Recitation 3: Understanding Marginals

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Recitation Takeaways

1. Marginal treatment effects as framework for policy extrapolation (applied to MH, AS, and selection on MH)
   - Accessible review article:
     Cornellison et al. (2016) Labour Economics
   - More advanced treatment on non-continuous instruments:
     Brinch, Mogstad, Wiswall (2017) JPE; Mogstad and Torgovitsky (2018) ARE
2. Alternative ways to characterize marginal compliers
   - Derive gap between marginal and average characteristics using regression equation
     Gruber, Levine, and Staiger (1999) QJE
   - Derive any functional of IV complier characteristics or potential outcomes
     Abadie (2003) JoE

Outline

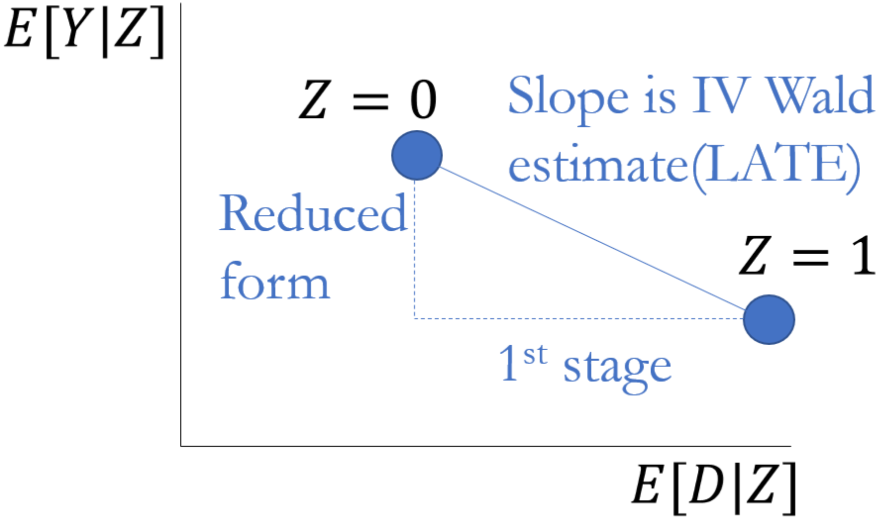
Marginal Treatment Effects

Characteristics of the Marginals

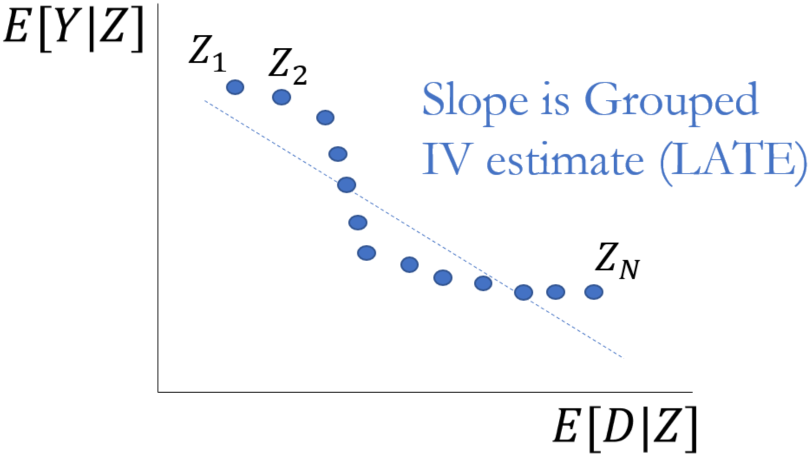
MTE vs. LATE

- **Rough intuition**: MTE is the continuous version of the LATE
- **Usefulness**: Various treatment effects of interest–ATE, ATT, ATUT, LATE, etc.–can be expressed as averages of MTEs
  - Selection on gains for different IV's deliver internally valid LATE that may not be useful for extrapolating to ATE
- **Notation**: Outcome Y, (binary endogenous) treatment D, instrument Z
  - E.g. $Y \equiv$ healthcare utilization, $D \equiv$ health insurance coverage, $Z \equiv$ (randomly assigned) insurance premium
- See Cornellison et al. (2016) Labour Economics for more details

Visual IV for LATE (Binary Z)



Visual IV for LATE (Non-binary Z)

