Section 7: Optimal Tax Insights

Jon Cohen

November 13, 2021

Recitation Takeaways

- Traditional welfare analysis is about compensated elasticities

Recitation Takeaways

- Traditional welfare analysis is about compensated elasticities
- Optimal income taxation is about "type" incentive compatibility constraints preventing full redistribution

Recitation Takeaways

- Traditional welfare analysis is about compensated elasticities
- Optimal income taxation is about "type" incentive compatibility constraints preventing full redistribution
- Optimal (non-Pigouvian) commodity "taxation" is about whether consumption choices have residual info about "type"

Outline

Harberger-Style DWL Analsyis

Optimal Nonlinear Income Taxation

Optimal Commodity Taxation

Old School Analysis

- Contrast with the MVPF framework at each step
- Requires hard to estimate objects (e.g. compensated demands)...
- ...but highlights useful insights using standard micro theory
- Builds straw men for optimal commodity/nonlinear income taxation to dunk on

Back to Basics: Welfare Effect of Price Change

- (Indirect) utility: $v(p_1, w) - v(p_0, w)$
- Challenge: Utility is ordinal, not cardinal

Back to Basics: Welfare Effect of Price Change

- (Indirect) utility: $v(p_1, w) - v(p_0, w)$
- Challenge: Utility is ordinal, not cardinal
- Solution: Use money-metric utility
  - i.e. expenditure function $e(\bar{p}, v(p, w))$
  - Valid representation of $v(p, w)$ because $e(p, u)$ strictly incr. in u

Back to Basics: Welfare Effect of Price Change

- (Indirect) utility: $v(p_1, w) - v(p_0, w)$
- Challenge: Utility is ordinal, not cardinal
- Solution: Use money-metric utility
  - i.e. expenditure function $e(\bar{p}, v(p, w))$
  - Valid representation of $v(p, w)$ because $e(p, u)$ strictly incr. in u
- New Challenge: $\bar{p} = p_1$ or $\bar{p} = p_0$?

Compensating Variation

- *Compensate* at new prices
- Define $u_t = v(p_t, w)$ for $t \in \{0, 1\}$

$$CV = e(p_1, u_1) - e(p_1, u_0) \tag{1}$$

(2)

(3)

(4)

Compensating Variation

- *Compensate* at new prices
- Define $u_t = v(p_t, w)$ for $t \in \{0, 1\}$

$$
\begin{aligned}
CV &= e(p_1, u_1) - e(p_1, u_0) && (1) \\
&= w - e(p_1, u_0) && (2) \\
& && (3) \\
& && (4)
\end{aligned}
$$

Compensating Variation

- *Compensate* at new prices
- Define $u_t = v(p_t, w)$ for $t \in \{0, 1\}$

$$\begin{aligned} CV &= e(p_1, u_1) - e(p_1, u_0) && (1) \\ &= w - e(p_1, u_0) && (2) \\ &= e(p_0, u_0) - e(p_1, u_0) && (3) \\ & && (4) \end{aligned}$$

Compensating Variation

- *Compensate* at new prices
- Define $u_t = v(p_t, w)$ for $t \in \{0, 1\}$

$$
\begin{aligned}
CV &= e(p_1, u_1) - e(p_1, u_0) && (1)\\
&= w - e(p_1, u_0) && (2)\\
&= e(p_0, u_0) - e(p_1, u_0) && (3)\\
&= \int_{p_1}^{p_0} h(p, u_0) dp && (4)
\end{aligned}
$$

where the last line follows Shephard's Lemma

Compensating Variation

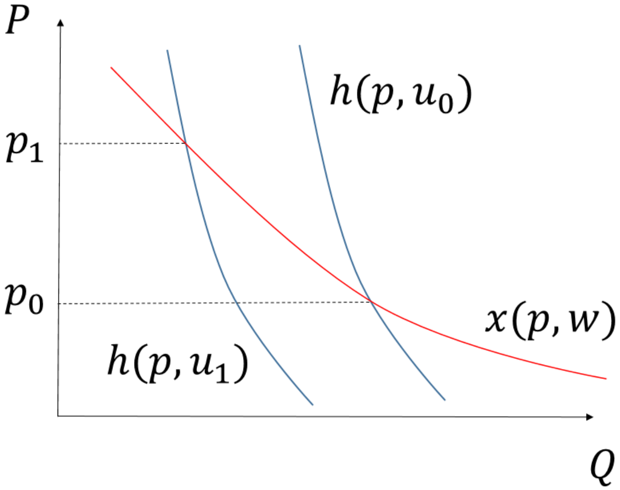
- *Compensate* at new prices
- Define $u_t = v(p_t, w)$ for $t \in \{0, 1\}$

$$
\begin{aligned}
CV &= e(p_1, u_1) - e(p_1, u_0) && (1)\\
&= w - e(p_1, u_0) && (2)\\
&= e(p_0, u_0) - e(p_1, u_0) && (3)\\
&= \int_{p_1}^{p_0} h(p, u_0) dp && (4)
\end{aligned}
$$

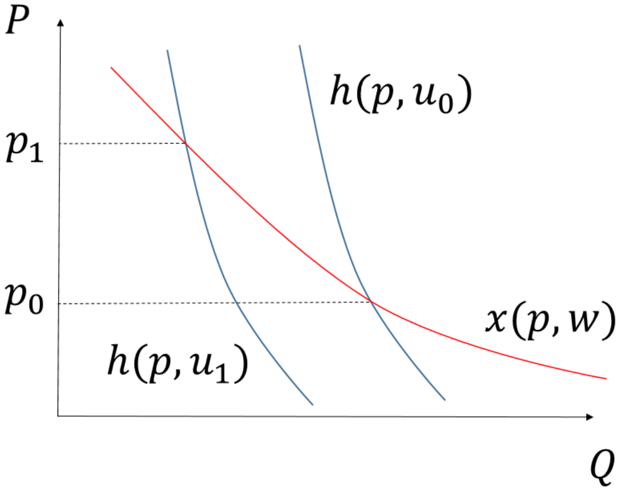
where the last line follows Shephard's Lemma

