Section 4: Selection Market Unraveling

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Overview

• Formalize competitive insurance selection market unraveling
     → Pooling with worse types breaks down trade
  2. Rothschild Stiglitz (1976)
     → Cream-skimming/pooling motives prevent pure strategy Nash equilibrium

• “Contract space”: fixed in Akerlof, endogenous in RS

• Today: Develop model showing both mechanisms w/ non-price contract response
Walk away from this recitation understanding:

1. Modeling distinction between Akerlof and Rothschild Stiglitz
2. "Equilibrium of unraveling" vs. "unraveling of market equilibrium"
3. Graphical analysis demonstrating the two
Disclaimers

- This recitation is largely based on Hendren (2014) “Unraveling versus Unraveling: A Memo on Competitive Equilibriums and Trade in Insurance Markets" and Alex Wolitzky’s 124 lecture notes
- It is heavily based on graphical intuition
- The Rothschild Stiglitz graphs can get cluttered, but try to see the big picture of strategic firms adjusting contracts and strategic consumers selecting into contracts
Outline

Canonical Akerlof (1970)

General Insurance Setup

Akerlof Unraveling

Rothschild and Stiglitz Unraveling
Why do cars lose resale value right after leaving the car dealership?

- Sellers know hard to observe car-specific quality while buyers do not
- Negotiation is over only price
Market for Lemons Intuition

Why do cars lose resale value right after leaving the car dealership?

- Sellers know hard to observe car-specific quality while buyers do not
- Negotiation is over **only** price
- *Key idea*: Before listing for sale, what does a seller know? For a car listed at a given price, what does a buyer know? And then?
Market for Lemons Math

- Quality of good $\theta \sim f(\theta)$ on $[\theta, \bar{\theta}]$ observed only by seller
- Buyer utility $\theta - p$
- Seller utility $p - r(\theta)$
- Competitive equilibrium is price $p^*$ s.t. all sellers value good less than $p^*$ and buyers expect to receive $p^*$ quality good

Formally:

$$\Theta^* = \{\theta : r(\theta) \leq p^*\}$$  \hspace{1cm} (1)

$$p^* = \mathbb{E}[\theta | \theta \in \Theta^*]$$  \hspace{1cm} (2)
Market for Lemons Pictures

\[ \theta \]

Gains from trade!

\[ r(\theta) \]
Market for Lemons Pictures

Consider lowest price to attract seller of highest quality...

\[ \Theta^* = \{\theta | r(\theta) < p'\} \]

\[ E[\theta | r(\theta) < p'] \]

...still too high!
Market for Lemons Pictures

Okay, so the price drops. So what?

Uh oh…

\[ p'' = E[\theta | r(\theta) < p'] \]
\[ E[\theta | r(\theta) < p''] \]

\[ \Theta'' = \{ \theta | r(\theta) < p'' \} \]
Intuition Checks

- How could different utilities/graph shapes lead to trade in equilibrium?
- In what sense was the “contract space fixed”?
- In what sense is this plotting supply and demand? (Brace yourself for EFC...?)
Aside: Seeing Akerlof as Supply/Demand Analysis

Subtle point: Upward sloping demand curve?
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Model Environment

- **Consumers**: Unit mass of facing binary loss $L$ w.p. $p \sim F(p)$
  - Status quo: consume $c_L = w - L$ w.p. $p$, $c_{NL} = w$ w.p. $1 - p$
  - EU preferences: $U(c_L, c_{NL}) = pu(c_L) + (1 - p)u(c_{NL})$, where $u(\cdot)$ strictly concave
  - Individual draw $P$ from $F(p)$ is **private information**

- **Insurers**: Risk-neutral firms $j \in J$ offer contracts indexed by $i$: $c^j_L(i)$, $c^j_{NL}(i)$
Model Environment

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- **Insurers**: Risk-neutral firms $j \in J$ offer contracts indexed by $i$: $c^j_L(i), c^j_{NL}(i)$
  - Fixed contract space in original Akerlof: $i$ is a singleton;
    $c_L = w - L - \text{premium+payout}, c_{NL} = w - \text{premium}$; competition over only premium

Sanity checks:
1. What is an allocation in this model?
2. In what sense is the "contract space endogenous"?
3. What are the simplifying assumptions? Are they approximately true?
Model Environment

