Section 4: Selection Market Unraveling

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Overview

- Formalize competitive ~~insurance~~ selection market unraveling
  1. Akerlof (1970)

     $\rightarrow$ Pooling with worse types breaks down trade
  2. Rothschild Stiglitz (1976)

     $\rightarrow$ Cream-skimming/pooling motives prevent pure strategy Nash equilibrium
- “Contract space": fixed in Akerlof, endogenous in RS
- **Today**: Develop model showing both mechanisms w/ non-price contract response

Walk away from this recitation understanding:

1. Modeling distinction between Akerlof and Rothschild Stiglitz
2. “Equilibrium of unraveling" vs. "unraveling of market equilibrium"
3. Graphical analysis demonstrating the two

Disclaimers

- This recitation is largely based on Hendren (2014) "Unraveling versus Unraveling: A Memo on Competitive Equilibriums and Trade in Insurance Markets" and Alex Wolitzky's 124 lecture notes
- It is heavily based on graphical intuition
- The Rothschild Stiglitz graphs can get cluttered, but try to see the big picture of strategic firms adjusting contracts and strategic consumers selecting into contracts

Outline

Market for Lemons Intuition

Why do cars lose resale value right after leaving the car dealership?

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- Negotiation is over **only** price

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- *Key idea*: Before listing for sale, what does a seller know? For a car listed at a given price, what does a buyer know? And then?

Market for Lemons Math

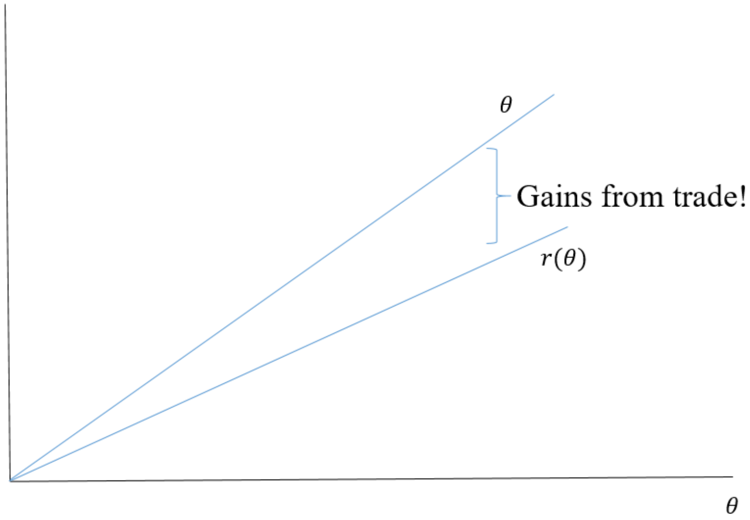
- Quality of good $\theta \sim f(\theta)$ on $[\underline{\theta}, \bar{\theta}]$ observed only by seller
- Buyer utility $\theta - p$
- Seller utility $p - r(\theta)$
- Competitive equilibrium is price p^* s.t. all sellers value good less than p^* and buyers expect to receive p^* quality good

Formally:

