# Online Appendix for "The Elusive Pro-Competitive Effects of Trade"

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**Abstract**

This online appendix provides the proofs for various theoretical results (Section A) as well as additional information regarding the empirical estimation and the quantitative exercises in the main paper (Section B).

# A Proofs

## A.1 Section 2.1

**Additively Separable Utility.** We first establish that our demand system under $\beta = 0$ encompasses the case of additively separable utility functions considered in Krugman (1979). Using our notation, his model corresponds to a situation in which preferences are represented by a utility function, $U = \int_{\omega\in\Omega} u(q_\omega)d\omega$. The first-order conditions associated with utility maximization imply $u'(q_\omega) = \lambda p_\omega$, where $\lambda$ is the Lagrangian multiplier associated with the budget constraint. Inverting the first-order conditions implies

$$q_\omega = u'^{-1}(\lambda p_\omega), \tag{A.1}$$

together with the budget constraint,

$$\int_{\omega\in\Omega} p_\omega q_\omega d\omega = y. \tag{A.2}$$

Under $\beta = 0$, equations (2) and (3) in the main text are equivalent to equation ($A.2$) and $Q = 1$, respectively. In turn, equation (1) in the main text and $Q = 1$ imply $q_\omega = D(p_\omega/P)$. Thus, setting $P \equiv 1/\lambda$ and $D(\cdot) \equiv u'^{-1}(\cdot)$, we see that if utility functions are additively separable, then the associated demand must satisfy equations (1)-(3) in the main text.

When $\beta = 0$, one can further show that the converse also holds. That is, if the demand system satisfies equations (1)-(3) in the main text, then the utility function of the representative agent must be additively separable. To see this, note that since $D(\cdot)$ is strictly decreasing, equation (1) in the main text implies

$$p_\omega = PD^{-1}(q_\omega).$$

From the first-order conditions associated with utility maximization we know that

$$dU/dq_\omega = \lambda p_\omega.$$

The two previous expressions imply that for any pair of goods, $\omega_1$and $\omega_2$,

$$\frac{dU/dq_{\omega_1}}{dU/dq_{\omega_2}} = \frac{D^{-1}(q_{\omega_1})}{D^{-1}(q_{\omega_2})}.$$

Thus the Leontief-Sono condition for separability (Blackorby et al. (1978), p.53) is satisfied:

$$\frac{d}{dq_{\omega_3}}\left(\frac{dU/dq_{\omega_1}}{dU/dq_{\omega_2}}\right) = 0 \text{ for any } \omega_3 \neq \omega_1, \omega_2.$$

The fact that $U$ is additively separable, up to a monotonic transformation, then follows from the Representation Theorem 4.8 in Blackorby et al. (1978), p. 136.

**Kimball Preferences.** We now show that our demand system under $\beta = 1$ encompasses the case of Kimball preferences. Under Kimball preferences, utility $Q$ from consuming $\{q_\omega\}_{\omega\in\Omega}$ is implicitly given by

$$\int \Upsilon\left(\frac{q_\omega}{Q}\right) d\omega = 1, \tag{A.3}$$

for some function $\Upsilon$ that satisfies $\Upsilon' > 0$ and $\Upsilon'' < 0$. The utility maximization program of the consumer is to $\max_{Q,\{q_\omega\}} Q$ subject to equations ($A.3$) and ($A.2$). Let $\gamma$ and $\lambda$ denote the Lagrange multipliers associated with these two constraints. Manipulating the first-order conditions of this problem we get

$$q_\omega = Q\Upsilon'^{-1}\left(\frac{\lambda \int q_\omega \Upsilon'\left(\frac{q_\omega}{Q}\right) d\omega}{Q} p_\omega\right) \text{ for all } \omega. \tag{A.4}$$

The demand system under Kimball preferences is characterized by equations ($A.2$)-($A.4$). Under $\beta = 1$, equations (2) and (3) in the main text are equivalent to $\int_{\omega\in\Omega} H(p_\omega/P)\, d\omega = 1$ and equation ($A.2$), respectively. Thus, setting $P \equiv Q/\left(\lambda \int q_\omega \Upsilon'\left(\frac{q_\omega}{Q}\right) d\omega\right)$, $D(\cdot) \equiv \Upsilon'^{-1}(\cdot)$, and $H(\cdot) \equiv \Upsilon(D(\cdot))$, our demand system with $\beta = 1$ replicates the demand system under Kimball preferences.

**QMOR Expenditure.** Finally, we show that our demand system under $\beta = 1$ also encompasses the demand system corresponding to QMOR expenditure functions in Feenstra (2014). The QMOR demand system entails $q_\omega = QD(p_\omega/P)$ with

$$D(x) \equiv \begin{cases} \varsigma x^{r-1}\left[1 - x^{-r/2}\right] & if \quad x \leq 1 \\ 0 & if \quad x > 1 \end{cases}, \tag{A.5}$$

where $P$ acts as a choke price defined implicitly by

$$P = \left( \left( \frac{N}{N - \left(\widetilde{N} - \varsigma/\varrho\right)} \right)^{r/2} \int_{p_\omega \leq P} \frac{1}{N} p_\omega^{r/2} d\omega \right)^{2/r}, \tag{A.6}$$

and where $Q$ is determined such that the budget constraint ($A.2$) is satisfied.[A.1] In the previous expressions, $\varsigma$ and $\varrho$ are parameters, $\widetilde{N} \equiv \int_\Omega d\omega$ is the measure of all possible goods, $N \equiv \int_{p_\omega \leq P} d\omega$ is the measure of the set of goods with prices equal or below the choke price $P$. To proceed, note that equation ($A.6$) can be rearranged as

$$1 = \frac{1}{N - \left(\widetilde{N} - \varsigma/\varrho\right)} \int_{p_\omega \leq P} \left(\frac{p_\omega}{P}\right)^{r/2} d\omega. \tag{A.7}$$

To conclude, let us show that this is equivalent to equation (2) in the main text under $\beta = 1$ if one sets

$$H\left(\frac{p_\omega}{P}\right) \equiv \frac{1}{\varsigma\left(\varsigma/\varrho - \widetilde{N}\right)} \left(\frac{p_\omega}{P}\right)^{1-r/2} D\left(\frac{p_\omega}{P}\right).$$

Together with the definition of $D(\cdot)$ in equation (A.5), the previous definition implies

$$\int_\Omega H\left(\frac{p_\omega}{P}\right) d\omega = \frac{1}{\varsigma/\varrho - \widetilde{N}} \int_{p_\omega \leq P} \left[\left(\frac{p_\omega}{P}\right)^{r/2} - 1\right] d\omega.$$

Thus, as argued above, $\int_\Omega H\left(\frac{p_\omega}{P}\right) d\omega = 1$ is equivalent to equation ($A.7$).[A.2]

[A.1] Equations ($A.5$) and ($A.6$) are the counterparts of equations (7) and (2) in Feenstra (2014), respectively.

[A.2] Since the translog expenditure system is a special case of QMOR expenditure functions, as shown in Feenstra (2014), this establishes that our demand system encompasses the translog case. But it is useful to show directly that our demand system leads to translog demand if we set $D(x) \equiv \zeta x^{-1} \ln x^{-1}$ for $x \leq 1$ and $D(x) = 0$ otherwise, with $\zeta$ some positive constant, and $H(x) \equiv xD(x)$. Equation (2) in the main text with $\beta = 1$ then implies $\int_{p_\omega \leq P} \zeta \ln(p_\omega/P)^{-1} d\omega = 1$, which is equivalent to

$$\ln P = \frac{1}{\zeta N} + \frac{1}{N} \int_{p_\omega \leq P} \ln p_\omega d\omega,$$

which is the condition that determines $P$ in the translog demand; see equation (8) in Feenstra (2014). Equa-

**Homothetic Preferences.** In Section 2.1 we have also argued that if $D(\cdot)$ satisfies Assumption A1, then consumers have homothetic preferences if and only if $\beta = 1$. We now establish this result formally. Throughout this proof we will repeatedly use the fact that preferences are homothetic if and only if the income elasticity, $\partial \ln q_\omega(\boldsymbol{p}, y)/\partial \ln y$, is equal to one for all goods $\omega \in \Omega$.

