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QUANTIFYING CONFIDENCE

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We develop a tractable method for augmenting macroeconomic models with autonomous variation in higher-order beliefs. We use this to accommodate a certain type of waves of optimism and pessimism that can be interpreted as the product of frictional coordination and, unlike the one featured in the news literature, regards the short-term economic outlook rather than the medium- to long-run prospects. We show that this enrichment provides a parsimonious explanation of salient features of the data; it accounts for a significant fraction of the business-cycle volatility in estimated models that allow for various competing structural shocks; and it captures a type of fluctuations that have a Keynesian flavor but do not rely on nominal rigidities.

KEYWORDS: Higher-order uncertainty, coordination failure, aggregate demand, sentiments, business cycles.

1. INTRODUCTION

AS A RECESSION SETS IN, confidence in the prospects of the economy sinks. Firms cut down on employment and investment as they turn pessimistic about the demand for their products; consumers reduce spending as they turn pessimistic about their job and income prospects; and the pessimism of one economic agent appears to justify, if not feed, that of others.

Workhorse macroeconomic models, especially those used for quantitative purposes, interpret such phenomena as the coordinated response to aggregate shifts in payoff-relevant fundamentals such as the general level of know-how (technology shocks) or the efficacy of the financial sector (financial shocks). This leaves little room for expectations and coordination to play an autonomous role in driving the business cycle.

This in turn is because such models assume away, not only coordination failures in the form of multiple equilibria, but also frictional coordination in the form of higher-order

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uncertainty.[1] Formally, the economy is modeled as a game in which all players share a common prior and the same information at all times, face no uncertainty about one another's beliefs and behavior conditional on the fundamentals, and reach a perfect consensus about the current state and the future prospects of the economy.

These are strong assumptions, which are at odds with the heterogeneity of expectations evident in surveys. Once these assumptions are relaxed, the expectations of economic outcomes—for instance, firms' expectations of consumer spending and consumers' expectations of employment and income—can diverge from the expectations of fundamentals. This provides a novel explanation of the discrepancies between the predictions of the baseline RBC model and the data. It also accommodates phenomena akin to coordination failures and self-fulfilling fluctuations despite the uniqueness of equilibrium. In this paper, we provide a tractable formalization of these ideas and explore their quantitative potential.

Two Contributions

We make two contributions, one methodological and one applied. We first develop a flexible method for enriching dynamic models with a tractable form of aggregate variation in higher-order beliefs (i.e., the beliefs of the beliefs of others). We then use this method to explore the implications of a certain type of waves of optimism and pessimism, namely, one that can be entirely disconnected from expectations of TFP, can be interpreted as the product of frictional coordination, and, most crucially, regards the short-term prospects of the economy rather than its medium- or long-term potential.

We refer to these waves as variation in "confidence" and explore their quantitative implications within RBC and New Keynesian models of either the textbook or the DSGE variety.[2] We show that they offer a parsimonious yet potent explanation of the business-cycle data. We also argue that they help capture a form of demand-driven fluctuations that does not rely on nominal rigidities and does not have to manifest as co-movement between inflation and real economic activity.

Background and Methodological Contribution

We build heavily upon a large literature that studies the macroeconomic implications of higher-order uncertainty. This literature goes back at least to Phelps (1971) and Townsend (1983) and has been revived recently by Morris and Shin (2001, 2002) and Woodford (2002).[3] Within this literature, the closest precursor to our paper is Angeletos and La'O (2013), which has shown how higher-order uncertainty can help unique-equilibrium models accommodate forces akin to animal spirits.

We borrow from this literature the insight that higher-order beliefs can deviate from first-order beliefs, but use heterogeneous priors instead of complex learning dynamics to engineer fluctuations in the gap between first- and higher-order beliefs. This approach entails a certain departure from Rational Expectations. But it also allows us to bypass the

[1]Higher-order uncertainty refers to uncertainty about the beliefs of others.

[2]RBC is acronym for Real Business Cycles, DSGE for Dynamic Stochastic General Equilibrium.

[3]See Angeletos and Lian (2016) for a survey. The broader literature on informational frictions is complementary but does not always focus on the role of higher-order uncertainty. For instance, Lucas (1972) considered a setting in which higher-order uncertainty is present because information is heterogeneous, but it is inconsequential because there is no strategic complementarity. Sims (2003), on the other hand, abstracted from whether and how rational inattention can be conductive to higher-order uncertainty.

computational complications that have hindered progress in this literature on the quantitative front,[4] and to develop a general method for augmenting macroeconomic models with rich, yet tractable, higher-order beliefs.

To illustrate this point, consider the baseline RBC model. Its equilibrium dynamics can be summarized by a policy rule of the form $X_t = G(K_t, A_t)$, where A_t is the technology shock (the exogenous state variable), K_t is the capital stock (the endogenous state variable), and $X_t = (Y_t, N_t, C_t, K_{t+1})$ is a vector that collects the relevant macroeconomic outcomes, namely, output, employment, consumption, and investment or, equivalently, the next-period capital stock. Adding incomplete information to this model allows higher-order beliefs to diverge from first-order beliefs but also increases the model's state space and considerably complicates its solution. By contrast, our heterogeneous-prior formulation allows one to capture the relevant belief dynamics with only a minimal change in the state space: the equilibrium policy rule takes the form

$$X_t = G(K_t, A_t, \xi_t),$$

where ξ_t is an exogenous random variable which, by construction, encapsulates the deviation of higher-order beliefs from first-order beliefs.

This gain in tractability is not limited to the baseline RBC model. For a large, essentially arbitrary, class of linear DSGE models, our approach guarantees a minimal increase in the state space and delivers the solution of the beliefs-augmented model as a relatively simple transformation of the solution of the original model. The beliefs-augmented model can thus be simulated, calibrated, and estimated with essentially the same facility as the original one.[5]

Applied Contribution

By construction, the ξ_t shock represents variation in the gap between the first- and the higher-order beliefs of the exogenous fundamental (TFP). In equilibrium, this translates into waves of optimism and pessimism about aggregate output, employment, spending, and so on. We refer to these waves as variation in "confidence" and to ξ_t as the "confidence shock."

A distinct attribute of these waves is that they regard the short-term economic outlook. For instance, a negative innovation in ξ_t causes the firms to become pessimistic about profitability and returns over the next few quarters, and the consumers to become pessimistic about wages and income over the same horizon, without any change in expectations of either the exogenous fundamentals at any horizon or the endogenous outcomes in the medium to long run.

This property underlies our preferred interpretation of the ξ_t shock as a vehicle for autonomous variation in expectations about the short-term economic outlook. It also distinguishes our contribution from the literature on news and noise shocks (Beaudry and

[4]These complications were first highlighted by Townsend (1983). They include the need for large state spaces in order to keep track of the dynamics of higher-order beliefs and the fixed point between the law of motion of the state and the agents' filtering problem. For detailed expositions of these complications and complementary attempts to make progress on the quantitative front, see Nimark (2017) and Huo and Takayama (2015a, 2015b).

[5]The aforementioned gain may carry a cost: we abstract from the restrictions that the common-prior assumption, together with appropriate evidence, may impose on the magnitude and persistence of higher-order uncertainty. We elucidate this issue in Section 3.3 and argue that it may not matter for the applied contribution of our paper.

Portier (2006), Jaimovich and Rebelo (2009), Lorenzoni (2009), Barsky and Sims (2012)). This literature stresses beliefs of productivity and income in the medium to long run, a feature that induces strong wealth effects on consumption and labor supply. This in turn prevents the aforementioned shocks from generating realistic business cycles in the absence of appropriate bells and whistles. Our mechanism does not face this problem precisely because of its emphasis on expectations about the short run.

To understand why, augment the RBC model with our mechanism and consider a negative innovation in ξ_t. As firms expect the demand for their products to be weak in the short run, they find it optimal to lower their demand for labor and capital. In the eyes of households, this translates into a transitory fall in wages, capital returns, and overall income. Because this entails relatively weak wealth effects and relatively strong substitution effects, households react by working less and by reducing both consumption and saving. Variation in "confidence" thus generates strong positive co-movement between employment, output, consumption, and investment at the business-cycle frequency, without commensurate movements in labor productivity and TFP at any frequency.

These predictions are in line with the comovements observed in the U.S. data and cannot be easily replicated by alternative theories. We provide support for these claims by carrying out two empirical exercises.

In the first, we consider the conditional moments in the data after removing the effects of an empirical proxy of the technology shock. One can think of the filtered data as representing the "residuals" between the data and the predictions of the baseline RBC model. Our theory does well on this front: not only does it capture the comovements in these residuals, but it also outperforms other parsimonious extensions of the RBC model. A similar picture emerges when considering the wedges along the lines of Chari, Kehoe, and McGrattan (2007).

In the second exercise, we estimate medium-scale DSGE models that include our confidence shock alongside several other shocks and also contain familiar bells and whistles from the DSGE literature, such as the specific types of habit persistence in consumption and adjustment costs in investment popularized by Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). This exercise lacks parsimony—in particular, it allows business-cycle comovements to be accounted for by the combination of a plethora of shocks—but corresponds more closely to standard practice.

Despite the presence of multiple, competing shocks, the confidence shock emerges as the main driver of the business cycle. It accounts for about one half of the volatility in the key macroeconomic quantities (GDP, hours, investment, consumption) and for an even larger fraction of their comovements.

This finding is robust across two specifications. The first includes sticky prices, lets monetary policy follow a realistic Taylor rule, and is estimated using both real and nominal variables. The second assumes flexible prices, abstracts from monetary policy and inflation, and is estimated using only real quantities. Irrespective of the specification, the posterior odds of the model that excludes the confidence shock are considerably smaller than those of the model that contains it. Last but not least, our mechanism allows for fluctuations that resemble those produced by aggregate demand shocks but do not require commensurate movements in inflation, a feature that seems consistent with the data and helps bypass the empirical failures of old and new Phillips curves.

Because a direct, empirical counterpart to the confidence shock is hard, if possible at all, to obtain,[6] these findings only provide indirect support for our theory. They nev-

[6]A well-known empirical measure of expectations is the University of Michigan Index of Consumer Sentiment. This index comoves with, and in fact leads, the key macroeconomic quantities over the business cycle.

ertheless indicate the quantitative potential of three elements that are missing from the DSGE literature: frictional coordination in the form of higher-order uncertainty; a prominent role for waves of optimism and pessimism about the short-term economic outlook; and demand-driven fluctuations outside the inflation-output nexus of the New Keynesian framework. Our contribution combines all three of these elements. Future work may narrow the focus to one or another of these elements.

Layout

The rest of the paper is organized as follows. Section 2 sets up the baseline model. Section 3 explains the recursive formulation of the equilibrium and our solution method. Section 4 derives, evaluates, and interprets the empirical properties of the baseline model. Section 5 extends the analysis to two richer, estimated, models. Section 6 concludes.

2. AN RBC PROTOTYPE WITH TRACTABLE HIGHER-ORDER BELIEFS

In this section, we set up our baseline model: an RBC prototype, augmented with a tractable form of higher-order belief dynamics. We first describe the physical environment, which is quite standard. We then specify the structure of beliefs, which constitutes the main novelty of our approach.

Geography, Markets, and Timing

There is a continuum of islands, indexed by i, and a mainland. Each island is inhabited by a firm and a household, which interact in local labor and capital markets. The firm uses the labor and capital provided by the household to produce a differentiated intermediate good. A centralized market for these goods operates in the mainland, alongside a market for a final good. The latter is produced with the use of the intermediate goods and is itself used for consumption and investment. All markets are competitive.

Time is discrete, indexed by $t \in \{0, 1, \dots\}$, and each period contains two stages. The labor and capital markets of each island operate in stage 1. At this point, the firm decides how much labor and capital to demand—and, symmetrically, the household decides how much of these inputs to supply—on the basis of incomplete information regarding the choices made in other islands. In stage 2, the centralized markets for the intermediate and the final goods operate, the actual level of economic activity is publicly revealed, and the households make their consumption and saving decisions on the basis of this information.

Households

Consider the household on island i. Her preferences are given by

$$\sum_{t=0}^{\infty} \beta^t U(c_{it}, n_{it}),$$

Furthermore, this index is essentially uncorrelated with utilization-adjusted TFP at all leads and lags. While these facts are in line with our theory, they do not rule out the possibility that the co-movement of that index with the business cycle is driven by some other fundamental. The inherent difficulty is that the definition of what is a fundamental and what is not depends on the model under consideration.

where $\beta \in (0, 1)$ is the discount factor, c_{it} is consumption, n_{it} is employment (hours worked), and U is the per-period utility function, given by

$$U(c, n) = u(c) - v(n) = \frac{c^{1-\gamma} - 1}{1-\gamma} - \frac{n^{1+\nu}}{1+\nu},$$

where $\gamma \geq 0$ is the inverse of the elasticity of intertemporal substitution and $\nu \geq 0$ is the inverse of the Frisch elasticity of labor supply. Balanced growth requires $\gamma = 1$, a restriction that we impose in our quantitative exercises; letting $\gamma \neq 1$ helps accommodate a useful example in Section 3.

The household's budget constraint is

$$P_t c_{it} + P_t i_{it} = w_{it} n_{it} + r_{it} k_{it} + \pi_{it},$$

where P_t is the price of the final good, i_{it} is investment, w_{it} is the local wage, r_{it} is the local rental rate on capital, and π_{it} is the profit of the local firm. The law of motion for capital is $k_{i,t+1} = (1 - \delta)k_{it} + i_{it}$, where $\delta \in (0, 1)$ is the depreciation rate.

Intermediate-Good Producers

The output of the firm on island i is given by

$$y_{it} = A_t n_{it}^{1-\alpha} k_{it}^{\alpha},$$

where A_t is the aggregate TFP level and k_{it} is the local capital stock. The firm's profit is

$$\pi_{it} = p_{it} y_{it} - w_{it} n_{it} - r_{it} k_{it}.$$

For future reference, note that variation in expectations of p_{it} translates in variation in expectations of the returns to both capital and labor.

