Optimal Monetary Policy with Informational Frictions

George-Marios Angeletos Jennifer La'O

July 2017

How should fiscal and monetary policy

respond to business cycles

when firms have imperfect information about the world?

What is the relevant informational friction?

is it uncertainty about fundamentals?

- representative agent models, single-agent decision problem
- can feature rich first-order beliefs about future fundamentals

  news shocks: Beaudry Portier (2006), Jaimovich Rebelo (2009)

What is the relevant informational friction?

is it uncertainty about fundamentals?

- representative agent models, single-agent decision problem
- can feature rich first-order beliefs about future fundamentals

  news shocks: Beaudry Portier (2006), Jaimovich Rebelo (2009)

... or incomplete info about the actions of others?

- beauty contests with strategic complementarity

  $\longrightarrow$ info friction impedes coordination among agents

  Morris and Shin (1998, 2002)
- Movements in Higher-order beliefs $\longrightarrow$ Sentiment-driven Fluctuations

  Angeletos La'O (2013), Benhabib et al (2015)

What is the relevant informational friction?

do informational frictions affect nominal choices?

- info friction may be the source of nominal rigidity

  $\longrightarrow$ sluggish price adjustment & monetary non-neutrality

- Mankiw Reis (2003), Woodford (2003), Mackowiak Wiederholt (2008) Paciello Wiederholt (2014)

What is the relevant informational friction?

do informational frictions affect nominal choices?

- info friction may be the source of nominal rigidity

  $\longrightarrow$ sluggish price adjustment & monetary non-neutrality

- Mankiw Reis (2003), Woodford (2003), Mackowiak Wiederholt (2008) Paciello Wiederholt (2014)

... or real quantity decisions?

- info friction may impede firms' real choices

  $\longrightarrow$ generate inertia to fundamentals,

  $\longrightarrow$ amplify aggregate response to noise or common errors

- beliefs- or noise-driven aggregate fluctuations

  Lorenzoni (2009), Angeletos La'O (2009, 2013)

What is the relevant informational friction?

what type of signals do agents receive?

- sticky info (Mankiw and Reis 2003)
- Gaussian dispersed info (Woodford 2003, Angeletos La'O 2009)
- binary signals, non-Gaussian signals, fat-tailed posteriors, etc.

What is the relevant informational friction?

what type of signals do agents receive?

- sticky info (Mankiw and Reis 2003)
- Gaussian dispersed info (Woodford 2003, Angeletos La'O 2009)
- binary signals, non-Gaussian signals, fat-tailed posteriors, etc.

is there endogenous information acquisition?

- given some cost, agents optimally choose their information
- rational inattention

  (Sims 2003, Mackowiak Wiederholt 2008, Paciello Wiederholt 2014)
- what is the exact shape of the cost function?

What is the relevant informational friction?

what type of signals do agents receive?

- sticky info (Mankiw and Reis 2003)
- Gaussian dispersed info (Woodford 2003, Angeletos La'O 2009)
- binary signals, non-Gaussian signals, fat-tailed posteriors, etc.

is there endogenous information acquisition?

- given some cost, agents optimally choose their information
- rational inattention

  (Sims 2003, Mackowiak Wiederholt 2008, Paciello Wiederholt 2014)
- what is the exact shape of the cost function?

informational constraint or cognitive limitations?

- limits on cognitive capacity (Woodford 2016, Gabaix 2014, Tirole 2015)

What we do

We study Optimal Fiscal and Monetary Policy
when firms face both nominal and real informational frictions

What we do

We study Optimal Fiscal and Monetary Policy
when firms face both nominal and real informational frictions

Micro-founded business cycle model with the following features:

1. Nominal and real decisions subject to informational frictions
   - Firms face uncertainty about the world
   - Must set prices and real inputs before observing demand

What we do

We study Optimal Fiscal and Monetary Policy
when firms face both nominal and real informational frictions

Micro-founded business cycle model with the following features:

1. Nominal and real decisions subject to informational frictions
   - Firms face uncertainty about the world
   - Must set prices and real inputs before observing demand
2. Flexible, General Information structure
   - remain agnostic about informational frictions (baseline: exogenous)
   - extension: endogenous information/rational inattention

What we do

We study Optimal Fiscal and Monetary Policy
when firms face both nominal and real informational frictions

Micro-founded business cycle model with the following features:

1. Nominal and real decisions subject to informational frictions
   - Firms face uncertainty about the world
   - Must set prices and real inputs before observing demand
2. Flexible, General Information structure
   - remain agnostic about informational frictions (baseline: exogenous)
   - extension: endogenous information/rational inattention
3. Multiple sources of aggregate fluctuations
   - technology, government spending shocks
   - news, noise, higher-order beliefs, sentiments

Methodological Contribution

- The Ramsey Problem
  - Optimal Policy without Informational Frictions:
    Lucas and Stokey (1983), Chari, Christiano, Kehoe (1994)
  - with Sticky Prices: Correia, Nicolini, Teles (2008)
- The Primal Approach
  - characterize set of allocations implementable as equilibria
  - identify welfare-maximizing allocation within that set
  - back-out policies that implement the Ramsey optimum
- We extend primal approach to heterogeneous info. environments
  - study normative properties while completely bypassing an explicit solution for the equilibrium

What we show

1. Flexible-price allocations remain optimal, despite info frictions
   - optimal taxes as in Lucas Stokey; Chari, Christiano, Kehoe
   - tax final goods and labor, zero taxation of capital
   - tax smoothing (constant taxes if utility is homothetic)

What we show

1. Flexible-price allocations remain optimal, despite info frictions
   - optimal taxes as in Lucas Stokey; Chari, Christiano, Kehoe
   - tax final goods and labor, zero taxation of capital
   - tax smoothing (constant taxes if utility is homothetic)
2. Despite nominal frictions, Price Stability is Suboptimal
3. Optimal Policy: Negative Correlation between Prices and GDP

The Model

The Model

- continuum of monopolistic firms, $i \in I$
- managers make decisions under incomplete info
  - nominal pricing decision
  - real intermediate good and investment decision

The Model

- continuum of monopolistic firms, $i \in I$
- managers make decisions under incomplete info
  - nominal pricing decision
  - real intermediate good and investment decision
- representative household
  - continuum of workers
  - continuum of managers
  - representative consumer

Intermediate Good Firms

$$y_{it} = A_t F\left(k_{it}, h_{it}, \ell_{it}\right)$$

$$k_{i,t+1} = (1-\delta) k_{i,t} + x_{it}$$

- for today

$$y_{it} = A_t g\left(k_{it}, h_{it}\right) \ell_{it}^{\alpha}$$

- firm faces a revenue tax and a capital income tax

$$\frac{\Pi_{it}}{P_t} = \left(1-\tau_t^k\right)\left[\left(1-\tau_t^r\right)\frac{p_{it} y_{it}}{P_t} - \left(h_{it} + W_t \ell_{it}\right)\right] - x_{it}$$

Final Good Firm and the Household

- final good firm

$$Y_t = \left[\int_I y_{it}^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}}$$

- household

$$\mathbb{E} \sum_{t=0}^{\infty} \beta^t \left[U(C_t) - V(L_t) \right]$$

$$(1 + \tau_t^c) P_t C_t + B_t \leq \left(1 - \tau_t^{\ell} \right) P_t W_t L_t + R_t B_{t-1}$$

- labor market clearing

$$\int_I \ell_{it} di = L_t$$

Government and Resource Constraints

- government budget constraint
  - exogenous government spending shocks, no lump sum taxes
  - must finance expenditure with proportional taxes and nominal debt
  - debt has a one-period maturity and a state-contingent return

$$
\begin{aligned}
R_t B_{t-1} + P_t G_t \quad \leq \quad & \tau_t^r P_t Y_t + \tau_t^c P_t C_t + \tau_t^\ell P_t W_t L_t \\
& + \tau_t^k \int_I e_{it} di + \int_I \Pi_{it} di + B_t
\end{aligned}
$$

- resource constraints

$$
C_t + H_t + X_t + G_t = Y_t
$$

$$
H_t = \int_I h_{it} di \quad \text{and} \quad X_t = \int_I x_{it} di
$$

Shocks and Information Structure

Shocks and Information

1. Nature draws $s_t \in S_t$ according to $s_t \sim \mu(s_t)$

Shocks and Information

1. Nature draws $s_t \in S_t$ according to $s_t \sim \mu(s_t)$
   - ◇ aggregate "real" shocks A_t, G_t
   - ◇ cross-sectional distribution of information sets Ω^t
   - ◇ thereby contains shocks to beliefs (noise, sentiments)
   - ◇ history: $s^t = (s_t, s_{t-1}, \ldots)$

