Fast and Slow Technological Transitions*

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Abstract

Do economies adjust slowly to certain technological innovations and more rapidly to others? We argue that the adjustment is slower when innovations mainly benefit production activities requiring skills which are more different from those used in the rest of the economy. The reason is that, when such skill specificity is stronger, the adjustment of labor markets is driven less by the fast reallocation of older incumbent workers and more by the gradual entry of younger generations to the benefitted activities. We begin by documenting that the U.S. labor market adjusted differently to the arrival of Information & Communications Technologies (ICT) in the late twentieth century than it did to innovations in manufacturing at the beginning of that century. We then build an overlapping generations model of technological transitions. It allows us to sharply characterize the effects of skill specificity on equilibrium dynamics, match the evidence in a parsimonious way, and study its welfare implications. We show that stronger skill specificity helps to explain why the ICT transition was more unequal and slower, driven entirely by the gradual entry of younger generations who accrued more of the welfare gains from ICT innovations.

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1 Introduction

Technological transitions follow the arrival of major innovations. In the past, technologies like the steam engine, electricity and computers have fundamentally changed the organization of economic activity. In the future, advances in artificial intelligence and robotics hold the promise to do so once again. When such innovations are biased towards specific skills, inequality can rise fast (Katz and Murphy, 1992). The adjustment of labor markets, however, can take decades, as workers reallocate and younger generations acquire new skills (Chari and Hopenhayn, 1991).

But do economies adjust slowly to certain technological innovations and more rapidly to others? Studies on this question are scarce. The literature has mainly focused on explaining facts that are common across episodes of adjustment, as opposed to how different technologies shape them.1 This is surprising for two reasons. First, much evidence shows that technologies differ remarkably both in their impact on workers and speed of diffusion.2 Second, understanding when economies adjust faster is central for weighing the benefits of technological innovations against their distributional consequences, and for the optimal design of government policies.3

In this paper, we show that technological transitions are not all alike. Labor markets adjust slowly to some technological innovations and more rapidly to others. We argue that one important reason is that certain innovations may benefit production activities requiring skills which are rather different from those used by incumbent workers in the rest of the economy. When such skill specificity is stronger, incumbent workers find it harder to reallocate, and the relative wage of activities that benefited from the innovation increases more. As a result, younger generations of workers entering the labor market have stronger incentives to enter these activities.4 Therefore, a technological transition is slower when skill specificity is stronger because it is driven less by the fast reallocation of older incumbent workers and more by the gradual entry of younger generations.

To support this argument, we first provide evidence that the U.S. labor market adjusted differently to the arrival of Information & Communications Technologies (ICT) in the late twentieth century than it did to innovations in manufacturing at the beginning of that century. We then build an overlapping generations model of technological transitions. It allows us to sharply characterize the effects of skill specificity on equilibrium

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1See Helpman (1998) for a review on the diffusion of general purpose technologies, and Herrendorf et al. (2014) for facts about structural transformation.


3Guerreiro et al. (2017) and Beraja and Zorzi (2022) show that optimal policy depends on the speed of adjustment.

4Younger generations can do so, for example, because they can acquire skills more easily or face lower mobility costs compared to older workers (Lagakos et al., 2018; Kambourov and Manovskii, 2008).
dynamics, match the evidence in a parsimonious way, and study its welfare implications.

We begin in Section 2 by establishing three differences between the transitions triggered by the recent innovations in ICT and the manufacturing innovations of the early 1900s. First, the relative wage of ICT-intensive occupations increased fast between 1980 and 2000, but their relative employment only slowly increased after 2000. In contrast, relative employment in manufacturing-intensive occupations increased shortly after 1900. Second, the employment growth of ICT-intensive occupations was entirely driven by younger workers; older workers did not reallocate towards these occupations. In contrast, older and younger workers contributed to the expansion of manufacturing-intensive occupations after 1900, and did so to a similar extent. Third, skill specificity was stronger for innovations in ICT than manufacturing: older incumbent workers in other occupations performed tasks which were more similar to those required by manufacturing-intensive occupations in 1900 than they were to ICT-intensive occupations in 1980.\footnote{We further show that the task distance between occupations affects responses to technological innovations. Given its exposure to the technological innovation, an occupation with a higher task distance from the rest of the economy experiences responses that are smaller for relative employment but larger for relative wages.}

Motivated by the evidence, Section 3 presents a model of technological transitions. Firms use two technologies to produce a final good. There is a continuum of worker skill types. A type determines workers’ effective labor units when employed in each technology. This allows us to formalize the notion of skill specificity as a type’s difference in productivity between the two technologies. There are overlapping generations of workers, and each generation forms a large household. The household assigns workers to technologies by comparing their wage in each one, similar to Roy (1951). This allows the model to reproduce the reallocation of older incumbent workers following changes in relative wages across technologies. Lastly, at birth, the household chooses how much labor of each skill type to supply, given the future path of relative wages. This allows the model to replicate differences across generations in relative employment when generations face distinct future paths of relative wages at birth.

In Section 4, we turn to the dynamic adjustment of our economy to an innovation that permanently increases the productivity of one of the technologies, therefore raising its labor demand. We characterize the equilibrium adjustment dynamics by first establishing an equivalence result. To a first order approximation, the equilibrium dynamics behave as if they were generated by a reduced-form model of relative labor demand and supply, where the relative labor supply of each generation combines time- and generation-specific components. The generation-specific term captures the effect of changes in the present discounted value of the relative wage (which we dub \( q \)) that a generation faces at birth. As such, it governs the relative labor supply elasticity at longer horizons. In the structural model, this elasticity is higher when the cost of adjusting the
skill supply of a new generation is lower. The time-specific term captures the impact that changes in the relative wage at a point in time have on the relative labor supply of every generation. As such, it governs the short-run elasticity of relative labor supply. In the structural model, this short-run elasticity is lower when skill types are more different in terms of their productivity in each technology (i.e., when skill specificity is stronger).

The adjustment dynamics follow the economic logic of the seminal \( q \)-theory (Hayashi, 1982), albeit applied to labor markets. On impact, the technological innovation raises the relative wage, which induces the reallocation of older incumbent generations towards the improved technology. The increase in current and future relative wages (summarized in \( q \)) causes entering generations of households to increase their relative labor supply compared to older incumbents. Through the lens of the structural model, this gap results from younger generations choosing to supply more labor of skill types that are more complementary to the improved technology. Along the transition, as younger generations replace older ones, the economy’s relative employment slowly increases and the relative wage declines. One interpretation of this labor reallocation process is that innovations slowly diffuse as more workers employ them over time.

Section 5 establishes our main theoretical result by comparing the adjustment in two economies with varying short-run labor supply elasticities. We interpret this as a comparison between two episodes: one in which the innovation improves a technology intensive in skills which are markedly different from those used by the rest of the economy (skill specificity is strong) and another one in which they are more similar (skill specificity is weak). Stronger skill specificity leads to a slower adjustment process, in which the increase in relative employment is more back-loaded and the rise in the relative wage is more front-loaded. On impact, stronger skill specificity directly weakens the reallocation of older incumbent workers. This results in a stronger increase in the present discounted value of the relative wage (\( q \)), which in turn endogenously strengthens the incentives of younger generations to increase their relative labor supply. Ultimately, transitional dynamics in relative employment and wages become more important because the adjustment is driven more by the slow entry of young generations.

Returning to our evidence, Section 6 shows that our parsimonious model can quantitatively match the different adjustment dynamics following both ICT and manufacturing innovations purely as a consequence of varying the short-run elasticity of relative labor supply (or skill specificity in our structural model). In particular, our baseline calibration sets this elasticity to zero, in line with the lack of reallocation of older workers during the ICT transition, and feeds a labor demand innovation of a magnitude that matches the cumulative relative employment increase in ICT-intensive occupations after 40 years. We externally calibrate the few remaining parameters using standard values from the literature (e.g., the discount and death rates). The model successfully repli-
icates the ICT transition’s fast increase in the relative wage, delayed relative employment increase, muted reallocation of older workers, and large differences in relative employment responses between younger and older workers. Lastly, in line with the smaller task distance between manufacturing-intensive occupations and the rest of the economy, we repeat the analysis for a larger short-run labor supply elasticity, while keeping all other parameters fixed. The model now can reproduce the transition following the manufacturing innovations of the early 1900s; its muted relative wage change, fast increase in relative employment and reallocation of older workers, and lack of differences in relative employment responses between younger and older workers.

We conclude the paper with a normative analysis in Section 7. Since the model can match the evidence on relative responses, this gives us some confidence that we can use it to study how skill specificity affects aggregate dynamics following technological innovations. We show that the welfare gains from technological innovations are smaller when the adjustment is slower due to stronger skill specificity, as aggregate consumption increases take longer to materialize and mostly benefit future generations. In terms of magnitudes, moving from a zero short-run labor supply elasticity (as in our baseline calibration for the ICT transition) to, for example, a larger elasticity of 3 implies that the welfare loss from slow transitional dynamics falls by 1/3.

**Related literature.** Our paper is related to a literature studying slow adjustment dynamics. Close to our paper, the within and between generations margins of adjustment have also been shown to be important for changes in U.S. employment composition (Murphy and Topel, 1987; Autor and Dorn, 2009) and wage inequality (Violante, 2002), as well as structural transformation (Hobijn et al., 2019; Porzio et al., 2020). As in our model, Violante (2002) also emphasizes skill specificity in determining the reallocation of older incumbent workers, and Porzio et al. (2020) highlights skill acquisition in explaining differences between young and old generations. We add to this literature by analyzing the dynamics of adjustment after technological innovations, as well as how skill specificity shapes them by varying the strength of the within and between generations margins.

More generally, a strand of the literature has emphasized slow changes in the supply of labor to particular sectors or occupations (Matsuyama, 1992; Heckman et al., 1998; Lee and Wolpin, 2006; Dvorkin and Monge-Naranjo, 2019; Traiberman, 2019). To the best of our knowledge, our theoretical results linking how slow the adjustment is to skill specificity is new to this literature, as is our evidence for the two technological transitions. That said, we note that stronger skill specificity would also lead to a slower adjustment in quantitative dynamic Roy models where wages are determined in equilibrium, but not in models of labor reallocation where wages are exogenous. The reason is that, in
the latter models, stronger skill specificity would only dampen the reallocation of older incumbent workers, but leave differences across generations unchanged because the incentives of entering generations to change their supply of skills would remain the same. A complementary literature has analyzed slow adjustment dynamics coming from gradual changes in labor productivity due to firm learning-by-doing (Atkeson and Kehoe, 2007), knowledge diffusion (Lucas and Moll, 2014), the creation of new worker tasks (Acemoglu and Restrepo, 2018), capital accumulation (Dix-Carneiro and Kovak, 2017), or automation (Jaimovich et al., 2021; Beraja and Zorzi, 2022). We show that gradual changes in labor productivity along the transition cannot alone account for the differences across technological transitions that we document. In particular, they cannot jointly explain the fast rise in relative wage and the slow increase in relative employment observed during the ICT transition, neither can they speak to relative employment changes which are larger for younger workers and weaker for occupations requiring more specific skills. In contrast, our paper highlights slow adjustment dynamics due to changes in the relative supply of labor within and between generations, and show that our parsimonious model can match the features of the two transitions that we study.

Lastly, a literature has focused on the consequences of ICT and manufacturing innovations in the U.S. Jovanovic and Rousseau (2005) show that, despite similar rates of price decline for ICT innovations and Electricity (used in manufacturing), diffusion was slower and productivity gains weaker following the ICT arrival. Adding to these facts, our findings show that the speed of labor reallocation was different in the two episodes too; and that varying skill specificity helps to explain why. A number of papers have shown evidence of capital-skill complementarity during both the expansion of manufacturing in the early 1900s and ICT-intensive occupations later in the twentieth century (Goldin and Katz, 1998; Autor et al., 1998, 2003; Goldin and Katz, 2009). Our paper also rests on the idea that these new technologies complemented certain skills, thereby raising their demand when they arrived. However, we show that manufacturing and ICT innovations impacted economic activities that differed in their task distance from other activities in the economy; varying the degree to which older incumbent workers could supply the specific skills whose demand increased and, as a consequence, how slow the adjustment was.

2 Evidence from Two Technological Transitions

In this section, we document three novel facts about how exposure to technological innovations affected employment and wages across occupations in the U.S. during two periods: the introduction of manufacturing-enhancing technologies in the early twentieth century, and the arrival of ICT innovations in the latter part of that century.
2.1 Exposure to new technologies and occupational outcomes

We begin by analyzing the dynamic responses of employment and wages in occupations more exposed to the new technologies in each of the two episodes. Our focus on occupations follows a recent literature studying the labor market consequences of new technologies — for a review, see Acemoglu and Autor (2011). Specifically, we estimate the following specification for different years $t$ during the period starting at $t_0$ in which new technologies became available:

$$\log Y_{o,t} - \log Y_{o,t_0} = \beta_t \text{Exposure}_o + \gamma_t + \epsilon_{o,t},$$  

(1)

where $Y_{o,t}$ is either the employment or the average wage in occupation $o$ at $t$, and Exposure$_o$ is a standardized measure of occupation $o$’s exposure to the main new technologies introduced during the period. Thus, $\beta_t$ is the estimate of how much higher the $t_0$ to $t$ growth in employment or wages was for an occupation with one standard-deviation higher shock exposure. We weigh occupations by their $t_0$ employment to obtain representative economy-wide estimates.

To define occupation $o$’s exposure in each episode, we rely on the idea that the relative demand for occupation $o$ should increase when a higher share of its workers are initially employed in industries that intensively use the main technologies that became available:

$$\text{Exposure}_o \equiv \sum_k \ell_{k,0} S^k,$$

where $\ell_{o,t_0}^k$ is the share of industry $k$ in the total employment of occupation $o$ at $t_0$, and $S^k$ is the importance for industry $k$ of the main new technologies of each episode.

For the early twentieth century, we define $S^k$ as the dummy variable that equals one if $k$ is a manufacturing industry. This captures the fact that manufacturing productivity increased substantially in this period due to several innovations such as the introduction of electricity and assembly lines (David, 1990; Gordon, 2000; Jovanovic and Rousseau, 2005). It yields the intuitive implication that the most exposed occupations were associated with blue collar manufacturing jobs; like millwrights, machinists, tool makers, and electrotypers. To address concerns that estimates capture other simultaneous shocks to manufacturing demand — for example, due to the Great Depression or the WWI effort — Section 2.4 discusses the robustness of our conclusions to defining exposure based on the electricity intensity of manufacturing industries. Note however that, as we discuss below, our main insights remain valid even if technology is not the sole driver of the relative demand increase for manufacturing-intensive occupations in this period.

For the late twentieth century, $S^k$ is a measure of $k$’s share of cost spent on products and services associated with ICT in the 1997 detailed input-output table. This measure...
builds on the idea that the decline in the ICT cost of the late twentieth century benefited industries using ICT intensively; and implies that the most exposed occupations include instructors, scientists, and managers. Our exposure measure is highly correlated with other measures used in the literature reviewed by Acemoglu and Autor (2011) to study the labor market consequences of recent skill-biased technological innovations and, for this reason, conclusions are similar when we use them.

We measure occupation outcomes using a sample extracted from the IPUMS U.S. Census database of males aged 16-64 years participating in the labor force. We use the 3-digit 1950 occupation classification for the early twentieth century, and the 3-digit occupation classification in Autor and Dorn (2013) for the more recent episode. Our wage measure is the average annual wage income of full-time employed individuals in each occupation. Appendix A.1 presents more details about the data construction.

Figure 1 reports our estimates of $\beta_t$ for the two transitions. We set $t_0$ as the earliest year of the U.S. Census following the arrival of the main technologies in each episode, as defined in Jovanovic and Rousseau (2005) and Comin and Mestieri (2018): 1900 for the early episode and 1980 for the latter episode. The black dots depict responses to ICT exposure for each decade in the 40-year period following 1980. The gray diamonds are the analog for manufacturing exposure after 1900. The vertical bars are the 90% confidence intervals associated with our estimates.

Panel A of Figure 1 shows that the relative employment growth in more exposed occupations was both slower and weaker overall following the ICT innovations of the late twentieth century compared to the manufacturing innovations of the early 1900s. In particular, higher ICT exposure was not associated with higher relative employment between 1980 and 2000, as indicated by the small and non-significant estimates in the black dots for the first two decades of the episode. In contrast, relative employment in manufacturing-intensive occupations rapidly increased between 1900 and 1920, as indicated by the large and statistically significant estimates in the gray diamonds. Employment only slowly reallocated towards ICT-intensive occupations after the year 2000, but the overall magnitude of reallocation during the entire period was half as large compared to manufacturing-intensive occupations. Specifically, over the forty-year period, a one standard deviation higher exposure was associated with a 0.1% increase in the relative employment of ICT-intensive occupations, whereas the corresponding increase was 0.2% for manufacturing-intensive occupations, both statistically significant.

We then turn to the wage response to ICT exposure in Panel B. Our estimates indicate that the path of the relative wage of ICT-intensive occupations has an inverted-U shape. The relative wage in ICT intensive occupations rapidly increased between 1980 and 1990, 

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6The final year of the second episode is 2019 to avoid the COVID pandemic in 2020. As in Autor and Dorn (2013), to increase sample size and measurement precision, we compute outcomes in 2019 by pooling the years of 2017-2019 of the American Community Survey.
Figure 1: Exposure to Technological Innovations and Occupation Outcomes in the Two Episodes

Panel A: Relative Employment

Panel B: Relative Wages

Panel C: Relative Employment, Older Workers

Panel D: Relative Employment, Younger Workers

Note. Panels A and B report estimates of $\beta_t$ obtained from (1) for a single group of all males aged 16-64yrs. Panels C and D report $\beta_{g,t}$ obtained from (2) for two groups: a Younger group including male workers aged 16-29yrs, and an Older group including male workers aged 30-64 yrs. Black dots are the estimates of the response to ICT exposure obtained from a sample of 310 occupations in the 40-year period following $t_0 = 1980$. Gray diamonds are the estimated responses to manufacturing exposure in the sample of 201 occupations in the 40-year period following $t_0 = 1900$. Estimates are weighted by the occupation’s employment at $t_0$. Dependent variable is the change in the occupation’s average wage in Panel B and employment in Panels A, C and D. The ICT exposure is the occupation’s average ICT-intensity across industries and the manufacturing exposure is the occupation’s average manufacturing-intensity across industries. Exposure measures are normalized to have zero mean and unit standard deviation. Vertical bars denote 90% confidence intervals clustered by occupation.

as can be seen from the positive and statistically significant estimates for the first decade of the episode. After 2000, the impact on relative wages slowly declines towards zero (albeit being more imprecisely estimated). Since the U.S. Census did not collect income information before 1940, we cannot implement our occupation-level specification for the first episode. However, we can analyze the evolution of the U.S. manufacturing wage premium using the 1975 Historical Statistics of the United States. Figure A2 in the Appendix shows that the relative wage in manufacturing did not increase much between 1900 and 1930.

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Due to the lack of employment responses before 1990, this increase is unlikely to be biased by selection based on unobserved worker attributes.

This is consistent with the evidence in Goldin and Margo (1992) that, in the decades prior to 1940, the strong relative expansion of skilled occupations happened while their wage premium was either relatively constant or mod-
In all, the joint dynamics of employment and wages suggest the following: responses observed in the early 1900s are consistent with a labor demand increase in manufacturing-intensive occupations that was matched by a rapid increase in labor supply. In contrast, during the more recent period, wage responses suggest a rapid increase in the demand for ICT-intensive occupations in 1980-1990, which only induced labor supply responses after 2000. As such, the slower employment adjustment to the arrival of ICT in the latter episode is not consistent with stories that emphasize a slower labor demand increase — for example, because ICT or other complementary technologies diffused more slowly. The reason is that a slower labor demand increase would have implied slower increases in both relative wages and employment, which is not what occurred as relative wages increased decades before employment did.