Final-Good Sector

The final good is produced with a Cobb–Douglas technology, which means that aggregate output satisfies

$$\log Y_t = \int_0^1 \log y_{it} \, \mathrm{d}i.$$

By implication, the demand for the good of island i satisfies

$$\frac{p_{it}}{P_t} = \frac{Y_t}{y_{it}}. \tag{2.1}$$

Without any loss, we henceforth normalize the price level so that $P_t = 1$.[7]

[7]This only applies to the present model, which abstracts from nominal rigidity. In the New Keynesian variant of Section 5, P_t is determined jointly with the real allocations.

The Technology Shock

TFP follows a random walk:

$$\log A_t = \log A_{t-1} + v_t,$$

where v_t is the period-t innovation. The latter is drawn from a Normal distribution with mean 0 and variance σ_a^2.

A Tractable Form of Higher-Order Uncertainty

We open the door to a gap between first- and higher-order beliefs by removing common knowledge of A_t in stage 1 of period t: each island i observes only a private signal of the form

$$z_{it} = \log A_t + \varepsilon_{it},$$

where ε_{it} is an island-specific error. We then engineer the desired variation in higher-order beliefs by departing from the common-prior assumption and letting each island believe that the signals of others are biased: for every i, the prior of island i is that $\varepsilon_{it} \sim \mathcal{N}(0, \sigma^2)$ and that $\varepsilon_{jt} \sim \mathcal{N}(\xi_t, \sigma^2)$ for all $j \neq i$, where ξ_t is a random variable that becomes commonly known in stage 1 of period t and that represents the perceived bias in one another's signals. These priors are commonly known: the agents "agree to disagree."

We have in mind a sequence of models in which first- and higher-order beliefs converge to Dirac measures as $\sigma \to 0$. But instead of studying the case with $\sigma \approx 0$, we only study the case with $\sigma = 0$. This guarantees that the agents act as if they were perfectly informed about the underlying state of Nature and that the pair (A_t, ξ_t) is a sufficient statistic for the entire hierarchy of beliefs about both current and future fundamentals. Together with the assumption that the aggregate capital stock (the endogenous state variable) becomes common knowledge at the end of each period, this guarantees that the model admits a tractable recursive solution, as shown in Section 3.

The Confidence Shock

We finally let ξ_t follow an AR(1) process:

$$\xi_t = \rho \xi_{t-1} + \zeta_t,$$

where $\rho \in [0, 1)$ and ζ_t is drawn from a Normal distribution with mean 0 and variance σ_ξ^2. This helps mimic an elementary property of common-prior settings: in such settings, any innovation in the gap between first- and higher-order beliefs can last for a while but must eventually vanish as old information gets replaced by new. See Section 3.3 for an example that illustrates this point.

Remarks and Interpretation

Our heterogeneous-prior specification puts strains on the rationality of the agents. First, it lets the impact of ξ_t on nth-order beliefs increase with n. Second, it ties the persistence of higher-order beliefs to the persistence of ξ_t. Finally, it implies a systematic bias in equilibrium expectations: although the firms and the consumers predict correctly the sign of the equilibrium impact of ξ_t on the relevant economic outcomes, they systematically overestimate its magnitude, and they also fail to learn from their past mistakes.

One does not have to take these properties literally. Common-prior settings such as those studied in Angeletos and La'O (2013), Benhabib, Wang, and Wen (2015), Huo and Takayama (2015a), Nimark (2017), and Rondina and Walker (2014) can accommodate similar fluctuations in higher-order beliefs. In effect, what is "bias" in our setting becomes "rational confusion" in those settings. Furthermore, higher-order beliefs can be persistent in both cases, although the persistence is endogenous to the learning that takes place over time in the latter case. We illustrate these points in Section 3.3 by establishing an observational equivalence, from the point of view of aggregate data, between a special case of our model and a common-prior variant.

Most importantly, the subsequent analysis will reveal that the empirical performance of our theory hinges, not on the precise micro-foundations of the belief waves considered, but rather on the property that the beliefs regard firm profitability and household income in the short run as opposed to the medium and long run. We thus encourage the reader to adopt a flexible interpretation of ξ_t as a modeling device that helps capture more generally this kind of belief waves.[8]

3. EQUILIBRIUM CHARACTERIZATION AND SOLUTION METHOD

In this section, we characterize the equilibrium of the model and present our solution method. We also use an example to illustrate the idea that our heterogeneous-prior formulation can be seen as a convenient proxy for belief fluctuations in common-prior settings with rich information structures.

3.1. *Recursive Equilibrium*

As behavior is forward looking, the optimal choices any agent (or island) makes at any point of time depend on her beliefs, not only of the concurrent behavior of others, but also of their behavior in the future. This suggests a high-dimensional fixed-point relation between actual behavior and the expectations that agents form at any time about future economic outcomes, including expectations of the future terms of trade (the prices of the island-specific goods), wages, and interest rates. In general, the introduction of higher-order uncertainty can perturb this kind of expectations in a sufficiently significant manner as to render a low-dimensional recursive representation infeasible. With our formulation, however, such a representation is feasible and, indeed, relatively straightforward.

To start with, note that the equilibrium allocations on any given island can be obtained by solving the problem of a fictitious local planner. The latter chooses local employment, output, consumption, and savings so as to maximize local welfare subject to the following resource constraint:

$$c_{it} + k_{i,t+1} = (1-\delta)k_{it} + p_{it}y_{it}. \tag{3.1}$$

Note that this constraint depends on p_{it} and, thereby, on aggregate output, objects that are endogenous in general equilibrium but are taken as given by the fictitious local plan-

[8]We can imagine at least two variants of our framework that can also capture such waves. The one replaces our heterogeneous-prior specification with Knightean uncertainty (ambiguity) about the information of others and allows for, possibly endogenous, variation in the level of this uncertainty; such a model could build a bridge between our work and a literature on ambiguity and robust control (Hansen and Sargent (2007, 2012), Woodford (2010)). The other allows directly for irrational shifts in expectations of profitability and income in the short run; such a model would fit well with the narratives in Akerlof and Shiller (2009) and Burnside, Eichenbaum, and Rebelo (2016).

ner (or, equivalently, in the partial equilibrium of the given island). This dependence captures the type of aggregate-demand externalities and other general-equilibrium effects that are at the core of DSGE models.

To make her optimal decisions at any point of time, the aforementioned planner must form beliefs about the value of p_{it} (or, equivalently, of Y_t) at all future points of time. These beliefs encapsulate the beliefs that the local firm forms about the evolution of the demand for its product and of the costs of its inputs, as well as the beliefs that the local consumer forms about the dynamics of local income, wages, and capital returns. The fact that the various beliefs are tied together underscores the cross-equations restrictions that discipline the exercises conducted in this paper: if expectations were "completely" irrational, the beliefs of different endogenous objects would not be tied together. The observable implications of these restrictions will be revealed in what follows. For now, we emphasize that ξ_t matters for equilibrium outcomes because, and only because, it triggers co-movement in the expectations of the various actors in our model.

In a recursive equilibrium, these expectations can be tracked with the help of a small number of functions, which themselves encapsulate the fixed-point relation between behavior and beliefs. For the model under consideration, this means that we can define a recursive equilibrium as a collection of four functions, denoted by $\mathbf{P}$, $\mathbf{G}$, V_1, and V_2, such that the following are true:

(i) $\mathbf{P}(z, \xi, K)$ captures the price (the terms of trade, or equivalently, the demand) expected by an island in stage 1 of any given period when the local signal is z, the confidence shock is ξ, and the capital stock is K; and $\mathbf{G}(A, \xi, K)$ gives the aggregate capital stock next period when the current realized value of the aggregate state is (A, ξ, K).

(ii) V_1 and V_2 solve the following Bellman equations:

$$\begin{cases} V_1(k; z, \xi, K) = \max\limits_{n} V_2(\hat{m}; z, \xi, K) - v(n), \\ \qquad\qquad \text{s.t.} \quad \hat{m} = \hat{p}\hat{y} + (1-\delta)k, \\ \qquad\qquad\qquad \hat{y} = zk^{\alpha}n^{1-\alpha}, \\ \qquad\qquad\qquad \hat{p} = \mathbf{P}(z, \xi, K), \end{cases} \tag{3.2}$$

$$\begin{cases} V_2(m; A, \xi, K) = \max\limits_{c,k'} u(c) + \beta \int V_1(k'; A', \xi', K')\,\mathrm{d}f(A', \xi' | A, \xi), \\ \qquad\qquad \text{s.t.} \quad c + k' = m, \\ \qquad\qquad\qquad K' = \mathbf{G}(A, \xi, K). \end{cases} \tag{3.3}$$

(iii) $\mathbf{P}$ and $\mathbf{G}$ are consistent with the policy rules that solve the local planning problem in (3.2)–(3.3).

To interpret (3.2) and (3.3), note that V_1 and V_2 denote the local planner's value functions in, respectively, stages 1 and 2; m denotes the quantity of the final good acquired in stage 2; and the *hat* symbol over a variable indicates the stage-1 belief of that variable. Next, note that the last constraint in (3.2) embeds the belief that the price of the local good is governed by the function $\mathbf{P}$, while the other two constraints are the local production function and the local resource constraint. The problem in (3.2) therefore describes the optimal employment and output choices in stage 1, when the local capital stock is k, the local signal of the aggregate state is (z, ξ, K), and the local beliefs of "aggregate demand" are captured by the function $\mathbf{P}$. The problem in (3.3), in turn, describes the optimal consumption and saving decisions in stage 2, when the available quantity of the final good is m, the realized aggregate state is (A, ξ, K), and the island expects aggregate capital to follow the policy rule $\mathbf{G}$.

The decision problem of the local planner treats the functions **P** and **G** as exogenous. In equilibrium, however, these functions must be consistent with the policy rules that solve this problem. Let $\mathbf{n}(k, z; \xi, K)$ be the optimal choice for employment that obtains from (3.2) and $\mathbf{g}(m; A, \xi, K)$ be the optimal policy rule for capital that obtains from (3.3). Next, let $\mathbf{y}(z; A, \xi, K) \equiv A\mathbf{n}(z, \xi, K)^{1-\alpha}K^{\alpha}$ be the output level that results from the aforementioned employment strategy where the realized TFP is A and the local capital stock coincides with the aggregate one. Equilibrium consistency can then be expressed as follows:

$$\mathbf{P}(z, \xi, K) = \frac{\mathbf{y}(z+\xi, z, \xi, K)}{\mathbf{y}(z, z, \xi, K)}, \tag{3.4}$$

$$\mathbf{G}(A, \xi, K) = \mathbf{g}\big(\mathbf{y}(A, A, \xi, K) + (1-\delta)K; A, \xi, K\big). \tag{3.5}$$

To interpret condition (3.4), recall that in stage 1 each island believes that, with probability 1, TFP satisfies $A = z$ and the signals of all other islands satisfy $z' = A + \xi = z + \xi$. Together with the fact that all islands make the same choices in equilibrium and that the function $\mathbf{y}$ captures their equilibrium production choices, this implies that the local beliefs of local and aggregate output are given by, respectively, $\hat{y} = \mathbf{y}(z, z, \xi, K)$ and $\hat{Y} = \mathbf{y}(z+\xi, z, \xi, K)$. By the demand function in (2.1), it then follows that the local belief of the price must satisfy $\hat{p} = \hat{Y}/\hat{y}$, which gives condition (3.4). To interpret condition (3.5), recall that all islands end up making identical choices in equilibrium, implying that the available resources of each island in stage 2 coincide with $Y + (1-\delta)K$, where Y is the aggregate quantity of the final good (aggregate GDP). Note next that the realized production level of *all* islands is given by $\mathbf{y}(A, A, \xi, K)$ and, therefore, Y is also given by $\mathbf{y}(A, A, \xi, K)$. Together with the fact that $\mathbf{g}$ is the optimal savings rule, this gives condition (3.5).

Summing up, an equilibrium is a fixed point of the Bellman equations (3.2)–(3.3) and the consistency conditions (3.4)–(3.5). In principle, one can obtain the global, nonlinear, solution of this fixed-point problem with numerical methods. As in the DSGE literature, however, we find it useful to concentrate on the log-linear approximation of the solution around the deterministic steady state. This makes it possible to obtain the equilibrium dynamics of the beliefs-augmented model as a tractable transformation of the equilibrium dynamics of the underlying complete-information model.

3.2. *Log-Linear Solution*

To obtain the log-linear solution, we first log-linearize the equilibrium equations around the deterministic steady state. With a slight abuse of notation, we henceforth reinterpret all the variables in terms of the log-deviations of these variables from their steady-state values.

The terms of trade faced by island i are $p_{it} = Y_t - y_{it}$. The associated marginal revenue products of labor and capital are, respectively, $MRPL_{it} \equiv p_{it} + y_{it} - n_{it}$ and $MRPK_{it} \equiv p_{it} + y_{it} - k_{it}$. The optimal behavior of island i is thus characterized by the following system:

$$\nu n_{it} = \mathbb{E}_{it}[MRPL_{it}] - \gamma \mathbb{E}_{it} c_{it}, \tag{3.6}$$

$$\gamma\big(\mathbb{E}'_{it} c_{i,t+1} - c_{it}\big) = \big(1 - \beta(1-\delta)\big)\mathbb{E}'_{it}[MRPK_{i,t+1}], \tag{3.7}$$

$$p_{it} + y_{it} = (1-s)c_{it} + s\iota_{it}, \tag{3.8}$$

$$y_{it} = A_t + \alpha k_{it} + (1-\alpha) n_{it}, \tag{3.9}$$

$$k_{i,t+1} = \delta i_{it} + (1-\delta) k_{it}, \tag{3.10}$$

where $s \equiv \frac{\alpha\beta\delta}{1-\beta(1-\delta)}$ denotes the steady-state investment-to-GDP ratio. The interpretation of these conditions is straightforward: (3.6) is the labor-supply condition; (3.7) is the Euler condition; (3.8) is the resource constraint; (3.9) is the production function; and (3.10) is the law of motion for capital.