Suppose first that $\beta = 1$. Then equation (2) in the main text implies $\int_{\omega\in\Omega} H(p_\omega/P)\, d\omega = 1$, so $P(\boldsymbol{p}, y)$ is independent of $y$. Differentiating equation (1) in the main text, we therefore get:

$$\frac{\partial \ln q_\omega(\boldsymbol{p}, y)}{\partial \ln y} = \frac{\partial \ln Q(\boldsymbol{p}, y)}{\partial \ln y}.$$

But Equation (3) in the main text implies $\frac{\partial \ln Q(\boldsymbol{p}, y)}{\partial \ln y} = 1$, hence the income elasticity is equal to one for all goods $\omega \in \Omega$, so preferences are homothetic.

Now suppose that $\beta = 0$. As established above, this requires additively separable utility functions. From Bergson (1936), we also know that such functions are homothetic only if they are CES. Since Assumption A1 rules out the CES case, we conclude that preferences cannot be homothetic if $\beta = 0$.

## A.2 Section 3.1

In Section 3.1 we have argued that more efficient firms charge higher markups, $\mu' > 0$, if and only if $\varepsilon_D' > 0$.

Suppose first that $\varepsilon_D' > 0$. Let $f(m, v) \equiv m - \frac{\varepsilon_D(m/v)}{\varepsilon_D(m/v)-1}$. Equation (5) in the main text entails $f(m, v) = 0$. Differentiating with respect to $m$ and $v$, we obtain

$$\begin{aligned}
\frac{\partial f(m,v)}{\partial m} &= 1 + \frac{\varepsilon_D'(m/v)}{(\varepsilon_D(m/v)-1)^2}\frac{1}{v} > 0, \\
\frac{\partial f(m,v)}{\partial v} &= -\frac{\varepsilon_D'(m/v)}{(\varepsilon_D(m/v)-1)^2}\frac{m}{v^2} < 0,
\end{aligned}$$

where the two inequalities derive from $\varepsilon_D' > 0$. By the Implicit Function Theorem, equation (5) therefore implies $\mu'(v) = -\left(\partial f(m,v)/\partial v\right) / \left(\partial f(m,v)/\partial m\right) > 0$.

Now suppose that $\mu' > 0$. We proceed by contradiction. If $\varepsilon_D' \leq 0$, then $\mu'(v) = -\left(\partial f(m,v)/\partial v\right) / \left(\partial f(m,v)/\partial m\right) > 0$ implies

$$1 + \frac{\varepsilon_D'(m/v)}{(\varepsilon_D(m/v)-1)^2}\frac{1}{v} < 0.$$

tion (3) in the main text with $\beta = 1$ is just the budget constraint, which given equation (2) in the main text immediately implies $Q = w/P$.

Using the fact that $m = \frac{\varepsilon_D(m/v)}{\varepsilon_D(m/v)-1}$, this can be rearranged as

$$\varepsilon_D(m/v)\left(\varepsilon_D(m/v)-1\right)+(m/v)\varepsilon_D'(m/v)<0.$$

By definition, $\varepsilon_D(x) = -xD'(x)/D(x)$, which implies

$$\varepsilon_D'(x) = -\frac{D''(x)x}{D(x)} - \frac{D'(x)}{D(x)} + \frac{\left(D'(x)\right)^2 x}{(D(x))^2}, \text{ for all } x.$$

Using this expression, we can rearrange the above inequality as

$$2\left(D'(m/v)\right)^2 - D(m/v)D''(m/v) < 0.$$

From the second-order condition of the firm's profit maximization problem, we know that

$$2(\partial q(p,Q,P)/\partial p) + (p-c)(\partial^2 q(p,Q,P)/\partial p^2) \leq 0$$

Together with the first-order condition, $(p-c)/p = -1/(\partial \ln q(p,Q,P)/\partial \ln p)$, this implies

$$2\left(\partial q(p,Q,P)/\partial p\right)^2 - q(p)(\partial^2 q(p,Q,P)/\partial p^2) \geq 0.$$

Using equation (4) in the main text, $m = p/c$, and $v = P/c$, we therefore have

$$2\left(D'\left(m/v\right)\right)^2 - D\left(m/v\right)D''\left(m/v\right) \geq 0,$$

a contradiction.

## A.3 Section 3.3

In Section 3.3, we have argued that once models with variable markups considered in this paper are calibrated to match the trade elasticity $\theta$ and the observed trade flows $\{X_{ij}\}$, they must predict the exact same changes in wages and trade flows for any change in variable trade costs as gravity models with CES utility, such as Krugman (1980), Eaton and Kortum (2002), Anderson and Van Wincoop (2003), and Eaton et al. (2011). We now establish this result formally.

Relative to ACR, their restriction R1 follows from equation (15), R2 from equation (10), and R3' from equation (16) in the main text. Combining these three conditions, we obtain

$$\begin{aligned} \lambda_{ij} &= \frac{N_i b_i^{\theta}\left(w_i \tau_{ij}\right)^{-\theta}}{\sum_k N_k b_k^{\theta}\left(w_k \tau_{kj}\right)^{-\theta}}, \\ w_i L_i &= \sum_j \lambda_{ij} w_j L_j, \end{aligned}$$

with $N_i$ invariant to changes in trade costs, as established in equation (12) in the main text. These are the same equilibrium conditions as in gravity models with CES utility in ACR. To show that counterfactual changes in wages and trade flows only depend on trade flows and expenditures in the initial equilibrium as well as the value of the trade elasticity, we can use the same argument as in the proof of Proposition 2 in ACR. Consider a counterfactual change in variable trade costs from $\tau \equiv \left\{\tau_{ij}\right\}$ to $\tau' \equiv \left\{\tau'_{ij}\right\}$. Let $\hat{x} \equiv x'/x$ denote the change in any variable $x$ between the initial and the counterfactual equilibrium. Since $N_i$ is fixed for all $i$, one can show that $\{\hat{w}_i\}_{i \neq j}$ are implicitly given by the solution of

$$\hat{w}_i = \sum_{j'=1}^{n} \frac{\lambda_{ij'} \hat{w}_{j'} Y_{j'} \left(\hat{w}_i \hat{\tau}_{ij'}\right)^{-\theta}}{Y_i \sum_{i'=1}^{n} \lambda_{i'j'} \left(\hat{w}_{i'} \hat{\tau}_{i'j'}\right)^{-\theta}}. \tag{A.8}$$

where $\hat{w}_j = 1$ by choice of numeraire. Given changes in wages, $\{\hat{w}_i\}$, changes in expenditure shares are then given by

$$\hat{\lambda}_{ij} = \frac{\left(\hat{w}_i \hat{\tau}_{ij}\right)^{-\theta}}{\sum_{i'=1}^{n} \lambda_{i'j} \left(\hat{w}_{i'} \hat{\tau}_{i'j}\right)^{-\theta}}. \tag{A.9}$$

Equations ($A.8$) and ($A.9$) imply $\{\hat{w}_i\}$ and $\{\hat{\lambda}_{ij}\}$ only depend on the value of trade flows and expenditures in the initial equilibrium as well as the trade elasticity. Once changes in expenditure shares, $\{\hat{\lambda}_{ij}\}$, are known, changes in bilateral trade flows can be computed using the identity, $\hat{X}_{ij} = \hat{\lambda}_{ij} \hat{w}_j$. Thus the same observation applies to changes in bilateral trade flows, which concludes the argument.