- Analogous EV is the transfer to get *equivalent* utility at old prices

Visualizing Compensated Demands

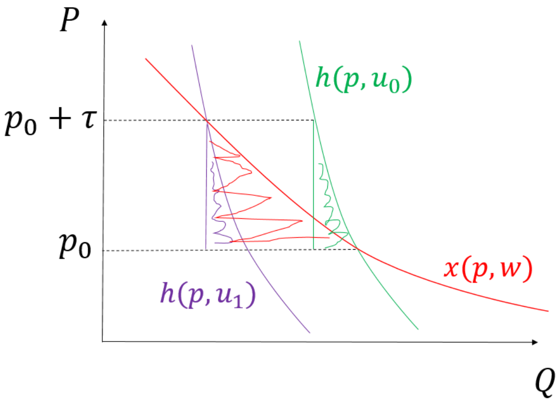


Visualizing Compensated Demands

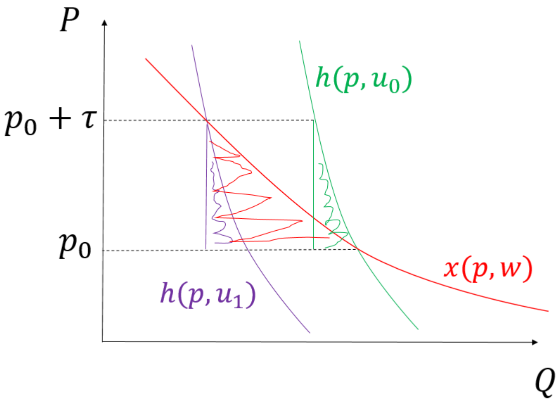


Aside: When is uncompensated demand flatter than compensated, as depicted above?

Visualizing Compensated vs. Uncompensated DWL

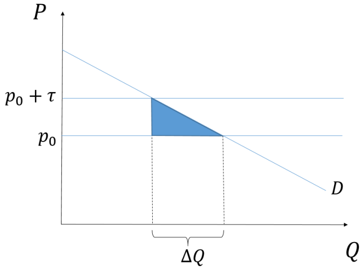


Visualizing Compensated vs. Uncompensated DWL

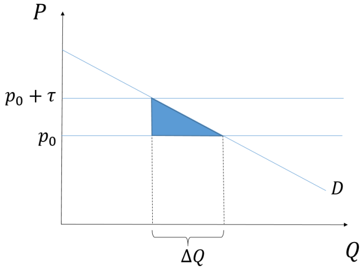


- Substitution effects (not income effects) matter for DWL
- Efficiency lost from forgone transactions due to relative price Δ, not income Δ
- Most applied papers assume away income effects. When is this more reasonable?

Partial Equilibrium DWL: New tax τ w/ pre-tax prices p_0 fixed

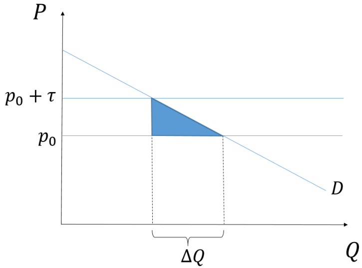


Partial Equilibrium DWL: New tax τ w/ pre-tax prices p_0 fixed



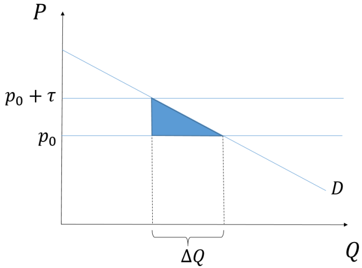
- $DWL = CV(p_0 + \tau, p_0, u_0) - \underbrace{R(p_0 + \tau, p_0, u_0)}_{\text{Compensated rev.} = \sum_j \tau_j h_j(p_0 + \tau, u_0)}$

Partial Equilibrium DWL: New tax τ w/ pre-tax prices p_0 fixed



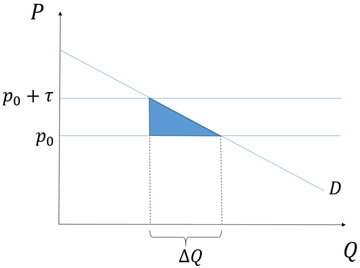
- $DWL = CV(p_0 + \tau, p_0, u_0) - \underbrace{R(p_0 + \tau, p_0, u_0)}_{\text{Compensated rev.}=\sum_j \tau_j h_j(p_0+\tau, u_0)}$
- Harberger approximation shown above: $DWL = \frac{1}{2}\Delta Q \Delta P$

Partial Equilibrium DWL: New tax τ w/ pre-tax prices p_0 fixed



- $DWL = CV(p_0 + \tau, p_0, u_0) - \underbrace{R(p_0 + \tau, p_0, u_0)}_{\text{Compensated rev.}=\sum_j \tau_j h_j(p_0+\tau, u_0)}$
- Harberger approximation shown above: $DWL = \frac{1}{2}\Delta Q \Delta P$
- Fixed pre-tax price $\Delta P = \tau$, new tax $\tau = \Delta\tau$, $\eta^D \equiv \frac{\Delta Q^D}{\Delta P}\frac{P}{Q^D} \Rightarrow \Delta Q^D = \tau \eta^D \frac{Q^D}{P}$