Shocks and Information

1. Nature draws $s_t \in S_t$ according to $s_t \sim \mu(s_t)$
   - aggregate "real" shocks A_t, G_t
   - cross-sectional distribution of information sets Ω^t
   - thereby contains shocks to beliefs (noise, sentiments)
   - history: $s^t = (s_t, s_{t-1}, \ldots)$
2. Nature draws $\omega_{it} \in \Omega^t$, $\omega_{it} \sim \mu\left(\omega_i^t | s^t\right)$, $\forall i \in I$
3. Information of manager i is $\omega_i^t = (\omega_{it}, \omega_{i,t-1}, \ldots)$
   - ω_i^t is manager's "Harsanyi type"

Examples of Info Structures

- sticky info (Mankiw Reis 2003)

$$\omega_{it} = \begin{cases} s^t & \text{with prob } \mu \\ \omega_i^{t-1} & \text{with prob } 1-\mu \end{cases}$$

- noisy info (Woodford 2003, Angeletos La'O 2009)

$$\omega_{it} = (x_{it}, z_t) = \begin{cases} x_{it} = \log A_t + \nu_{it} \\ z_t = \log A_t + \varepsilon_t \end{cases}$$

- may also construct examples with "sentiments"

  (Angeletos La'O 2013)

Informational Frictions and Market Clearing

1. Managers make nominal and real decisions with incomplete info

$$\text{thus } p_{it}, h_{it}, x_{it} \;\; \text{contingent on} \;\; \omega_i^t$$

Informational Frictions and Market Clearing

1. Managers make nominal and real decisions with incomplete info

$$\text{thus } p_{it}, h_{it}, x_{it} \ \text{ contingent on } \ \omega_i^t$$

2. All other market outcomes/choices/wages adjust to aggregate state

   - given prices, household chooses consumptionr
   - thus hours ℓ_{it}, y_{it} are contingent on (ω_i^t, s^t)
     must adjust so that supply = demand
   - govt policy, household consumption, savings contingent on s^t

Info Friction is both Nominal and Real

- standard in the literature: info friction = nominal friction

$$p \text{ contingent on } \omega_i^t$$

$$\text{but all real choices adjust to } s^t$$

  - Ball, Mankiw, Reis (2005), Adam (2007), Lorenzoni (2010), Paciello Wiederholt (2014)

Info Friction is both Nominal and Real

- standard in the literature: info friction = nominal friction

$$p \text{ contingent on } \omega_i^t$$

$$\text{but all real choices adjust to } s^t$$

  - Ball, Mankiw, Reis (2005), Adam (2007), Lorenzoni (2010), Paciello Wiederholt (2014)

- our generalization: info friction = both nominal and real

$$p \text{ and } h, x \text{ contingent on } \omega_i^t$$

$$\ell \text{ adjusts to } s^t$$

  - info friction still relevant even under flexible prices

Feasibility

Let ξ denote an allocation

$$\xi\left(s^t\right) \equiv \left\{ \begin{array}{c} Y\left(s^t\right), C\left(s^t\right), L\left(s^t\right), \\ \left(x\left(\omega_i^t\right), k\left(\omega_i^t\right), h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right), y\left(\omega_i^t, s^t\right)\right)_{i \in I} \end{array} \right\}$$

Definition

An allocation ξ is feasible if and only if it satisfies the following:

$$C\left(s^t\right) + \int_I h\left(\omega_i^t\right) di + \int_I x\left(\omega_i^t\right) di + G\left(s^t\right) = Y\left(s^t\right) = \left[\int_I \left(y\left(\omega_i^t, s^t\right)\right)^{\frac{\rho-1}{\rho}} di\right]^{\frac{\rho}{\rho-1}}$$

$$\begin{aligned} y\left(\omega_i^t, s^t\right) &= A\left(s^t\right) F\left(k\left(\omega_i^{t-1}\right), h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right)\right), \\ k\left(\omega_i^t\right) &= (1-\delta) k\left(\omega_i^{t-1}\right) + x\left(\omega_i^t\right) \end{aligned}$$

Equilibrium

We Analyze Two Scenarios

1. sticky-price equilibrium. firm chooses

$$p\left(\omega_i^t\right), h\left(\omega_i^t\right), x\left(\omega_i^t\right) \quad \text{conditional on } \omega_i^t$$

   both real and nominal informational friction

2. flexible-price equilibrium. firm chooses

$$h\left(\omega_i^t\right), x\left(\omega_i^t\right) \quad \text{conditional on } \omega_i^t,$$

$$\text{but } p\left(\omega_i^t, s^t\right) \quad \text{adjusts to realized } s^t$$

   only the real informational friction

Equilibrium Definitions

Let θ denote a government policy

$$\theta\left(s^t\right) \equiv \left\{\tau^r\left(s^t\right), \tau^c\left(s^t\right), \tau^\ell\left(s^t\right), \tau^k\left(s^t\right), R\left(s^t\right)\right\}$$

