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# REPUTATIONAL BARGAINING WITH MINIMAL KNOWLEDGE OF RATIONALITY

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Two players announce bargaining postures to which they may become committed and then bargain over the division of a surplus. The share of the surplus that a player can guarantee herself under first-order knowledge of rationality is determined (as a function of her probability of becoming committed), as is the bargaining posture that she must announce in order to guarantee herself this much. This "maxmin" share of the surplus is large relative to the probability of becoming committed (e.g., it equals 30% if the commitment probability is 1 in 10 and equals 13% if the commitment probability is 1 in 1000), and the corresponding bargaining posture simply demands this share plus compensation for any delay in reaching agreement.

KEYWORDS: Bargaining, knowledge of rationality, posturing, reputation.

## 1. INTRODUCTION

ECONOMISTS HAVE LONG BEEN INTERESTED in how individuals split gains from trade. Recently, "reputational" models of bargaining have been developed that make sharp predictions about the division of surplus independently of many details of the bargaining procedure (Myerson (1991), Abreu and Gul (2000), Kambe (1999), Compte and Jehiel (2002), Abreu and Pearce (2007)). In these models, players may be committed to a range of possible bargaining strategies, or "postures," before the start of bargaining, and bargaining consists of each player attempting to convince her opponent that she is committed to a strong posture. These models assume that the probabilities with which the players are committed to various bargaining postures (either ex ante or after a stage where players strategically announce bargaining postures) are common knowledge and that play constitutes a (sequential) equilibrium. In this paper, I study reputational bargaining while assuming only that the players know that each other is rational (so that, in particular, players do not know each other's beliefs or strategies). I show that each player can guarantee herself a share of the surplus that is large relative to her probability of being committed by announcing the posture that simply demands this share plus compensation for any delay in reaching agreement. Furthermore, announcing any other posture does not guarantee her as much.

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More precisely, I assume that there is a positive number $\varepsilon$ such that if a player announces any bargaining posture (i.e., any infinite path of demands) at the beginning of the game, she then becomes committed to that posture with probability at least $\varepsilon$ (or, equivalently, she convinces her opponent that she is committed to that posture with probability at least $\varepsilon$). Player 1's "maxmin" payoff is then the highest payoff $u_1$ with the property that there exists a corresponding posture (the "maxmin posture") and bargaining strategy such that player 1 receives at least $u_1$ whenever she announces this posture and follows this strategy, and player 2 plays *any* best response to *any* belief about player 1's strategy that assigns probability at least $\varepsilon$ to player 1 following her announced posture.

The main result of this paper characterizes the maxmin payoff and posture when only one player may become committed to her announced posture; as discussed below, a very similar characterization applies when both players may become committed. The maxmin payoff equals $1/(1-\log\varepsilon)$. This equals 1 when $\varepsilon=1$ (i.e., when the player makes a take-it-or-leave-it offer) and goes to 0 very slowly as $\varepsilon$ goes to 0. For example, a bargainer can guarantee herself approximately 30% of the surplus if her commitment probability is 1 in 10, 13% if it is 1 in 1000, and 7% if it is 1 in 1 million. In addition, the unique bargaining posture that guarantees this share of the surplus simply demands this share in addition to compensation for any delay; that is, the demand increases at a rate equal to the common discount rate, $r$.[2] This compensation amounts to the entire surplus after a long enough delay, so the unique maxmin posture demands

$$\min\{e^{rt}/(1-\log\varepsilon), 1\}$$

at every time $t$. This posture is depicted in Figure 1 for commitment probability $\varepsilon=1/1000$ and discount rate $r=1$.

The intuition for why the maxmin payoff is large relative to $\varepsilon$ is that when player 1's demand is small, player 2 must accept unless he believes that he will be quickly rewarded for rejecting. In the latter case, if player 1 does not reward player 2 for rejecting, then player 2 quickly updates his belief toward player 1's being committed to her announced posture (i.e., player 1 builds reputation at a high rate), and player 2 accepts when he becomes convinced that she is committed.[3] Hence, player 1 is able to compensate for having a small commitment

[2]To my knowledge, this is the first bargaining model that predicts that such a posture will be adopted, though it seems like a reasonable bargaining position to stake out. For example, in most U.S. states, defendants must pay "prejudgment interest" on damages in tort cases, which amounts to plaintiffs demanding the initial damages in addition to compensation for any delay (e.g., Knoll (1996)); similarly, unions sometimes include payment for strike days among their demands.

[3]This is related to the argument in the existing reputational bargaining literature that player 1 builds reputation more quickly *in equilibrium* when her demand is small, though my analysis is not based on equilibrium.

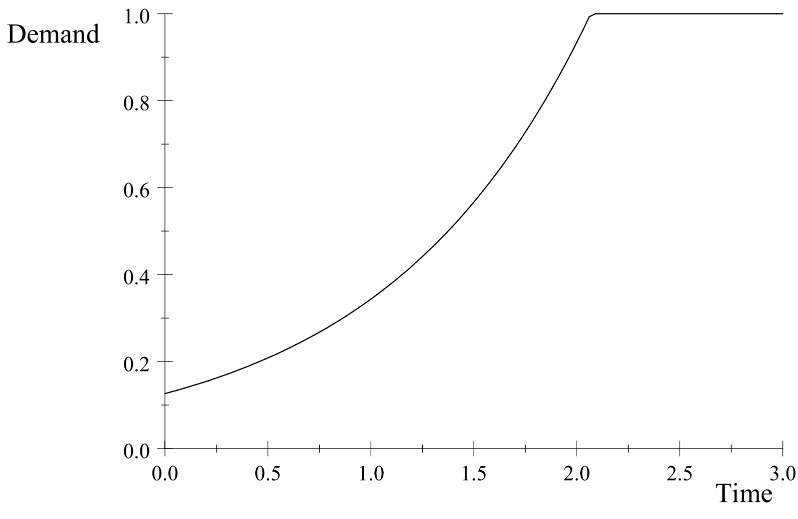


FIGURE 1.—The unique maxmin bargaining posture for $\varepsilon = 1/1000$ and $r = 1$.

probability by reducing her demand and thereby increasing the rate at which she builds reputation. This exponentially reduces the cost of the delay before her demand is accepted and thus guarantees her a relatively large payoff.

The intuition for why the unique maxmin posture demands compensation for delay involves two key ideas. First, as I have argued, player 1's demand is accepted sooner when it is lower (when player 2's beliefs are those that lead him to reject for as long as possible). Second, the maxmin posture can never make demands that would give player 1 less than her maxmin payoff if they were accepted, because otherwise player 2 could simply accept some such demand and give player 1 a payoff below her maxmin payoff, which was supposed to be guaranteed to player 1 (though it must be verified that such behavior by player 2 is rational). Combining these ideas implies that player 1 must always demand at least her maxmin level of *utility* (hence, compensation for delay), but no more.

I also characterize the maxmin payoffs and postures when both players may become committed to their announced postures. Each player's maxmin posture is exactly the same as in the one-sided commitment model, and each player's maxmin payoff is close to her maxmin payoff in the one-sided commitment model as long as her opponent's commitment probability is small. Thus, the one-sided commitment analysis applies to each player separately.

The paper most closely related to mine is Kambe (1999), which endogenizes the "behavioral types" of Abreu and Gul (2000) by having players strategically announce postures to which they may become committed (as in my model).[4,5] There are two differences between Kambe's model and mine. First, Kambe required that players announce postures that demand a constant share of the surplus (as did Abreu and Gul), while I allow players to announce nonconstant postures (and players do benefit from announcing nonconstant postures in my model). Second, and more fundamentally, Kambe studied sequential equilibria (as does the rest of the existing reputational bargaining literature), while I study maxmin payoffs and postures. My approach entails weaker assumptions on knowledge of commitment probabilities (i.e., second-order knowledge that commitment probabilities are at least $\varepsilon$, rather than common knowledge of exact commitment probabilities) and on behavior (i.e., first-order knowledge of rationality, rather than sequential equilibrium), but does not yield unique predictions about the division of surplus or about the details of how bargaining will proceed. One motivation for this complementary approach is that behavioral types are sometimes viewed as "perturbations" reflecting the fact that a player (or an outside observer) cannot be sure that the model captures all of the other player's strategic considerations, and assuming that the distribution over perturbations is common knowledge goes against the spirit of introducing perturbations.

This paper is related more broadly to the literature on commitment tactics in bargaining dating back to Schelling (1956), who discussed observable factors that make announced postures more credible, corresponding to a higher value of $\varepsilon$ in my model.[6] It is also related to the literature on bargaining with incomplete information either without common priors (Yildiz (2003, 2004), Feinberg and Skrzypacz (2005)) or with rationalizability rather than equilibrium (Cho (1994), Watson (1998)), in that players may disagree about the distribution over outcomes of bargaining. Finally, this paper weakens the solution concept from equilibrium to knowledge of rationality in reputational bargaining models in the same way that Watson (1993) and Battigalli and Watson (1997) weakened the solution concept from equilibrium to knowledge of rationality in Fudenberg and Levine's (1989) model of reputation in repeated games. They found that Fudenberg and Levine's equilibrium predictions also apply under knowledge of rationality, whereas my predictions differ dramatically from the existing reputational bargaining literature. The reason for this difference is that in repeated games with one long-run player and one short-run player or

[4]Other important antecedents include Kreps and Wilson (1982) and Milgrom and Roberts (1982), who pioneered the incomplete information approach to reputation formation, and Chatterjee and Samuelson (1987, 1988), who studied somewhat simpler reputational bargaining models.

[5]It is not essential for the main points of my paper that commitment comes from strategic announcements rather than exogenous behavioral types. What matters is the posterior probability with which a player's opponent thinks she is committed after she stakes out a posture.

[6]Subsequent contributions include Crawford (1982), Fershtman and Seidmann (1993), Muthoo (1996), Compte and Jehiel (2004), and Ellingsen and Miettinen (2008).

in bargaining where one player is infinitely more patient than her opponent, the long-run or patient player receives close to her Stackelberg payoff (which in bargaining equals the entire surplus) under either knowledge of rationality or equilibrium. Thus, the predictions of Watson and Battigalli and Watson coincide with those of Fudenberg and Levine, just as my predictions coincide with those of Abreu and Gul (and others) *in the special case where one player is infinitely more patient than the other* (Section 4.2). However, the main focus of this paper is on the case of equally (or at least comparably) patient players, where predictions under equilibrium and knowledge of rationality differ for both repeated games and bargaining.

The paper proceeds as follows: Section 2 presents the model and defines maxmin payoffs and postures. Section 3 analyzes the baseline case with one-sided commitment and presents the main characterization of maxmin payoffs and postures. Section 4 presents three brief extensions. Section 5 considers two-sided commitment. Section 6 concludes. Omitted proofs are provided in the Appendix. The Supplemental Material (Wolitzky (2012)) shows that the main characterization is robust to details of the bargaining procedure such as the order and relative frequency of offers (as long as offers are frequent), as well as to strengthening the solution concept from knowledge of normal-form rationality to iterated conditional dominance.[7]

## 2. MODEL AND DEFINITION OF MAXMIN PAYOFF AND POSTURE

### 2.1. *Model*

Two players ("she," "he") bargain over one unit of surplus in two phases: a "commitment phase" followed by a "bargaining phase." I describe the bargaining phase first. It is intended to capture a continuous bargaining process where players can change their demands and accept their opponents' demands at any time, but to avoid well known technical issues that emerge when players can condition their play on the "instantaneous" actions of their opponents (Simon and Stinchcombe (1989), Bergin and MacLeod (1993)), I assume that players can revise their paths of demands only at integer times (while letting them accept their opponents' demands at any time).[8]

Time runs continuously from $t = 0$ to $\infty$. At every integer time $t \in \mathbb{N}$ (where $\mathbb{N}$ is the natural numbers), each player $i \in \{1, 2\}$ chooses a path of demands

[7]The results of Rubinstein (1982) imply that the main result is *not* robust to *simultaneously* specifying a discrete-time bargaining procedure and strengthening the solution concept to iterated conditional dominance. See the Supplemental Material for details.

[8]In this paper, working in continuous time not only allows the players more flexibility than does discrete time, but also yields simpler results. In particular, the continuous-time maxmin posture demands exact compensation for delay and thus changes over time in a simple way, while I conjecture that the discrete-time maxmin posture demands approximate compensation for delay, but that the details of the approximation are complicated and depend on the exact timing of offers.

for the next length-1 period of time, $u_i^t : [t, t+1) \to [0, 1]$, which is required to be the restriction to $[t, t+1)$ of a continuous function on $[t, t+1]$. Let $\mathcal{U}^t$ be the set of all such functions and let $\Delta(\mathcal{U}^t)$ be the space of probability measures on the Borel $\sigma$-algebra of $\mathcal{U}^t$ endowed with the product topology. The interpretation is that $u_i^t(\tau)$ is the demand that player $i$ makes at time $\tau$ (this is simply denoted by $u_i(\tau)$ when $t$ is understood; note that $u_i : \mathbb{R}_+ \to \mathbb{R}_+$ can be discontinuous at integer times, but is everywhere right-continuous with left limits). Even though player $i$'s path of demands for $[t, t+1)$ is decided at $t$, player $j$ only observes demands as they are made. Intuitively, each player $i$ may *accept* her opponent's demand $u_j(t)$ at any time $t$, which ends the game with payoffs $(e^{-rt}(1 - u_j(t)), e^{-rt}u_j(t))$, where $r > 0$ is the common discount rate (throughout, $j = -i$). Formally, every instant of time $t$ is divided into three *dates*, $(t, -1)$, $(t, 0)$, and $(t, 1)$ (except for time 0, which is divided only into dates $(0, 0)$ and $(0, 1)$), with the following timing: First, at date $(t, -1)$, each player $i$ announces *accept* or *reject*. If both players reject, the game continues; if only player $i$ accepts, the game ends with payoffs $(e^{-rt}(1 - \lim_{\tau\uparrow t} u_j(\tau)), e^{-rt} \lim_{\tau\uparrow t} u_j(\tau))$; and if both players accept, the games ends with payoffs determined by the average of the two demands, $\lim_{\tau\uparrow t} u_1(\tau)$ and $\lim_{\tau\uparrow t} u_2(\tau)$. Next, at date $(t, 0)$, both players simultaneously announce their time-$t$ demands $(u_1(t), u_2(t))$ (which were determined at the most recent integer time); if $t$ is an integer, this is also the date where each player $i$ chooses a path of demands for the next length-1 period, $u_i^t$. Finally, at date $(t, 1)$, each player $i$ again announces *accept* or *reject*. If both players reject, the game continues; if only player $i$ accepts, the game ends with payoffs $(e^{-rt}(1 - u_j(t)), e^{-rt}u_j(t))$; and if both players accept, the game ends, and the demands $u_1(t)$ and $u_2(t)$ are averaged. This timing ensures that there is a first and last date at which each player can accept each of her opponent's demands. In particular, at integer time $t$, player $i$ may accept either her opponent's "left" demand, $\lim_{\tau\uparrow t} u_j(\tau)$, or her time-$t$ demand, $u_j(t)$. I say that *agreement is reached at time $t$* if the game ends at time $t$ (i.e., at date $(t, -1)$ or $(t, 1)$). Both players receive payoff 0 if agreement is never reached.

The public history up to time $t$ excluding the time-$t$ demands is denoted by $h^{t-} = (u_1(\tau), u_2(\tau))_{\tau<t}$, and the public history up to time $t$ including the time-$t$ demands is denoted by $h^{t+} = (u_1(\tau), u_2(\tau))_{\tau\le t}$ (with the convention that this corresponds to all offers having been rejected, as otherwise the game would have ended). A generic time-$t$ history is denoted by $h^t$. Since $\lim_{\tau\uparrow t} u_i(t) = u_i(t)$ for noninteger $t$, I generally distinguish between $h^{t-}$ and $h^{t+}$ only for integer $t$. Formally, *a bargaining phase (behavior) strategy* for player $i$ is a pair $\sigma_i = (F_i, G_i)$, where $F_i$ is a map from histories $h^t$ into $[0, 1]$ with the properties that $F_i(h^t) \le F_i(h^{t'})$ whenever $h^{t'}$ is a successor of $h^t$ and $F_i(h^{t+})$ is a right-continuous function of $t$, and $G_i$ is a map from histories $h^{t-}$ with $t \in \mathbb{N}$ into $\Delta(\mathcal{U}^t)$. Let $\Sigma_i$ be the set of player $i$'s bargaining phase strategies. The interpretation is that $F_i(h^{t-})$ is the probability that player $i$ accepts player $j$'s demand at or before date $(t, -1)$, $F_i(h^{t+})$ is the probability that player $i$ accepts player $j$'s demand at or before date $(t, 1)$, and $G_i(h^{t-})$ is the probability

distribution over paths of demands $u_i^t : [t, t+1) \to [0,1]$ chosen by player $i$ at date $(t, 0)$. This formalism implies that player $i$'s *hazard rate of acceptance at history* $h^t$, $f_i(h^t)/(1 - F_i(h^t))$, is well defined at any time $t$ at which the realized distribution function $F_i$ admits a density $f_i$, and, in addition, player $i$'s *probability of acceptance at history* $h^{t+}$ (resp., $h^{t-}$), $F_i(h^{t+}) - F_i(h^{t-})$ (resp., $F_i(h^{t-}) - \lim_{\tau \uparrow t} F_i(h^\tau)$), is well defined for all times $t$. However, as long as one bears in mind these formal definitions, it suffices for the remainder of the paper to omit the notation $(F_i, G_i)$ and instead simply view a (bargaining phase) strategy $\sigma_i \in \Sigma_i$ as a function that maps every history $h^t$ to a hazard rate of acceptance, a discrete probability of acceptance, and (if $h^t = h^{t-}$ for $t \in \mathbb{N}$) a probability distribution over paths of demands $u_i^t$. A *pure bargaining phase strategy* is a strategy $\sigma_i$ such that $F_i(h^t) \in \{0, 1\}$ for all $h^t$ and $G_i(h^{t-})$ is a degenerate distribution for all $h^{t-}$.