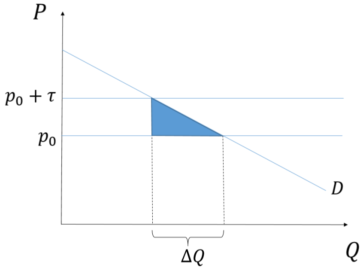
Partial Equilibrium DWL: New tax τ w/ pre-tax prices p_0 fixed



- $DWL = CV(p_0+\tau, p_0, u_0) - \underbrace{R(p_0+\tau, p_0, u_0)}_{\text{Compensated rev.}=\sum_j \tau_j h_j(p_0+\tau, u_0)}$
- Harberger approximation shown above: $DWL = \frac{1}{2}\Delta Q \Delta P$
- Fixed pre-tax price $\Delta P = \tau$, new tax $\tau = \Delta\tau$, $\eta^D \equiv \frac{\Delta Q^D}{\Delta P}\frac{P}{Q^D} \Rightarrow \Delta Q^D = \tau\eta^D \frac{Q^D}{P}$

$$\Rightarrow DWL = \frac{1}{2}\tau^2\eta^D\frac{Q^D}{P}$$

Partial Equilibrium DWL: New tax τ w/ pre-tax prices p_0 fixed



- $DWL = CV(p_0+\tau, p_0, u_0) - \underbrace{R(p_0+\tau, p_0, u_0)}_{\text{Compensated rev.}=\sum_j \tau_j h_j(p_0+\tau, u_0)}$
- Harberger approximation shown above: $DWL = \frac{1}{2}\Delta Q \Delta P$
- Fixed pre-tax price $\Delta P = \tau$, new tax $\tau = \Delta\tau$, $\eta^D \equiv \frac{\Delta Q^D}{\Delta P}\frac{P}{Q^D} \Rightarrow \Delta Q^D = \tau\eta^D \frac{Q^D}{P}$

$$\Rightarrow DWL = \frac{1}{2}\tau^2\eta^D\frac{Q^D}{P}$$

How could we think about the MVPF of this new tax?

(More) General DWL

What if pre-tax prices can adjust?

(More) General DWL

What if pre-tax prices can adjust?

- $D(p + \tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change

(More) General DWL

What if pre-tax prices can adjust?

- $D(p+\tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change
    - Differentiate equality to get $\frac{\partial p}{\partial \tau}$ and rewrite in terms of elasticities
      $\Rightarrow \frac{\partial p}{\partial \tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D}$ where $\eta_D < 0 < \eta_S$

(More) General DWL

What if pre-tax prices can adjust?

- $D(p+\tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change
  - Differentiate equality to get $\frac{\partial p}{\partial \tau}$ and rewrite in terms of elasticities

    $\Rightarrow \frac{\partial p}{\partial \tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D}$ where $\eta_D < 0 < \eta_S$
- Need to translate producer incidence into equil. Q response w/ S curve

(More) General DWL

What if pre-tax prices can adjust?

- $D(p+\tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change
  - Differentiate equality to get $\frac{\partial p}{\partial \tau}$ and rewrite in terms of elasticities
    $\Rightarrow \frac{\partial p}{\partial \tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D}$ where $\eta_D < 0 < \eta_S$
- Need to translate producer incidence into equil. Q response w/ S curve
  - Chain rule: $\frac{\partial Q^S}{\partial \tau} = \frac{\partial Q^S}{\partial p} \frac{\partial p}{\partial \tau}$

(More) General DWL

What if pre-tax prices can adjust?

- $D(p+\tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change
  - Differentiate equality to get $\frac{\partial p}{\partial \tau}$ and rewrite in terms of elasticities
    $\Rightarrow \frac{\partial p}{\partial \tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D}$ where $\eta_D < 0 < \eta_S$
- Need to translate producer incidence into equil. Q response w/ S curve
  - Chain rule: $\frac{\partial Q^S}{\partial \tau} = \frac{\partial Q^S}{\partial p}\frac{\partial p}{\partial \tau}$
- Substitute into DWL calculation from before
  - Recall $DWL = \frac{1}{2}\Delta P \Delta Q$
  - For simplicity suppose there was no pre-existing tax

(More) General DWL

What if pre-tax prices can adjust?

- $D(p+\tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change
  - Differentiate equality to get $\frac{\partial p}{\partial \tau}$ and rewrite in terms of elasticities
    $\Rightarrow \frac{\partial p}{\partial \tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D}$ where $\eta_D < 0 < \eta_S$
- Need to translate producer incidence into equil. Q response w/ S curve
  - Chain rule: $\frac{\partial Q^S}{\partial \tau} = \frac{\partial Q^S}{\partial p}\frac{\partial p}{\partial \tau}$
- Substitute into DWL calculation from before
  - Recall $DWL = \frac{1}{2}\Delta P \Delta Q$
  - For simplicity suppose there was no pre-existing tax

$$\Rightarrow DWL = \frac{1}{2}\tau^2 \frac{\eta_D \eta_S}{\eta_S - \eta_D}\frac{Q}{P}$$

(More) General DWL

What if pre-tax prices can adjust?