$$\Theta^* = \{\theta : r(\theta) \leq p^*\} \tag{1}$$

$$p^* = E[\theta | \theta \in \Theta^*] \tag{2}$$

Market for Lemons Pictures



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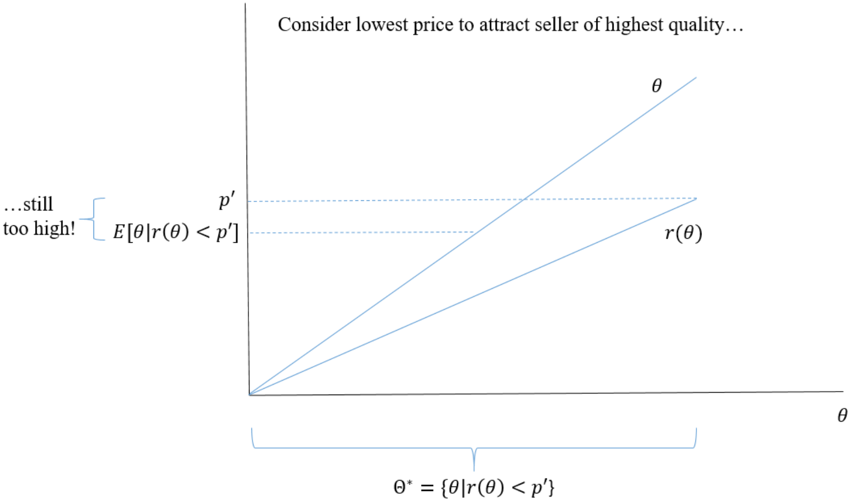

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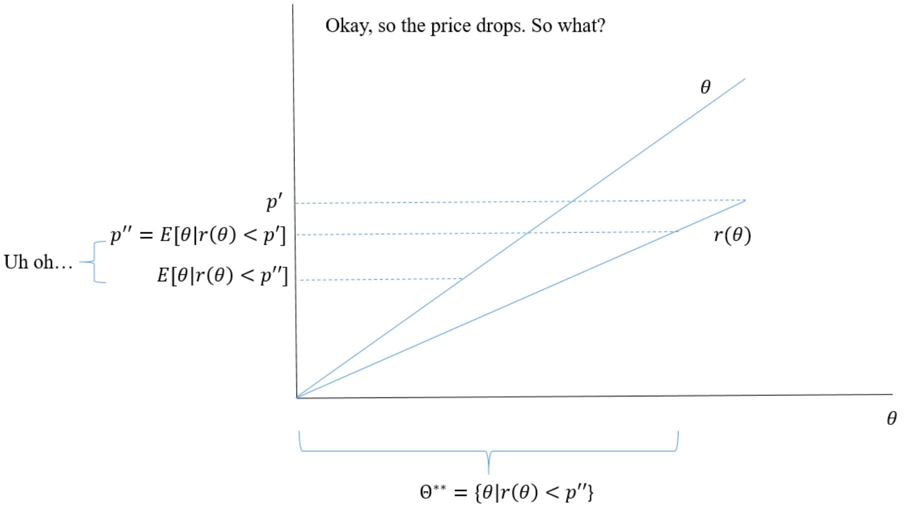
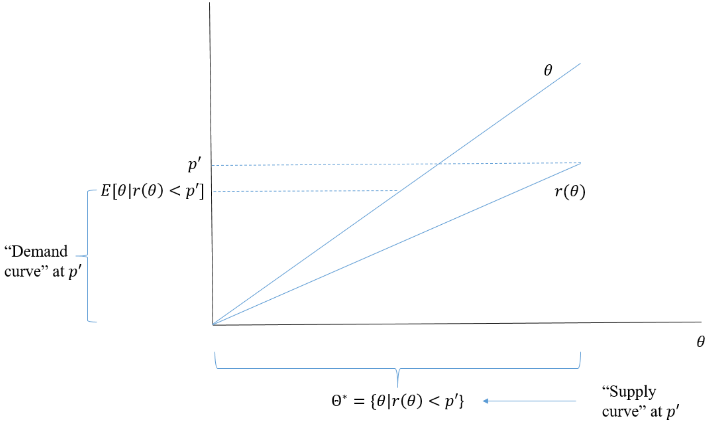


Intuition Checks

- How could different utilities/graph shapes lead to trade in equilibrium?
- In what sense was the “contract space fixed”?
- In what sense is this plotting supply and demand? (Brace yourself for EFC...)