2.2 Responses in occupation outcomes across worker generations

We established above that relative labor supply across occupations adjusted at a slower pace to the demand shock induced by post-1980 ICT innovations than to the shock induced by the manufacturing innovations of the early 1900s. This is perhaps surprising given that many sources of reallocation frictions were likely more severe during the earlier episode. For instance, search and matching frictions declined over time (Martellini and Menzio, 2020), reallocation towards manufacturing jobs in the early 1900s also involved a costly move from traditional rural areas to new urban areas (Michaels et al., 2012), and consumption smoothing during job transitions likely improved over time due to better access to (social and private) insurance instruments (Chetty, 2008).

To investigate further what underlies the difference across the two episodes in the observed patterns of labor supply adjustment over time, we next separately estimate employment responses for worker groups representing younger and older generations. Intuitively, we would expect the overall labor supply response to be faster when generations already present in the labor market move to more exposed occupations compared to when the response is mostly driven by younger entrant generations. Specifically, we estimate the following specification:

$$\log Y_{og,t} - \log Y_{og,t_0} = \beta_{g,t} \text{Exposure}_o + \gamma_{g,t} + \epsilon_{og,t}.$$  (2)

The dependent variable is the log-change in employment for individuals of group $g$ in occupation $o$ between $t_0$ and $t$. The group-period fixed-effect $\gamma_{g,t}$ absorbs any group-specific shock with a common effect on all occupations. Accordingly, $\beta_{g,t}$ measures how much an increase of one standard-deviation in shock exposure affects occupation outcomes among workers of group $g$. We estimate this specification in a pooled sample of

\footnote{erately increasing — at least in comparison to the strong increase in the returns to skill observed in the 1980s. Similarly, Piketty and Saez (2003) find that “top income and wages shares display a U-shaped pattern over the century.”}
all groups, and compute standard errors clustered by occupation to account for correlated shocks affecting all groups in the same occupation. In our baseline, the older group includes males aged 30-64, and the younger group is composed of males aged 16-29.

Panels C and D of Figure 1 report the relative employment response for older and younger workers, respectively. The expansion of ICT-intensive occupations after 1980 was almost entirely driven by younger workers. Older workers did not reallocate towards ICT-intensive occupations: the estimates in the black dots of Panel C are small and non-significant during the entire period. For younger generations however, the black dots in Panel D show that the relative employment increase in ICT-intensive occupations was significant after forty years. In terms of magnitudes, the response for younger workers was 124% larger than that for older workers. In contrast, older workers significantly contributed to the expansion of manufacturing-intensive occupations after 1900, and did so to a similar extent than younger workers. The gray diamond estimates for both generations are positive and statistically significant in every decade of the early episode. In terms of magnitude, the relative employment response for younger workers was only 21% larger than that of older workers between 1900 and 1940.\(^9\)

Overall, our results suggest that the adjustment in labor supply across occupations in response to the demand shock triggered by ICT innovations was limited and delayed due to the muted reallocation of older generations. In fact, the post-2000 labor supply adjustment in ICT-intensive occupations was almost entirely driven by the response in the relative employment of younger entrant generations. However, in the early twentieth century, both younger and older generations contributed to the fast labor supply adjustment to the shock in the relative demand for manufacturing-intensive occupations. These patterns are consistent with at least three explanations. The first is that mobility frictions were less severe in the early 1900s (especially for older workers), regardless of which occupations workers moved towards. However, as mentioned above, many sources of mobility frictions likely were more, not less, severe in the earlier episode. Clearly this explanation is unappealing, though it cannot be entirely discarded. A second explanation is that ICT innovations increased the demand for younger workers (but not the old) in more exposed occupations. It is however hard to see a reason why, given the same skill set, ICT technologies would benefit younger workers more than older ones: ICT innovations were not biased towards age-specific attributes like physical force or health conditions. In line with this argument, Figure A6 in the Appendix shows that, conditional on a worker’s occupational exposure to ICT, older and younger work-\(^{9}\)Table A2 in the Appendix reports the difference in the relative employment response of younger and older workers in the two episodes. The between-generation gap is small and non-significant in the case of manufacturing-intensive occupations, whereas it is positive and statistically significant in the case of ICT-intensive occupations after 2000.
ers report similar usage of two important ICT innovations, computers and internet.\textsuperscript{10,11} The last explanation is that it was easier for older incumbent workers to reallocate towards manufacturing-intensive occupations than it was for them to reallocate towards ICT-intensive occupations. This could be, for instance, because their skills were more transferable to the manufacturing-intensive occupations that were expanding in the early episode. The next section provides evidence that this mechanism helps to explain the differences across the two episodes.

2.3 Differences in task content between occupations

We now investigate how the two episodes differed in terms of the ability of older incumbent workers in the rest of economy to supply the skills required by the occupations whose demand increased. We do so by measuring, for each episode, how different the task requirements of the occupations in high demand were in comparison to those of other occupations. Our approach builds on a recent literature showing that, both in the U.S. and other countries, gross job-to-job flows are higher between occupations that require similar tasks (Gathmann and Schönb erg, 2010; Traiberman, 2019; Schubert et al., 2021). Intuitively, task requirements capture the common component of the activities that workers need to perform on the job and, consequently, reallocation should be easier across occupations requiring workers to perform similar activities (for a given change in relative wages). Formally, for any two occupations \( o \) and \( o' \), we measure the distance between the distribution of their task requirements using an entropy metric:

\[
D_{o, o'} = \sum_v \chi_{o, v} \log \frac{\chi_{o, v}}{\chi_{o', v}},
\]

where \( \chi_{o, v} \) is occupation \( o \)'s intensity in task \( v \).\textsuperscript{12} We follow Autor and Dorn (2013) to measure \( \chi_{o, v} \) for \( v \in \{ \text{Manual, Routine, Cognitive} \} \) among the occupations in our sample in each of the two episodes. By using the same occupation task content in the two periods, we implicitly assume that the task intensity ranking across occupations in 1900 has a high correlation with that of 1980. We do so for lack of comprehensive task content data for the early twentieth century.\textsuperscript{13}

We start by measuring the average task distance between the occupations with the highest and the lowest levels of shock exposure in each of the two episodes. We consider

\textsuperscript{10}We use on-the-job time-use data from Germany (and not the U.S.) because of the availability of this type of data.

\textsuperscript{11}We also note that, if ICT innovations were biased towards young workers (given their skills), one would expect to see an increase in the young-old wage gap within ICT-intensive occupations. Panel B of Table A2 in the Appendix shows that this did not happen: higher ICT exposure had a similar impact on relative wages for the two groups. Moreover, this story is not consistent with the finding in Section 2.1 that the relative wage in ICT-intensive occupations increased decades before younger workers started to reallocate towards these occupations.

\textsuperscript{12}This is a standard metric of the distance between two distributions. Results are similar when using alternatives such as the Euclidean metric.

\textsuperscript{13}We do not require the task content of occupations to be the same in 1900 and 1980. Instead we assume that, for instance, occupations that required relatively more cognitive-intensive tasks than the average in 1980 also required relatively more cognitive-intensive tasks in 1900. Spitz-Oener (2006) shows that the task intensity ranking across occupations did not change much over the period of her sample (as the majority of changes happened within occupations).
two sets of occupations in each episode: those in the top 25 percent ($O_H$) and those in the bottom 75 percent ($O_L$) of the empirical exposure distribution. We then compute for each occupation $o \in O_L$ its average task distance from the occupations in $O_H$, weighted by their employment at $t_0$.

The left panel of Figure 2 depicts two histograms, one for each episode, of such distance measure across occupations. The key takeaway from the figure is that the task distance distribution for ICT exposure in the latter episode has more mass on higher distance values than the distribution for manufacturing exposure in the earlier episode. In fact, we obtain a p-value of 0.00 for the test of whether the distribution of distances in the latter episode first-order stochastically dominates that of the earlier episode. Thus, occupations with low and high exposure to the manufacturing innovations of the early 1900s were much more similar in their task contents than occupations with low and high exposure to the latter ICT innovations.

To further investigate the relationship between shock exposure and task distance in the two episodes, we estimate how an occupation’s exposure was associated with its (employment-weighted) average task distance from all other occupations. Column (1) of Table 1 shows that occupations with higher ICT exposure in 1980 required tasks that were significantly different from the tasks used in the rest of the economy. However, as column (4) of Table 1 shows, no such relation existed between manufacturing exposure and task distance in 1900, since the estimated coefficient is close to zero (with a similar
Table 1: Exposure, Task Distance, and Responses in Occupation Outcomes in the Two Episodes

| Dependent Variable: | ICT exposure, 1980-2019 | | Manufacturing exposure, 1900-1940 | |
|---------------------|--------------------------|-----------------------------|-----------------------------|
|                     | Avg. Task distance | Log-change in avg. wage | Avg. Task distance | Log-change in avg. wage |
| Exposure<sub>o</sub> | 0.144 | 0.119 | -0.021 | -0.006 | 0.187 |
|                      | (0.082) | (0.047) | (0.031) | (0.075) | (0.064) |
| Exposure<sub>o</sub> × TaskDistance<sub>o</sub> | -0.074 | 0.049 | -0.137 | |
|                      | (0.043) | (0.024) | (0.107) | |

Note. Sample of 291 occupations for columns (1)-(3) and 193 occupations for columns (4)-(5) with positive employment and task content information in each episode. Occupation outcomes built for a single group of all makes aged 16-64yrs. Estimates are weighted by the occupation’s employment at t<sub>0</sub>. Dependent variable indicated in each column. The occupation exposure in columns (1)-(3) is the occupation’s average ICT-intensity across industries and in columns (4)-(5) is the occupation’s average manufacturing-intensity across industries. The occupation “Task Distance” is its average task distance from all other occupations, weighted by those occupations employment at t<sub>0</sub>. Exposure and Task Distance are normalized to have zero mean and unit standard deviation. Standard errors in parentheses are clustered by occupation.

We thus conclude that the new technologies of the early 1900s raised the relative demand for manufacturing-intensive occupations that required workers to perform activities using skills that were relatively similar to those used in other occupations. The opposite was true during the second episode: ICT technologies increased the relative demand for occupations requiring skills that were remarkably different from those used in the rest of the economy.

To the extent that relative labor supply is more elastic between occupations with a similar task content (Traiberman, 2019), this distinction between the two episodes helps to explain why the reallocation of older incumbent workers was more muted during the adjustment to ICT innovations. The right panel of Figure 2 provides evidence that indeed this mechanism played a role. It shows that the employment expansion (contraction) of the occupations with the highest (lowest) levels of exposure in each episode happened almost entirely from the occupations with a similar task content to those in the rest of the economy; that is, those with a low average task distance whose employment growth is denoted by the dashed bars.

We now complement this evidence by estimating how the responses of relative employment and wages in more exposed occupations vary with the occupation’s task distance from the rest of the economy:

\[
\log Y_{o,t} - \log Y_{o,t_0} = \beta_t \text{Exposure}_o + \beta_t^{PD} \text{Exposure}_o \times \text{TaskDistance}_o + \gamma_t + \epsilon_{o,t}. \tag{3}
\]

where Exposure<sub>o</sub> is the same exposure measure defined above, and TaskDistance<sub>o</sub> is occupation o’s average task distance from all other occupations, weighted by their employment at t<sub>0</sub>. We normalize shock exposure and task distance to have zero mean and unit standard deviation across occupations in each episode. Hence, \( \beta_t^{PD} \) measures how much higher or lower was the response to shock exposure in an occupation with one
standard deviation higher task distance from the rest of the economy.

Columns (2) and (3) of Table 1 indicate that, for an occupation with a higher task distance from others, the impact of higher ICT exposure on relative employment was weaker but the impact on relative wages was stronger. These effects are economically large: the impact of exposure on employment is close to zero for occupations with one standard deviation higher average task distance. For the early twentieth century, the point estimate in column (5) is even more pronounced, but also less precise.

These response patterns are consistent with a lower elasticity of relative labor supply in occupations requiring tasks that are more different from those used in the rest of the economy. Together with the fact that ICT-intensive occupations had a higher task distance from the rest of economy, we show below that this can jointly explain the different employment responses of older and younger workers in the two episodes (as documented in Section 2.2), and, in turn, the different dynamic adjustments of overall relative employment and wages (as documented in Section 2.1). Finally, while our empirical findings alone do not speak to aggregate outcomes, we show that, when combined with the model below, the distinct observed patterns of adjustment imply remarkably distinct welfare gains from the new technologies introduced in the two episodes.

2.4 Additional results

We now summarize additional evidence complementing the empirical findings above. Appendix A.2 contains all tables and figures discussed below.

Robustness of Figure 1. We note first that our conclusions do not depend on the exact source of the shock driving the relative demand increase for manufacturing- and ICT-intensive occupations in each of the episodes as long as responses are not confounded with the impact of shocks to the relative supply of workers in such occupations. Table A2 in the Appendix shows that responses are similar when we consider other measures of exposure to the main technological innovations in each period; specifically, we consider alternative measures of occupation exposure based on electricity-intensity in the first episode, or task requirements in the second episode.14 More importantly, we also attest that our conclusions are robust to shocks in the occupation supply of immigrant and non-white workers. In particular, we show that estimates are essentially identical when we restrict the sample to only include natives or white males. Thus, our conclusions are not driven by changes in the share of these demographic groups across occupations, since the relative responses in employment and wages of incumbents should have been weaker if this was the case. We also show that responses are similar when we account for

14We note that, in line with the less precise estimated wage responses after 2000, point estimates across specifications vary in the timing and magnitude of the slow down in relative wage growth following the ICT arrival.
changes in job amenities by controlling for job attributes (e.g., hours, self-employment). Finally, Figure A3 shows that the young-old gap in relative employment responses in the two episodes is qualitatively robust to the age cutoff used to define the two generations. However, the stronger reallocation of younger workers to ICT-intensive occupations becomes more pronounced as we decrease the age cutoff defining the young group (that is, as we decrease the average age of the young group). This is not true for relative employment in manufacturing-intensive occupations in the early 1900s.

**Decomposition of employment reallocation across cohorts.** An alternative to estimating responses by age groups is to implement a cohort decomposition of the transition. While somewhat harder to interpret, we do find that our conclusions are robust to such an approach. In Table A3 in the Appendix, we extract the common and cohort-specific components of the reallocation towards highly exposed occupations over the decades of each episode. Our results show that relative employment growth in highly exposed occupations was entirely driven by cohort-specific effects in the more recent ICT-driven episode, but period effects common to all cohorts explain almost entirely the reallocation towards manufacturing-intensive occupations in the early episode.

**Cross-occupation evidence from other countries.** In Figure A4 in the Appendix, we use cross-country data on employment by broad occupation and age groups to document that, in all seventeen developed countries in our sample, recent growth in the relative employment of ICT-intensive occupations was stronger for younger than older workers.

**Robustness of Figure 2.** Figure A5 in the Appendix establishes that the takeaway from Figure 2 is robust to alternative choices of distance metric, exposure measures, weighting schemes, and thresholds for defining highly exposed occupations. In all cases, the distribution of task distances between the most and least exposed occupations to ICT in 1980 first-order stochastically dominates that implied by manufacturing exposure in 1900.

**Training in ICT-intensive occupations and relative employment of younger entrant generations.** One natural question is why younger entrant generations were able to increase their relative employment in ICT-intensive occupations. In Adão et al. (2020), we shed light on this question by leveraging data from Germany; a country with a unique large-scale vocational training program. We showed that occupations with a higher ICT exposure were associated with stronger growth in the number of young trainees. In terms of magnitude, the impact of exposure on the relative employment of younger generations was similar to the impact on the relative number of trainees. These findings indicate that acquisition of occupation-specific human capital by young
workers contributed to the observed between-generation gap in employment responses to ICT innovations.

3 A Model of Technological Transitions

We now present a model of technological transitions that can generate the patterns of adjustment across occupations and generations documented above for the two episodes. We consider a closed economy in continuous time. There is a single final good whose production uses two intermediate inputs, the high-tech \((k = H)\) and the low-tech \((k = L)\) goods. Input production uses only labor. There is a continuum of worker skill types \(i \in [0, 1]\). The worker’s skill type determines her effective labor supply when employed with the \(H\) and \(L\) production technologies. There are overlapping generations of workers, and each generation forms a large household. The household consumes the final good, assigns workers into a technology, and chooses how much labor of each type to supply at birth. Labor and good markets are competitive.

**Firms.** Production of the final good is a CES aggregator of the two inputs:

\[
Y_t = \left[ \left( A_t X_{Ht} \right)^{\frac{\theta-1}{\theta}} + \left( X_{Lt} \right)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}
\]  

(4)

where \(\theta > 0\) is the elasticity of substitution between the low-tech and the high-tech inputs, and \(A_t\) is a shifter of the relative productivity of the high-tech input.

A skill type determines the effective labor units that a worker supplies when employed in the production of each of the two inputs.\(^{15}\) Specifically, the production functions of \(L\) and \(H\) are respectively

\[
X_{Lt} = \int_0^1 \alpha(i) s_{Lt}(i) di,
\]

(5)

\[
X_{Ht} = \int_0^1 \alpha(i) \sigma(i) s_{Ht}(i) di,
\]

(6)

where \(s_{kt}(i)\) is the mass of workers of type \(i\) employed with technology \(k\) at time \(t\), \(\alpha(i)\) is the number of efficiency labor units of type \(i\), and \(\sigma(i)\) is \(i\)'s differential number of efficiency units in high-tech production. Therefore, \(\alpha(i)\) governs vertical differences in productivity across skill types (i.e, absolute advantage), whereas \(\sigma(i)\) governs horizontal productivity differences across technologies for a given type (i.e., comparative advantage). Without loss of generality, we assume that \(\sigma(i)\) is increasing in \(i\), so that the skills of

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\(^{15}\)Consistent with the evidence discussed above, we assume that the productivity of a worker when operating any of the two technologies only depends on their skills and not on their generation.
higher $i$ types are more complementary to the $H$ technology than to the $L$ technology.\textsuperscript{16}

Let $w_{kt}$ denote the wage rate of one efficiency unit of labor employed with technology $k$ at time $t$. In a competitive environment in which firms take wages as given, the labor earnings of type $i$ when employed with the $H$ and $L$ technologies are respectively

$$ w_{Ht}(i) = \omega_t \sigma(i) \alpha(i) \quad \text{and} \quad w_{Lt}(i) = \alpha(i), $$

(7)

where $\omega_t \equiv \omega_{Ht} / \omega_{Lt}$ is the relative wage per efficiency unit in high-tech good. In the rest of the paper, we refer to $\omega_t$ simply as the relative wage, and specify the economy’s numeraire as $w_{Lt} \equiv 1$.

Under perfect competition, $\omega_t$ is also the relative price of the high-tech input faced by final producers. Thus, their cost minimization problem implies that the relative demand for effective labor units in $H$ (or simply the relative demand in $H$) is

$$ x_t \equiv \frac{X_{Ht}}{X_{Lt}} = \omega_t^{-\theta} A_t^{\theta-1}, $$

(8)

and their zero profit condition implies that the final good price is

$$ P_t = (1 + \omega_t x_t)^{1-\sigma}. $$

(9)

**Workers.** We consider overlapping generations of workers whose birth and death follow a Poisson process with rate $\delta$. We assume that each generation $\tau$ forms a large household at birth.\textsuperscript{17}

At each time $t$, the household chooses how to assign workers to technologies and uses their labor income to buy the final good. The optimal assignment maximizes static labor income, so that a worker of skill type $i$ earns

$$ w_t(i) = \max\{\omega_t \sigma(i), 1\} \alpha(i). $$

(10)

At birth, the household of generation $\tau$ chooses how much labor of each skill type to supply. It does so to maximize the present discounted value of total household log-consumption, net of the utility cost from labor supply. Specifically, the problem of the

\textsuperscript{16}Appendix C.1 provides one economic interpretation for a skill type. We show that (5)--(6) arise when production combines individual-level output of each worker’s ”cognitive” and ”non-cognitive” task input. Differences in productivity across types are a consequence of differences in their relative ability to perform cognitive tasks, and differences in the importance of cognitive tasks for the two technologies.