At the beginning of the bargaining phase, player $i$ has an initial belief $\pi_i$ about the behavior of her opponent. Formally, $\pi_i \in \Delta(\Sigma_j)$, the set of finite-dimensional distributions over $\Sigma_j$, so $\pi_i$ is a finite-dimensional distribution over behavior strategies $\sigma_j$; note that $\pi_i$ can alternatively be viewed as an element of $\Sigma_j$ by reducing lotteries over behavior strategies. Let $\operatorname{supp}(\pi_i) \subseteq \Sigma_j$ be the support of $\pi_i$, let $u_i(\sigma_i, \sigma_j)$ be player $i$'s expected utility given strategy profile $(\sigma_i, \sigma_j)$, let $u_i(\sigma_i, \pi_i)$ be player $i$'s expected utility given strategy $\sigma_i$ and belief $\pi_i$, and let $\Sigma_i^*(\pi_i) \equiv \arg\max_{\sigma_i} u_i(\sigma_i, \pi_i)$ be the set of player $i$'s (normal-form) best responses to belief $\pi_i$ (which may be empty; see footnote 9). An action (accepting, rejecting, or choosing a demand path $u_i^t$) is *optimal* at history $h^t$ under belief $\pi_i$ if there exists a pure strategy $\sigma_i$ that prescribes that action at $h^t$ such that $\sigma_i \in \Sigma_i^*(\pi_i)$.

At the beginning of the game (prior to time 0), player 1 (but not player 2) publicly announces a *bargaining posture* $\gamma : [0, \infty) \to [0, 1]$, which must be continuous at noninteger times $t$ and be everywhere right-continuous with left limits. Slightly abusing notation, a posture $\gamma$ is identified with the strategy of player 1 that demands $\gamma(t)$ for all $t \in \mathbb{R}_+$ and always rejects player 2's demand; with this notation, $\gamma \in \Sigma_1$. In other words, a posture is a pure bargaining phase strategy that does not condition on player 2's play or accept player 2's demand. After announcing posture $\gamma$, player 1 becomes committed to $\gamma$ with some probability $\varepsilon > 0$, meaning that she must play strategy $\gamma$ in the bargaining phase. With probability $1 - \varepsilon$, she is free to play any strategy in the bargaining phase. Whether or not player 1 becomes committed to $\gamma$ is observed only by player 1.

### 2.2. *Definition of Maxmin Payoff and Posture*

This subsection defines player 1's maxmin payoff and posture. Intuitively, player 1's maxmin payoff is the highest payoff she can guarantee herself when all she knows about player 2 is that he is rational (i.e., maximizes his expected payoff given his belief about her behavior) and that he believes that she follows her announced posture $\gamma$ with probability at least $\varepsilon$.

Formally, that player 2 is rational and assigns probability at least $\varepsilon$ to player 1 following her announced posture $\gamma$ means that his strategy satisfies the following condition:

DEFINITION 1: A strategy $\sigma_2$ of player 2 is *rational given posture* $\gamma$ if there exists a belief $\pi_2$ of player 2 such that $\pi_2(\gamma) \geq \varepsilon$ and $\sigma_2 \in \Sigma_2^*(\pi_2)$.

I assume that player 1's belief $\pi_1$ is *consistent with knowledge of rationality given posture* $\gamma$ in that every strategy $\sigma_2 \in \operatorname{supp}(\pi_1)$ is rational given posture $\gamma$.[9] Let $\Pi_1^\gamma \equiv \Delta\{\sigma_2 : \sigma_2 \text{ is rational given posture } \gamma\}$ be the set of beliefs $\pi_1$ that are consistent with knowledge of rationality given posture $\gamma$. Then the highest payoff that player 1 can guarantee herself after announcing posture $\gamma$ is the following:

DEFINITION 2: Player 1's *maxmin payoff given posture* $\gamma$ is

$$u_1^*(\gamma) \equiv \sup_{\sigma_1} \inf_{\pi_1 \in \Pi_1^\gamma} u_1(\sigma_1, \pi_1).$$

A strategy $\sigma_1^*(\gamma)$ of player 1 is a *maxmin strategy given posture* $\gamma$ if

$$\sigma_1^*(\gamma) \in \arg\max_{\sigma_1} \inf_{\pi_1 \in \Pi_1^\gamma} u_1(\sigma_1, \pi_1).$$

Equivalently, $u_1^*(\gamma)$ is the highest payoff player 1 can receive when she chooses a strategy $\sigma_1$ and then player 2 chooses a rational strategy $\sigma_2$ that minimizes $u_1(\sigma_1, \sigma_2)$; that is,

$$u_1^*(\gamma) = \sup_{\sigma_1} \inf_{\sigma_2 : \sigma_2 \text{ is rational given posture } \gamma} u_1(\sigma_1, \sigma_2).$$

In particular, to guarantee herself a high payoff, player 1 must play a strategy that does well against *any* rational strategy of player 2.[10]

Finally, I define player 1's maxmin payoff, the highest payoff that player 1 can guarantee herself before announcing a posture, as well as the corresponding maxmin posture.

[9] A subtlety here is that the set $\Sigma_2^*(\pi_2)$ may be empty for some beliefs $\pi_2$. It is inevitable in bargaining models that players do not have best responses to all beliefs; for example, a player has no best response to the belief that her opponent will accept any strictly positive offer but will refuse an offer of 0. Thus, the assumption that player 2 plays a strategy $\sigma_2 \in \Sigma_2^*(\pi_2)$ for some belief $\pi_2$ is in fact a *joint* assumption on his belief and strategy. While the implied assumption on beliefs is certainly not without loss of generality, it is extremely natural and indeed fundamental, as it says precisely that player 2's choice set is nonempty. It is also weaker than any equilibrium assumption, as players play best responses in equilibrium.

[10] A potential criticism of the concept of the maxmin payoff given posture $\gamma$ is that it appears to neglect the fact that in the event that player 1 does become committed to posture $\gamma$, she is guaranteed only $\inf_{\pi_1 \in \Pi_1^\gamma} u_1(\gamma, \pi_1)$ in the bargaining phase, rather than $\sup_{\sigma_1} \inf_{\pi_1 \in \Pi_1^\gamma} u_1(\sigma_1, \pi_1)$. However, I show in Section 3.3 that these two numbers are actually identical in my model.

DEFINITION 3: Player 1's *maxmin payoff* is

$$u_1^* \equiv \sup_{\gamma} u_1^*(\gamma).$$

A posture $\gamma^*$ is a *maxmin posture* if there exists a sequence of postures $\{\gamma_n\}$ such that $\gamma_n(t) \to \gamma^*(t)$ for all $t \in \mathbb{R}_+$ and $u_1^*(\gamma_n) \to u_1^*$.

I sometimes emphasize the dependence of $u_1^*$ and $\gamma^*$ on $\varepsilon$ by writing $u_1^*(\varepsilon)$ and $\gamma_\varepsilon^*$.[11] Both the set of maxmin strategies given any posture $\gamma$ and the set of maxmin postures are nonempty, though at this point, this is not obvious.

Note that Definitions 2 and 3 are "non-Bayesian" in that they characterize the largest payoff that player 1 can guarantee herself, rather than the maximum payoff that she can obtain given some belief. However, repeating the analysis with the "Bayesian" version of these definitions (with the order of the sup and inf reversed) would yield the same results (see footnote 23).

Another reason for studying player 1's maxmin payoff is that it determines the entire range of payoffs that are consistent with knowledge of rationality, as shown by the following proposition.

PROPOSITION 1: *For any posture $\gamma$ and any payoff $u_1 \in [u_1^*(\gamma), 1)$, there exists a belief $\pi_1 \in \Pi_1^\gamma$ such that* $\max_{\sigma_1} u_1(\sigma_1, \pi_1) = u_1$.

## 3. CHARACTERIZATION OF MAXMIN PAYOFF AND POSTURE

This section states and proves Theorem 1, the main result of the paper, which solves for player 1's maxmin payoff and posture. Section 3.1 states Theorem 1 and provides intuition, and Sections 3.2–3.4 provide the proof.

### 3.1. *Main Result*

The main result is the following.

THEOREM 1: *Player* 1*'s maxmin payoff is*

$$u_1^*(\varepsilon) = 1/(1 - \log \varepsilon)$$

*and the unique maxmin posture $\gamma_\varepsilon^*$ is given by*

$$\gamma_\varepsilon^*(t) = \min\{e^{rt}/(1 - \log \varepsilon), 1\} \quad \textit{for all } t \in \mathbb{R}_+.$$

[11]The notation $\gamma^*(\cdot)$ is already taken by the time-$t$ demand of posture $\gamma^*$. I apologize for abusing notation in writing $u_1^*(\gamma)$ and $u_1^*(\varepsilon)$ for different objects, and hope that this will not cause confusion.

Theorem 1 shows that player 1's maxmin payoff is large relative to her commitment probability $\varepsilon$ and that her unique maxmin posture is simply demanding the maxmin payoff plus compensation for any delay in reaching agreement.[12] I first give intuition for why the maxmin posture demands the maxmin payoff plus compensation for delay and then give intuition for why the maxmin payoff equals $1/(1 - \log \varepsilon)$.

The first step (Sections 3.2 and 3.3) is solving the "min" in maxmin: that is, determining the worst belief that player 2 can have after player 1 announces an arbitrary posture $\gamma$. This belief is called the *$\gamma$-offsetting belief* and plays an important role in the analysis. A preliminary observation is that player 1 should mimic her announced posture $\gamma$ forever in order to guarantee herself as much as possible (however ill-chosen $\gamma$ may be). This is because player 1 is not guaranteed a positive payoff at histories following a deviation from her announced posture, because at such histories, player 2's beliefs and strategy are unrestricted. It follows from this observation that the $\gamma$-offsetting belief is whatever belief leads player 2 to rejects player 1's demand for as long as possible when player 1 mimics $\gamma$.[13]

What belief leads player 2 to reject for as long as possible? It is the belief that player 1 is committed to $\gamma$ with the smallest possible probability (i.e., $\varepsilon$), and that player 1 concedes the entire surplus to him at the rate that makes him (player 2) indifferent between accepting and rejecting (call this rate $\lambda(t)$). This is because if player 1 conceded more slowly, player 2 would accept, and if player 1 conceded more quickly, player 1 would build reputation more quickly when mimicking $\gamma$ and would thus eventually have her demand accepted sooner.[14] Thus, the $\gamma$-offsetting belief is that player 1 is committed to $\gamma$ with probability $\varepsilon$ and concedes the entire surplus at rate $\lambda(t)$ (which depends on $\gamma$).

The second step (Section 3.4) is solving the "max" in maxmin: that is, determining the posture $\gamma$ that maximizes player 1's payoff when player 2 has $\gamma$-offsetting beliefs. A key observation here is that $\lambda(t)$ is higher when player 1's demand is smaller, because when player 1's demand is smaller, player 2 is more tempted to accept. Hence, player 1 builds reputation more quickly when

[12]The importance of nonconstant postures is a difference between this paper and existing reputational bargaining models, where it is usually assumed that players may only be committed to strategies that demand a constant share of the surplus. A notable exception is Abreu and Pearce (2007), where players may be committed to nonconstant postures that can also condition their play on their opponents' behavior. However, Abreu and Pearce's main result is that a particular posture that demands a constant share of the surplus is approximately optimal in their model when commitment probabilities are small.

[13]This is not true if $\gamma$ ever increases so quickly that delay benefits player 1. For this intuitive discussion, consider instead the "typical" case where delay hurts player 1.

[14]This point is clearest in the extreme case where player 2 thinks that player 1 will surely concede in 1 second. Then player 1 needs only to mimic $\gamma$ for 2 seconds to convince player 2 that she is committed.

her demand is smaller, so she benefits from demanding as little as possible, *subject to the constraint that she always demands at least her maxmin payoff plus compensation for delay* (as otherwise player 2 might rationally accept at a time where she demands less than this, leaving her with less than her maxmin payoff). This implies that the maxmin posture demands exactly the maxmin payoff plus compensation for delay, until player 1's reputation reaches 1 (i.e., until player 2 becomes certain that she is committed to $\gamma$).[15] It can also be shown that under the maxmin posture, player 1's reputation reaches 1 at the same time at which her demand reaches 1, and that her demand can never subsequently drop below 1. So the maxmin posture demands compensation for delay until player 1's demand reaches 1 and, subsequently, demands 1 forever.

It remains to describe why the maxmin payoff equals $1/(1-\log\varepsilon)$. Consider a posture $\gamma$ given by $\gamma(t)=\min\{e^{rt}u_1, 1\}$ for all $t\in\mathbb{R}_+$, for arbitrary $u_1\in\mathbb{R}_+$; that is, $\gamma$ demands $u_1$ plus compensation for delay. Observe that if player 1's reputation reaches 1 before her demand reaches 1, then player 2 must accept by the time her reputation reaches 1, as at that time he is certain that player 1's demand will only increase if he rejects further. Since $\gamma$ demands compensation for delay until player 1's demand reaches 1, it follows that if player 1's reputation reaches 1 before her demand reaches 1, then player 1 is guaranteed a payoff equal to her initial demand $u_1$. I now argue that player 1's reputation reaches 1 before her demand does whenever $u_1 < 1/(1-\log\varepsilon)$, which proves that player 1 can guarantee herself up to $1/(1-\log\varepsilon)$.

I first compute the concession rate $\lambda(t)$ that makes player 2 indifferent between accepting and rejecting player 1's demand $\gamma(t)$. For player 2, accepting yields flow payoff $r(1-\gamma(t))$, while rejecting yields flow payoff $\lambda(t)\gamma(t)-\gamma'(t)$, and equalizing these flow payoffs gives[16]

$$\lambda(t)=\frac{r(1-\gamma(t))+\gamma'(t)}{\gamma(t)}.$$

Since $\gamma(t)=e^{rt}u_1$ until $\gamma(t)$ reaches 1, it follows that $\lambda(t)=r/(e^{rt}u_1)$ until $\gamma(t)$ reaches 1. Now player 1's reputation reaches 1 at the time $T$ such that the probability that player 1 has not conceded by $T$ equals $\varepsilon$, since if player 1

[15] A subtlety is that some postures $\gamma$ that always demand more than the maxmin posture $\gamma^*$ may have $\gamma'(t)>\gamma^{*\prime}(t)$ for some $t$, and thus allow player 1 to build reputation more quickly for some $t$ (i.e., $\lambda(t)>\lambda^*(t)$; see the equation for $\lambda(t)$ below). However, it can be shown that this advantage in the derivative term $\gamma'$ is always more than offset by the disadvantage in the level term $\gamma$ when integrating $\lambda$ over an interval, so that player 1's reputation is always greater with posture $\gamma^*$ than with $\gamma$.

[16] Two remarks are in order. First, the formal definition of $\lambda(t)$ is provided in Section 3.2 (the current definition assumes that $\gamma$ is differentiable and that $r(1-\gamma(t))+\gamma'(t)\geq 0$, for example). Second, a slightly more rigorous derivation of $\lambda(t)$ comes from considering the equation for player 2 to be indifferent between accepting at $t$ and $t+dt$, $1-\gamma(t)=\lambda(t)\,dt+(1-\lambda(t)\,dt)(1-r\,dt)(1-\gamma(t+dt))$, and taking a first-order expansion in $dt$.

does not concede by this time, then she must be committed to $\gamma$ (as $\gamma$ never concedes). This time $T$ is given by[17]

$$\exp\left(-\int_0^T \lambda(t)\,dt\right) = \varepsilon.$$

Substituting $r/(e^{rt}u_1)$ for $\lambda(t)$, this becomes

$$T = -\log(1 + u_1 \log\varepsilon)/r.$$

On the other hand, player 1's demand reaches 1 at the time $T^1$ given by $e^{rT^1}u_1 = 1$, or

$$T^1 = -\log(u_1)/r.$$

Hence, player 1's reputation reaches 1 before her demand does if and only if $T < T^1$, that is, if and only if $u_1 < 1/(1 - \log\varepsilon)$. So player 1 can guarantee herself up to $1/(1 - \log\varepsilon)$.

Finally, player 1 cannot guarantee herself more than $1/(1 - \log\varepsilon)$, because it can be shown that any posture that guarantees close to $1/(1 - \log\varepsilon)$ must be close to the maxmin posture $\gamma^*$.

### 3.2. *Offsetting Beliefs and Strategies*

This subsection solves the problem

$$\inf_{(\pi_2,\sigma_2):\pi_2(\gamma)\geq\varepsilon,\sigma_2\in\Sigma_2^*(\pi_2)} u_1(\gamma, \sigma_2). \tag{1}$$

The resulting *(belief-strategy)* pair $(\pi_2^\gamma, \sigma_2^\gamma)$ are the $\gamma$*-offsetting belief* and *strategy*.

The key step in solving (1) is to compute the smallest time $T$ by which agreement must be reached under strategy profile $(\gamma, \sigma_2)$ for $\sigma_2 \in \Sigma_2^*(\pi_2)$. I then show that the value of (1) is simply $\min_{t\leq T} e^{-rt}\underline{\gamma}(t)$, where $\underline{\gamma}(t) \equiv \min\{\lim_{\tau\uparrow t}\gamma(\tau), \gamma(t)\}$.

Toward computing $T$, let $v(t, -1)$ be the continuation value of player 2 from best-responding to $\gamma$ starting from date $(t, -1)$ (for integer $t$), and let $v(t)$ be the corresponding continuation value starting from date $(t, 1)$ (for any $t \in \mathbb{R}_+$),

$$\begin{aligned} v(t, -1) &\equiv \max_{\tau\geq t} e^{-r(\tau-t)}\big(1 - \underline{\gamma}(\tau)\big), \\ v(t) &\equiv \max\Big\{1 - \gamma(t), \sup_{\tau>t} e^{-r(\tau-t)}\big(1 - \underline{\gamma}(\tau)\big)\Big\}, \end{aligned} \tag{2}$$

[17] A more general definition of $T$ is provided in Section 3.2.

where $\underline{\gamma}(\tau) \equiv \min\{\lim_{s\uparrow\tau}\gamma(s), \gamma(\tau)\}$. Thus, the difference between $v(t,-1)$ and $v(t)$ is that only $v(t,-1)$ gives player 2 the opportunity to accept the demand $1-\lim_{\tau\uparrow t}\gamma(\tau)$. In particular, $v(t,-1) = v(t)$ if $\gamma$ (or $v$) is continuous at $t$. Note that $\max_{\tau\geq t} e^{-r(\tau-t)}(1-\underline{\gamma}(\tau))$ is well defined because $\underline{\gamma}$ is lower semicontinuous and $\lim_{\tau\to\infty} e^{-r(\tau-t)}(1-\underline{\gamma}(\tau)) = 0$, and that $v$ is continuous at all noninteger times $t$. Let $\{s_1, s_2, \ldots\} \equiv S \subseteq \mathbb{N}$ be the set of discontinuity points of $v$. Finally, note that $v$ can increase at a rate no faster than $r$; that is, $v(t) \geq e^{-r(t'-t)}v(t')$ for all $t' \geq t$, because if $v(t') = e^{-r(\tau-t')}(1-\underline{\gamma}(\tau))$ for some $\tau \geq t'$, then $v(t) \geq e^{-r(\tau-t)}(1-\underline{\gamma}(\tau)) = e^{-r(t'-t)}v(t')$. This implies that $v$ is continuous but for downward jumps[18] and that $v$ is differentiable almost everywhere.[19] These are but two of the useful properties of the function $v$ (which are not shared by $\gamma$) that reward working with $v$ rather than $\gamma$ in the subsequent analysis.

