# Conformity Concerns: A Dynamic Perspective
# Supplemental Appendix

Roi Orzach*

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This supplemental appendix contains two sections. The first provides additional results within the linear Gaussian environment and the second discusses extensions to more general environments.

## 1 Additional Linear Gaussian Analyses

In this subsection, I conduct four additional analyses within the linear Gaussian environment: shifted conformist preferences, long-lived players, changing preference types, and environments without variation in $\gamma_t$.

### 1.1 Shifted Preferences:

Let us begin with an analysis with conformist preferences where individuals want their perceived preference type to be $c$ units higher than the average in the population. Further, I consider the environment of Section 3 without uncertainty allowing us to drop any time-dependence. A player with preference type $v$'s utility is,

$$-(a-v)^2 - \kappa\big(\hat{v}(a) + c\big)^2. \tag{1}$$

**Proposition A1** (**Shifted Preferences**)
*Suppose the preferences of the players are as in Equation* (1)*, then in the linear equilibrium where $\hat{v}(a) = \alpha \cdot a + \beta$, $\alpha$ is independent of $c$.*

*Department of Economics, Boston University, 270 Bay State Road, Boston MA 02215 (e-mail: orzach@bu.edu).

This result shows that while choosing a value $c \neq 0$ will lead to different equilibrium decisions, the level of adaptiveness remains the same. This result can easily be generalized to show that the results regarding social learning extend to $c \neq 0$.

*Proof of Proposition A1.* Conjecture equilibrium beliefs $\hat{v}(a) = \alpha a + \beta$. Given $\hat{v}(\cdot)$ the first-order condition of a player is,

$$a - v + \kappa\alpha(\alpha a + \beta + c) = 0 \iff v = (1 + \kappa\alpha^2)a + \kappa\alpha(\beta + c).$$

In equilibrium the conjectures are correct which implies,

$$\alpha = 1 + \kappa\alpha^2 \text{ and } \beta = \kappa\alpha(\beta + c).$$

Finally, whether a solution to this system of equations exists is independent of $c$. $\square$

## 1.2 Long-Lived Players

This subsection analyzes the incentives of a longer-lived player. To model such a phenomena, I continue to index time by $t \in \{0, 1, \dots\}$, but label the players by $i \in \{0, 1, \dots\}$. I denote by $t(i)$ the first period in which player $i$ appears. Further, each player, $i$, continues onto the next period with uniform probability $p$ and with probability $1 - p$ is replaced by player $i + 1$. Each player has a discount factor $\delta < p$, which includes both the probability of continuation and inter-temporal discounting. The model in the primary analysis considers $\delta = 0$. When the players are long-lived, the utility for player $i$ is as follows:

$$u^{t(i)} + \delta u^{t(i)+1} + \delta^2 u^{t(i)+2} + \dots.$$

This game featuring persistent private information entails well-known non-trivial modeling choices. Assuming $\theta$ is common knowledge greatly simplifies the analysis. Further, if $\theta$ is common knowledge, it is without loss of generality to consider $\theta = 0$.

The final simplification I make is a restriction to the following class of equilibria that mimic the characterization in the primary analysis. In period $t(i)$, player $i$ utilizes a linear decision rule. If there exists a linear decision rule with revelation, such a decision rule is utilized in period $t(i)$. As player $i$'s preference type is then fully revealed on-path, in all subsequent periods player $i$ utilizes a pooling decision rule as determined by $\mathbb{E}(v_i|h_t, a_t)$. On path, this equals $v_i$ and is hence socially optimal. When there does not exist a revealing linear decision rule that constitutes an equilibrium in period $t(i)$, the decision rule is fully pooling in period $t(i)$. As no new information is learned, period $t(i)+1$ is equivalent to $t(i)$, and thus a pooling decision rule will be used in period $t(i)+1$, and by induction, for all subsequent periods for player $i$. Note that this set of equilibrium refinements prescribes a unique decision rule for each player because the pooling decision rule is always an equilibrium.