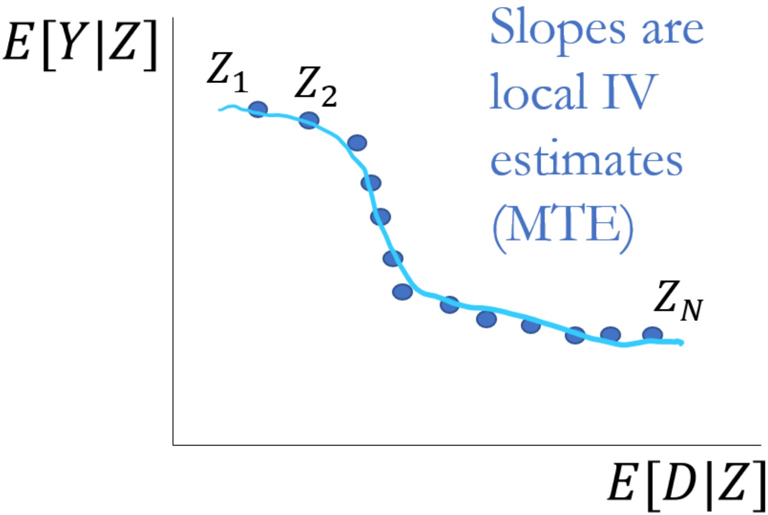


See Figure 1 of Angrist (1990) AER on Vietnam draft lottery

Recasting Instrument as Revealing Unobservables

- Translate x-axis to propensity score: $E[D|Z] = P(D = 1|Z) \equiv P(Z) \in [0, 1]$
- Z traces out unobserved willingness to select into treatment
- Slope at a given point reveals *marginal* treatment effect at a given quantile of the willingness to select into treatment distribution

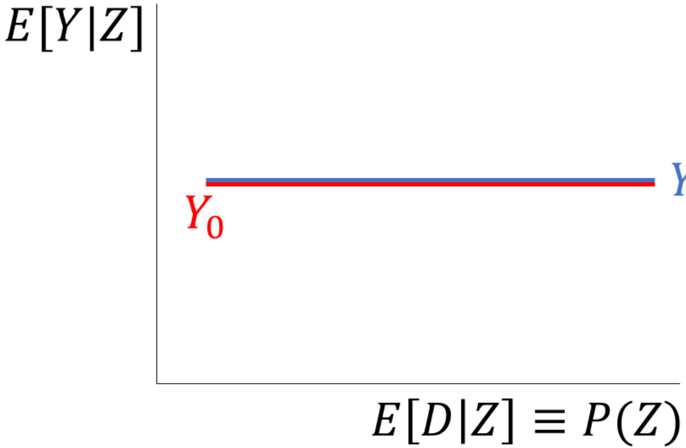
Visual IV for MTE (Non-binary Z)



Understanding MTE's Using Potential Outcomes

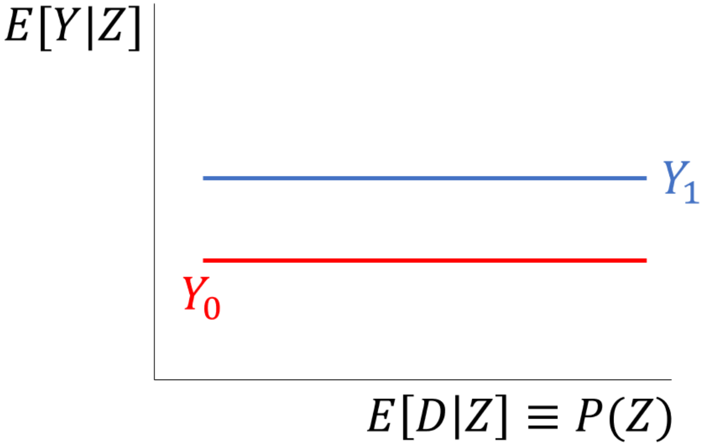
- Previous graphs showed outcomes of both $D = 1$ and $D = 0$ at each Z
- Instead, we can separately show (potential) outcomes for $D = 1$ and $D = 0$ by Z
  - **EFC**: Y_1 as utilization w/ insurance, Y_0 as utilization w/o insurance, Z as randomly assigned price
  - See Brinch, Mogstad, Wiswall (2017) JPE for more details