To convey the basic idea behind our solution method, consider momentarily a special case that can be solved by hand: let utility be linear in consumption and assume away capital ($\gamma = \alpha = 0$). In this case, the equilibrium can be reduced to the following fixed-point relation:

$$n_{it} = \mathbb{E}_{it}[\chi A_t + \omega N_t], \tag{3.11}$$

where $\chi \equiv \frac{1}{1+\nu}$ and $\omega \equiv \frac{1}{1+\nu} \in (0, 1)$.[9] Equilibrium employment can therefore be understood as the solution to a static "beauty contest," namely, a coordination game with linear best responses and incomplete information, of the type found in Morris and Shin (2002) and Angeletos and Pavan (2007). In this game, a player is an island, her action is local employment, the fundamental is the underlying TFP, χ measures the direct effect of the fundamental on individual outcomes holding constant the aggregate outcomes, and ω measures the degree of strategic complementarity.[10] Importantly, an island responds to ξ_t because, and only because, this shock influences its beliefs about aggregate employment (and thereby its beliefs about its terms of trade).

To solve (3.11), guess the following policy rule at the individual level:

$$n_{it} = \Lambda_z z_{it} + \Lambda_\xi \xi_t. \tag{3.12}$$

Aggregation gives $N_t = \Lambda_z \bar{z}_t + \Lambda_\xi \xi_t$. Due to our specification of priors,

$$\mathbb{E}_{it}[A_t] = z_{it} \quad \text{and} \quad \mathbb{E}_{it}[\bar{z}_t] = \mathbb{E}_{it}[A_t + \xi_t] = z_{it} + \xi_t.$$

It follows that the individual belief of aggregate employment is

$$\mathbb{E}_{it}[N_t] = \Lambda_z z_{it} + \left(\Lambda_\xi + \Lambda_z^n\right)\xi_t.$$

Using this fact in (3.11), we infer that whenever i expects the others to play according to the rule given by (3.12), his best response is to set

$$n_{it} = (\chi + \omega\Lambda_z) z_{it} + \omega(\Lambda_\xi + \Lambda_z)\xi_t.$$

Matching the coefficients obtained above with those in the proposed policy rule implies that the latter is part of an equilibrium if and only if the following is true:

$$\Lambda_z = (\chi + \omega\Lambda_z) \quad \text{and} \quad \Lambda_\xi = \omega(\Lambda_\xi + \Lambda_z).$$

[9]To obtain condition (3.11), note first that, when $\alpha = 0$, investment is zero, output is given by $y_{it} = A_t + n_{it}$, and the resource constraint reduces to $c_{it} = p_{it} + y_{it}$, using these facts in the labor-supply condition (3.8) gives $(1+\nu)n_{it} = \mathbb{E}_{it}[p_{it} + y_{it}]$. Finally, using $p_{it} = Y_t - y_{it}$ and $Y_t = A_t + N_t$ results into condition (3.11).

[10]In the example under consideration, ω happens to coincide with χ, but this is not generally true. It is therefore best to think of ω and χ as two distinct objects.

Solving these two equations for the coefficients Λ_z and Λ_ξ gives

$$\Lambda_z = \frac{\chi}{1-\omega} = \frac{1}{\nu} \quad \text{and} \quad \Lambda_\xi = \frac{\omega}{1-\omega}\Lambda_z = \frac{1}{\nu^2}. \tag{3.13}$$

We infer that there exists a unique equilibrium and that the equilibrium policy rule for local employment is given by (3.12) along with (3.13). Finally, using the fact that $z_{it} = A_t$ with probability 1, we conclude that the realized aggregate level of output is given by

$$Y_t = A_t + N_t = \Lambda_A A_t + \Lambda_\xi \xi_t,$$

with $\Lambda_A = 1 + \Lambda_z$ and with (Λ_z, Λ_ξ) given as in (3.13).

Two properties of this solution are worth noting. First, the coefficient Λ_A, which governs the response of Y_t to A_t, is the same as the one in the version of the model that imposes common knowledge of A_t and shuts down the ξ_t shock. Second, the coefficient Λ_ξ, which governs the effect of the ξ_t shock, is proportional to Λ_A by a factor that is itself increasing in ω. That is, the impact of the confidence shock relative to that of the technology shock increases with the degree of strategic complementarity. This is because ξ_t matters only by influencing the beliefs of the actions of others.

Go back now to the general case $(\alpha, \gamma > 0)$. The presence of an endogenous state variable (capital) and of forward-looking behavior implies that the economy can be thought of as a *dynamic* game in which the best response of a player (or island) today depends both on past outcomes and on expectations of future outcomes. This complicates the fixed-point problem that needs to be solved. The essence, however, is similar to that in the above example.

We thus start by guessing the following policy rules at the island level:

$$n_{it} = \Lambda_k^n (k_{it} - K_t) + \Lambda_K^n K_t + \Lambda_z^n z_{it} + \Lambda_\xi^n \xi_t, \tag{3.14}$$

$$c_{it} = \Gamma_k^c (k_{it} - K_t) + \Gamma_K^c K_t + \Gamma_z^c z_{it} + \Gamma_{\bar{z}}^c \bar{z}_t + \Gamma_a^c A_t + \Gamma_\xi^c \xi_t, \tag{3.15}$$

$$k_{it+1} = \Omega_k^k (k_{it} - K_t) + \Omega_K^k K_t + \Omega_z^k z_{it} + \Omega_{\bar{z}}^k \bar{z}_t + \Omega_a^k A_t + \Omega_\xi^k \xi_t, \tag{3.16}$$

where Λ^n, Γ^c, and Ω^k are coefficients that remain to be determined. We then proceed to solve for the equilibrium values of these coefficients by solving the fixed-point problem between the individual policy rules and the associated aggregate outcomes imposed by conditions (3.6)–(3.10).

To generate data from the model, we set $z_{it} = \bar{z}_t = A_t$ and compute the aggregate outcomes implied by (3.14)–(3.16). This gives N_t, C_t, and K_{t+1} as functions of the vector (K_t, A_t, ξ_t), verifying that the latter is the state variable for the aggregate outcomes. Note that setting $z_{it} = \bar{z}_t = A_t$ corresponds to invoking the objective truth. However, to solve the fixed-point problem between the individual policy rules (or strategies) and the aggregate outcomes, we have to treat z_{it}, $\bar{z}_t$, and A_t as distinct objects. This is necessary in order to keep track of the difference between the first- and the higher-order beliefs of the underlying fundamental and, thereby, between objective and subjective beliefs.

The details are spelled out in Supplemental Material Appendix S.5 (Angeletos, Collard, and Dellas (2018)). The bottom line is that we can obtain the solution of our model as a tractable transformation of that of the standard RBC model. Furthermore, this solution has two key properties. First, the coefficients $(\Lambda_K^n, \Gamma_K^c, \Omega_K^k)$ and $(\Lambda_A^n, \Gamma_A^c, \Omega_A^k)$, which determine the impact of the capital stock and the technology shock on aggregate outcomes, coincide with those in the standard RBC model. Second, the coefficients $(\Lambda_\xi^n, \Gamma_\xi^c, \Omega_\xi^k)$, which determine the impact of the confidence shock, can be solved as functions of the aforementioned coefficients and a few other coefficients, which themselves capture the degree of strategic complementarity in the economy. These properties mirror those noted in the example above.

The solution strategy described above and the aforementioned properties extend to a large class of linear DSGE models; see Supplemental Material Appendix S.5. The beliefs-augmented model can thus be simulated and estimated with the same ease as the original model. This explains the broader methodological conntribution of our paper[11] and facilitates the quantitative explorations conducted in Sections 4 and 5.

3.3. *Heterogeneous versus Common Priors*

As already mentioned, the main advantage of our approach relative to common-prior, incomplete-information models is its flexibility and its straightforward applicability to macroeconomic models. A potential cost is that it bypasses the restrictions that the common-prior assumption imposes on the size and dynamics of higher-order uncertainty. We now use an example to illustrate this tradeoff and to corroborate the claim that our heterogeneous-prior specification can be thought of as a proxy for higher-order uncertainty in common-prior settings.

In particular, we show that the tractable example considered in the previous section is observationally equivalent to a common-prior variant, in a sense that will be made precise below. We then derive the restrictions that the common-prior variant imposes on the volatility and the persistence of the kind of belief-driven fluctuations we are interested in.

We start by showing that the special case of our model that was solved by hand in the previous section (namely, the one with $\alpha = \gamma = 0$) is observationally equivalent, in a sense that will be made precise below, to a common-prior variant. This variant is obtained by introducing heterogeneity in TFP and letting trade be done according to random, pairwise, matching across the islands. As in Angeletos and La'O (2013), these modifications allow fluctuations to obtain from correlated noise in the rational beliefs that islands form about their pairwise terms of trade.

Let us fill in the details. TFP in island i is given by $A_{it} = A_t + a_i$, where A_t is the aggregate TFP shock and a_i is an island-specific fixed effect. The former follows a random walk with the same variance as in the heterogeneous-prior economy; the latter is distributed in the cross-section of islands according to a Normal distribution with mean zero and variance $\tilde{\sigma}_a^2$. The aggregate TFP shock is common knowledge. Nevertheless, higher-order uncertainty is present because each island is uncertain about the productivity and the information of its trading partner when choosing employment and production.

In particular, the information that island i has in the morning of period t about its current-period match is summarized by the following two signals:

$$z_{it} = a_{m(i,t)} + \tilde{\xi}_t \quad \text{and} \quad w_{it} = \tilde{\xi}_t + u_{i,t},$$

where $m(i,t)$ denotes the trading partner of island i in period t, $u_{i,t}$ is orthogonal to $a_{m(i,t)}$, i.i.d. across islands and unpredictable on the basis of past information, and $\tilde{\xi}_t$ is an aggregate shock that is orthogonal to the aggregate TFP shock and that follows an AR(1) process. More specifically,

$$\tilde{\xi}_t = \tilde{\rho}\tilde{\xi}_{t-1} + \tilde{\sigma}_\xi \tilde{\zeta}_t,$$

[11]We are currently developing a user-friendly toolkit that may allow other researchers to apply our method to their prefered model. Once completed, this toolkit will become publicly available in our webpages.

where $\tilde{\zeta}_t \rightsquigarrow \mathcal{N}(0,1)$, $\tilde{\sigma}_\xi > 0$, and $\tilde{\rho} \in [0,1]$. Literally taken, z_{it} is i's private signal about the idiosyncratic TFP of its trading partner; this signal is contaminated by common noise, given by $\tilde{\xi}_t$; and w_{it} is a private signal that is informative about this noise.[12] Clearly, the $\tilde{\xi}_t$ shock plays the same role in this common-prior setting as the ξ_t shock in our heterogeneous-prior setting.

In the absence of the aforementioned shocks, the two economies reduce to the same underlying RBC benchmark and thus give rise, in equilibrium, to the same observables at the aggregate level. Let Y_t^* denote the level of output in that benchmark. From the results of the previous subsection, we have that the equilibrium level of output in the heterogeneous-prior economy is given by

$$Y_t = Y_t^* + \Lambda_\xi \xi_t,$$

with Λ_ξ as in (3.13). And since ξ_t is an AR(1) process, we conclude that the difference $Y_t - Y_t^*$, which represents the "output gap" relative to the frictionless RBC benchmark, is also an AR(1) process. In particular,

$$Y_t - Y_t^* = \varphi\big(Y_{t-1} - Y_{t-1}^*\big) + \psi \varepsilon_t, \tag{3.17}$$

where $\varepsilon_t \rightsquigarrow \mathcal{N}(0,1)$ is i.i.d. over time and independent of the technology shock,[13] and where

$$\varphi = \rho \quad \text{and} \quad \psi = \frac{\omega \sigma_\xi}{(1-\omega)^2}. \tag{3.18}$$

Consider next the common-prior economy and let $\tilde{\theta} \equiv (\tilde{\rho}, \tilde{\sigma}_\xi, \tilde{\sigma}_u, \tilde{\sigma}_a)$ collect its informational parameters. Its solution is far from trivial, but can be obtained by adapting Theorem 1 in Huo and Takayama (2015b).[14] We thus have that the output gap in this economy also follows an AR(1) process as in (3.17), except that now φ and ψ are given by the following:

$$\begin{aligned} \varphi &= \Phi(\tilde{\theta}, \omega) \\ &\equiv \frac{1}{2}\left(\frac{1}{\tilde{\rho}} + \tilde{\rho} + \frac{1-\omega}{\tilde{\rho}} \frac{\tilde{\sigma}_a^2 + \tilde{\sigma}_u^2}{\tilde{\sigma}_a^2 \tilde{\sigma}_u^2} \tilde{\sigma}_\xi^2\right) \\ &\quad - \frac{1}{2}\sqrt{\left(\frac{1}{\tilde{\rho}} + \tilde{\rho} + \frac{1-\omega}{\tilde{\rho}} \frac{\tilde{\sigma}_a^2 + \tilde{\sigma}_u^2}{\tilde{\sigma}_a^2 \tilde{\sigma}_u^2} \tilde{\sigma}_\xi^2\right)^2 - 4}, \end{aligned} \tag{3.19}$$

$$\psi = \Psi(\tilde{\theta}, \omega) \equiv \frac{\omega \Phi(\tilde{\theta}, \omega)}{\tilde{\rho}\left(1 - \omega^2 \dfrac{\tilde{\rho}\tilde{\sigma}_a^2 + \Phi(\tilde{\theta}, \omega)\tilde{\sigma}_u^2}{\tilde{\rho}\tilde{\sigma}_a^2 + \tilde{\rho}\tilde{\sigma}_u^2}\right)} \tilde{\sigma}_a. \tag{3.20}$$

By comparing (3.18) to (3.19)–(3.20), we can readily prove that the two economies are observationally equivalent in the following sense.