## A.4 Section 4.2

**Invariance of Distribution of Markups.** In Section 4.2, we have argued that if markups are an increasing function of firm-level productivity, then the univariate distribution of markups is independent of the level of trade costs. We now establish this result formally. Let $M_{ij}(m;\tau)$ denote the distribution of markups set by firms from country $i$ in country $j$ in a trade equilibrium if trade costs are equal to $\tau \equiv \left\{\tau_{ij}\right\}$. Since firm-level markups only depend on the relative efficiency of firms, we can express

$$M_{ij}(m;\tau) = \Pr\{\mu(v) \le m | v \ge 1\},$$

where the distribution of $v$ depends, in principle, on the identity of both the exporting and the importing country. Recall that $v \equiv P/c$ and $c = c_{ij}/z$. Thus for a firm with productivity $z$ located in $i$ and selling in $j$, we have $v = P_j z/c_{ij} = z/z^*_{ij}$. Combining this observation with Bayes' rule, we can rearrange the expression above as

$$M_{ij}(\mu;\tau) = \frac{\Pr\left\{\mu(z/z^*_{ij}) \le m, z^*_{ij} \le z\right\}}{\Pr\left\{z^*_{ij} \le z\right\}}.$$

Using Assumption A2 and the fact that $\mu(\cdot)$ is monotone, we can rearrange the previous expression as

$$M_{ij}(m;\tau) = \frac{\int_{z^*_{ij}}^{z^*_{ij}\mu^{-1}(m)} dG_i(z)}{\int_{z^*_{ij}}^{\infty} dG_i(z)} = 1 - \left(\mu^{-1}(m)\right)^{-\theta}.$$

Since the function $\mu(\cdot)$ is identical across countries and independent of $\tau$, by equation (5) in the main text, this establishes that for any exporter $i$ and any importer $j$, the distribution of markups $M_{ij}(\cdot;\tau)$ is independent of the identity of the exporter $i$, the identity of the importer $j$, and the level of trade costs $\tau$. As a result, the overall distribution of markups in any country $j$ is also invariant to changes in trade costs.

**Domestic Markups and Misallocation.** In Section 4.2, we have argued that changes in domestic markups, $\rho\lambda_{jj} d\ln P_j$, are proportional to the opposite of the covariance between firm-level markups on the domestic market and changes in firm-level employment shares for that market. We now establish this result formally.

Let us denote by $L_{jj}(z)$ the number of workers allocated by a firm with productivity $z$ in country $j$ to production of goods for market $j$. We must have

$$L_{jj}(z) = \tau_{jj} q_{jj}(z)/z,$$

where $q_{jj}(z)$ is such that

$$q_{jj}(z) = Q_j D\left(z^*_{jj}\mu(z/z^*_{jj})/z\right).$$

Similarly, let us denote by $\sigma_{jj}(z) \equiv L_{jj}(z)/L_{jj}$ denote the employment share that goes to a firm with productivity $z$. We have

$$\sigma_{jj}(z) = \frac{D\left(z_{jj}^{*}\mu(z/z_{jj}^{*})/z\right)/z}{\int_{z_{jj}^{*}}^{\infty} N_j D\left(z_{jj}^{*}\mu(z'/z_{jj}^{*})/z'\right)/z' dG_j(z')}.$$

Let us now compute the average of markups, $\bar{m}_{jj} \equiv \int_{z_{jj}^{*}}^{\infty} m_{jj}(z)\,\sigma_{jj}(z)\,N_j dG_j(z)$, for firms from country $j$ selling in country $j$ weighted by employment. We have:

$$\bar{m}_{jj} = \int_{z_{jj}^{*}}^{\infty} m_{jj}(z) \frac{D\left(z_{jj}^{*}m_{jj}(z)/z\right)/z}{\int_{z_{ij}^{*}}^{\infty} D\left(z_{jj}^{*}m_{jj}(z')/z'\right)/z' dG_j(z')} dG_j(z).$$

Under Assumption A2, we can rearrange the previous expression as

$$\bar{m}_{jj} = \int_{1}^{\infty} \mu(v) \frac{D(\mu(v)/v)\,v^{-\theta-2}dv}{\int_{1}^{\infty} D(\mu(v')/v')\,(v')^{-\theta-2}\,dv'}.$$

This implies

$$\frac{d\bar{m}_{jj}}{dz_{jj}^{*}} = \int_{z_{jj}^{*}}^{\infty} \frac{dm_{jj}(z)}{dz_{jj}^{*}} \sigma_{jj}(z)\,N_j dG_j(z) + \int_{z_{jj}^{*}}^{\infty} m_{jj}(z) \frac{d\sigma_{jj}(z)}{dz_{jj}^{*}} N_j dG_j(z) = 0,$$

where we have used the fact that $\sigma_{jj}\left(z_{jj}^{*}\right) = 0$. The first term can be rearranged as

$$\int_{z_{jj}^{*}}^{\infty} \frac{dm_{jj}(z)}{dz_{jj}^{*}} \sigma_{jj}(z)\,N_j dG_j(z) = -\frac{\rho\bar{m}_{jj}}{z_{jj}^{*}}.$$

By construction, $\int_{z_{jj}^{*}}^{\infty} \sigma_{jj}(z)\,N_j dG_j(z) = 1$. Using again $\sigma_{jj}\left(z_{jj}^{*}\right) = 0$, we therefore have $\int_{z_{jj}^{*}}^{\infty} \frac{d\sigma_{jj}(z)}{z_{jj}^{*}} N_j dG_j(z) = 0$. Thus the second term can be rearranged as

$$\int_{z_{jj}^{*}}^{\infty} m_{jj}(z) \frac{d\sigma_{jj}(z)}{dz_{jj}^{*}} N_j dG_j(z) = \int_{z_{jj}^{*}}^{\infty} \left(m_{jj}(z) - \bar{m}_{jj}\right) \left(\frac{d\sigma_{jj}(z)}{dz_{jj}^{*}} - 0\right) N_j dG_j(z),$$

Combining the three previous expressions we therefore get

$$\frac{\rho\bar{m}_{jj}}{z_{jj}^{*}} = \int_{z_{jj}^{*}}^{\infty} \left(m_{jj}(z) - \bar{m}_{jj}\right) \left(\frac{d\sigma_{jj}(z)}{dz_{jj}^{*}} - 0\right) N_j dG_j(z).$$

To conclude note that $z_{jj}^{*} = 1/P_j$, by our choice of numeraire. Thus the previous expression implies

$$\rho\lambda_{jj} d\ln P_j = -\left(\frac{\lambda_{jj}}{\bar{m}_{jj}}\right)\left(\int_{z_{jj}^*}^{\infty}\left(m_{jj}(z)-\bar{m}_{jj}\right)\left(d\sigma_{jj}(z)-0\right)N_j dG_j(z)\right),$$

where the integral on the right-hand side is equal to the covariance between firm-level markups on the domestic market and changes in firm-level employment shares for that market.

**Pro-Competitive Effects in Krugman (1979).** In Section 4.2, we have argued that, ceteris paribus, the pro-competitive effects in Krugman (1979) are positive if an increase in country size raises output per firm, and firms were producing too little before market integration, or it lowers output, and they were producing too much. We now establish this result formally.