- $D(p + \tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change
  - Differentiate equality to get $\frac{\partial p}{\partial \tau}$ and rewrite in terms of elasticities
    $\Rightarrow \frac{\partial p}{\partial \tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D}$ where $\eta_D < 0 < \eta_S$
- Need to translate producer incidence into equil. Q response w/ S curve
  - Chain rule: $\frac{\partial Q^S}{\partial \tau} = \frac{\partial Q^S}{\partial p} \frac{\partial p}{\partial \tau}$
- Substitute into DWL calculation from before
  - Recall $DWL = \frac{1}{2}\Delta P \Delta Q$
  - For simplicity suppose there was no pre-existing tax

$$\Rightarrow DWL = \frac{1}{2}\tau^2 \frac{\eta_D \eta_S}{\eta_S - \eta_D} \frac{Q}{P}$$

(More) General DWL

What if pre-tax prices can adjust?

- $D(p+\tau) = S(p)$
- Need to calculate producer incidence $\frac{\partial p}{\partial \tau}$ for price change
  - Differentiate equality to get $\frac{\partial p}{\partial \tau}$ and rewrite in terms of elasticities

    $\Rightarrow \frac{\partial p}{\partial \tau} = \frac{\eta_D}{\frac{p+\tau}{p}\eta_S - \eta_D}$ where $\eta_D < 0 < \eta_S$
- Need to translate producer incidence into equil. Q response w/ S curve
  - Chain rule: $\frac{\partial Q^S}{\partial \tau} = \frac{\partial Q^S}{\partial p}\frac{\partial p}{\partial \tau}$
- Substitute into DWL calculation from before
  - Recall $DWL = \frac{1}{2}\Delta P \Delta Q$
  - For simplicity suppose there was no pre-existing tax

$$\Rightarrow DWL = \frac{1}{2}\tau^2 \frac{\eta_D \eta_S}{\eta_S - \eta_D}\frac{Q}{P}$$

How does your previous answer change about thinking of the tax change's MVPF?

(Even More) General DWL

What if there's already an existing tax?

(Even More) General DWL

What if there's already an existing tax?

- Recall $DWL(\tau) = \underbrace{[e(p+\tau,u) - e(p,u)]}_{\text{CV}} - \underbrace{\tau h(p+\tau,u)}_{\text{Tax Revenue}}$

(Even More) General DWL

What if there's already an existing tax?

- Recall $DWL(\tau) = \underbrace{[e(p+\tau, u) - e(p,u)]}_{\text{CV}} - \underbrace{\tau h(p+\tau, u)}_{\text{Tax Revenue}}$
- Take 2nd order Taylor expansion around $DWL(\tau)$ to get *marginal* DWL:

$$MDWL(\tau) = \frac{\partial DWL(\tau)}{\partial \tau}\Delta\tau + \frac{1}{2}\frac{\partial^2 DWL(\tau)}{\partial \tau^2}\tau^2$$

(Even More) General DWL

What if there's already an existing tax?

- Recall $DWL(\tau) = \underbrace{[e(p+\tau,u) - e(p,u)]}_{\text{CV}} - \underbrace{\tau h(p+\tau,u)}_{\text{Tax Revenue}}$
- Take 2nd order Taylor expansion around $DWL(\tau)$ to get *marginal* DWL:

$$MDWL(\tau) = \frac{\partial DWL(\tau)}{\partial \tau}\Delta\tau + \frac{1}{2}\frac{\partial^2 DWL(\tau)}{\partial \tau^2}\tau^2$$

Note $\frac{\partial DWL(\tau)}{\partial \tau} = \underbrace{h - h}_{\text{=0 by env. thm.}} - \tau\frac{\partial h}{\partial \tau} = -\tau\frac{\partial h}{\partial \tau}$ and $\frac{\partial^2 DWL(\tau)}{\partial \tau^2} = -\frac{\partial h}{\partial \tau} - \tau\underbrace{\frac{\partial^2 h}{\partial \tau^2}}_{\text{assume =0 for simplicity}}$

(Even More) General DWL

What if there's already an existing tax?

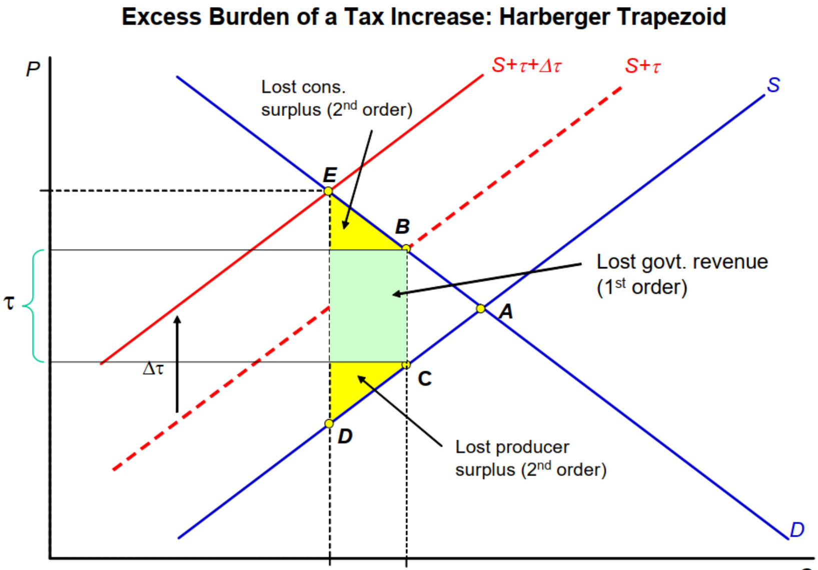
- Recall $DWL(\tau) = \underbrace{[e(p+\tau,u) - e(p,u)]}_{\text{CV}} - \underbrace{\tau h(p+\tau,u)}_{\text{Tax Revenue}}$
- Take 2nd order Taylor expansion around $DWL(\tau)$ to get *marginal* DWL:

$$MDWL(\tau) = \frac{\partial DWL(\tau)}{\partial \tau}\Delta\tau + \frac{1}{2}\frac{\partial^2 DWL(\tau)}{\partial \tau^2}\tau^2$$