[12]This signal can be recast as information extracted from past trades; see Angeletos and La'O (2013) for details.

[13]Note that $\varepsilon_t \equiv \zeta_t / \sigma_\xi$, with ζ_t being the innovation in the confidence shock.

[14]The result contained in Huo and Takayama (2015b) abstracts from the aggregate TFP shock. By adding such a shock but assuming that it is always common knowledge, we guarantee that the same solution applies to the gap $Y_t - Y_t^*$.

PROPOSITION 1: *Let* $\theta \equiv (\rho, \sigma_\xi)$, $\tilde{\Theta} \equiv [0,1] \times \mathbf{R}_+^3$, *and* $\Theta \equiv [0,1] \times \mathbf{R}_+$; *and let* $\mathcal{C}(\tilde{\theta})$ *and* $\mathcal{H}(\theta)$ *denote, respectively, the common-prior economy parameterized by* $\tilde{\theta}$ *and the heterogeneous-prior economy parameterized by* θ.

(i) *For any* $\tilde{\theta} \in \tilde{\Theta}$, *there exists a* $\theta \in \Theta$ *such that* $\mathcal{H}(\theta)$ *generates the same stochastic process for all the macroeconomic quantities as* $\mathcal{C}(\tilde{\theta})$.

(ii) *The converse is also true: for any* $\theta \in \Theta$, *there exists a* $\tilde{\theta} \in \tilde{\Theta}$ *such that* $\mathcal{C}(\tilde{\theta})$ *generates the same stochastic process for all the macroeconomic quantities as* $\mathcal{H}(\theta)$.

The intuition behind this result is that the two economies feature exactly the same variation in the expectations of the relevant economic outcomes: in either economy, a positive (resp., negative) output gap obtains if and only if the firms and the households of each island are optimistic (resp., pessimistic) about the terms of trade, or the demand, that their island is likely to face in the short run.

What differs between the two economies is the way these waves of optimism and pessimism are captured: in one economy, they are engineered with the help of a specific departure from rational expectations; in the other, they are instead sustained by rational confusion. Accordingly, whereas the higher-order belief shock is allowed to be common knowledge in the heterogeneous-prior economy, it has to be imperfectly observed in the common-prior one. Nevertheless, by choosing the parameters that govern the dynamics of that shock and of the quality of learning in the latter economy, we can always match the stochastic process for the aforementioned expectations in the former economy, and can therefore also generate the same observables at the aggregate level.

This result is subject to the following qualification: replicating a heterogeneous-prior economy with a common-prior one relies on the freedom to choose a sufficiently high $\tilde{\sigma}_a$ in the latter. This is because the level of fundamental, or first-order, uncertainty in the common-prior economy—parameterized here by $\tilde{\sigma}_a$—imposes certain bounds on the persistence and the volatility of higher-order beliefs and, equivalently, on φ and ψ. For the heterogeneous-prior economy to respect the same bounds, ρ and σ_ξ must satisfy certain restrictions. Proposition 2 below describes the bounds on φ and ψ; Corollary 1 gives the corresponding restrictions on ρ and σ_ξ.

PROPOSITION 2: *For any* $\varphi \in [0, 1)$ *and any* $\omega \in (0, 1)$, *let*

$$B(\varphi, \omega) \equiv \max_{\hat{\rho} \in [0,1], \hat{\sigma}_u \geq 0, \hat{\sigma}_\xi \geq 0} \left\{ \Psi(\hat{\rho}, \hat{\sigma}_u, \hat{\sigma}_\xi, 1, \omega) \text{ s.t. } \Phi(\hat{\rho}, \hat{\sigma}_u, \hat{\sigma}_\xi, \omega) = \varphi \right\}.$$

A process for the output gap as in condition (3.17) *can be obtained in the equilibrium of a common-prior economy* $\mathcal{C}(\tilde{\theta})$ *if and only if* (i) $0 \leq \varphi < 1$ *and* (ii) $0 \leq \psi \leq B(\varphi, \omega)\tilde{\sigma}_a$.

COROLLARY 1: *A heterogeneous-prior economy* $\mathcal{H}(\theta)$ *can be replicated by a common-prior economy* $\mathcal{C}(\tilde{\theta})$, *in the sense of sharing the same stochastic process for* (Y_t, A_t, ξ_t), *if and only if* (i) $0 \leq \rho < 1$ *and* (ii) $\sigma_\xi \leq \frac{\omega}{(1-\omega)^2} B(\rho, \omega)\tilde{\sigma}_a$.

Part (i) of Proposition 2 states that the beliefs-driven fluctuations in the common-prior economy are necessarily transitory. This would be true even if we allowed $\tilde{\rho} > 1$, meaning an explosive process for the $\tilde{\xi}_t$ shock. The reason is that these fluctuations are sustained only by rational confusion, which itself fades away as additional information arrives over time. Part (ii), on the other hand, provides a tight upper bound on the volatility of these fluctuations. This bound is proportional to $\tilde{\sigma}_a$, because, as already explained, this parameter pins down the level of first-order uncertainty, which in turn binds the level of higher-order uncertainty.

Corollary 1 converts the above properties into restrictions on the parameters of the heterogeneous-prior specification. Part (i) justifies our earlier assertion that letting $\rho < 1$ helps capture within our framework the property that the fluctuations sustained by higher-order uncertainty have to be transient. Part (ii), on the other hand, provides an upper bound on σ_ξ.

To recap, we have established two lessons. First, the heterogeneous-prior setting is observationally equivalent to a common-prior variant in terms of beliefs-driven fluctuations. Second, the common-prior setting imposes a joint restriction between the magnitude and persistence of these fluctuations and the underlying fundamental uncertainty. Translating this restriction to the heterogeneous-prior setting yields a bound on σ_ξ.

How tight is this bound? In Supplemental Material Appendix S.1, we use a back-of-the-envelope exercise to argue the following: if we were to approach the U.S. data with the exceedingly simple model considered in this subsection, the bound would be large enough to allow for the entire business cycle to be driven by the confidence shock. And although a similar result is not readily available for the estimated models of Section 5, we suspect that our quantitative findings are consistent with realistic common-prior models. The recent work of Huo and Takayama (2015b) seems to corroborate this conjecture.

4. EMPIRICAL PROPERTIES OF THE CONFIDENCE SHOCK

We now use a parameterized version of our model to illustrate the business-cycle properties of the confidence shock. We also explain why these properties are consistent with salient features of the data and why they are not shared by other *parsimonious* business-cycle models. (Note the emphasis on parsimony; the performance of our mechanism within richer, medium-scale, DSGE models is addressed in Section 5.) We finally elaborate on the sense in which the confidence shock can be thought of as an aggregate demand shock whose ability to generate realistic business cycles does not require either the presence of nominal rigidities or the co-movement of the real quantities with inflation.

4.1. *Parameterization and IRFs*

The parameters are set as follows: the discount factor is 0.99; the elasticity of intertemporal substitution is 1; the Frisch elasticity of labor supply is 2; the capital share in production is 0.3; the depreciation rate is 0.015; and the persistence of the confidence shock is $\rho = 0.75$. The last choice is somewhat arbitrary, but can be motivated as follows. First, the implied forecast errors have a half life of less than a year, which is broadly in line with survey evidence in Coibion and Gorodnichenko (2012). Second, the value of ρ assumed here is close to the one estimated in the next section in the context of two medium-scale, DSGE models. Finally, to the extent that the fluctuations induced by ξ_t in our model resemble either the "demand shock" identified in Blanchard and Quah (1989) or the "main business-cycle shock" identified in Angeletos, Collard, and Dellas (2017), our parameterization is consistent with the evidence in those papers as well.[15]

[15]Note that we have not specified σ_a and σ_ξ, the standard deviations of the two shocks. This is not necessary for the purposes of this section, because we focus on co-movement patterns and do not attempt to match the overall volatility in the data. See, however, the remarks in footnote 20.

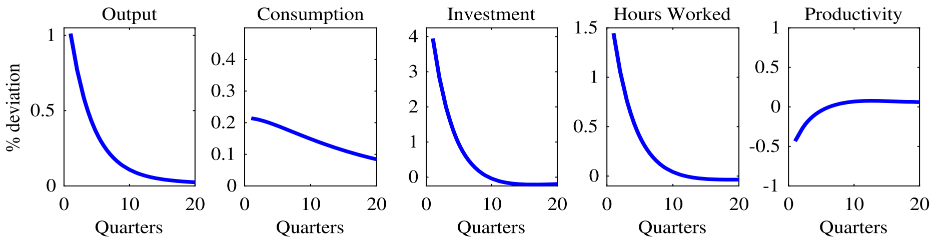

FIGURE 1.—Impulse responses to a positive confidence shock.

Figure 1 reports the Impulse Response Functions (IRFs) of the model's key quantities to a positive innovation in ξ_t. Clearly, the shock causes output, hours, consumption, and investment to move in the same direction. But why?

We address this question in two steps. In the rest of this subsection, we explain how the variation in higher-order beliefs of the exogenous fundamental (TFP) translates into variation in the expectations of the aggregate economic activity and the terms of trade. In the next subsection, we clarify how the empirical performance of the theory hinges on the horizon of the latter kind of expectations.

Start by inspecting conditions (3.6)–(3.10), which determine the equilibrium behavior. The following property is evident: the optimal behavior of an island depends on its higher-order beliefs of aggregate TFP *only* through its first-order beliefs of its terms of trade, which in turn coincide with its first-order beliefs of aggregate output. This reveals the ultimate modeling role of the ξ_t shock, which is to capture extrinsic variation in the expectations of the relevant economic outcomes.

This perspective applies more generally. In the class of models we are interested in, the equilibrium expectations of the endogenous outcomes can be expressed as a function of the hierarchy of beliefs about the underlying fundamentals regardless of the information structure. However, different assumptions about the information structure lead to different predictions about the stochastic properties of the expectations of economic outcomes. In the standard practice, these expectations are spanned by the expectations of fundamentals because higher-order beliefs collapse to first-order beliefs. By contrast, our approach leaves room for autonomous variation in the expectations of economic outcomes by letting the higher-order beliefs deviate from the first-order beliefs.

4.2. *The Key Mechanism: Beliefs About the Short-Term Economic Outlook*

So far, we have argued that it is best to think of the assumed shock to higher-order beliefs as a modeling device for introducing autonomous variation in the expectations of the relevant economic outcomes. This is important, but it is not the whole story. Because behavior is forward looking, the horizon of these expectations is a crucial determinant of how actual outcomes respond to shifts in them. We now build on this basic observation to explain why the co-movement patterns seen in Figure 1 hinge on the property that the assumed shock captures expectations of the short-term economic outlook, as opposed to expectations of the medium- or long-run prospects.

To reveal the short-term nature of the belief waves triggered by the ξ_t shock, we present the effects of the shock on the "term structure of expectations." Consider, in particular, the forecasts that island i forms in period t about its terms of trade k periods ahead, namely, $\mathbb{E}_{i\tau}[p_{i,\tau+k}]$, for all $k \geq 1$. As already noted, these forecasts are tied to the forecasts that the firms make about their sales, that the households form about their income,

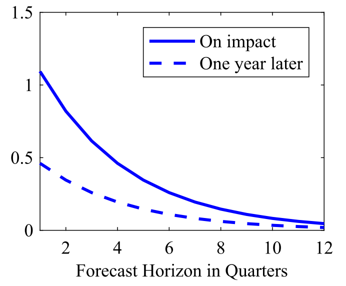

FIGURE 2.—Forecasts of terms of trade, following a confidence shock.

and that everybody forms about aggregate output. Figure 2 draws the average forecast at different horizons (namely, for $k \in \{1, \ldots, 12\}$), both at the moment the shock hits the economy (solid line) and four quarters later (dashed line).

As is evident in the figure, a positive innovation in ξ_t raises the expected terms of trade in the next few quarters *without* moving the corresponding expectations at longer horizons. The same point applies to the forecasts of the aggregate levels of output, hours, consumption, and investment. As time passes, the optimism fades away and the curve in Figure 2 shifts down. Nevertheless, the curve remains downward-sloping, underscoring that the waves of optimism (and pessimism) accommodated in our paper regard *exclusively* the short-term economic outlook.

This property is key to understanding the co-movement patterns documented in Figure 1. In the eyes of the firms, a positive innovation in ξ_t means a short-lived increase in the expected demand for their product. To take advantage of this, the firms raise their demand for both labor and capital, pushing the wage and the rental rate of capital up. As a result, the households experience a transitory increase in their income and in the returns to labor and capital. Because this entails only a small increase in permanent income, the wealth effect on labor supply is easily dominated by the competing substitution effect. This guarantees that hours, and hence also output and income, increase in equilibrium. Finally, because the boom is expected to be transitory, the households find it optimal to consume only a fraction of the realized increase in their income and to save the rest. All in all, the shock therefore causes a joint increase in hours, output, consumption, and investment, and without a commensurate shift in TFP and labor productivity, just as seen in Figure 1.