\textsuperscript{17}This assumption implies that technological innovations will only have distributional consequences across generations in terms of welfare. In Adão et al. (2020), we explored an alternative model where individuals workers chose lotteries over skill types. As such, they were exposed to ex-post risk from the realization of the lottery, thus creating welfare differences within generations. This alternative model, however, delivers identical predictions for the approximate equilibrium dynamics we characterize in the following sections.
household of generation $\tau$ is

$$U_\tau \equiv \max_{\tilde{s}_\tau(i), C_{\tau,t}, L_\tau} \int_\tau^\infty e^{-\rho(t-\tau)} \log(C_{\tau,t}) \, dt - L_\tau$$

s.t.

$$P_t C_{\tau,t} = \int_0^1 w_i(i) \tilde{s}_\tau(i) \, di$$

$$L_\tau = \left( \int_0^1 \tilde{s}_\tau(i) \frac{1}{\nu+1} \, di \right)^{\frac{1}{\nu+1}} \leq 1$$

where $C_{\tau,t}$ is consumption at time $t$ of generation $\tau$, $L_\tau$ is generation $\tau$’s labor supply, and $\tilde{s}_\tau(i)$ is the labor supply of type $i$ from generation $\tau$. Households consume their labor earnings at each point in time — we already impose that the budget constraint is binding. In addition, the second constraint summarizes the household’s cost of allocating its labor endowment to supply different skill types: It specifies a CES aggregator of the mass of different types that must be below the household’s labor endowment which we normalize to one. This constraint will be binding in equilibrium.

Discussion. At this point, it is worth discussing how three features of the model relate to the evidence in Section 2. First, skill heterogeneity coupled with the skill-technology assignment in (10) implies that, when faced with different relative wages over time, each generation will choose to reallocate workers across technologies (given their skills). This allows the model to rationalize the type of within-generation worker reallocation that we documented for the manufacturing innovations of the early twentieth century. Second, the skill supply decision of entering generations implies that, when faced with different future paths of relative wages upon entry, different generations will choose different skills to supply and will, therefore, allocate their workers differently across technologies. This allows our model to rationalize the type of between-generation differences in employment reallocation that we documented for the ICT innovations of the late twentieth century. It is also consistent with the observed increase in vocational training in ICT-intensive occupations by younger workers in Adão et al. (2020). Finally, as discussed in more detail below, the slope of $\sigma(i)$ measures how different skill types are in terms of relative productivity across technologies and, thus, controls the elasticity of relative labor supply of incumbent workers. This will allow us to study how different the economy’s behavior is when the distance between skill requirements across technologies is different, such as the differences across the two episodes documented in Section 2.3.

Our preferred economic interpretation of these assumptions is that changes in relative

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18Our baseline model assumes that only new generations choose the supply of specific skills and that this decision is independent of the skills from previous generations. We relax both assumptions in Section 6.4, allowing older generations to re-train and new generations to learn-from-others when choosing their skills supply.
wages induce older workers to move towards sectors or occupations that require similar skills and thus entail minimal retraining. Intuitively, in contrast to younger workers, they face a high cost of fundamentally changing career paths by acquiring completely different skills. For tractability, we collapse into a one-time decision upon entry the skill investment that in reality occurs through formal schooling, major college choice, on-the-job learning, or vocational training. Note, however, that this decision goes beyond traditional unidimensional decisions to acquire more human capital (or whether to attend college) that allow workers to vertically differentiate. Instead, it is a multidimensional decision to supply different types of skills, driven both by vertical considerations — expressed in absolute advantage $\alpha(i) —$ as well as horizontal considerations — expressed through comparative advantage $\sigma(i)$ and relative wages $\omega_t$.

**Equilibrium.** Given our overlapping generations structure, the aggregate labor supply of a skill type, $s_t(i)$, follows the Kolmogorov-Forward equation,

$$\frac{\partial s_t(i)}{\partial t} = -\delta s_t(i) + \delta \tilde{s}_t(i). \quad (12)$$

Finally, an equilibrium must satisfy market clearing for all $t$. By Walras’ law, it suffices that the relative demand for effective labor in $H$ production in (8) is equal to the relative supply of effective labor units in $H$, $x_t$ implied by the ratio of (5)–(6) under the equilibrium assignment:

$$x_t = \omega_t^{-\theta} A_t^{\theta-1} = \frac{\int_{G_t(i) = H} \alpha(i) \sigma(i) s_t(i) \, di}{\int_{G_t(i) = L} \alpha(i) s_t(i) \, di} \quad (13)$$

where $\{G_t(i) : i \in [0, 1] \rightarrow \{H, L\}\}$ is the equilibrium technology-skill assignment implied by (10). In the rest of the paper, we refer to the equilibrium $x_t$ as the relative employment of effective labor units in $H$ or, simply, as relative employment.

**Definition 1 (Competitive Equilibrium)** Given an initial $s_0(i)$ and a path for $\{A_t\}_{t \geq 0}$, a competitive equilibrium is a path of the technology-skill assignment $\{G_t(i) : i \in [0, 1] \rightarrow \{H, L\}\}_{t \geq 0}$, the skill type supply $\{s_t(i)\}_{t \geq 0}$, a generation $\tau$’s skill type supply $\{\tilde{s}_\tau(i)\}_{\tau \geq 0}$, the relative wage, employment and final price index $\{\omega_t, x_t, P_t\}_{t \geq 0}$, such that

1. Given $\{\omega_t\}_{t \geq 0}$, $\{G_t(i), \tilde{s}_t(i)\}_{t \geq 0}$ are determined by (10) and (11).
2. Given $s_0(i)$ and $\{\tilde{s}_t(i)\}_{t \geq 0}$, $\{s_t(i)\}_{t \geq 0}$ is determined by (12).
3. For all $t \geq 0$, the market clearing condition (13) is satisfied and $P_t$ is given by (9).

In principle, the characterization of the equilibrium dynamics involves solving a complex infinite-dimensional fixed-point problem. To see this, consider a conjectured path for the relative wage $\{\omega_t\}_{t \geq 0}$. This path determines the skill supply decisions of new
generations \(\{\tilde{s}_\tau(i)\}_{\tau \geq 0}\) from (11) and, consequently, the path for the labor supply of skill types \(\{s_t(i)\}_{t \geq 0}\) from (12) given \(s_0(i)\). The relative wage path also determines the assignment of workers from (10). Taken together, the skill supply and assignment decisions determine the relative supply of effective labor units in \(H\). In an equilibrium, the implied relative supply needs to be equal to relative demand at the conjectured path for the relative wage — i.e., they need to be consistent with market clearing in (13). The next section shows that, up to a first order approximation, the equilibrium dynamics can be sharply characterized.

4 Equilibrium Dynamics

In this section, we begin by showing that, up to a first order, the equilibrium dynamics of relative employment and wages in our structural model are equivalent to those from a reduced-form model of the relative labor supply and demand across technologies. We then use this characterization to study the economy’s adjustment to a technological innovation.

4.1 A reduced-form equivalence

In order to provide an analytical characterization of the equilibrium dynamics, we consider a log-linear expansion around the stationary equilibrium.\(^{19}\) We let “” denote variables in log-deviations from their levels in the stationary equilibrium. The following proposition establishes our main equivalence result for equilibrium dynamics in relative employment and wages.

**Proposition 1 (Reduced-form equivalence)** The equilibrium dynamics of relative employment and wages \(\{\hat{x}_t, \hat{\omega}_t\}\) are identical to those from a reduced-form model where relative labor demand at time \(t\) is

\[
\hat{x}_t = -\theta \hat{\omega}_t + (\theta - 1) \hat{A}_t, \tag{14}
\]

and relative labor supply at time \(t\) is a population-weighted average of the labor supply of different generations \(\tau \leq t\) at time \(t\), \(\hat{x}_{\tau,t}\),

\[
\hat{x}_t = \delta \int_{-\infty}^{t} e^{-\delta(t-\tau)} \hat{x}_{\tau,t} d\tau, \tag{15}
\]

where

\[
\hat{x}_{\tau,t} = \eta \hat{\omega}_t + \psi \hat{q}_{\tau} \quad \text{with} \quad \hat{q}_{\tau} \equiv \int_{\tau}^{\infty} e^{-\rho(t - \tau)} \hat{\omega}_t dt. \tag{16}
\]

\(^{19}\)In Adão et al. (2020), we also numerically solve for the equilibrium and show that our main insights are not driven by the equilibrium approximation.
Proof. See Appendix B.1. 

The proposition shows that relative labor supply across technologies in the structural model of Section 3 behaves as if it was coming from a reduced-form model in which the relative labor supply of each generation combines time- and generation-specific components. These components determine how the relative labor supply elasticity varies over different horizons. We note that, conditional on the value of the reduced-form parameters, all structural models admitting the reduced-form representation in Proposition 1 generate isomorphic transitional dynamics following technological innovations. In this sense, the structural model of Section 3 is only one of many microfoundations that yield the positive implications about technological transitions outlined in Sections 4.2 to 6. 

Our structural model however provides a clear link between the mechanisms driving the economy’s adjustment margins and the parameters in the reduced-form representation, which we leverage below when analyzing how skill specificity shapes the economy’s adjustment to technological innovations. We discuss such a link in more detail next.

The generation-specific term of relative labor supply, $\psi \hat{q}_\tau$, is a forward-looking component capturing the impact that changes in the present discounted value of the relative wage at birth ($\hat{q}_\tau$) have on a generation’s relative labor supply, permanently. In the reduced-form model, $\psi$, together with $\delta$, thus govern the elasticity of relative labor supply over long horizons. In the structural model, this generation-specific component arises from the skill supply choices at birth. In particular, $\psi$ is inversely proportional to the cost of adjusting the skill supply as governed by $1/\nu$,

$$\psi = \nu \rho.$$ 

Intuitively, when the cost of adjusting the skill supply is higher (i.e., $1/\nu$ is higher), skill supply is less responsive to changes in the path of future relative wages, and so the elasticity of labor supply at longer horizons is smaller.

The time-specific term of relative labor supply, $\eta \hat{\omega}_t$, captures the instantaneous impact that changes in the relative wage ($\hat{\omega}_t$) have on the relative labor supply of every generation. In the reduced-form model, $\eta$ thus governs the short-run elasticity of relative labor supply. In the structural model, this short-run elasticity is intrinsically linked to how similar skill types are in terms of their productivity in each technology (as governed by the function $\sigma(i)$). Formally,

$$\eta \propto \left( \frac{\partial \log \sigma(i)}{\partial \log i} \bigg|_{i=l} \right)^{-1}.$$ 

20See for example Adão et al. (2020) for a model featuring individual-level skill investment choices and Beraja and Zorzi (2022) for a model featuring occupation-specific costs of job transitions. However, the particular structural model of Section 3 does matter for analyzing normative implications of technological innovations, as we do in Section 7.
with \( l \) denoting the marginal skill type that earns the same wage in both technologies at the stationary equilibrium (as implicitly defined by (10)). We will say that skill specificity is stronger when \( \partial \log \sigma(i)/\partial \log i \bigg|_{i=l} \) is larger because a worker’s productivity changes more when deployed to the \( H \)- rather than to the \( L \)-technology. Intuitively, when skill specificity is stronger, skill types are less similar and workers become less transferable across technologies. As such, the household requires larger changes in relative wages to switch them across technologies, and the short-run elasticity of relative labor supply \( \eta \) is lower. In what follows, we thus associate stronger skill specificity with a lower \( \eta \).

The proof of the proposition shows that, up to a first order approximation, the elasticity \( \partial \log \sigma(i)/\partial \log i \bigg|_{i=l} \) summarizes the heterogeneity between marginal types around the threshold \( l \) and, thus, how skill heterogeneity shapes the equilibrium dynamics of relative labor supply \( x_t \). The reason is that most workers never switch technologies along an equilibrium path in which the relative wage remains around its stationary level. Therefore, when evaluating changes in relative labor supply over time, the effect of changes in the supply of the marginal types that switch technologies along the transition are of second order.

### 4.2 The economy’s adjustment to a technological innovation \( \Delta \log(A) \)

We are now ready to characterize the dynamic adjustment of our economy to a permanent, unanticipated increase in the relative productivity of high-tech production in the structural model of Section 3 or, equivalently, an increase in the relative labor demand in the reduced-form model of Section 4.1.

We consider a shock \( \Delta \log(A) > 0 \) at \( t = 0 \) in an economy starting from a stationary equilibrium immediately prior to the shock at time \( t = 0^- \). The next proposition characterizes the transitional dynamics of \( \{\hat{x}_t, \hat{q}_t, \hat{\omega}_t\} \) after the shock. Appendix B.2 completes the characterization by deriving the short-run and long-run responses.

**Proposition 2 (\( q \)-theory transitional dynamics)** Given the initial condition \( \hat{x}_0 \) and the terminal condition \( \lim_{t \to \infty} \hat{x}_t = 0 \), the equilibrium transitional dynamics of \( \{\hat{x}_t, \hat{q}_t, \hat{\omega}_t\} \) are described by the following system of ordinary differential equations:

\[
\begin{align*}
\frac{\partial \hat{x}_t}{\partial t} &= -\delta \hat{x}_t + \frac{\theta \psi \delta}{\eta + \theta \hat{q}_t} \tag{17} \\
\rho \hat{q}_t &= \hat{\omega}_t + \frac{\partial \hat{q}_t}{\partial t} \tag{18} \\
\hat{\omega}_t &= -\frac{1}{\theta} \hat{x}_t. \tag{19}
\end{align*}
\]

The equilibrium \( \{\hat{x}_t, \hat{q}_t, \hat{\omega}_t\}_{t \geq 0} \) is saddle-path stable with a rate of convergence \( \lambda \).

**Proof.** See Appendix B.1. ■
The system of ordinary differential equations is a rather standard one in macroeconomics: it has one control variable \((q_t)\) and one predetermined variable \((x_t)\), as well as one Kolmogorov-forward equation (17) and one Kolmogorov-backward equation (18). The system is in fact mathematically isomorphic to the one in the \(q\)-theory of capital investment. In our model, the present discounted value of relative wages \(\hat{q}_t\) represents the shadow price of the human capital “asset” associated with having one additional effective unit of labor in high-tech production. As in the \(q\)-theory, we can see in (17) that parameters governing the ”costs of adjustment” in the economy \((\delta \text{ and } \psi)\) affect the sensitivity of changes in the state variable \(\frac{\partial \hat{x}_t}{\partial t}\) to the control variable \(\hat{q}_t\). Moreover, since \(\eta \text{ and } \theta\) affect how changes in relative wages translate into changes in labor supply and demand, they also mediate the impact of \(q_t\) on \(x_t\). The last part of the proposition shows that (locally) the equilibrium exists and is unique — a consequence of saddle-path stability. Given an initial condition \(\hat{x}_0, \hat{x}_t\) and \(\hat{q}_t\) converge at a constant rate to the stationary equilibrium, where \(-\lambda\) is the negative eigenvalue of the system of differential equations.

Figure 3 illustrates the economy’s dynamic response relative to the initial stationary equilibrium after the technological shock at \(t = 0\). For example, the response of relative employment is \(\Delta \log(x_t) \equiv \log(x_t / x_0^-)\). We do so for the case of a demand elasticity \(\theta\) larger than 1.

In the short-run, the increase in the relative labor demand causes the relative wage to rise \(\Delta \log(\omega_0) > 0\). The higher relative wage induces the reallocation of existing worker generations from the \(L\) to the \(H\) technology \(\Delta \log(x_{0,0}^0) > 0\), as governed by the elasticity \(\eta\). In turn, relative employment increases too \(\Delta \log(x_{0}^0) > 0\).

The persistent increase in the relative wage implies that younger worker generations entering after the shock face a higher present discounted value of the relative wage \(q_t\) compared to that faced by older generations born before the shock. This induces younger generations to increase their relative labor supply in comparison to older generations. To illustrate this, Panel D in the figure shows a positive gap \(\Delta \log(x_{0,0}^0) > \Delta \log(x_{0,-,0}^0)\) in relative employment between the initial generation born right after shock \((\tau = 0)\) and those born before \((\tau = 0^-)\). Through the lens of the structural model, such a gap results from younger generations of households choosing to supply more labor of skill types that are more complementary to the \(H\) technology.

Finally, along the transition, relative employment \(x_t\) increases over time as older generations are replaced with younger generations at rate \(\delta\), and the relative wage \(\omega_t\) declines in turn. In the long-run, relative employment and wages are higher than in the initial steady state.

We note that the only source of dynamics in our model is the \textit{differential} labor supply responses of younger and older generations. If households do not respond to changes
in the present discounted value of wages ($\psi = 0$) or relative wages do not change (as in partial equilibrium; see section Section 6.3), there are no transitional dynamics and the responses in the long- and short-run are identical.

**Diffusion.** One interpretation of the above process is that the technological innovation slowly diffuses in the economy as more workers with the appropriate skills operate the $H$ technology over time. Our model thus formalizes the idea in Rosenberg (1972) that the accumulation of complementary skills shapes the diffusion process. However, this interpretation overlooks the fact that our model does not give rise to an S-curve, which is a common observed feature in practice (Griliches, 1957; Comin and Mestieri, 2018). Instead, as the figure above shows, relative employment jumps on impact. The reason is that we have assumed that both the innovation increases productivity immediately and existing workers can relocate instantaneously. We have done so for simplicity; to highlight the dynamics induced by changes in relative labor supply over a time frame of generations. Without qualitatively changing any of our main insights, one could “smooth-out” the initial jump of our economy (and generate an S-curve) by enriching our model with any of the well-studied elements that produce a sluggish adoption at relatively shorter horizons, such as the slow increase in productivity due to firm-level learning (Atkeson and Kehoe, 2007) or the slow reallocation of existing workers (Mat-
Through the lens of our model, the evidence in Section 2 can be interpreted as showing that the short-run labor supply elasticity $\eta$ was lower during the ICT transition in part because of stronger skill specificity. Motivated by this, we next analyze the effect of skill specificity on equilibrium dynamics. In particular, Section 5.2 studies the effect of skill specificity on how slow technological transitions are. This is our main outcome of interest for two reasons. First, we wish to speak to the evidence showing a slower adjustment following ICT innovations. Second, as Section 7 shows, the welfare implications substantially differ between fast and slow technological transitions.

Our main theoretical result establishes that a technological transition is slower whenever skill specificity is stronger. The reason is that the adjustment is driven more by the slow entry of new generations of workers instead of the relatively faster reallocation of existing generations that are already present in the economy when the technological innovation arrives.

5.1 The adjustment in high and low $\eta$ economies

We begin by comparing the adjustment of two economies featuring a high and a low short-run labor supply elasticity $\eta$. We interpret this as a comparison between two episodes of adjustment triggered by distinct types of technological innovations that impact different sets of economic activities. In one episode, the innovation augments the productivity of the $H$ technology by $\Delta \log(A)$, where $H$ corresponds to occupations or sectors intensive in skills that are markedly different from those used in the rest of the economy (represented by the $L$ technology). That is, this episode is associated with an economy featuring strong skill specificity (in our structural model, the elasticity of $\sigma(i)$ is high) where the short-run labor supply elasticity $\eta$ is low. In the other episode, the productivity of the $H$ technology is again augmented by $\Delta \log(A)$ but $H$ instead corresponds to occupations or sectors intensive in similar skills to those used in the rest of the economy. That is, this episode is instead associated with an economy featuring weak skill specificity (in our structural model, the elasticity of $\sigma(i)$ is low) where the short-run labor supply elasticity $\eta$ is high.