Consider a closed economy with a measure $L$ of identical agents with additively separable preferences over a continuum of symmetric varieties,

$$U = Nu(q/L)$$

where $N$ is the measure of available varieties and $q$ is total output per variety. Let $c(q)$ denote the total labor cost of producing $q$ units of a given variety. In Krugman (1979), $c(q) = f + q$ if $q > 0$ and zero otherwise. Let $\pi(q, L)$ denote the profit of a representative firm given total output, $q$, and market size, $L$. In Krugman (1979), $\pi(q, L) = \frac{\epsilon_D(q/L)}{\epsilon_D(q/L)-1}c'(q)q - c(q)$, where $\epsilon_D(q/L) \equiv -\frac{u'(q/L)}{(q/L)u''(q/L)}$ denotes the elasticity of demand faced by each firm as a function of consumption per capita, $q/L$.[A.3]

To study the welfare implications of an increase in market size, it is convenient to focus on the following constrained planning problem:

$$V(L, W) = \max_{N,q} Nu(Wq/L)$$

subject to

$$Nc(q) = L, \tag{A.10}$$

$$\pi(q, L) = 0 \tag{A.11}$$

Equations (A.10) and (A.11) correspond to the resource constraint and the free entry condition, respectively. By construction, $(q, N)$ in the decentralized equilibrium is equal to the solution to the constrained planning problem for $W = 1$.

[A.3] By definition, we have $\epsilon_D(q/L) = \varepsilon_D(u'(q/L))$, where $\varepsilon_D$ is the elasticity of demand as function of price used in the main text.

We are interested in computing the percentage change in income, $d \ln W$, equivalent to a percentage change in market size, $d \ln L$, i.e.,

$$d \ln W = (\frac{L}{W} \frac{dV/dL}{dV/dW})_{W=1} d \ln L.$$

Let $q(L)$ denote the output level that solves equation (A.11). By the Envelope Theorem, we have

$$\frac{dV}{dL} = -\frac{NWq(L)u'(Wq(L)/L)}{L^2} + \lambda + q'(L)(\frac{NWu'(Wq(L)/L)}{L} - \lambda N c'(q(L)))$$
$$\frac{dV}{dW} = \frac{Nq(L)u'(Wq(L)/L)}{L},$$

where $\lambda$ is the Lagrange multiplier associated with equation (A.10). This leads to

$$d \ln W = \left( -1 + \frac{\left(\lambda + q'(L)\left(\frac{NWu'(Wq(L)/L)}{L} - \lambda N c'(q(L))\right)\right) L^2}{Nq(L)u'(q(L)/L)} \right) d \ln L. \tag{A.12}$$

The first-order condition with respect to $N$, evaluated at $W = 1$, further implies $u(q/L) = \lambda c(q)$. Together with the resource constraint, we therefore have $\lambda = Nu(q(L)/L)/L$. Substituting for the Lagrange multiplier, $\lambda$, in equation (A.12), we therefore obtain, after simplifications,

$$d \ln W = \left( \frac{1 - \epsilon_u}{\epsilon_u} + \epsilon_q \vartheta \right) d \ln L, \tag{A.13}$$

where $\epsilon_u(x) \equiv (\frac{d \ln u}{d \ln x})_{x=q(L)/L}$, $\epsilon_q \equiv \frac{d \ln q(L)}{d \ln L}$, and $\vartheta = \frac{u'(q(L)/L) - Nu(q(L)/L)c'(q(L))}{u'(q(L)/L)}$ captures the wedge between the marginal benefit of increasing output per variety, $(N/L)u'(q(L)/L)$, and its marginal cost, $\lambda c'(q(L)) = (N^2/L)u(q(L)/L)c'(q(L))$.

In the case with constant markups and CES utility considered by Krugman (1980), the decentralized equilibrium is efficient, $\vartheta = 0$. Thus gains from market integration only reflects gains from new varieties, as captured by $(1 - \epsilon_u)/\epsilon_u$.[A.4] Accordingly, we can express the pro-competitive effects from trade, defined as the differential impact of trade liberalization on welfare when markups vary and when they do not, as $\Delta(\frac{1-\epsilon_u}{\epsilon_u}) + \epsilon_q \vartheta$, where $\Delta(\frac{1-\epsilon_u}{\epsilon_u})$ denotes the difference between the welfare gains from new varieties in models with and without variable markups (a difference that depends, in general, on which moments one chooses to hold fix when comparing these models). For a given value of $\Delta(\frac{1-\epsilon_u}{\epsilon_u})$, the previous analysis establishes that welfare gains from market integration will be higher if an increase in market size raises output per firm, $\epsilon_q > 0$, and firms were producing too little

[A.4] In the CES case, one can also check that $\epsilon_u = \frac{\epsilon_D - 1}{\epsilon_D}$. Thus, the gains from market integration can be rearranged in a familiar way as $d \ln W = d \ln L/(\epsilon_D - 1)$.

before market integration, $\vartheta > 0$, or it lowers output,$\epsilon_q < 0$, and they were producing too much, $\vartheta < 0$.

## A.5 Section 4.3

In the multi-sector case, and ignoring for now the country sub-index, the expenditure minimization problem of the representative consumer is given by

$$e(\boldsymbol{p}, U) \equiv \min_{\boldsymbol{q}} \sum_k \int_{\Omega^k} p^k(\omega) q^k(\omega) d\omega$$
$$\text{s.t.} \quad U(C^1(\boldsymbol{q}^1), ..., C^K(\boldsymbol{q}^K)) \geq U.$$

Since preferences are weakly separable, the solution to the previous problem can be computed in two stages. At the lower stage, the optimal consumption of varieties within each sector solves

$$e^k(\boldsymbol{p}^{\boldsymbol{k}}, C^k) \equiv \min_{\boldsymbol{q}^k} \int_{\Omega^k} p^k(\omega) q^k(\omega) d\omega$$
$$\text{s.t.} \quad C^k(\boldsymbol{q}^k) \geq C^k.$$

At the upper stage, the optimal level of consumption between sectors solves

$$e(\boldsymbol{p}, U) \equiv \min_{C^1,...,C^K} \sum_k e^k(\boldsymbol{p}^{\boldsymbol{k}}, C^k)$$
$$\text{s.t.} \quad U(C^1, ..., C^K) \geq U.$$

We are interested in $d \ln W = d \ln y - d \ln e$, with $y$ being per-capita income. By Shephard's lemma, we know that a foreign shock implies that

$$d \ln e = \sum_k s^k d \ln e^k. \tag{A.15}$$

To compute $d \ln y$ and $d \ln e^k$, we consider separately the cases of restricted and free entry.

**Restricted entry.** Under restricted entry equation (17) in the main text remains valid at the sector level. So we can use the exact same approach as in the one-sector case to derive

$$d \ln e_j^k = \left(1 - \rho^k\right) \sum_i \lambda_{ij}^k d \ln c_{ij}^k + \rho^k d \ln P_j^k. \tag{A.16}$$

To compute $d \ln P_j^k$, we use the sector-level counterpart of equations (22)-(23) in the main text, which imply

$$\begin{aligned}\kappa^k \left(Q_j^k\right)^{1-\beta^k} \left(P_j^k\right)^{\theta^k+1-\beta^k} \left(\sum_i N_i^k \left(b_i^k\right)^{\theta^k} \left(c_{ij}^k\right)^{-\theta^k}\right) &= \left(y_j^k\right)^{1-\beta^k}, \\ \left(\chi^k\right)^{\beta^k} Q_j^k \left(P_j^k\right)^{\beta^k\left(1+\theta^k\right)} \left(\sum_i N_i^k \left(b_i^k\right)^{\theta^k} \left(c_{ij}^k\right)^{-\theta^k}\right)^{\beta^k} &= \left(y_j^k\right)^{\beta^k},\end{aligned}$$

with

$$\begin{aligned}\kappa^k &\equiv \theta^k \int_1^\infty \left[H^k\left(\mu^k(v)/v\right)\right]^{\beta^k} \left[\left(\mu^k(v)/v\right) D^k(\mu^k(v)/v)\right]^{1-\beta^k} v^{-1-\theta^k} dv, \\ \chi^k &\equiv \theta^k \int_1^\infty \left(\mu^k(v)/v\right) D^k(\mu^k(v)/v) v^{-\theta^k-1} dv.\end{aligned}$$