As noted in the Introduction, this mechanism is different from the one in the literature on news and noise shocks (Beaudry and Portier (2006), Jaimovich and Rebelo (2009), Lorenzoni (2009)). To illustrate the difference, consider Barsky and Sims (2012), an example of that literature that accommodates both news and noise shocks. The process of aggregate TFP is specified as follows:

$$A_t = A_{t-1} + \gamma_{t-1} + \varepsilon_{a,t} \quad \text{and} \quad \gamma_t = \rho_\gamma \gamma_{t-1} + \varepsilon_{\gamma,t},$$

where $\rho_\gamma \in (0, 1)$, and $\varepsilon_{a,t} \sim \mathcal{N}(0, \sigma_a^2)$ and $\varepsilon_{\gamma,t} \sim \mathcal{N}(0, \sigma_\gamma^2)$ are independent of one another and serially uncorrelated. Furthermore, the representative agent observes A_t perfectly, but only receives a noisy signal of γ_t. Finally, this signal is given by $z_t = \gamma_t + \eta_t$, where $\eta_t \sim \mathcal{N}(0, \sigma_\eta^2)$ is uncorrelated over time and independent of the current and past values of the innovations $\varepsilon_{a,t}$ and $\varepsilon_{\gamma,t}$. In this formulation, $\varepsilon_{\gamma,t}$ moves both the expectations and the actual realizations of future TFP, whereas $\varepsilon_{\eta,t}$ moves the expectations

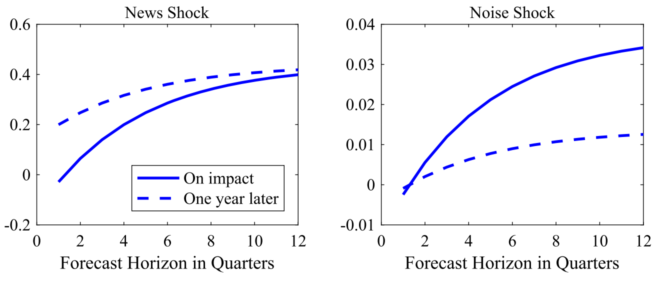

FIGURE 3.—Forecasts of output at different horizons, following a news and a noise shock.

without moving the actual realizations. The former represents a news shock, the latter a noise shock.

Figure 3 reports the impact of these shocks on the expectations of aggregate output at different horizons, both right after the realization of the shock (solid line) and four quarters later (dashed line). The left panel corresponds to the news shock, the right to the noise shock. By comparing the two panels, we see that the two shocks have qualitatively similar effects on impact. As time passes and more information arrives, the agents can tell whether the initial shift in their beliefs was due to a true increase in the long-run level of TFP or due to noise. This explains why the effects of the news shock get reinforced with the passage of time, while those of the noise fade away. The nature of optimism, however, is the same across these two cases—and it is very different from the one seen in Figure 2. While the confidence shock shifts the expectations of the short-term economic outlook, the news and noise shocks shift expectations of the medium- and long-run prospects.

It is precisely this difference that accounts for the superior quantitative performance of our mechanism. As already explained, the confidence shock triggers small shifts in expected permanent income and large shifts in the expected short-run returns to capital and labor. The opposite is true with the kind of news and noise shocks studied in the extant literature. When a positive news or noise shock hits the economy, the firms do not change their demand for labor and capital because they perceive no immediate change in their short-term returns, but the households reduce both their supply of labor and their saving because they expect higher wages and higher income in the future: a positive news shock is a good time both to consume more and to take a vacation. As a result, the equilibrium levels of employment and investment move in the opposite direction than that of consumption, which in turn explains why these shocks fail to generate realistic business cycles within baseline versions of either the RBC or the New Keynesian model.[16]

4.3. *Conditional Moments*

We have shown that the confidence shock produces transitory comovements in the key macroeconomic quantities, without commensurate movement in TFP and labor productivity. We have also offered the economic intuition for this result. But is this prediction consistent with the data?

[16]To overcome this challenge, Jaimovich and Rebelo (2009) augmented the baseline RBC model with adjustment costs that makes investment today increase in anticipation of higher investment in the future; and with a particular form of internal habit that generates a negative income effect on leisure in the short run. Lorenzoni (2009), on the other hand, abstracted from investment, added nominal rigidity, and let monetary policy induce pro-cyclical output gaps.

One could imagine answering this question by obtaining an empirical counterpart of ξ_t from surveys of higher-order beliefs of TFP. However, such surveys are not available. But even if they were available, they would only help under a literal, narrow interpretation of ξ_t, which is not our preferred way to think about the *applied* contribution of our paper. Instead, we believe that this contribution is maximized by interpreting ξ_t as a proxy for autonomous variation in the *first-order* beliefs of the endogenous economic outcomes over the business cycle—think of the expectations that the firms form about the demand for their products, or those that the consumers in turn form about their employment and income.

Because these expectations are part of the equilibrium and are jointly determined with the actual outcomes, it is unclear how one could identify ξ_t through, say, a SVAR approach analogous to those used in the identification of technology and monetary shocks. Lacking a better alternative, we thus proceed to evaluate the empirical performance of our theory in a more indirect way, by comparing two sets of conditional moments: those generated in our model by the confidence shock alone; and those observed in the data after filtering them from the effects of an empirical proxy of the technology shock. We view this comparison of conditional moments as a crucial test of any parsimonious theory that aspires to improve upon the baseline RBC model: if such a theory fails to account for the TFP-filtered "residuals" of the data, then it fails to achieve this objective.

We obtain the relevant component of the data in one of two ways. In the one, we regress each variable of interest on the current level and the four lags of TFP, as measured by Fernald (2014), and extract the residuals. In the other, we include all the variables in a SVAR; identify the technology shock as in Galí (1999), that is, as the only shock that exerts an effect on labor productivity in the long run; and then take the residuals from the projection of the data on the identified technology shock.

Although none of these approaches offers a bullet-proof identification of the technology shock, they generate macroeconomic variables that can be used to test parsimonious theories that seek to explain the business cycle with a single shock besides the standard technology shock.

The first two columns of Table I report the relevant moments in the data, namely, the business-cycle correlations and the relative volatilities of the aforementioned residuals, under the two specifications described above.[17] The third column reports the relevant moments in our model, namely, the correlations and relative volatilities induced by the confidence shock. The information contained in this column is, of course, the same as the one contained in the IRFs of the confidence shock: the shock causes hours, output, consumption, and investment to co-move, without commensurate co-movement in labor productivity. The next three columns report the corresponding moments for three other candidate shocks, which are often used in the literature as proxies for demand shocks: a discount-rate, or consumption-specific, shock; an investment-specific shock; and a news shock.[18] The last column considers a transitory shock to the efficiency wedge; this can be thought of as a proxy for the supply-side effects of financial or uncertainty shocks.[19]

[17]The moments have been computed on bandpass-filtered series at frequencies corresponding to 6–32 quarters. This filter is preferable to the simpler HP filter because it removes not only low-frequency trends but also high-frequency "noise" such as seasonal fluctuations and measurement error; see Stock and Watson (1999). Note, though, that the picture that emerges from Table I is not sensitive to the choice of the filter.

[18]To obtain the predictions of each of these alternative shocks, we maintain the parameterization of preferences and technologies and merely replace the confidence shock with the considered alternative.

[19]Such a shock is not removed by the specification used by Galí (1999), because that approach identifies only permanent technology shocks. It may also not be removed by our specification based on regressing the macroe-

TABLE I

CONDITIONAL COMOVEMENTS (6–32 QUARTERS)[a]

	Filtered Data		Our Theory	Alternative Theories			
	(a)	(b)	ξ Shock	I Shock	C Shock	News Shock	E Shock
σ_n/σ_y	1.05	1.29	1.43	1.44	1.44	0.74	0.59
σ_c/σ_y	0.63	0.41	0.25	1.21	1.19	0.35	0.19
σ_i/σ_y	3.35	4.04	3.92	8.93	8.93	5.06	4.26
$\sigma_{y/h}/\sigma_y$	0.45	0.63	0.44	0.51	0.49	0.37	0.43
$\mathrm{corr}(c, y)$	0.86	0.80	0.85	−0.92	−0.94	−0.17	0.65
$\mathrm{corr}(i, y)$	0.94	0.95	0.99	0.98	0.99	0.97	0.99
$\mathrm{corr}(n, y)$	0.91	0.88	0.99	0.98	0.98	0.95	0.99
$\mathrm{corr}(c, n)$	0.80	0.93	0.81	−0.98	−0.99	−0.46	0.51
$\mathrm{corr}(i, n)$	0.86	0.82	0.99	0.99	0.99	0.99	0.99
$\mathrm{corr}(c, i)$	0.73	0.76	0.78	−0.98	−0.98	−0.40	0.55
$\mathrm{corr}(y, y/n)$	0.12	−0.23	−0.96	−0.79	−0.84	0.78	0.97
$\mathrm{corr}(n, y/n)$	−0.31	−0.66	−0.98	−0.91	−0.92	0.56	0.92

[a]Columns (a) and (b) refer to the residuals that obtain, respectively, from the projection of the data on current and past TFP and from the removal of the technology shock identified in the same way as in Galí (1999). All other columns refer to theoretical predictions.

The main lesson that emerges from inspection of Table I is that the confidence shock does a good job in matching the conditional patterns in the data both absolutely and relatively to the other shocks. This is because none of the aforementioned demand shocks is able to generate positive co-movement between hours, consumption, and investment within the baseline RBC model; and the efficiency-wedge shock can generate such co-movement only by predicting a positive co-movement between hours and labor productivity, which is exactly the opposite of what is observed in the data.

As shown in Supplemental Material Appendix S.1, the same picture emerges if we consider a New Keynesian variant that adds sticky prices and lets monetary policy follow a realistic Taylor rule. In principle, these modifications help improve the empirical performance of the aforementioned kind of demand shocks by letting these shocks induce pro-cyclical output gaps, that is, by letting output increase relative to its flexible-price counterpart in response to positive demand shock. Yet, unless one adds various bells and whistles, the predicted output gaps are not large enough to undo the counterfactual co-movement properties of the underlying flexible-price allocations.

Of course, these findings do not mean that no other model can match the moments reported in the first two columns. For instance, DSGE models such as Smets and Wouters (2007) are able to do so by attributing the aforementioned residuals to the joint contribution of several shocks, despite the fact that none of these shocks can by itself generate the right co-movement patterns. Nevertheless, these findings illustrate in a simple and transparent manner that our theory does better relative to a number of comparable, parsimonious formalizations of either demand- or supply-driven fluctuations—a property that we view as valuable.

Additional support is provided by the evidence in a companion paper (Angeletos, Collard, and Dellas (2017)), where we use a SVAR approach to document that the bulk of the business-cycle volatility in output, hours, investment, and consumption in U.S. data

conomic quantities on current and past TFP to the extent that there is measurement error in the available TFP measure.

can be accounted for by a single shock whose IRFs look very much like those seen in Figure 1. A similar picture is also painted in Section 5, where the confidence shock emerges as the main driver of the business cycle within medium-scale DSGE models that contain multiple other shocks.[20]

4.4. *Wedges, Output Gaps, and Aggregate Demand*

We conclude this section by offering two additional perspectives on the empirical performance of our theory and its interpretation.

Suppose first that one approaches the data generated by our model through the lenses of the RBC model augmented with various wedges, as suggested by Chari, Kehoe, and McGrattan (2007). In Supplemental Material Appendix S.1, we show that the confidence shock manifests itself as a combination of wedges in the equilibrium conditions that characterize the behavior of the households and of the firms. This is true whether we consider the total wedges between the marginal rates of substitution and the corresponding marginal rates of transformation, or their household-side and firm-side components. What is more, the predicted wedges are consistent with those estimated in the data.

These findings speak to our theory's ability to capture the "residuals" between the data and the predictions of the baseline RBC model. More generally, they illustrate how higher-order uncertainty offers a theory of beliefs-driven wedges. The wedges emerge because, and only because, the agents use a distorted expectations operator relative to the complete-information, common-prior, fully-rational benchmark. The magnitude and correlation structure of these wedges are tied to the underlying structure of the market interactions and the degree of strategic complementarity. For instance, were we to shut down trade across islands in our own model, strategic complementarity and wedges would vanish.[21]

Suppose next that one tries to interpret the data generated by our model through the lenses of the New Keynesian framework. In our setting, prices are flexible. Yet, because firms make their input choices prior to observing the demand for their products, a drop in confidence can manifest itself as an increase in the realized markup. Furthermore, the resulting recession will register as a negative output gap insofar as the latter is measured

[20]Throughout this section, we have focused attention on comparing features of the data to theoretical counterparts that do not require us to parameterize the standard deviation of either the confidence shock or the technology shock: the IRFs seen in Figure 1, and the conditional correlations and relative volatilities reported in Table I, are invariant to the choice of σ_ξ and σ_a. But what about the ability of our baseline model to capture the unconditional moments of the data? Clearly, this depends on the choice of σ_ξ and σ_a. Suppose we pick σ_ξ and σ_a so as to minimize the distance between the unconditional volatilities of hours, output, consumption, and investment predicted by our baseline model from those found in the data. This exercise yields $\sigma_a = 0.79$ and $\sigma_\xi = 5.77$; it also attributes almost all of the volatility of hours to the confidence shock. We find these properties of our baseline model problematic for two reasons. First, σ_ξ is too large compared to σ_a, a property that questions the plausibility of the interpretation of ξ_t as a bias in the signals of aggregate TFP. (We thank a referee for pointing this out.) Second, our prior is that coordination frictions cannot possibly explain so much of the business cycle. In Section 5, we alleviate the first concern, not only by allowing for other shocks to absorb part of the volatility in the data, but also by modifying the degree of strategic complementarity. This concern can also be alleviated by reinterpreting ξ_t as a shock to higher-order beliefs of idiosyncratic fundamentals, and thereby to first-order beliefs of idiosyncratic terms of trade, along the lines discussed in Angeletos and La'O (2013), Huo and Takayama (2015b), and Section 3.3 of our paper. Regarding the second concern, we are open to the idea that our mechanism is proxying for other forces, whose effects are similar to those of the confidence shock but whose micro-foundations remain to be discovered.

[21]These points indicate the relation between our paper and recent work that considers other forms of belief distortions, such as Ilut and Saijo (2017), Bhandari, Borovicka, and Ho (2016), and Pei (2017).

relative to the frictionless RBC benchmark, a property clearly illustrated by the example in Section 3.3. Consequently, an adverse confidence shock in our setting looks like a negative demand shock in the New Keynesian model.

