Information Design, Informational Robustness and Non-Linear Pricing

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recent developments in information economics

1. information economy
   - information central to digital economy

2. second generation of information economics...
   - non parametric approach to modelling information
   - three related questions:
     1. Bayesian persuasion or information design: what information structure is optimal for an agent who can control it?
     2. informationally robust predictions: what bounds can the analyst put on outcomes without knowing the information structure?
     3. revenue guarantees: what mechanism provides the best guarantee of revenue (or some other objective), whatever the information structure?
my talk today

- illustrate the three questions and the relation to the information economy in a single economic setting: the non-linear pricing model of Mussa-Rosen (1978)
- in particular, I will.....
  1. sketch a slightly non-standard and informal treatment of the classic screening / mechanism design problem of Mussa-Rosen (1978)
  2. introduce the three questions by asking what happens if we vary the information structure - instead of or in addition to choosing the mechanism?
- talk is based on work with Dirk Bergemann (Yale University) and Tibor Heumann (Pontificia Universidad Catolica de Chile), as well as other co-authors and authors
- will post slides and with references...
setting 1

- continuum of buyers
  - the "value" of $t$th quantile buyer is $v = V(t)$, where increasing $V : [0, 1] \rightarrow [\underline{v}, \bar{v}]$
  - i.e., c.d.f. of values on $[\underline{v}, \bar{v}]$ is $V^{-1}$

- seller has fixed inventory of different quality goods
  - the "quality" of the $t$th quantile good is $q = Q(t)$, where increasing $Q : [0, 1] \rightarrow [\underline{q}, \bar{q}]$
  - i.e., c.d.f. of qualities on $[\underline{q}, \bar{q}]$ is $Q^{-1}$

- mass of buyers and goods both normalized to 1
setting 2

- utility of consumer of value \( v \) paying \( p \) for good of quality \( q \) is
  \[ v \cdot q - p \]

- can also
  1. endogenize inventory with convex cost (as in Mussa-Rosen 78)
  2. consider finite bidder auction with single unit demand

- we will first solve for the optimal selling mechanism assuming buyers know their values....

- ... and then discuss various scenarios where buyers’ information is varied
incentive compatibility implies that expected quality of the good must be an increasing function of buyer’s expected value

therefore, there must be assortative matching of expected qualities and expected values in any incentive compatible mechanism

so all that matters is the distribution of expected qualities sold

so a mechanism assigns expected quality \( x = X(t) \) to the \( t \)th quantile buyer, where \( X : [0, 1] \rightarrow [\underline{q}, \overline{q}] \)
modelling mechanism 2

- what mechanisms $X$ can the seller choose among?
- seller can (i) pool qualities (offer lotteries) and (ii) exclude buyers (which we model as setting quality to 0)
- equivalently, the seller can induce a mean preserving contraction of the distribution of qualities (by pooling) and a first-order stochastic dominance shift downwards (by exclusion)
- equivalently, the seller can choose any $X$ such that $Q$ weakly majorizes $X$ ($Q \succeq_w X$) i.e.,

$$\int_x^1 X(t) \, dt \leq \int_x^1 Q(t) \, dt$$

for all $x \in [0, 1]$
information rent

- write $U(t)$ for the rent (utility) of quantile $t$ buyer; by envelope theorem

$$U'(t) = V'(t) X(t)$$

- information rent by quantile is

$$U(t) = \int_{s=0}^{t} V'(s) X(s) \, ds$$

- total information rent is

$$U(V, X) = \int_{t=0}^{1} \left[ \int_{s=0}^{t} V'(s) X(s) \, ds \right] \, dt$$
total surplus and profit

- total surplus is

\[
S(V, X) = \int_{t=0}^{1} V(t)X(t) \, dt
\]

- a famous profit formula:

\[
\Pi(V, X) = \int_{t=0}^{1} V(t)X(t) \, dt - \int_{t=0}^{1} \left( \int_{s=0}^{t} V'(s)X(s) \, ds \right) \, dt
\]

\[
= V(0)X(0) + \int_{t=0}^{1} V(t)(1-t)X'(t) \, dt
\]
mechanism design: regular case

- what is the optimal mechanism for the seller (maximizing profits), taking as given the distribution of values?
- choose $X \leq_w Q$ to maximize $\Pi(V, X)$
- "regular" case: if virtual value $V(t) - V'(t)(1 - t)$ is increasing (revenue $V(t)(1 - t)$ is concave), then
  - optimal to exclude buyers with negative virtual values (marginal revenue), i.e., if
  \[
  t \leq t_m = \arg \max_{t \in [0,1]} V(t)(1 - t) - \]
  - allocate remaining inventory at higher quantiles without pooling
mechanism design: irregular case

- more generally,

$$\max_{X \prec Q} \Pi(V, X) = V(t_m) (1 - t_m) + \int_{t_m}^{1} \text{cav}[V(t)(1 - t)] dQ(t)$$

