Cross-Sectional Efficiency and Labour Hoarding in a Matching Model of Unemployment

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We study positive and normative aspects of steady-state equilibrium in a market where firms of endogenous size experience idiosyncratic shocks and undergo a costly search process to hire their workers. The stylized model we propose highlights interactions between job-security provisions and sectoral shocks in determining the natural rate of unemployment, the allocation of labour, and the extent of labour hoarding, and rationalizes cross-sectional asymmetries of gross employment flows at the firm level. In our model, where productivity and search costs are dynamically heterogeneous across firms, decentralized wage bargains imply important cross-sectional inefficiencies, which overshadow the static search inefficiencies on which simpler models focus.

1. INTRODUCTION

Unemployment is usually viewed as a waste of potentially productive resources. Times of cyclically high unemployment are indeed associated with low output and welfare, but economists have always recognized that it is "natural" to expect positive unemployment even in normal times: if movement from one job to the next is not instantaneous and such mobility is socially desirable, then unemployed workers do contribute to an economy's dynamic efficiency. One important formalization of this idea is due to Lucas and Prescott (1974). In their model, inflows into unemployment are the endogenous result of optimal responses to sector specific shocks. Lucas and Prescott note that a lump-sum cost of changing job would imply lower unemployment, but less efficient labor allocation in equilibrium, a tradeoff we study in detail in this paper.

In the Lucas and Prescott framework, a relocating worker's new wage is determined by Walrasian market clearing within each sector. The assumption of centralized market clearing has been relaxed in this context by Mortensen (1982), Pissarides (1985, 1987), and Blanchard and Diamond (1990), among others, following the seminal Diamond (1982a, b) papers. In these authors' models firms post vacancies and workers search for jobs; when an unemployed worker and an open vacancy (randomly) match with each other, workers and firms bargain over the rents which must be associated to successful matches if vacancy-posting and/or search are costly.

While wage-bargaining and costly matching processes are arguably more realistic than Walrasian determination of wage rates, most existing contributions to the matching literature let established matches dissolve at an exogenously given rate, and are therefore unable to address some aspects of the unemployment-efficiency tradeoff identified by Lucas and Prescott. Further, most matching models feature simple or degenerate cross-sectional distributions of productivity and search intensity. This simplifies derivation of the equilibrium's efficiency properties, but also eliminates important dimensions of the labour-allocation problem.[1]

This paper takes a first step towards endogenizing the separation rate and relaxing the assumption that all active jobs are similar. At a methodological level, our work integrates aspects of the matching models by Blanchard and Diamond (1989) and others, which emphasize the role of idiosyncratic uncertainty but are based on very stylized modeling of "jobs" and "firms", with aspects of models by Pissarides (1987) and others, where decreasing returns to scale at the firm's level drive aggregate dynamics but all firms are alike. The structure of the resulting model is closely related to that of Lucas and Prescott (1974), albeit with an important role for non-Walrasian matching mechanisms in labour allocation. As in Lucas and Prescott, labour mobility costs make it sub-optimal for labour to fully relocate to high productivity sectors. Low-productivity firms tend to hoard labour, waiting for better times; higher mobility costs yield lower equilibrium unemployment, but also decrease the economy's efficiency.[2] Crucially, our model also features indirect mobility costs: matching unemployed workers and vacant jobs takes time and uses resources, and optimizing firms take at least part of such costs into account when deciding how much labour to hoard through times of low productivity. Realistic interactions between job-security provisions and workers' bargaining strength interact with the character of the matching technology and with sectoral shocks in determining the natural rate of unemployment, the level of output, wage distributions, and the extent of labour hoarding.

We relate the main positive implications of our framework of analysis to available evidence on employment flows. We argue that our admittedly very stylized model does capture several cross-sectional features of labour markets: in particular, the observed asymmetric behaviour of gross employment flows at the firm level, and several cross-sectional aspects of such asymmetry, are easily rationalized in our framework of analysis.[3] We also discuss the efficiency properties of the decentralized equilibrium we characterize. Heterogeneity and endogenous flows into unemployment considerably complicate the analysis of efficiency properties. While a single parameter (the Nash-bargaining share of workers) determines the decentralized outcome's efficiency in the simpler models considered by Hosios (1990), we find that decentralizing the socially efficient outcome would require more intricate subsidies and taxes in our case. In our model, where productivity and search costs have non-degenerate cross-sectional distributions, the outcome of continuous Nash bargaining over wages features cross-sectional inefficiencies which overshadow the standard search inefficiencies of simpler homogeneous or two-type firm models.