From the two previous equations, we obtain

$$P_j^k = \left(\frac{\kappa^k \sum_i N_i^k \left(b_i^k\right)^{\theta^k} \left(c_{ij}^k\right)^{-\theta^k}}{\left(y_j^k\right)^{1-\beta^k}}\right)^{-1/\left(\theta^k+1-\beta^k\right)}, \tag{A.17}$$

and in turn, under restricted entry,

$$d\ln P_j^k = \frac{\theta^k}{\theta^k+1-\beta^k} \sum_i \lambda_{ij}^k d\ln c_{ij}^k + \frac{1-\beta^k}{\theta^k+1-\beta^k} d\ln y_j^k.$$

Together with equations (A.15) and (A.16), the previous expression yields

$$d\ln e_j = \sum_{i,k} s_j^k \lambda_{ij}^k \left(1-\eta^k\right) d\ln c_{ij}^k + \sum_k s_j^k \eta^k d\ln y_j^k,$$

with $\eta^k \equiv \rho^k \left((1-\beta^k)/(1-\beta^k+\theta^k)\right)$. Using the fact that $y_j^k = s_j^k y_j$, we can rearrange the second term on the right-hand side as

$$\sum_k s_j^k \eta^k \left(d\ln s_j^k + d\ln y_j\right) = \sum_k \eta^k ds_j^k + \eta_j d\ln y_j,$$

with $\eta_j \equiv \sum_k s_j^k \eta^k$. Since $d\ln W_j = d\ln y_j - d\ln e_j$, we get

$$d\ln W_j = \left(1-\eta_j\right) d\ln y_j - \sum_{i,k} \left(1-\eta^k\right) s_j^k \lambda_{ij}^k d\ln c_{ij}^k - \sum_k \eta^k ds_j^k. \tag{A.18}$$

Proceeding as in the one sector case, one can show that $\sum_i \lambda_{ij}^k d\ln c_{ij}^k$ is equal to $d\ln\lambda_{jj}^k/\theta^k$. To establish equation (30) in the main text, we therefore only need to solve for $d\ln y_j$. Under restricted entry, per-capita income in country $j$ is given by $y_j = 1 + \sum_{i,k}\Pi_{ji}^k/L_j$, where we have set $w_j = 1$ by choice of numeraire. As in the one-sector case, sector-level profits are such that $\Pi_{ji}^k = \zeta^k X_{ji}^k$, with

$$
\begin{aligned}
&\zeta^k \equiv \pi^k/\chi^k, \\
&\pi^k \equiv \theta^k \int_1^\infty \left(\mu^k(v) - 1\right) D^k(\mu^k(v)/v) v^{-\theta^k - 2} dv > 0.
\end{aligned}
$$

As in the one-sector case, under restricted entry and with $w_j = 1$, sector-level employment is such that $L_j^k = (1-\zeta^k)(\sum_i X_{ji}^k)$. Combining the previous observations, we obtain

$$
d\ln y_j = d\ln(\sum_k L_j^k/(1-\zeta^k)).
$$

Plugging into (A.18), we obtain equation (30) in the main text. Proposition 2 derives from this expression and the joint observation that $\eta^k = \eta$ for all $k$ implies $\sum_k \eta^k ds_j^k = \eta\sum_k ds_j^k = 0$ whereas $\zeta^k = \zeta$ for all $k$ implies $d\ln y_j = d\ln L_j/(1-\zeta) = 0$.

**Free Entry.** Under free entry, equation (17) in the main text is no longer valid since we may have $d\ln N_i^k \neq 0$ for some $i$ and $k$. To capture the welfare implications of the previous changes, we restrict ourselves to the three examples of demand functions discussed in Section 2.1: $(i)$ additively separable utility functions; $(ii)$ quadratic mean of order $r$ (QMOR) expenditure functions; and $(iii)$ Kimball preferences.

We first consider the case of additively separable utility functions and Kimball preferences. Under both cases, using Assumption A2, we can write the sector-level expenditure function as

$$
\begin{aligned}
e_j^k &= \min_{\boldsymbol{q}_j^k} \sum_i \int_{b_i^k}^\infty p_{ij}^k(z) q_{ij}^k(z) \theta^k \left(b_i^k\right)^{\theta^k} N_i^k z^{-\theta^k - 1} dz \\
s.t. &\sum_i \int_{b_i^k}^\infty \Psi_j^k\left(q_{ij}^k(z)/\left(C_j^k\right)^{\beta^k}\right) \theta^k \left(b_i^k\right)^{\theta^k} N_i^k z^{-\theta^k-1} dz \geq \left(C_j^k\right)^{1-\beta^k},
\end{aligned}
$$

where $p_{ij}^k(z)$ is the price in country $j$ of a variety with productivity $z$ in sector $k$ produced in country $i$ and $q_{ij}^k(z)$ is the corresponding quantity. In the case of additively separable utility functions, we have $\beta_j^k = 0$ and the function $\Psi_j^k$ is country $j's$ sub-utility function $u_j^k$, while in the case of Kimball preferences we have $\beta_j^k = 1$ and the function $\Psi_j^k$ is the sector-level counterpart of the function $\Upsilon$ in Appendix A.1. Using the change of variable $\tilde{z} = N_i^k(b_i^k/z)^{\theta^k}$

and letting $\tilde{N}_i^k \equiv N_i^k \left(b_i^k\right)^{\theta^k}$, we now have

$$e_j^k = \min_{q_j^k} \sum_i \int_0^{\tilde{N}_i^k \left(b_i^k\right)^{-\theta^k}} p_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) q_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) d\tilde{z}$$

$$s.t. \sum_i \int_0^{\tilde{N}_i^k \left(b_i^k\right)^{-\theta^k}} \Psi_j^k \left(q_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) / \left(C_j^k\right)^{\beta^k}\right) d\tilde{z} \geq \left(C_j^k\right)^{1-\beta^k}.$$

Applying the Envelope Theorem and using the fact that demand is zero for the least productive firm, we get

$$d \ln e_j^k = \sum_i \int_0^{(\tilde{z}_{ij}^k)^*} \lambda_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) d \ln p_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) d\tilde{z}, \tag{A.19}$$

where $(\tilde{z}_{ij}^k)^* = \tilde{N}_i^k \left((z_{ij}^k)^*\right)^{-\theta^k}$ is the (rank) productivity cut-off; $\lambda_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right)$ is the expenditure share,

$$\lambda_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) = x^k \left(c_{ij}^k \left(\tilde{N}_i^k/\tilde{z}\right)^{-1/\theta^k}, \left((\tilde{z}_{ij}^k)^*/\tilde{z}\right)^{1/\theta^k}, Q_j^k, L_j^k\right) / y_j^k;$$

and $d \ln p_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right)$ is the total derivative of the log price, including both the change in the price schedule conditional on productivity and the change in the normalized measure of entrants, $\tilde{N}_i^k$. To compute the latter, note that $p_{ij}^k(z) = (c_{ij}^k/z)\mu^k(z/z_{ij}^{k*})$, which implies

$$p_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) = c_{ij}^k \left(\tilde{N}_i^k/\tilde{z}\right)^{-1/\theta^k} \mu^k \left(\left((\tilde{z}_{ij}^k)^*/\tilde{z}\right)^{1/\theta^k}\right),$$

and, in turn,

$$d \ln p_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) = d \ln \left(c_{ij}^k \left(\tilde{N}_i^k\right)^{-1/\theta_k}\right) + \rho^k \left(\left((\tilde{z}_{ij}^k)^*/\tilde{z}\right)^{1/\theta^k}\right) d \ln \left((\tilde{z}_{ij}^k)^*\right)^{1/\theta^k},$$

with $\rho^k(z) \equiv d \ln \mu^k(z)/d \ln z$. Noting that $\int_0^{(\tilde{z}_{ij}^k)^*} \lambda_{ij}^k \left(\left(\tilde{N}_i^k/\tilde{z}\right)^{1/\theta^k}\right) d\tilde{z} = \lambda_{ij}^k$ and substituting into equation (A.19), we get

$$d \ln e_j^k = \sum_i \lambda_{ij}^k d \ln \left(c_{ij}^k \left(\tilde{N}_i^k\right)^{-1/\theta_k}\right) + \sum_i \rho^k \lambda_{ij}^k d \ln \left((\tilde{z}_{ij}^k)^*\right)^{1/\theta^k}, \tag{A.20}$$

with

$$\rho^k = \int_0^{(\tilde{z}_{ij}^k)^*} \rho^k \left( \left( (\tilde{z}_{ij}^k)^* / \tilde{z} \right)^{1/\theta^k} \right) \frac{\lambda_{ij}^k \left( \left( \tilde{N}_i^k / \tilde{z} \right)^{1/\theta^k} \right)}{\lambda_{ij}^k} d\tilde{z}.$$