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Competitive Equilibrium

Allocation $A = \{c_L(p), c_{NL}(p)\}$ is an equilibrium if:

1. **Incentive compatibility**
   - $pu(c_L(p)) + (1 - p)u(c_{NL}(p)) \geq pu(c_L(\tilde{p})) + (1 - p)u(c_{NL}(\tilde{p})) \ \forall p, \tilde{p}$
   - Intuition: insurer can’t be surprised in equilibrium/revelation principle

2. **Individual rationality**

3. **Non-negative profits**

4. **No profitable deviations**

Aside: Which two pairs are alike? Could any plausibly not hold in the real world?
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2. **Individual rationality**
   - $pu(c_L(p)) + (1 - p)u(c_{NL}(p)) \geq pu(w - L) + (1 - p)u(w)$ $\forall p$

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   - $pu(c_L(p)) + (1 - p)u(c_{NL}(p)) \geq pu(w - L) + (1 - p)u(w) \ \forall p$

3. Non-negative profits
   - $\int_p p(w - L - c_L(p)) + (1 - p)(w - c_{NL}(p)) \geq 0$
   - Competition will end up implying zero profits
Competitive Equilibrium

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   - $pu(c_L(p)) + (1 - p)u(c_{NL}(p)) \geq pu(w - L) + (1 - p)u(w) \quad \forall p$

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   - $\int_p p(w - L - c_L(p)) + (1 - p)(w - c_{NL}(p)) \geq 0$
   - Competition will end up implying zero profits

4. No profitable deviations
   - $\forall \hat{A} = \{\hat{c}_L(p), \hat{c}_{NL}(p)\}$, $\int_{p \in \hat{D}} p(w - L - \hat{c}_L(p)) + (1 - p)(w - \hat{c}_{NL}(p)) \leq 0$
   - where $\hat{D} = \{$types $p$ who prefer $\hat{A}$ to $A\}$
Competitive Equilibrium

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4. **No profitable deviations**
   - \( \forall \hat{A} = \{\hat{c}_L(p), \hat{c}_{NL}(p)\}, \int_{p \in \hat{D}} p(w - L - \hat{c}_L(p)) + (1 - p)(w - \hat{c}_{NL}(p)) \leq 0, \)
     where \( \hat{D} = \{\text{types } p \text{ who prefer } \hat{A} \text{ to } A\} \)

Aside: Which two pairs are alike? Could any plausibly not hold in the real world?
Orienting Yourself

\[ c_L \]

\[ c_{NL} \]

Direction of higher profits

Direction of higher utility
Outline

Canonical Akerlof (1970)

General Insurance Setup

Akerlof Unraveling

Rothschild and Stiglitz Unraveling
The following example will demonstrate the notion of market unraveling due to adversely selected consumers “repeatedly” driving marginal consumers out of the market.

*Equilibrium of market unraveling*

Of course, I could have drawn the curves differently so that the market does not completely unravel if the gains from trade are large relative to the degree of adverse selection.
General Akerlof: Verbal Derivation

- Consider a contract transferring $1 to $L$
  - What is type $p$’s WTP from $NL$?

\[ \begin{align*}
  \text{MRS} &= \frac{pu'(w - L)}{(1 - p)u'(w)} \\
  \text{What is the insurer isocost for type } p? \\
  p^* &= \frac{1 - p}{\text{profit}} \rightarrow \text{actuarially fair premium} = p \frac{1 - p}{\text{premium}}.
\end{align*} \]

Suppose $\hat{p}$ is the lowest type that wants to trade the amount in $NL$ for $1 in $L$. Who else will do so?

Akerlof unraveling occurs when nobody is willing to pay an insurance premium large enough to cover the average costs of riskier people.
• Consider a contract transferring $1 to $L$
  
  What is type $p$'s WTP from $NL$?

  \[ MRS = pu'(w - L)/[(1 - p)u'(w)] \]
Consider a contract transferring $1 to $L$

- What is type $p$'s WTP from $NL$?
  \[ MRS = pu'(w - L)/[u'(w)(1 - p)] \]
- What is the insurer's isocost for type $p$?