Aside: Seeing Akerlof as Supply/Demand Analysis



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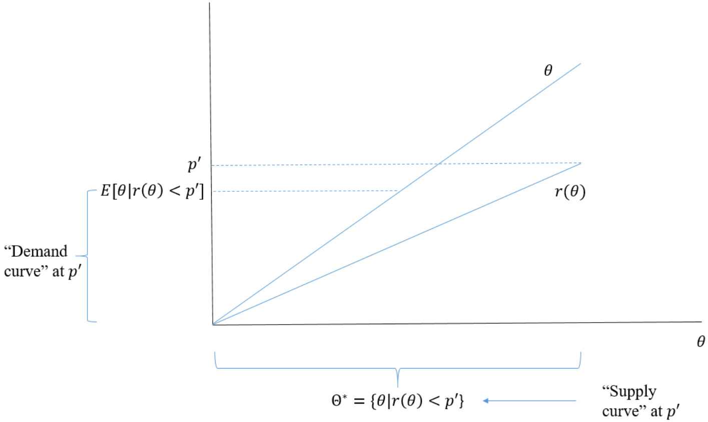

Subtle point: Upward sloping demand curve?

Outline

Model Environment

- Consumers: Unit mass of facing binary loss L w.p. $p \sim F(p)$
  - Status quo: consume $c_L = w - L$ w.p. p, $c_{NL} = w$ w.p. $1 - p$
  - EU preferences: $U(c_L, c_{NL}) = pu(c_L) + (1 - p)u(c_{NL})$, where $u(\cdot)$ strictly concave
  - Individual draw P from $F(p)$ is **private information**
- **Insurers**: Risk-neutral firms $j \in J$ offer contracts indexed by i: $c_L^j(i), c_{NL}^j(i)$

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- **Insurers**: Risk-neutral firms $j \in J$ offer contracts indexed by i: $c_L^j(i), c_{NL}^j(i)$
  - Fixed contract space in original Akerlof: i is a singleton;

    $c_L = w - L -$ premium+payout,$c_{NL} = w -$ premium; competition over only premium

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Sanity checks:

1. What is an allocation in this model?
2. In what sense is the "contract space endogenous"?
3. What are the simplifying assumptions? Are they approximately true?

Competitive Equilibrium

Allocation $A = \{c_L(p), c_{NL}(p)\}$ is an equilibrium if:

1. Incentive compatibility
   - $pu(c_L(p)) + (1-p)u(c_{NL}(p)) \geq pu(c_L(\tilde{p})) + (1-p)u(c_{NL}(\tilde{p})) \;\; \forall p, \tilde{p}$
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   - $\int_p p(w - L - c_L(p)) + (1-p)(w - c_{NL}(p)) \geq 0$
   - Competition will end up implying zero profits

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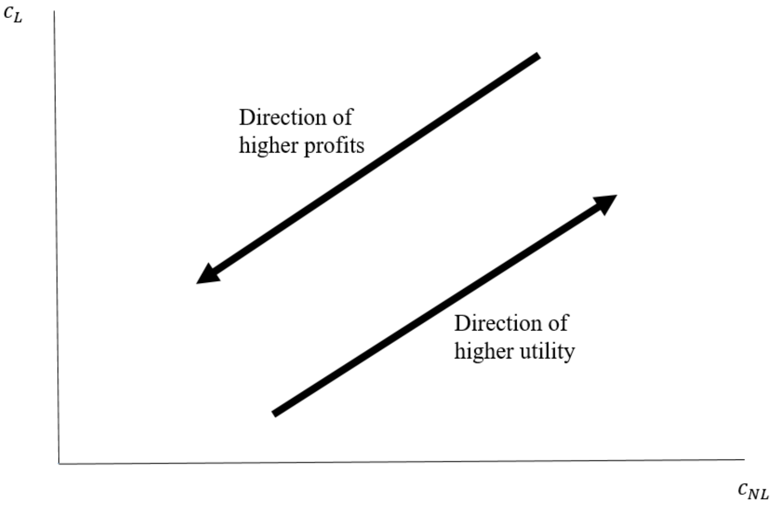
Aside: Which two pairs are alike? Could any plausibly not hold in the real world?