Note that in line with equation (20) in Section 4.1, a simple change of variable, $v = \left( (\tilde{z}_{ij}^k)^* / \tilde{z} \right)^{1/\theta^k}$, implies

$$\rho^k = \int_1^\infty \frac{d \ln \mu^k(v)}{d \ln v} \frac{\left( \mu^k(v)/v \right) D^k(\mu^k(v)/v) v^{-1-\theta^k}}{\int_1^\infty \left( \mu^k(v')/v' \right) D^k \left( \mu^k(v')/v' \right) (v')^{-1-\theta^k} dv'} dv.$$

Since $(\tilde{z}_{ij}^k)^* = \tilde{N}_i^k \left( z_{ij}^{k*} \right)^{-\theta^k}$ and $z_{ij}^{k*} = c_{ij}^k / P_j^k$, equation (A.20) further implies

$$d \ln e_j^k = \sum_i \left( 1 - \rho^k \right) \lambda_{ij}^k d \ln \left( c_{ij}^k \left( \tilde{N}_i^k \right)^{-1/\theta^k} \right) + \sum_i \rho^k \lambda_{ij}^k d \ln P_j^k.$$

To compute $d \ln P_j^k$, we can start from equation (A.17), which remains valid under free entry. Log-differentiation yields

$$d \ln P_j^k = \frac{\theta^k}{\theta^k + 1 - \beta^k} \sum_i \lambda_{ij}^k d \ln \left( c_{ij}^k \left( \tilde{N}_i^k \right)^{-1/\theta^k} \right) + \frac{1 - \beta^k}{\theta^k + 1 - \beta^k} d \ln y_j^k. \tag{A.21}$$

Combining the two previous expressions, we obtain

$$d \ln e_j^k = \sum_i \left( 1 - \eta^k \right) \lambda_{ij}^k d \ln \left( c_{ij}^k \left( \tilde{N}_i^k \right)^{-1/\theta^k} \right) + \eta^k d \ln y_j^k. \tag{A.22}$$

Combined with equations (A.15) and (A.22), we then have

$$d \ln W_j = d \ln y_j - \sum_{i,k} s_j^k \left( 1 - \eta^k \right) \lambda_{ij}^k d \ln \left( c_{ij}^k \left( \tilde{N}_i^k \right)^{-1/\theta^k} \right) - \sum_k s_j^k \eta^k d \ln y_j^k.$$

Under free entry, we know that $y_j = 1$, where we have again set $w_j = 1$ by choice of numeraire. This immediately implies $d \ln y_j = 0$. Given that $y_j^k = s_j^k y_j$, this further implies that $\sum_k s_j^k \eta^k d \ln y_j^k = \sum_k \eta^k d s_j^k$, and hence

$$d \ln W_j = - \sum_{i,k} s_j^k \left( 1 - \eta^k \right) \lambda_{ij}^k d \ln \left( c_{ij}^k \left( \tilde{N}_i^k \right)^{-1/\theta^k} \right) - \sum_k \eta^k d s_j^k. \tag{A.23}$$

To conclude, note that sector-level trade flows still satisfy gravity,

$$\lambda_{ij}^k = \frac{\tilde{N}_i^k \left(c_{ij}^k\right)^{-\theta^k}}{\sum_l \tilde{N}_l^k \left(c_{lj}^k\right)^{-\theta^k}},$$

which implies

$$\sum_i \lambda_{ij}^k d\ln\left(\tilde{N}_i^k \left(c_{ij}^k\right)^{-\theta^k}\right) = d\ln \tilde{N}_j^k - d\ln \lambda_{jj}^k. \tag{A.24}$$

Combining this result with equation (A.23) and noting that $N_j^k = \zeta^k (L_j^k / F_j^k)$ implies $d\ln \tilde{N}_j^k = d\ln N_j^k = d\ln L_j^k$, we get

$$d\ln W_j = -\sum_k s_j^k \left(1-\eta^k\right)\left(d\ln \lambda_{jj}^k - d\ln L_j^k\right)/\theta^k - \sum_k \eta^k ds_j^k.$$

If $\eta^k = \eta$ for all $k$, this simplifies into equation (31) in the main text.

Finally, consider the case of the QMOR expenditure functions analyzed by Feenstra (2014). Lemma 1 in Feenstra (2014) and the fact that the Herfindahl index is constant when productivity is distributed Pareto together imply that (in our notation) $d\ln e_j^k = d\ln P_j^k$. Combining this observation with equations (A.21) and (A.24), which remain valid in this case, and using the fact that $\beta^k = 1$ and $\eta = 0$ for this case, we again obtain equation (31) from the main text.

# B Estimation and quantitative exercises

## B.1 Section 5.1

This section describes a number of details behind the procedure used to estimate $\eta$ from micro trade data that was described in Section 5.1.

**From theory to data.** We aim to estimate a parametric demand system that satisfies equations (1)-(3) in the main text. Our choice of a particular parameterization is motivated by parsimony, as well as the two following considerations. First, we want to nest the case of CES demand because of its prominence in prior work and because it provides a reference point in which markups will be constant under monopolistic competition. And second, we want to allow the average elasticity of markups—and hence $\eta$—to be positive or negative, so that data can speak to whether the existence of variable markups increases or decreases the gains from trade liberalization. In order to achieve these goals, we restrict attention to additively separable preferences in the "Pollak family"; see Pollak (1971) and Mrazova and Neary (2016a). This implies the following parametric restriction on $D(\cdot)$:

$$D(p_\omega/P) = (p_\omega/P)^{1/\gamma} - \alpha,$$

where $\alpha$ and $\gamma$ are the two structural parameters to be estimated.[B.1] In turn, the parameter $\beta$ in equations (2) and (3) in the main text is equal to 0 if $\alpha \neq 0$ and to either 0 or 1 if $\alpha = 0$. Assumption A1 is only satisfied if $\alpha > 0$ but we do not impose this restriction on the estimation.

When $\alpha = 0$, the previous demand system reduces to the CES case, with elasticity of substitution given by $-1/\gamma$. In this case, trade liberalization has no effects on markups and $\eta = 0$. In contrast, when $\alpha > 0$, the demand elasticity is decreasing with the level of consumption, and hence increasing with the level of prices, $\varepsilon'_D > 0$, which implies $\rho > 0$ and $\eta = \rho/(1+\theta) > 0$. Finally, when $\alpha < 0$, the opposite happens, and hence $\varepsilon'_D < 0$ and $\rho < 0$.