No Selection and No Causal Effects



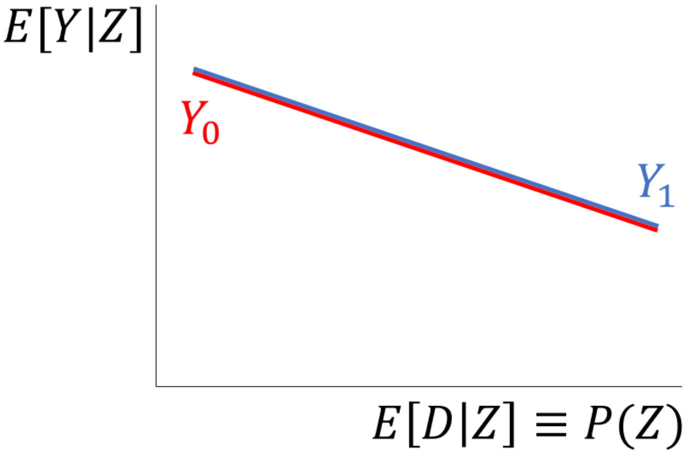
- “No MH or AS"
- $ATE = ATT = ATUT = E[Y|D=1] - E[Y|D=0] = 0$

Causal Effects but No Selection on Levels or Slopes



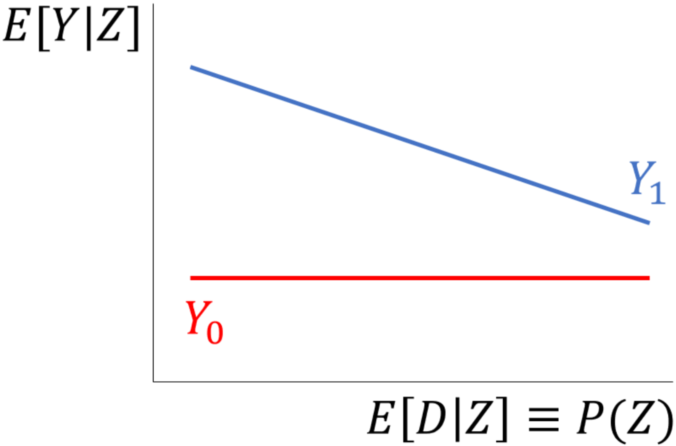
- “MH but not AS"
- $ATE = ATT = ATUT = LATE = E[Y|D = 1] - E[Y|D = 0] \neq 0$

No Causal Effects but Selection on Levels



- "AS but not MH"
- $ATE = ATT = ATUT = LATE = 0 \neq E[Y|D=1] - E[Y|D=0]$

Causal Effect with Selection on Levels and Slopes



- “Selection on MH"
- $ATE \neq ATT, ATUT, LATE$ varies by Z

Aside: Estimating MTE's with Binary Z

- Previous graphs suggest that you can implement MTE's with a binary Z assuming linearity of potential outcomes
  - Test with linearity assumption for $LATE \neq ATE$ is testing for unequal slopes by D
- More variation in Z allows you to relax assumptions

Aside: Selection Bias Formula for ATE

- Formula for ATT should be familiar:

$$\underbrace{E[Y_1|D=1]-E[Y_0|D=0]}_{\text{Observed diff. in outcomes}} = \underbrace{E[Y_1-Y_0|D=1]}_{ATT} + \underbrace{E[Y_0|D=1]-E[Y_0|D=0]}_{\text{Selection bias}}$$

- ATE decomposition has additional term of treatment effect heterogeneity:

$$\begin{aligned}\underbrace{E[Y_1|D=1]-E[Y_0|D=0]}_{\text{Observed diff. in outcomes}} = {} & \underbrace{E[Y_1-Y_0]}_{ATE} + \\ & \underbrace{E[Y_0|D=1]-E[Y_0|D=0]}_{\text{Selection bias}} + \\ & \underbrace{(1-P(D=1))}_{\text{Share untreated}}(\underbrace{E[Y_1-Y_0|D=1]}_{ATT} - \underbrace{E[Y_1-Y_0|D=0]}_{ATUT})\end{aligned}$$