Orienting Yourself



Outline

Section Overview

- The following example will demonstrate the notion of market unraveling due to adversely selected consumers “repeatedly” driving marginal consumers out of the market
  - *Equilibrium of market unraveling*
- Of course, I could have drawn the curves differently so that the market does not completely unravel if the gains from trade are large relative to the degree of adverse selection

General Akerlof: Verbal Derivation

- Consider a contract transferring \$1 to L
  - What is type p's WTP from NL?

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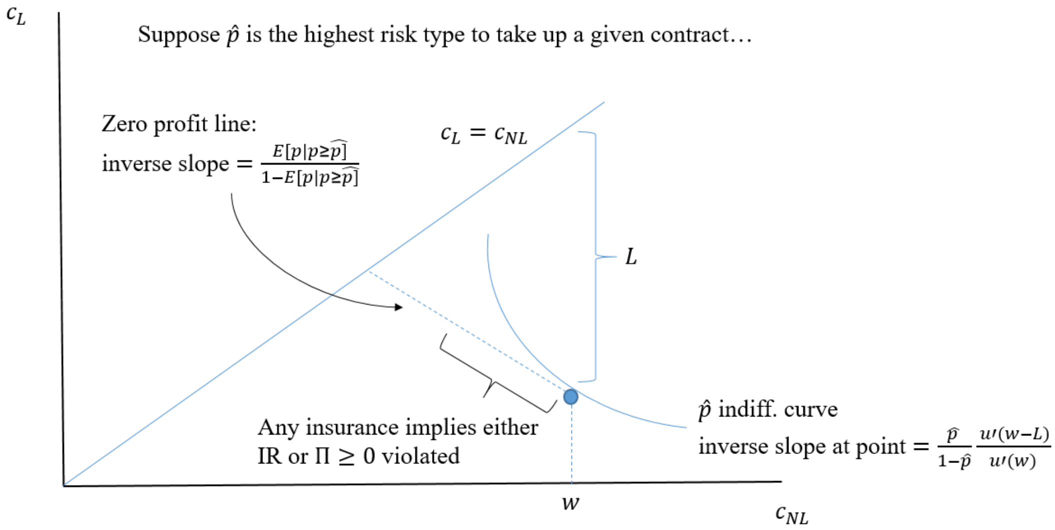
Akerlof unraveling occurs when nobody is willing to pay an insurance premium large enough to cover the average costs of riskier people

General Akerlof: Pictures



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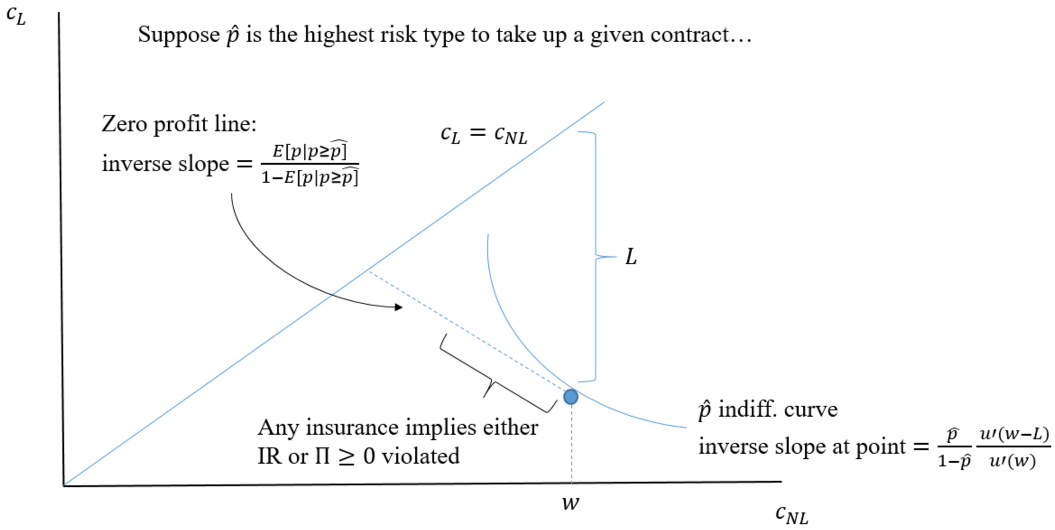

Recall "uh-oh" from earlier Akerlof slide...