**Proposition A2** (**Long-Lived Players**)
*When players are long-lived with a discount factor, $\delta$, have a conformity weight, $\kappa$, and $\theta$ is common knowledge, then an equilibrium with revelation exists if and only if:*

$$\kappa < \kappa^{c.k.}(1-\delta)\frac{\tau_{\mu,t}+1}{\tau_{\mu,t}}, \tag{2}$$

*where $\tau_{\mu,t}$ denotes the precision about the public beliefs about the average preference type.*

*Proof of Proposition A2.* I will denote by $\tau_{v_{\text{ind}}}$ the precision of $\nu_t$ which was normalized to one, where $\nu_t$ was defined to satisfy $v_t := \mu + \nu_t$. Further, sufficient statistics for $h_{t(i)}$ are the current mean and precision of $\mu(t)$ which are denoted as $\bar{\mu}(t)$ and $\tau_{\mu,t}$. I will now consider whether the decision rule for player $i$ can involve revelation in period $t(i)$, which will be denoted as $t$ for the rest of the proof. A linear equilibrium with revelation, when it exists, is of the form,

$$a_t = \alpha_t v_i + \beta_t.$$

The public perception of $v_i$ given $a_t$ is equal to $(a_t - \beta_t)/\alpha_t$. Therefore, one can write the utility as a function of $a_t$ as follows,

$$\begin{aligned} u_i(a_t) = &-(a_t - v_i)^2 - \kappa\Big(\frac{a_t-\beta_t}{\alpha_t} - \frac{\bar{\mu}(t)\tau_{\mu,t} + v_i\tau_{v_{\text{ind}}}}{\tau_{\mu,t}+\tau_{v_{\text{ind}}}}\Big)^2 \\ &- \frac{\delta}{1-\delta}\Big(\frac{a_t-\beta_t}{\alpha_t} - v_i\Big)^2 - \frac{\delta}{1-\delta}\kappa\Big(\frac{a_t-\beta_t}{\alpha_t} - \frac{\bar{\mu}(t)\tau_{\mu,t} + v_i\tau_{v_{\text{ind}}}}{\tau_{\mu,t}+\tau_{v_{\text{ind}}}}\Big)^2, \end{aligned}$$

where the first line denotes the payoffs in period $t$ given player $i$'s perception of $\mu$ and the second line denotes the continuation payoff in all subsequent periods. Taking a first-order condition implies:

$$0 = a_t - v_i + \frac{\kappa}{\alpha_t}\frac{1}{1-\delta}\Big(\frac{a_t - \beta_t}{\alpha_t} - \frac{\bar{\mu}(t)\tau_{\mu,t} + v_i\tau_{v_{\text{ind}}}}{\tau_{\mu,t} + \tau_{v_{\text{ind}}}}\Big) + \alpha_t\frac{\delta}{1-\delta}\big(\frac{a_t - \beta_t}{\alpha_t} - v_i\big).$$

Finally, in equilibrium, the beliefs are correct which implies $v_t = (a_t - \beta_t)/\alpha_t$. Simplifying the remaining terms implies:

$$\begin{aligned} 0 &= a_t - v_i + \frac{\kappa}{\alpha_t}\frac{1}{1-\delta}\Big(v_i - \frac{\bar{\mu}(t)\tau_{\mu,t} + v_i\tau_{v_{\text{ind}}}}{\tau_{\mu,t} + \tau_{v_{\text{ind}}}}\Big) \\ \iff a_t &= v_i\big(1 - \frac{\kappa}{\alpha_t}\frac{1}{1-\delta}\frac{\tau_{\mu,t}}{\tau_{\mu,t} + \tau_{v_{\text{ind}}}}\big) + \frac{\kappa}{\alpha_t}\frac{1}{1-\delta}\frac{\tau_{\mu,t}\bar{\mu}(t)}{\tau_{\mu,t} + \tau_{v_{\text{ind}}}}. \end{aligned} \tag{3}$$