Our estimation of this demand system draws on detailed data on bilateral U.S. merchandise imports within narrowly defined product codes to estimate the representative U.S. consumer's demand parameters. In particular, we use annual data (from 1989-2009) at the 10-digit HS level.[B.2] In mapping these data to our model we assume that a variety $\omega$ in the model corresponds to a particular 10-digit HS product, indexed by $g$, from a particular exporting country, indexed by $i$; that is, a "variety" $\omega$ in the model is a "product-country" pair $gi$ in the data.[B.3] There are 13,746 unique products and 242 unique exporters. Because the demand system in equation (4) in the main text is intended to represent demand for varieties within a differentiated sector, we assume that a "sector", which we index by $k$, in the data is a level of product aggregation that is higher than the 10-digit level and in practice take this to be the 4-digit HS category (of which there are 1387) level . In what follows, we

[B.1] Simonovska (2015) uses the log-version of this demand system to analyze the relationship between income and prices across countries.

[B.2] We download this dataset from Peter Schott's homepage and use the concordances provided in Pierce and Schott (2009) to adjust for changes in 10-digit HS codes over this time period. The July 2015 version of this paper reported results from an earlier dataset spanning 1989-2005 only.

[B.3] While this practice is standard in the literature (e.g. Broda and Weinstein 2006), we note that the issue of "hidden varieties" is more problematic here than in the CES case. Under the assumption of CES demand, the fact that an unobserved number of firms from the same country may be producing a particular 10-digit HS product simply acts as an unobserved quality shifter. This is no longer true if $\alpha \neq 0$. We are unaware of a study that documents the extent of firm-level concentration at the country-HS10-digit level for US imports. But Feenstra and Weinstein (2017) estimate that for US imports in 1998 (the closest among their tabulated years to the mid-point of our sample) the trade-weighted average of the Herfindahl index within exporter-HS 4-digit product groups was 0.190. This would imply, for equally sized firms, about five firms per exporting country within each 4-digit industry. By comparison, on average there are approximately ten 10-digit HS products within each 4-digit group. There is therefore ample scope for the possibility that most exporter-HS10-digit product cells are served by only one firm.

let the price aggregator $P_t^k$ vary across sectors and over time, but restrict (in our baseline analysis) the demand parameters $\alpha$ and $\gamma$ to be common across all sectors.

We focus on the following empirical demand equation:

$$q_{git}^k = \left(\varepsilon_{git}^k p_{git}^k / P_t^k\right)^{1/\gamma} - \alpha, \tag{B.1}$$

where $p_{git}^k$ is the price paid by U.S. importers when buying quantity $q_{git}^k$ for a product $g$ in sector $k$ from an exporting country $i$ in year $t$. The import data contain measures of total (that is, aggregated across all importers) expenditure, i.e., the empirical analogue of $q_{git}^k \times p_{git}^k$, and measures of total quantities purchased, which we take as our measure of $q_{git}^k$. To construct a measure of prices $p_{git}^k$ we therefore simply use the ratio of expenditure to quantity. The variety-specific demand shifter, $\varepsilon_{git}^k$, captures the fact that physical units in the data may differ from the choice of units in Section 2, under which all varieties are implicitly assumed to enter utility in a symmetric fashion. Such differences in units of account can be interpreted as unobserved quality differences; see e.g. Baldwin and Harrigan (2011).

**Estimation procedure.** There are two key challenges involved in estimating equation (*B*.1): (*i*) the price aggregator $P_t^k$ is unobserved and correlated with $p_{git}^k$; and (*ii*) the demand shifter $\varepsilon_{git}^k$ is unobserved and correlated with $p_{git}^k$. We describe below, in turn, a procedure to estimate the demand parameters, $\alpha$ and $\gamma$, that overcomes these challenges.

First, consider the problem that the price aggregator $P_t^k$ is unobserved and correlated with $p_{git}^k$. The key restriction imposed in equation (*B*.1), however, is that the demand for all varieties depends symmetrically on this aggregator; that is, the price aggregator does not vary across products $g$ and exporters $i$ within sector $k$. This suggests that identification of the demand parameters, $\alpha$ and $\gamma$, can be achieved through a differencing procedure designed to eliminate the unobserved and endogenous $P_t^k$ term in equation (*B*.1). Specifically, inverting our demand function and taking logs, we have

$$\ln p_{git}^k = \gamma \ln(q_{git}^k + \alpha) - \ln P_t^k + \ln \varepsilon_{git}^k.$$

Taking differences with respect to one reference product-country within the same sector $k$, we then obtain

$$\Delta_{gi} \ln p_{git}^k = \gamma \Delta_{gi} \ln(q_{git}^k + \alpha) + \Delta_{gi} \ln \varepsilon_{git}^k, \tag{B.2}$$

where $\Delta_{gi}$ denotes the corresponding difference operator. While in principle the difference $\Delta_{gi}$ could be taken across any two product-country $gi$ observations within a sector-year $kt$, we use the convention of mean differencing such that, for any variable $Z$, $\Delta_{gi} Z_{git}^k = Z_{git}^k -$

$\frac{1}{M_{kt}} \sum_{gi \in \mathcal{I}_{kt}} Z^k_{git}$ where $\mathcal{I}_{kt}$ is the set of product-country pairs $gi$ in sector $k$ and year $t$ and $M_{kt}$ is the number of observations in this set.

Second, consider the problem posed by the correlation between $p^k_{git}$ and the unobserved demand-shifter, $\varepsilon^k_{git}$. We first follow the literature on demand system estimation using international trade data—e.g. Broda and Weinstein (2006) and Feenstra and Weinstein (2017)—and decompose this demand-shifter into two terms:

$$\ln \varepsilon^k_{git} = \ln \delta^k_{gi} + \ln \epsilon^k_{git}.$$

In this decomposition, the first term, $\ln \delta^k_{gi}$, reflects systematic differences in quality or units of account across products from different countries within a sector, whereas the second term, $\ln \epsilon^k_{gct}$, reflects idiosyncratic determinants of demand that are free to vary over time. To eliminate systematic unobserved differences in quality, we take a second difference of equation (*B*.2), now across time periods, to obtain

$$\Delta_t \Delta_{gi} \ln p^k_{git} = \gamma \Delta_t \Delta_{gi} \ln(q^k_{git} + \alpha) + \Delta_t \Delta_{gi} \ln \epsilon^k_{git}, \tag{B.3}$$

where $\Delta_t$ denotes the corresponding difference operator. Again, while the difference $\Delta_t$ could be taken across any two time periods we use mean differencing, as in $\Delta_{gi}$ defined above. While this double-differencing procedure will remove cross-sectional sources of bias due to unobserved quality shifters, endogeneity bias concerns due to potentially time-varying quality shifters (or measurement error in prices) remain. A natural solution is to use an instrumental variable (IV) approach, where here the instrument must be exogenous with respect to the error term $\Delta_t \Delta_{gc} \ln \epsilon^k_{git}$ and must be correlated with the endogenous variable, i.e. the double-demeaned quantity $\Delta_t \Delta_{gi} \ln(q^k_{git} + \alpha)$, for any value of $\alpha$. In our model a natural candidate for such an instrument is trade costs. For this purpose we use the (log of one plus the) value of tariff duties charged, expressed as a percentage of import value, as a measure of trade costs; this variable is reported in the US 10-digit HS imports data. This procedure of using trade costs as exogenous demand shifters in an international trade setting is commonly employed in the empirical gravity literature; see e.g. Head and Mayer (2014).