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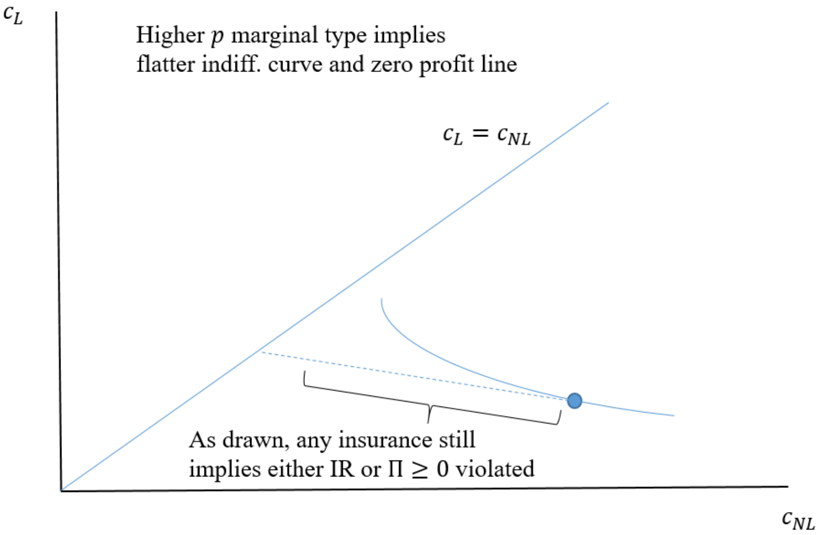

Taking Stock

- *Econ 101*: WTP always less than WTA $\Rightarrow$ no trade
- *Econ 472*: $\forall \hat{p}$, $MRS(\hat{p}) <$ Pooled price ratio at $\hat{p}$

  $\Rightarrow$ endowment is unique competitive Nash equilibrium
- Turns out you can also show the other direction

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  Order of the following slides:

  1. Very contrived parameters to illustrate these mechanics
  2. More realistic regularity conditions under which the market is guaranteed to unravel in either the Akerlof or Rothschild-Stiglitz sense
  3. What a possible market equilibrium with insurance can look like if the regularity conditions do not hold

Setup

Same as before, but suspend disbelief and consider two types: $0 < p_L < p_H = 1$

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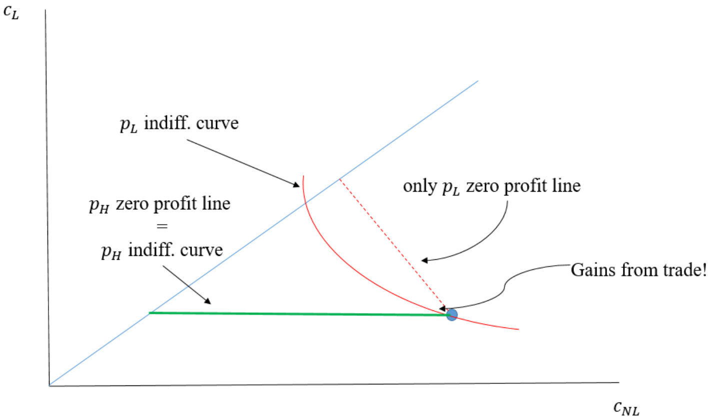
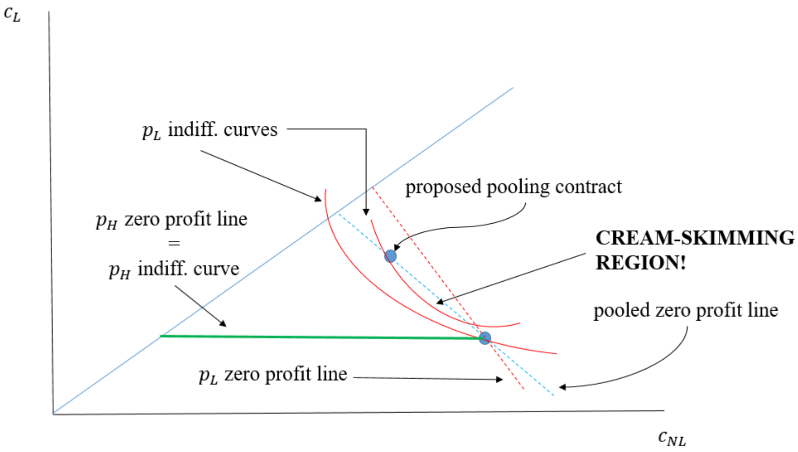
Denote L and H by **level of risk**