In equilibrium, $\alpha_t$ must equal the coefficient on $v_i$ in the decision rule:

$$\alpha_t = 1 - \frac{\kappa}{\alpha_t}\frac{1}{1-\delta}\frac{\tau_{\mu,t}}{\tau_{\mu,t} + \tau_{v_{\text{ind}}}} \iff \alpha_t = \frac{1 \pm \sqrt{1 - 4\kappa\frac{1}{1-\delta}\frac{\tau_{\mu,t}}{\tau_{\mu,t}+\tau_{v_{\text{ind}}}}}}{2}.$$

Further, note that given $\alpha_t$, the solution to $\beta_t$ is uniquely determined by Equation (3). Hence, a solution exists if and only if the condition in the text holds because this condition corresponds to the term in the square root being non-negative. □

Proposition A2 generates the stark prediction that the degree of misperceptions about the average preference type, $\mu$, is shaped by the players' discount factor. Ignoring integer constraints, the players cease utilizing decision rules with revelation when $\tau_{\mu,t}$ binds Equation (2). Solving Equation (2) implies that when the players' discount factor is higher, the players learn less about $\mu$. This result adds a competing force to those suggested by the social learning literature. In that literature, long-lived players have an ability to observe more data and thus make more accurate decisions. This extension highlights that when conformity concerns are present, long-lived players might make worse decisions.

## 1.3 Changing Preferences and Peer-Oriented Interventions

The social psychology literature notes that "a society's perception of itself tends to lag behind actual changes in people's private beliefs and values," and argues this lag necessitates peer-oriented interventions to improve decision-making (Miller, 2023). To address these phenomena, I generalize the main model to allow $\mu$ to be time dependent and follow an autoregressive process parameterized as,

$$\mu^*(t) = \rho\mu^*(t-1) + (1-\rho)\psi_t, \tag{4}$$

where $\psi_t$ is independent across time and distributed as $N(0, \tau_\mu)$, with $\tau_\mu$ being the same $\tau_\mu$ as in Section 2 defined implicitly by $\mu \sim N(0, \tau_\mu)$. Making this equivalence allows the ex-ante uncertainty about $\mu^*(t)$ to be equal to that of $\mu$ for all $t$. The first implication of changing preference types is that switches to pooling decision rules are temporary.

In the primary analysis, once the players switch to a pooling decision rule, the players pool for all subsequent periods. However, given the changing average preference type, if the players pool in all subsequent periods, the public beliefs eventually converge back to beliefs where revelation occur. Finally, in this extension, the players utilize the decision rule with revelation for arbitrarily many periods and $\theta$ remains fixed, implying that it is without loss to assume that $\theta$ is common knowledge and fixed at zero when analyzing the asymptotic behavior of the community. The following proposition analyzes the asymptotic behavior of the players as a function of the conformity concerns.

**Proposition A3** (**Shifting Preferences**)
*Suppose $\mu$ follows an autoregressive process as defined in Equation* (4) *and denote by $\tau_{\mu,t}$ the precision about the public beliefs about $\mu^*(t)$ in period $t$. The signaling equilibrium in period $t$ involves revelation if and only if*

$$\tau_{\mu,t} < \frac{\kappa^{c.k.}}{\kappa - \kappa^{c.k.}}. \tag{5}$$

*As a result, there exists a threshold $\kappa^* \in \left(\kappa^{c.k.}, \kappa^{c.k.} \cdot (\tau_\mu + 1)/\tau_\mu\right)$ such that*

1. *If $\kappa \leq \kappa^*$ the signaling equilibrium involves revelation in all periods and $\tau_{\mu,t} \to \tau(\rho)$. Further, $\tau'(\rho) < 0$ and $\lim_{\rho\to 0} \tau(\rho) = \infty$.*