Note $\frac{\partial DWL(\tau)}{\partial \tau} = \underbrace{h - h}_{\text{=0 by env. thm.}} - \tau\frac{\partial h}{\partial \tau} = -\tau\frac{\partial h}{\partial \tau}$ and $\frac{\partial^2 DWL(\tau)}{\partial \tau^2} = -\frac{\partial h}{\partial \tau} - \tau\underbrace{\frac{\partial^2 h}{\partial \tau^2}}_{\text{assume =0 for simplicity}}$

$$\Rightarrow MDWL(\tau) = \underbrace{-\tau\Delta\tau\frac{\partial h}{\partial \tau}}_{\text{New 1st order distortion}} - \underbrace{\frac{1}{2}\tau(\Delta\tau)^2\frac{\partial h}{\partial \tau}}_{\text{Standard Harberger triangle}}$$

Visualizing the Harberger Triangle (Courtesy of Chetty)



(More-ish) General DWL

How does this interact with other markets?

(More-ish) General DWL

How does this interact with other markets?

- Analogous to the analysis when there are existing taxes
- Behavioral response has 1^{st} order welfare effect if there's a pre-existing distortion

(More-ish) General DWL

How does this interact with other markets?

- Analogous to the analysis when there are existing taxes
- Behavioral response has 1st order welfare effect if there's a pre-existing distortion
- Tax τ_i in market i

$$\Rightarrow DWL = \frac{1}{2}\tau_i dh_i + \sum_{j \neq i} \tau_i \tau_j \frac{\mathrm{d}h_j}{\mathrm{d}\tau_i}$$

(More-ish) General DWL

How does this interact with other markets?

- Analogous to the analysis when there are existing taxes
- Behavioral response has 1st order welfare effect if there's a pre-existing distortion
- Tax τ_i in market i

$$\Rightarrow DWL = \frac{1}{2}\tau_i dh_i + \sum_{j \neq i} \tau_i \tau_j \frac{\mathrm{d}h_j}{\mathrm{d}\tau_i}$$

- Atkinson-Stiglitz Theorem says uniform commodity taxation is optimal under certain conditions
- The above formula foreshadows a "Law of the 2nd Best" application that you might want to subsidize/tax goods that have spillover effects on already distorted markets

Outline

The 16th Amendment was Ratified in 1913...Get With the Times!

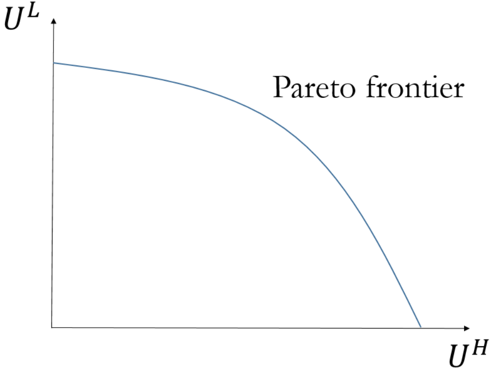
- (Income taxation)
- Potentially more natural to redistribute using this
- Focus on two type case (Stiglitz 1982) to highlight intuition
    1. Incentive compatibility
    2. Impossibility of Laffer effects at optimum

Mechanism Design Approach to Income Taxation: Stiglitz (1982) Two Types

- **Worker preferences**: $U^i(c, Y) = U(c, Y; \theta^i)$
  - consumption c, heterogeneous productivity θ^i, labor $l = \frac{Y}{\theta^i}$
  - $\theta^H > \theta^L$
- **Aggregate resource constraint**: $\sum_i c(\theta^i) \leq \sum_i Y(\theta^i)$
- **Work budget constraint**: $B = \{(c, Y) | c \leq Y - T(Y)\}$
  - Potentially nonlinear income tax schedule $T(Y)$
- **Objective**: Redistribution across types θ^i
- **Challenge**: θ^i unobserved

What If Worker Type IS Observed?

- With perfect observability, can use type-specific policy (i.e. lump-sum transfers)
- Only constraint is the aggregate resource constraint



But alas...

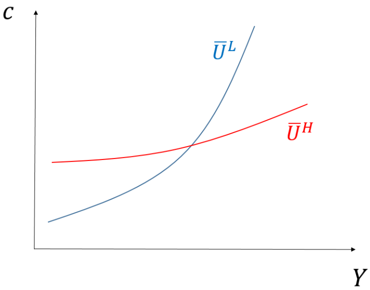
I'm Going to Make Him an Offer He Can't Refuse

- **Key idea**: Unobservability of type $\Rightarrow$ any type-specific policy will have to get each type to *willingly* reveal themselves
  - Lets you consider allocations as function of θ
  - Related to *revelation principle* from mechanism design
- **Incentive compatibility (IC) constraint**: $u(c(\theta), Y(\theta); \theta) \geq u(c(\tilde{\theta}), Y(\tilde{\theta}); \theta) \;\; \forall \theta, \tilde{\theta}$
- **Worker FOC**:

$$MRS(c, Y; \theta) \equiv -\frac{U_Y(c,Y;\theta)}{U_c(c,Y;\theta)} = 1 - T'(Y)$$