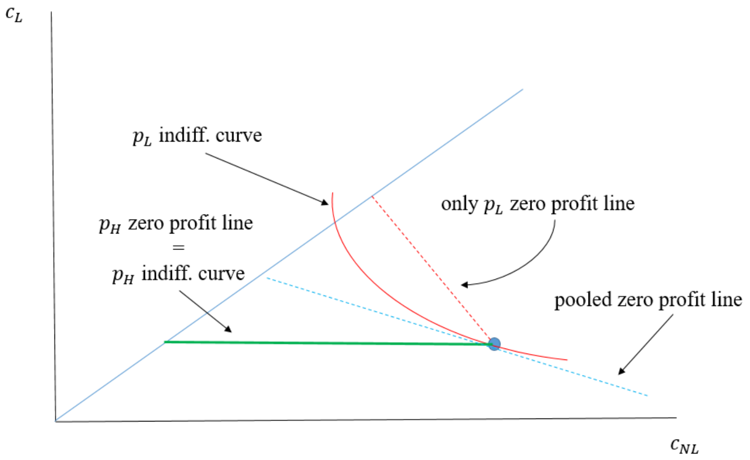


Possibility 1: Many L relative to H



Market unravels in the sense endowment is the only pure strategy Nash equilibrium

Possibility 2: Many H relative to L



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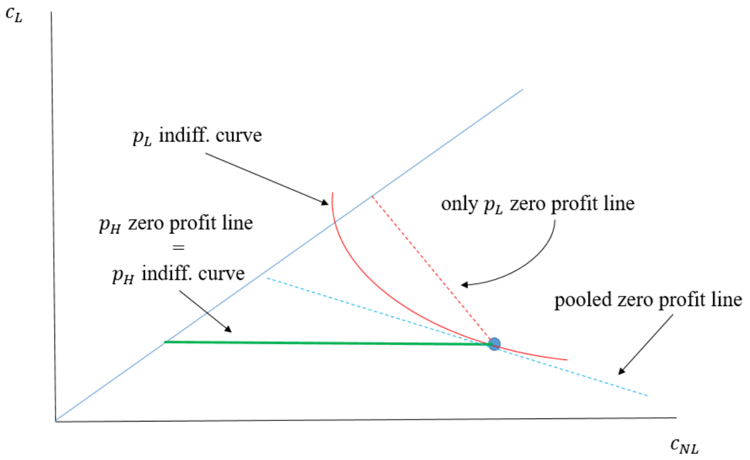

Back to Akerlof unraveling!

Summarizing the Two Possibilities

With $p_H = 1$, only way for L to get insurance is by subsidizing H

1. L willing to subsidize for a potential equilibrium

   $\Rightarrow$ Rothschild Stiglitz unraveling

2. L unwilling to subsidize for any potential equilibrium

   $\Rightarrow$ Akerlof unraveling

“But Jon, $p_H = 1$ is silly!"

It is. But it turns out the result’s mechanics hold as long as either (i) $f(p)$ has positive mass on $p = 1$ or (ii) the support of p contains an interval.