Nonetheless, there is an important difference: in our setting, fluctuations in this output gap can arise without any variation in inflation. This is because our mechanism does not need to satisfy the restriction between the output gap and inflation imposed by the New Keynesian Phillips Curve, or its ancestors. We view this as an advantage of our theory because the aforementioned restriction receives little support from the data, as the empirical literature on Phillips curves has demonstrated; see Mavroeidis, Plagborg-Møller, and Stock (2014) for a review.

The evidence provided in a companion paper (Angeletos, Collard, and Dellas (2017)) also speaks against a Phillips-curve-centric explanation of the business cycle and in favor of a mechanism like the one accommodated here: in that paper, we use a SVAR approach to document that the bulk of the business-cycle variation in employment, output, investment, and consumption can be explained by a single shock that can be thought of as a "non-inflationary demand shock" in the sense that it triggers strong co-movement between the aforementioned quantities at the business-cycle frequencies without commensurate co-movements in either TFP and labor productivity or inflation at any frequency. What is more, the empirical IRFs of the shock identified in that paper are actually quite similar to the theoretical IRFs of the confidence shock in the present paper.

With this backdrop, we like to interpret our confidence shock as a form of demand shock that does not hinge either on nominal rigidity or on the inflation-output nexus of the Keynesian paradigm.[22] One may, however, object to this interpretation on the following grounds. In our model, employment and output are fixed in the morning of each period, whereas consumption and investment are determined in the afternoon. In this sense, supply is determined first and prices adjust to make demand meet supply. By contrast, the Keynesian paradigm assumes that prices are determined first and supply adjusts to meet demand.

We now demonstrate that reversing the timing of decisions in our model does not change the nature of the business-cycle fluctuations generated by the confidence shock. In particular, we consider a variant of our model that has consumption and investment be fixed in the morning of each period and that lets employment and output adjust in the afternoon. This variant permits us to capture the Keynesian concept that "demand drives supply."

Consider Figure 4. The solid red lines repeat the IRFs of the baseline model (previously reported in Figure 1). The blue crosses report the IRFs of the present variant. With the exception of consumption, where there is only a modest difference, the IRFs of the two models line up almost perfectly on top of each other. Not surprisingly, this similarity extends to the kind of business-cycle moments we reported earlier in Table I.

Let us explain why. In the baseline, supply-first version of our model, a positive confidence shock causes employment and output to increase in the morning. The overall spending therefore *has* to increase in the afternoon. Its composition, however, is free to adjust. The reason that consumption and investment co-move at that point is that the optimism applies only to the short run—which is also the reason why employment increases in the first place during the morning. In the demand-first variant, consumption and investment are determined first. The only reason that they both increase in response to a

[22]In this regard, our work is related to that of Beaudry and Portier (2013), which offers a different theory of non-inflationary demand shocks.

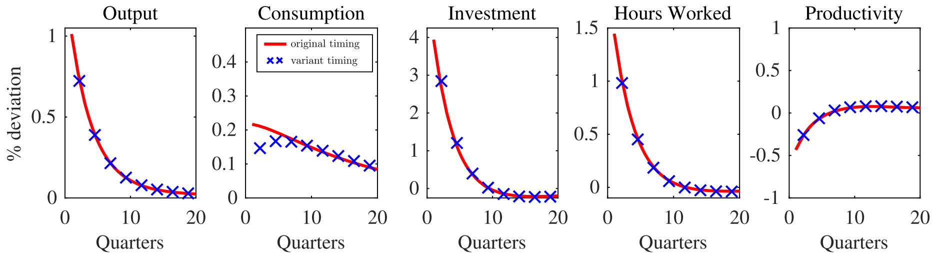

FIGURE 4.—Impulse responses to a positive confidence shock, under different timing protocols.

positive confidence shock is, once again, that the shock causes the agents to become optimistic about the short run. If, instead, the shock caused the agents to become optimistic about income in the medium to long run, the agents would like to borrow against their future income, so consumption and investment would move in the opposite direction.

We conclude that our mechanism is not unduly sensitive to the timing protocol and, in this sense, to whether output is supply- or demand-determined. Each protocol, however, is useful for different purposes. On the one hand, the protocol used in our baseline model is the same as the one assumed in the related works of Angeletos and La'O (2013), Benhabib, Wang, and Wen (2015), Huo and Takayama (2015b), and Ilut and Saijo (2017). On the other hand, the variant introduced here better captures the concept of demand-driven fluctuations. It also has two advantages in the context of the medium-scale DSGE models considered in the next section. First, it is more suitable in the presence of adjustment frictions in consumption and investment. And second, it boosts the degree of strategic complementarity, helping generate larger macroeconomic fluctuations out of the same variation in higher-order beliefs.[23]

5. EXTENSION AND ESTIMATION

In this section, we apply our method to two medium-scale DSGE models, which are estimated using U.S. data. This requires the introduction of various bells and whistles, which are standard in the DSGE literature but are at odds with our modeling tastes as well as with the microeconomic evidence. The main goal of this section is therefore, not to write and estimate our preferred models, but rather to illustrate the potential robustness of our theoretical mechanism as we move from the baseline RBC model to richer DSGE models, and as we switch on and off the role of nominal rigidities and monetary policy.

5.1. *Two Medium-Scale Models*

We start with a brief description of the main features of the two models. A more detailed description and the relevant equations can be found in the Appendix.

In order to accommodate price-setting behavior, we let each island contain a large number of monopolistic firms, each of which produces a differentiated commodity. These

[23]To understand the first point, note that, if it is very costly to adjust consumption and investment, a period of temporarily high returns to labor could be a good time to take vacation. This raises the possibility that optimism could trigger a recession in the supply-first version. To understand the second point, consider the knife-edge case in which the income and the substitution effects on labor supply cancel each other out. This rules out belief-driven fluctuations in the supply-first version. The demand-first version avoids both issues by having expectations drive demand, which in turn drives supply.

commodities are combined through a CES aggregator into an island-specific composite good, which in turn enters the production of the final good in the mainland through another CES aggregator. The elasticity parameter in the first aggregator is denoted by η and pins down the monopoly markup; the one in the second aggregator is denoted by ϱ and controls, in conjunction with all the other preference and technology parameters, the degree of strategic complementarity across the islands.[24]

In one of the two models, firms are free to adjust their price in each and every period, after observing the realized demand for their product (the flexible-price model). In the other, firms can only adjust prices infrequently, in the familiar Calvo fashion (the sticky-price model). The latter model also contains a conventional Taylor rule for monetary policy.

In order to let other business-cycle drivers compete with our mechanism, we include several additional shocks: a permanent and a transitory TFP shock; a permanent and a transitory investment-specific shock; a news shock regarding future productivity; a transitory discount-rate shock; a government-spending shock; and, in the sticky-price model, a monetary shock.[25]

We finally introduce adjustment costs in investment and habit persistence in consumption, of the type assumed in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). These features lack supporting microeconomic evidence. They have nevertheless become standard in the DSGE literature because they serve, not only as sources of persistence, but also as mechanisms that help improve the co-movement implications of certain shocks, including investment-specific, discount-rate, and news shocks. Their inclusion makes our results more easily comparable to those in the literature and gives these competing shocks a better chance to outperform the confidence shock.

5.2. *Estimation*

We estimate our models using Bayesian maximum likelihood in the frequency domain, focusing on business-cycle frequencies. The method is described in the Appendix. Here, we discuss briefly the rationale behind this empirical strategy, the data used, and the priors and the posteriors.

Rationale

The models described above—like other business-cycle models—cater to business-cycle phenomena and therefore omit shocks and mechanisms that may account for medium- to long-run phenomena, such as trends in demographics and labor-market participation, structural transformation, regime changes in productivity growth or inflation, and so on.

[24]The baseline model is nested with $\eta = 0$ and $\varrho = 1$. Letting $\eta > 0$ accommodates monopoly power. Letting $\varrho \neq 1$ helps parameterize the degree of strategic complementarity.

[25]The motivation for the inclusion of these particular shocks is as follows. First, previous research has argued that investment-specific technology shocks are at least as important as neutral, TFP shocks (Fisher (2006)). Second, monetary, fiscal, and transitory discount-rate or investment-specific shocks, as well as news shocks, have been proposed as formalizations of the notion of "aggregate demand shocks" within the New Keynesian framework. Third, transitory TFP, investment-specific, or discount-rate shocks are often used as proxies for financial frictions that lead to, respectively, misallocation, a wedge in the firm's investment decisions, or a wedge in the consumer's saving decisions; see Christiano, Eichenbaum, and Trabandt (2015) for a recent example of these shortcuts. Fourth, the introduction of multiple transitory shocks, whatever their interpretation, increases the chance that these shocks will pick up the transitory fluctuations in the data, leaving less to be accounted for by our mechanism.

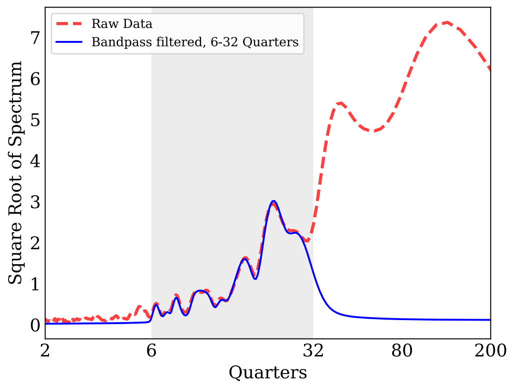

FIGURE 5.—Spectral density of hours, 1960Q1–2007Q4.

Because of this omission, estimating our models by simple maximum likelihood is likely to lead to erroneous inferences about their business-cycle properties. This is because the estimation will guide the parameters of the model towards matching all the frequencies of the data, as opposed to only those that pertain to business-cycle phenomena. In a nutshell, there is a risk of contamination of the estimates of a model by frequencies that the model was not designed to capture.

This problem was first discussed by Hansen and Sargent (1993) and Sims (1993) in the context of seasonal adjustment, but the logic applies more generally. Sala (2015) has recently documented the relevance of this problem for standard DSGE practice: estimating the model of Smets and Wouters (2007) over different frequency bands leads to different estimates of the model's impulse responses and of the underlying parameters, a fact that underscores the importance of making a judicious selection of the band of frequencies used to estimate the model.

Figure 5 indicates that this concern may be particularly relevant in the context of the exercise carried out in this section. This figure inspects the spectral density of hours.[26] The red line corresponds to the raw data; the blue line results from application of a bandpass filter that keeps only the business-cycle frequencies, namely, those ranging from 6 to 32 quarters. The figure reveals substantial movements at the medium- and long-run frequencies. Such movements may originate from changes in demographics or in the labor-market participation of women, structural transformation, and other mechanisms which our models have neither hope nor ambition to capture.

There are two possible ways to try to mitigate the problem. One is to add the missing mechanisms that would enable the model(s) to account for all the frequencies at once. Another is to estimate the model(s) on the basis of only the business-cycle frequencies. We follow the latter route because of two reasons. First, while we believe that our mechanism and the models considered in this paper are useful for understanding business-cycle phenomena, we are relatively less confident about the "right" choice of mechanisms that can account for the medium- to long-term phenomena; adding the "wrong" mechanisms could aggravate the misspecification problem. Second, we believe that low frequencies

[26]The spectrum is computed as the smoothed periodogram, a Hamming window with a bandwidth parameter of 15 is used, and the x-axis is represented in periods rather than frequencies to ease interpretation. A similar figure appears in Beaudry, Galizia, and Portier (2015), although that paper uses it towards a different goal: to motivate a model that actually connects the short to the medium run.

of the data contain relatively little information about the business-cycle properties of the model, especially those that regard the confidence shock or any other transitory shock; inclusion of the low frequencies is therefore more likely to contaminate, than to improve, the estimation of the business-cycle properties.

Data

The data used in the estimation include GDP, consumption, investment, hours worked, the inflation rate, and the federal fund rate for the period 1960Q1 to 2007Q4; a detailed description is in Supplemental Material Appendix S.2. The first four variables are in logs and linearly de-trended; the remaining two are in percentage points. Our sticky-price model is estimated on the basis of all these six variables, while the flexible-price model is estimated on the basis of real quantities only (GDP, consumption, investment, and hours). The rationale is that the latter model is not designed to capture the properties of nominal data.

Remark on ϱ and σ_ξ

A challenge faced in the estimation of the two models is the following. Consider the parameter ϱ. Holding constant all the other parameters, this parameter governs the degree of strategic complementarity across the islands. In so doing, this parameter also governs the magnitude of the response of the macroeconomic quantities to the confidence shock, without, however, affecting their covariation structure. It follows that this parameter cannot be identified separately from σ_ξ, the standard deviation of the confidence shock, on the basis of the macroeconomic time series alone.

For our main estimation exercise, we fix ϱ exogenously at 0.75; this yields an estimate for σ_ξ that is lower than the estimated volatility in aggregate TFP. In Supplemental Material Appendix S.4, we motivate this choice with an exercise that tries to identify both parameters jointly by combining the macroeconomic time series with the time series of the University of Michigan Index of Consumer Sentiment and by making an assumption about how to extract the expectations that are relevant for our theory from that index. This leads to an estimate of ϱ that is in the neighborhood of 0.75 and to results that are similar to those reported below.

However, we do not wish to push this exercise too far, because it hinges on delicate assumptions about the mapping between that index and our theory. We thus invite the reader to adopt a broader perspective in thinking about what the estimation results mean for our theory, namely, that they illustrate that the considered models can match the data with plausible assumptions about the magnitude of the underlying higher-order uncertainty, but leave unanswered the delicate question of whether and how additional discipline in the estimation of the confidence shock could be provided from sources outside the standard macroeconomic time series.

Priors and Posteriors

The priors and the estimated values of all the parameters are reported in Table VIII in Supplemental Material Appendix S.3 and are broadly in line with the literature. Posterior distributions were obtained with the MCMC algorithm. The estimated values of the preference, technology, and monetary parameters are similar to those found in the literature, an indication that the only essential difference from the state of the art is the accommodation of the confidence shock.

