

## Fairness and Redistribution: Reply<sup>†</sup>

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The key contribution of Alesina and Angeletos (2005)—henceforth, AA—is to show how fairness considerations open the door to multiple equilibria in the level of taxation and redistribution. At the heart of their results is the dependence of the equilibrium tax rate on the “signal-to-noise” ratio in income inequality: the higher the fraction of income that is due to effort and talent (the “signal”) rather than luck (the “noise”), the lower the social demand for redistribution.

In their critique, Di Tella and Dubra (2013)—henceforth, DD—raise two separate issues about AA. First, they provide an example in which multiplicity emerges even though the aforementioned signal-to-noise ratio plays no role, and use this example to question the interpretations developed in AA. Second, they note that the equilibrium tax in AA need not be the Condorcet winner.

We agree on the second point but disagree on the first one. Quite simply, AA’s model admits two distinct types of multiplicity: one that is at the core of AA’s contribution, and a *separate* one that is at work in DD’s example.

We clarify this point in Section I of this note. In Section II, we then proceed to address the second point by showing how AA’s results are robust to alternative specifications of the voting mechanism.

### I. On the Origin of Multiplicity

At this point, it is useful to recall the meaning of three key exogenous parameters in AA’s model:

- $\sigma_\eta \geq 0$ , the standard deviation of the luck component of income, parameterizes the extent to which luck drives income inequality.
- $\gamma \geq 0$ , the weight with which  $\Omega$  enters voter preferences, parameterizes the “altruistic” motive for redistribution, or how much society cares about fairness (in the sense defined in AA).
- $\Delta \geq 0$ , the distance between the productivity of the pivotal and the mean agent, parameterizes the familiar “selfish” motive for redistribution as in Meltzer and Richard (1981).

With this notation in mind, we can establish the following proposition, which clarifies the two distinct types of multiplicity that can obtain in AA’s model.<sup>1</sup>

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<sup>1</sup>See Appendix A for the analytical details behind all the statements made in this section.

## PROPOSITION 1:

- (i) *Suppose the Meltzer-Richard effect is absent ( $\Delta = 0$ ). Then, multiple equilibria exist only if luck is present along with fairness concerns (both  $\sigma_\eta > 0$  and  $\gamma > 0$ ).*
- (ii) *Suppose luck is absent ( $\sigma_\eta = 0$ ). Then, multiple equilibria exist only if the Meltzer-Richard effect interacts with fairness concerns (both  $\Delta > 0$  and  $\gamma > 0$ ).*

Part (i) isolates the type of multiplicity that is at the core of AA's contribution; part (ii) isolates the alternative, and complementary, source of multiplicity that is highlighted by DD's example. Let us elaborate.

1. AA allow  $\Delta > 0$  in order to accommodate the Meltzer-Richard benchmark. However, the core of their contribution is orthogonal to this benchmark. Accordingly, the intuitions they develop and the particular numerical examples they consider in Figure 1 restrict attention to the case in which  $\Delta = 0$ . This restriction rules out the *familiar* Meltzer-Richard forces and permits AA to isolate the *novel* forces they are interested in.

2. For the case of interest ( $\Delta = 0$ ), AA thus establish the following properties: (i) the ex-post optimal tax decreases with the signal-to-noise ratio in the income distribution; (ii) the signal-to-noise ratio in turn decreases with the anticipated tax; (iii) this two-way feedback is the *sole* origin of multiplicity; and (iv) multiplicity emerges only if *both*  $\sigma_\eta$  and  $\gamma$  are positive, that is, only if luck is present and voters wish to correct for its impact on inequality.

3. One can further show that, for the case of interest,  $\sigma_\eta = 0$  guarantees that the unique equilibrium is such that  $\tau = 0$ , no matter  $\gamma$ . That is, as long as one isolates the AA fairness effect, both the possibility of multiple equilibria and the possibility of positive taxation hinge entirely on the presence of luck.

4. The points raised in DD do not contradict any of the aforementioned statements. Their example only highlights an *additional* type of multiplicity that emerges once one lets the AA notion of fairness interact with the familiar Meltzer-Richard motive for redistribution.

To explain the mechanism behind this second type of multiplicity, let us henceforth restrict  $\sigma_\eta = 0$  (which, as explained above, rules out the AA type of multiplicity), while also allowing  $\Delta > 0$  (which introduces the Meltzer-Richard effect).