Since the estimating equation (*B*.3) is linear in $\gamma$, but non-linear in $\alpha$, we separate our estimation procedure into an inner-loop and an outer-loop. In the inner-loop, we take the value of $\alpha$ as given and compute $\hat{\gamma}(\alpha)$ as the IV estimator of $\gamma$ with $\Delta_t \Delta_{gi} \ln(t^k_{git} + \alpha)$ the instrumental variable for $\Delta_t \Delta_{gi} \ln(q^k_{git} + \alpha)$, where $t^k_{git}$ denotes the tariff rate charged by the United States on imports of product $g$ in sector $k$ from country $i$ in year $t$. In the outer-loop, we then search for the value of $\alpha$ that minimizes the sum of the squared residuals across

| | $\gamma$ | $\alpha$ |
|---|---|---|
| Panel A: CES demand | | |
| | $-0.206^{***}$ | |
| | $(0.036)$ | |
| Panel B: Generalized CES demand | | |
| | $-0.347^{***}$ | $3.053^{***}$ |
| | $[-0.373, -0.312]$ | $[0.633, 9.940]$ |

Table 2: Demand Estimates. Panel A reports IV estimates of equation (B.3) with $\alpha = 0$ and standard errors clustered at the exporter level. Panel B reports IV estimates of equation (B.3) without restrictions and with 95 percent confidence intervals from a block-bootstrap procedure, with blocks at the exporter level. The number of observations in both panels is 3,563,993. *** indicates p<0.05.

all linear IV regressions, and denote this value $\hat{\alpha}$.[B.4] Our estimator of $\gamma$ is finally given by $\hat{\gamma} = \hat{\gamma}(\hat{\alpha})$.

**Demand estimation and welfare implications.** We begin by estimating the demand system in equation ($B.3$) under the restriction that $\alpha = 0$. This reduces equation ($B.3$) to the CES case, in which the estimating equation is linear. Our results are reported in Panel A of Table 2. In this restricted (CES) case, our IV estimate is $\widehat{\gamma} = -0.206$ with a standard error—clustered at the exporting country level to account for serial correlation over time and across products within exporters—that implies that the point estimate is statistically significantly different from zero at the 95% confidence level. This finding corresponds to an elasticity of substitution equal to $1/\widehat{\gamma} = -4.854$, which is in line with typical estimates of the CES demand parameter in international trade settings. This suggests that our particular instrumental variable, based on the reported value of tariff duties charged, isolates exogenous variation in trade costs that is similar to that used in the literature. Reassuringly, the F-statistic (again adjusted for clustering at the exporter level) on the instrumental variable in the first-stage is 27.28, implying that finite-sample bias due to a weak instrument is unlikely to be a first-order concern here.

We then estimate equation ($B.3$) without any restriction on $\alpha$—this corresponds to estimating unrestricted Pollak (rather than CES) demand. These results are reported in Panel B of Table 2. Our non-linear IV estimate of equation ($B.1$) results in estimates of $\widehat{\gamma} = -0.347$ and $\widehat{\alpha} = 3.053$, with 95% confidence intervals, block-bootstrapped at the exporting country level, with 200 bootstrap replications, shown in parentheses in the table. Notably, this

[B.4] In practice we conduct a grid search over $\alpha$ subject to the restriction that $q^k_{git} + \alpha$ must be strictly positive for $\ln(q^k_{git} + \alpha)$ to be well-defined. Namely, we require $\alpha$ to be greater than minus the lowest value of $q^k_{git}$ in our dataset, which is equal to 1 in all years. After first verifying with a coarse grid that the best-fitting value of $\alpha$ lies below 10, we consider a grid of 400 evenly-spaced values between $-1$ and 10.

estimate of $\alpha$ has a 95% confidence interval that excludes zero, suggesting that the departure from CES that is modeled in equation ($B.1$) is a statistically significant feature of these data.[B.5] Furthermore, $\hat{\alpha}$ is positive. As argued above, this implies that $\eta$ must be positive as well. So, regardless of the value of other structural parameters, Proposition 1 establishes that there cannot be any pro-competitive effect of trade in the sense that welfare gains from trade liberalization must be lower than those predicted by a model with constant markups.

As discussed in Section 5.1, the demand parameter estimates reported in Table 2, Panel B imply that $\hat{\rho} = 0.36$ and in turn $\hat{\eta} = \hat{\rho}/(1+\theta) = 0.06$.[B.6]

## B.2 Section 6.3

All models that we consider are calibrated so that the trade elasticity for a 1% change in trade costs is equal to 5 in the initial equilibrium. Except when the distribution of productivity is Pareto, however, this elasticity will vary with the level of trade costs. Figure 6 plots the trade elasticity as a function of trade costs in the case of Pareto, log-normal and bounded Pareto distributions. In both the log-normal and bounded Pareto cases, we see that the trade elasticity increases, in absolute value, with the level of trade costs, as noted in Section 6.3.

[B.5] We have also explored this by HS "section", the coarsest level of disaggregation for which the HS system is designed. Across 22 such sections (two of which we do not include since they do not have the required tariff variation), the median estimates are $\widehat{\gamma} = -0.321$ $[-0.358, -0.210]$ and $\widehat{\alpha} = 0.898$ $[-0.999, 20]$, the 25th percentile estimates are $\widehat{\gamma} = -0.372$ $[-0.530, -0.211]$ and $\widehat{\alpha} = -0.729$ $[-0.999, -0.143]$, and the 75th percentile estimates are $\widehat{\gamma} = -0.200$ $[-0.326, -0.168]$ and $\widehat{\alpha} = 6.153$ $[1.490, 23.898]$. For two sections the estimates fail to reject the null hypothesis of $\gamma = 0$, whereas for six sections the estimates reject the null of $\alpha = 0$ (two of which have a point estimate in the $\alpha < 0$ region). Because of the imprecision of many of these estimates, and in line with the theoretical analysis of Section 4.3, we abstract from misallocations associated with heterogeneity in the values of $\alpha$, $\gamma$, and, in turn, $\eta$ across sectors.

[B.6] Since we focus on non-zero trade flows, one may be concerned that the previous estimates are subject to selection bias. To explore the potential importance of the previous concern, we have rerun our baseline estimation on a subsample that only includes bilateral trade flow observations at or above the 15th percentile value. We find (with 95% confidence intervals given in brackets) $\widehat{\gamma} = -0.287$ $[-0.304, -0.236]$ and $\widehat{\alpha} = 6.212$ $[1.305, 16]$. This implies that $\widehat{\eta} = 0.05$, only slightly lower than our baseline estimate.

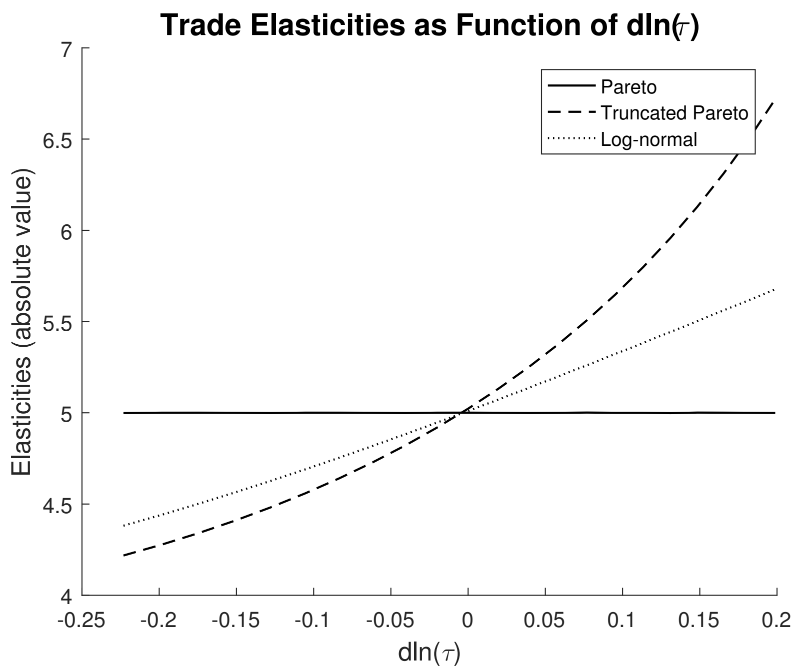


Figure 6: Trade elasticity