of outcomes she can induce
- Recall Auction Example
- Say that information structure S "is more incentive constrained than" (= more informed than) S' if it gives rise to a smaller set of BCE outcomes than S' in all games
  - in one player case, this ordering corresponds to Blackwell's sufficiency ordering
  - in many player case, corresponds to "individual sufficiency" ordering
- Bergemann-Morris 16, see also Lehrer et al 10 and 11

Nice Properties of Individual Sufficiency Ordering

- Reduces to Blackwell in one player case
- Transitive
- Neither implies nor implied by Blackwell on join of players' information
- Two information structures are each individually sufficient for each other if and only if they share the same higher order beliefs about Θ
- S is individually sufficient for S' if and only if giving extra signals to S' equals S plus an appropriate correlation device

3. Concavification

- We described two step procedure for solving information design problem (with one or many players):
  1. Characterize all implementable decision rules
  2. Pick the designer's favorite
- Concavification procedure (with one player) [Aumann-Maschler 95 and Kamenica-Gentzkow 11]
  - Identify information designer's utility for every belief of the single player
  - Identify utility from optimal design by concavification, identifying information design only implicitly
- Many player generalization: Mathevet al 16
- Always nice interpretation, sometimes (but not always) useful in solving information design problem

4. Adversarial Equilibrium Selection

- Suppose that an information designer gets to make a communication $\Phi : T \times \Theta \to \Delta(M)$; new game of incomplete information (G, S, Φ)
- Write $E(G, S, \Phi)$ for the set of Bayes Nash equilibria of (G, S, Φ) and write $V_S^*(\Phi, \beta)$ for the information designer's utility
- We have been studying the maxmax problem

$$\max_{C} \max_{\beta} V_S^*(\Phi, \beta)$$

using a revelation principle argument to show that this equals

$$\max_{\sigma \in BCE(G,S)} V_S(\sigma)$$

- The maxmin problem

$$\max_{C} \min_{\beta} V^*(S, \Phi, \beta)$$

does not have a revelation principle characterization
- Carroll 15, Taneva et al 16, Kajii-Morris 97

5. Incomplete Information Correlated Equilibrium

- Decision rule $\sigma : T \times \Theta \to A$ is *incentive compatible* for (G, S) if, for each i, t_i and a_i, we have

$$\sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(a_i, a_{-i}\right), \theta\right) \sigma\left(a | t, \theta\right) \pi\left(t | \theta\right) \psi\left(\theta\right) \tag{1}$$
$$\geq \sum_{a_{-i}, t_{-i}, \theta} u_i\left(\left(\delta\left(a_i\right), a_{-i}\right), \theta\right) \sigma\left(a | \left(t_i', t_{-i}\right), \theta\right) \pi\left(t | \theta\right) \psi\left(\theta\right);$$

for all t_i' and $\delta_i : A_i \to A_i$.

- Decision rule $\sigma : T \times \Theta \to A$ is *join feasible* for (G, S) if $\sigma(a|t, \theta)$ is independent of θ, i.e., $\sigma(a|t, \theta) = \sigma(a|t, \theta')$ for each $t \in T$, $a \in A$,and $\theta, \theta' \in \Theta$.
- Solution Concepts:
  - Bayes correlated equilibrium = obedience
  - Communication equilibrium = incentive compatibility (and thus obedience) and join feasibility
  - etc...

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- compare also informed principal literature