Visualizing MRS

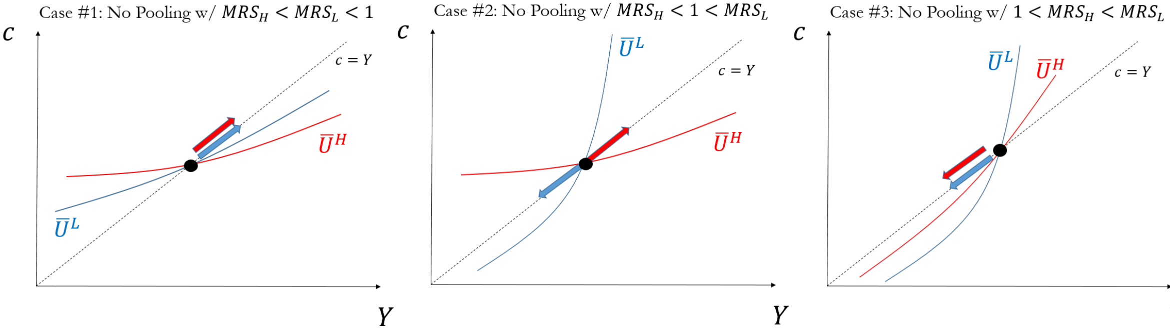
- Draw indifference curves for each type
  - Y implicitly defines labor
  - c, Y together implicitly define $\underbrace{T(Y)}_{\text{by budget/resource constraint}}$ and $\underbrace{T'(Y)}_{\text{by worker FOC}}$



Model can be generalized, but key is $MRS(c, Y; \theta)$ decreasing in θ (*single crossing*)

Non-Type Specific Policy Cannot Be Along Pareto Frontier

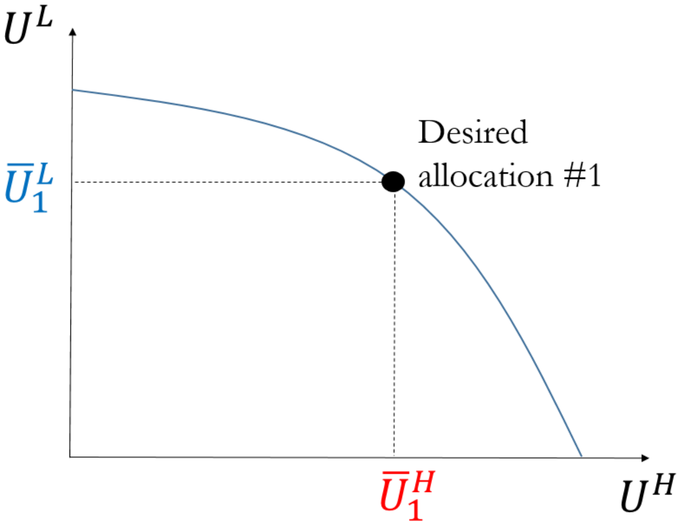
- Pooling equilibrium represented by both allocation at same point along $c = Y$
- **Key idea**: Check feasible Pareto improvements along $c = Y$



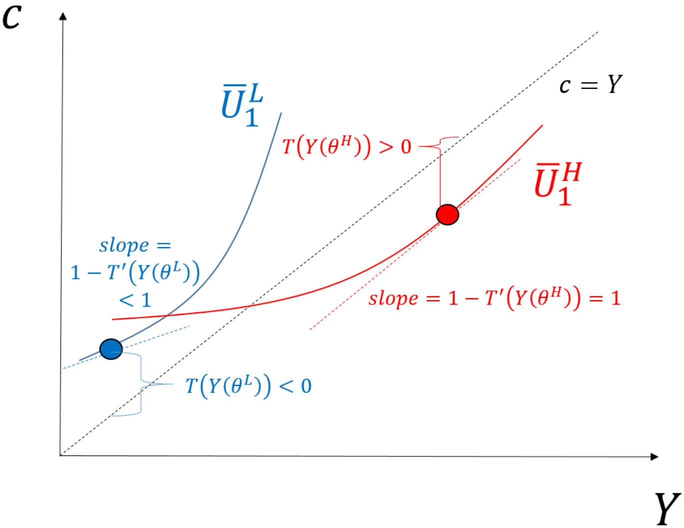
Above are graphs showing efficiency $\Rightarrow MRS(c, Y; \theta) = 1 - T'(Y)$ but $\theta^H > \theta^L$

Counterintuitive corollary: $T'(Y(\theta^H)) > 0$ is not optimal because of Laffer effects

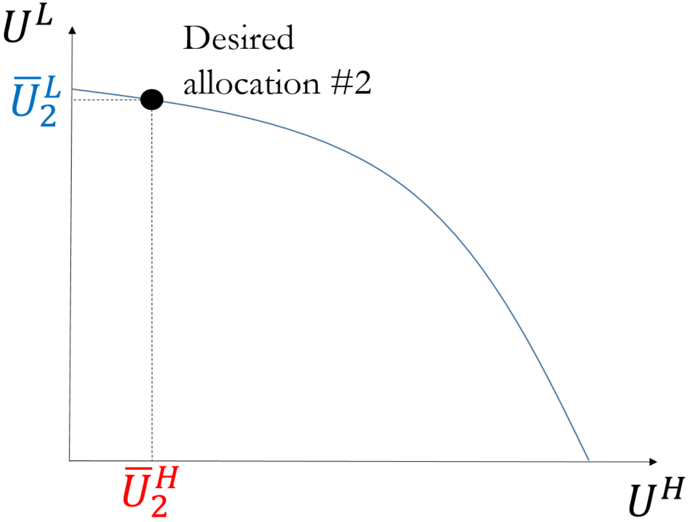
So What Can Be Pareto Optimal?