Definition

A sticky-price equilibrium is a policy θ, an allocation ξ, and prices

$$\left\{p\left(\omega_i^t\right)\right\}_{i\in I}, \text{ such that}$$

(i) the household and firms are at their respective optima

(ii) the government's budget constraint is satisfied, and

(iii) markets clear.

Definition

A flexible-price equilibrium is a policy θ, an allocation ξ, and prices

$$\left\{p\left(\omega_i^t, s^t\right)\right\}_{i\in I} \text{ such that (i)-(iii) hold.}$$

Flexible-Price Equilibrium

Household Optimization

$$
\begin{aligned}
V_\ell\left(s^t\right) &= U_c\left(s^t\right)\frac{\left(1-\tau^\ell\left(s^t\right)\right)}{\left(1+\tau^c\left(s^t\right)\right)}W\left(s^t\right) \\
\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)} &= \beta\mathbb{E}\left[\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}R\left(s^{t+1}\right)\middle|\ s^t\right]
\end{aligned}
$$

Intermediate Firm's Problem

Choose functions (h, x, ℓ) so as to maximize expected profits

$$\max \mathbb{E}\left[\mathcal{M}\left(s^t\right)\frac{\Pi\left(\omega_i^t, s^t\right)}{P\left(s^t\right)}\middle|\,\omega_i^t\right]$$

subject to

$$\begin{aligned}
\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)} &= \left(\frac{y(\omega_i^t, s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}} \quad \forall \omega_i^t, s^t \\
k\left(\omega_i^t\right) &= (1-\delta)k\left(\omega_i^{t-1}\right) + x\left(\omega_i^t\right) \quad \forall \omega_i^t \\
y\left(\omega_i^t, s^t\right) &= A\left(s^t\right)F\left(k\left(\omega_i^{t-1}\right), h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right)\right) \quad \forall \omega_i^t, s^t
\end{aligned}$$

where

$$\mathcal{M}\left(s^t\right) = U_c\left(s^t\right) / \left(1+\tau^c\left(s^t\right)\right)$$

Firm FOCs

intermediate goods demand optimality:

$$\mathbb{E}\left[\mathcal{M}\left(s^t\right)\left(\left(1-\tau^r\left(s^t\right)\right)\frac{\rho-1}{\rho}MP_h\left(\omega_i^t,s^t\right)-1\right)\middle|\,\omega_i^t\right]=0 \quad \forall\ \omega_i^t$$

labor demand optimality:

$$\left(1-\tau^r\left(s^t\right)\right)\frac{\rho-1}{\rho}MP_\ell\left(\omega_i^t,s^t\right)-W\left(s^t\right) \quad =0 \quad \forall\ \omega_i^t,s^t$$

where $MP_z\left(\omega_i^t,s^t\right)\equiv\left(\frac{y(\omega_i^t,s^t)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}}A(s^t)f_z\left(\omega_i^t,s^t\right)$ for any $z\in\{k,h,\ell\}$

Flexible Price Equlibrium Allocations

Proposition

A feasible allocation is implementable as a flexible-price equilibrium iff

$$\exists \text{ functions } \phi^r, \phi^c, \phi^\ell, \phi^k : \mathcal{S}^t \to \mathbb{R}_+, \quad \text{such that}$$

(i) equil. labor condition

$$\mathcal{M}\left(s^t\right)\phi^\ell\left(s^t\right)\phi^r(s^t)MP_\ell\left(\omega_i^t, s^t\right) - V_\ell\left(s^t\right) \quad = 0 \quad \forall\, \omega_i^t, s^t$$

$$\text{with } \ \mathcal{M}\left(s^t\right) = U_c\left(s^t\right)/\phi^c\left(s^t\right)$$

(ii) equil. intermediate goods condition

$$\mathbb{E}\left[\mathcal{M}\left(s^t\right)\left(\phi^r(s^t)MP_h\left(\omega_i^t, s^t\right) - 1\right)\middle|\, \omega_i^t\right] = 0 \quad \forall\, \omega_i^t$$