Figure 4 illustrates the adjustment in these two economies. The black solid lines show the responses of an economy with a low elasticity $\eta$ and the blue dashed lines with a high one. In Appendix B.3, we support the graphical representation in Figure 4 with Proposition A.1 formally establishing how skill specificity affects the short- and long-run responses, the cumulative impulse response, and the rate of convergence.
In the short-run, when skill specificity is weaker (higher $\eta$), older workers of generations born before shock reallocate more ($\Delta \log(x_{0,-0})$ is larger). That is, the within-generation adjustment margin is stronger. As a result, relative wages increase less in the short-run ($\Delta \log(\omega_0)$ is smaller) whereas relative employment increases more ($\Delta \log(x_0)$ is larger). The smaller and less persistent increase in relative wages (and so their present discounted value $q$) then implies that young entering generations increase their relative labor supply by less compared to older generations alive before the shock ($\Delta \log(x_{0,0}/x_{0,-0})$ is smaller). In other words, the cross-generation adjustment margin becomes endogenously weaker. In our structural model, this is because, when $q$ increases by less, younger entering households have weaker incentives to supply labor of skill types that are complementary to the $H$ technology.

Ultimately, transitional dynamics in relative employment $x_t$ and wages $\omega_t$ are less important because there are less persistent changes in the relative labor supply that take place as younger generations replace older generations. Formally, we measure this as the cumulative impulse response function being smaller — graphically, the blue shaded areas being smaller than the black shaded areas. The next section shows how the (discounted) cumulative impulse response connects to how slow the adjustment is.

We emphasize that the elasticity $\eta$ affects cross-generation employment differences
only because we solve our model in general equilibrium where the relative wage is endogenous. If we had instead taken the relative wage as exogenous, changes in $\eta$ would impact the labor supply responses of older workers but not the differential responses of younger generations (see Section 6.3). In this case, $\eta$ would also not affect transitional dynamics nor how slow the adjustment is.

5.2 How slow is the adjustment?

We conclude this section by showing how different degrees of skill specificity affect our main outcome of interest: how slow technological transitions are. We first define a measure that summarizes the importance of transitional dynamics and, therefore, gives a formal way of quantifying whether the economy’s adjustment is fast or whether it slowly plays out over many generations. Specifically, we define the discounted cumulative impulse response (DCIR).

Intuitively, the DCIR is the answer to the question: from the point of view of generations alive just before the arrival of a technological innovation, how different is the adjustment they expect to see during their lifetime compared to the long-run adjustment? We will say that the economy’s adjustment is slower when existing generations expect to miss more of the overall adjustment (i.e., the DCIR is larger).

**Definition 2 (Discounted Cumulative Impulse Response)** For any variable $z_t$ and innovation $\Delta \log(A)$, the discounted cumulative impulse response DCIR($z$) is:

$$DCIR(z) = \left| \int_0^\infty e^{-\delta t} \left( \frac{\Delta \log(z_t)}{\Delta \log(A)} \right) dt - \frac{\Delta \log(z_\infty)}{\Delta \log(A)} \right|.$$  

Formally, the DCIR is the distance between the long-run response and the expected response of $\log(z_t)$ during the initial generations’ lifetime, since all generations born before the innovation have exponentially distributed death probabilities with rate $\delta$. This is a convenient measure of the importance of transitional dynamics in our context for two reasons. First, it encodes not only the convergence rate $\lambda$, but also other relevant features of the impulse responses like how front-loaded they are. For instance, one could have an adjustment where the short- and long-run changes are almost identical — implying a DCIR close to zero — but the rate of convergence $\lambda$ from the short-run to the long-run is very low. According to the DCIR, we would intuitively say that it is a fast adjustment since almost all of the overall adjustment is completed on impact, whereas looking at $\lambda$ alone suggests a slow adjustment. Second, the DCIR does not mechanically scale with the replacement rate of generations. If $\delta$ is higher, this mechanically increases $\lambda$ (making the adjustment faster) but it also decreases the expected lifetime of a generation.

The next proposition shows that, according to this measure, technological transitions are slower whenever skill specificity is stronger. This is because, as Figure 4 illustrated,
stronger skill specificity mutes the reallocation of older generations of workers in the short-run causing larger endogenous changes in relative wages and, in turn, in the relative labor supply of younger entrant generations. As a result, when technological innovations are biased towards economic activities intensive in skills which differ more from those used in the rest of the economy, the adjustment in relative employment is slower since it is driven more by the reallocation across rather than within generations.

**Proposition 3 (DCIR comparative statics with respect to $\eta$)** Following a technological innovation $\Delta \log(A)$, relative employment ($x$) and wages ($\omega$) adjust slower in economies featuring stronger skill specificity (lower $\eta$). Formally,

$$\frac{\partial \text{DCIR}(x)}{\partial \eta} < 0, \quad \frac{\partial \text{DCIR}(\omega)}{\partial \eta} < 0.$$  

**Proof.** See Appendix B.4.

## 6 Back to the Evidence

Having established our main theoretical results, we are now ready to come back to the evidence in Section 2 about how the U.S. economy adjusted to ICT and manufacturing innovations. In Section 6.1, we show that the model can match the distinct adjustment paths in both episodes when the short-run elasticity of labor supply is assumed to be lower in the ICT transition. Section 6.2 shows that potential alternative explanations deliver predictions that are at odds with the evidence in one or many dimensions. Section 6.3 discusses the role of key model features in accounting for the evidence. Section 6.4 shows that our main results are robust to a number of extensions of our baseline model.

### 6.1 Model vs. evidence

We map the set of occupations mostly impacted by a technological innovation — i.e., ICT- and manufacturing-intensive occupations — to high-tech production (H) in the model. Likewise, in every time $t$, we map the group of “younger” workers to those with an age below 29 years in the model (i.e., born between $t$ and $t - 29$) and map the group of “older” workers to those older than 29. Therefore, the relative employment of older generations is $x_{t, \text{older}} \equiv \delta \int_{t - 29}^{t - \infty} e^{\delta(\tau - t)} x_{\tau, \text{old}} d\tau$ and that of younger generations is $x_{t, \text{younger}} \equiv \delta \int_{t - 29}^{t} e^{\delta(\tau - t)} x_{\tau, \text{young}} d\tau$.

Table 2 shows the calibration of the parameters in the reduced-form model. We set the discount rate to $\rho = 0.02$ so that future wages are discounted at an interest rate of 2%.

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21 We conjecture that this intuition holds more generally. After a shock, economies with a less mobile stock of a factor experience stronger changes in the flow of entrants because of larger relative price changes — e.g., if old vintages of physical capital are less adaptable to new sectors, then the flow of entrants with newer capital vintages will be larger.
Table 2: Parameter Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>0.02</td>
<td>Discount rate</td>
<td>2% annual interest rate</td>
</tr>
<tr>
<td>$\delta$</td>
<td>1/36</td>
<td>Death rate</td>
<td>Workers aged 29 expect to work until 65</td>
</tr>
<tr>
<td>$\theta$</td>
<td>3</td>
<td>Labor demand elasticity</td>
<td>Hsieh et al. (2019)</td>
</tr>
<tr>
<td>$\psi$</td>
<td>0.4</td>
<td>Long-run labor supply elasticity</td>
<td>Wiswall and Zafar (2015)</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0</td>
<td>Short-run labor supply elasticity</td>
<td>Figure 1 in Section 2.1</td>
</tr>
</tbody>
</table>

per year. We pick a death rate of $\delta = 1/36$ so that young workers aged 29 expect to work until age 65. We set the elasticity of relative labor demand to $\theta = 3$ to match the elasticity of substitution across occupations in Hsieh et al. (2019). We set the generation-specific elasticity of relative labor supply to $\psi = 0.4$. In the structural model, this elasticity is interpreted as the elasticity of occupational choices of young workers to changes in the present discounted value of relative earnings. Using experimental variation in beliefs about future earnings in different college majors, Wiswall and Zafar (2015) estimate that a 1% increase in expected relative earnings increases the log odds of majoring in a field by about 0.4% for freshmen and sophomores. Finally, for our baseline calibration, we set the short-run relative labor supply elasticity $\eta$ to zero, consistent with the lack of relative employment responses for older workers during the ICT episode reported in Figure 1 of Section 2.1. In the structural model, this corresponds to an economy with infinite skill specificity where worker skills cannot be transferred across sectors.

The left column of Figure 5 shows the empirical estimates of the impact of one standard deviation higher ICT exposure on relative outcomes across occupations over time (black dots), along with theoretical impulse responses for the same outcomes obtained from our calibrated model (red squares). We set the shock $\Delta \log(A)$ so that the model matches the relative employment response between 1980 and 2019. In the model, we interpret the period between 1980 and 1990 as “the short-run” when the technological innovation arrives. Thus, we associate the year 1990 to the impact responses at $t = 0$ when the shock $\Delta \log(A)$ hits the economy.

The model matches the magnitude and timing of the empirical responses remarkably well. It almost perfectly predicts the slow increase in relative employment (panel A.1) and the inverted-U shape response in relative wages (panel B.1). Crucially, the model achieves this through the right margins of adjustment: it replicates the lack of reallocation of older workers (panel C.1), and the increase in the relative employment of younger workers compared to older ones (panel D.1). The model only slightly overstates the differences in relative employment across generations, and somewhat understates the
Figure 5: Dynamic Responses to a Technological Innovation: Model v. Evidence

Note. In column 1, black dots correspond to the estimates of the impact of one standard deviation higher exposure to ICT on the log-change of outcomes across occupations — employment in panel A, wages in panel B, employment of older workers in panel C, and the difference in employment of younger and older workers in panel D. In column 2, gray diamonds correspond to the analogous estimates associated with higher manufacturing exposure, except in Panel B.2 where gray diamonds depict changes in the manufacturing wage premium (as reported in Figure A2). All estimates are obtained from specifications described in Section 2, with vertical bars denoting the associated 90% confidence intervals. Red squares are the transitional dynamics predicted by our model following a shock $\Delta \log(A)$ that we set to match the long-run relative employment response in Panel A.1. Column 1 uses parameters in Table 2, and column 2 the same parameters except that we set $\eta = \infty$.

increase for old generations (although these are not significantly different from zero). Finally, we find a $DCIR(x)$ of approximately 0.5, implying that generations born before 1980 expect to miss half of the overall adjustment because of how slow it was.

The right column of Figure 5 shows the empirical estimates of the impact of one standard deviation higher manufacturing exposure on relative outcomes across occupations between 1900 and 1940 (gray diamonds), together with the model-implied impulse responses (red squares) when the short-run relative labor supply elasticity is large ($\eta \to \infty$) instead of our baseline zero elasticity. In the structural model, a higher value of $\eta$ arises from weaker skill specificity, in line with the lower task distance between the most and least exposed occupations that we documented in Section 2.3 for manufacturing innova-
tions. The large value that we use corresponds to an economy with weak skill specificity where worker skills are easily transferable across sectors. We use the same values for all other parameters, as well as the magnitude of the shock \( \Delta \log(A) \).

For \( \eta \) large enough, the model predicts a fast increase in relative employment (panel A.2) coupled with weak changes in relative wages following the technological innovation (panel B.2). Again, the margins of adjustment are consistent with the evidence: the model generates a fast increase in the relative employment of older workers (panel C.2), and similar responses for younger and older generations (panel D.2). The faster transition implies a \( DCIR(x) \) close to zero, so that generations born prior to the shock expected to see the full adjustment within their lifetime.

Taken together, these results show that a lower short-run elasticity \( \eta \) can quantitatively explain why the adjustment to the ICT innovations of the late twentieth century was slower, more unequal, and mainly driven by the entry of young generations when compared to the adjustment triggered by the innovations in manufacturing of the early twentieth century. We note that the model needs a large difference in \( \eta \) between the two episodes to match the large difference in the magnitude of the reallocation of older generations. Our evidence in Section 2.3 shows that skill specificity contributed to reduce the labor supply elasticity of older workers into ICT-intensive occupations. However, we acknowledge that the exercise above does not quantify whether the higher task distance of ICT-intensive occupations (compared to manufacturing-intensive occupations) fully accounts for the lower \( \eta \) during the ICT transition.

### 6.2 Alternative explanations

We now use our model to evaluate alternative explanations for the differences across the transitions triggered by innovations in ICT and manufacturing. Specifically, we next re-do the analysis above for changes in model parameters that capture other potential reasons for the distinct adjustment patterns observed during the two transitions. We conclude that none of them can jointly explain the differences that we document.

**Slow-moving \( A_t \).** Technological transitions may be slower or faster due to factors entirely unrelated to specificity and skills. For instance, technologies themselves may only improve gradually and their price decline slowly. Or knowledge about how to use new technologies may be accumulated gradually. In our model, these type of stories would affect productivity and, thus relative labor demand, directly. They can be captured in reduced-form as changes in \( A_t \) happening gradually over time as opposed to our baseline of a permanent one-time shock \( \Delta \log(A) \). If technologies differ in the speed at which \( A_t \) evolves, so will the adjustment. This could in part explain why the ICT and
manufacturing transitions differed.\textsuperscript{22}

However, this explanation is at odds with two facts. First, a slower relative labor demand increase during the ICT transition would have implied a slower increase not only of relative employment but also of relative wages. This is not what we observed: the rise in the relative wage of ICT-intensive occupations happened decades before the rise in their relative employment. Second, in an economy without skill specificity ($\eta = 0$) or differences in relative labor supply across generations ($\psi = 0$), slow-moving changes in labor demand by themselves do not lead to differences in employment reallocation across generations. As such, these other sources of slow adjustment dynamics cannot explain the observed differences across younger and older generations that we document. This suggests that mechanisms based on slower dynamics of labor demand cannot alone explain the differences between the two technological transitions.

Changes in $\psi$. One potential reason behind the different responses in the two episodes is that it was easier for younger workers to acquire the skills required to operate the new technologies used in manufacturing in the early 1900s. That is, the elasticity $\psi$ used to be higher. Indeed, this narrative is in line with historical accounts in Goldin and Katz (2009) and Alon et al. (2018) showing that schooling costs, for example, used to be lower in the early twentieth century than in recent decades.

The left column of Figure 6 shows the model-implied responses for $\psi = 2$ instead of our baseline 0.4, again together with the empirical estimates for the impact of higher manufacturing exposure across occupations. A larger $\psi$ cannot explain the different short-run responses in the two episodes: namely, the larger employment response and the smaller wage response over the initial decades of the transition to manufacturing (panels A.1 and B.1). That is, a larger $\psi$ gets the timing of the adjustment wrong. The reason is that this amplifies the relative employment response of young generations (panel D.1), but it cannot generate the strong response observed for older incumbent generations (panel C.1). Thus, any explanation based on the cost of acquiring skills for younger workers is inconsistent with the evidence that employment responses were stronger for both older and younger workers during the manufacturing transition.\textsuperscript{23}

Changes in $\delta$. Another possibility is that, early in the twentieth century, older generations were replaced by younger ones more often. That is, the rate $\delta$ used to be higher because of the shorter life expectancy of workers in the early 1900s.

\textsuperscript{22}For example, ICT could have been held back by the slow rollout of distribution and network technologies that were key complementary inputs.

\textsuperscript{23}Note that, if $\psi$ was smaller instead, then the model could generate the small differences across generations that we observe in reallocation towards manufacturing-intensive occupations. However, this would imply a larger and more persistent relative wage increase and a smaller employment response at all horizons; the opposite of what we documented.
The middle column of Figure 6 shows the model-implied responses for $\delta = 1/20$ instead of our baseline 1/36. An increase in $\delta$ has an effect that is similar to an increase in $\psi$. Intuitively, both of these parameters govern the elasticity of relative labor supply at longer horizons. As such, they are unable to simultaneously generate the stronger reallocation of older workers and the smaller between-generation differences in relative employment responses that we observed during the transition triggered by innovations in manufacturing (panels C.2 and D.2). Consequently, they are also unable to generate the smaller (larger) relative wage (employment) increase that we document at short horizons (panels A.2 and B.2).

Changes in $\theta$. When the short-run labor supply elasticity $\eta$ is zero but the long-run elasticity $\psi$ is positive, as in our baseline calibration, our model is a variant of Katz and Murphy (1992) with an elastic labor supply, similar to Caselli (1999). In this context,
we can interpret the parameter $\theta$ as the elasticity of substitution between skilled and unskilled workers. The evidence in Goldin and Katz (2009) suggests that such an elasticity declined over the course of the twentieth century. In the early 1900s, the relevant distinction was between those workers with and without a high school degree, with an elasticity of substitution of 5. In the late 1900s, the relevant distinction was between workers with and without a college degree, with a lower elasticity of 1.6.

The right column of Figure 6 shows the model-implied responses for $\theta = 5$ instead of our baseline 3. An increase in $\theta$ can explain the larger relative employment response to the innovations in manufacturing of the early 1900s. However, at odds with the evidence, it also implies an even larger response of relative wages (panel B.3) and, as a result, of relative employment differences across generations (panel D.3). This follows from the usual demand-supply logic that, for the same demand shock $\Delta \log(A)$, the relative wage and employment increases are larger when the relative demand elasticity is larger.24

6.3 The role of key model features

In our model, differences in skill specificity help to explain the observed differences in the economy’s adjustment during the two technological transitions. This is because stronger skill specificity reduces the reallocation of older incumbent workers, while it endogenously increases that of younger entrant workers. We next discuss the ingredients of the model that are essential to these conclusions.

Endogenous relative wages. The results above are in contrast to quantitative dynamic Roy models where wages are sometimes taken to be exogenous due to the difficulties in computing the general equilibrium (e.g., Lee and Wolpin (2006)). In particular, changes in skill specificity affect the labor supply responses of older workers but not the differential responses of younger generations when the relative wage $\omega_t$ (and thus their present discounted value $q_t$) is exogenously given.

The intuition follows from the discussion around Figure 4. Cross-generation differences arise only when younger generations’ relative labor supply respond to changes in the present discounted value of wages. In a model with endogenous relative wages, a higher $\eta$ leads to a smaller relative wage increase following the technological innovation $\Delta \log(A)$ (due to the stronger reallocation for older workers), which then implies smaller cross-generation differences in relative employment and a faster overall employment adjustment. This effect is absent when changes in relative wages are exogenously given, since the incentives of households of entering generations to supply labor of skill types

24Suppose instead that $\theta$ used to be smaller. This could now rationalize the smaller relative wages and cross-generation differences in relative employment at longer horizons. However, in contrast with the evidence, it would also imply a smaller employment response at all horizons and no reallocation of older workers.
complementary to the $H$ technology would not be affected by $\eta$. Finally, we note that, for the same reason, differences in skill specificity do not affect how slow the adjustment is (as measured by the DCIR) in a model with exogenous changes in inequality.

**Endogenous labor supply of young generations.** Our results are also distinct from those implied by models with exogenous within-generation skill heterogeneity — as in static Roy models, e.g., Costinot and Vogel (2010) and Burstein et al. (2019). Our model features a fixed labor supply of young generations when the long-run labor supply elasticity $\psi$ is zero. This rules out the type of cross-generational employment differences implied by ICT innovations. In our model, it also implies that all responses are instantaneous (i.e., there are no transitional dynamics).