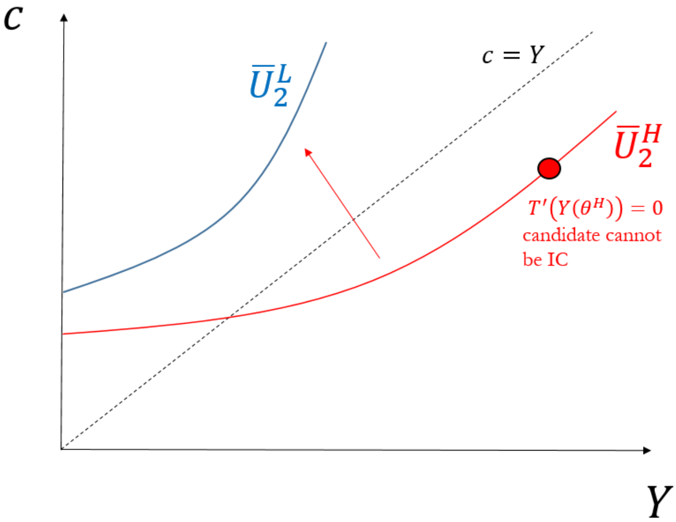
...Good to Go if IC's are Satisfied



Can Anything Be Pareto Optimal?

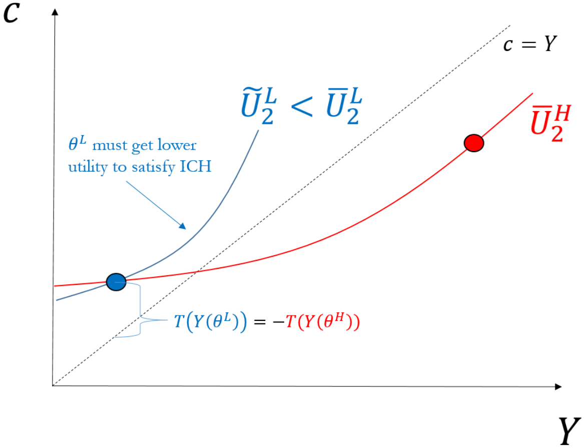


...Not If IC's Can't be Satisfied

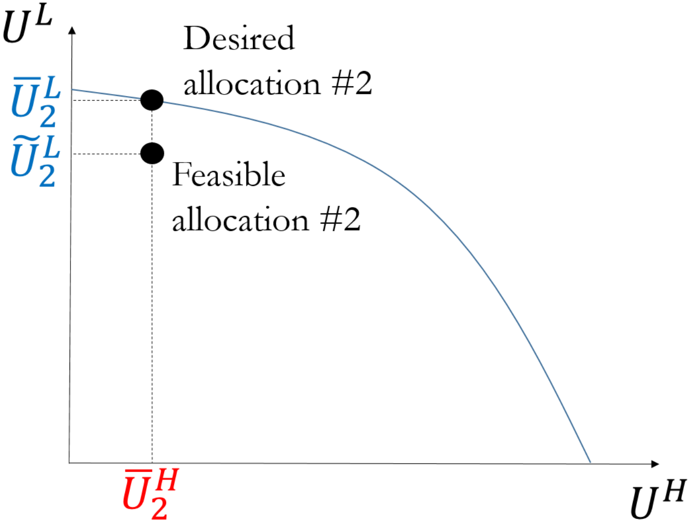


Adjustment Necessary for Feasibility (with Unobservability)

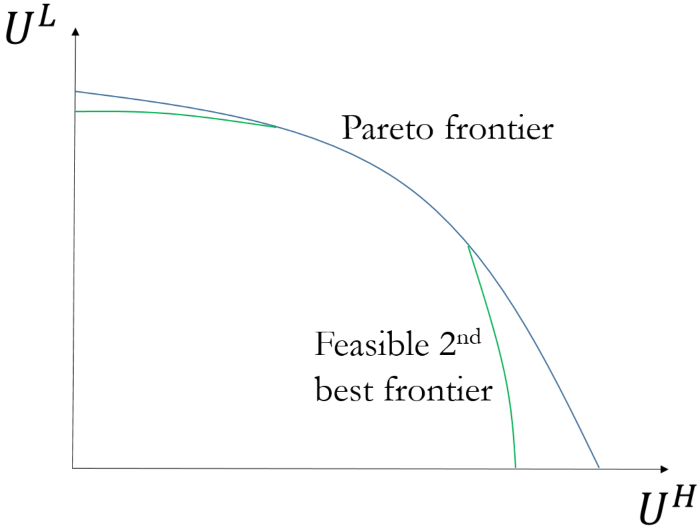
- Previous slide demonstrates ICH (incentive compatibility constraint for θ^H binds)
- **2nd best allocation**: Maximal $\bar{U}^L$ along $\bar{U}_2^H$ s.t. resource constraint holds



IC Constraint Prevents Reaching Pareto Frontier



More General 2nd Best Frontier (Due to Unobservability)



Super Aside: Alternative Variational Approach to Optimal Income Taxation

- Consider marginal increase in $T'(Y_0)$ at given Y_0
  1. **Mechanical Effect**: Increase $T(Y)\ \ \forall Y \geq Y_0$
     - Avg welfare weight $\bar{\lambda} \equiv E[\lambda(Y)|Y \geq \bar{Y}]$
     - Raising \$1 revenue good but comes at welfare cost of $\bar{\lambda}$
  2. **Behavioral Effect**: People previously with $Y \geq Y_0$ adjust earnings down
     - Fiscal externality with no 1^{st} order private welfare effect (*envelope theorem!*)
     - Sum of income and substitution effects (*Inverse elasticity rule!*)

Optimum equates effects $\Rightarrow (1 - \lambda(Y_0))M + B = 0$

Super Aside: Alternative Variational Approach to Optimal Income Taxation

- Consider marginal increase in $T'(Y_0)$ at given Y_0
  1. **Mechanical Effect**: Increase $T(Y) \ \forall Y \geq Y_0$
     - Avg welfare weight $\bar{\lambda} \equiv E[\lambda(Y)|Y \geq \bar{Y}]$
     - Raising $1 revenue good but comes at welfare cost of $\bar{\lambda}$
  2. **Behavioral Effect**: People previously with $Y \geq Y_0$ adjust earnings down
     - Fiscal externality with no 1st order private welfare effect (*envelope theorem!*)
     - Sum of income and substitution effects (*Inverse elasticity rule!*)

Optimum equates effects $\Rightarrow (1 - \lambda(Y_0))M + B = 0$

How can we see the MVPF in the above formula?