1. For a given separation rate and a homogeneous or two-types firms setup, Hosios (1990) shows that the matching/bargaining process is constrained-efficient if the surplus-sharing rule correctly internalizes to individual agents' search programmes the effect of aggregate unemployment and vacancies on the aggregate matching rate.

2. Less than the entire labour force is allocated to the most productive sectors in Lucas and Prescott's equilibrium but, strictly speaking, their model does not feature labour hoarding: as labour and goods markets are competitive within each sector, all mobility decisions are taken by workers.

3. As noted by Mortensen and Pissarides (1991), the intensity of match separations does not vary across markets and over time. Their model partly endogenizes such heterogeneity by allowing for a continuum of (exogenous) productivity levels, while maintaining the linearity assumptions or previous matching models.

The rest of the paper consists of six short sections. Section 2 outlines standard assumptions as to the matching process, and studies firms' hiring and firing decisions. Section 3 completes the model with standard assumptions as to workers' mobility decisions and wage determination, and Section 4 derives the market's steady-state equilibrium. Section 5 characterizes the equilibrium, discusses the cross-sectional empirical implications of our framework, and confronts them with available evidence. Section 6 derives optimality conditions for steady-state efficiency and discusses the sources of inefficiency in decentralized equilibrium. Section 7 concludes.

2. HIRING AND FIRING

We consider a labour market populated by a continuum of firms indexed by $i \in [0, 1]$, of unitary total mass, and by a continuously divisible labour force whose size is also normalized to unity. The marginal revenue product of labour at a firm i at time t is a function $\pi(\cdot, \cdot)$ of current employment, l_{it}, and of a business conditions index ε_{it}.

Firms cannot hire employees instantaneously. Rather, workers must be hired from unemployment through a costly and time-consuming process, and firms must actively search in the pool of unemployed workers if they want to increase their labour force. Following the literature, we formalize this idea assuming that firms post *vacancies*. Open vacancies are matched to unemployed workers at a rate that depends on aggregate unemployment U_t and aggregate vacancies V_t. For simplicity as well as realism (see Blanchard and Diamond (1989)), we assume constant-returns, Cobb–Douglas matching: the probability intensity per unit time that any open vacancy be matched with an unemployed worker is given by

$$p(\theta_t) = \xi \theta_t^\nu, \qquad -1 < \nu < 0, \qquad \theta_t \equiv \frac{V_t}{U_t}.$$

The scale parameter ξ (which has the dimension of 1/time) indexes the efficiency of the matching process.

Each vacancy posted by firm i is matched to an unemployed worker (and yields a unitary employment increase) with probability $p(\theta_t)$. For simplicity, we use a large-numbers approximation and let the total increase in employment be non-stochastic: if l_{it} denotes employment at the firm indexed by i and v_{it} denotes the number of vacancies it is posting, we have $\dot{l}_{it} = p(\theta_t) v_{it}$ whenever $v_{it} > 0$ and the firm is not firing.[4] As to employment reductions, we allow the firm to shed labour instantaneously if it finds it profitable to do so, and (for simplicity only) we neglect voluntary quits. Thus, employment at firm i evolves according to

$$l_{it+dt} = l_{it} + \xi \theta_t^\nu v_{it} dt - \Delta l_{it}, \tag{1}$$

where Δl_{it} denotes the mass firing which may occur at time t if firm i is hit by a negative business conditions shock. Of course, the aggregate vacancy stock is given by $V_t \equiv \int_0^1 v_{it} di$.