**Heterogenous skills and specificity.** A special case of our model without transferable skills — i.e., infinite skill specificity, $\eta = 0$ — implies that old generations do not reallocate after a technological innovation, which is inconsistent with the evidence in Section 2 for the manufacturing innovations of the early 1900s. This would also be the case in models without worker mobility (e.g., Guerreiro et al., 2017) or where the innovation is biased towards certain observable worker attributes like education or age (e.g., Katz and Murphy, 1992; Caselli, 1999; or Card and Lemieux, 2001) instead of impacting economic activities using specific skills.

At the other extreme, we can consider a model where skills are perfectly transferable — i.e., no skill specificity $\eta = \infty$. This case is similar to canonical structural transformation models where labor is homogeneous, reviewed by Herrendorf et al. (2014). It implies that there are no cross-generational employment differences, which is not consistent with our evidence for the ICT episode. This would be the case as well in models where the dynamics of (homogeneous) labor reallocation are driven by shifters of labor demand, like firms investment and entry decisions (e.g., Atkeson and Kehoe, 2007; Dix-Carneiro and Kovak, 2017). Note also that this class of models cannot explain our findings that, upon arrival in the 1980s, ICT innovations triggered a fast increase in relative wages that preceded relative employment responses by decades.

### 6.4 Additional determinants of adjustment dynamics

The theory so far has ignored several determinants of adjustment dynamics. In Appendix C.2, we present three extensions that relax some of the assumptions of our baseline model. For all extensions, our comparative static results with respect to $\eta$ in Proposition 3 and the numerical results in this section remain valid. However, as we discuss below, the extensions affect the *levels* of the DCIRs and the relative employment responses for older and younger generations.

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**Learning-from-others.** Our first extension considers a "learning-from-others" externality. Specifically, we relax the assumption that the labor supply choices of young generations depends only on the future path of wages. Instead, we assume that certain skill-types may be easier to supply because workers can "learn from others" when those types are already abundant in the economy. This extension introduces a backward-looking element into the household’s problem and complementarities in the labor supply decisions across generations. This is similar to the cross-generation complementarities that arise in the environment considered by Chari and Hopenhayn (1991). In this case, dynamic responses are qualitatively similar to those of our baseline economy when $\psi$ is higher and $\delta$ is lower. This makes the adjustment slower, the relative employment response of older workers at long horizons smaller, and the cross-generation differences in relative employment at long horizons larger.

**Re-training of old workers.** Our second extension relaxes the assumptions that households choose their supply of skill types only at birth. We allow an exogenous fraction of older generations that were present before the technological innovation to re-optimize their labor supply as if they were a young generation entering at time $t = 0$. One interpretation of this is that older workers can now re-train after the innovation arrives. This extension yields responses that are qualitatively similar to our baseline when the short-run labor supply elasticity $\eta$ is higher. Thus, it makes the adjustment faster, the employment responses of older generations stronger, and the cross-generation differences in employment lower.

**Population growth.** Our third extension allows for the birth rate to be higher than the death rate. This raises the convergence rate $\lambda$, resulting in a faster adjustment.

7 Welfare Implications of Skill Specificity

So far, we have shown that the model matches in a parsimonious way the evidence on relative responses across occupations and generations for the transitions triggered by innovations in ICT and manufacturing. This gives us confidence that we can use the model to study the aggregate implications of technological innovations. In particular, we next analyze how welfare is affected by the economy’s degree of skill specificity. This normative analysis requires us to move away from the reduced-form model of Section 4 and commit to the structural model of Section 3.

To obtain an average welfare measure across generations $W$, we take an utilitarian approach by considering a population-weighted average of the utility of different generations, where generation-$\tau$’s weight is $\delta e^{-\rho \tau}$ as in Calvo and Obstfeld (1988) and
their utility $U_\tau$ is defined in (11). This implies that aggregate welfare in consumption-equivalent units is

$$W = \rho \delta \int_{-\infty}^{\infty} e^{-\rho \tau} U_\tau d\tau. \quad (20)$$

We consider a second order approximation of the welfare function around the stationary equilibrium. As usual, this higher-order approximation allows us to capture households’ preference for smoother consumption paths.\footnote{For example, second order approximations are common in the business cycles literature when studying the welfare losses from fluctuations in output and inflation (Woodford, 2011).} For the relevant case of a small discount rate $\rho$, the following proposition shows that the welfare gains from technological innovations are smaller when skill specificity is stronger because the adjustment is slower.

**Proposition 4 (Welfare gains)** Let $\Delta W$ be the change in welfare from a technological innovation $\Delta \log(A)$ and assume that $\theta > 1$. Then,

$$\lim_{\rho \to 0} \frac{\partial \Delta W}{\partial \eta} > 0$$

**Proof.** See Appendix B.5.

To get a sense of magnitudes and provide intuition for the proposition, Figure 7 uses our calibrated model to compute welfare and the DCIR$(x)$ under different values of the elasticity $\eta$. This also allows us to show, numerically, that the result in the proposition still holds for reasonable values of $\rho$ away from the limit $\rho \to 0$.

The black solid line depicts how the elasticity $\eta$ affects the welfare loss from transitional dynamics. We compute this loss as $\Delta W - \Delta U_\infty$. That is, the difference between the actual welfare gains and the gains associated with the long-run change in utility between the initial and final steady states. We note that such long-run gains $\Delta U_\infty$ also correspond to an economy where the adjustment is instantaneous, such as when the elasticity $\eta$ is infinity. Moreover, to ease interpretation, we normalize the loss to 1 for the economy from our baseline calibration with infinite specificity ($\eta = 0$). The blue dotted line shows the DCIR$(x)$ as a measure of how slow the adjustment is. Lastly, as a summary measure of the responses across generations, the red dashed line in the figure also shows the ratio between the responses at impact in the relative employment of older and younger workers $\Delta \log(x_0^{\text{older}})/\Delta \log(x_0^{\text{younger}})$ as defined in Section 6.1.

For our baseline calibration with $\eta = 0$, the DCIR$(x)$ is 0.5 and older workers do not reallocate. As $\eta$ increases, the adjustment becomes faster, the reallocation of older workers (relative to younger workers) strengthens, and the welfare loss from transitional dynamics decreases. For example, when $\eta$ increases to 3, the DCIR$(x)$ is cut in half to about 0.25 and the relative log-employment response of older workers increases to about
0.3. In turn, this faster adjustment implies that the welfare losses are about 1/3 smaller compared to the baseline. Underlying this welfare calculation are significant consumption differences across generations.\textsuperscript{26} Early generations have much smaller consumption than later ones which entered the market when average wages were higher and the relative wage $\omega_t$ lower. Such differences across generations decrease as $\eta$ increases because there is a faster and stronger increase in average wages and a less persistent and smaller increase in relative wages. Taken together, these results imply that observing a more muted reallocation of older workers (relative to younger workers) shortly after a technological innovation is a signal of a high degree of skill specificity, and thus (everything else equal) a slower transition going forward where welfare gains are smaller.

8 Conclusions

Technological transitions are not always alike. Labor markets adjust rapidly to some technological innovations and more slowly to others. One important reason is that certain innovations benefit production activities that require more specific skills. Following such innovations, the adjustment is slower because it relies more on the gradual entry of young generations of workers to the benefitted activities and less by the faster reallocation of older incumbent workers. To support this argument, we first present evidence on how U.S. labor markets adjusted to manufacturing innovations in the early twentieth

\textsuperscript{26}Recall that total labor supply is fixed for each generation. Thus, in response to shocks, welfare changes are entirely driven by consumption changes.
century and ICT innovations later in that century. We then build a model of technological transitions that allows us to sharply characterize the impact of skill specificity on equilibrium dynamics, interpret our evidence, and analyze normative implications.

The evidence shows that the adjustment to ICT innovations was slower and more unequal. Differences in the reallocation of younger and older generations account for the slower expansion of ICT-intensive occupations. While the reallocation to manufacturing occupations in the early twentieth century was similar for young and old generations, older generations did not move towards ICT-intensive occupations. Instead, the ICT transition was entirely driven by the gradual entry of younger generations. Stronger skill specificity during the ICT transition can partly explain these facts.

Going forward, our results raise the question of whether recent advances in Artificial Intelligence (AI) — a new general-purpose technology that could transform multiple sectors and occupations — resemble past innovations in ICT or manufacturing. Will the adjustment be slow, mainly driven by the entry of younger generations, or will it be fast, as the skills possessed by older incumbent generations can easily be transferred to work in activities benefiting from AI?

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Online Appendix for:
Fast and Slow Technological Transitions

This appendix contains supplemental material for the article “Fast and Slow Technological Transitions.” We provide (i) further details on data construction and additional evidence, (ii) the proofs of all lemmas and propositions in the paper, and (iii) additional theoretical results.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded “A.” – “C.” refer to the main article.
Appendix A  Evidence from Two Technological Transitions: Data Construction and Additional Results

This section complements the results presented in Section 2. We provide details about data construction and then present additional results.

A.1  Data construction

This section outlines in detail the construction of the data used in Section 2.

Employment and average wage by occupation and age group. Our main data source is the historical data of the U.S. Census from IPUMS. For each Census year, we select the sample of males in the labor force aged 16-64 years old that have valid occupation information. Following the guidelines provided by IPUMS, we use the 1950 occupation classification before 1970, and the 1990 occupation classification after 1980. For the latter period, we further aggregate occupations into the classification used by Autor and Dorn (2013). We use sampling weights to compute outcomes by occupation and age group. Employment is the sum of the number of individuals in each occupation and group. The average wage is the aggregate annual income (“incwage”) of full-time workers in an occupation-group divided the total number of such individuals. We define full-time workers as those working at least 40 weeks in a year and 35 hours in a week.

Figure A1 plots the distribution of employment by occupation in each episode. We restrict the figure to the top 50 occupations by employment in each period for clarity. Employment in the early period is largely concentrated in agricultural and manual-intensive occupations, with nearly 40 percent of employment in “farmers” and “farm laborers, wage workers.” An additional 13 percent of employment is in “laborers, not elsewhere classified.” There is substantially less concentration in employment by occupation in the latter episode (although we note that there are more finely defined occupational categories in this period). The largest occupation in the second episode is “managers and administrators, not elsewhere classified” with nearly 7 percent of total employment. The top 50 occupations account for around 67 percent of employment in the second episode, and around 90 percent of employment in the first episode.

Occupation exposure to manufacturing industries. We use the U.S. Census classification to group industries into a manufacturing sector and a non-manufacturing sector. The occupation’s exposure is $\text{Exposure}_o \equiv \sum_k \ell_{o,t_0}^k S^k$, where $S^k$ is a dummy that equals one if industry $k$ is classified as manufacturing (codes 300-500 in the 1950 industry classification), and $\ell_{o,t_0}^k$ is the share of industry $k$ in the number of employed individuals in
occupation $o$ in the 1900 Census.

**Occupation exposure to electricity-intensive industries.** To construct an occupation’s electricity exposure, we begin by obtaining industry electricity exposure from the 1947 BEA input-output tables. These tables are the oldest available, and are coded in 1957 SIC codes. Our occupation-industry data from IPUMS uses Census industry codes. A concordance between any vintage of Census industry codes and 1957 SIC codes is not available, so we construct a concordance for this match. For this concordance, we map each 2-digit SIC code from the IO tables to corresponding Census industry codes. In the few cases of multiple SIC codes matching to a single Census 1950 industry code, we assign the simple average of the SIC electricity exposure to the Census industry code.
The SIC industry’s electricity exposure is defined as the sum of its input spending (as a share of its output) on “Purchases of Electrical Industrial Equipment and Apparatus,” “Purchases of Electric lighting and wiring,” “Purchases of Electric components and accessories,” and “Purchases of Misc. electrical machinery, equipment and supplies.” As the 1947 IO tables do not separate purchases of electricity from other utilities such as gas, water and sanitary services, we do not include direct electricity purchases in our exposure measure. Finally, to mitigate noise in the exposure measure, we then average the exposure measure of each Census 1950 industry in four bins – non-manufacturing, low electricity usage manufacturing, moderate electricity usage manufacturing, and high electricity usage manufacturing. The bins within the manufacturing sector are constructed as the bottom, middle and top 33% of manufacturing electricity exposure. Finally, we compute the occupation’s exposure to electricity as \( \text{Exposure}_o \equiv \sum_k \ell_{o,t_0}^k S_k \), where \( S_k \) is the electricity input exposure for industry \( k \) (computed as described above), and \( \ell_{o,t_0}^k \) is the share of industry \( k \) in the number of employed individuals in occupation \( o \) in the 1900 Census.

**Occupation exposure to ICT-intensive industries.** For industry ICT exposure, we first construct for each NAICS industry its spending (as a share of its output) on the following items related to ICT in the 1997 BEA input-output table: “Audio, visual and communications equipment,” “Semiconductors and electronic components,” “Software,” “Motion pictures and sound recordings,” “Radio and television broadcasting,” “Cable networks and program distribution,” “Telecommunications,” “Information services,” and “Data processing services.” We use the crosswalk provided by the Census Bureau to map the NAICS 1997 codes to the 1990 Census industry classification. For each Census industry, we use the simple average of the ICT exposures of all associated NAICS industries.

We compute the occupation’s exposure following the same steps described above for the construction of the exposure to electricity-intensive industries. We first group industries into four bins of ICT exposure based on quartiles of industry spending shares on ICT, and use the simple average in each bin as the exposure measure for the Census industries in that bin. We then compute the occupation’s exposure to ICT as \( \text{Exposure}_o \equiv \sum_k \ell_{o,t_0}^k S_k \), where \( S_k \) is the ICT input exposure for industry \( k \) (computed as described above), and \( \ell_{o,t_0}^k \) the share of industry \( k \) in the number of full-time employed individuals in occupation \( o \) in the 1980 Census.
Table A1: Most Exposed Occupations in the Two Episodes

<table>
<thead>
<tr>
<th>Rank</th>
<th>Manufacturing</th>
<th>Electricity</th>
<th>ICT</th>
<th>Cognitive</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Millwrights</td>
<td>Millwrights</td>
<td>Secondary school teachers</td>
<td>Chemical engineers</td>
</tr>
<tr>
<td>2</td>
<td>Typesetters and compositors</td>
<td>Millwrights</td>
<td>Insurance adjusters, examiners, and investigators</td>
<td>Physical scientists, n.e.c.</td>
</tr>
<tr>
<td>3</td>
<td>Slicing and cutting machine operators</td>
<td>Machinists</td>
<td>Primary school teachers</td>
<td>Metallurgical and materials engineers, variously phrased</td>
</tr>
<tr>
<td>4</td>
<td>Mixing and blending machine operatives</td>
<td>Boilermakers</td>
<td>Subject instructors (HS/college)</td>
<td>Chemists</td>
</tr>
<tr>
<td>5</td>
<td>Printing machine operators, n.e.c.</td>
<td>Sawing machine operators and sawyers</td>
<td>Insurance sales occupations</td>
<td>Aerospace engineers</td>
</tr>
<tr>
<td>6</td>
<td>Tool and die makers and die setters</td>
<td>Sawing machine operators and sawyers</td>
<td>Managers in education and related fields</td>
<td>Civil engineers</td>
</tr>
<tr>
<td>7</td>
<td>Engravers</td>
<td>Patternmakers and model makers</td>
<td>Clergy and religious workers</td>
<td>Petroleum, mining, and geological engineers</td>
</tr>
<tr>
<td>8</td>
<td>Patternmakers and model makers</td>
<td>Heat treating equipment operators</td>
<td>Kindergarten and earlier school teachers</td>
<td>Other health and therapy</td>
</tr>
<tr>
<td>9</td>
<td>Rollers, roll hands, and finishers of metal</td>
<td>Engravers</td>
<td>Actuaries</td>
<td>Actuaries</td>
</tr>
<tr>
<td>10</td>
<td>Molders, and casting machine operators</td>
<td>Structural metal workers</td>
<td>Hairdressers and cosmetologists</td>
<td>Dietitians and nutritionists</td>
</tr>
</tbody>
</table>

Note. Each column lists the top 10 occupations by a particular exposure category. The occupation exposure in column (1) is the occupation’s average manufacturing intensity, and in column (2) is its average electricity intensity. The exposure in column (3) is the average ICT-intensity while column (4) has the occupation cognitive intensity.
Occupation task requirements and task-based exposure. We use the replication data from Autor and Dorn (2013) to measure the intensity of each occupation (in the 1990 Census classification) on three types of tasks: manual, routine, and abstract/cognitive. Autor and Dorn (2013) follow the procedure in Autor et al. (2003) to measure task intensity using the job task requirements published in the fourth edition of the U.S. Department of Labor’s Dictionary of Occupational Titles (DOT) (U.S. Department of Labor 1977) – for details see Section II of Autor and Dorn (2013). For each occupation, the intensity index in a task is a number that varies between 0 and 10. We define $\text{Exposure}_o$ as occupation $o$’s standardized intensity index in abstract activities. Table A1 above lists the top 10 exposed occupations by each measure for each period.

A.2 Additional results

A.2.1 Robustness of Figure 1: Responses in occupation outcomes

Manufacturing wage premium, 1900-1926. To shed light on wage responses to the manufacturing innovations of the early twentieth century, we use wage indices obtained from the 1975 Historical Statistics of the United States published by the U.S. Census. This is the same primary data source in Goldin and Margo (1992). We define the manufacturing wage premium as the log of the ratio of wage indices for manufacturing workers (series D-781) and non-farm workers (series D-780). Figure A2 shows that the relative wage in manufacturing did not increase much between 1900 and 1926. Following an increase shortly after 1900, the manufacturing wage premium falls until WWI, and then returns to its 1900 level by the mid 1920s.

Alternative exposure measures. In Table A2, we investigate the robustness of the estimates reported in Figure 1. Panels A and B report respectively the responses of relative employment and wages in occupations more exposed to ICT innovations in the late twentieth century while Panel C reports the response of relative employment in occupations more exposed to manufacturing innovations in the early twentieth century. Columns (1)-(4) report estimates for the first two decades of each episode and columns (5)-(8) for the entire sample period. The heading of each column indicates whether estimates are for all workers aged 16-64yrs, older workers aged 30-64yrs, younger workers aged 16-29yrs, or the difference between older and younger workers. For ease of comparison, the first set of estimates in all panels replicates the baseline estimates reported in Figure 1.

We start by reporting estimates based on alternative exposure measures for the two episodes. For the ICT episode, the extensive literature reviewed in Acemoglu and Autor (2011) has suggested that technology-driven labor demand shocks after 1980 might have increased the productivity of jobs intensive in cognitive tasks (relative to those inten-
sive in routine tasks). Thus, as described above, we construct an alternative exposure measure based on each occupation’s cognitive task intensity. For the early episode, motivated by the link between the spread of electricity and the increase in manufacturing in this period (David, 1990; Jovanovic and Rousseau, 2005), we construct a measure of an occupation’s exposure to electricity usage. Appendix A.1 described in detail how we built these alternative exposure measures.