2. *If $\kappa \in \left(\kappa^*, \kappa^{c.k.}(\tau_\mu + 1)/\tau_\mu\right]$ the signaling equilibrium involves revelation (i.e., the precision in the beliefs is less than the right-hand side of Equation* (5)*) for infinitely many periods, involves pool for infinitely many periods (i.e., the precision in the beliefs is greater than the right-hand side of Equation* (5)*), and $\tau_{\mu,t}$ does not converge.*

3. *If* $\kappa > (\tau_\mu + 1)/\tau_\mu$*, the signaling equilibrium is pooling in all periods and* $\tau_{\mu,t} = \tau_\mu$ *in all periods.*

*Proof of Proposition A3.* Note that when $\theta$ is common knowledge the condition for the existence of a revealing equilibrium reduces to a function $\kappa^*(\tau_{\mu,t})$ whereby a decision rule with revelation exists if and only if $\kappa < \kappa^*(\tau_{\mu,t})$ (for a proof, set $\delta = 0$ in the characterization in Proposition A2).

The threshold $\kappa^*$ is determined by the value of $\kappa$ that binds Equation (5) when $\tau_{\mu,t} = \max_{t'} \tau_{\mu,t'} := \tau^*$ in an equilibrium where every period involves revelation. As $\tau_{\mu,t} < \tau_\mu/\rho^2$ and $\rho > 0$, then $\kappa^* > \kappa^{\text{c.k.}}$.

Statement (1) and (3) follow immediately from the criterion for when the decision rule in period $t$ involves revelation.

Finally, statement (2) follows from noting that when $\kappa > \kappa^*$, the equilibrium must have infinitely many periods of pooling. If the equilibrium had finitely many periods of pooling, then consider the final period of pooling. After this period, there will exist a period where the precision is $\tau^*$. Further, as $\kappa > \kappa^*$, the decision rule must involve revelation in this period deriving a contradiction. Similarly, if there are only finitely many periods of revelation, then after all the periods of pooling, $\tau_{\mu,t} \to \tau_\mu$. However, at this point, the decision rule must involve revelation. Finally, that there are infinitely periods of pooling and infinitely many periods of revelation implies $\tau_{\mu,t}$ is not Cauchy and thus does not converge. □

This proposition implies that, all else equal, groups who have stronger weights on conformity concerns wait longer to adapt to the underlying conditions. This result is broadly consistent with the social psychology literature on pluralistic ignorance and how norms change. In a review article, Miller (2023) states, "widespread changes in private attitudes change are not sufficient for social norm change. The group's recognition that its collective attitudes have changed is also necessary. Without this recognition, norm change will be impeded." One can interpret this through the model as follows: if the players are pooling in period $t$ on a low decision due to a low belief $\mu^*(t)$, despite a high value for $v_{t+1}$ signaling a change in private attitudes has occurred, this change is *not* sufficient to change the decision rule. Rather, enough periods must pass for the group to be certain that their collective attitudes have changed. Finally, note that changing preference types are an additional motivation behind the peer-oriented interventions considered in Section 5. Without these interventions, the social norms will lag behind the true attitudes in the population, causing inefficiencies.

## 1.4 Unstable Confounded Learning Outcomes

If $\gamma_t = 1$ for all $t$, there is a possibility that social learning fails despite a revealing equilibrium in every period. For instance, if $a_t = \lambda s_t + (1-\lambda)v_t$ in every period, the players' will never be able to disentangle $\theta$ and $\mu$. The following proposition formalizes this intuition.

**Proposition A4** *If there exists infinitely many periods of revelation, the variance matrix of the belief, $\Sigma_t$, either converges to the zero-matrix or to*

$$\begin{pmatrix} c_1 & -\sqrt{c_1 c_2} \\ -\sqrt{c_1 c_2} & c_2 \end{pmatrix} \tag{6}$$