Whenever  $\Delta > 0$ , the pivotal agent finds it optimal to vote for a strictly positive tax precisely because this entails a positive net transfer of income to him.

When in addition  $\gamma = 0$ , the equilibrium tax trades off the benefit of a higher such transfer against only the cost of higher effort distortion. Furthermore, the equilibrium is unique and it is such that the equilibrium tax rate increases with  $\Delta$ , exactly as in Meltzer and Richard (1981).

When, instead,  $\gamma > 0$ , the equilibrium tax now trades off the benefit of a higher transfer against also the (altruistic) cost of depriving others from what they deserved

on the basis of their effort and talent, that is, against the cost of raising  $\Omega$ . What is more, the magnitude of this cost crucially depends on the anticipated rate of taxation: the higher the anticipated rate of taxation, the lower the level of investment and the fair component of income, and hence the lower the aforementioned fairness cost. As a result, there can be multiple equilibria. In one equilibrium, the pivotal agent faces a small fairness-related cost from taxing away the mean agent and chooses a high rate of taxation; in another equilibrium, the pivotal agent faces a large such cost and chooses a low rate of taxation. The example in DD is an example of precisely this kind of multiplicity.

5. To recap, the model developed in AA allows for two types of equilibrium multiplicity: one that rests on the interaction of fairness ( $\gamma > 0$ ) with luck ( $\sigma_\eta > 0$ ), and another that rests on the interaction of fairness ( $\gamma > 0$ ) with the Meltzer-Richard motive for redistribution ( $\Delta > 0$ ). The core of AA's contribution rests on the former type of multiplicity. The example in DD highlights the latter kind, complementing but not contradicting any of the results and interpretations in AA.

6. We find this second type of multiplicity interesting on its own right. Nevertheless, if one wishes to eliminate this kind of multiplicity and isolate the core contribution of AA, one has at least two options. First, as AA already did, one can simply focus on the special case in which  $\Delta = 0$ . Alternatively, one can adjust the definition of  $\Omega$  so as to guarantee that the measure of fairness is "scale-free" in a sense that we make precise in Appendix B.

## II. Probabilistic Voting

As highlighted by DD, the preference structure in AA is sufficiently complex that, contrary to what is claimed by AA, a Condorcet winner does not necessarily coincide with the tax selected in AA. In fact, a Condorcet winner may not even exist in some cases, because preferences may fail to be single-peaked. In this section we show how these complications can be bypassed by considering a more flexible voting mechanism, namely the probabilistic voting model of Lindbeck and Weibull (1987, 1993), which does not require single-peakedness.

In the probabilistic-voting model, the voters have preferences for a party per se (party *A* or *B* affiliation) in addition to preferences over policy platforms (the tax rate in our case).<sup>2</sup> More specifically, let  $\varepsilon_i$  denote individual  $i$ 's degree of party *B* identity, independently and uniformly distributed in  $[-\frac{1}{2\varphi_i}, \frac{1}{2\varphi_i}]$ , with  $\varphi_i > 0$ , and  $i \in [0, 1]$ . Let  $U_i(\tau, \tau_e)$  denote the indirect utility of agent  $i$  as a function of the anticipated tax  $\tau_e$  and the ex post chosen tax  $\tau$ ; this is given by condition (24) in AA (and is repeated in DD). If party *A* commits to policy  $\tau^A$  and party *B* commits to policy  $\tau^B$  (before the realization of the  $\varepsilon_i$ s), individual  $i$  will vote for *A* if  $U_i(\tau^A, \tau_e) - U_i(\tau^B, \tau_e) > \varepsilon_i$ , which happens with probability  $[U_i(\tau^A, \tau_e) - U_i(\tau^B, \tau_e)] \varphi_i + \frac{1}{2}$ .

<sup>2</sup>See Persson and Tabellini (2000) for a textbook exposition and Lemma 1 in Alesina, Cozzi, and Mantovan (forthcoming) for an application to AA's dynamic setup.