The shadow value of a marginal worker at each of the firms in the labour market is a function $A(l_{it}, \varepsilon_{it}, t)$ of the variables determining labour's marginal profitability at that

4. Strictly speaking, we may view firm i as representative of a large number of firms that are, except for the current matching luck, indistinguishable from each other. Alternatively, we could view employment dynamics as the infinitesimal limit of those appropriate for finite sets of firms with similar (but not identical) initial employment.

firm, indexed by ε_{it}, and of aggreate state variables (such as vacancies and unemployment), summarized by t. If an optimal labour demand policy exists, it must be the case that

$$rA(l_{it}, \varepsilon_{it}, t) = \pi(l_{it}, \varepsilon_{it}) - \phi(l_{it}, \varepsilon_{it}) + \frac{1}{dt} E_t[dA(l_{it}, \varepsilon_{it}, t)] \tag{2}$$

where $r \geq 0$ denotes the rate of return on the firm's operation, and $\phi(\cdot, \cdot)$ denotes the marginal cost to the firm of employing an additional (marginal) worker, i.e., the derivative of its wage bill with respect to employment l_{it}. If firms could take the wage rate w_{it} as given and independent of their employment level then, of course, we would have $\phi(l_{it}, \varepsilon_{it}) = w_{it}$. In our model's equilibrium, conversely, higher employment will be associated with lower wage rates at the firm's level, and the notation in (2) makes it possible to take such monopsony power into account.

Consider next the shadow value of a vacancy posted by firm i, denoted A_{it}^v (it will not be necessary in what follows to explicitly write this as function of firm-specific and aggregate variables). With probability intensity $\xi \theta_t^\gamma dt$, the vacancy is filled and becomes a marginal job at the firm posting it. We denote with C_{it} the marginal flow cost to firm i of keeping v_{it} vacancies open at time t, and obtain the asset-valuation relationship

$$rA_{it}^v = -C_{it} + \frac{1}{dt} E_t[dA_{it}^v] + \xi \theta_t^\gamma (A(l_{it}, \varepsilon_{it}, t) - A_{it}^v). \tag{3}$$

The firms' managers decide how many vacancies to keep open if it is currently optimal to increase employment, and how many workers to fire if labour shedding is optimal. With homogeneous labour and no sunk-cost component for open vacancies, it will never be optimal to have open vacancies and fire at the same time. Hence, the optimality conditions for these decisions can be considered separately to characterize the marginal value of labour at an individual firm.

If it is costless to open and close vacancies, their shadow value must be zero at all firms which choose to post vacancies (Pissarides (1987); for firms which post no vacancies, of course, the shadow value of an open vacancy is less than its cost, namely zero). With $A^v = dA^v = 0$, (3) implies that if $v_{it} > 0$:

$$A(l_{it}, \varepsilon_{it}, t) = \frac{C_{it}}{\xi \theta_t^\gamma}. \tag{4}$$

Quite intuitively, the value of an additional worker for a vacancy-posting firm (on the left-hand side) is equal to the flow cost of posting a vacancy divided by the probability of matching that vacancy, or, to the expected hiring cost of an additional worker.

The other choice variable of firms is the number of workers fired. By complementary slackness, any firm that posts no vacancies and does employ positive labour must be indifferent between retaining and firing employees at the margin. The shadow value of a filled job at a firm which is not posting any vacancies (and may be firing) must then be equal to the cost (if any) of shedding one additional unit of labour, which we denote F_t and view as generic index of the technological and institutional labour mobility costs that would be present even if matching were instantaneous.[5] Thus, if $v_{it} = 0$ and $l_{it} > 0$ then

$$A(l_{it}, \varepsilon_{it}, t) = -F_t, \qquad dA(l_{it}, \varepsilon_{it}, t) = -dF_t. \tag{5}$$

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5. The parameter F collectively accounts for all non-matching mobility costs: transportation, relocation and retraining costs, and also "job security" constraints on firms' employment flexibility (for which see Piore, (1987) and Emerson (1989)). In reality, part of such costs are paid by the firm which hires the displaced worker or by the worker himself, rather than by the firing firm as in our notation. Under risk neutrality or perfect insurance, however, the distinction is unimportant in equilibrium.

Equation (5) does not hold for a firm (or a category of jobs) which is not posting vacancies *and* is not employing any workers. In this case, the value of an additional worker is more negative than $-F_t$, to imply that the non-negativity constraint on employment is binding and turns the equality in (5) into an inequality. This is in fact what happens in models that identify "bad" jobs with "idle" ones.