Flexible Price Equilibrium Allocations

Proposition

(iii) equil. capital investment condition

$$\mathbb{E}\left[\mathcal{M}\left(s^{t}\right)-\beta\mathcal{M}\left(s^{t+1}\right)\left\{1-\delta+\phi^{r}(s^{t+1})\phi^{k}\left(s^{t+1}\right)MP_{k}\left(\omega_{i},s^{t+1}\right)\right\}\,\middle|\,\omega_{i}^{t}\right]=0$$

and (iv) implementability condition for govt solvency:

$$\sum_{t,s^{t}}\beta^{t}\mu\left(s^{t}\right)\left[U_{c}\left(s^{t}\right)C\left(s^{t}\right)-V_{\ell}\left(s^{t}\right)L\left(s^{t}\right)\right]=\mathcal{M}\left(s^{0}\right)R_{b}\left(s^{0}\right)B_{-1}$$

Tax Wedges

- wedges result from taxes and markups

$$\phi^c(s^t) \equiv 1 + \tau^c\left(s^t\right), \quad \phi^\ell(s^t) \equiv 1 - \tau^\ell\left(s^t\right), \quad \phi^k(s^t) \equiv 1 - \tau^k\left(s^t\right)$$

$$\phi^r(s^t) \equiv \left(1 - \tau^r(s^t)\right)\left(\frac{\rho-1}{\rho}\right)$$

Sticky-Price Equilibrium

Intermediate Firm's Problem

Choose functions (p, h, x, ℓ) so as to maximize expected profits

$$\max \mathbb{E}\left[\mathcal{M}\left(s^t\right)\frac{\Pi\left(\omega_i^t, s^t\right)}{P\left(s^t\right)}\middle|\, \omega_i^t\right]$$

s.t. same technological constraints in flexible-price firm problem,

but faces one additional constraint when choosing nominal price:

$$A\left(s^t\right)F\left(k\left(\omega_i^{t-1}\right), h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right)\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho} Y\left(s^t\right) \quad \forall \omega_i^t, s^t$$

Sticky Price Equilibrium Allocations

Proposition

A feasible allocation is implementable as a sticky-price equilibrium iff

$$\exists \text{ functions } \phi^r, \phi^c, \phi^\ell, \phi^k : \mathcal{S}^t \to \mathbb{R}_+ \quad \text{and} \quad \chi : \Omega^t \times \mathcal{S}^t \to \mathbb{R}_+ \quad \text{such that}$$

(i) equil. labor condition

$$\mathcal{M}\left(s^t\right)\phi^\ell\left(s^t\right)\phi^r(s^t)\chi(\omega_i^t, s^t)MP_\ell\left(\omega_i^t, s^t\right) - V_\ell\left(s^t\right) \quad = 0 \quad \forall\ \omega_i^t, s^t$$

(ii) equil. intermediate goods condition

$$\mathbb{E}\left[\mathcal{M}\left(s^t\right)\left(\phi^r(s^t)\chi(\omega_i^t, s^t)MP_h\left(\omega_i^t, s^t\right) - 1\right)\middle|\,\omega_i^t\right] = 0 \quad \forall\ \omega_i^t$$

(iii) equil. capital investment condition

$$\mathbb{E}\left[\mathcal{M}\left(s^t\right) - \beta\mathcal{M}\left(s^{t+1}\right)\left(1 - \delta + \phi^r(s)\chi(\omega_i, s)\phi^k\left(s\right)MP_k\left(\omega_i, s\right)\right)\middle|\,\omega_i^t\right] = 0$$

Sticky Price Equilibrium Allocations

Proposition

(iv) firm optimality condition for the nominal price

$$\mathbb{E}\left[\mathcal{M}(s^t)Y\left(s^t\right)^{1/\rho}y\left(\omega_i^t,s^t\right)^{1-1/\rho}\phi^r(s^t)\left\{\chi(\omega_i^t,s^t)-1\right\}\middle|\,\omega_i^t\right]=0 \quad \forall\ \omega_i^t$$

and (v) implementability condition for govt solvency exactly the same as in flex-price equilibrium.