Next, I introduce two functions $\lambda : \mathbb{R}_+ \to \mathbb{R}_+$ and $p : \mathbb{R}_+ \to \mathbb{R}_+$ with the property that if player 1 mixes between mimicking $\gamma$ and conceding the entire surplus to player 2, then $\lambda(t)$ (resp., $p(t)$) is the smallest nonnegative hazard rate (resp., discrete probability) at which player 1 must concede for player 2 to be willing to reject player 1's time-$t$ demand $\underline{\gamma}(t)$. Let

$$\lambda(t) = \frac{rv(t) - v'(t)}{1 - v(t)} \tag{3}$$

if $v$ is differentiable at $t$ and $v(t) < 1$, and let $\lambda(t) = 0$ otherwise. Note that $\lambda(t) \geq 0$ for all $t$, because $v$ cannot increase at a rate faster than $r$.[20] Also, for integer $t$, let

$$p(t) = \frac{v(t,-1) - v(t)}{1 - v(t)} \tag{4}$$

if $v(t) < 1$ and let $p(t) = p(0) = 0$ otherwise.

When player 2 expects player 1 to accept his demand at rate (resp., probability) $\lambda$ (resp., $p$), he becomes convinced that player 1 is committed to posture $\gamma$ at the time $\tilde{T}$ defined in the following crucial lemma, which leads him to accept player 1's demand no later than the time $T$ defined in the lemma. In the

[18]A function $f : \mathbb{R} \to \mathbb{R}$ is *continuous but for downward jumps* if $\liminf f_{x\uparrow x^*}(x) \geq f(x^*) \geq \limsup_{x\downarrow x^*} f(x)$ for all $x \in \mathbb{R}$.

[19]To see this, let $f(t) = e^{-rt}v(t)$. Then $f$ is nonincreasing, which implies that $f$ is differentiable almost everywhere (e.g., Royden (1988, p. 100)). Hence, $v$ is differentiable almost everywhere.

[20]If $v'(t) = 0$, then $\lambda(t)$ becomes the concession rate that makes player 2 indifferent between accepting and rejecting the constant offer $v(t)$, which is familiar from the literatures on wars of attrition and reputational bargaining. However, in these literatures, $\lambda(t)$ is the rate at which player 1 concedes in equilibrium, while here it is the rate at which player 1 concedes according to player 2's $\gamma$-offsetting belief, as will become clear.

lemma, and throughout the paper, maximization or minimization over times $t$ should be read as taking place over $t \in \mathbb{R}_+ \cup \{\infty\}$ (i.e., as allowing $t = \infty$, with the convention that $e^{-r\infty}\underline{\gamma}(\infty) \equiv 0$ for all postures $\gamma$).

LEMMA 1: *Let*

$$\tilde{T} \equiv \sup\left\{t : \exp\left(-\int_0^t \lambda(s)\,ds\right) \prod_{s\in S\cap[0,t)} \bigl(1 - p(s)\bigr) > \varepsilon\right\}$$

*and let*

$$T \equiv \max \arg\max_{t\geq\tilde{T}} \begin{cases} e^{-rt}\bigl(1-\gamma(t)\bigr), & \text{if } t = \tilde{T}, \\ e^{-rt}\bigl(1-\underline{\gamma}(t)\bigr), & \text{if } t > \tilde{T}. \end{cases}$$

*Then for any $\pi_2$ such that $\pi_2(\gamma) \geq \varepsilon$ and any $\sigma_2 \in \Sigma_2^*(\pi_2)$, agreement is reached no later than time $T$ under strategy profile $(\gamma, \sigma_2)$. In particular,*

$$\inf_{(\pi_2,\sigma_2):\pi_2(\gamma)\geq\varepsilon,\sigma_2\in\Sigma_2^*(\pi_2)} u_1(\gamma, \sigma_2) \geq \min_{t\leq T} e^{-rt}\underline{\gamma}(t). \tag{5}$$

Thus, Lemma 1 shows that agreement is delayed for as long as possible when player 1's concession rate and probability are given by $\lambda$ and $p$. This gives a lower bound for (1).

The remainder of this subsection is devoted to showing that (5) holds with equality, which proves that (1) equals $\min_{t\leq T} e^{-rt}\underline{\gamma}(t)$. The idea is that player 2 may hold a belief that leads him to demand the entire surplus until time $t^* \equiv \min\arg\min_{t\leq T} e^{-rt}\underline{\gamma}(t)$ and then accept player 1's offer; this is the $\gamma$-offsetting belief.[21] I first define the $\gamma$-offsetting belief and then show that (5) holds with equality.

I begin by introducing a strategy $\tilde{\gamma}$, which is used in defining the $\gamma$-offsetting belief.[22] Let

$$\chi(t) = \max\left\{\frac{\exp\left(-\int_0^t \lambda(s)\,ds\right) \prod_{s\in S\cap[0,t)} (1-p(s)) - \varepsilon}{\exp\left(-\int_0^t \lambda(s)\,ds\right) \prod_{s\in S\cap[0,t)} (1-p(s))}, 0\right\}, \tag{6}$$

[21]Note that $\min\arg\min_{t\leq T} e^{-rt}\underline{\gamma}(t)$ is well defined, because $\underline{\gamma}(t)$ is lower semicontinuous (though it may equal $\infty$, if $T = \infty$). Note also that $\gamma(t) > 0$ for all $t < t^*$. This property of $t^*$ makes it more convenient to define the $\gamma$-offsetting belief with reference to $t^*$, rather than some other element of $\arg\min_{t\leq T} e^{-rt}\underline{\gamma}(t)$.

[22]This approach is related to a construction in Wolitzky (2011).

let

$$\hat{\lambda}(t) = \frac{\lambda(t)}{\chi(t)} \tag{7}$$

if $\chi(t) > 0$ and $\hat{\lambda}(t) = 0$ otherwise, and let

$$\hat{p}(t) = \min\left\{\frac{p(t)}{\chi(t)}, 1\right\} \tag{8}$$

if $\chi(t) > 0$ and $\hat{p}(t) = 0$ otherwise. Thus, $\chi(t)$ is the posterior probability that player 2 assigns to player 1's playing a strategy other than $\gamma$ at time $t$ when player 1's *unconditional* concession rate and probability are $\lambda(t)$ and $p(t)$, and $\hat{\lambda}(t)$ and $\hat{p}(t)$ are the *conditional* (on not playing $\gamma$) concession rate and probability needed for the unconditional concession rate and probability to equal $\lambda(t)$ and $p(t)$.

DEFINITION 4: $\tilde{\gamma}$ is the strategy that demands $\gamma(t)$ at all $t$, accepts with hazard rate $\hat{\lambda}(t)$ at all $t < t^*$, accepts with probability $\hat{p}(t)$ at date $(t, 1)$ for all $t < t^*$, and rejects for all $t \geq t^*$, for all histories $h^t$.

I now define the $\gamma$-offsetting belief. Throughout, a history $h^{t-}$ (resp., $h^{t+}$) is *consistent with posture* $\gamma$ if $u_1(\tau) = \gamma(\tau)$ for all $\tau < t$ (resp., $\tau \leq t$).

DEFINITION 5: The *$\gamma$-offsetting belief*, denoted $\pi_2^\gamma$, is given by $\pi_2^\gamma(\gamma) = \varepsilon$ and $\pi_2^\gamma(\tilde{\gamma}) = 1 - \varepsilon$. The *$\gamma$-offsetting strategy*, denoted $\sigma_2^\gamma$, is the strategy that always demands 1 and accepts or rejects player 1's demand as follows:

(i) If $h^t$ is consistent with $\gamma$, then reject if $t < t^*$, accept at date $(t^*, -1)$ if and only if $\lim_{\tau\uparrow t^*} \gamma(\tau) \leq \gamma(t^*)$, accept at date $(t^*, 1)$ if and only if $\lim_{\tau\uparrow t^*} \gamma(\tau) > \gamma(t^*)$, and reject if $t > t^*$.

(ii) If $h^t$ is inconsistent with $\gamma$, then reject.

Finally, I show that (5) holds with equality and also that the $\gamma$-offsetting *(belief, strategy)* pair $(\pi_2^\gamma, \sigma_2^\gamma)$ is a solution to (1). If $t^* = \infty$, then the following statement that agreement is reached at time $t^*$ means that agreement is never reached.

LEMMA 2: *Agreement is reached at time $t^*$ under strategy profile $(\gamma, \sigma_2^\gamma)$, and $\sigma_2^\gamma \in \Sigma_2^*(\pi_2^\gamma)$. In particular, the pair $(\pi_2^\gamma, \sigma_2^\gamma)$ is a solution to (1), and $u_1(\gamma, \sigma_2^\gamma) = \min_{t\leq T} e^{-rt}\underline{\gamma}(t)$.*

PROOF: It is immediate from Definition 5 that agreement is reached at $t^*$ under strategy profile $(\gamma, \sigma_2^\gamma)$, which implies that $u_1(\gamma, \sigma_2^\gamma)$ equals $\min_{t\leq T} e^{-rt}\underline{\gamma}(t)$, the right-hand side of (5). Since $\pi_2^\gamma(\gamma) \geq \varepsilon$, it remains only to show that $\sigma_2^\gamma \in \Sigma_2^*(\pi_2^\gamma)$.

If $t < \min\{\tilde{T}, t^*\}$ and $h^t$ is consistent with $\gamma$, then, by construction of $\tilde{\gamma}$, player 1 accepts player 2's demand of 1 with unconditional hazard rate $\lambda(t)$ and unconditional discrete probability $p(t)$ under $\pi_2^\gamma$. The proof of Lemma 1 establishes that it is optimal for player 2 to demand $u_2(t) = 1$ and reject at any time $t < \min\{\tilde{T}, t^*\}$ when player 1 accepts his demand of 1 at rate $\lambda$ and probability $p$ until time $\tilde{T}$, and that, in addition, if $t^* < \tilde{T}$, then player 2 is indifferent between accepting and rejecting at time $t^*$ when player 1 accepts with this rate and probability until time $\tilde{T}$. Therefore, it is optimal for player 2 to demand $u_2(t) = 1$ and reject at time $t$ when player 1 accepts with this rate and probability only until time $\min\{\tilde{T}, t^*\}$.

If $t \in [\tilde{T}, t^*)$ and $h^t$ is consistent with $\gamma$, then under $\pi_2^\gamma$, player 2 is certain that player 1 is playing $\gamma$ at $h^t$. Since $t^* \leq T$, this implies that it is optimal for player 2 to reject. If a history $h^t$ is not reached under strategy profile $(\pi_2^\gamma, \sigma_2^\gamma)$ (as is the case if $h^t$ is inconsistent with $\gamma$ or if $t > t^*$), then any continuation strategy of player 2 is optimal. Finally, to see that accepting $\underline{\gamma}(t^*)$ (i.e., accepting at the more favorable of dates $(t^*, -1)$ and $(t^*, 1)$) is optimal, note that the fact that $t^* \in \arg\min_{t \leq T} e^{-rt}\underline{\gamma}(t)$ implies that $\underline{\gamma}(t) \geq \underline{\gamma}(t^*)$ for all $t \in [t^*, T]$. Hence, $t^* \in \arg\max_{t \in [t^*, T]} e^{-rt}(1 - \underline{\gamma}(t))$. Because $\tilde{\gamma}$ coincides with $\gamma$ after time $t^*$, it follows that, conditional on having reached time $t^*$, player 2 receives at most $\sup_{t \in (t^*, T]} e^{-rt}(1 - \underline{\gamma}(t))$ if he rejects and receives $e^{-rt^*}(1 - \underline{\gamma}(t^*))$ if he accepts, which is weakly more. Therefore, $\sigma_2^\gamma \in \Sigma_2^*(\pi_2^\gamma)$. *Q.E.D.*

### 3.3. *Maxmin Strategies*

This subsection shows that $\gamma$ itself is a maxmin strategy given posture $\gamma$. Henceforth, I write $\tilde{T}(\gamma)$ and $T(\gamma)$ for the times defined in Lemma 1, making the dependence on $\gamma$ explicit.

LEMMA 3: *For any posture* $\gamma$, $u_1^*(\gamma) = u_1(\gamma, \sigma_2^\gamma) = \min_{t \leq T(\gamma)} e^{-rt}\underline{\gamma}(t)$.

PROOF: By Lemma 2, $(\pi_2^\gamma, \sigma_2^\gamma)$ is a solution to (1), so

$$\sigma_2^\gamma \in \arg\min_{\pi_1 \in \Pi_1^\gamma} u_1(\gamma, \pi_1). \tag{9}$$

Under strategy $\sigma_2^\gamma$, player 2 always demands 1 and only accepts player 1's demand if she conforms to $\gamma$ through time $t^*$. Hence, $\sup_{\sigma_1} u_1(\sigma_1, \sigma_2^\gamma) = e^{-rt^*}\underline{\gamma}(t^*) = u_1(\gamma, \sigma_2^\gamma)$ and, therefore,

$$\gamma \in \arg\max_{\sigma_1} u_1(\sigma_1, \sigma_2^\gamma). \tag{10}$$

Now (9) and (10) imply the chain of inequalities

$$\begin{aligned}\sup_{\sigma_1}\inf_{\pi_1\in\Pi_1^\gamma} u_1(\sigma_1,\pi_1) &\geq \inf_{\pi_1\in\Pi_1^\gamma} u_1(\gamma,\pi_1)\\ &= u_1\bigl(\gamma,\sigma_2^\gamma\bigr) \quad \text{(by (9))}\\ &= \sup_{\sigma_1} u_1\bigl(\sigma_1,\sigma_2^\gamma\bigr) \quad \text{(by (10))}\\ &\geq \sup_{\sigma_1}\inf_{\pi_1\in\Pi_1^\gamma} u_1(\sigma_1,\pi_1).\end{aligned}$$

This is possible only if both inequalities hold with equality (and the supremum and infimum are attained at $\gamma$ and $\sigma_2^\gamma$, respectively).[23] Therefore, $u_1^*(\gamma) = u_1(\gamma,\sigma_2^\gamma) = \min_{t\leq T(\gamma)} e^{-rt}\underline{\gamma}(t)$. *Q.E.D.*

### 3.4. *Proof of Theorem 1*

I now sketch the remainder of the proof of Theorem 1. The details of the proof are deferred to the Appendix.

The first part of the proof is to construct a sequence of postures $\{\gamma_n\}$ such that $\lim_{n\to\infty} u_1^*(\gamma_n) = 1/(1-\log\varepsilon)$ and $\{\gamma_n(t)\}$ converges to $\gamma^*(t) \equiv \min\{e^{rt}/(1-\log\varepsilon), 1\}$ for all $t\in\mathbb{R}_+$. Define $\gamma_n$ by

$$\gamma_n(t) = \min\left\{\left(\frac{n}{n+1}\right)\frac{e^{rt}}{1-\log\varepsilon}, 1\right\} \quad \text{for all } t\in\mathbb{R}_+.$$

Let $T_n^1$ be the time where $\gamma_n(t)$ reaches 1. It can be shown that $T_n^1 > \tilde{T}(\gamma_n)$ for all $n\in\mathbb{N}$. Hence, $\gamma_n(t) = (\frac{n}{n+1})\frac{e^{rt}}{1-\log\varepsilon}$ for all $t\leq\tilde{T}(\gamma_n)$, and $\gamma_n(\tilde{T}(\gamma_n)) < 1$. Since $\gamma_n(t)$ is nondecreasing and $\gamma_n(\tilde{T}(\gamma_n)) < 1$, it follows from the definition of $T(\gamma_n)$ that $T(\gamma_n) = \tilde{T}(\gamma_n)$. Thus, by Lemma 3,

$$\begin{aligned}u_1^*(\gamma_n) = \min_{t\leq T(\gamma_n)} e^{-rt}\underline{\gamma_n}(t) &= \min_{t\leq\tilde{T}(\gamma_n)}\left(\frac{n}{n+1}\right)\frac{1}{1-\log\varepsilon}\\ &= \left(\frac{n}{n+1}\right)\frac{1}{1-\log\varepsilon}.\end{aligned}$$

Therefore, $\lim_{n\to\infty} u_1^*(\gamma_n) = 1/(1-\log\varepsilon)$.

The second part is to show that no posture $\gamma$ guarantees more than $1/(1-\log\varepsilon)$. Here, the crucial observation is that any posture $\gamma$ such that $\gamma(t)\geq e^{rt}/(1-\log\varepsilon)$ for all $t\leq T(\gamma)$ satisfies $\tilde{T}(\gamma)\geq T^1$, where $T^1$ is the time at which

[23] The same argument applies with the order of the sup and inf reversed, which is why Theorem 1 continues to hold when the order of the sup and inf is reversed in Definitions 2 and 3.

$\gamma^*(t)$ reaches 1. Since $u_1^*(\gamma) = \min_{t \le T(\gamma)} e^{-rt}\underline{\gamma}(t)$, this implies that any posture $\gamma$ that guarantees at least $1/(1 - \log\varepsilon)$ must satisfy $\tilde{T}(\gamma) \ge T^1$; in particular, player 2 may reject $\gamma(t)$ until time $T^1$. But receiving the entire surplus at $T^1$ is worth only $1/(1 - \log\varepsilon)$, so it follows that no posture guarantees more than $1/(1 - \log\varepsilon)$. The Appendix shows that, in addition, the maxmin posture is unique.

## 4. EXTENSIONS

This section presents three extensions of Theorem 1: Section 4.1 restricts player 1 to announcing constant postures; Section 4.2 allows for heterogeneous discounting; Section 4.3 considers higher-order knowledge of rationality.

### 4.1. *Constant Postures*

Theorem 1 shows that the unique maxmin posture is nonconstant. In this subsection, I determine how much lower a player's maxmin payoff is when she is required to announce a constant posture. This establishes that value of announcing nonconstant postures and also facilitates comparison with the existing reputational bargaining literature, in which, typically, players can only announce constant postures.