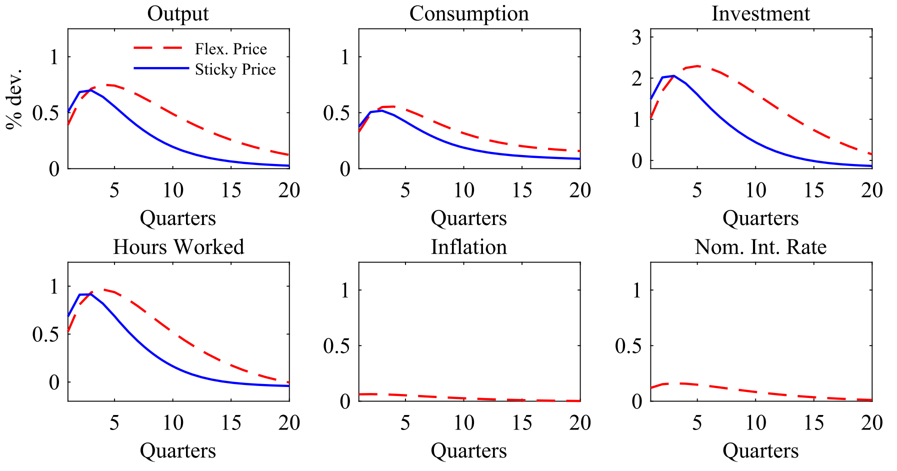

FIGURE 6.—Theoretical IRFs to confidence shock.

5.3. *Results*

We now review the main findings. A few additional results are presented in Supplemental Material Appendix S.3.

The Confidence Shock

Figure 6 reports the estimated IRFs to a positive confidence shock. The solid blue lines correspond to the flexible-price model, the red dashed lines to the sticky-price model.

As far as real quantities are concerned, the IRFs are similar across the two models, as well as similar to those in our baseline model. The introduction of investment-adjustment costs and consumption habit adds a hump but does not alter the co-movement patterns found in the baseline model. This underscores the robustness of the key positive implications of our mechanism as we move across RBC and New Keynesian settings, or, as we add various bells and whistles.

The top half of Table II reports the estimated contribution of the confidence shock to the volatility of the key macroeconomic variables at business-cycle frequencies (6–32 quarters). Despite all the competing shocks, the confidence shock emerges as the single most important source of volatility in real quantities. For example, the confidence shock

TABLE II

CONTRIBUTION OF CONFIDENCE SHOCK (6–32 QUARTERS)

Variances	Y	C	I	N	π	R
Flexible prices	54.72	70.21	41.60	68.32	–	–
Sticky prices	51.28	61.95	38.50	64.15	11.64	40.84
Covariances	(Y, N)	(Y, I)	(Y, C)	(N, I)	(N, C)	(I, C)
Flexible prices	74.88	53.74	78.49	66.41	105.08	94.73
Sticky prices	68.10	50.83	70.95	58.29	104.26	94.89

accounts for 55% of the business-cycle volatility in output in the flexible-price model, and for 51% in the sticky-price model.

The bottom half of Table II completes the picture by reporting the estimated contribution of the confidence shock to the *covariances* of output, hours, investment, and consumption. The confidence shock is, by a significant margin, the main driving force behind the co-movement of all these variables, underscoring once again the ability of our theory to capture this co-movement. In particular, confidence shock explains *more* than one hundred percent of the covariance between hours and consumption, precisely because, as anticipated in the previous section, many of the other structural shocks tend to generate the opposite co-movement than the one seen in the data.

Are these findings too good to be true? It depends on how one reads them. In our eyes, they do not mean that our theory is the "true" explanation of the business cycle. They nevertheless reinforce the lessons of the previous section: not only is our theory consistent with salient features of the data, but it is also more potent than other, more familiar, structural interpretations of the data.

Business-Cycle Moments

Table III reports some key moments of the data (first column); those predicted by our estimated models (second and third column); and, for comparison purposes, those predicted by the model in Smets and Wouters (2007) (fourth column). Inspection of this table leads to the following conclusions. First, both of our models do a good job on the real side of the economy. Second, our sticky-price model does a good job in matching also the nominal side of the data. Finally, our sticky-price model appears to outperform the model of Smets and Wouters (2007) in terms of matching the moments of the real quantities as well as the correlations of the nominal variables with output and hours. Of course, this does not mean that our model is as good as theirs in, say, matching the responses to identified monetary shocks or in out-of-sample forecasting. It nevertheless indicates that the inclusion of our mechanism in New Keynesian models does not interfere with their

TABLE III

MOMENTS (6-32 QUARTERS)[a]

	Data	FP	SP	SW	Data	FP	SP	SW
		Standard Deviations				*Correlations With Output*		
Y	1.41	1.28	1.36	1.42				
I	5.12	4.46	4.88	4.86	0.94	0.88	0.86	0.74
N	1.56	1.59	1.66	0.97	0.87	0.82	0.83	0.81
C	0.76	0.82	0.91	1.11	0.85	0.78	0.77	0.67
Y/N	0.76	0.91	0.90	0.84	0.07	−0.02	−0.03	0.74
π	0.23	–	0.25	0.34	0.21	–	0.37	0.13
R	0.35	–	0.34	0.35	0.33	–	0.54	0.06
		Correlations With Investment				*Correlations With Hours*		
N	0.82	0.79	0.82	0.67				
C	0.73	0.56	0.47	0.30	0.83	0.65	0.58	0.59
Y/N	0.07	−0.14	−0.21	0.47	−0.43	−0.58	−0.56	0.22
π	0.09	–	0.41	0.18	0.44	–	0.48	0.23
R	0.23	–	0.60	0.23	0.61	–	0.70	0.21

[a] FP and SP stand for our estimated flexible- and sticky-price models, respectively. SW stands for the model in Smets and Wouters (2007).

TABLE IV

POSTERIOR ODDS OF MODEL A VS MODEL B

	Model A	
Model B	Sticky, without	Sticky, with
Flexible prices, without confidence	1.00	1.00
Flexible prices, with confidence	0.36	0.84
Sticky prices, without confidence	–	0.90

ability to capture the nominal side of the data and that our mechanism itself is robust to the introduction of realistic nominal rigidities.

5.4. *On Demand-Driven Business Cycles*

We now explore how our mechanism, viewed as a formalization of demand-driven business cycles, compares to that of the New Keynesian model.

To this goal, Table IV reports the posterior odds of four models, starting from a uniform prior and estimating them on the real data only. The models differ on whether they assume flexible or sticky prices, and on whether they contain the confidence shock or not. We concentrate on the real data, not only because the flexible-price models are not designed to capture the nominal variables, but also because we wish to evaluate both kinds of models on the basis of the co-movements of the real quantities. Once we drop the nominal data for this exercise, the nominal parameters of the sticky-price models are not well identified. We have thus chosen to fix these parameters at the values that obtained when the models were estimated on both real and nominal data. We nevertheless re-estimate the preference and technology parameters and the shock processes in order to give each model a fair chance to match the data on the real quantities.

Consider first the models that abstract from the confidence shock. In this case, the sticky-price model wins: the posterior odds that the data are generated by that model are nearly 100%. But once the flexible-price model is augmented with the confidence shock, the odds of the sticky-price model fall below 50%, to 36%. By this metric, our mechanism appears to be more potent than the New Keynesian mechanism when the two are viewed in isolation. Finally, the sticky-price model that contains the confidence shock wins 90–10 over the sticky-price model that excludes it. By this metric, the inclusion of our mechanism improves significantly the performance of the New Keynesian model.

We interpret these results as follows. Insofar as we abstract from monetary phenomena, our approach emerges as a potent substitute for the New Keynesian formalization of demand-driven fluctuations. Perhaps more fruitfully, our approach can complement the New Keynesian framework by offering what, in our view, is a more appealing structural interpretation of the observed business cycles—one that attributes the "deficiency in aggregate demand" during a recession in part to a coordination failure and to lack of confidence.

6. CONCLUSION

By relying on the rational-expectations solution concept together with the auxiliary assumption that all agents share the same information about the aggregate state of the economy, standard macroeconomic models impose a rigid structure on how agents form beliefs about endogenous economic outcomes and how they coordinate their behavior. In this paper, we propose a certain relaxation of this structure and explore its quantitative implications.

In particular, we develop a method for augmenting macroeconomic models with a tractable form of higher-order belief dynamics. We argue that this method helps proxy for the effects of incomplete information and frictional coordination and can be used to accommodate a certain kind of waves of optimism and pessimism about the short-term outlook of the economy. We document the quantitative importance of such waves within the context of RBC and New Keynesian models of both the textbook and the medium-scale variety.

We believe that our paper adds to the understanding of business-cycle phenomena along the following dimensions:

- It highlights the distinct role played by expectations of the short-run prospects of the economy, as opposed to expectations of productivity and growth in the medium to long run.
- It offers a parsimonious explanation of salient features of the macroeconomic data and does so in a manner that appears to outperform alternative narratives found in the literature.
- It offers a formalization of the notion of demand-driven fluctuations that is both conceptually and empirically distinct from the one found in the New Keynesian paradigm.
- It leads to a structural interpretation of the observed recessions that attributes a significant role to "coordination failures," "lack of confidence," or "market sentiment."

These findings naturally raise the question of where the variation in confidence comes from. Having attributed this variation to an extrinsic shock, we cannot offer a useful answer to this question. Nevertheless, our analysis has revealed the potential importance of two previously overlooked forces, namely, frictional coordination and belief waves regarding the short-term economic outlook, and so it can provide the impetus for future research on these subjects.

There is an emerging literature in this area. Ilut and Saijo (2017) and Angeletos and Lian (2018) considered models that feature a similar kind of belief-driven wedges as the one found here, except that these wedges are allowed to co-vary with conventional structural shocks; this has the interesting implication that a drop in confidence may be triggered by an adverse financial shock, while a boost in confidence may be accomplished by a fiscal stimulus. Huo and Takayama (2015b) obtained quantitative findings that are broadly consistent with ours while maintaining the common-prior assumption. Angeletos, Collard, and Dellas (2017) provided VAR-based evidence that the business cycle in the U.S. data can be explained by a shock that has similar properties to the one we have accommodated in our theory. Levchenko and Pandalai-Nayar (2017) provided additional corroborating evidence in an international context.

Finally, it is worth iterating how the belief waves formalized and quantified in this paper compare to those found in the existing literature on news and noise shocks. In that literature, recessions are periods in which the agents expect the economy to do badly for a long time, and more so in the long run than in the short run; in our paper, they are periods in which the agents expect the economy to recover after a few years. Future work could shed further light on which kind of expectations—those regarding the long run versus those regarding the short run—is more relevant empirically.

APPENDIX: ESTIMATED MODELS

In this appendix, we fill in the details of the two models studied in Section 5. We next describe the estimation method, the assumed priors, and the obtained posteriors. We finally review a few additional findings that were omitted from the main text.

The Details of the Two Models

As mentioned in the main text, the two models share the same backbone as our baseline model, but add a number of structural shocks along with certain forms of habit persistent in consumption and adjustment costs in investment, as in Christiano, Eichenbaum, and Evans (2005) and Smets and Wouters (2007). To accommodate monopoly power and sticky prices, we also introduce product differentiation within each island. We finally assume that there exists a lump-sum transfer that eliminates the effects of the markup rate in steady state.

Fix an island $i \in [0, 1]$. Index the firms in this island by $j \in [0, 1]$ and let y_{ijt} denote the output produced by firm j in period t. The composite output of the island is given by

$$y_{it} = \left(\int_0^1 y_{ijt}^{\frac{1}{1+\eta}} \, \mathrm{d}j \right)^{1+\eta},$$

where $\eta > 0$ is a parameter that pins down the monopoly power. The aggregate quantity of the final good, on the other hand, is given by

$$Y_t = \left(\int_0^1 y_{it}^{1-\varrho} \, \mathrm{d}i \right)^{\frac{1}{1-\varrho}},$$

where $\varrho > 0$ is a parameter that ultimately governs the degree of strategic complementarity.

The technology is the same as before, so that firm j's output in island i is

$$y_{ijt} = \exp(\zeta_t^{\mathrm{A}}) (u_{ijt} k_{ijt})^{\alpha} n_{ijt}^{1-\alpha};$$

but now TFP is given by the sum of a permanent and a transitory component. More specifically,

$$\zeta_t^{\mathrm{A}} = a_t^{\tau} + a_t^{p},$$

where a_t^{τ} is the transitory component, modeled as an AR(1), and a_t^{p} is given by

$$a_t^{p} = a_{t-1}^{p} + a_{t-1}^{n} + \varepsilon_t^{\mathrm{P}},$$

where ε_t^{p} is the unanticipated innovation and a_{t-1}^{n} captures all the TFP changes that agents anticipated in earlier periods. The latter is given by an AR(1) process of the form

$$a_t^{n} = \rho_n a_{t-1}^{n} + \varepsilon_t^{n},$$

where ε_t^n is the innovation to the anticipated component of TFP.[27] In line with our baseline model, the confidence shock is now modeled as a shock to higher-order beliefs of a_t^p.