Comparing Flexible and Sticky Allocations

- In sticky price equilibrium allocations we have the new wedge:

$$\chi(\omega_i^t, s^t) = \text{realized markup due to monetary policy \& sticky prices}$$

- In any flexible price equilibrium,

$$\chi(\omega_i^t, s^t) = 1 \quad \text{for all} \quad \omega_i^t, s^t$$

Comparing Flexible and Sticky Allocations

- In sticky price equilibrium allocations we have the new wedge:

$$\chi(\omega_i^t, s^t) = \text{realized markup due to monetary policy \& sticky prices}$$

- In any flexible price equilibrium,

$$\chi(\omega_i^t, s^t) = 1 \quad \text{for all} \quad \omega_i^t, s^t$$

- Let Φ^f denote the set of implementable allocations under flexible prices
- Let Φ^s denote the set of implementable allocations under sticky prices.
- Then

$$\Phi^f \subset \Phi^s.$$

The Ramsey Problem

The Ramsey Problem

Definition

The Ramsey Planner's Problem is to maximize welfare over Φ^s, the set of sticky-price allocations.

A Ramsey Optimal allocation is a solution to this problem.

The Relaxed Set

Definition

The Relaxed set Φ^R is the set of all feasible allocations in which the implementability condition for govt solvency holds.

Definition

A Relaxed Ramsey Optimal allocation is an allocation ξ^* which maximizes household ex-ante utility subject to

$$\xi^* \in \Phi^R$$

- Note that the relaxed planner still respects informational feasibility
  - measurability constraints = technological constraints
- relaxed planner also respects government solvency constraint

Why look at the Relaxed Ramsey Problem?

- Clearly the relaxed set is a larger set

$$\Phi^f \subset \Phi^s \subset \Phi^R$$

Why look at the Relaxed Ramsey Problem?

- Clearly the relaxed set is a larger set

$$\Phi^f \subset \Phi^s \subset \Phi^R$$

- We show the following:

$$\zeta^* \in \Phi^f$$

which further implies,

$$\zeta^* \in \Phi^s$$

- Therefore ζ^* solves the (non-relaxed) Ramsey problem!

The Relaxed Ramsey Optimum

Proposition

The Relaxed Ramsey optimal allocation satisfies

$$\begin{aligned}
\tilde{U}_c\left(s^t\right) MP_\ell\left(\omega_i^t, s^t\right) - \tilde{V}_\ell\left(s^t\right) &= 0 \quad \forall\, \omega_i^t, s^t \\
\mathbb{E}\left[\, \tilde{U}_c\left(s^t\right)\left(MP_h\left(\omega_i^t, s^t\right) - 1\right) \;\middle|\; \omega_i^t \,\right] &= 0 \quad \forall\, \omega_i^t \\
\mathbb{E}\left[\, \tilde{U}_c\left(s^t\right) - \beta \tilde{U}_c\left(s^{t+1}\right)\left\{1 - \delta + MP_k\left(\omega_i^{t+1}, s^{t+1}\right)\right\} \;\middle|\; \omega_i^t \,\right] &= 0 \quad \forall\, \omega_i^t
\end{aligned}$$

with

$$\begin{aligned}
\tilde{U}(C\left(s^t\right)) &\equiv U(C\left(s^t\right)) + \Gamma U_c\left(s^t\right) C\left(s^t\right) \\
\tilde{V}\left(L\left(s^t\right)\right) &\equiv V\left(L\left(s^t\right)\right) + \Gamma V_\ell\left(s^t\right) L\left(s^t\right)
\end{aligned}$$

and Γ *is the Lagrange-multiplier on the implementability condition*

The Relaxed Ramsey Optimum

Proposition

There exists a set of state-contingent taxes

$$\phi^c(s^t) = \frac{U_c(s^t)}{\tilde{U}_c(s^t)}, \quad \phi^\ell(s^t) = \frac{V_\ell(s^t)}{\tilde{V}_\ell(s^t)}, \quad \phi^k(s^t) = 1, \text{ and } \phi^r(s^t) = 1, \text{ for all } s^t$$

such that the Relaxed Ramsey optimum is implemented under flexible prices.

$$\xi^* \in \Phi^f$$

The Relaxed Ramsey Optimum

Proposition

There exists a set of state-contingent taxes

$$\phi^c(s^t) = \frac{U_c(s^t)}{\tilde{U}_c(s^t)}, \quad \phi^\ell(s^t) = \frac{V_\ell(s^t)}{\tilde{V}_\ell(s^t)}, \quad \phi^k(s^t) = 1, \quad \text{and} \quad \phi^r(s^t) = 1, \quad \text{for all} \quad s^t$$

such that the Relaxed Ramsey optimum is implemented under flexible prices.