- in the "irregular case" (where virtual value $V(t) - V'(t)(1 - t)$ is not increasing), optimal to pool intervals whenever

$$\text{cav}[V(t)(1 - t)] > V(t)(1 - t)$$

- $X$ is "monotone partitional", alternating pooling and full revelation

- Myerson (1981) ironing; also Kleiner et al (2022)
modelling information

- recall that the "value" of $t$th quantile buyer is $\nu = V(t)$, where $V : [0, 1] \rightarrow [\underline{\nu}, \overline{\nu}]$.
- an information structure for buyers will give rise to a distribution over expected values.
- let expected value of the $t$th quantile buyer be $\omega = W(t)$, where $W : [0, 1] \rightarrow [\underline{\nu}, \overline{\nu}]$.
- expected values must be a mean preserving contraction of the (ex post) values.
- equivalently, the seller can choose any $W$ such that $V$ majorizes $W$ ($V \succeq W$) i.e.,

$$\int_{x}^{1} W(t) \, dt \leq \int_{x}^{1} V(t) \, dt$$

for all $x \in [0, 1]$, with equality if $x = 0$. 
pure information design

- now suppose that the seller can choose an information structure $W$ to maximize profits but the inventory must be sold efficiently (so $X = Q$).
- thus the seller chooses $W \preceq V$ to maximize $\Pi(W, Q)$
- "regular" case: if inverse hazard rate $(1 - t) Q'(t)$ is increasing, optimal to fully reveal values
- if not, concavification argument gives optimal policy
- as in mechanism design problem, $W$ is "monotone partitional", alternating pooling and full revelation
- under reasonable conditions, pooling at the top, separation at the bottom
- in particular, in second price auction, optimal to pool high valuation bidders, separate low valuation bidders
suppose seller can choose both mechanism and information (to maximize profits)

thus the seller chooses $W \leq V$ and $X \leq Q$ to maximize $\Pi(W, X)$

maximization subject to two majorization constraints

optimal $W$ has finite number of expected values in support, and mechanism has corresponding finite expected qualities

intuition: if there was ever full revelation, pooling a small interval would give rise to third order decrease in total surplus, second order decrease in information rent

under mild conditions, it is optimal to provide no information, sell the uniform lottery to all buyers and extract full surplus
suppose a seller must decide how many virtually differentiated variants of a good to sell and at what prices

every variant is available to all buyers at common price (no personalized pricing)

but seller can target consumers with a particular variant, i.e., make a (perhaps implicit) recommendation

this gives implementation of previous direct mechanism

but suggests that recommendation systems may not be optimal for vertically differentiated goods
varying information: more questions

1. **information design I**: what information structure maximizes profits (given efficient or optimal mechanism)?

2. **information design II**: what information structure maximizes information rent?

3. **revenue guarantee**: what information structure minimizes profits?

4. **robust predictions**: what bounds can one put on profits and information rent if you don’t know the information structure
question 2: maximizing information rent

- what information structure maximizes information rent given that the seller will choose an optimal mechanism given the information structure?
- thus "the buyers" choose the chooses $W \leq V$ to maximize $U(W, X)$ subject $X \leq_w Q$ maximizing $\Pi(W, X)$
- compare Roessler and Szentes (2017) for the homogenous quality case
- continuous information structure is optimal
- equalize virtual values with generalized Pareto distribution but must also deter seller from exclusion
question 3: minimizing profits

- what information structure minimizes profits given that the seller will choose an optimal mechanism given the information structure?
- thus an "adversary" chooses $W \leq V$ to minimize $\Pi(W, X)$ subject $X \leq_w Q$ maximizing $\Pi(W, X)$
- or consider zero sum game where (i) seller chooses $X \leq_w Q$ and (ii) adversary chooses $W \leq V$ to maximize/minimize revenue respectively
- saddle point $(X, W)$
- compare Du (2018) for the homogenous quality case
- solution is revenue guarantee for the seller and $X$ is the mechanism that attains it
question 4: robust predictions

- what bounds can the analyst put on profits and information rent if you don’t know the information structure?
- to answer question, suppose a (metaphorical) informational designer maximizes a weighted sum of profits and information rent, anticipating that a seller would choose the profit maximizing mechanism
- thus the information designer chooses $W \leq V$ to maximize $\lambda \Pi (W, X) + \mu U (W, X)$ (for positive and negative $\lambda$ and $\mu$) subject $X \leq_w Q$ maximizing $\Pi (W, X)$
- finite support solution if weight on profits exceeds weight on information rent (for same reason as earlier)
- continuous solution otherwise
- see picture
conclusion

- you may have heard a lot about (and maybe written about) Bayesian persuasion and information design in recent years (also well represented at SAET Paris)
- to a significant extent, this comes from an internal dynamic in the theory community
- two external drivers of interest:
  - the information economy
  - the sensitivity of first generation information economics to parametric information structures
- the way forward is surely integration on second generation information economics and information economy applications

"Screening with Persuasion"

"The Consumer Optimal Information Structure in Optimal Auctions"