To accommodate for a form of habit in consumption as well as discount-rate shocks, we let the per-period utility be as follows:

$$u(c_{it}, n_{it}; \zeta_t^c, C_{t-1}) = \exp(\zeta_t^c)\left(\log(c_{it} - bC_{t-1}) - \theta \frac{n_{it}^{1+\nu}}{1+\nu}\right),$$

where ζ_t^c is a transitory preference shock, modeled as an AR(1), $b \in (0, 1)$ is a parameter that controls for the degree of habit persistence, and C_{t-1} denotes the aggregate consumption in the last period.[28]

To accommodate permanent shocks to the relative price of investment as well as transitory shocks to government spoending, we let the resource constraint of the island be given by the following:

$$c_{it} + \exp(\zeta_t^{\mathrm{IP}})i_{it} + G_t + \exp(\zeta_t^{\mathrm{IP}})\Psi(u_{it})k_{it} = p_{it}y_{it},$$

where ζ_t^{IP} measures the cost of investment, G_t is government spending, and $\exp(\zeta_t^{\mathrm{IP}})\Psi(u_{it})$ is the cost of utilization per unit of capital. The latter is scaled by $\exp(\zeta_t^{\mathrm{IP}})$ in order to transform the units of capital to units of the final good, and thereby also guaranteed a balanced-growth path. ζ_t^{IP} is modeled as a random walk: $\zeta_t^{\mathrm{IP}} = \zeta_{t-1}^{\mathrm{IP}} + \varepsilon_t^{\mathrm{IP}}$. Literally taken, this represents an investment-specific *technology* shock. But since our estimations do not include data on the relative price of investment, this shock can readily be reinterpreted as a demand-side shock. The utilization-cost function satisfies $u\Psi''(u)/\Psi'(u) = \frac{\psi}{1-\psi}$, with $\psi \in (0, 1)$, and government spending is given by $G_t = \bar{G}\exp(\tilde{G}_t)$, where $\bar{G}$ is a constant and $\tilde{G}_t = \zeta_t^g + \frac{1}{1-\alpha}a_t^p - \frac{\alpha}{1-\alpha}\zeta_t^{\mathrm{IP}}$. In this equation, ζ_t^g denotes a transitory shock, modeled as an AR(1), and the other terms are present in order to guarantee a balanced-growth path.

Finally, to accommodate adjustment costs to investment as well as transitory investment-specific shocks, we let the law of motion of capital on island i take the following form:

$$k_{it+1} = \exp(\zeta_t^{\mathrm{IT}})i_{it}\left(1 - \Phi\left(\frac{i_{it}}{i_{it-1}}\right)\right) + (1-\delta)k_{it}.$$

We impose $\Phi'(\cdot) > 0$, $\Phi''(\cdot) > 0$, $\Phi(1) = \Phi'(1) = 0$, and $\Phi''(1) = \varphi$, so that φ parameterizes the curvature of the adjustment cost to investment. ζ_t^{IT} is a temporary shock, modeled as an AR(1) and shifting the demand for investment, as in Justiniano, Primiceri, and Tambalotti (2010).

This completes the description of the flexible-price model of Section 5. The sticky-price model is then obtained by embedding the Calvo friction and a Taylor rule for monetary policy. In particular, the probability that any given firm resets its price in any given period is given by $1-\chi$, with $\chi \in (0, 1)$. As for the Taylor rule, the reaction to inflation is given by $\kappa_\pi > 1$, the reaction to the output gap is given by $\kappa_y > 0$, and the parameter that controls the degree of interest-rate smoothing is given by $\kappa_R \in (0, 1)$; see condition (6.12) below.

[27]We have experimented with alternative forms of diffusion, as well as with specifications such as $\zeta_t^n = \varepsilon_{t-4}^n$, and we have found very similar results.

[28]Note that we are assuming that habit is external. We experimented with internal habit, as in Christiano, Eichenbaum, and Evans (2005), and the results were virtually unaffected.

In the sticky-price model, the log-linear version of the set of the equations characterizing the equilibrium is thus given by the following:

$$\begin{aligned}\mathbb{E}_{it}\big[\zeta_t^c+\nu\tilde{n}_{it}\big]&=\zeta_t^c-\frac{1}{1-b}\tilde{c}_{it}\\&\quad+\frac{b}{1-b}\tilde{C}_{t-1}\\&\quad+\mathbb{E}_{it}\big[\tilde{s}_{it}+\varrho\tilde{Y}_t+(1-\varrho)\tilde{y}_{it}-\tilde{n}_{it}\big],\end{aligned}\tag{6.1}$$

$$\begin{aligned}\mathbb{E}_{it}[\tilde{\lambda}_{it}+\tilde{q}_{it}]&=\mathbb{E}_{it}\big[\tilde{\lambda}_{it+1}+\beta(1-\delta)\tilde{q}_{it+1}\\&\quad+\big(1-\beta(1-\delta)\big)\big(\tilde{s}_{it+1}+\varrho\tilde{Y}_{t+1}\\&\quad+(1-\varrho)\tilde{y}_{it+1}-\tilde{u}_{it+1}-\tilde{k}_{it+1}\big)\big],\end{aligned}\tag{6.2}$$

$$\tilde{y}_{it}=a_t+\alpha(\tilde{u}_{it}+\tilde{k}_{it})+(1-\alpha)\tilde{n}_{it},\tag{6.3}$$

$$Z_t+\frac{1}{1-\psi}\tilde{u}_{it}=\tilde{s}_{it}+\varrho\tilde{Y}_t+(1-\varrho)\tilde{y}_{it}-\tilde{k}_{it},\tag{6.4}$$

$$\varrho\tilde{Y}_t+(1-\varrho)\tilde{y}_{it}=s_c\tilde{c}_{it}+(1-s_c-s_g)\big(\zeta_t^{\mathrm{IP}}+\tilde{\imath}_{it}\big)+s_g\tilde{G}_t+\alpha\tilde{u}_{it},\tag{6.5}$$

$$\tilde{k}_{it+1}=\delta\big(\zeta_t^{\mathrm{IT}}+\tilde{\imath}_{it}\big)+(1-\delta)\tilde{k}_{it},\tag{6.6}$$

$$\tilde{q}_{it}=(1+\beta)\varphi\tilde{\imath}_{it}-\varphi\tilde{\imath}_{t-1}-\beta\varphi\mathbb{E}'_{it}\tilde{\imath}_{it+1}+\zeta_t^{\mathrm{IP}}-\zeta_t^{\mathrm{IT}},\tag{6.7}$$

$$\tilde{\lambda}_{it}=\zeta_t^c-\frac{1}{1-b}\tilde{c}_{it}+\frac{b}{1-b}\tilde{C}_{t-1},\tag{6.8}$$

$$\tilde{R}_t=\zeta_t^c-(1+\nu)\tilde{n}_{it}-\tilde{s}_{it}-\varrho Y_t-(1-\varrho)y_{it}\tag{6.9}$$

$$\quad-\mathbb{E}'_{it}[\tilde{\lambda}_{it+1}-\tilde{\pi}_{it+1}],\tag{6.10}$$

$$\tilde{x}_{it}=s_c\tilde{c}_{it}+(1-s_c-s_g)\big(\zeta_t^{\mathrm{IP}}+\tilde{\imath}_{it}\big)+s_g\tilde{G}_t,\tag{6.11}$$

$$\tilde{R}_t=\kappa_R\tilde{R}_{t-1}+(1-\kappa_R)\big(\kappa_\pi\tilde{\pi}_{it}+\kappa_y\big(\tilde{x}_{it}-\tilde{x}_{it}^F\big)\big)+\zeta_t^m,\tag{6.12}$$

$$\chi\big(1+\chi(1-\beta)\big)\tilde{\pi}_{it}=(1-\chi)(1-\beta\chi)\tilde{s}_{it}+\beta\chi(1-\chi)\tilde{\Pi}_t+\beta\chi\mathbb{E}'_{it}\tilde{\pi}_{it+1},\tag{6.13}$$

where uppercases stand for aggregate variables, λ_{it} and s_{it} denote, respectively, the marginal utility of consumption and the realized markup in island i, $\tilde{\pi}_{it}\equiv\tilde{p}_{it}-\tilde{p}_{it-1}$ and $\tilde{\Pi}_t\equiv\tilde{P}_t-\tilde{P}_{t-1}$ denote, respectively, the local and the aggregate inflation rate, x_{it} denotes the measured of GDP on island i, X_{it}^F denotes the GDP that would be attained in a flexible-price allocation, and s_c and s_g denote the steady-state ratios of consumption and government spending to output.

The interpretation of the above system is straightforward. Conditions (6.1) and (6.2) give, respectively, the consumption and investment decisions. Conditions (6.3) and (6.4) characterize the equilibrium employment and utilization levels. Condition (6.5) gives the local resource constraint. Conditions (6.6) and (6.7) give the local law of motion of capital and the equilibrium price of capital. Conditions (6.8) and (6.10) give the marginal utility of consumption and the optimal bond holdings decision. Condition (6.11) gives the measured aggregate GDP. Condition (6.12) gives the Taylor rule for monetary policy. Finally, condition (6.13) gives the inflation rate in each island; aggregating this condition across islands gives our model's New Keynesian Phillips Curve. The only essential novelty in all the above is the presence of the subjective expectation operators in the conditions characterizing the local equilibrium outcomes of each island.

Finally, the flexible-price allocations are obtained by the same set of equations, modulo the following changes: we set $s_{it} = 0$, meaning that the realized markup is always equal to the optimal markup; we restate the Euler condition (6.10) in terms of the real interest rate; and we drop the nominal side of this system, namely, conditions (6.12) and (6.13).

Estimation

As mentioned in the main text, we follow Christiano and Vigfusson (2003) and Sala (2015) and estimate the model using a Bayesian maximum likelihood technique in the frequency domain. This method amounts to maximizing the following posterior likelihood function:

$$\mathcal{L}(\theta|\mathcal{Y}_T) \propto f(\theta) \times L(\theta|\mathcal{Y}_T),$$

where $\mathcal{Y}_T$ denotes the set of data (for $t = 1, \dots, T$) used for estimation, θ is the vector of structural parameters to be estimated, $f(\theta)$ is the joint prior distribution of the structural parameters, and $L(\theta|\mathcal{Y}_t)$ is the likelihood of the model expressed in the frequency domain. Note that the log-linear solution of the model admits a state-space representation of the following form:

$$Y_t = M_y(\theta)X_t,$$
$$X_{t+1} = M_x(\theta)X_t + M_e\varepsilon_{t+1}.$$

Here, Y_t and X_t denote, respectively, the vector of observed variables and the underlying state vector of the model; ε is the vector of the exogenous structural shocks, drawn from a Normal distribution with mean zero and variance-covariance matrix $\Sigma(\theta)$; $M_y(\theta)$ and $M_x(\theta)$ are matrices whose elements are (nonlinear) functions of the underlying structural parameters θ; and finally, M_e is a selection matrix that describes how each of the structural shocks impacts on the state vector. As shown in Whittle (1951), Hannan (1970), and Harvey (1991), the likelihood function is asymptotically given by

$$\log\bigl(L(\theta|\mathcal{Y}_T)\bigr) \propto -\frac{1}{2}\sum_{j=1}^{T} \gamma_j(\log\bigl(\det S_Y(\omega_j, \theta) + \operatorname{tr}\bigl(S_Y(\omega_j, \theta)^{-1} I_Y(\omega_j)\bigr)\bigr),$$

where $\omega_j = 2\pi j/T$, $j = 1, \dots, T$, and where $I_Y(\omega_j)$ denotes the periodogram of $\mathcal{Y}_T$ evaluated at frequency ω_j. $S_Y(\omega, \theta)$ is the model spectral density of the vector Y_t, given by

$$S_Y(\omega, \theta) = \frac{1}{2\pi} M_y(\theta)\bigl(I - M_x(\theta)e^{-i\omega}\bigr)^{-1} M_e\Sigma(\theta)M_e'\bigl(I - M_x(\theta)'e^{i\omega}\bigr)^{-1} M_y(\theta)''.$$

Following Christiano and Vigfusson (2003) and Sala (2015), we include a weight γ_j in the computation of the likelihood in order to select the desirable frequencies: this weight is 1 when the frequency falls between 6 and 32 quarters, and 0 otherwise.

Priors

The following parameters are estimated in both models: the inverse labor supply elasticity, ν; the capital share, α; the utilization elasticity parameter, ψ; the habit persistence parameter, b; the parameter governing the size of investment adjustment costs, φ; and the standard deviations and persistences of all the structural shocks. In the sticky-price model, the Calvo parameter, χ, and parameters of the Taylor rule, κ_R, κ_π, and κ_y, are also estimated. The priors used for all these parameters are reported in Table VIII in Supplemental Material Appendix S.3 and are broadly consistent with those used in the DSGE literature. The prior for the confidence shock was set in line with the other shocks. Finally, the following parameters are fixed: the discount factor, β, is 0.99; the depreciation rate, δ, is 0.025; the parameter, η, is such that the monopoly markup is 15%; and the parameter ϱ is 0.75 for the reasons explained in the main text.

Posteriors

Posterior distributions were obtained with the MCMC algorithm, with an acceptance rate of 37%. We generated two chains of 200,000 observations each. The posteriors for all the parameters are reported in the last four columns of Table VIII in Supplemental Material Appendix S.3. The posteriors for the preference, technology, and monetary parameters are broadly consistent with other estimates in the literature.

IRFs and Variance/Covariance Decompositions

With the exception of the confidence shock, which is novel, the IRFs to all the other shocks are comparable to those found in the literature. See Figures 8 and 9 in Supplemental Material Appendix S.3.

The estimated contribution of the shocks to, respectively, the variances and the covariances of the key variables at business-cycle frequencies is reported in Tables IX and X in the aforementioned appendix. For comparison purposes, we also include the estimated contributions that obtain in the variants of the models that remove the confidence shock. Three findings are worth mentioning.

First, unlike the case of the confidence shock, the variance/covariance contributions of some of the other shocks changes significantly as we move from the flexible-price to the sticky-price model.

Second, in the models that assume away the confidence shocks, the combination of permanent and transitory investment shocks emerge as the main driver of the business cycle. This is consistent with existing findings in the DSGE literature (e.g., Justiniano, Primiceri, and Tambalotti (2010)) and confirms that, apart from the inclusion of the confidence shock, our exercises are quite typical.

Finally, in all models, neither the investment-specific shocks nor the news or discount-rate shocks are able to contribute to a positive covariation between all of the key real quantities (output, consumption, investment, hours) at the same time. This illustrates, once again, the superior ability of our mechanism to generate the right kind of comovement patterns.

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