I.e. under the above regularity condition, Akerlof and RS unraveling are **mutually exhaustive**

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Intuition for (ii) is cream-skimming motive at upper end of interval support. See Theorem 3 in Riley (1979) “Informational Equilibrium" for painful details.

“Okay, so what about $0 < p_L < p_H < 1$"

Fair enough, that seems like a nice simple case (where the previous slide’s regularity condition doesn’t hold!)

First, some intermediate results:

1. No pooling equilibrium exists
   - *Intuition*:

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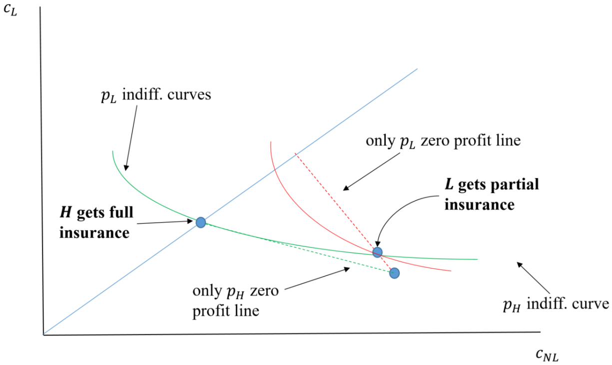
   - *Intuition*: same “cream-skimming" motive but on both types
3. Incentive compatibility constraints:
   - Both types want full insurance...
   - ...but one type wants to “masquerade" as the other type

Candidate Separating Equilibrium



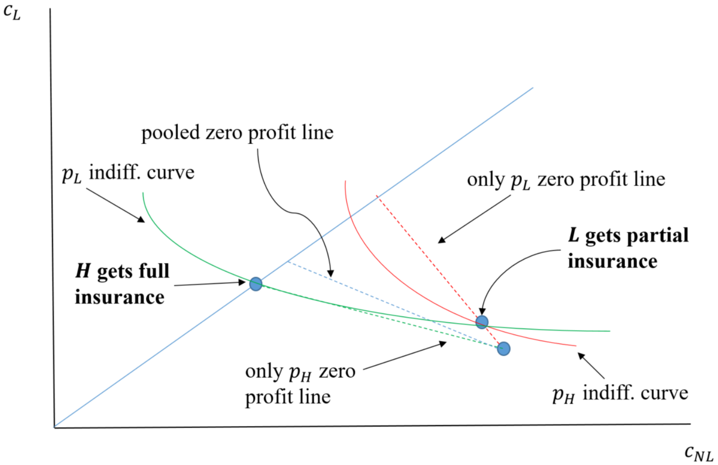
Intuition: Recalling that each type's contract must lie on its zero profit line:

1. H's allocation can't change because they strictly prefer full insurance
2. $c_L(p_L)$ can't decrease because L wouldn't select that
3. $c_L(p_L)$ can't increase because H would select that contract

All that's left to do is check robustness to pooling equilibrium

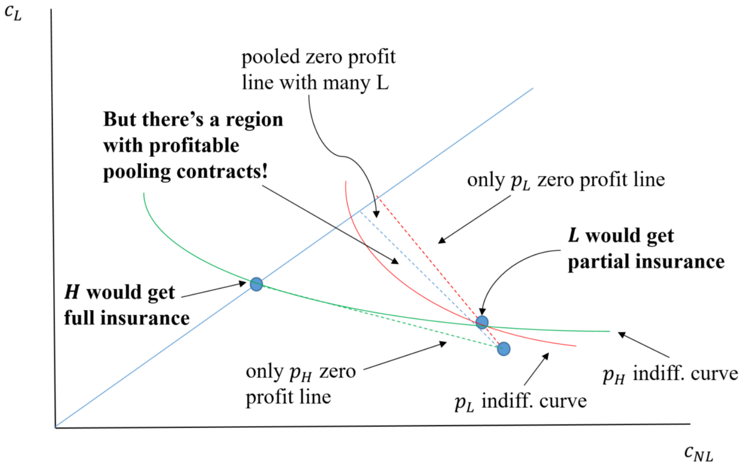
Separating Equilibrium ✓

Under the candidate separating equilibrium with many H, any deviating contract that pools both types earns strictly negative profits