Party  $A$  will maximize its votes<sup>3</sup> by choosing  $\tau^A \in [0, 1]$ , given  $\tau^B \in [0, 1]$ , and vice versa for party  $B$ . It follows that the political outcome maximizes a weighted average of the utilities in the cross section of the population:<sup>4</sup>

$$\tau^*(\tau_e) = \arg \max_{\tau \in [0, 1]} \int_0^1 \varphi_i U_i(\tau, \tau_e) di.$$

Consider now the benchmark case of homogenous degrees of party identification over the population, that is, let  $\varphi_i = \varphi$  across all  $i \in [0, 1]$ . We then have

$$\tau^*(\tau_e) = \arg \max_{\tau \in [0, 1]} \int_0^1 U_i(\tau, \tau_e) di.$$

Inspecting condition (24) in AA, one can see that

$$\int_0^1 U_i(\tau, \tau_e) di = U_{mean}(\tau, \tau_e),$$

where  $i = mean$  identifies the agent with the mean luck ( $\eta_i = \bar{\eta}$ ) and the mean talent ( $\delta_i = \bar{\delta}$ ). A variant of AA based on probabilistic voting is therefore isomorphic to a special case of AA's original model, namely the case in which the distributions of luck and talent are symmetric (so that the means coincide with the respective medians). It then follows directly from Theorem 1 in AA that the probabilistic-voting variant considered here admits a unique equilibrium if  $\gamma = 0$ , while it robustly admits multiple equilibria if  $\gamma > 0$ .

Furthermore, one can find a distribution  $\phi_i$  such that  $\int_0^1 \varphi_i U_i(\tau, \tau_e) di = U_{median}(\tau, \tau_e)$  where  $i = median$  identifies what AA incorrectly called the “median voter.” Modulo a reinterpretation of the voting mechanism, all the results of AA therefore remain intact.

Finally, note that the agents in AA *unanimously* agree on how to rank different tax rates from a fairness perspective (that is, according to  $\Omega$ ). This underscores that, by design, AA's results do not hinge on the precise details of the voting (or social-choice) mechanism.

## APPENDIX A

In this Appendix we elaborate on the distinct type of multiplicity featured in DD's example by restricting AA's model to the case with no luck (i.e.,  $\eta_i = 0$  for all  $i \in [0, 1] \Rightarrow \sigma_\eta^2 = 0$ ).

<sup>3</sup> Which, by the law of large numbers, are equal to  $\int_0^1 [U_i(\tau^A, \tau_e) - U_i(\tau^B, \tau_e)] \varphi_i di + \frac{1}{2}$ .

<sup>4</sup> Without serious loss, we are assuming away degenerate cases where there are multiple global maximizers.

Let  $i$  be the pivotal voter, whoever that might be.<sup>5</sup> This agent decides the optimal rate of taxation by maximizing, with respect to  $\tau$ , the following indirect utility function:

$$(A1) \quad U_i(\tau, \tau_e) = \frac{\delta_i}{2} [1 - \alpha \tau_e^2 - (1 - \alpha) \tau^2] + \tau \Delta [1 - \alpha \tau_e - (1 - \alpha) \tau] - \gamma \tau^2 [1 - \alpha \tau_e - (1 - \alpha) \tau]^2 \sigma_\delta^2,$$

where  $\Delta_i \equiv \bar{\delta} - \delta_i \geq 0$ ,  $\alpha \in (0, 1)$ ,  $\sigma_\delta > 0$ , and  $\tau, \tau_e \in [0, 1]$ . The three summands in this expression capture the three effects of  $\tau$  on individual utility: (i) the negative incentive effect of  $\tau$  on effort; (ii) the gains from redistribution for a voter who is poorer than the mean (the Meltzer-Richard effect); and (iii) the effect of  $\tau$  on the (un)fairness of the income distribution.

We can then establish the following result, which proves Proposition 1 and remarks 3 and 4 in the main text.

**PROPOSITION 2:** *Suppose  $\eta_i = 0$  for all  $i \in [0, 1]$ . The following are then true:*

- (i)  $\tau_e = \tau = 1$  is never an equilibrium.
- (ii) For any expectation  $\tau_e$  that is consistent with equilibrium, the measure of unfairness  $\Omega$  is minimized by, and only by,  $\tau = 0$ .
- (iii) If  $\Delta = 0$ , there is a unique equilibrium and it is such that  $\tau_e = \tau = 0$ , no matter  $\gamma$ .
- (iv) If  $\gamma = 0$ , there is a unique equilibrium, no matter  $\Delta$ .