A posture $\gamma$ is *constant* if $\gamma(t) = \gamma(0)$ for all $t$. If $\gamma$ is constant, I slightly abuse notation by writing $\gamma$ for the constant demand $\gamma(t)$ in addition to the posture itself. The constant posture $\gamma$ that maximizes $u_1^*(\gamma)$ is the *maxmin constant posture*, denoted $\bar{\gamma}^*$, and the corresponding payoff is the *maxmin constant payoff*, denoted $\bar{u}_1^*$. These can be derived using Lemmas 1–3, leading to the following proposition.

PROPOSITION 2: *For all $\varepsilon < 1$, the unique maxmin constant posture is $\bar{\gamma}_\varepsilon^* = \frac{2-\log\varepsilon-\sqrt{(\log\varepsilon)^2-4\log\varepsilon}}{2}$ and the maxmin constant payoff is $\bar{u}_1^*(\varepsilon) = \exp(-(1-\bar{\gamma}_\varepsilon^*))\bar{\gamma}_\varepsilon^*$.*

Proposition 2 solves for $\bar{\gamma}_\varepsilon^*$ and $\bar{u}_1^*(\varepsilon)$, but it does not yield a clear relationship between the maxmin constant payoff, $\bar{u}_1^*(\varepsilon)$, and the (overall) maxmin payoff, $u_1^*(\varepsilon)$. Figure 2 graphs $u_1^*(\varepsilon)$ and $\bar{u}_1^*(\varepsilon)$. In addition, the following result regarding the ratio of $u_1^*(\varepsilon)$ to $\bar{u}_1^*(\varepsilon)$ is straightforward.

COROLLARY 1: *$u_1^*(\varepsilon)/\bar{u}_1^*(\varepsilon)$ is decreasing in $\varepsilon$, $\lim_{\varepsilon\to1} u_1^*(\varepsilon)/\bar{u}_1^*(\varepsilon) = 1$, and $\lim_{\varepsilon\to0} u_1^*(\varepsilon)/\bar{u}_1^*(\varepsilon) = e$.*

The most interesting part of Corollary 1 is that a player's maxmin payoff is approximately $e \approx 2.72$ times greater when she can announce nonconstant postures than when she can only announce constant postures, when her commitment probability is small. Thus, there is a large advantage to announcing

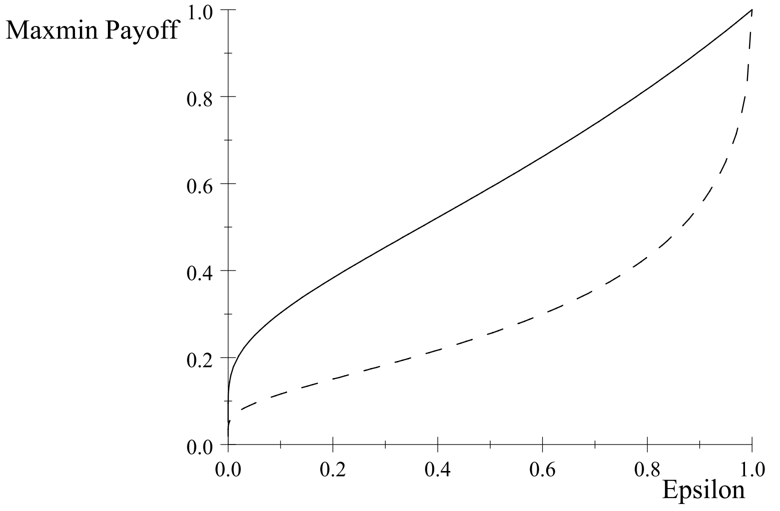


FIGURE 2.—$u_1^*(\varepsilon)$ (solid line) and $\bar{u}_1^*(\varepsilon)$ (dashed line).

nonconstant postures. However, a player can still guarantee herself a substantial share of the surplus when she can only announce constant postures, and her maxmin payoff goes to 0 with $\varepsilon$ at the same rate in either case.

### 4.2. *Heterogeneous Discounting*

The assumption that the players have the same discount rate has simplified notation and led to simple formulas for $u_1^*(\varepsilon)$ and $\gamma_\varepsilon^*$ in Theorem 1. However, it is straightforward to let player $i$ have discount rate $r_i$, with $r_i \neq r_j$, and doing so yields interesting comparative statics with respect to the players' relative patience, $r_1/r_2$ (as will become clear, $u_1^*$ depends on $r_1$ and $r_2$ only through $r_1/r_2$). First, the standard result in the reputational bargaining literature that player 1's sequential equilibrium payoff converges to 1 as $r_1/r_2$ converges to 0 and converges to 0 as $r_1/r_2$ converges to $\infty$ also applies to player 1's maxmin payoff. This is analogous to the finding of Watson (1993) and Battigalli and Watson (1997) that the limit uniqueness result of Fudenberg and Levine (1989) holds under knowledge of rationality. However, I also derive player 1's maxmin payoff for fixed $r_1/r_2$ (rather than only in the limit). This leads to a second comparative static result, which indicates that a geometric change in relative patience has a similar effect on the maxmin payoff as does an exponential change in commitment probability. An analogous result holds in equilibrium in existing reputational bargaining models.

I first present the analog of Theorem 1 for heterogeneous discount rates and then state the two comparative statics results as corollaries, omitting their proofs.

PROPOSITION 3: *If player $i$'s discount rate is $r_i$, then player* 1*'s maxmin payoff*, $u_1^*(\varepsilon)$, *is the unique number* $u_1^*$ *that solves*

$$u_1^* = \frac{1}{1 - \frac{r_1}{r_2}\log\varepsilon - \left(\frac{r_1}{r_2} - 1\right)\log u_1^*}. \tag{11}$$

Corollary 2 shows that the standard limit comparative statics on $r_1/r_2$ in reputational bargaining models require only first-order knowledge of rationality.

COROLLARY 2:

$$\lim_{r_1/r_2\to 0} u_1^*(\varepsilon) = 1.$$

*If* $\varepsilon < 1$, *then*, *in addition*,

$$\lim_{r_1/r_2\to\infty} u_1^*(\varepsilon) = 0.$$

Corollary 3 shows that the commitment probability $\varepsilon$ must decrease exponentially to (approximately) offset a geometric increase in relative patience $(r_1/r_2)^{-1}$. The result is stated for the case $r_1/r_2 \le 1$, where even an exponential decrease in $\varepsilon$ does not fully offset a geometric increase in $(r_1/r_2)^{-1}$. If $r_1/r_2 > 1$, then an exponential decrease in $\varepsilon$ can more than offset a geometric increase in $(r_1/r_2)^{-1}$.

COROLLARY 3: *Suppose that* $r_1/r_2 \le 1$, *and that* $r_1/r_2$ *and* $\varepsilon$ *both decrease while* $(r_1/r_2)\log\varepsilon$ *remains constant*. *Then* $u_1^*(\varepsilon)$ *increases*.

### 4.3. *Rationalizability*

Theorem 1 derives the highest payoff that player 1 can guarantee herself under first-order knowledge of rationality, the weakest epistemic assumption consistent with the possibility of reputation-building. I now show that player 1 cannot guarantee herself more than this under the much stronger assumption of normal-form rationalizability (or under any finite-order knowledge of rationality), which reinforces Theorem 1 substantially.[24]

I consider the following definition of (normal-form) rationalizability:

DEFINITION 6: A set of bargaining phase strategy profiles $\Omega = \Omega_1 \times \Omega_2 \subseteq \Sigma_1 \times \Sigma_2$ is *closed under rational behavior given posture* $\gamma$ if for all $\sigma_1 \in \Omega_1$, there

[24]The Supplemental Material shows that Theorem 1 also extends to iterated conditional dominance.

exists some belief $\pi_1 \in \Delta(\Omega_2)$ such that $\sigma_1 \in \Sigma_1^*(\pi_1)$, and for all $\sigma_2 \in \Omega_2$, there exists some belief $\pi_2 \in \Delta(\Omega_1 \cup \{\gamma\})$ such that $\pi_2(\gamma) \geq \varepsilon$, with strict inequality only if $\gamma \in \Omega_1$ and $\sigma_2 \in \Sigma_2^*(\pi_2)$.

The set of *rationalizable strategies given posture* $\gamma$ is

$$\Omega^{\mathrm{RAT}}(\gamma) \equiv \bigcup \big\{\Omega : \Omega \text{ is closed} \\ \text{under rational behavior given posture } \gamma\big\}.$$

Player 1's *rationalizable maxmin payoff given posture* $\gamma$ is $u_1^{\mathrm{RAT}}(\gamma) \equiv \sup_{\sigma_1} \inf_{\sigma_2 \in \Omega_2^{\mathrm{RAT}}(\gamma)} u_1(\sigma_1, \sigma_2)$. Player 1's *rationalizable maxmin payoff* is $u_1^{\mathrm{RAT}} \equiv \sup_\gamma u_1^{\mathrm{RAT}}(\gamma)$. A posture $\gamma^{\mathrm{RAT}}$ is a *rationalizable maxmin posture* if there exists a sequence of postures $\{\gamma_n\}$ such that $\gamma_n(t) \to \gamma^{\mathrm{RAT}}(t)$ for all $t \in \mathbb{R}_+$ and $u_1^{\mathrm{RAT}}(\gamma_n) \to u_1^{\mathrm{RAT}}$.

The result is the following proposition.

PROPOSITION 4: *Player* 1*'s rationalizable maxmin payoff equals her maxmin payoff, and the unique rationalizable maxmin posture is the unique maxmin posture; that is,* $u_1^{\mathrm{RAT}} = u_1^*$ *and the unique rationalizable maxmin posture is* $\gamma^{\mathrm{RAT}} = \gamma^*$.

Any rationalizable strategy given posture $\gamma$ is also rational given posture $\gamma$. Therefore, Lemma 1 applies under rationalizability. The only additional fact used in the proof of Theorem 1 is that $u_1^*(\gamma) = \min_{t \leq T(\gamma)} e^{-rt} \underline{\gamma}(t)$ for any posture $\gamma$ (Lemma 3). Supposing that the analogous equation holds under rationalizability (i.e., that $u_1^{\mathrm{RAT}}(\gamma) = \min_{t \leq T(\gamma)} e^{-rt} \underline{\gamma}(t)$), the proof of Theorem 1 goes through as written. Hence, to prove Proposition 4, it suffices to prove the following lemma, the proof of which shows that the $\gamma$-offsetting belief and strategy are not only rational but rationalizable:

LEMMA 4: *For any posture* $\gamma$, $u_1^{\mathrm{RAT}}(\gamma) = \min_{t \leq T(\gamma)} e^{-rt} \underline{\gamma}(t)$.

## 5. TWO-SIDED COMMITMENT

This section introduces the possibility that both players may announce—and become committed to—postures prior to the start of bargaining. I show that each player $i$'s maxmin payoff is close to that derived in Section 3 when her opponent's commitment probability, $\varepsilon_j$, is small in absolute terms (even if $\varepsilon_j$ is large relative to $\varepsilon_i$). In addition, each player's maxmin posture is exactly as in Section 3. This shows that the analysis of Section 3 provides a two-sided theory of reputational bargaining. The results of this section contrast with the existing reputational bargaining literature, which emphasizes that *relative* commitment probabilities are crucial for determining *equilibrium* behavior and payoffs.

Formally, modify the model of Section 2 by assuming that in the announcement stage players simultaneously announce postures $(\gamma_1, \gamma_2)$, to which they become committed with probabilities $\varepsilon_1$ and $\varepsilon_2$, respectively. The bargaining phase is unaltered. Thus, at the beginning of the bargaining phase, player $i$ believes that player $j$ is committed to posture $\gamma_j$ with probability $\varepsilon_j$ and is rational with probability $1-\varepsilon_j$ (though this fact is not common knowledge). The following definitions are analogs of Definitions 1–3 that allow for the fact that both players may become committed to the postures they announce:

DEFINITION 7: A belief $\pi_i$ of player $i$ is *consistent with knowledge of rationality given postures* $(\gamma_i, \gamma_j)$ if $\pi_i(\gamma_j) \geq \varepsilon_j$; $\pi_i(\gamma_j) > \varepsilon_j$ only if there exists $\pi_j$ such that $\pi_j(\gamma_i) \geq \varepsilon_i$ and $\gamma_j \in \Sigma_j^*(\pi_j)$; and, for all $\sigma_j \neq \gamma_j$, $\sigma_j \in \operatorname{supp}(\pi_i)$ only if there exists $\pi_j$ such that $\pi_j(\gamma_i) \geq \varepsilon_i$ and $\sigma_j \in \Sigma_j^*(\pi_j)$. Let $\Pi_i^{\gamma_i,\gamma_j}$ be the set of player $i$'s beliefs that are consistent with knowledge of rationality given postures $(\gamma_i, \gamma_j)$. Player $i$'s *maxmin payoff given postures* $(\gamma_i, \gamma_j)$ is

$$u_i^*(\gamma_i, \gamma_j) \equiv \sup_{\sigma_i} \inf_{\pi_i \in \Pi_i^{\gamma_i,\gamma_j}} u_i(\sigma_i, \pi_i).$$

Player $i$'s *maxmin payoff* is

$$u_i^* \equiv \sup_{\gamma_i} \inf_{\gamma_j} u_i^*(\gamma_i, \gamma_j).$$

A posture $\gamma_i^*$ is a *maxmin posture (of player i)* if there exists a sequence of postures $\{\gamma_n\}$ such that $\gamma_n(t) \to \gamma_i^*(t)$ for all $t \in \mathbb{R}_+$ and $\inf_{\gamma_j} u_i^*(\gamma_n, \gamma_j) \to u_i^*$.

I now show that $u_i^*(\varepsilon_i, \varepsilon_j)$ (player $i$'s maxmin payoff with commitment probabilities $\varepsilon_i$ and $\varepsilon_j$) is approximately equal to $u_i^*(\varepsilon_i)$ (her maxmin payoff in the one-sided commitment model) whenever $\varepsilon_j$ is small, and that the maxmin posture is exactly as in the one-sided commitment model. This is simply because player $i$ cannot guarantee herself anything in the event that player $j$ is committed (e.g., if player $j$'s announced posture always demands the entire surplus), which implies that player $i$ guarantees herself as much as possible by conditioning on the event that player $j$ is not committed. In this event, which occurs with probability $1-\varepsilon_j$, player $i$ can guarantee herself $u_i^*(\varepsilon_i)$, and the only way she can guarantee herself this much is by announcing $\gamma_{\varepsilon_i}^*$.

THEOREM 2: *Player i's maxmin payoff is* $u_i^*(\varepsilon_i, \varepsilon_j) = (1-\varepsilon_j)u_i^*(\varepsilon_i)$, *and player i's unique maxmin posture is* $\gamma_{i,(\varepsilon_i,\varepsilon_j)}^* = \gamma_{\varepsilon_i}^*$.

PROOF: Let $\gamma_j^0$ be the posture of player $j$ given by $\gamma_j^0(t) = 1$ for all $t$. Note that $u_i(\sigma_i, \gamma_j^0) = 0$ for all $\sigma_i$. Therefore, $\inf_{\gamma_j} u_i(\sigma_i, \gamma_j) = 0$ for all $\sigma_i$.

Next, let $\Pi_i^{\gamma_i,\gamma_j}(\varepsilon_i, \varepsilon_j)$ be the set of beliefs $\pi_i$ that are consistent with knowledge of rationality for commitment probabilities $(\varepsilon_i, \varepsilon_j)$, and let $\Pi_i^{\gamma_i}(\varepsilon_i)$

be the analogous set in the one-sided commitment model. I claim that if $\pi_i \in \Pi_i^{\gamma_i,\gamma_j}(\varepsilon_i, \varepsilon_j)$, then there exists $\pi_i' \in \Pi_i^{\gamma_i}(\varepsilon_i)$ such that $\pi_i$ puts probability $1-\varepsilon_j$ on strategy $\pi_i'$ and puts probability $\varepsilon_j$ on strategy $\gamma_j$. To see this, note that $\pi_i(\gamma_j) \geq \varepsilon_j$, so there exists a strategy $\pi_i'$ such that $\pi_i$ puts probability $1-\varepsilon_j$ on $\pi_i'$ and puts probability $\varepsilon_j$ on $\gamma_j$. Furthermore, by definition of $\Pi_i^{\gamma_i,\gamma_j}(\varepsilon_i, \varepsilon_j)$, $\sigma_j \in \operatorname{supp}(\pi_i')$ only if there exists $\pi_j$ such that $\pi_j(\gamma_i) \geq \varepsilon_i$ and $\sigma_j \in \Sigma_j^*(\pi_j)$ (whether or not $\sigma_j$ equals $\gamma_j$). By definition of $\Pi_i^{\gamma_i}(\varepsilon_i)$, this implies that $\pi_i' \in \Pi_i^{\gamma_i}(\varepsilon_i)$.

Combining the above observations gives

$$
\begin{aligned}
\inf_{\gamma_j} u_i^*(\gamma_i, \gamma_j) &= \inf_{\gamma_j} \sup_{\sigma_i} \inf_{\pi_i \in \Pi_i^{\gamma_i,\gamma_j}(\varepsilon_i,\varepsilon_j)} u_i(\sigma_i, \pi_i) \\
&= \inf_{\gamma_j} \sup_{\sigma_i} \inf_{\pi_i' \in \Pi_i^{\gamma_i}(\varepsilon_i)} (1-\varepsilon_j) u_i(\sigma_i, \pi_i') + \varepsilon_j u_i(\sigma_i, \gamma_j) \\
&= \sup_{\sigma_i} \inf_{\pi_i' \in \Pi_i^{\gamma_i}(\varepsilon_i)} (1-\varepsilon_j) u_i(\sigma_i, \pi_i') + \varepsilon_j(0) \\
&= (1-\varepsilon_j) u_i^*(\gamma_i).
\end{aligned}
$$

Therefore, the definitions of $u_i^*(\varepsilon_i, \varepsilon_j)$ and $u_i^*(\varepsilon_i)$ imply that $u_i^*(\varepsilon_i, \varepsilon_j) = \sup_{\gamma_i}(1-\varepsilon_j) u_i^*(\gamma_i) = (1-\varepsilon_j) u_i^*(\varepsilon_i)$. Similarly, the definition of a maxmin posture in the one-sided commitment model implies that $\gamma^*_{i,(\varepsilon_i,\varepsilon_j)}$ is a maxmin posture in the two-sided commitment model if and only if it is a maxmin posture in the one-sided commitment model with $\varepsilon = \varepsilon_i$. *Q.E.D.*

Note that the assumption that both players best-respond to beliefs that are consistent with knowledge of rationality does not determine how bargaining proceeds. However, it is interesting to compare a player's opponent's worst-case conjecture (i.e., offsetting belief) about her strategy with her equilibrium strategy in, for example, Abreu and Gul (2000). Both in equilibrium and in her opponent's worst-case conjecture, a player mixes between mimicking her announced posture and (in effect) conceding. In the worst-case conjecture, a player concedes at the rate that makes her opponent indifferent between accepting and rejecting her demand when she follows her maxmin posture and he demands the entire surplus. In equilibrium, a player concedes at the rate that makes her opponent indifferent between accepting and rejecting when both players make their equilibrium demands. There is no general way to order the concession rates in equilibrium and in the worst-case conjecture, because a player's demand is often higher in equilibrium (implying a lower concession rate), while her opponent's demand is always lower in equilibrium (implying a higher concession rate). Thus, a player's concession rate in equilibrium and in her opponent's worst-case conjecture are determined by similar indifference conditions, but one cannot predict whether agreement will be reached sooner in equilibrium or under knowledge of rationality (for either the players' true strategies or for their opponents' worst-case conjectures).