$$\xi^* \in \Phi^f$$

Corollary

$$\xi^* \in \Phi^s$$

The Relaxed Ramsey optimum is implemented under sticky prices with the same taxes as above and

$$\chi(\omega_i^t, s^t) = 1, \quad \text{for all} \quad \omega_i^t, s^t.$$

Optimal Policy

Optimal Fiscal and Monetary Policy

Theorem

ζ^* is implemented as part of sticky-price equilibrium with

(i) a monetary policy that replicates flexible prices; and

(ii) a tax policy that satisfies the following:

$$1+\tau^c\left(s^t\right)=\frac{U_c\left(s^t\right)}{\tilde{U}_c\left(s^t\right)},\quad 1-\tau^\ell\left(s^t\right)=\frac{V_\ell\left(s^t\right)}{\tilde{V}_\ell\left(s^t\right)},\ 1-\tau^k\left(s^t\right)=1,$$

$$1-\tau^r(s^t)=\left(\frac{\rho-1}{\rho}\right)^{-1}$$

Fiscal Policy

Lemma

Suppose preferences are homothetic

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad \textit{and} \quad V(L) = \frac{L^{1+\epsilon}}{1+\epsilon}$$

Then the optimal consumption and labor tax rates are constant:

$$1+\tau^c = \frac{1}{1+\Gamma(1-\gamma)}, \; 1-\tau^\ell = \frac{1}{1+\Gamma(1+\epsilon)}, \; \tau^k = 0,$$

$$1-\tau^r(s^t) = \left(\tfrac{\rho-1}{\rho}\right)^{-1}$$

- Taxes as in Lucas Stokey (1983), Chari Christiano Kehoe (1994)

Monetary Policy

Lemma

There exist functions Ψ^{ω}, Ψ^{s} such that in any sticky-price equilibrium, firm output is log-separable

$$y\left(\omega_i^t, s^t\right) \;=\; \Psi^{\omega}(\omega_i^t)\,\Psi^{s}(s^t), \text{ where } \;\Psi^{\omega}(\omega) \;= g\left(k\left(\omega\right), h\left(\omega\right)\right)^{\zeta}$$

Monetary Policy

Lemma

There exist functions Ψ^{ω}, Ψ^{s} such that in any sticky-price equilibrium, firm output is log-separable

$$y\left(\omega_i^t, s^t\right) = \Psi^{\omega}(\omega_i^t)\, \Psi^{s}(s^t), \text{ where } \Psi^{\omega}(\omega) = g\left(k\left(\omega\right), h\left(\omega\right)\right)^{\zeta}$$

- stickiness implies relative prices must be independent of s^t

$$\frac{p(\omega_i^t)}{p(\omega_j^t)} = \left[\frac{y(\omega_i^t, s^t)}{y(\omega_j^t, s^t)} \right]^{-1/\rho} = \left[\frac{\Psi^{\omega}(\omega_i^t)}{\Psi^{\omega}(\omega_j^t)} \right]^{-1/\rho}$$

  - further implies relative output must be independent of s^t

- a sticky-price allocation may be implemented with nominal prices

$$p(\omega_i^t) = \Psi^{\omega}(\omega_i^t)^{-1/\rho}$$

Optimal Monetary Policy

$$\text{let } \mathcal{B}(s^t) \equiv \left[\int \Psi^{\omega} \left(\omega_i^t\right)^{\frac{\rho-1}{\rho}} d{\preceq}(\omega_i^t | s^t) \right]^{\frac{\rho}{\rho-1}}$$

Theorem

Along any equilibrium that implements the Ramsey optimal allocation,

$$\log P(s) - \log P(s') = -\frac{1}{\rho} \left[\log \mathcal{B}(s) - \log \mathcal{B}(s') \right] \quad \forall s, s' \in \mathcal{S}^t, \forall t$$

What does this theorem mean?

$$\mathcal{B}(s^t) = \left[\int \Psi^{\omega} \left(\omega_i^t \right)^{\frac{\rho-1}{\rho}} d{\preceq}(\omega_i^t | s^t) \right]^{\frac{\rho}{\rho-1}} \quad \text{where} \quad \Psi^{\omega}\left(\omega\right) = g\left(k\left(\omega\right), h\left(\omega\right)\right)^{\zeta}$$

Proposition

Along any implementable allocation,

$$Y(s^t) = A\left(s^t\right) \mathcal{B}(s^t)^{1-\alpha} L\left(s^t\right)^{\alpha}$$

where, up to a first-order log-linear approximation,

$$\log \mathcal{B}(s^t) = \zeta_K \log K(s^t) + \zeta_H \log H(s^t),$$

- $\mathcal{B}$ is a proxy for aggregate beliefs
- variation in $\mathcal{B}$ related to variation in aggregate labor productivity
- inherits the cyclical properties of capital and intermediate goods

Countercyclical Price

Corollary

Suppose that capital and intermediate goods investment are procyclical along the Ramsey optimal allocation. Then, the optimal monetary policy targets a countercyclical price level.