- See here for full derivation

Taking Stock

- Growing recognition of treatment effect heterogeneity
- MTEs provide a formal framework for:
    1. Aggregating heterogeneous treatment effects to policy-relevant parameters
    2. Considering how treatment effect heterogeneity interacts with selection into treatment

Outline

Marginal Treatment Effects

Characteristics of the Marginals

(P)review of EFC (2010) Strategy

- **Specification**: $c_i = \gamma + \delta p_i + u_i$
- **Sample**: i who select into coverage at price p_i (of measure $D(p)$)
- **Variation**: p_i randomly assigned
- **Intuition**: p_i has no causal effect on c_i so $\delta \neq 0$ is due to sample selection
- **Translating to marginal outcome**: Use chain rule to express marginal (costs at p) in terms of average (costs at p) and total number ($D(p)$)

Gruber, Levine, and Staiger (1999): Gap Between Marginal and Average

- **Research question**: What is the impact of abortion on average living standards due to selection?
- **Specification**: $O_{st}/B_{st} = \alpha \ln(B_{st}) + controls$
  - E.g. $O/B \equiv$ % infants under FPL

$$\alpha = \frac{\partial O/B}{\partial \ln(B)} = B\frac{\partial O/B}{\partial B} = \underbrace{\frac{\partial O}{\partial B}}_{\text{marginal}} - \underbrace{\frac{O}{B}}_{\text{average}}$$

- **Variation**: Instrument for state births B_{st} using abortion law repeal
- **Intuition**: Same as EFC

Abadie (2003): Extending LATE Theorem Logic

- Binary instrument Z, binary treatment D, outcome Y
- Potential outcomes Y_{zd} and D_z for $d \in \{0, 1\}$, $z \in \{0, 1\}$
- Standard IV assumptions:
  - Independence: $(Y_{00}, Y_{01}, Y_{10}, Y_{11} \perp Z)$
  - Exclusion: $P(Y_{1d} = Y_{0d}) = 1$ for $d \in \{0, 1\}$
  - 1[st] stage: $0 < P(Z = 1) < 1$ and $P(D_1) > P(D_0)$
  - Monotonicity: $P(D_1 \geq D_0) = 1$

Abadie's κ in words

- Split population into compliers (C), always-takers (AT), and never-takers (NT)
- Use law of total probability to decompose any observable into those for C, AT, NT
- Observables for AT revealed by $(D, Z) = (1, 0)$ and NT by $(D, Z) = (0, 1)$
- "Subtract off" AT and NT by reweighting based on realized (D, Z)
  - Applicable for any function[1] $g(\cdot)$ (e.g. quantile) applied to any observable (i.e. outcome Y, treatment D, or covariate X)
  - Applicable for complier $Y(1)$ $[Y(0)]$ by subtracting off AT $[NT]$ from treated [untreated] outcomes

Abadie's κ in math

Complier observables

- Define $\kappa = 1 - \underbrace{\frac{D(1-Z)}{P(Z=0)}}_{\text{Subtract off } AT} - \underbrace{\frac{(1-D)Z}{P(Z=1)}}_{\text{Subtract off } NT}$
- $\underbrace{E[g(Y,D,X)|D_1 > D_0]}_{\text{complier observables}} = \underbrace{\frac{1}{P(D_1 > D_0)}}_{\text{scale by size of } C} \underbrace{E[\kappa g(Y,D,X)]}_{\text{weight each observation}}$

Complier Treated Potential Outcomes

- Define $\kappa_{(1)} = 1 - \underbrace{D}_{NT \text{ get 0 weight}} - \underbrace{\frac{(Z - P(Z=1))}{P(Z=0)P(Z=1)}}_{AT \text{ get weight < 0} NT}$
- $E[g(Y_1,X)|D_1 > D_0] = \frac{1}{P(D_1 > D_0)} E[\kappa_{(1)} g(Y,D,X)]$

Analogous for Complier Untreated Potential Outcomes

Comparing Approaches

- **EFC**: Clear mapping to visual plots of potential outcomes
- **Gruber et al.**: Derivation from regression specification
- **Abadie**: Derivation from LATE theorem logic
    - Powerful to be able to estimate any function of potential outcomes (and therefore treatment effects on those functions)