## 6. CONCLUSION

This paper analyzes a model of reputational bargaining in which players initially announce postures to which they may become committed and then bargain over a unit of surplus. It shows that under first-order knowledge of rationality, a player can guarantee herself a share of the surplus that is large relative to her probability of becoming committed, and that the unique bargaining posture that guarantees this much is simply demanding this share in addition to compensation for any delay in reaching agreement. These insights apply for one- or two-sided commitment, for heterogeneous discounting, for any level of knowledge of rationality or iterated conditional dominance, and for any bargaining procedure with frequent offers. In addition, if a player could only announce postures that always demand the same share of the surplus (as in most of the existing literature), her maxmin payoff would be approximately $e$ times lower.

These results are intended to complement the existing equilibrium analysis of reputational bargaining models. Consider the fundamental question, "What posture should a bargainer stake out?" In equilibrium analysis, the answer to this question depends on her opponent's beliefs about her continuation play following every possible announcement. Yet it may be impossible for either the bargainer or an outside observer to learn these beliefs, especially when bargaining is one shot. Hence, an appealing alternative approach is to look for a posture that guarantees a high payoff against *any* belief of one's opponent and for the highest payoff that each player can guarantee herself. This paper shows that this alternative approach yields sharp and economically plausible results.

## APPENDIX: OMITTED PROOFS

PROOF OF PROPOSITION 1[25]: Fix a posture $\gamma$ and payoff $u_1 \in [u_1^*(\gamma), 1)$. If $u_1 \neq \gamma(0)$, then let $\hat{\sigma}_2^\gamma$ be identical to the $\gamma$-offsetting strategy defined in Definition 5, with the modification that player 1's demand is accepted at any history $h^t$ at which player 1 has demanded $u_1$ at all previous dates.[26] If $u_1 = \gamma(0)$, then let $\hat{\sigma}_2^\gamma$ be identical to the $\gamma$-offsetting strategy defined in Definition 5, with the modification that player 1's demand is accepted at date $(-\log(u_1)/r, -1)$ if player 1 has demanded 1 at all previous dates. In either case, let $\pi_2^\gamma$ be as in Definition 5 and note that $\pi_2^\gamma(\gamma) \geq \varepsilon$. If $u_1 \neq \gamma(0)$, no strategy under which $u_1(0) = u_1$ is in the support of $\pi_2^\gamma$; similarly, if $u_1 = \gamma(0)$, no strategy under which $u_1(0) = 1$ is in the support of $\pi_2^\gamma$ (since $u_1 < 1$). Therefore, the same argument as in the proof of Lemma 2 shows that $\hat{\sigma}_2^\gamma \in \Sigma_2^*(\pi_2^\gamma)$. Hence, the belief

[25]The proof uses results from Section 3 and, therefore, should not be read before reading Section 3.

[26]That is, modify the second part of Definition 5 to include this contingency.

$\hat{\pi}_1$ given by $\hat{\pi}_1(\hat{\sigma}_2^{\gamma}) = 1$ is an element of $\Pi_1^{\gamma}$. Furthermore, under strategy $\hat{\sigma}_2^{\gamma}$, player 2 always demands 1 and only accepts player 1's demand if player 1 has either conformed to $\gamma$ through time $t^*$ (defined in Section 3.2) or has always demanded $u_1$ (in the $u_1 \neq \gamma(0)$ case) or 1 (in the $u_1 = \gamma(0)$ case). Note that $\exp(-r(-\log(u_1)/r)) = u_1$. Hence, in either case, $u_1(\sigma_1, \hat{\pi}_1) \in \{0, u_1^*(\gamma), u_1\}$ for every strategy $\sigma_1$. Let $\hat{\sigma}_1$ be the strategy of player 1 that always demands $u_1$ (if $u_1 \neq \gamma(0)$) or 1 (if $u_1 = \gamma(0)$) and never accepts player 2's demand. Then $u_1(\hat{\sigma}_1, \hat{\pi}_1) = u_1 = \max_{\sigma_1} u_1(\sigma_1, \hat{\pi}_1)$, completing the proof. *Q.E.D.*

PROOF OF LEMMA 1: I prove the result for pure strategies $\sigma_2$, which immediately implies the result for mixed strategies.

Fix $\pi_2$ such that $\pi_2(\gamma) \geq \varepsilon$ and pure strategy $\sigma_2 \in \Sigma_2^*(\pi_2)$. The plan of the proof is to show that if agreement is not reached by $\tilde{T}$ under strategy profile $(\gamma, \sigma_2)$, then player 2 must be certain that player 1 is playing $\gamma$ at any time $t > \tilde{T}$. This suffices to prove the lemma, because $\sigma_2 \in \Sigma_2^*(\pi_2)$ implies that player 2 accepts $\underline{\gamma}(t)$ no later than time $t = \bar{T}$ if at any time $t > \tilde{T}$ agreement has not been reached and he is certain that player 1 is playing $\gamma$.

I begin by introducing some notation. Let $\chi^{(\pi_2,\sigma_2)}(t)$ be the probability that player 2 assigns to player 1 *not* playing $\gamma$ at date $(t, -1)$ when his initial belief is $\pi_2$ and play up until date $(t, -1)$ is given by player 1 following strategy $\gamma$ and player 2 following (pure) strategy $\sigma_2$; this is determined by Bayes' rule, because $\pi_2(\gamma) \geq \varepsilon > 0$. By convention, if agreement is reached at time $\tau$, let $\chi^{(\pi_2,\sigma_2)}(t) = \chi^{(\pi_2,\sigma_2)}(\tau)$ for all $t > \tau$. Let $t(\gamma, \sigma_2)$ be the time at which agreement is reached under strategy profile $(\gamma, \sigma_2)$ (with the convention that $t(\gamma, \sigma_2) \equiv \infty$ if agreement is never reached under $(\gamma, \sigma_2)$) and let

$$\hat{t}(\gamma, \sigma_2, \pi_2) \equiv \sup\{t : \chi^{(\pi_2,\sigma_2)}(t) > 0\}$$

be the latest time at which player 2 is not certain that player 1 is playing $\gamma$ under strategy profile $(\gamma, \sigma_2)$ with belief $\pi_2$. Let

$$\hat{T} \equiv \sup_{(\pi_2,\sigma_2):\pi_2(\gamma)\geq\varepsilon,\sigma_2\in\Sigma_2^*(\pi_2),t(\gamma,\sigma_2)\geq\hat{t}(\gamma,\sigma_2,\pi_2)} \hat{t}(\gamma, \sigma_2, \pi_2); \tag{12}$$

that is, $\hat{T}$ is the latest possible time $t$ at which player 2 is not certain that player 1 is following $\gamma$ and agreement is not reached by $t$. The remainder of the proof consists of showing that $\hat{T} = \tilde{T}$.

There are three steps involved in showing that $\hat{T} = \tilde{T}$; that is, that the value of the program (12) is $\tilde{T}$. Step 1 shows that in solving (12), one can restrict attention to a simple class of belief–strategy pairs $(\pi_2, \sigma_2)$. Step 2 reduces the constraints that $\sigma_2 \in \Sigma_2^*(\pi_2)$ and $t(\gamma, \sigma_2) \geq \hat{t}(\gamma, \sigma_2, \pi_2)$ to an infinite system of inequalities involving player 1's concession rate and probability. Step 3 solves the reduced program.

STEP 1: *In the definition of* $\hat{T}$ *it is without loss of generality to restrict attention to* $(\pi_2, \sigma_2)$ *such that* $\sigma_2$ *always demands* 1, $\pi_2$ *puts probability* 1 *on player* 1 *conceding at any history* $h^{t+}$ *at which* $u_1(t) \neq \gamma(t)$, *and* $\pi_2$ *puts probability* 0 *on player* 1 *conceding at any history* $h^{t-}$; *that is*, *that the right-hand side of* (12) *continues to equal* $\hat{T}$ *when this additional constraint is imposed on* $(\pi_2, \sigma_2)$.

PROOF: Suppose that $(\pi_2', \sigma_2')$ satisfies $\pi_2'(\gamma) \geq \varepsilon$, $\sigma_2' \in \Sigma_2^*(\pi_2')$, and $t(\gamma, \sigma_2') \geq \hat{t}(\gamma, \sigma_2', \pi_2')$ (the constraints of (12)). Let $\pi_2$ be the belief under which player 1 demands $\gamma(t)$ for all $t \in \mathbb{R}_+$; accepts player 2's demand at every history of the form $(\gamma(\tau), 1)_{\tau \leq t}$ at the same rate and probability at which player 1 deviates from $\gamma$ at time $t$ (i.e., at date $(t, -1)$, $(t, 0)$, or $(t, 1)$) under strategy profile $(\pi_2', \sigma_2')$; and rejects player 2's demand at every other history. Clearly, there exists a strategy $\sigma_2 \in \Sigma_2^*(\pi_2)$ that always demands 1 and that, in addition, rejects player 1's demand whenever rejection is optimal under belief $\pi_2$. Note that player 1's rate and probability of deviating from $\gamma$ at history $(\gamma(\tau), 1)_{\tau \leq t}$ under belief $\pi_2$ is the same as at time $t$ under strategy profile $(\pi_2', \sigma_2')$, and that player 2's continuation payoff after such a deviation is weakly higher in the former case. Recall that strategy $\gamma$ never accepts player 2's demand, so agreement is reached only if player 2 accepts player 1's demand or if player 1 has deviated from $\gamma$. Therefore, since rejecting player 1's demand $\gamma(t)$ under strategy profile $(\pi_2', \sigma_2')$ is optimal for all $t < t(\gamma, \sigma_2')$, it follows that rejecting player 1's demand $\gamma(t)$ at history $(\gamma(\tau), 1)_{\tau \leq t}$ is optimal under belief $\pi_2$ for all $t < t(\gamma, \sigma_2')$. Since $\sigma_2$ prescribes rejection whenever it is optimal, this implies that $t(\gamma, \sigma_2) \geq t(\gamma, \sigma_2')$. Furthermore, $\chi^{(\pi_2, \sigma_2)}(t) = \chi^{(\pi_2', \sigma_2')}(t)$ for all $t \in \mathbb{R}_+$, so $\hat{t}(\gamma, \sigma_2, \pi_2) = \hat{t}(\gamma, \sigma_2', \pi_2')$. Hence, $t(\gamma, \sigma_2) \geq \hat{t}(\gamma, \sigma_2, \pi_2)$. Finally, $\pi_2(\gamma) \geq \varepsilon$. Therefore, $(\pi_2, \sigma_2)$ satisfies the constraints of (12), $\sigma_2$ always demands $u_2(t) = 1$, $\pi_2$ puts probability 1 on player 1 conceding at any history $h^{t+}$ at which $u_1(t) \neq \gamma(t)$, $\pi_2$ puts probability 0 on player 1 conceding at any history $h^{t-}$, and $\hat{t}(\gamma, \sigma_2, \pi_2) \geq \hat{t}(\gamma, \sigma_2', \pi_2')$. So the right-hand side of (12) continues to equal $\hat{T}$ when the additional constraint is imposed. *Q.E.D.*

Step 2 of the proof builds on Step 1 to further simplify the constraint set of (12). For any belief $\pi_2$ satisfying the conditions of Step 1, let $\lambda^{\pi_2}(t)$ and $p^{\pi_2}(t)$ be the concession rate and probability of player 1 at history $(\gamma(\tau), 1)_{\tau \leq t}$ when her strategy is given by $\pi_2$; let $S^{\pi_2}$ be the (countable) set of times $s$ such that $p^{\pi_2}(s) > 0$; and let $\hat{t}(\pi_2) \equiv \hat{t}(\gamma, \sigma_2^0, \pi_2)$, where $\sigma_2^0$ is the strategy that always demands 1 and always rejects player 1's demand. Fixing a strategy $\sigma_2 \in \Sigma_2^*(\pi_2)$ that always demands 1 (which exists, by Step 1), note that $(\gamma, \sigma_2)$ and $(\gamma, \sigma_2^0)$ induce the same path of play until time $t(\gamma, \sigma_2)$ (at which point player 2 accepts under $\sigma_2$, but not under $\sigma_2^0$), and, therefore, $t(\gamma, \sigma_2) \geq \hat{t}(\gamma, \sigma_2, \pi_2)$ if and only if $t(\gamma, \sigma_2) \geq \hat{t}(\pi_2)$. Hence, there exists a strategy $\sigma_2 \in \Sigma_2^*(\pi_2)$ that always demands 1 and satisfies $t(\gamma, \sigma_2) \geq \hat{t}(\gamma, \sigma_2, \pi_2)$ if and only if it is optimal for player 2 to reject player 1's offer until time $\hat{t}(\pi_2)$ when he always demands

1, player 1 plays $\gamma$, and his initial belief is $\pi_2$. I now use this observation to simplify the constraints of (12).

STEP 2A: *For any belief $\pi_2$ satisfying the conditions of Step 1, there exists a strategy $\sigma_2 \in \Sigma_2^*(\pi_2)$ that always demands 1 and satisfies $t(\gamma, \sigma_2) \geq \hat{t}(\gamma, \sigma_2, \pi_2)$ if and only if*

$$
\begin{aligned}
(13)\qquad 1-\gamma(t) \leq {} & \int_t^{\hat{t}(\pi_2)} \exp\biggl(-r(\tau - t) - \int_t^{\tau} \lambda^{\pi_2}(s)\,ds\biggr) \\
& \times \biggl(\prod_{s\in S^{\pi_2}\cap(t,\tau)} \bigl(1 - p^{\pi_2}(s)\bigr)\biggr)\lambda^{\pi_2}(\tau)\,d\tau \\
& + \sum_{s\in S^{\pi_2}\cap(t,\hat{t}(\pi_2))} \exp\biggl(-r(s-t) - \int_t^{s} \lambda^{\pi_2}(q)\,dq\biggr) \\
& \times \biggl(\prod_{q\in S^{\pi_2}\cap(t,s)} \bigl(1 - p^{\pi_2}(q)\bigr)\biggr) p^{\pi_2}(s) \\
& + \exp\biggl(-r\bigl(\hat{t}(\pi_2) - t\bigr) - \int_t^{\hat{t}(\pi_2)} \lambda^{\pi_2}(s)\,ds\biggr) \\
& \times \biggl(\prod_{s\in S^{\pi_2}\cap(t,\hat{t}(\pi_2))} \bigl(1 - p^{\pi_2}(s)\bigr)\biggr) v\bigl(\hat{t}(\pi_2)\bigr)
\end{aligned}
$$

*for all $t < \hat{t}(\pi_2)$.*

PROOF: By the above discussion, it suffices to show that (13) holds if and only if it is optimal for player 2 to reject player 1's offer until time $\hat{t}(\pi_2)$ when he always demands 1, player 1 plays $\gamma$, and his initial belief is $\pi_2$. The left-hand side of (13) is player 2's payoff from accepting player 1's demand at date $(t, 1)$ when $p^{\pi_2}(t) = 0$. The right-hand side of (13) is player 2's continuation payoff from rejecting player 1's demand until time $\hat{t}(\pi_2)$ when $p^{\pi_2}(t) = 0$. Thus, (13) must hold if it is optimal for player 2 to reject until time $\hat{t}(\pi_2)$, and (13) implies that it is optimal for player 2 to reject at times before $\hat{t}(\pi_2)$ where $p^{\pi_2}(t) = 0$. It remains to show that (13) implies that it is optimal for player 2 to reject at times before $\hat{t}(\pi_2)$ where $p^{\pi_2}(t) > 0$. Suppose that $p^{\pi_2}(t) > 0$. At date $(t, -1)$, the fact that $S^{\pi_2}$ is countable and (13) holds at all times before $t$ that are not in $S^{\pi_2}$ implies that $\lim_{\tau\uparrow t}(1 - \gamma(\tau))$ is weakly less than player 2's continuation payoff from rejecting player 1's demand until time $\hat{t}(\pi_2)$. Furthermore, the fact that player 1 concedes with probability 0 at date $(t, -1)$ (as $\pi_2$ satisfies the conditions of Step 1) implies that $\lim_{\tau\uparrow t}(1-\gamma(\tau))$ is indeed player 2's payoff from accepting at date $(t, -1)$. Thus, rejecting is optimal at date $(t, -1)$. At date $(t, 1)$, player 2's payoff from accepting is $(1-p^{\pi_2}(t)/2)(1-\gamma(t))+(p^{\pi_2}(t)/2)(1)$, while his continuation payoff from rejecting until time $\hat{t}(\pi_2)$ is $1-p^{\pi_2}(t)$ times the right-hand side of (13) plus $p^{\pi_2}(t)(1)$. Hence, (13) implies that rejecting is optimal at date $(t,1)$ as well. So (13) implies that it is optimal for player 2 to reject at times before $\hat{t}(\pi_2)$ where $p^{\pi_2}(t)>0$ (when he always demands 1, player 1 plays $\gamma$, and his initial belief is $\pi_2$). *Q.E.D.*

Step 2b shows that one can replace $1-\gamma(t)$ with $v(t)$ in Step 2a.