Takeaways from Mechanism Design Approach

- Desire to redistribute to one type is constrained by an incentive compatibility condition on the other
- It would be really great if we could relax that...

Outline

What should commodity taxes be?

- You have nonlinear income taxes. Should you *differentially* tax commodities, too?
  - If it relaxes information constraints, then yes!
  - If it doesn't, then you can generate a Pareto improvement by removing the commodity tax distortion and compensating through the income tax

What should commodity taxes be?

- You have nonlinear income taxes. Should you *differentially* tax commodities, too?
  - If it relaxes information constraints, then yes!
  - If it doesn't, then you can generate a Pareto improvement by removing the commodity tax distortion and compensating through the income tax
- What is the formal condition determining whether consumption choices reveal information?
  - **Preference restriction**: Weak separability of all consumption choices with respect to labor supply
  - **Key PF result**: Atkinson-Stiglitz

What should commodity taxes be?

- You have nonlinear income taxes. Should you *differentially* tax commodities, too?
  - If it relaxes information constraints, then yes!
  - If it doesn't, then you can generate a Pareto improvement by removing the commodity tax distortion and compensating through the income tax
- What is the formal condition determining whether consumption choices reveal information?
  - **Preference restriction**: Weak separability of all consumption choices with respect to labor supply
  - **Key PF result**: Atkinson-Stiglitz
- If there's a revenue requirement, is the DWL analysis from before relevant?
  - Uniform taxation on *all goods* is like a lump-sum tax!

Weak Separability

- *(Semi-)formal definition*: $u(x_1, x_2, ..., x_n) = u(x_1, G(x_2, ..., x_n)) \Rightarrow u(\cdot)$ weakly separable between x_1 and $(x_2, ..., x_n)$
- *Intuition*: Two-stage budgeting (weak separability is necessary and sufficient for this)
  - First decide upper-level (i.e. \$ towards x_1 vs. \$ towards $(x_1, ..., x_n)$)
  - Next decide lower-level (i.e. \$ towards x_2 vs. x_3 vs. x_4 and so on)
  - MRS between goods in subutility $G(\cdot)$ unaffected by level of x_1

Super Aside: How Can You Test For Weak Separability

- Goldman and Uzawa (1964) derived that weak separability $\Rightarrow$ Slutky substitution terms $\propto$ income effects
- Afriat (1970) and Varian (1983) develop non-parametric tests
  - *Very limited intuition*: Similar to GARP tests about whether choice data can be rationalized with certain preferences

Weak Separability as Required by Atkinson-Stiglitz

- Consider $u^i(x_1, ..., x_n, l)$ over goods $(x_1, ..., x_n)$ and labor l
- Require $u^i(x_1, ..., x_n, l) = u^i(G(x_1, ..., x_n), l)$
  - Allow heterogeneity in consumption vs. leisure decisions, but not in MRS's between common $G(\cdot)$ with respect to labor

Proof Sketch of Uniform Commodity Taxation Under WS

By contradiction (see Kaplow 2006 for details)

1. Remove differential commodity taxes and adjust (arbitrary) nonlinear income tax so that indirect utility is constant for everyone
2. By weak separability, labor is also constant for everyone
3. Show old consumption bundle now isn't affordable
4. Therefore government gain revenue leaving everyone indifferent
5. PROFIT!

Atkinson-Stiglitz Intuition

- Weak separability implies, conditional on income, relative consumption decisions reveal no information about type
- Therefore *differential* taxation doesn't relax IC constraints but does introduce distortions

How Atkinson-Stiglitz Fails

- Just because a benchmark is useful doesn't mean it's always true
- Intuitive violations of weak separability (Saez 2002):
  1. Conditional on income, owning a yacht reveals info about hidden assets (i.e. income Y) or that you're an insufferable person (i.e. welfare weight λ)
     $\Rightarrow$ tax it!
  2. Child care is a complement to labor and thus, conditional on income, reveals info on unobserved productivity θ
     $\Rightarrow$ subsidize it!
     - Intuition related to multi-market DWL from slide 12 (Corlett and Hague 1953)
- More generally, any induced relaxation of information constraints is efficient
- How is this related to the MVPF?

Fitting in-kind provision into each MVPF term

$$MVPF = \bar{\eta}\frac{\text{WTP}}{1 + FE}$$

Relation to Mysterious Nichols and Zeckhauser Figure

- A is “advantaged" and B is “broke"
- Plot indifference curve of both A and B w.r.t consumption of indicator good
  - Use **residual** income to consume “everything else"
- Different slopes at same level of indicator good (i.e. MRS heterogeneity) is a violation of weak separability
  - Consumption choice reveals info on type
  - Distorting consumption relaxes info constraint
  - Redistribution with the indicator good can improve on redistribution through cash tax and transfers alone

Nichols and Zeckhauser Figure!