*for $c_1, c_2 > 0$.*

*Proof of Proposition A4.* Suppose that $\Sigma_t \not\to_p 0$, then Lemma 1 implies that $\theta(t) \not\to_p \theta$ and $\mu(t) \not\to_p \mu$. Further, for all $t$, $a_t$ is a linear combination of $s_t$ and $v_t$ plus a constant. As a result, for all $t$ a sufficient statistic for $a_t$ is, $\lambda_t s_t + (1-\lambda_t)v_t$, for some constant $\lambda_t \in (0,1)$. If $\lambda_t$ does not converge, then there exist two different convergent subsequences that converge to $\lambda^1, \lambda^2$, respectively. By the Law of Large Numbers, the players learn $\lambda^1\theta + (1-\lambda^1)\mu$ and $\lambda^2\theta + (1-\lambda^2)\mu$. However, $\lambda^1 \neq \lambda^2$ implies the players learn $\theta$ and $\mu$. Thus, if the beliefs do not converge to the zero matrix, $\lambda_t \to_p \lambda$.

Hence, assume $\lambda_t \to_p \lambda$. As social learning fails, $\lambda \in (0,1)$. Denote by $\tau_t$ the precision matrix of the beliefs at time $t$ with the following parametrization.

$$\tau_t := \begin{pmatrix} \tau_{1,t} & \tau_{2,t} \\ \tau_{2,t} & \tau_{3,t} \end{pmatrix}.$$

The beliefs update as follows in any equilibrium with revelation,

$$\tau_{1,t+1} = \tau_{1,t} + \phi_t \ , \tau_{2,t} = \tau_{2,t} + \sqrt{(1-\phi_t)\phi_t} \ , \text{ and } \tau_{3,t} = \tau_{3,t} + 1 - \phi_t,$$

where $\phi_t = \dfrac{\lambda_t^2}{\lambda_t^2 + (1-\lambda_t)^2}$.

Further, $\phi_t \to_p \phi := \lambda^2/(\lambda^2 + (1-\lambda)^2) \in (0,1)$. Therefore, $\rho_t$, the correlation in the variance matrix, is equal to the negative square root of $\tau_{2,t}^2/(\tau_{1,t}\tau_{3,t})$ as stated below:

$$\rho_t = -\sqrt{\frac{(\sum_{i=1}^t \phi_t(1-\phi_t)}{(\sum_{i=1}^t \phi_t)(\sum_{i=1}^t 1-\phi_t)}}.$$

As $\phi_t \to_p \phi$, one can re-write $\rho_t$ asymptotically as follows,

$$\rho_t = -\sqrt{\frac{t^2 \lim \phi_t(1-\phi_t) + o(t^2)}{t^2 \lim \phi_t(1-\phi_t) + o(t^2)}} \to_p -1.$$

Finally, the variance of $\theta$ and $\mu$ are each bounded and decreasing sequences, and thus both variances converge completing the proof. □

I will call the above such beliefs "confounded learning outcomes." In this model, such beliefs are not asymptotically stable (in the dynamical-system sense), because after a small perturbation (such as decreasing the variance of $\theta$) the beliefs will never return to this confounded learning outcome. Generally, for confounded learning outcomes to be stable, there must exist multiple preference types who have opposing preferences (cf. Smith and Sørensen, 2000). By introducing variance through $\gamma_t$, this ensures that if the signaling equilibrium involves revelation in each period, then $\lambda_t$ does not converge.

# 2 Beyond Linear Gaussian

I first conduct a parallel analysis to that in Subsection 6.2 in the main text. Next, I discuss why such an analysis is intractable for general multivariate distributions.

## 2.1 Learning the Fundamental State with Generalized Uncertainty

Subsection 6.2 analyzed the impact of conformity concerns on social learning about $\mu$ when $\theta$ was common knowledge for a general class of distributions. This subsection will analyze the impact of conformity concerns on social learning about $\theta$ when $\mu$ is common knowledge for a general class of distributions.