STEP 2B: *For any belief $\pi_2$ satisfying the conditions of Step 1, there exists a strategy $\sigma_2 \in \Sigma_2^*(\pi_2)$ that always demands 1 and satisfies $t(\gamma,\sigma_2) \geq \hat{t}(\gamma,\sigma_2,\pi_2)$ if and only if*

$$
\begin{aligned}
(14)\qquad v(t) \leq & \int_t^{\hat{t}(\pi_2)} \exp\left(-r(\tau-t)-\int_t^{\tau}\lambda^{\pi_2}(s)\,ds\right) \\
& \times \left(\prod_{s\in S^{\pi_2}\cap(t,\tau)}\bigl(1-p^{\pi_2}(s)\bigr)\right)\lambda^{\pi_2}(\tau)\,d\tau \\
& + \sum_{s\in S^{\pi_2}\cap(t,\hat{t}(\pi_2))} \exp\left(-r(s-t)-\int_t^{s}\lambda^{\pi_2}(q)\,dq\right) \\
& \times \left(\prod_{q\in S^{\pi_2}\cap(t,s)}\bigl(1-p^{\pi_2}(q)\bigr)\right)p^{\pi_2}(s) \\
& + \exp\left(-r\bigl(\hat{t}(\pi_2)-t\bigr)-\int_t^{\hat{t}(\pi_2)}\lambda^{\pi_2}(s)\,ds\right) \\
& \times \left(\prod_{s\in S^{\pi_2}\cap(t,\hat{t}(\pi_2))}\bigl(1-p^{\pi_2}(s)\bigr)\right)v\bigl(\hat{t}(\pi_2)\bigr)
\end{aligned}
$$

*for all $t<\hat{t}(\pi_2)$.*

PROOF: By Step 2a, it suffices to show that (13) is equivalent to (14). Note that (14) immediately implies (13) because $v(t)\geq 1-\gamma(t)$ for all $t$. For the converse, suppose that (13) holds and that $v(t)>1-\gamma(t)$ (as (13) and (14) are identical at $t$ if $v(t)=1-\gamma(t)$). Now $v(t)>1-\gamma(t)$ implies that $v(t)=e^{-r(\tau-t)}(1-\underline{\gamma}(\tau))$ for some $\tau>t$ such that $v(\tau,-1)=1-\underline{\gamma}(\tau)$. Therefore, $v(\tau,-1)$ is weakly less than the limit as $s\uparrow\tau$ of the right-hand side of (14) evaluated at time $s$ (with the convention that the right-hand side of (14) equals $v(s)$ if $s\geq\hat{t}(\pi_2)$). But the right-hand side of (14) at time $t$ is at least $e^{-r(\tau-t)}$ times as large as this limit, which implies that the right-hand side of (14) at time $t$ is at least $e^{-r(\tau-t)}v(\tau,-1)=v(t)$. Hence, (14) holds. *Q.E.D.*

Let $\chi^{\pi_2}(t) \equiv \chi^{(\pi_2, \sigma_2^0)}$. By Steps 1 and 2b (and the use of Bayes' rule to compute $\chi^{\pi_2}(t)$), (12) may be rewritten as

$$(15)\qquad \hat{T} = \sup_{\substack{\pi_2:\pi_2(\gamma)\geq\varepsilon,\\ (14)\text{ holds}}} \sup\left\{ t: \vphantom{\int} \right.$$

$$\left. \chi^{\pi_2}(t) = \frac{\exp\left(-\int_0^t \lambda^{\pi_2}(s)\,ds\right) \prod_{s\in S^{\pi_2}\cap[0,t)} (1-p^{\pi_2}(s)) - \varepsilon}{\exp\left(-\int_0^t \lambda^{\pi_2}(s)\,ds\right) \prod_{s\in S^{\pi_2}\cap[0,t)} (1-p^{\pi_2}(s))} > 0 \right\}.$$

The last step of the proof is solving this simplified program. Steps 3a and 3b show that there exists some belief $\pi_2$ that both attains the (outer) supremum in (15) (with the convention that the supremum is attained at $\pi_2$ if $\hat{t}(\pi_2) = \hat{T} = \infty$) and also maximizes $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t)$ over all beliefs $\pi_2$ that attain the supremum (note that this limit exists for all $\pi_2$, because $\chi^{\pi_2}(t)$ is nonincreasing). I then show that (14) must hold with equality (at all $t < \hat{T}$) under any such belief $\pi_2$, which implies that (15) may be solved under the additional constraint that (14) holds with equality. This final program is also solved in Step 3a and has value $\tilde{T}$.

STEP 3A: *There exists a belief $\pi_2$ that attains the supremum in* (15). *In addition, the value of* (15) *under the additional constraint that* (14) *holds with equality equals $\tilde{T}$.*

PROOF: The plan of the proof is as follows. I first construct a belief $\pi_2$ such that $\pi_2(\gamma) \geq \varepsilon$ and (14) holds at all times $t < \hat{T}$ under $\pi_2$. I then show that if there exists a time $t < \hat{T}$ at which (14) holds with strict inequality under $\pi_2$, then there exists an alternative belief $\pi_2'$ that attains the supremum in (15). Finally, I show that if (14) holds with equality at all times $t < \hat{T}$ under $\pi_2$, then $\pi_2$ itself attains the supremum in (15), which equals $\tilde{T}$.

Fix a sequence $\{\chi^{\pi_2^n}\}$ such that $\hat{t}(\pi_2^n) \uparrow \hat{T}$, $\pi_2^n(\gamma) \geq \varepsilon$ for all $n$, and (14) holds for all $n$. Note that $\chi^{\pi_2^n}(t)$ is nonincreasing in $t$ for all $n$. Since the space of monotone functions from $\mathbb{R}_+$ to $[0, 1]$ is sequentially compact (by Helly's selection theorem; see, e.g., Billingsley (1995, Theorem 25.9)), there exists a subsequence $\{\chi^{\pi_2^m}\}$ that converges pointwise to some (nonincreasing) function $\chi^{\pi_2}$.[27] Furthermore, $\chi^{\pi_2}(0) \leq 1-\varepsilon$, because $\chi^{\pi_2^m}(0) \leq 1-\varepsilon$ for all $m$. Let $\pi_2 \in \Delta(\Sigma_1)$ be a belief such that $\pi_2(\gamma) \geq \varepsilon$ and player 1 demands $\gamma(t)$ for all $t$, concedes at rate $\lambda^{\pi_2}(t) = -\frac{\chi^{\pi_2 \prime}(t)}{1-\chi^{\pi_2}(t)}$ if $\chi^{\pi_2}$ is differentiable at $t$ and $\chi^{\pi_2}(t) > 0$, and concedes at rate $\lambda^{\pi_2}(t) = 0$ otherwise, and concedes with discrete probability $p^{\pi_2}(t) = \frac{\lim_{\tau \uparrow t} \chi^{\pi_2}(\tau) - \chi^{\pi_2}(t)}{(1-\chi^{\pi_2}(t)) \lim_{\tau \uparrow t} \chi^{\pi_2}(\tau)}$ if $\lim_{\tau \uparrow t} \chi(t) > 0$ and concedes with probability $p^{\pi_2}(t) = 0$ otherwise. Note that such a belief exists, because it can be easily verified that for any belief corresponding to concession rate and probability $\lambda^{\pi_2}$ and $p^{\pi_2}$,

$$\chi^{\pi_2}(t) = \frac{\exp\left(-\int_0^t \lambda^{\pi_2}(s)\,ds\right) \prod_{s \in S^{\pi_2} \cap [0,t)} (1-p^{\pi_2}(s)) - (1-\chi^{\pi_2}(0))}{\exp\left(-\int_0^t \lambda^{\pi_2}(s)\,ds\right) \prod_{s \in S^{\pi_2} \cap [0,t)} (1-p^{\pi_2}(s))} \tag{16}$$

for all $t$,

and, therefore, player 1 never concedes with probability at least $1-\chi^{\pi_2}(0) \geq \varepsilon$ under any belief with this concession rate and probability.

Observe that (14) holds at all times $t < \hat{T}$ under $\pi_2$. To see this, note that the fact that $\chi^{\pi_2^m}(t) \to \chi^{\pi_2}(t)$ for all $t$ implies that

$$\begin{aligned} &\exp\left(-\int_0^t \lambda^{\pi_2^m}(s)\,ds\right) \prod_{s \in S^{\pi_2} \cap [0,t)} \left(1-p^{\pi_2^m}(s)\right) \\ &\quad \to \exp\left(-\int_0^t \lambda^{\pi_2}(s)\,ds\right) \prod_{s \in S^{\pi_2} \cap [0,t)} \left(1-p^{\pi_2}(s)\right) \end{aligned}$$

for all $t$. Since for all $t < \hat{T}$, there exists $M > 0$ such that (14) holds at time $t$ under $\pi_2^m$ for all $m > M$, this implies that (14) holds at all times $t < \hat{T}$ under $\pi_2$.

I now show that if there exists a time $t < \hat{T}$ at which (14) holds with strict inequality under belief $\pi_2$, then there exists an alternative belief $\pi_2'$ that attains the supremum in (15). Suppose such a time $t$ exists. I claim that there then exists a time $t_1 < \hat{t}(\pi_2)$ at which (14) holds with strict inequality and, in addition,

[27] Showing that the space of monotone functions from $\mathbb{R}_+ \to [0,1]$ is sequentially compact requires a slightly different version of Helly's selection theorem than that in Billingsley (1995), so here is a direct proof: If $\{f_n\}$ is a sequence of monotone functions $\mathbb{R}_+ \to [0,1]$, then there exists a subsequence $\{f_m\} \subseteq \{f_n\}$ that converges on $\mathbb{Q}_+$ to a monotone function $f: \mathbb{Q}_+ \to [0,1]$. Let $\tilde{f}: \mathbb{R}_+ \to [0,1]$ be given by $\tilde{f}(x) = \lim_{l \to \infty} f(x_l)$, where $\{x_l\}_{l=1}^{\infty} \uparrow x$ and $x_l \in \mathbb{Q}_+$ for all $l$. Then $\tilde{f}$ is monotone, which implies that there is a countable set $\mathcal{S}$ such that $\tilde{f}$ is continuous on $\mathbb{R}_+ \setminus \mathcal{S}$. Since $\mathcal{S}$ is countable, there exists a subsubsequence $\{f_k\} \subseteq \{f_m\}$ such that $\{f_k\}$ converges on $\mathcal{S}$. Finally, let $\hat{f}(x) = \tilde{f}(x)$ if $x \in \mathbb{R}_+ \setminus \mathcal{S}$ and $\hat{f}(x) = \lim_{k \to \infty} f_k(x)$ if $x \in \mathcal{S}$. Then $\{f_k\} \to \hat{f}$.

either $\int_{t_1}^{t_1+\Delta}\lambda^{\pi_2}(s)\,ds>0$ for all $\Delta>0$ or $\sum_{s\in S^{\pi_2}\cap[t_1,t_1+\Delta)}p^{\pi_2}(s)>0$ for all $\Delta>0$. To see this, note that there must exist a time $t'\in(t,\hat{t}(\pi_2))$ such that either $\int_{t'}^{t'+\Delta}\lambda^{\pi_2}(s)\,ds>0$ for all $\Delta>0$ or $p^{\pi_2}(t')>0$ (because otherwise (14) could not hold with strict inequality at $t$). Let $t_1$ be the infimum of such times $t'$, and note that either $\int_{t_1}^{t_1+\Delta}\lambda^{\pi_2}(s)\,ds>0$ for all $\Delta>0$ or $\sum_{s\in S^{\pi_2}\cap[t_1,t_1+\Delta)}p^{\pi_2}(s)>0$ for all $\Delta>0$. Then the fact that (14) holds with strict inequality at time $t$ implies that (14) holds with strict inequality at time $t_1$, because otherwise the fact that $\int_t^{t_1}\lambda^{\pi_2}(s)\,ds=0$ and $p^{\pi_2}(t'')=0$ for all $t''\in[t,t_1)$ would imply that (14) could not hold with strict inequality at time $t$. This proves the claim.

Thus, let $t_0<\hat{t}(\pi_2)$ be such that (14) holds with strict inequality at time $t_0$ and, in addition, $\int_{t_0}^{t_0+\Delta}\lambda^{\pi_2}(s)\,ds>0$ for all $\Delta>0$ (the case where $\sum_{s\in S^{\pi_2}\cap[t_0,t_0+\Delta)}p^{\pi_2}(s)>0$ is similar and thus omitted). Since $v$ is continuous but for downward jumps, there exist $\eta>0$ and $\Delta>0$ such that (14) holds with strict inequality at $t$ for all $t\in[t_0,t_0+\Delta)$ when $\lambda^{\pi_2}(t)$ is replaced by $(1-\eta)\lambda^{\pi_2}(t)$ for all $t\in[t_0,t_0+\Delta)$. Define $\lambda^{\pi_2'}(t)$ by $\lambda^{\pi_2'}(t)\equiv\lambda^{\pi_2}(t)$ for all $t\notin[t_0,t_0+\Delta)$ and $\lambda^{\pi_2'}(t)\equiv(1-\eta)\lambda^{\pi_2}(t)$ for all $t\in[t_0,t_0+\Delta)$. Next, I claim that at time $t_0$, player 2's continuation payoff from rejecting $\gamma$ until $\hat{t}(\pi_2)$ is strictly lower when player 1's concessions are given by $(\lambda^{\pi_2}(t),p^{\pi_2}(t))$ than when they are given by $(\lambda^{\pi_2'}(t),p^{\pi_2''}(t))$, where $p^{\pi_2''}(t)$ is defined by $p^{\pi_2''}(t)\equiv p^{\pi_2}(t)$ for all $t\neq t_0$ and $p^{\pi_2''}(t_0)\equiv 1-\exp(-\eta\int_t^{t+\Delta}\lambda^{\pi_2}(s)\,ds)(1-p^{\pi_2}(t_0))>0$. This follows because the total probability with which player 1 concedes in the interval $[t_0,t_0+\Delta)$ is the same under $(\lambda^{\pi_2}(t),p^{\pi_2}(t))$ and under $(\lambda^{\pi_2'}(t),p^{\pi_2''}(t))$, and some probability mass of concession is moved earlier to $t_0$ under $(\lambda^{\pi_2'}(t),p^{\pi_2''}(t))$. Therefore, there exists $\zeta\in(0,p^{\pi_2''}(t_0))$ such that at time $t_0$, player 2's continuation payoff from rejecting $\gamma$ until $\hat{t}(\pi_2)$ is the same when player 1's concessions are given by $(\lambda^{\pi_2}(t),p^{\pi_2}(t))$ and when they are given by $(\lambda^{\pi_2'}(t),p^{\pi_2'}(t))$, where $p^{\pi_2'}(t)$ is defined by $p^{\pi_2'}(t)\equiv p^{\pi_2}(t)$ for all $t\neq t_0$ and $p^{\pi_2'}(t_0)\equiv p^{\pi_2''}(t_0)-\zeta<p^{\pi_2''}(t_0)$. The fact that (14) holds at all $t<\hat{T}$ when player 1's concessions are given by $(\lambda^{\pi_2}(t),p^{\pi_2}(t))$ now implies that (14) holds at all $t<\hat{T}$ when player 1's concessions are given by $(\lambda^{\pi_2'}(t),p^{\pi_2'}(t))$. Furthermore, $\exp(-\int_0^{\hat{T}}\lambda^{\pi_2'}(t)\,dt)\prod_{s\in S^{\pi_2}\cap[0,\hat{T})}(1-p^{\pi_2'}(s))>\exp(-\int_0^{\hat{T}}\lambda^{\pi_2}(t)\,dt)\prod_{s\in S^{\pi_2}\cap[0,\hat{T})}(1-p^{\pi_2}(s))\geq\varepsilon$. Therefore, $\sup\{t:\chi^{\pi_2'}(t)>0\}\geq\hat{T}$, so by the definition of $\hat{T}$, it must be that $\sup\{t:\chi^{\pi_2'}(t)>0\}=\hat{T}$. Thus, $\pi_2'$ attains the supremum in (15).

Finally, suppose that (14) holds with equality at all $t<\hat{T}$ under belief $\pi_2$ (defined above). Then (14) holds with equality at all $t<\hat{t}(\pi_2)$ under belief $\pi_2$, because $\hat{t}(\pi_2)\leq\hat{T}$ (by definition of $\hat{T}$). Let $t<\hat{t}(\pi_2)$ be a time at which $v$ is differentiable. Then the derivative of the right-hand side of (14) at $t$ must exist and equal $v'(t)$. This implies that $p^{\pi_2}(t) = 0$, and, by Leibniz's rule, the derivative of the right-hand side of (14) equals $-\lambda^{\pi_2}(t) + (r + \lambda^{\pi_2}(t))v(t)$. Hence,[28]

$$\lambda^{\pi_2}(t) = \frac{rv(t) - v'(t)}{1 - v(t)}.$$

Since $v$ is differentiable almost everywhere, this implies that

$$\int_0^\tau \lambda^{\pi_2}(s)\,ds = \int_0^\tau \lambda(s)\,ds \quad \text{for all } \tau < \hat{t}(\pi_2), \tag{17}$$

where $\lambda$ is defined by (3). Similarly, if (14) holds with equality, then the difference between the limit as $s \uparrow t$ of the right-hand side of (14) evaluated at $s$ and the limit as $s \downarrow t$ of the right-hand side of (14) evaluated at $s$ must equal $v(t, -1) - v(t)$ for all $t < \hat{t}(\pi_2)$. By inspection, this difference equals $p^{\pi_2}(t) - p^{\pi_2}(t)v(t)$. Hence,

$$p^{\pi_2}(t) = \frac{v(t, -1) - v(t)}{1 - v(t)}$$

for all $t < \hat{t}(\pi_2)$ at which $v$ is discontinuous, and $p^{\pi_2}(t) = 0$ otherwise.[29] Therefore,

$$\prod_{s \in S^{\pi_2} \cap [0,\tau)} \bigl(1 - p^{\pi_2}(s)\bigr) = \prod_{s \in S \cap [0,\tau)} \bigl(1 - p(s)\bigr) \tag{18}$$

for all $\tau < \hat{t}(\pi_2)$, where $S$ is the set of discontinuity points of $v$ and $p$ is defined by (4). Combining (17) and (18), I conclude that if (14) holds with equality under belief $\pi_2$, then

$$\hat{t}(\pi_2) = \sup\left\{ t : \exp\left(-\int_0^t \lambda(s)\,ds\right) \prod_{s \in S \cap [0,t)} \bigl(1 - p(s)\bigr) > \varepsilon \right\},$$

which equals $\tilde{T}$. In addition, $\chi^{\pi_2}(t) \leq 0$ for all $t > \tilde{T}$ (by (16), recalling that $1 - \chi^{\pi_2}(0) \geq \varepsilon$), so $\hat{T} = \tilde{T}$ and the supremum in (15) is attained by $\pi_2$. *Q.E.D.*

STEP 3B: *There exists a belief that both attains the supremum in* (15) *and maximizes* $\lim_{t \uparrow \hat{T}} \chi^{\pi_2}(t)$ *over all beliefs* $\pi_2$ *that attain the supremum in* (15).