In what follows we focus on (aggregate) steady-state relations, which allows us to remove the time sub-index from aggregate variables. For simplicity, we assume that ε_{it} takes one of only two possible values, ε_g (good state) and ε_b (bad state), letting an individual firm's business conditions switch between the two according to a semi-continuous Markov chain process. Firms in good business conditions (or good firms for short) turn bad with constant probability intensity δ, and firms in bad business conditions turn good with probability intensity γ. We also linearize the marginal revenue function, so that

$$\pi(l_i, \varepsilon_g) = \eta_g - \sigma l_i \quad \text{if good,} \qquad \pi(l_i, \varepsilon_b) = \eta_b - \sigma l_i \quad \text{if bad}$$

where η_i is short for $\eta(\varepsilon_i)$. We assume $\eta_g > \eta_b$, so that optimal employment levels and revenues are higher for good firms than for bad firms.

As labour shedding is instantaneous, all firms in bad business conditions look alike and have the same employment level.[6] Conversely, since vacancies are slowly filled, good firms' employment levels depend on the time elapsed since they last turned good. In particular, firms cannot hire instantaneously when switching from bad to good, and the level of employment when bad must coincide with a new good firm's initial level of employment. We denote with τ the length of a continuing spell of good business conditions, or a good firm's "age", and we seek a function $l(\tau)$ for employment at good firms, with $l(0)$ denoting employment at bad firms or hoarded labour.

Using (5) and (4) in (2), and denoting with $C(0)$ the marginal flow cost of posting a vacancy for a firm that just turned good (or a bad firm), we obtain

$$-rF \geqq \eta_b - \phi(l(0), \varepsilon_b) - \sigma l(0) + \gamma\left(\frac{C(0)}{\xi\theta^\nu} + F\right), \tag{6}$$

with equality if $l(0) > 0$, i.e., if bad firms have positive employment, as we think is realistic. It is interesting to note that even when $F=0$ bad firms pay workers more than their marginal revenue product as long as $\gamma C(0)/\xi\theta^\nu > 0$. If matching is costly and time-consuming, then labour is a scarce factor for a firm in good business conditions; moreover, decreasing marginal revenue product of labour implies that the shadow value of a filled good job is large at small employment levels. Thus, it is optimal for firms in bad business conditions to hoard labour, as this lowers their opportunity-cost losses when business conditions improve.

Similar steps yield a condition for a firm that has enjoyed continuing good business conditions for τ periods:

$$r\frac{C(\tau)}{\xi\theta^\nu} = \eta_g - \phi(l(\tau), \varepsilon_g) - \sigma l(\tau) + \frac{d}{d\tau}\frac{C(\tau)}{\xi\theta^\nu} - \delta\left(\frac{C(\tau)}{\xi\theta^\nu} + F\right). \tag{7}$$

6. This need not be true in more general models, of course. In particular, we might allow for costless voluntary quits (e.g. retirements), which firms would exploit to achieve costless employment reductions for a while after experiencing a negative shock. Saint-Paul (1990) characterizes these aspects, but allows firms to hire instantaneously from the pool of unemployed workers.

We depart from the standard vacancy cost setup by letting the marginal cost of posting vacancies be increasing with respect to the number of vacancies posted by the firm: $C(\tau) = cv(\tau)$, where $v(\tau)$ denotes the vacancies.posted by a good firm of age τ. Such convexity is necessary to obtain a well-defined vacancy-posting equilibrium when productivity is heterogeneous across firms, as firms with high productivity and low employment levels would tend to post infinitely many vacancies for arbitrarily short intervals of time if such policies were not made prohibitively costly by convexity of total vacancy-posting costs.[7] More intense search efforts on an individual firm's part may indeed entail more than proportionally increasing total costs. Alternatively, vacancies posted by individual firms might have a decreasing rather than constant yield in terms of matching intensity: as our heterogeneous firms are differentiated across geographical or sectoral dimensions, it may well be realistic to assume that the pool of workers likely to respond to vacancies posted by a given firm is also limited along the same dimensions.

3. WORKERS' BEHAVIOUR

Having established that the firm's vacancy-posting policy must satisfy (6) and (7) with $C(\tau) = cv(\tau)$, we now turn to consider workers' behaviour. Our treatment of workers' mobility decisions is completely standard, and based on asset-valuation relationships for the present discounted value of worker j's labour income if unemployed or employed at time t, denoted with J_{jt}^u and J_{jt}^e, respectively. We let all individual labour market participants enjoy the same income-equivalent flow z from leisure and unemployment benefits when not working, and take them to bear no cost of searching for a job when not employed. We denote every worker's discount rate with r, the same as the firm's, which is appropriate if workers and firms have access to the same financial market but could harmlessly be relaxed.