Akerlof unraveling occurs when nobody is willing to pay an insurance premium large enough to cover the average costs of riskier people.
Consider a contract transferring $1 to $L$

- What is type $p$'s WTP from $NL$?
  
  $$MRS = pu'(w - L)/[(1 - p)u'(w)]$$

- What is the insurer's isocost for type $p$?
  
  $$p * 1 - (1 - p)\text{premium} = \text{profit} \rightarrow \text{actuarially fair premium} = \frac{p}{1 - p}$$
General Akerlof: Verbal Derivation

- Consider a contract transferring $1 to $L$
  - What is type $p$'s WTP from $NL$?
    \[ MRS = pu'(w - L)/(1 - p)u'(w) \]
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- Suppose $\hat{p}$ is the lowest type that wants to trade the amount in $NL$ for $1 in $L. Who else will do so?
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Suppose $\hat{p}$ is the lowest type that wants to trade the amount in $NL$ for $1 in $L$. Who else will do so?

Akerlof unraveling occurs when nobody is willing to pay an insurance premium large enough to cover the average costs of riskier people.
Suppose $\hat{p}$ is the highest risk type to take up a given contract...

Zero profit line:
inverse slope $= \frac{E[p|p \geq \hat{p}]}{1 - E[p|p \geq \hat{p}]}$

Any insurance implies either IR or $\Pi \geq 0$ violated

$\hat{p}$ indifference curve
inverse slope at point $= \frac{\hat{p}}{1 - \hat{p}} \frac{w(w-L)}{w(w)}$
Suppose $\hat{p}$ is the highest risk type to take up a given contract...

Zero profit line:
inverse slope $= \frac{E[p|p\geq \hat{p}]}{1-E[p|p\geq \hat{p}]}$
Higher $p$ marginal type implies flatter indiff. curve and zero profit line

$c_L = c_{NL}$

As drawn, any insurance still implies either IR or $\Pi \geq 0$ violated
• *Econ 101*: WTP always less than WTA $\Rightarrow$ no trade
• *Econ 472*: $\forall \hat{\mathbf{p}}, MRS(\hat{\mathbf{p}}) <$ Pooled price ratio at $\hat{\mathbf{p}}$
  $\Rightarrow$ endowment is unique competitive Nash equilibrium
• Turns out you can also show the other direction
Outline

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Rothschild and Stiglitz Unraveling
The following example will demonstrate the notion of market unraveling due to insurers repeatedly adjusting contracts to pick off safe buyers/avoid risky buyers.

- Unraveling of market equilibrium
The following example will demonstrate the notion of market unraveling due to insurers repeatedly adjusting contracts to pick off safe buyers/avoid risky buyers:

- *Unraveling of market equilibrium*

Order of the following slides:

1. Very contrived parameters to illustrate these mechanics
2. More realistic regularity conditions under which the market is guaranteed to unravel in either the Akerlof or Rothschild-Stiglitz sense
3. What a possible market equilibrium with insurance can look like if the regularity conditions do not hold
Setup

Same as before, but suspend disbelief and consider two types: $0 < p_L < p_H = 1$

- Why $\not\exists$ separating equilibrium with $p_H = 1$?
Setup

Same as before, but suspend disbelief and consider two types: \(0 < p_L < p_H = 1\)

- Why \(\not\exists\) separating equilibrium with \(p_H = 1\)?

Denote \(L\) and \(H\) by **level of risk**

![Diagram of indifference curves and profit lines](image)
Possibility 1: Many $L$ relative to $H$

Market unravels in the sense endowment is the only pure strategy Nash equilibrium
Possibility 2: Many $H$ relative to $L$
Possibility 2: Many $H$ relative to $L$

Back to Akerlof unraveling!
Summarizing the Two Possibilities

With $p_H = 1$, only way for $L$ to get insurance is by subsidizing $H$

1. $L$ willing to subsidize for a potential equilibrium
   ⇒ Rothschild Stiglitz unraveling

2. $L$ unwilling to subsidize for any potential equilibrium
   ⇒ Akerlof unraveling
“But Jon, $p_H = 1$ is silly!"

It is. But it turns out the result’s mechanics hold as long as either (i) $f(p)$ has positive mass on $p = 1$ or (ii) the support of $p$ contains an interval.

I.e. under the above regularity condition, Akerlof and RS unraveling are **mutually exhaustive**
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Intuition for (ii) is cream-skimming motive at upper end of interval support. See Theorem 3 in Riley (1979) “Informational Equilibrium” for painful details.
“Okay, so what about $0 < p_L < p_H < 1$"

Fair enough, that seems like a nice simple case (where the previous slide’s regularity condition doesn’t hold!)