The second set of estimates in each panel of Table A2 relies on these alternative exposure measures. Overall, results attest the robustness of our conclusions. In Panels A and C, we show that the estimated responses in the relative employment of more exposed occupations are similar to our baseline estimates in both episodes. Panel B shows that the task-based exposure measure also yields a significant increase in the relative wage of more exposed occupations shortly after 1980, but this increase is larger and more persistent, with the slow down only starting in 2010.27

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27We note that the alternative exposure measure implies that younger workers actually had lower relative wage growth in more exposed occupations. This reinforces our conclusion that relative demand did not increase more for younger than older workers conditional on their occupation’s exposure.
Table A2: Exposure to Technological Innovations and Occupation Outcomes for Younger and Older Workers in the Two Episodes – Robustness

<table>
<thead>
<tr>
<th>Change between</th>
<th>Worker group:</th>
<th>All 16-64yrs</th>
<th>Younger 16-29yrs</th>
<th>Older 30-64yrs</th>
<th>Younger - Older</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>All 16-64yrs</td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td></td>
<td>Younger 16-29yrs</td>
<td>(5)</td>
<td>(6)</td>
<td>(7)</td>
<td>(8)</td>
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</tbody>
</table>

**Panel A: Effect of ICT exposure on relative employment**

Baseline

<table>
<thead>
<tr>
<th></th>
<th>0.040</th>
<th>0.052</th>
<th>0.015</th>
<th>0.037</th>
<th>0.099**</th>
<th>0.141***</th>
<th>0.062</th>
<th>0.078***</th>
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<tbody>
<tr>
<td>(0.034)</td>
<td>(0.044)</td>
<td>(0.032)</td>
<td>(0.025)</td>
<td>(0.044)</td>
<td>(0.050)</td>
<td>(0.043)</td>
<td>(0.029)</td>
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</tbody>
</table>

Alternative exposure measure

Cognitive exposure

<table>
<thead>
<tr>
<th>0.041</th>
<th>0.018</th>
<th>-0.018</th>
<th>0.036*</th>
<th>0.141**</th>
<th>0.163**</th>
<th>0.054</th>
<th>0.109***</th>
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<tr>
<td>(0.046)</td>
<td>(0.052)</td>
<td>(0.046)</td>
<td>(0.020)</td>
<td>(0.062)</td>
<td>(0.066)</td>
<td>(0.064)</td>
<td>(0.031)</td>
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Alternative sample

Only U.S. natives

<table>
<thead>
<tr>
<th>0.046</th>
<th>0.057</th>
<th>0.021</th>
<th>0.036</th>
<th>0.099**</th>
<th>0.134***</th>
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<th>0.067**</th>
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</thead>
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<td>(0.049)</td>
<td>(0.042)</td>
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<table>
<thead>
<tr>
<th>0.044</th>
<th>0.050</th>
<th>0.017</th>
<th>0.034</th>
<th>0.090**</th>
<th>0.131***</th>
<th>0.050</th>
<th>0.082***</th>
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<td>(0.032)</td>
<td>(0.024)</td>
<td>(0.043)</td>
<td>(0.048)</td>
<td>(0.042)</td>
<td>(0.028)</td>
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</table>

Alternative controls

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<thead>
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<th>0.030</th>
<th>0.003</th>
<th>0.027</th>
<th>0.105**</th>
<th>0.120**</th>
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<th>0.063**</th>
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<td>(0.047)</td>
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**Panel B: Effect of ICT exposure on relative wage**

Baseline

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<tr>
<th>0.025*</th>
<th>0.039***</th>
<th>0.021</th>
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<th>0.006</th>
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<td>(0.013)</td>
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<td>(0.012)</td>
<td>(0.032)</td>
<td>(0.028)</td>
<td>(0.035)</td>
<td>(0.018)</td>
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</table>

Alternative exposure measure

Cognitive exposure

<table>
<thead>
<tr>
<th>0.076***</th>
<th>0.074***</th>
<th>0.086***</th>
<th>-0.011</th>
<th>0.136***</th>
<th>0.101***</th>
<th>0.162***</th>
<th>-0.061***</th>
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<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.016)</td>
<td>(0.040)</td>
<td>(0.034)</td>
<td>(0.042)</td>
<td>(0.020)</td>
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Alternative sample

Only U.S. natives

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<thead>
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<th>0.016</th>
<th>0.021*</th>
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<th>0.006</th>
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<td>(0.012)</td>
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<td>(0.030)</td>
<td>(0.036)</td>
<td>(0.018)</td>
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<thead>
<tr>
<th>0.022</th>
<th>0.036***</th>
<th>0.018</th>
<th>0.017</th>
<th>-0.007</th>
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<td>(0.015)</td>
<td>(0.012)</td>
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<td>(0.031)</td>
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<td>(0.018)</td>
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Alternative controls

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<thead>
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<th>0.018*</th>
<th>0.021</th>
<th>0.029**</th>
<th>0.026*</th>
<th>0.003</th>
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<tr>
<td>(0.011)</td>
<td>(0.012)</td>
<td>(0.011)</td>
<td>(0.011)</td>
<td>(0.013)</td>
<td>(0.014)</td>
<td>(0.014)</td>
<td>(0.013)</td>
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</tbody>
</table>

**Panel C: Effect of Manufacturing exposure on relative employment**

Baseline

<table>
<thead>
<tr>
<th>0.153***</th>
<th>0.167**</th>
<th>0.121***</th>
<th>0.046</th>
<th>0.190***</th>
<th>0.205***</th>
<th>0.169**</th>
<th>0.036</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.053)</td>
<td>(0.070)</td>
<td>(0.031)</td>
<td>(0.062)</td>
<td>(0.069)</td>
<td>(0.076)</td>
<td>(0.069)</td>
<td>(0.058)</td>
</tr>
</tbody>
</table>

Alternative exposure measure

Electricity exposure

<table>
<thead>
<tr>
<th>0.150***</th>
<th>0.167**</th>
<th>0.118***</th>
<th>0.050</th>
<th>0.150**</th>
<th>0.153*</th>
<th>0.134*</th>
<th>0.020</th>
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</thead>
<tbody>
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<td>(0.034)</td>
<td>(0.060)</td>
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<td>(0.089)</td>
<td>(0.071)</td>
<td>(0.070)</td>
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</tbody>
</table>

Alternative sample

Only U.S. natives

<table>
<thead>
<tr>
<th>0.153**</th>
<th>0.173**</th>
<th>0.115***</th>
<th>0.058</th>
<th>0.219***</th>
<th>0.242***</th>
<th>0.195***</th>
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<tbody>
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<td>(0.063)</td>
<td>(0.075)</td>
<td>(0.080)</td>
<td>(0.072)</td>
<td>(0.058)</td>
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</table>

Only white

<table>
<thead>
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<th>0.153***</th>
<th>0.163**</th>
<th>0.125***</th>
<th>0.038</th>
<th>0.188**</th>
<th>0.208***</th>
<th>0.168**</th>
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<tbody>
<tr>
<td>(0.052)</td>
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<td>(0.033)</td>
<td>(0.060)</td>
<td>(0.073)</td>
<td>(0.079)</td>
<td>(0.072)</td>
<td>(0.058)</td>
</tr>
</tbody>
</table>

Note. Sample based on 310 occupations for Panels A-B and 201 occupations for Panel C with positive employment in the initial and final years of each episode for the U.S. Census sample of males aged 16-64yrs in the labor force. Estimates are weighted by the occupation’s employment at \( t_0 \). Dependent variable is the change in the outcome indicated in each panel between years \( t_0 \) and \( t_0 + 20 \) in columns (1)-(4) and between years \( t_0 \) and \( t_0 + 40 \) in columns (5)-(8). Estimates in columns (1) and (5) based on equation (1) for a single group of all males aged 16-64yrs. Estimates in columns (2)-(4) and (6)-(8) based on equation (2) for two groups: a Younger group including males aged 16-29yrs, and an Older group including males aged 30-64 yrs. Columns (4) and (8) report the difference between the coefficients for younger and older workers. The occupation exposure in Panels A-B is the occupation’s average ICT-intensity across industries, except in the second row where it is the occupation cognitive intensity, and in Panel C is the occupation’s average manufacturing-intensity across industries, except in the second row where it is the occupation’s average electricity-intensity across industries. Exposure measures are normalized to have a unit standard deviation. Estimates in the middle two rows of each panel use occupation outcomes computed from the sub-sample of U.S. native and white males. Estimates in the last row of Panels A and B control for the following occupation variables measured in 1980: hours worked, self employment rate, and share of full-time workers. Standard errors in parentheses are clustered by occupation. * \( p < 0.1 \), ** \( p < 0.05 \), *** \( p < 0.01 \).
Alternative sample. The third set of estimates in each panel of Table A2 addresses concerns that two sources of aggregate labor supply shocks during these episodes had heterogeneous effects across occupations that could have been correlated with our exposure measures. Both the early and the late twentieth centuries were periods in which immigration changed substantially. To the extent that immigrants differ from natives in their occupation allocation (Card, 2005), changes in the stock of immigrants may have affected relative employment in more exposed occupations. We show that this was not the case, since our estimates are similar when we restrict our sample to include only U.S. natives. Both periods were also marked by changes in social norms that affected the employment allocation of minorities across occupations (Hsieh et al., 2019). We show that this does not affect our conclusions as estimates are similar when we restrict our sample to include only white males.

Alternative controls. The fourth set of estimates in Panels A and B of Table A2 address concerns that our estimates could be biased by job amenities that varied across occupations in a manner correlated with their shock exposure. The U.S. Census did not collect much information on job attributes in 1900-1940, so we only report estimates with additional controls for the more recent ICT episode. Specifically, we estimate equation (1) including controls for the following occupation variables measured in 1980: hours worked, self employment rate, and share of full-time workers. Estimates show that our conclusions are robust to the inclusion of these controls.

Alternative generation definition. We next turn to the robustness of the between-generation gap in employment responses with respect to the definition of the two worker generations. We do so by estimating (2) with alternative age cutoffs to define the groups of younger and older workers. Figure A3 reports our estimates of $\beta_{\text{younger},t} - \beta_{\text{older},t}$ for different age cutoffs defining the younger worker generation. For the employment response to ICT exposure between 1980 and 2019, the black dots show that the young-old gap becomes smaller as we increase the maximum age of workers included in the younger group. For the employment response to manufacturing exposure in 1900-1940, the gray diamonds show that, if anything, the differential reallocation of younger workers becomes stronger as we increase the age cutoff.

A.2.2 Decomposition of employment reallocation across cohorts

We now relate the different patterns of occupation reallocation for older and younger workers in the two episodes to the occupational choices of successive cohorts. We do so by decomposing the reallocation of cohorts towards highly exposed occupations in each decade of an episode into a common decade effect for all cohorts ($\zeta_t$) and cohort-
specific deviations from that common effect ($e_{c,t}$). In particular, we estimate the following regression:

$$\log s_{Emp_{Hc,t}}^c - \log s_{Emp_{Hc,t-1}}^c = \bar{\zeta}_t + e_{c,t}, \quad (A.1)$$

where the dependent variable is the ten-year log-change in the share of individuals of cohort $c$ that are employed in occupations with the 25% highest values of exposure (i.e., the set of highly exposed occupations that we denote by $O_{H}$).\footnote{Formally, $s_{Emp_{Hc,t}} = \sum_{o \in O_{H}} Emp_{oc,t} / \sum_{o} Emp_{oc,t}$, where $Emp_{oc,t}$ is the number of individuals of cohort $c$ employed in occupation $o$ at year $t$.} We consider a sample that includes each of the five U.S. Decennial Census years in the corresponding episode and, in any given year, each of the 49 cohorts aged 16-64 years old.

When the pattern of reallocation towards highly exposed occupations is more similar across cohorts, the cohort-period residuals $e_{c,t}$ have a lower dispersion (i.e., the standard deviation of $e_{c,t}$ is lower), and account for a smaller share of the variation in the dependent variable (i.e., $1 - R^2$ of the regression is lower). In fact, in the extreme case in which all cohorts were to exhibit exactly the same employment growth in highly exposed occupations, we would have $e_{c,t} = 0$ and thus $1 - R^2 = 0$.

In Table A3, we report the statistics associated with the importance of the cohort-specific components in explaining the employment reallocation towards the highly ex-
Table A3: Importance of Cohort-specific Effects in Reallocation towards Highly Exposed Occupations in the Two Episodes

<table>
<thead>
<tr>
<th></th>
<th>Reallocation towards occupations intensive in</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ICT, 1980-2019</td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td></td>
<td>manufacturing, 1900-1940</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Panel A: Importance of cohort-specific components

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1 − $R^2$</td>
<td>0.969</td>
<td>0.359</td>
</tr>
<tr>
<td>St. Dev. of $\epsilon_{c,t}$</td>
<td>0.200</td>
<td>0.087</td>
</tr>
</tbody>
</table>

Panel B: Importance of cohort-specific components (w/ cohort linear trend controls)

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>1 − $R^2$</td>
<td>0.620</td>
<td>0.082</td>
</tr>
<tr>
<td>St. Dev. of $\epsilon_{c,t}$</td>
<td>0.186</td>
<td>0.050</td>
</tr>
</tbody>
</table>

Cohort-period obs. 156 156
Cohort obs. 69 69

Note. Statistics obtained from the estimation of (A.1) in Panel A, and (A.1) with cohort-specific linear time trends in Panel B. For each column, the set of high-exposure occupations, $O_H$, contains the top 25 percent of occupations in terms of the indicated exposure measure. The occupation exposure in column (1) is the occupation’s average ICT-intensity across industries and in column (2) is the occupation’s average manufacturing-intensity across industries. Data from U.S. Census sample of males aged 16-64yrs in the labor force.

posed occupations in each of the two episodes. Panel A shows that, compared to the recent reallocation to ICT-intensive jobs, cohort-specific effects explain less of the reallocation towards manufacturing-intensive jobs in the early 1900s; that is, both 1 − $R^2$ and the standard deviation of $\epsilon_{c,t}$ are lower in the earlier episode. In Panel B, we show that these results remain valid when we control for cohort-specific time trends in the estimation of the decomposition in (A.1). Thus, our findings are not driven by changes across cohorts in their life-cycle of transitions to highly exposed occupations.

A.2.3 Cross-occupation evidence from other countries.

We now investigate whether other developed countries experienced patterns of relative employment growth in ICT-intensive occupations that are similar to those observed for the United States in recent decades. We rely on data from Eurostat for European countries and IPUMS International for Canada. Because of the coarseness of the data that is available for several countries, we consider nine occupation groups (2-digit ISCO occupations) and define “Younger” workers as those aged 15-39 years (with “Older” workers defined as those aged 40-64 years). Because direct measures of ICT exposure are not available for occupations across all countries, we follow Spitz-Oener (2006) in constructing the cognitive intensity of occupations using the BERUFNET data, which is highly correlated with our measures of ICT exposure across occupations in the United States.²⁹ The ICT-intensive occupations are the top three in this ranking: Managers,

²⁹The BERUFNET dataset is based on the knowledge of experts about the skills required to perform tasks in each occupation. The occupation’s cognitive intensity is the simple average of the time spent on analytical non-routine and
Figure A4: Recent Trends in ICT-intensive Employment Growth in Developed Countries


Professionals, Technicians and Associate Professionals.30

Figure A4 displays the recent trends of employment in ICT-intensive occupations for seventeen developed countries. The dashed bars indicate that relative employment in these occupations has been expanding in almost every country in our sample. This trend is a reflection of the occupation polarization process documented by Goos et al. (2009) and Autor and Dorn (2013). The figure also shows that, while older workers increased their relative employment in more exposed occupations in most countries, this increase was substantially stronger for younger generations. Across all countries, the annualized growth in the relative employment of younger workers was 73% higher than that of older workers. The between-generation gap was higher whenever overall reallocation was higher: there is a correlation of 0.43 between the young-old gap in relative employment growth and that of all workers.

A.2.4 Robustness of Figure 2: Differences in task content between occupations

Figure A5 assesses whether the patterns in the left panel of Figure 2 are similar when we use alternative exposure thresholds, alternative distance metrics, and alternative exposure measures. Panel A defines the set of occupations exposed to the shock $O_H$ in each episode as the top 50% of all occupations in terms of exposure, with $O_L$ then defined as the remaining occupations. Panel B is the histogram of the bilateral distance, $D_{o,o'}$,

---

30The other occupation groups are Elementary occupations, Plant and machine operators, Craft and related trades workers, Skilled agricultural and fishery workers, Service and sales workers, and Clerks.
Figure A5: Task Content Distance Between Occupations with The Lowest and The Highest Levels of Exposure to the New Technologies in the Two Episodes – Alternative Specifications

A: $O_H$ defined for top 50% exposed occ.

B: Bilateral distance between all occ.

C: Alternative exposure measures $O_H$ defined for top 25% exposed occ.

D: Unweighted task distance $O_H$ defined for top 25% exposed occ.

FOSD test: P-value 0.00, Difference 0.35***  
FOSD test: P-value 0.00, Difference 0.06***

Note. The histograms plot the distributions of the average distance from high-exposure occupations in $O_H$ for each low-exposure occupation in $O_L$, except in Panel B that plots the bilateral distance between each pair of occupations in $O_H$ and $O_L$. Data from U.S. Census for employed males aged 16-64yrs. The gray bars for manufacturing exposure use the 1950 Census occupation definitions with positive employment in 1900 and 1940. The black bars for ICT exposure use the 1990 occupation definition in Autor and Dorn (2009) with positive employment in 1980 and 2018. Distance between a pair of occupations is computed using an Entropy metric based on distance between the skill content of the pair of occupations using the Autor and Dorn (2013) measure of skills as discussed in Section 2. The sets of high and low exposure occupations, $O_H$ and $O_L$, are respectively defined as the top 25 percent and the bottom 75 percent of occupations in terms of exposure in Panels B, C and D. In Panel A, the sets are defined as the top 50% and bottom 50% in terms of exposure. Panel C uses the alternative exposure measure based on electricity-intensity for manufacturing and cognitive-intensity for ICT. Panel D uses an unweighted average distance for each $O_L$ with all occupations in $O_H$, while Panels A and C report the employment weighted average distance. The reported test for first-order stochastic dominance is the Kolmogorov-Smirnov test.

between each occupation $o \in O_L$ and $o' \in O_H$. Panel C presents the histogram of task distances obtained with the alternative exposure measures in terms of electricity- and cognitive-intensity for the two episodes. Panel D presents results from considering the unweighted average task distance, with $O_H$ defined as the top 25% in terms of exposure.

In all cases, it is evident that the distribution of task distance for ICT exposure in 1980 has more mass on higher values when compared to that implied by manufacturing exposure in 1900. For all specifications in Figure A5, we obtain a p-value of zero for
the formal test of the first-order stochastic dominance of the distribution implied by ICT exposure in 1980.

A.2.5 Technology usage across occupation and worker generations

We investigate next how the usage of two important ICT innovations, computers and internet, differed between younger and older workers conditional on the ICT-intensity of their occupation. This analysis requires time-use information for workers by age and occupation. While such data is not readily available for the United States, it exists for Germany. The 2012 Working Condition Survey provides information on whether German workers intensively use computers and internet on their jobs, as well as the occupation and the age of each worker. We use this information to build the share of individuals intensively using computers and internet for each occupation and generation. We again consider two worker generations, younger and older, but we now define the younger group as those aged below forty.\footnote{We choose a higher threshold than in our baseline for the United States because of the small sample size of workers below 29 in the data caused by the higher age of labor market entry in Germany.} As a measure of ICT exposure, we follow the same steps described in Appendix A.2.3 to build the cognitive intensity of the 85 occupations in the 2012 Working Condition Survey.

Figure A6 graphically depicts how on-the-job usage of computers and internet varies with ICT exposure across occupations. It shows that the share of workers in an occupation intensively using both technologies is strongly correlated with the occupation's cognitive intensity. Notice that internet and computer usages are not systematically different for younger and older workers conditional on their occupation’s exposure.\footnote{These results complement the finding in Spitz-Oener (2006) that there were small cross-cohort differences in the change of the task content of German occupations in the 1990s.} This suggests that the stronger reallocation of younger workers towards more exposed occupations that we documented in Section 2.2 was not driven by differential technology usage by age conditional on the skills used in each occupation.
Figure A6: Internet and Computer On-the-job Usage across Occupation

Panel A: Intensive Internet Use by Occupation
Panel B: Intensive Computer Use by Occupation

Note. Sample of 85 occupations in the 2012 Working Condition Survey. For each occupation, we compute the share of individuals reporting intensive internet and computer usage on their job. Figure reports the local polynomial smooth fit.
Appendix B  Proofs

B.1 Proof of Proposition 1 and Proposition 2

Part 1. Optimal assignment and labor supply of skill types. The optimal assignment in (10) is characterized by a threshold $l_t$ corresponding to a skill type that is indifferent between working in the two technologies. The threshold satisfies

\[ 1 = \omega_t \sigma(l_t) = A_t \frac{\int_{l_t}^{1} a(i)\sigma(i)s_t(i)di}{\sigma(l_t)^\theta \int_0^{l_t} a(i)s_t(i)di} \]

where the last equality comes from market clearing. The right-hand side is strictly increasing in $l_t$, converges to infinity as $l_t \to 1$, and converges to zero as $l_t \to 0$. Then, existence and uniqueness of a solution follows from applying Bolzano’s theorem.