A total of $\xi \theta_t^\gamma V_t$ new matches are formed per unit time if the aggregate vacancy rate is V_t. Hence, any one of U_t job seekers finds a job with probability intensity $\xi \theta_t^\gamma V_t / U_t = \xi \theta_t^{\gamma+1}$, and the present discounted value of labour income of a currently unemployed worker satisfies

$$rJ_{jt}^u = z_{jt} + \frac{1}{dt} E_t[dJ_{jt}^u] + \xi \theta_t^{\gamma+1}(J_{jt}^e - J_{jt}^u). \tag{8}$$

If worker j is employed, and receives a wage w_{jt}, his human wealth symmetrically satisfies

$$rJ_{jt}^e = w_{jt} + \frac{1}{dt} E_t[dJ_{jt}^e] + q_{jt}(J_{jt}^u - J_{jt}^e) \tag{9}$$

if q_{jt} denotes the probability intensity of job loss. As quits are ruled out, mobility is

7. More formally, consider equations (6) and (7) with $C(\tau) = C$ for all τ: if $w_g(\tau) = w_b$ for all τ, then these optimality conditions could be simultaneously satisfied by the same (aggregate) θ only with a constant employment schedule, $l(\tau) = l(0)$ for all τ, which would obviously be inconsistent with optimal (interior) vacancy posting behaviour. Allowing good wages to depend on τ, as we do later, would introduce additional equilibrium conditions and would in general not resolve the inconsistency between positive labour reallocation and labour hoarding. Of course, constant vacancy-posting costs could be accommodated by setting $\sigma = 0$: but then either equation (6) would not be satisfied (no labour hoarding), or no reallocation would take place in equilibrium as all jobs would be permanently occupied, regardless of their current productivity level (see Blanchard and Diamond (1989, fn. 10, p. 9)). In the Mortensen and Pissarides (1991) model, labour hoarding (at the individual job's rather than at the firm's level) is consistent with linear cost and revenue functions, because each job's productivity follows a persistent stochastic process taking more than two possible values.

involuntary and, in general, the last term is a negative capital gain for the worker in question.

Such asset-valuation equations hold for any worker $j \in [0, 1]$. To combine them with those pertaining to firms indexed by $i \in [0, 1]$ in full generality, it would be necessary to track the two-dimensional distribution of firm/worker pairs as well as the vacancy and unemployment rates resulting from aggregation of optimal policies for firms' vacancy-posting and workers' search. Fortunately, our simple assumptions on the cross-sectional and time-series behaviour of firm- and worker-specific state variables make it possible to bypass such complications. All worker-specific quantitites can be simply indexed by their employment status and, if they work at a good firm, by the "good-time age" τ (as defined above) of their employer. In what follows, J^u denotes the value function of the representative unemployed worker; w_b and J^b denote the wage rate and value function of bad firms' employees (both firms and workers are alike in this group); $w(\tau)$ and $J^g(\tau)$ denote the corresponding quantities for workers employed by a firm which has remained in the good state for τ consecutive periods.

Since employment cannot be immediately adjusted (upwards) to drive the value of an additional worker down to zero, the rents from the marginal job/worker match need to be split between the firm and the worker. We follow the literature in invoking a Nash bargaining device, and let labour receive a fraction β, $0 \leq \beta \leq 1$ of the asset-value surplus from marginal worker-job matches. Labour's marginal contribution to revenues and to a firm's value is heterogeneous in the market under study, and this implies wage dispersion in equilibrium. With a matching technology of the urn/ball type and complete capital markets (or risk neutrality), unemployed workers need only be concerned with the mean of open vacancies' values, which fully determines the reservation value of continued search. In equilibrium, then, workers who are matched with an open vacancy always prefer work at the Nash-bargained wage to continued unemployment and search (or else, all workers being the same, some vacancies would have no chance of ever being filled and should not be open in equilibrium).[8] In the market's steady-state equilibrium, the asset-valuation relationship (8) for the value of being unemployed reads

$$rJ^u = z + \xi \theta^{v+1}(\bar{J}^g - J^u), \tag{10}$$

where $\bar{J}^g$ is the average value of (good) jobs for which vacancies are posted, and will need to be computed on the basis of the equilibrium wage and vacancy distributions found below.