[28]Note that $v(t) = 1$ is impossible. To see this, $v(\tau) \in [0, 1]$ implies that $v'(t) = 0$ if $v'(t)$ exists and $v(t) = 1$. But then the equation $v'(t) = -\lambda^{\pi_2}(t) + (r + \lambda^{\pi_2}(t))v(t)$ would reduce to $0 = r$, violating the assumption that $r > 0$.

[29]The fact that $v$ cannot jump up rules out $v(t) = 1$.

PROOF: Let $\chi \in [0, 1]$ be the supremum of $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t)$ over all beliefs $\pi_2$ that attain the supremum in (15). If $\chi = 0$, then any belief $\pi_2$ that attains the supremum in (15) also satisfies $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t) = \chi$. Thus, suppose that $\chi > 0$. Let $\{\pi_2^n\}$ be a sequence of beliefs that all attain the supremum in (15) such that $\lim_{t\uparrow\hat{T}} \chi^{\pi_2^n}(t) \uparrow \chi$. The sequential compactness argument in Step 3a implies that there exists a subsequence $\{\pi_2^m\} \subseteq \{\pi_2^n\}$ and a belief $\pi_2$ that satisfies the constraints of (15) such that $\chi^{\pi_2^m}(t) \to \chi^{\pi_2}(t)$ for all $t$. Furthermore, $\chi^{\pi_2}(t)$ is nonincreasing, so $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t)$ exists. Because $\pi_2$ satisfies the constraints of (15), $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t) \le \chi$. Now suppose, toward a contradiction, that $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t) < \chi$. Then there exists $\eta > 0$ and $t' \le \hat{T}$ such that $\chi^{\pi_2}(t') < \chi - \eta$. Since $\lim_{m\to\infty} \lim_{t\uparrow\hat{T}} \chi^{\pi_2^m}(t) = \chi$, there exists $M > 0$ such that, for all $m > M$, $\lim_{t\uparrow\hat{T}} \chi^{\pi^m}(t) > \chi - \eta$. In addition, $\chi^{\pi_2^m}(t)$ is nonincreasing for all $m$, so this implies that $\chi^{\pi_2^m}(t') > \chi - \eta$ for all $m > M$. Now $\chi^{\pi_2^m}(t') \to \chi^{\pi_2}(t')$ implies that $\chi^{\pi_2}(t') \ge \chi - \eta$, a contradiction. Therefore, $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t) = \chi$. Finally, the fact that $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t) > 0$ implies that $\pi_2$ attains the supremum in (15). *Q.E.D.*

I now complete the proof of Lemma 1. If (14) holds with strict inequality at some time $t < \hat{T}$ under a belief $\pi_2$ such that $\hat{t}(\pi_2) = \hat{T}$, then the procedure for modifying $\pi_2$ described in the fifth paragraph of the proof of Step 3a yields a belief $\pi_2'$ such that $\hat{t}(\pi_2') = \hat{T}$ and $\lim_{t\uparrow\hat{T}} \chi^{\pi_2'}(t) > \lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t)$. This implies that the only beliefs $\pi_2$ that both attain the supremum in (15) and maximize $\lim_{t\uparrow\hat{T}} \chi^{\pi_2}(t)$ (over all beliefs that attain the supremum in (15)) satisfy the additional constraint that (14) holds with equality. Since such a belief exists by Step 3b, the value of (15) equals the value of (15) under this additional constraint, which equals $\tilde{T}$ by Step 3a. *Q.E.D.*

PROOF OF THEOREM 1: Let $\gamma_n$ and $\gamma^*$ be defined as in Section 3.4. To show that $\lim_{n\to\infty} u_1^*(\gamma_n) = 1/(1 - \log\varepsilon)$, it remains only to show that $T_n^1 > \tilde{T}(\gamma_n)$ for all $n \in \mathbb{N}$. To see this, note that $T_n^1 = \frac{1}{r}\log(\frac{n+1}{n}(1 - \log\varepsilon))$. Since $\gamma_n(t) = (\frac{n}{n+1})\frac{e^{rt}}{1-\log\varepsilon}$ for all $t \le T_n^1$ and $\gamma_n$ is nondecreasing, it follows that $v(t) = 1 - (\frac{n}{n+1})\frac{e^{rt}}{1-\log\varepsilon}$ for all $t \le T_n^1$. Therefore,

$$\begin{aligned}
&\exp\left(-\int_0^{T_n^1} \frac{rv(t) - v'(t)}{1 - v(t)}\,dt\right) \prod_{s\in S\cap[0,T_n^1]} \left(\frac{1 - v(s, -1)}{1 - v(s)}\right) \\
&\quad = \exp\left(-\int_0^{T_n^1} r\left(\frac{n+1}{n}\right)(1 - \log\varepsilon)e^{-rt}\,dt\right) \\
&\quad = \exp\left(-\frac{1}{n}(1 - \log\varepsilon)\right)\varepsilon \\
&\quad < \varepsilon.
\end{aligned}$$

Hence, by the definition of $\tilde{T}(\gamma_n)$, $T_n^1 \geq \tilde{T}(\gamma_n)$. Furthermore, the fact that $\exp(-\int_0^\tau \frac{rv(t)-v'(t)}{1-v(t)}\,dt)$ is strictly decreasing in $\tau$ for all $\tau \in [0, T_n^1]$ implies that $T_n^1 > \tilde{T}(\gamma_n)$.

I now complete the proof of Theorem 1 by showing that $\gamma^*$ is the unique maxmin posture[30]; that is, I show that if $\{\gamma_n\}$ is any sequence of postures converging pointwise to some posture $\gamma$ satisfying $u_1^*(\gamma_n) \to u_1 \geq 1/(1-\log\varepsilon)$, then $\gamma = \gamma^*$. There are two steps. First, letting $\{v_n\}$ be the continuation value functions corresponding to the $\{\gamma_n\}$ and letting $v^*$ be the continuation value function corresponding to $\gamma^*$ (that is, $v^*(t) = \max\{1 - e^{rt}/(1-\log\varepsilon), 0\}$), I show that $\sup_{t\in\mathbb{R}_+} e^{-rt}|v^*(t) - v_n(t)| \to 0$. This step is divided into showing first that $\sup_{t\leq T(\gamma)}|v^*(t)-v_n(t)| \to 0$ (Step 1a) and then that $\sup_{t\in\mathbb{R}_+} e^{-rt}|v^*(t) - v_n(t)| \to 0$ (Step 1b). Second, I show that this implies that $\gamma = \gamma^*$.

STEP 1A: *For all $\delta > 0$, there exists $\zeta > 0$ such that if $u_1^*(\gamma) \geq 1/(1-\log\varepsilon) - \zeta$, then $\sup_{t\leq\tilde{T}(\gamma)}|v^*(t) - v(t)| \leq \delta$ (where $v$ is the continuation value function corresponding to $\gamma$).*

PROOF: The plan is first to note that any posture $\gamma$ that guarantees close to $1/(1-\log\varepsilon)$ must demand close to $e^{rt}(1/(1-\log\varepsilon))$ (or more) for all $t \leq T(\gamma)$ and must also have $\tilde{T}(\gamma)$ close to $T^1$ (the time at which $\gamma^*(t)$ reaches 1), and then to show that any posture with these two properties must correspond to a continuation value function that is close to $v^*$ until time $\tilde{T}(\gamma)$.

Formally, suppose that $u_1^*(\gamma) \geq 1/(1-\log\varepsilon) - \zeta$ for some posture $\gamma$ and some $\zeta \in (0, 1/(1-\log\varepsilon))$. Let $T^1 \equiv (1/r)\log(1-\log\varepsilon)$. Then it must be that $\tilde{T}(\gamma) \leq T^1 - (1/r)\log(1-\zeta(1-\log\varepsilon))$, for otherwise it would follow from $T(\gamma) \geq \tilde{T}(\gamma)$ that

$$\begin{aligned} u_1^*(\gamma) &= \min_{t\leq T(\gamma)} e^{-rt}\underline{\gamma}(t) \leq e^{-r\tilde{T}(\gamma)}\underline{\gamma}\bigl(\tilde{T}(\gamma)\bigr) \\ &< \exp\bigl(-rT^1 + \log\bigl(1-\zeta(1-\log\varepsilon)\bigr)\bigr)(1) = \frac{1}{1-\log\varepsilon} - \zeta. \end{aligned}$$

Furthermore, if $u_1^*(\gamma) \geq 1/(1-\log\varepsilon) - \zeta$, it must also be that $\gamma(t) \geq e^{rt}(1/(1-\log\varepsilon) - \zeta)$ for all $t \leq T(\gamma)$, for otherwise $\min_{t\leq T(\gamma)} e^{-rt}\underline{\gamma}(t)$ would be strictly less than $1/(1-\log\varepsilon) - \zeta$. I will show that, for all $\delta > 0$, there exists $\zeta \in (0, 1/(1-\log\varepsilon))$ such that if $\gamma(t) \geq e^{rt}(1/(1-\log\varepsilon) - \zeta)$ for all $t \leq T(\gamma)$ and $\sup_{t\leq\tilde{T}(\gamma)}|v^*(t) - v(t)| > \delta$, then $\tilde{T}(\gamma) > T^1 - (1/r)\log(1-\zeta(1-\log\varepsilon))$. This completes the proof of Step 1a.

Fix $\delta > 0$ and $\zeta \in (0, 1/(1-\log\varepsilon))$, and suppose that $\gamma(t) \geq e^{rt}(1/(1-\log\varepsilon) - \zeta)$ for all $t \leq T(\gamma)$ and $\sup_{t\leq\tilde{T}(\gamma)}|v^*(t) - v(t)| > \delta$. A straightforward

[30]Technically, I must also show that $u_1^*(\gamma^*) \leq 1/(1-\log\varepsilon)$. In fact, $u_1^*(\gamma^*) = 0$ by Lemma 3 and the observation that $T(\gamma^*) = \infty$ (which follows because $\gamma^*(t) = 1$ for all $t \geq \tilde{T}(\gamma^*)$).

implication is that $v(t) \le 1 - e^{rt}(1/(1-\log\varepsilon) - \zeta)$ for all $t \le T(\gamma)$. Also, if $\tilde{T}(\gamma)$ is finite, then

$$\exp\left(-\int_0^{\tilde{T}(\gamma)} \frac{rv(t) - v'(t)}{1 - v(t)}\,dt\right) \prod_{s\in S\cap[0,\tilde{T}(\gamma)]} \left(\frac{1 - v(s,-1)}{1 - v(s)}\right) \le \varepsilon.$$

Thus, if $\tilde{T}(\gamma)$ is finite, then it must be that

$$\inf_{\substack{v:\mathbb{R}_+\to\mathbb{R}_+:\\ v(t)\le 1-e^{rt}(1/(1-\log\varepsilon)-\zeta),\\ \sup_{t\le\tilde{T}(\gamma)}|v^*(t)-v(t)|>\delta}} \exp\left(-\int_0^{\tilde{T}(\gamma)} \frac{rv(t) - v'(t)}{1 - v(t)}\,dt\right) \times \prod_{s\in S\cap[0,\tilde{T}(\gamma)]} \left(\frac{1 - v(s,-1)}{1 - v(s)}\right) \le \varepsilon, \tag{19}$$

where $\tilde{T}(\gamma)$ is viewed as a parameter and the infimum is taken over all functions $v:\mathbb{R}_+ \to \mathbb{R}_+$ satisfying the constraints. I will show that if $\zeta > 0$ is sufficiently small, then (19) can hold only if $\tilde{T}(\gamma) > T^1 - (1/r)\log(1-\zeta(1-\log\varepsilon))$.

I first show that any attainable value of the program on the left-hand side of (19) can be arbitrarily closely approximated by the value attained by a *continuous* function $v(t)$ satisfying the constraints of (19); hence, in calculating the infimum over such values, attention may be restricted to continuous functions. To see this, fix $\eta \in (0, 1)$ and let

$$S^\eta \equiv \bigcup_{s\in S\cap[0,\tilde{T}(\gamma)]} [s-\eta, s].$$

Define the function $v^\eta(t)$ by $v^\eta(t) \equiv v(t)$ for all $t \notin S^\eta$ and by

$$v^\eta(t) \equiv \left(1 - \frac{t-(s-\eta)}{\eta}\right)v(s-\eta) + \frac{t-(s-\eta)}{\eta}v(s) \quad \text{for all } t \in S^\eta.$$

Observe that $v^\eta$ is continuous and that $v^\eta$ satisfies the constraints of (19) if $\eta$ is sufficiently small.[31] Furthermore, for all $s \in S$,

$$\exp\left(\int_{s-\eta}^{s} \frac{v^{\eta\prime}(t)}{1 - v^\eta(t)}\,dt\right) = \frac{1 - v^\eta(s-\eta)}{1 - v^\eta(s)} = \frac{1 - v(s-\eta)}{1 - v(s)}.$$

[31] To see this, note that the constraint $v(t) \le 1 - e^{rt}(\frac{1}{1-\log\varepsilon} - \zeta)$ specifies that the graph of $v(t)$ lies in a convex subset of $\mathbb{R}_+^2$. Since the graph of $v^\eta(t)$ lies in the convex hull of the graph of $v(t)$, the graph of $v^\eta(t)$ lies in any convex subset of $\mathbb{R}_+^2$ that contains the graph of $v(t)$. Hence, $v^\eta(t) \le 1 - e^{rt}(\frac{1}{1-\log\varepsilon} - \zeta)$ (for all $\eta > 0$). In addition, the fact that $\sup_{t\le\tilde{T}(\gamma)}|v^*(t) - v(t)| > \delta$ and $v^\eta(t) \to v(t)$ for all $t \in \mathbb{R}_+$ implies that $\sup_{t\le\tilde{T}(\gamma)}|v^*(t) - v^\eta(t)| > \delta$ for sufficiently small $\eta$.

Also, since $v^{\eta}(t) \leq 1 - (1/(1-\log\varepsilon) - \zeta) < 1$ for all $t \in [0, \tilde{T}(\gamma)]$ and the measure of $S^{\eta}$ goes to 0 as $\eta \to 0$, it follows that

$$\lim_{\eta \to 0} \exp\left(-\int_{S^{\eta}} \frac{rv^{\eta}(t)}{1 - v^{\eta}(t)}\, dt\right) = 1.$$

Therefore,

$$\begin{aligned}
&\lim_{\eta \to 0} \exp\left(-\int_0^{\tilde{T}(\gamma)} \frac{rv^{\eta}(t) - v^{\eta\prime}(t)}{1 - v^{\eta}(t)}\, dt\right) \\
&\quad = \lim_{\eta \to 0} \exp\left(-\int_{[0,\tilde{T}(\gamma)]\setminus S^{\eta}} \frac{rv^{\eta}(t) - v^{\eta\prime}(t)}{1 - v^{\eta}(t)}\, dt\right) \\
&\qquad \times \exp\left(-\int_{S^{\eta}} \frac{rv^{\eta}(t)}{1 - v^{\eta}(t)}\, dt\right) \prod_{s \in S \cap [0,\tilde{T}(\gamma)]} \left(\frac{1 - v(s-\eta)}{1 - v(s)}\right) \\
&\quad = \exp\left(-\int_0^{\tilde{T}(\gamma)} \frac{rv(t) - v'(t)}{1 - v(t)}\, dt\right) \prod_{s \in S \cap [0,\tilde{T}(\gamma)]} \left(\frac{1 - v(s,-1)}{1 - v(s)}\right).
\end{aligned}$$

Thus, the value of the program in (19) attained by any function $v$ is arbitrarily closely approximated by the value attained by the continuous function $v^{\eta}$ as $\eta \to 0$, and for sufficiently small $\eta > 0$, this function also satisfies the constraints of (19).

I now derive a lower bound on the left-hand side (19) under the additional constraint that $v$ is continuous. Using the fact that $v(s,-1) = v(s)$ for all $s$ when $v$ is continuous and integrating the $v'(t)/(1-v(t))$ term, this constrained program can be rewritten as

$$\inf_{\substack{v:\mathbb{R}_+ \to \mathbb{R}_+ \text{ continuous:} \\ v(t) \leq 1 - e^{rt}(1/(1-\log\varepsilon) - \zeta), \\ \sup_{t \leq \tilde{T}(\gamma)} |v^*(t) - v(t)| > \delta}} \exp\left(-\int_0^{\tilde{T}(\gamma)} \frac{rv(t)}{1 - v(t)}\, dt\right)\left(\frac{1 - v(0)}{1 - v(\tilde{T}(\gamma))}\right).$$

Since $v(t) \geq 0$ for all $t$, the value of this program is bounded from below by the value of the program

$$\inf_{\substack{v:\mathbb{R}_+ \to \mathbb{R}_+ \text{ continuous:} \\ v(t) \leq 1 - e^{rt}(1/(1-\log\varepsilon) - \zeta), \\ \sup_{t \leq \tilde{T}(\gamma)} |v^*(t) - v(t)| > \delta}} \exp\left(-\int_0^{\tilde{T}(\gamma)} \frac{rv(t)}{1 - v(t)}\, dt\right)\big(1 - v(0)\big). \tag{20}$$

Note that the value of the program (20) is continuous and nonincreasing in $\tilde{T}(\gamma)$.

Let $\tilde{T}_\zeta \equiv 0$ if the value of the program (20) is less than $\varepsilon$ when $\tilde{T}(\gamma) = 0$, let $\tilde{T}_\zeta \equiv \infty$ if the value of the program is greater than $\varepsilon$ for all $\tilde{T}(\gamma) \in \mathbb{R}_+$, and otherwise let $\tilde{T}_\zeta$ be the value of the parameter $\tilde{T}(\gamma)$ such that (20) equals $\varepsilon$ (which exists by the intermediate value theorem). I will show that $\tilde{T}_\zeta > T^1 - (1/r)\log(1 - \zeta(1 - \log\varepsilon))$ for sufficiently small $\zeta \in (0, 1/(1 - \log\varepsilon))$.

I first show that this inequality holds for $\zeta = 0$, that is, that $\tilde{T}_0 > T^1$. To see this, note that (20) decreases whenever the value of $v(t)$ is increased on a subset of $[0, \tilde{T}(\gamma)]$ of positive measure. Hence, the unique solution to the program (20) without the constraint $\sup_{t\leq\tilde{T}(\gamma)} |v^*(t) - v(t)| > \delta$ is $v(t) = 1 - e^{rt}/(1 - \log\varepsilon) = v^*(t)$ for all $t \leq \tilde{T}(\gamma)$ (when $\zeta = 0$). Using this observation, it is straightforward to check that the value of $\tilde{T}(\gamma)$ such that the value of the program (20) without the constraint $\sup_{t\leq\tilde{T}(\gamma)} |v^*(t) - v(t)| > \delta$ equals $\varepsilon$ is $T^1$. Therefore, the constraint $\sup_{t\leq\tilde{T}(\gamma)} |v^*(t) - v(t)| > \delta$ binds in (20) and $\tilde{T}_0 > T^1$.