The optimal labor supply of skill types in (11) is characterized by the FOC

\[ \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \frac{w_t(i)}{\int_0^{1} w_t(x)\tilde{s}_\tau(x)dx} dt - (1 + \lambda_\tau) \left( \int_{0}^{1} \tilde{s}_\tau(x)^{1+\frac{1}{\nu}}dx \right)^{-1} \tilde{s}_\tau(i)^{\frac{1}{\nu}} = 0 \]

\[ \left( \int_{0}^{1} \tilde{s}_\tau(x)^{1+\frac{1}{\nu}}dx \right)^{\frac{1}{1+\frac{1}{\nu}}} = 1. \]

Multiplying by $\tilde{s}_\tau(i)$ on both sides and integrating over $i \in [0,1]$, we obtain $1 + \lambda_\tau = \frac{1}{\rho}$. Therefore,

\[ \tilde{s}_\tau(i) = \left( \rho \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \frac{w_t(i)}{\int_0^{1} w_t(x)\tilde{s}_\tau(x)dx} dt \right)^{\nu}. \]

Part 2. The q-theory system of ODEs. We begin by solving for the stationary distribution $s(i)$ implied by (12) and (B.2):

\[ s(i) = \tilde{s}(i) \frac{w(i)^{\nu}}{\left( \int_0^{1} w(x)^{1+\nu}dx \right)^{1+\nu}} \]

We then take a first order approximation around the stationary equilibrium of equa-
tions (12) and (B.1). We obtain

\[
\frac{d\hat{s}_t(i)}{dt} = -\delta \hat{s}_t(i) + \delta \hat{\hat{s}}_t(i) \quad \text{(B.4)}
\]

\[
\hat{\hat{s}}_t = \frac{1}{\kappa} \frac{\eta}{\theta} \hat{s}_t \quad \text{(B.5)}
\]

\[
\hat{\hat{s}}_t = \frac{1}{\kappa} \frac{\eta}{\theta} \left( \int_l^1 \hat{s}_t(i) \frac{\alpha(i)\sigma(i)s(i)}{\int_l^1 \alpha(i)\sigma(i)s(i)di} di - \int^l_1 \hat{s}_t(i) \frac{\alpha(i)s(i)}{\int_0^l \alpha(i)s(i)di} di \right) \quad \text{(B.6)}
\]

where

\[
\eta = \kappa \left( \frac{d\log(\sigma(x))}{d\log(x)} \right) \bigg|_{x=\hat{\hat{s}}_t}^{-1}
\]

\[
\kappa \equiv \frac{\alpha(l)s(l)l}{\int_l^1 \alpha(i)s(i)di} + \frac{\alpha(l)\sigma(l)s(l)l}{\int_l^1 \alpha(i)\sigma(i)s(i)di}.
\]

Differentiating (B.6) with respect to time, we get that

\[
\frac{d\hat{\hat{s}}_t}{dt} = \frac{1}{\kappa} \frac{\eta}{\theta} \left( \int_l^1 \frac{d\hat{s}_t(i)}{dt} \frac{\alpha(i)\sigma(i)s(i)}{\int_l^1 \alpha(i)\sigma(i)s(i)di} di - \int^l_1 \frac{d\hat{s}_t(i)}{dt} \frac{\alpha(i)s(i)}{\int_0^l \alpha(i)s(i)di} di \right).
\]

Applying (B.4) to this expression and using the equations (10) and (B.6), we obtain

\[
\frac{d\hat{\hat{s}}_t}{dt} = -\delta \hat{s}_t + \frac{1}{\kappa} \frac{\eta}{\theta} \left( \int_l^1 \frac{\hat{s}_t(i)}{\int_l^1 w(i)s(i)di} \frac{w(i)s(i)}{\int_l^1 w(i)s(i)di} di - \int^l_1 \frac{\hat{s}_t(i)}{\int_0^l w(i)s(i)di} \frac{w(i)s(i)}{\int_0^l w(i)s(i)di} di \right) \quad \text{(B.7)}
\]

Log-linearizing (B.2), we obtain

\[
\hat{s}_\tau(i) = \nu \rho \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \left( \hat{\hat{s}}_t(i) - \frac{1}{\int_0^1 w(x)s(x)dx} \int_0^1 w(x)s(x) \left( \hat{\hat{s}}_t(x) + \hat{s}_\tau(x) \right) dx \right) dt
\]

\[
= \nu \rho \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \left( \hat{\hat{s}}_t(i) - \frac{1}{\int_0^1 w(x)s(x)dx} \int_0^1 w(x)s(x) \hat{\hat{s}}_t(x) dx \right) dt,
\]

where the last equality follows from the fact that \( \int_0^1 w(x)s(x)\hat{s}_\tau(x)dx = 0 \), since (B.3) holds and \( \left( \int_0^1 \hat{s}_\tau(x)x^{1+\frac{1}{\rho}}dx \right)_{1+\frac{1}{\rho}} = 1 \) for all \( \tau \).

Next, to obtain expressions for \( \hat{\hat{s}}_t(i) \), we guess and verify that \( l_t \) converges monotonically along the equilibrium path. We establish this starting from \( \hat{\hat{s}}_0 < 0 \). We omit the analogous proof for \( \hat{l}_0 > 0 \). Whenever \( \hat{l}_0 < 0 \) and increases monotonically along the equilibrium path, we have that for all \( s > t \), types \( i < l_t \) are employed in technology \( L \) and types \( i > l_t \) are employed in technology \( H \). Also, for workers with \( i \in (l_t, l) \), there
exist a $\tilde{\tau}(i)$ such that they work in $H$ for all $\tau < t < \tau + \tilde{\tau}(i)$ and in $L$ for all $t > \tau + \tilde{\tau}(i)$. This implies that

$$\hat{\omega}_t(i) = \left( \mathbb{I}_{t \geq l} + \mathbb{I}_{i \in (l_i, l]} \mathbb{I}_{t \in (\tau, \tau + \tilde{\tau}(i))} \right) \hat{\omega}_t.$$  

Letting $q_\tau$ be the present discounted value of log-wages $\int_{t}^{\infty} e^{-\rho(t-\tau)} \log(\omega_i) dt$, we have that

$$\hat{q}_\tau = \int_{t}^{\infty} e^{-\rho(t-\tau)} \hat{\omega}_t dt,$$

and we obtain

$$\dot{s}_\tau(i) = n \rho \left( \mathbb{I}_{t \geq l} - \int_{0}^{1} w(x)s(x) dx \right) \hat{q}_\tau$$

$$+ n \rho \left( \mathbb{I}_{i \in (l_i, l]} \left( \hat{q}_\tau - \hat{q}_{\tau + \tilde{\tau}(i)} \right) - \frac{1}{\int_{0}^{1} w(x)s(x) dx} \int_{l_i}^{l} w(x)s(x) \left( \hat{q}_\tau - \hat{q}_{\tau + \tilde{\tau}(x)} \right) dx \right).$$

Replacing this expression (evaluated at $\tau = t$) in equation (B.7), we obtain

$$\frac{\partial \hat{l}_t}{\partial t} = -\delta \hat{l}_t + \frac{1}{\kappa} \frac{\eta}{\kappa + \theta} \rho \left( \hat{q}_t - \int_{l_i}^{l} \frac{w(x)s(x)}{\int_{0}^{1} w(i)s(i) di} \hat{l}_t \hat{q}_t \right).$$

Then, given our guess that $l_t$ increases monotonically along the equilibrium path, from (B.1) we see that $\omega_t$ decreases monotonically along the equilibrium path. This implies that $\hat{q}_t > \hat{q}_{l_t + l(i)} > 0$ for all $i$ and all $t$. So, we can show that the term inside the integral is of second order:

$$0 \leq \int_{l_i}^{l} \frac{w(x)s(x)}{\int_{0}^{1} w(i)s(i) di} \left( \hat{q}_t - \hat{q}_{l_t + l(i)} \right) \leq \int_{l_i}^{l} \frac{w(x)s(x)}{\int_{0}^{1} w(i)s(i) di} \hat{q}_t \leq \frac{\max_{x \in (l_i, l]} w(x)s(x)l}{\int_{0}^{1} w(i)s(i) di} \hat{l}_t \hat{q}_t \approx 0.$$

Using (B.6) to replace $\hat{l}_t$, we obtain the law of motion for $\hat{x}_t$

$$\frac{\partial \hat{x}_t}{\partial t} = -\delta \hat{x}_t + \frac{\theta \rho}{\eta + \theta} \hat{q}_t$$

(B.8)

Finally, we differentiate the definition of $\hat{q}_t$ with respect to time

$$\rho \hat{q}_t = \omega_t + \frac{\partial \hat{q}_t}{\partial t}.$$

**Part 3. The equivalence.** We are now ready to show the equivalence of approximate equilibrium dynamics between the reduced-form and the structural models. By defi-
nition, (18) and (19) hold in both models. It then remains to be shown that (17) holds too.

Differentiating with respect to time and log-linearizing the relative labor supply equation in the reduced-form model, we obtain
\[
\frac{\partial \hat{x}_t}{\partial t} = -\delta \hat{x}_t + \delta \hat{x}_{t,t} + e^{-\delta t} \frac{\partial \hat{x}_{0-t}}{\partial t} + \delta \int_0^t e^{-\delta(t-\tau)} \frac{\partial \hat{x}_{\tau,t}}{\partial t} d\tau
\]
\[
= -\delta \hat{x}_t + \delta (\eta \hat{\omega}_t + \psi \hat{q}_t) + \eta \frac{\partial \hat{\omega}_t}{\partial t}
\]
\[
= -\delta \hat{x}_t + \delta \left(-\frac{\eta}{\theta} \hat{x}_t + \psi \hat{q}_t\right) - \frac{\eta}{\theta} \frac{\partial \hat{x}_t}{\partial t}
\]
\[
= -\delta \hat{x}_t + \frac{\theta \psi \delta}{\eta + \theta} \hat{q}_t
\]
where the second equality uses (16) and third equality uses (19). The last equation is (17) and is identical to (B.8) when \(\psi = \nu \rho\); which completes the proof of equivalence.

Part 4. Saddle-path stability. We now show that the equilibrium is saddle-path stable, and verify that \(l_t\) increases monotonically along the equilibrium path.

We start by guessing that the policy functions are given by \(\frac{\partial \hat{x}_t}{\partial t} = -\lambda \hat{x}_t\) and \(\hat{q}_t = \zeta \hat{x}_t\). By replacing this guess into (17)–(19), we obtain the following system:
\[
-\lambda = -\delta + \frac{\theta}{\eta + \theta} \delta \psi \zeta
\]
\[
\rho \zeta = -1 - \zeta \lambda.
\]

The second equation immediately yields the expression for \(\zeta\). To get the expression for \(\lambda\), notice that substituting the expression for \(\zeta\) into the first equation implies that
\[
(\delta - \lambda)(\rho + \lambda) + \frac{\psi \delta}{\eta + \theta} = 0,
\]
which yields the following solutions
\[
\lambda_{1,2} = -\frac{\rho - \delta}{2} \pm \sqrt{\left(\frac{\rho - \delta}{2}\right)^2 + \delta \left(\rho + \frac{\psi}{\eta + \theta}\right)}.
\]

Because the term inside the square root is always positive, two solutions always exist with one being positive and the other negative. This implies that the equilibrium is saddle-path stable. The positive solution is the speed of convergence of \(l_t\). With some abuse of notation, we will just denote the positive solution \(\lambda\).

Finally, the equilibrium threshold is \(\hat{l}_t = \hat{l}_0 e^{-\lambda t}\). Then, if \(\hat{l}_0 < 0\), this implies that \(l_t\)
increases monotonically along the equilibrium path, which verifies our initial guess and completes the proof of the proposition.

B.2 Derivation of dynamic responses

Using the definitions $x_t$ and $q_t$ together with Proposition 2, we have

$$\Delta \log(x_t) = (\theta - 1)\Delta \log(A) - \theta (\Delta \log(\omega) + \hat{\omega}t) \tag{B.10}$$

$$\Delta \log(q_t) = \frac{1}{\rho} \Delta \log(\omega) + \frac{1}{\rho + \lambda} \hat{\omega}t \tag{B.11}$$

**Long-run.** In this case, the stationary skill distribution is given by (B.3), so that the equilibrium threshold solves

$$A^{\theta - 1} \sigma(l)^{\theta} \int_0^l \alpha(i)(\alpha(i))^\nu di = \int_l^1 \alpha(i)\sigma(i) \left( \frac{\sigma(i)}{\sigma(l)} \right)^\nu di$$

A log-linear approximation around the stationary equilibrium gives

$$(\theta - 1)\Delta \log(A) + \left( \left( \theta + \frac{\psi}{\rho} \right) \frac{\kappa}{\eta} + \kappa \right) \Delta \log(l) = 0$$

Thus,

$$\Delta \log(l) = -\frac{\eta}{\theta + \frac{\psi}{\rho} + \eta \kappa} (\theta - 1) \Delta \log(A)$$

From equation (B.1), $\Delta \log(\omega) = -\frac{\kappa}{\eta} \Delta \log(l)$ and, therefore,

$$\Delta \log(\omega) = \frac{1}{\theta + \frac{\psi}{\rho} + \eta} (\theta - 1) \Delta \log(A) \tag{B.12}$$

**Transition.** We start by deriving the change in the skill supply across steady states using (B.3) and the fact that $\psi = \nu \rho$. We obtain $\hat{s}_0(i) = \hat{s}_0(l)$ if $i < l$ and $\hat{s}_0(i) = \hat{s}_0(l) - \frac{\psi}{\rho} \Delta \log(\omega)$ if $i > l$. Along the transition, the change in the assignment threshold is determined by (B.1) given the change in the skill distribution:

$$\left( \frac{\theta \kappa}{\eta} + \kappa \right) \hat{t}_0 = -\frac{\psi}{\rho} \Delta \log(\omega)$$

Then,

$$\hat{\omega}_0 = -\frac{\kappa}{\eta} \hat{t}_0 = \frac{\psi}{\theta + \eta} \Delta \log(\omega) \tag{B.13}$$
Dynamic responses. We now use the derivations above to show that

\[ \Delta \log(x_t) = \left( \eta + \frac{\theta \psi}{\theta + \eta + \frac{\psi}{\rho}} \left(1 - e^{-\lambda t}\right) \right) \frac{\theta - 1}{\theta + \eta} \Delta \log(A) \]  
(B.14)

\[ \Delta \log(\omega_t) = \left(1 - \frac{\psi}{\theta + \eta + \frac{\psi}{\rho}} \left(1 - e^{-\lambda t}\right) \right) \frac{\theta - 1}{\theta + \eta} \Delta \log(A) \]  
(B.15)

\[ \Delta \log(x_{0-t}) = \eta \Delta \log(\omega_t) \]  
(B.16)

\[ \Delta \log(x_{t-t}) = \eta \Delta \log(\omega_t) + \frac{\psi}{\rho} \left(1 + \frac{\lambda - \delta}{\delta} e^{-\lambda t}\right) \Delta \log(\omega) \]  
(B.17)

where, with some abuse of notation, \( \Delta \log(x_{t-t}) \equiv \log(x_{t-t}) - \log(x_{0-0}) \).

### B.3 Comparative Statics with respect to \( \eta \)

**Proposition A.1 (Comparative statics with respect to \( \eta \))** Assume that \( \theta > 1 \). Then,

1. **Short-run adjustment**

   \[ \frac{\partial \Delta \log(x_0)}{\partial \eta} > 0, \quad \frac{\partial |\Delta \log(\omega_0)|}{\partial \eta} < 0, \quad \frac{\partial \Delta \log(x_{0-0})}{\partial \eta} > 0, \quad \frac{\partial \Delta \log(x_{0,0}/x_{0-0})}{\partial \eta} < 0. \]

2. **Long-run adjustment**

   \[ \frac{\partial \Delta \log(x_\infty)}{\partial \eta} > 0, \quad \frac{\partial |\Delta \log(\omega_\infty)|}{\partial \eta} < 0, \quad \frac{\partial \Delta \log(x_{0-\infty})}{\partial \eta} > 0, \quad \frac{\partial \Delta \log(x_{0,\infty}/x_{0-\infty})}{\partial \eta} < 0. \]

3. **Rate of convergence**

   \[ \frac{\partial \lambda}{\partial \eta} < 0 \]

4. **Cumulative impulse response**

   \[ \frac{\partial \left( \int_0^\infty |\tilde{x}_t| \, dt \right)}{\partial \eta} < 0, \quad \frac{\partial \left( \int_0^\infty |\tilde{\omega}_t| \, dt \right)}{\partial \eta} < 0. \]

Next, we prove each of the items of the proposition above.
1. Short-run adjustment

\[ \Delta \log(x_0) = \frac{\eta}{\theta + \eta} (\theta - 1) \Delta \log(A) \]
\[ \Delta \log(\omega_0) = \frac{\theta - 1}{\theta + \eta} \Delta \log(A) \]
\[ \Delta \log(x_{0-}) = \frac{\eta}{\theta + \eta} (\theta - 1) \Delta \log(A) \]
\[ \Delta \log(x_{0,0} / x_{0-}) = \frac{\lambda}{d} \frac{\psi}{\theta + \eta + \psi} (\theta - 1) \Delta \log(A) \]

The first and third lines immediately show that \( \Delta \log(x_0), \Delta \log(x_{0-}) \) increase with \( \eta \), and the second line that \( \Delta \log(\omega_0) \) decreases with it. Since \( \lambda \) is decreasing in \( \eta \), the last line shows that \( \Delta \log(x_{0,0} / x_{0-}) \) decreases with \( \eta \).

2. Long-run adjustment

\[ \Delta \log(x_\infty) = \frac{\eta + \psi}{\theta + \eta + \psi} (\theta - 1) \Delta \log(A) \]
\[ \Delta \log(\omega_\infty) = \frac{\theta - 1}{\theta + \eta + \psi} \Delta \log(A) \]
\[ \Delta \log(x_{0-\infty}) = \frac{\eta}{\theta + \eta + \psi} (\theta - 1) \Delta \log(A) \]
\[ \Delta \log(x_{0,\infty} / x_{0-\infty}) = \frac{\psi}{\theta + \eta + \psi} (\theta - 1) \Delta \log(A) \]

It is straightforward to see that \( \Delta \log(x_\infty), \Delta \log(x_{0-\infty}) \) are increasing in \( \eta \), while the opposite holds for \( \Delta \log(\omega_\infty) \) and \( \Delta \log(x_{0,\infty} / x_{0-\infty}) \).

3. Rate of convergence

From the implicit equation for \( \lambda \) in (B.9), it is straightforward to see that is decreasing in \( \eta \).