Intuition for Countercyclical Price

- consider two firms: ω and ω'. efficiency requires that

$$\frac{y(\omega, s)}{y(\omega', s)} \text{ increases in belief } \omega$$

Intuition for Countercyclical Price

- consider two firms: ω and ω'. efficiency requires that

$$\frac{y(\omega, s)}{y(\omega', s)} \text{ increases in belief } \omega$$

- implementability: demand implies

$$\frac{p(\omega)}{p(\omega\prime)} = \left[\frac{y(\omega, s)}{y(\omega', s)} \right]^{-1/\rho} = \left[\frac{\Psi^{\omega}(\omega)}{\Psi^{\omega}(\omega')} \right]^{-1/\rho}$$

  - relative price must fall in belief ω

Intuition for Countercyclical Price

- consider two firms: ω and ω'. efficiency requires that

$$\frac{y(\omega, s)}{y(\omega', s)} \text{ increases in belief } \omega$$

- implementability: demand implies

$$\frac{p(\omega)}{p(\omega\prime)} = \left[\frac{y(\omega, s)}{y(\omega', s)} \right]^{-1/\rho} = \left[\frac{\Psi^{\omega}(\omega)}{\Psi^{\omega}(\omega')} \right]^{-1/\rho}$$

  - relative price must fall in belief ω

- relative price falls iff

$$p(\omega) \text{ falls with belief } \omega$$

$$P\left(s^t\right) \text{ falls in aggregate belief } \mathcal{B}(s^t)$$

Simple Example

Simple Example

$$U(C) = \frac{C^{1-\gamma}}{1-\gamma} \quad \text{and} \quad V(L) = \frac{L^{1+\epsilon}}{1+\epsilon}$$

- assume capital is fixed at 1 for all firms
- no government spending shocks
- variant with aggregate and idiosyncratic productivity shocks

$$y_{it} = A_{it} \left(h_{it}^{\eta} \right)^{1-\alpha} \ell_{it}^{\alpha},$$

$$A_{it} = A_t \exp v_{it}$$

Gaussian Information Structure

$$\omega_{it} = (x_{it}, z_t)$$

$$x_{it} = \log A_{it} = a_t + v_{it}, \quad v_{it} \sim \mathcal{N}(0, 1/\kappa_v) \text{ iid}$$

$$z_t = a_t + u_t, \quad u_t \sim \mathcal{N}(0, 1/\kappa_u)$$

- u_t introduces correlated noise in beliefs
  - common shock orthogonal to aggregate productivity
  - source of beliefs-driven aggregate fluctuations

The Power of Tax Instruments

$$\log\left(1-\tau^{r}\left(A_t, Y_t\right)\right) = \hat{\tau}_0 + \hat{\tau}_A \log A_t + \hat{\tau}_Y \log Y_t$$

Proposition

Under flexible prices, equilibrium GDP satisfies

$$\log GDP\left(s^t\right) = \gamma_0 + \gamma_a \log A_t + \gamma_u u_t$$

for some scalars

$$\gamma_0, \gamma_Z, \gamma_z \in \mathbb{R}$$

which are determined by the tax contingencies

$$\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y \in \mathbb{R}$$

Optimal Monetary Policy with Correlated Noise

Proposition

In any equilibrium that implements the Ramsey optimal allocation,

$$\log C\left(s^t\right) = \Delta_{ca} \log A\left(s^t\right) + \Delta_{cu} u_t,$$

$$\log P(s^t) = -\Delta_{pa} \log A\left(s^t\right) - \Delta_{pu} u_t,$$

where

$$\frac{\Delta_{pa}}{\Delta_{ca}} > 0 \qquad \textit{and} \qquad \frac{\Delta_{pu}}{\Delta_{cu}} > 0.$$

Conclusion: Policy Lessons

Despite informational frictions and beliefs-driven fluctuations,

- Flexible-price allocations remain optimal
    - optimal taxes as in Lucas Stokey (1983)

- In order to implement Flex-price allocations:

  Negative Correlation between prices and output