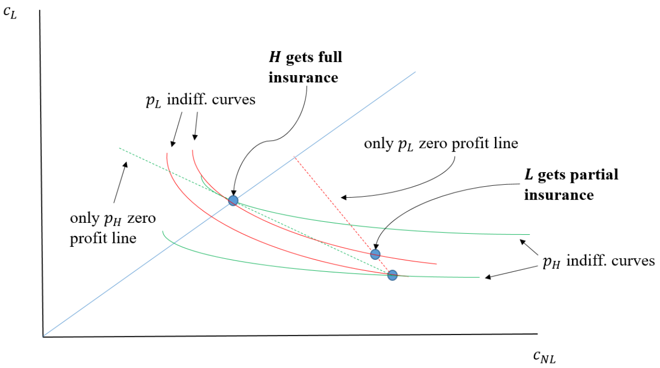


Not a Separating Equilibrium ✗

Under the candidate separating equilibrium with many L, there are deviating pooling contracts with non-negative profits



Intuition Check: Can L's IC constraint bind?



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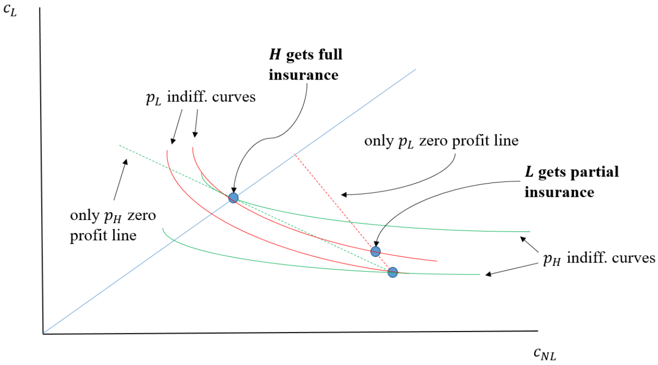

- Notice can't be separating equilibrium because there's a region that is...

1. Below H's full insurance indiff. curve
2. Above H's partial insurance indiff. curve
3. Below the only L zero profit line

(1) and (2) ensure *only* H will take up a deviating contract in that region, while (3) implies the deviating contract is profitable

Possible Nonexistence of Separating Equilbrium

- Summary: Depending on relative type masses/size of L distortion, there might be a profitable deviation to a pooling equilibrium (that itself can't be an equilibrium)
- Resolutions from 2-stage contract posting game between insurers:

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    2. **Riley equilibrium**: Don't worry about deviations that become unprofitable once *new* contracts are *added*
        - i.e. pooling deviation excluded because it's unprofitable once new cream-skimming contract added
        - Aside: Stronger form of the "Intuitive Criterion". For folks who've seen the Spence signaling model: this is what rules out no-education pooling PBE based on employers inferring education deviators are low-types.

Speculative Thoughts

- Theory delivers a lot of nonexistence results despite what we see in the real-world. What gives?
- Do you find anything unsatisfying about Rothschild Stiglitz unraveling in the real world?
- Are there other equilibrium concepts that could apply?
- How can you map the general insurance model to other selection markets?

Overall Takeaways

- Almost all markets have asymmetric information. What is it about ~~insurance~~ selection markets that causes problems?
- Akerlof: *equilibrium of market unraveling*
- Rothschild Stiglitz: *unraveling of market equilibrium*

Connections to Some Papers on the Syllabus

1. **EFC (2010)**: Akerlof unraveling but w/ a fixed contract space
2. **FM 2006**: Not only unobserved heterogeneity in risk type p but also risk preference $u(\cdot)$
3. **Shepard 2016**: Cream-skimming on non-price characteristics
4. **Hendren 2017**: Akerlof unraveling w/ general contracts due to private info making 1$^{\text{st}}$ \$ of supplemental UI too costly