**PROOF:**

Part (i). Note that  $U_i(0, 1) > U_i(1, 1)$  no matter the values of  $\delta_i > 0$ ,  $\Delta \geq 0$ , and  $\gamma \geq 0$ . That is, when  $\tau_e = 1$ ,  $\tau = 1$  is strictly dominated by  $\tau = 0$ , which proves that  $\tau = 1$  cannot be an equilibrium.

Part (ii). Note that

$$\Omega = \tau^2 [1 - \alpha \tau_e - (1 - \alpha) \tau]^2 \sigma_\delta^2.$$

If  $\tau_e < 1$ , the above is minimized only at  $\tau = 0$ . If instead  $\tau_e = 1$ , the above is also minimized at  $\tau = 1$ . However, part (i) already ruled out  $\tau_e = 1$  from equilibrium.

Part (iii). Using  $\Delta = 0$  in (1) gives

$$U_i = \frac{\delta_i}{2} [1 - \alpha \tau_e^2 - (1 - \alpha) \tau^2] - \gamma \Omega.$$

<sup>5</sup>Notice that we are not restricting voter  $i$  to be the “median” according to any specific ordering.

No matter  $\tau_e$ , the first term is obviously minimized at  $\tau = 0$ . Along with part (ii), this proves that  $\tau = 0$  is the unique equilibrium.

Part (iv). Using  $\gamma = 0$  in (1) and taking the derivative of  $U_i$  with respect to  $\tau$  gives

$$\frac{\partial U_i(\tau, \tau_e)}{\partial \tau} = -\bar{\delta}(1 - \alpha)\tau + \Delta[1 - \alpha\tau_e - (1 - \alpha)\tau].$$

It follows that the pivotal voters ideal tax rate  $\tau^*$  is zero if  $\Delta \leq 0$ . If instead  $\Delta > 0$ , her ideal tax rate is<sup>6</sup>

$$\tau^* = \frac{\Delta(1 - \alpha\tau_e)}{(\bar{\delta} + \Delta)(1 - \alpha)}.$$

In equilibrium,  $\tau^* = \tau_e$ , and hence the unique equilibrium tax rate is

$$\tau^* = \frac{\Delta}{\bar{\delta}(1 - \alpha) + \Delta}.$$

Therefore, it does not matter whether  $\Delta = 0$  or  $\Delta > 0$ , the equilibrium is necessarily unique.

Parts (i) and (ii) of Proposition 1 in the main text follow directly from, respectively, parts (iv) and (iii) of the above result. Part (iii) of the above result further explains that, as long as luck is absent, the AA measure of fairness is necessarily minimized at  $\tau = 0$ . Finally, when luck is present, AA have already proved that the tax rate  $\tau$  that minimizes  $\Omega$  is a monotone function of the signal-to-noise ratio in the income distribution. Combined, these facts verify the interpretations developed in AA and the statements made in the main text of this note.

## APPENDIX B

In this Appendix we propose a variation of the AA measure of (un)fairness that rules out the type of multiplicity highlighted by DD.

Let  $\bar{y}$  denote the mean level of income in the economy and, instead of the definition used in AA, let  $\Omega$  be defined as follows:

$$(B1) \quad \Omega = \int_0^1 \left( \frac{y_j^D - \hat{y}_j}{\bar{y}} \right)^2 dj.$$

The only difference from AA's original definition of  $\Omega$  is that the above measure scales up all sources of income for all agents in the economy by the mean level of income  $\bar{y}$ . This scale-free property seems a priori appealing. With this modification, the multiplicity that is featured in DD disappears, and multiple equilibria emerge only because of the interaction of fairness with luck.

<sup>6</sup>Notice that  $\frac{\partial^2 U_i(\tau, \tau_e)}{\partial \tau^2} = -(\bar{\delta} + \Delta)(1 - \alpha) < 0$ , which guarantees a point of minimum.