First, some intermediate results:

1. No pooling equilibrium exists
   
   - **Intuition:**
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   - *Intuition*: same cream-skimming motive as above with $p_H = 1$
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2. Insurers earn zero profit on both contracts
   - i.e. no cross-subsidization
   - *Intuition*: same “cream-skimming" motive but on both types
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   - **Intuition**: same “cream-skimming” motive but on both types

3. Incentive compatibility constraints:
   - Both types want full insurance...
   - ...but one type wants to “masquerade” as the other type
Candidate Separating Equilibrium

Intuition: Recalling that each type’s contract must lie on its zero profit line:
1. $H$’s allocation can’t change because they strictly prefer full insurance
2. $c_L(p_L)$ can’t decrease because $L$ wouldn’t select that
3. $c_L(p_L)$ can’t increase because $H$ would select that contract

All that’s left to do is check robustness to pooling equilibrium
Under the candidate separating equilibrium with many $H$, any deviating contract that pools both types earns strictly negative profits.
Not a Separating Equilibrium $\times$

Under the candidate separating equilibrium with many $L$, there are deviating pooling contracts with non-negative profits.
Intuition Check: Can L’s IC constraint bind?

- Notice can’t be separating equilibrium because there’s a region that is...
Intuition Check: Can L’s IC constraint bind?

- Notice can’t be separating equilibrium because there’s a region that is...
  1. Below $H$’s full insurance indiff. curve
  2. Above $H$’s partial insurance indiff. curve
  3. Below the only $L$ zero profit line

(1) and (2) ensure only $H$ will take up a deviating contract in that region, while (3) implies the deviating contract is profitable.
Possible Nonexistence of Separating Equilibrium

- Summary: Depending on relative type masses/size of $L$ distortion, there might be a profitable deviation to a pooling equilibrium (that itself can’t be an equilibrium).
- Resolutions from 2-stage contract posting game between insurers:
Possible Nonexistence of Separating Equilibrium

• Summary: Depending on relative type masses/size of $L$ distortion, there might be a profitable deviation to a pooling equilibrium (that itself can’t be an equilibrium)

• Resolutions from 2-stage contract posting game between insurers:
  1. **Wilson equilibrium**: Don’t worry about deviations that become unprofitable once pre-existing contracts are withdrawn
     - i.e. cream-skimming deviation excluded because it’s unprofitable once previous pooling contract removed
Possible Nonexistence of Separating Equilibrium

• Summary: Depending on relative type masses/size of $L$ distortion, there might be a profitable deviation to a pooling equilibrium (that itself can’t be an equilibrium)

• Resolutions from 2-stage contract posting game between insurers:
  1. **Wilson equilibrium**: Don’t worry about deviations that become unprofitable once pre-existing contracts are withdrawn
     - i.e. cream-skimming deviation excluded because it’s unprofitable once previous pooling contract removed
  2. **Riley equilibrium**: Don’t worry about deviations that become unprofitable once new contracts are added
     - i.e. pooling deviation excluded because it’s unprofitable once new cream-skimming contract added
     - Aside: Stronger form of the “Intuitive Criterion”. For folks who’ve seen the Spence signaling model: this is what rules out no-education pooling PBE based on employers inferring education deviators are low-types.
Speculative Thoughts

- Theory delivers a lot of nonexistence results despite what we see in the real-world. What gives?
- Do you find anything unsatisfying about Rothschild Stiglitz unraveling in the real world?
- Are there other equilibrium concepts that could apply?
- How can you map the general insurance model to other selection markets?
Overall Takeaways

- Almost all markets have asymmetric information. What is it about insurance selection markets that causes problems?
- Akerlof: *equilibrium of market unraveling*
- Rothschild Stiglitz: *unraveling of market equilibrium*
Connections to Some Papers on the Syllabus

1. **EFC (2010)**: Akerlof unraveling but w/ a fixed contract space
2. **FM 2006**: Not only unobserved heterogeneity in risk type $p$ but also risk preference $u(\cdot)$
3. **Shepard 2016**: Cream-skimming on non-price characteristics
4. **Hendren 2017**: Akerlof unraveling w/ general contracts due to private info making 1st $ of supplemental UI too costly