4. Cumulative impulse response

\[ \int_0^\infty |\dot{x}_t| dt = \theta \int_0^\infty |\dot{\omega}_t| dt \]
\[ \int_0^\infty |\dot{\omega}_t| dt = \frac{1}{\lambda} \frac{\psi}{\theta + \eta + \psi} \theta - 1 \Delta \log(A) \]
To show that both are decreasing in $\eta$, we show that:

$$\frac{\partial \log \left( \frac{\varphi}{\lambda \varphi + \varphi \varphi \theta + \eta + \rho \theta} \right)}{\partial \eta} = \frac{1}{\lambda + \rho} \psi \frac{\delta}{\theta} - \frac{1}{\theta + \eta} \psi \frac{\delta}{\rho + \delta} - \frac{1}{\theta + \eta} \lambda$$

$$= - \left( 1 - \frac{\lambda - \delta}{\lambda + \rho} \right) \frac{1}{\theta + \eta} + \frac{1}{\theta + \eta} \psi \frac{\delta}{\rho + \delta} < 0$$

### B.4 Proof of Proposition 3

We have that $DCIR(\omega) = \frac{\lambda \delta}{\lambda + \delta} \left| \int_0^\infty \hat{\omega}_t dt \right|$. From Proposition A.1 in Appendix B.3, we know that $\lambda$ and $\left| \int_0^\infty \hat{\omega}_t dt \right|$ are both smaller when $\eta$ is larger. Thus, $\frac{\partial DCIR(\omega)}{\partial \eta} < 0$. Since $DCIR(x) = \theta DCIR(\omega)$, we also have that $\frac{\partial DCIR(x)}{\partial \eta} < 0$.

### B.5 Proof of Proposition 4

We begin by showing the first order approximation to flow consumption utility $\log(C_{t,T})$. This equals the total flow utility $\log(C_{t,T}) - \rho L_T$ since $L_T = 1$.

$$\log(C_{t,T}) = \log \left( \int_0^1 c_{t,T}(i) di \right)$$

$$= \log \left( \int_0^1 w_t(i) s_t(i) di \right) - \log(P_t)$$

$$\approx \log(C) + \int_0^1 \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} (\hat{\omega}_t(i) + \hat{s}_t(i)) di - \hat{P}_t$$

$$= \log(C) + \int_0^1 \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \hat{\omega}_t(i) di - \hat{P}_t$$

$$= \log(C) + \int_0^1 \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} di \hat{\omega}_t - \hat{P}_t$$

$$= \log(C) + \int_l^1 \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \frac{dx}{1 + \omega x} \hat{\omega}_t - \frac{\omega x}{1 + \omega x} \hat{\omega}_t$$

$$= \log(C) + \frac{\omega x}{1 + \omega x} \hat{\omega}_t - \frac{\omega x}{1 + \omega x} \hat{\omega}_t = \log(C),$$
where $\Delta \log(C) = \frac{\omega x}{1 + \omega x} \Delta \log(A)$. For the older generations, we have

$$
\log(C_{0,t}) \approx \log(C) + \int_0^1 \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} (\hat{\omega}_t(i) + \hat{s}_0(i) - \hat{P}_t) di
$$

$$
= \log(C) + \int_0^1 \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \hat{s}_0(i) di
$$

$$
= \log(C) - \psi \int_0^1 \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \left( \mathbb{I}_{i>t} - \frac{\int_0^1 w(i)s(i) di}{\int_0^1 w(x)s(x)dx} \right) di \Delta \log(\omega)
$$

$$
= \log(C).
$$

The second order approximation is

$$
\log \left( \int_0^1 c_{\tau,t}(i) di \right) \approx \log(C) + \int_0^1 \frac{c(i)}{C} \hat{c}_{\tau,t}(i) di - \frac{1}{2} \int_0^1 \left( \frac{c(i)}{C} \right)^2 \hat{c}_{\tau,t}(i)^2 di
$$

first order is zero $\Rightarrow = \log(C) - \frac{1}{2} \int_0^1 \left( \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \right)^2 (\hat{\omega}_t(i) + \hat{s}_\tau(i) - \hat{P}_t)^2 di$.

This implies

$$
\hat{c}_{\tau,t} = -\frac{1}{2} \int_0^1 \left( \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \right)^2 \left( \mathbb{I}_{i>t} - \mathbb{I}_{i>\tau} - \frac{\omega x}{1 + \omega x} \right) di \hat{\omega}_t + \frac{\omega x}{1 + \omega x} \hat{\omega}_t
$$

$$
\hat{c}_{\tau,t} = -\frac{1}{2} \int_0^1 \left( \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \right)^2 \left( \mathbb{I}_{i>t} - \frac{\omega x}{1 + \omega x} \right) di (\hat{\omega}_t + \hat{\psi}_\tau)^2
$$

$\propto - (\hat{\omega}_t + \hat{\psi}_\tau)^2$.

For older generations, we have

$$
\hat{c}_{0,t} \approx -\frac{1}{2} \int_0^1 \left( \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \right)^2 (\hat{\omega}_t(i) + \hat{s}_0(i) - \hat{P}_t)^2 di
$$

$$
= -\frac{1}{2} \int_0^1 \left( \frac{w(i)s(i)}{\int_0^1 w(x)s(x)dx} \right)^2 \left( \mathbb{I}_{i>t} - \frac{\omega x}{1 + \omega x} \right)^2 di \left( \hat{\omega}_t - \frac{\psi}{\rho} \Delta \log(\omega) \right)^2
$$

$\propto - \left( \hat{\omega}_t - \frac{\psi}{\rho} \Delta \log(\omega) \right)^2$. 

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Then, aggregate welfare for all generations is

\[
\Delta W = \rho(U_0 - U_0^-) + \rho\delta \int_0^\infty e^{-\rho \tau} U_\tau d\tau
\]

\[
= \Delta \log(C) + \rho \int_0^\infty e^{-\rho \tau} \hat{C}_{0^-} d\tau + \rho\delta \int_0^\infty e^{-\rho \tau} \left( \log(C) + \int_\tau^\infty e^{-\rho(t-\tau)} \hat{C}_{\tau, t} dt \right) d\tau
\]

\[
= \Delta \log(C)(1 + \delta) + \rho \left( \int_0^\infty e^{-\rho \tau} \hat{C}_{0^-} d\tau + \delta \int_0^\infty e^{-\rho \tau} \int_\tau^\infty e^{-\rho(t-\tau)} \hat{C}_{\tau, t} dt d\tau \right).
\]

Using the expressions above, we thus have that

\[
\Delta W - \Delta \log(C) \frac{\rho + \delta}{\rho} \alpha - (\hat{\omega}_0)^2 \rho \int_0^\infty e^{-\rho t} \left( e^{-\lambda t} - (\theta + \eta) \right)^2 dt
\]

\[
- (\hat{\omega}_0)^2 \rho \delta \int_0^\infty e^{-\rho \tau} e^{-2\lambda t} \left( \int_\tau^\infty e^{-\rho(t-\tau)} \left( e^{-\lambda(t-\tau)} + \frac{\psi}{\rho + \lambda} \right)^2 dt \right) d\tau
\]

\[
= -(\hat{\omega}_0)^2 \left( \frac{\rho}{2\lambda + \rho} - \frac{2\rho}{\rho + \lambda} (\theta + \eta) + (\theta + \eta)^2 \right)
- (\hat{\omega}_0)^2 \frac{\rho \delta}{2\lambda + \rho} \left( \frac{1}{2\lambda + \rho} + \frac{2\rho}{\rho + \lambda} \frac{\psi}{\rho + \lambda} + \frac{1}{\rho} \left( \frac{\psi}{\rho + \lambda} \right)^2 \right)
\]

Finally, taking the limit \(\rho \to 0\), we obtain

\[
\lim_{\rho \to 0} \frac{\partial \Delta W}{\partial \eta} = \lim_{\rho \to 0} \frac{\partial}{\partial \eta} \left( \Delta W - \Delta \log(C) \frac{\rho + \delta}{\rho} \right)
\]

\[
= -\frac{\partial}{\partial \eta} \left( (\hat{\omega}_0)^2 \left( (\theta + \eta)^2 + \frac{\delta}{2\lambda} \left( \frac{\psi}{\lambda} \right)^2 \right) \right)
\]

\[
= -\frac{\partial}{\partial \eta} \left( ((\theta - 1)\Delta \log(A))^2 \left( 1 + \frac{\delta}{2\lambda} \left( \frac{\lambda - \delta}{\delta} \right)^2 \right) \right)
\]

\[
= - ((\theta - 1)\Delta \log(A))^2 \frac{1}{2\delta^2} \left( -\frac{1}{\lambda^2} (\lambda - \delta)^2 + \frac{1}{\lambda} 2 (\lambda - \delta) \right) \frac{\partial \lambda}{\partial \eta}
\]

\[
= - ((\theta - 1)\Delta \log(A))^2 \frac{1}{2\delta^2} \frac{(\lambda - \delta)}{\lambda^2} (\lambda + \delta) \frac{\partial \lambda}{\partial \eta} > 0,
\]

which completes the proof of the proposition.
Appendix C  Additional Theoretical Results

C.1 Microfoundation of the production functions in (5)-(6)

Consider two firms: high-tech \((k = H)\) and low-tech \((k = L)\). Assume that the output of firm \(k\) at time \(t\) aggregates per-worker output \(x_{kt}(i)\),

\[
X_{kt} = \int_0^1 x_{kt}(i)s_{kt}(i)di,
\]

where \(s_{kt}(i)\) is the quantity demanded of workers of type \(i\) at time \(t\) by firm \(k\).

The output of workers of type \(i\) depends on their skills to perform cognitive and noncognitive tasks, \(\{a_C(i), a_{NC}(i)\}\), as well as how intensely each task is used in the firm’s production process:

\[
x_{kt}(i) = a_C(i)\phi k a_{NC}(i)^{1-\phi k},
\]

where \(\phi k\) denotes the production intensity of firm \(k\) on cognitive tasks.

In our model, technology-skill specificity arises whenever firms are heterogeneous in terms of task intensity and workers are heterogeneous in terms of their skills bundle. To see this, suppose that firm \(H\)’s technology uses cognitive tasks more intensely than firm \(L\)’s technology, \(\phi H > \phi L\), and that a worker of type \(i\) is able to produce a higher cognitive-noncognitive task ratio than a worker of type \(j\), \(a_C(i)/a_{NC}(i) > a_C(j)/a_{NC}(j)\). In this case, \(i\) has a higher relative output with the cognitive-intensive technology \(H\) than \(j\), \(x_{Hi}(i)/x_{Li}(i) > x_{Hi}(j)/x_{Li}(j)\), and, therefore, type \(i\) is more complementary to the cognitive-intensive technology \(H\) than type \(j\).

To map this setting to the production functions in equations (5)-(6), we assume that high-tech production is more intensive in cognitive tasks than low-tech production, \(\phi H > \phi L\). We also assume that types differ in terms of their skill bundle and, without loss of generality, impose that high-\(i\) types are relatively better in performing cognitive-intensive tasks.

1. High-tech technology \(H\) uses cognitive tasks more intensely than Low-tech technology \(L\): \(\phi H > \phi L\).

2. Define \(\sigma(i) \equiv \left(\frac{a_C(i)}{a_{NC}(i)}\right)^{\phi H - \phi L}\) and \(\alpha(i) \equiv a_C(i)^{\phi L}a_{NC}(i)^{1-\phi L}\). Assume that high-\(i\) types have higher cognitive-noncognitive task ratio: \(\sigma(i)\) is increasing in \(i\).

C.2 Extensions

This section discusses the extensions described in Section 6.4.
C.2.1 Learning-from-others

We now assume that certain skills may be easier to supply because workers can "learn from others" when such types are already abundant in the economy. Formally, we assume that

\[ L_\tau = \left( \int_0^1 \bar{s}_\tau(i)^{-\frac{1}{\nu}} \tilde{s}_\tau(i)^{\frac{1}{\nu}+1} \, di \right)^{\frac{1}{\frac{1}{\nu}+1}} \]

where \( \bar{s}_\tau(i) \) is a geometric average of a fixed distribution \( \bar{\epsilon}(i) \) and the current skill type distribution in the economy \( s_\tau(i) \) at the time where generation \( \tau \) is born,

\[ \bar{s}_\tau(i) = s_\tau(i)^{\gamma} \bar{\epsilon}(i)^{1-\gamma}, \quad \gamma \in [0, 1). \]

Note that as \( \gamma \) increases it becomes easier for workers to supply skill types that are already abundant in the economy. As opposed to our benchmark case (\( \gamma = 0 \)), this extension with \( \gamma > 0 \) introduces a backward-looking element to the labor supply problem and complementarities in decisions across generations.

In what follows, we reproduce the key steps that change in the proofs in Appendix B.1. First, the optimal labor supply is

\[ \bar{s}_\tau(i) = \bar{s}_\tau(i) \left( \rho \int_{\tau}^{\infty} e^{-\rho(t-\tau)} \frac{w_t(i)}{\int_0^1 w_t(x)\bar{s}_\tau(x) \, dx} \, dt \right)^{\nu}, \]

and the stationary distribution exist and is

\[ s(i) = \frac{\epsilon(i) w(i)^{\frac{\nu}{\nu-\gamma}}}{\left( \int_0^1 \epsilon(x) w(x)^{1+\frac{\nu}{\nu-\gamma}} \, dx \right)^{\frac{\nu}{\nu-\gamma}}}. \]

Following the same steps as in Appendix B.1, we show that

\[ \frac{\partial \hat{l}_t}{\partial t} = -\delta(1-\gamma)\hat{l}_t + \frac{1}{\kappa \eta + \theta} (\delta(1-\gamma)) \frac{\psi}{1-\gamma} \hat{q}_t. \]

Fourth, since the law of motion for \( \hat{q}_t \) is the same as in the benchmark model, this implies that the equilibrium is saddle-path stable where the new \( \lambda \) in the economy with learning-from-others is the positive solution to

\[ (\delta(1-\gamma) - \lambda)(\rho + \lambda) + \frac{\psi}{1-\gamma} \frac{\delta(1-\gamma)}{\eta + \theta} = 0. \]

Next, we reproduce the key steps that change in Appendices B.2 and B.3. First, from
the expression for the stationary distribution above, note that the long-run skill supply elasticity in the learning-from-others economy is \( \frac{\psi}{1 - \gamma} \) as opposed to simply \( \psi \). Likewise, in the expression for \( \lambda \), the death rate is \( \delta (1 - \gamma) \) instead of just \( \delta \). In all, the learning-from-others economy behaves as if it had a higher \( \psi \) and lower \( \delta \).

### C.2.2 Re-training of old workers

We now let a fraction of workers that were present before the shock re-optimize their labor supplies "as if" they were a young generation entering at time \( t = 0 \). Formally, the skill type distribution on impact now becomes

\[
s_0(i) = (1 - \beta)s_{0-}(i) + \beta \tilde{s}_0(i),
\]

where \( \beta \) is the fraction of workers in the generation present before the shock that can re-optimize.

The first thing to note is that this does not change any of the transitional dynamics given the new initial skill distribution on impact. As such Proposition 2 is unchanged. However, the initial conditions and the dynamic responses do change. Next, we reproduce the key steps that change in Appendix B.2.

The deviation from the skill distribution on impact from the new stationary distribution is now

\[
\hat{s}_0(i) = s_{0-}(i) + \beta \left( \hat{s}_0(i) - \hat{s}_{0-}(i) \right)
\]

\[
= (1 - \beta) \left( \hat{s}_0(l) - \mathbb{I}_{i > l} \frac{\psi}{\rho} \Delta \log(\omega) \right) + \beta \left( \mathbb{I}_{i > l} - \int_{l}^{1} s(i)di \right) \psi \hat{q}_0
\]

where the long-run change \( \Delta \log(\omega) \) is the same as in the benchmark model.

Following the same steps as in the benchmark proof, this then implies that

\[
\frac{\theta + \eta}{\eta} \hat{\kappa}_0 = \int_{l}^{1} \frac{\sigma(i)\alpha(i)s(i)}{\int_{l}^{1} \sigma(i)\alpha(i)s(i)} \hat{s}_0(i)di - \int_{0}^{l} \frac{\alpha(i)s(i)}{\int_{0}^{l} \alpha(i)s(i)} \hat{s}_0(i)di
\]

\[
= - (1 - \beta) \frac{\psi}{\rho} \Delta \log(\omega) + \beta \psi \hat{q}_0.
\]
Thus,
\[
\hat{\omega}_0 = -\frac{\kappa}{\eta} \hat{t}_0 \\
= \frac{1}{\theta + \eta} \left( \frac{\psi}{\rho} \Delta \log(\omega) - \beta \left( \frac{\psi}{\rho} \Delta \log(\omega) + \psi \hat{q}_0 \right) \right) \\
= \frac{1}{\theta + \eta} \left( \frac{\psi}{\rho} \Delta \log(\omega) - \beta \left( \frac{\psi}{\rho} \Delta \log(\omega) + \psi \frac{1}{\rho + \lambda} \hat{\omega}_0 \right) \right) \\
= \frac{1 - \beta}{1 + \beta \frac{\psi}{\rho + \lambda} \theta + \eta} \frac{1}{\theta + \eta} \psi \Delta \log(\omega).
\]

Finally, using the above together with the expression for \(\Delta \log(\omega)\), we obtain:
\[
\Delta \log(x_t) = \left( \eta + \frac{\theta \psi}{\theta + \eta + \frac{\psi}{\rho}} \left( 1 - \frac{1 - \beta}{1 + \beta \frac{\lambda - \delta}{\delta} e^{-\lambda t}} \right) \right) \frac{\theta - 1}{\theta + \eta} \Delta \log(A) \\
\Delta \log(\omega_t) = \left( 1 - \frac{\psi}{\theta + \eta + \frac{\psi}{\rho}} \left( 1 - \frac{1 - \beta}{1 + \beta \frac{\lambda - \delta}{\delta} e^{-\lambda t}} \right) \right) \frac{\theta - 1}{\theta + \eta} \Delta \log(A) \\
\Delta \log(x_{0-t}) = \eta \Delta \log(\omega_t) \\
\Delta \log(x_{t,t}) = \eta \Delta \log(\omega_t) + \frac{\psi}{\rho} \left( 1 + \frac{\lambda - \delta}{\delta} \frac{1 - \beta}{1 + \beta \frac{\lambda - \delta}{\delta} e^{-\lambda t}} \right) \Delta \log(\omega)
\]

Then, mathematically, the dynamic responses in the economy where old generations can re-train are similar to those in the benchmark economy except that the function \(e^{-\lambda t}\) is now multiplied by \(\frac{1 - \beta}{1 + \beta \frac{\lambda - \delta}{\delta} e^{-\lambda t}} < 1\). This immediately implies that: the long-run responses are the same in both economies, the short-run responses of \(x(\omega)\) are now larger (smaller), and the DCIR of all variables is now smaller. Hence, in many ways, this new economy behaves similar to an economy with a lower degree of skill specificity (higher \(\eta\)), except that long-run responses are unchanged.

**C.2.3 Population growth**

We now assume that the size of entering generations is \(\mu\) as opposed to \(\delta\). This implies that the population growth rate is \(\mu - \delta\). The Kolmogorov-Forward equation describing the evolution of the skill type distribution becomes
\[
\frac{\partial e^{(\mu - \delta)t} s_t(i)}{\partial t} = -\delta e^{(\mu - \delta)t} s_t(i) + \mu e^{(\mu - \delta)t} s_t(i).
\]
Then, we have that

$$\frac{\partial s_t(i)}{\partial t} = -\mu s_t(i) + \mu \tilde{s}_t(i).$$

The remaining elements in the model remain the same. Hence, the economy with population growth is identical to our benchmark economy except that the convergence rate $\lambda$ is higher if, and only if, $\mu > \delta$ since it is now the positive solution to:

$$(\lambda - \mu)(\rho + \lambda) = \frac{\psi \mu}{\theta + \eta}.$$ 

Then, if $\mu > \delta$, the short- and long-run responses for $y_t, l_t$ remain unchanged, the short-run response of $q$ is smaller in magnitude, and the DCIR of all variables is lower. The opposite holds when $\mu < \delta$. 