**PROPOSITION 3:** *No matter the value of  $\Delta$ , multiple equilibria cannot exist if either there is no luck ( $\eta_i = 0$  for all  $i \in [0, 1] \Rightarrow \sigma_\eta^2 = 0$ ) or there is no demand for fairness ( $\gamma = 0$ ).*

**PROOF:**

Following similar steps as in AA, one can show that

$$\tilde{\Omega} = \frac{1}{\bar{\delta}^2} \left[ \tau^2 \sigma_\delta^2 + \frac{\sigma_\eta^2}{\left( \alpha \frac{1-\tau_e}{1-\tau} + 1 - \alpha \right)^2} \right]$$

and, by implication,

$$U_i(\tau, \tau_e) = \frac{\delta_i}{2} [1 - \alpha \tau_e^2 - (1 - \alpha) \tau^2] + \tau(\bar{\delta} - \delta_i) [1 - \alpha \tau_e - (1 - \alpha) \tau] - \frac{\gamma}{\bar{\delta}^2} \left[ \tau^2 \sigma_\delta^2 + \frac{\sigma_\eta^2}{\left( \alpha \frac{1-\tau_e}{1-\tau} + 1 - \alpha \right)^2} \right].$$

Let  $i$  be the pivotal agent and consider the case in which luck is absent ( $\eta_i = 0$  and  $\sigma_\eta = 0$ ). The derivative of  $U$  with respect to  $\tau$  then reduces to

$$\left. \frac{\partial U_i(\tau, \tau_e)}{\partial \tau} \right|_{\tau_e=\tau} = -\delta_i(1 - \alpha)\tau + \Delta(1 - 2\tau + \tau\alpha) - \frac{2\gamma}{\bar{\delta}^2} \tau \sigma_\delta^2,$$

which is non-positive if and only if

$$\tau \geq \max \left[ \frac{\Delta}{\bar{\delta}(1 - \alpha) + \Delta + \frac{2\gamma}{\bar{\delta}^2} \sigma_\delta^2}, 0 \right].$$

The right-hand side of the above condition thus identifies the unique equilibrium tax rate.

We now proceed to show that the decreasing relationship between the equilibrium tax rate and the signal-to-noise ratio highlighted in AA survives in the present variant. In equilibrium, fair income is given by  $\hat{y}_i = y_i - \eta_i = \delta_i [1 - \alpha \tau_e - (1 - \alpha) \tau]$ .

With  $\tau_e = \tau$ , this means that the “signal” in the income distribution is given by

$$(B2) \quad \sigma_{\hat{y}}^2 = \sigma_\delta^2 (1 - \tau)^2,$$

while the “noise” is  $\sigma_\eta^2$ . Consider now the tax that minimizes  $\tilde{\Omega}$ . Assuming an interior solution, the optimality condition for the tax is given by the following:

$$\left. \frac{\partial \tilde{\Omega}(\tau, \tau_e)}{\partial \tau} \right|_{\tau_e=\tau} = \frac{2\gamma}{\delta^2} \left( \tau \sigma_\delta^2 - \frac{\alpha \sigma_\eta^2}{1-\tau} \right) = 0.$$

Using (B2), the above optimality condition can be rewritten as follows:

$$\frac{\sigma_y^2}{\sigma_\eta^2} = \frac{\alpha(1-\tau)}{\tau},$$

which proves that the tax rate that minimizes  $\Omega$  is a negative function of the signal-to-noise ratio in the income distribution, similarly to AA.

## REFERENCES

- Alesina, Alberto, and George-Marios Angeletos.** 2005. “Fairness and Redistribution.” *American Economic Review* 95 (4): 960–80.
- Alesina, Alberto, Guido Cozzi, and Noemi Mantovan.** Forthcoming. “The Evolution of Ideology, Fairness and Redistribution.” *The Economic Journal*.
- Di Tella, Rafael, and Juan Dubra.** 2013. “Fairness and Redistribution: Comment.” *American Economic Review* 103 (1): <http://dx.doi.org/10.1257/aer.103.1.549>.
- Lindbeck, Assar, and Jörgen Weibull.** 1987. “Balanced-Budget Redistribution as the Outcome of Political Competition.” *Public Choice* 52 (3): 273–97.
- Lindbeck, Assar, and Jörgen W. Weibull.** 1993. “A Model of Political Equilibrium in a Representative Democracy.” *Journal of Public Economics* 51 (2): 195–209.
- Meltzer, Allan H., and Scott F. Richard.** 1981. “A Rational Theory of the Size of Government.” *Journal of Political Economy* 89 (5): 914–27.
- Persson, Torsten, and Guido Tabellini.** 2000. *Political Economics: Explaining Economic Policy*. Cambridge, MA: MIT Press.