Suppose each player has a preference type $v_t \sim F$ where $F$ admits a continuous and differentiable density $f(\cdot)$ with support equal to the real line and mean normalized to zero. Further, denote by $\omega_t$ the player $t$'s posterior expectation of $\theta$ in period $t$ after observing $s_t$. I will assume the distribution of $\omega_t$ admits a continuous and differentiable density. The only decision relevant information for the player is $v_t + \omega_t := \tilde{v}_t$ and it is without loss to consider decision rules that only condition $\tilde{v}_t$. Further, by an identical argument to Bernheim (1994), the only equilibria satisfying D1 are central pooling equilibria as a function of $\tilde{v}_t$.

Outside the central pool the decision rule will be strictly increasing in $\tilde{v}_t$ which implies the existence of an inverse. Denote by $x$ an arbitrary decision where $a^{-1}(x) = \tilde{v}_t$, then the conformity loss upon choosing the decision $x$ is

$$\kappa\Big(\tilde{v}_t - h_1\big(\tilde{v}_t\big)\Big)^2 + \kappa h_2\big(\tilde{v}_t\big). \tag{7}$$

Note that a player has concerns over the second moment of their perceived preference type. This second moment has two components: the mean and the variance. The mean of the perceived preference type is simply $\tilde{v}_t$ less $h_1\big(\tilde{v}_t\big)$, where $h_1(t)$ is defined such to make such an equality hold. Without uncertainty, $h_1(\tilde{v}_t) = 0$, however, in general $h_1(\tilde{v}_t) \neq 0$ because a high value of $\tilde{v}_t$ may stem from either a high value of $v_t$ or a high value of $\omega_t$. Further, $h_2(\tilde{v}_t)$ represents the impact of the variance of players with different preference types choosing the same decision on the conformity loss. Formally, $h_1\big(\tilde{v}_t\big) = \tilde{v}_t - \mathbb{E}(v_t|v_t + \omega_t = \tilde{v}_t)$ and $h_2\big(v_t|v_t + \omega_t = \tilde{v}_t)\big) = \mathbf{Var}(v_t|v_t + \omega_t = \tilde{v}_t)$.

Rather than dealing with the distributions of $v_t$ and $\omega_t$, I will work with $h_1(\cdot)$ and $h_2(\cdot)$. Differentiating the utility function given the conformity loss in Equation (7) implies

$$\begin{aligned} 0 &= 2(a_t - \tilde{v}_t) + 2\kappa\Big(\tilde{v}_t - h_1(\tilde{v}_t)\Big)\frac{1}{a'(\tilde{v}_t)}\big(1 - h_1'(\tilde{v}_t)\big) + \frac{\kappa h_2'(\tilde{v}_t)}{a'(\tilde{v}_t)} \\ \iff a'(v_t) &= \frac{\kappa\Big(\big(\tilde{v}_t - h_1(\tilde{v}_t)\big)\big(1 - h_1'(\tilde{v}_t)\big) + \frac{1}{2}h_2'(\tilde{v}_t)\Big)}{a_t - v_t}. \end{aligned} \tag{8}$$

However, such an expression has a similar formulation to Equation 19 from the main text before Proposition 7. One can replace $g_t(v_t)$ in Equation 19 with the expression in Equation (8) to recover similar results.

## 2.2 Learning About Both the Preferences of Others and the Fundamental State:

This analysis is qualitatively different to the previous analysis. Now $a_t$ is a function of two-dimensional private information: $s_t, v_t$. Thus, the equilibrium decision rule in general will be determined by a partial differential equation. Further, for any conjectured non-linear decision rule, $\phi(b, a_t)$ will be non-Gaussian. As one must integrate $\phi(b, a_t)$ to determine the conformity loss, one cannot solve (let alone, write down) the partial differential equation in closed form as was done for the uni-dimensional analyses.

# References


**Bernheim, B Douglas**, "A theory of conformity," *Journal of political Economy*, 1994, *102* (5), 841–877.

**Miller, Dale T**, "A century of pluralistic ignorance: what we have learned about its origins, forms, and consequences," *Frontiers in Social Psychology*, 2023, *1*, 1260896.

**Smith, Lones and Peter Sørensen**, "Pathological outcomes of observational learning," *Econometrica*, 2000, *68* (2), 371–398.