Next, $\tilde{T}_\zeta$ is continuous in $\zeta$ by the maximum theorem (which implies that the value of (20) is continuous in $\zeta$ for fixed $\tilde{T}(\gamma)$) and the implicit function theorem, and $T^1 - (1/r)\log(1 - \zeta(1 - \log\varepsilon))$ is continuous in $\zeta$ as well. Hence, the fact that $\tilde{T}_0 > T^1$ implies that $\tilde{T}_\zeta > T^1 - (1/r)\log(1 - \zeta(1 - \log\varepsilon))$ for some $\zeta \in (0, 1/(1 - \log\varepsilon))$.

By (19), if $\gamma(t) \geq e^{rt}(1/(1 - \log\varepsilon) - \zeta)$ for all $t \leq T(\gamma)$ and $\sup_{t\leq\tilde{T}(\gamma)} |v^*(t) - v(t)| > \delta$, then $\tilde{T}_\zeta$ is a lower bound on $\tilde{T}(\gamma)$. Thus, the fact that $\tilde{T}_\zeta > T^1 - (1/r)\log(1 - \zeta(1 - \log\varepsilon))$ for some $\zeta \in (0, 1/(1 - \log\varepsilon))$ completes the proof. *Q.E.D.*

STEP 1B: *For all* $\delta > 0$, *there exists* $\zeta > 0$ *such that if* $u_1^*(\gamma) \geq 1/(1 - \log\varepsilon) - \zeta$, *then* $\sup_{t\in\mathbb{R}_+} e^{-rt}|v^*(t) - v(t)| \leq \delta$.

PROOF: Step 1a implies that for any $\delta > 0$ and $K > 1$, there exists $\zeta(K) \in (0, 1/(1 - \log\varepsilon))$ such that if $u_1^*(\gamma) \geq 1/(1 - \log\varepsilon) - \zeta(K)$, then $\sup_{t\leq\tilde{T}(\gamma)} e^{-rt} \times |v^*(t) - v(t)| \leq \delta/K$. I now argue that for $K$ sufficiently large, there exists $\zeta' \in (0, \zeta(K))$ such that if $u_1^*(\gamma) \geq 1/(1 - \log\varepsilon) - \zeta'$, then, in addition, $\sup_{t>\tilde{T}(\gamma)} e^{-rt}|v^*(t) - v(t)| \leq \delta$. To see this, note that as $K \to \infty$, $\tilde{T}(\gamma) \to T^1$ uniformly over all postures $\gamma$ such that $\sup_{t\leq\tilde{T}(\gamma)} e^{-rt}|v^*(t) - v(t)| \leq \delta/K$. Choose $K^* > 1$ such that $|e^{-r\tilde{T}(\gamma)} - e^{-rT^1}| < \delta/2$ and $v^*(\tilde{T}(\gamma)) \leq e^{r\tilde{T}(\gamma)}\delta$ for any such posture $\gamma$, and suppose that a posture $\gamma$ is such that $\sup_{t\leq\tilde{T}(\gamma)} e^{-rt}|v^*(t) - v(t)| \leq \delta/K^*$ but $e^{-rt_0}|v^*(t_0) - v(t_0)| > \delta$ for some $t_0 > \tilde{T}(\gamma)$. Now $v^*(t_0) \leq v^*(\tilde{T}(\gamma)) \leq e^{r\tilde{T}(\gamma)}\delta \leq e^{rt_0}\delta$, so it follows that $e^{-rt_0}v(t_0) > \delta + e^{-rt_0}v^*(t_0)$. Therefore,

$$\max_{t\geq\tilde{T}(\gamma)} e^{-rt}\bigl(1 - \underline{\gamma}(t)\bigr) \geq e^{-rt_0}v(t_0) \geq \delta.$$

By the definition of $T(\gamma)$, this implies that there exists $t_1 \in [\tilde{T}(\gamma), T(\gamma)]$ such that $e^{-rt_1}(1-\underline{\gamma}(t_1)) \geq \delta$ or, equivalently, $\underline{\gamma}(t_1) \leq 1 - e^{rt_1}\delta$. Hence,

$$u_1^*(\gamma) = \min_{t \leq T(\gamma)} e^{-rt}\underline{\gamma}(t) \leq e^{-rt_1}\big(1 - e^{rt_1}\delta\big) \leq e^{-r\tilde{T}(\gamma)}\big(1 - e^{r\tilde{T}(\gamma)}\delta\big)$$
$$= e^{-r\tilde{T}(\gamma)} - \delta < e^{-rT^1} - \delta/2 = 1/(1 - \log\varepsilon) - \delta/2.$$

Therefore, taking $\zeta' \equiv \min\{\zeta(K^*), \delta/2\}$, it follows that if $u_1^*(\gamma) \geq 1/(1 - \log\varepsilon) - \zeta'$, then $\sup_{t \leq \tilde{T}(\gamma)} e^{-rt}|v^*(t) - v(t)| \leq \delta/K^*$ and $\sup_{t > \tilde{T}(\gamma)} e^{-rt}|v^*(t) - v(t)| \leq \delta$, and hence $\sup_{t\in\mathbb{R}_+} e^{-rt}|v^*(t) - v(t)| \leq \delta$. *Q.E.D.*

STEP 2: *If $\gamma_n(t) \to \gamma(t)$ for all $t \in R_+$ for some posture $\gamma$ and* $\sup_{t\in\mathbb{R}_+} e^{-rt} \times |v^*(t) - v_n(t)| \to 0$, *then $\gamma = \gamma^*$.*

PROOF: First, note that if $\gamma(t) < \gamma^*(t)$ for some $t \in \mathbb{R}_+$, then there exist $N > 0$ and $\eta > 0$ such that $\gamma_n(t) < \gamma^*(t) - \eta$ for all $n > N$. Since $v_n(t) \geq 1 - \gamma_n(t)$, this implies that $v_n(t) \geq 1 - \gamma^*(t) + \eta = v^*(t) + \eta$ for all $n > N$, a contradiction.

It is more difficult to rule out the possibility that $\gamma(t) > \gamma^*(t)$ for some $t \in \mathbb{R}_+$. Suppose that this is so. Since $\gamma$ and $\gamma^*$ are right-continuous, there exist $\eta > 0$ and a non-degenerate closed interval $I_0 \subseteq \mathbb{R}_+$ such that $\gamma(t) > \gamma^*(t) + \eta$ for all $t \in I_0$. If it were the case that $\gamma_n(t) \geq \gamma^*(t) + \eta/2$ for all $t \in I_0$ and $n$ sufficiently large, then the condition $\sup_{t\in\mathbb{R}_+} e^{-rt}|v^*(t) - v_n(t)| \to 0$ would fail, so this is not possible.[32] Hence, there exists $t_1 \in I_0$ and $n_1 \geq 0$ such that $\gamma_{n_1}(t_1) < \gamma^*(t_1) + \eta/2$. Since $\gamma_{n_1}$ and $\gamma^*$ are right-continuous, there exists a nondegenerate closed interval $I_1 \subseteq I_0$ such that $\gamma_{n_1}(t) < \gamma^*(t) + \eta/2$ for all $t \in I_1$. Next, it cannot be the case that $\gamma_n(t) \geq \gamma^*(t) + \eta/2$ for all $t \in I_1$ and $n > n_1$ (by the same argument as above), so there exists$t_2 \in I_1$ and $n_2 > n_1$ such that $\gamma_{n_2}(t_2) < \gamma^*(t_2) + \eta/2$. As above, this implies that there exists a nondegenerate closed $I_2 \subseteq I_1$ such that $\gamma_{n_2}(t) < \gamma^*(t) + \eta/2$ for all $t \in I_2$. Proceeding in this manner yields an infinite sequence of nondegenerate closed intervals $\{I_m\}$ and integers $\{n_m\}$ such that $I_{m+1} \subseteq I_m$, $n_{m+1} > n_m$, and $\gamma_{n_m}(t) < \gamma^*(t) + \eta/2$ for all $t \in I_m$ and $m \in \mathbb{N}$. Let $I \equiv \bigcap_{m\in\mathbb{N}} I_m$, a nonempty set (possibly a single point), and fix $t \in I$. Then $\gamma_{n_m}(t) < \gamma^*(t) + \eta/2$ for all $m \in \mathbb{N}$. Since $n_{m+1} > n_m$ for all $m \in \mathbb{N}$, this contradicts the assumption that $\gamma_n(t) \to \gamma(t)$. *Q.E.D.*

*Q.E.D.*

[32]For the proof, suppose that $\sup_{t\in\mathbb{R}_+} e^{-rt}|v^*(t) - v_n(t)| \to 0$. Fix $N > 0$, suppose that $\gamma_n(t) \geq \gamma^*(t) + \eta/2$ for all $t \in I_0$ and $n > N$, and denote the length of $I_0$ by $2\Delta$ and the midpoint of $I_0$ by $t_0$. Then player 2 cannot receive a payoff above $e^{-rt_0}(1 - \gamma^*(t_0) - \eta/2)$ from accepting at any time $t \in [t_0, t_0 + \Delta]$ when facing posture $\gamma_n$ for any $n > N$. Hence, $v_n(t_0) \leq \max\{1 - \gamma^*(t_0) - \eta/2, e^{-r\Delta}v_n(t_0 + \Delta)\}$ for all $n > N$. In addition, noting that $\gamma^*(t_0) < 1 - \eta/2$, there exists $N' > 0$ such that $v_n(t_0 + \Delta) < v_n^*(t_0 + \Delta) + (e^{r\Delta} - 1)(1 - \gamma^*(t_0) - \eta/2) = 1 - \gamma^*(t_0 + \Delta) + (e^{r\Delta} - 1)(1 - \gamma^*(t_0) - \eta/2)$ for all $n > N'$. Therefore, $v_n(t_0) \leq \max\{1 - \gamma^*(t_0) - \eta/2, e^{-r\Delta}(1 - \gamma^*(t_0 + \Delta)) + (1 - e^{-r\Delta})(1 - \gamma^*(t_0) - \eta/2)\} \leq \max\{1 - \gamma^*(t_0) - \eta/2, 1 - \gamma^*(t_0) - (1 - e^{-r\Delta})\eta/2\} = v^*(t_0) - (1 - e^{-r\Delta})\eta/2$ for all $n > \max\{N, N'\}$, which contradicts the hypothesis that $\sup_{t\in\mathbb{R}_+} e^{-rt}|v^*(t) - v_n(t)| \to 0$.

PROOF OF PROPOSITION 2: Lemmas 1–3 apply to any posture, whether or not it is constant. In addition, if $\gamma$ is constant, then $T(\gamma) = \tilde{T}(\gamma)$. Thus, Lemma 3 implies that $u_1^*(\gamma) = \min_{t \le T(\gamma)} e^{-rt}\gamma = e^{-r\tilde{T}(\gamma)}\gamma$. Furthermore, $\lambda(t) = r(1-\gamma)/\gamma$ and $p(t) = 0$ for all $t$, so it follows by the definition of $\tilde{T}(\gamma)$ that $\exp(-r(\frac{1-\gamma}{\gamma})\tilde{T}(\gamma)) = \varepsilon$. Hence, $\tilde{T}(\gamma) = -\frac{1}{r}(\frac{\gamma}{1-\gamma})\log\varepsilon$ if $\gamma < 1$ and $\tilde{T}(\gamma) = \infty$ if $\gamma = 1$. Therefore,

$$\bar{u}_1^* = \max_{\gamma\in[0,1]} e^{-r\tilde{T}(\gamma)}\gamma = \max_{\gamma\in[0,1)} \exp\left(\frac{\gamma}{1-\gamma}\log\varepsilon\right)\gamma. \tag{21}$$

Note that (21) is concave in $\gamma$. The first-order condition is

$$1 = -\frac{\bar{\gamma}_\varepsilon^*}{(1-\bar{\gamma}_\varepsilon^*)^2}\log\varepsilon, \tag{22}$$

which has a solution if $\varepsilon < 1$. Solving this quadratic equation yields the formula for $\bar{\gamma}_\varepsilon^*$. Finally, substituting (22) into (21) yields $\bar{u}_1^* = \exp(-(1-\bar{\gamma}_\varepsilon^*))\bar{\gamma}_\varepsilon^*$. *Q.E.D.*

PROOF OF PROPOSITION 3: Lemmas 1–3 continue to hold, replacing $r$ with $r_1$ or $r_2$ as appropriate. In particular, $\lambda(t) = \frac{r_2 v(t) - v'(t)}{1-v(t)}$, and the same argument as in the proof of Theorem 1 implies that the unique maxmin posture $\gamma^*$ satisfies $\gamma^*(t) = \min\{e^{r_1 t}u_1^*, 1\}$, where $u_1^*$ is the (unique) number such that the time at which $\gamma^*(t)$ reaches 1 equals $\tilde{T}(\gamma^*)$. Thus, given posture $\gamma^*$, it follows that $\lambda(t) = \frac{r_2(1-e^{r_1 t}u_1^*) + r_1 e^{r_1 t}u_1^*}{e^{r_1 t}u_1^*} = r_2\frac{e^{-r_1 t}}{u_1^*} + r_1 - r_2$. Now

$$\begin{aligned}
&\exp\left(-\int_0^{\tilde{T}(\gamma^*)}\left(r_2\frac{e^{-r_1 t}}{u_1^*} + r_1 - r_2\right)dt\right)\\
&\quad = \exp\left(-\frac{1}{u_1^*}\left(\frac{r_2}{r_1}\right)\left(1 - e^{-r_1\tilde{T}(\gamma^*)}\right) + (r_1 - r_2)\tilde{T}\left(\gamma^*\right)\right).
\end{aligned}$$

Setting this equal to $\varepsilon$ and rearranging implies that $\tilde{T}(\gamma^*)$ is given by

$$e^{-r_1\tilde{T}(\gamma^*)} - \frac{r_1}{r_2}u_1^*\log\varepsilon + \left(\frac{r_1}{r_2} - 1\right)u_1^* r_1\tilde{T}\left(\gamma^*\right) = 1. \tag{23}$$

Using the condition that $e^{r_1\tilde{T}(\gamma^*)}u_1^* = 1$, this can be rearranged to yield (11). Finally, there is a unique pair $(u_1^*, \tilde{T}(\gamma^*))$ that satisfies both (23) and $e^{r_1\tilde{T}(\gamma^*)}u_1^* = 1$, because the curve in $(u_1^*, \tilde{T}(\gamma^*))$ space defined by (23) is upward-sloping, while the curve defined by $e^{r_1\tilde{T}(\gamma^*)}u_1^* = 1$ is downward-sloping. *Q.E.D.*

PROOF OF LEMMA 4: The fact that $\Omega_2^{\mathrm{RAT}}(\gamma) \subseteq \Pi_1^\gamma$ immediately implies that $u_1^{\mathrm{RAT}}(\gamma) \geq u_1^*(\gamma) = \min_{t \leq T(\gamma)} e^{-rt}\underline{\gamma}(t)$. Therefore, it suffices to show that $u_1^{\mathrm{RAT}}(\gamma) \leq \min_{t \leq T(\gamma)} e^{-rt}\underline{\gamma}(t)$.

Let $\dot{T} \equiv \min \arg\max_t e^{-rt}(1-\underline{\gamma}(t))$. Note that $\dot{T}$ is well defined and finite because $\underline{\gamma}(t)$ is lower semicontinuous and $\lim_{t\to\infty} e^{-rt}(1-\underline{\gamma}(t)) = 0$. Let $\dot{\sigma}_2 \in \Sigma_2$ be the strategy that always demands 0, rejects up to time $\dot{T}$, accepts at date $(\dot{T}, -1)$ if and only if $\lim_{\tau\uparrow\dot{T}} \gamma(\tau) \leq \gamma(\dot{T})$, and accepts at all dates $(\dot{T}, 1)$ and later (for all histories). Let $\dot{\pi}_2^\gamma \in \Sigma_1$ be identical to the $\gamma$-offsetting belief $\pi_2^\gamma$, with the modification that $\dot{\pi}_2^\gamma$ always accepts demands of 0. Since $p(0) = 0$ for any posture $\gamma$, it follows that $u_1(\dot{\pi}_2^\gamma, \dot{\sigma}_2) = 1$ and, therefore, $\dot{\pi}_2^\gamma \in \Sigma_1^*(\dot{\sigma}_2)$. In addition, it is clear that $u_2(\pi_2^\gamma, \sigma_2) \geq u_2(\dot{\pi}_2^\gamma, \sigma_2)$ for all $\sigma_2 \in \Sigma_2$, so the observations that $\sigma_2^\gamma \in \Sigma_2^*(\pi_2^\gamma)$ (by Lemma 2) and $u_2(\pi_2^\gamma, \sigma_2^\gamma) = u_2(\dot{\pi}_2^\gamma, \sigma_2^\gamma)$ imply that $\sigma_2^\gamma \in \Sigma_2^*(\dot{\pi}_2^\gamma)$. Finally, it is clear that $\dot{\sigma}_2 \in \Sigma_2^*(\gamma)$ and $\gamma \in \Sigma_1^*(\sigma_2^\gamma)$ by Lemma 3. Summarizing, I have established that the arrows in the following diagram can be read as "is a best response to":

$$\begin{array}{ccc} \gamma & \rightarrow & \sigma_2^\gamma \\ \uparrow & & \downarrow \\ \dot{\sigma}_2 & \leftarrow & \dot{\pi}_2^\gamma \end{array}.$$

Therefore, the set $\{\gamma, \dot{\pi}_2^\gamma\} \times \{\sigma_2^\gamma, \dot{\sigma}_2\}$ is closed under rational behavior given posture $\gamma$, which implies that $\{\sigma_2^\gamma, \dot{\sigma}_2\} \subseteq \Omega_2^{\mathrm{RAT}}(\gamma)$. Hence, $u_1^{\mathrm{RAT}}(\gamma) \leq \sup_{\sigma_1} u_1(\sigma_1, \sigma_2^\gamma) = u_1(\gamma, \sigma_2^\gamma) = \min_{t\leq T(\gamma)} e^{-rt}\underline{\gamma}(t)$. *Q.E.D.*

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