With no centralized labour-market clearing, we need to specify carefully what is the object of negotiation upon the inception of a new employment relationship, and which are the parties to such negotiations. We shall assume that long-term contracts are not enforceable, so that spot wage rates are continuously renegotiated by firms with individual workers. Implementing the familiar Nash bargaining device, the surplus from a completed match is split between a worker and an employer according to

$$\beta(A^g(\tau) - A^v) = (1 - \beta)(J^g(\tau) - J^u). \tag{11}$$

By implication, all employees of each good, vacancy-posting firm receive equal pay. Incumbents may threaten to leave and cause a costly vacancy if their wage is not increased when

8. In the model proposed by Albrecht and Axell (1984), conversely, workers' reservation utilities are heterogeneous as well. As a result, some wage offers are rejected in equilibrium, and offering higher wages makes it possible for high-productivity firm to fill posted vacancies more quickly. Our detailed microeconomic framework would allow for a realistic implementation of the Albrecht-Axell mechanism, but we defer this refinement to future research.

the firm's business conditions improve and, in general, their bargaining position in bilateral negotiations will never be different from that of newly-hired workers.[9] Then, the value of being at a currently bad firm is enhanced by the probability of the firm turning good:

$$rJ^b = w_b + \gamma(J^g(0) - J^b), \tag{12}$$

where J^b denotes the value of working at a bad firm (and is constant in steady state), and $J^g(0)$ denotes the value of being matched with a firm that has just turned good ($\tau = 0$) and begins to post positive vacancies.

As we assume that wage rates are continuously and individually renegotiated between individual workers and employers, the employees of a firm hit by a negative business-conditions shock (some of whom will be laid off) must be indifferent between unemployment and continued employment at lower wages. Indeed, the surplus-sharing rule requires that $J^b - J^u = \beta(A^b + J^b - A^v - J^u)$ when wages are negotiated between a bad firm (which obtains A^b from agreement, and would have to post a vacancy if negotiations break down) and its current employees (who obtain J^b from agreement and have outside option J^u). Of course, $J^b - J^u \geq 0$ is also necessary for workers to participate in the bargain. In equilibrium, $A^b = -F$ and $A^v = 0$, so an interior bargaining solution would have $(1-\beta)(J^b - J^u) = -\beta F \leq 0$, and the workers' participation constraint binds:[10]

$$J^u = J^b. \tag{13}$$

Consider next the value of being matched to a firm that has enjoyed continuing good business conditions for τ periods, and pays the wage rate $w(\tau)$ to its employees. With constant probability intensity δ the firm may experience a negative business-conditions shock. Since only a fraction $l(0)/l(\tau)$ of the current labour force is retained in this contingency, current employees face unemployment with probability $1 - l(0)/l(\tau)$, or employment in a bad firm with the complementary probability. As both outcomes have the same value J^b by (13), equation (9) implies the following relationship between $J^g(\tau)$ and $w(\tau)$:[11]

$$rJ^g(\tau) = w(\tau) + \dot{J}^g(\tau) - \delta(J^g(\tau) - J^u). \tag{14}$$

4. WAGE SCHEDULE AND FIRM'S DYNAMICS

Under our parametric assumptions, we need to compute the wage rate paid by bad firms as well as the wage schedule $w(\tau; 0 \leq \tau \leq \infty)$ resulting from bargains struck with firms which have been good for τ periods. Firms with higher τ have been able to hire more labour and, given the downward-sloping marginal revenue product of labour, attach lower value to marginal employment increases. Hence, wage bargains are struck at lower wage rates with "older" good firms, and $w'(\tau) \leq 0$; wage rates are uniform in equilibrium only if $\beta = 0$: with $\beta > 0$ and decreasing marginal revenues, conversely, the model implies that

9. More complex labour contracts may be written: for example, "two-tier" provisions may specify different wages for new hires and incumbents. In general, these would have real consequences in the absence of a centralized market for labour.

10. We are grateful to a referee for suggesting this argument.

11. We assume that none of the firing/relocation cost F is paid directly to the worker. More general settings would complicate notation with no gain in generality, since only the *net* cost of a separation to workers and employers as a group matters for employment and wage determination. See Burda (1989), Lazear (1990), and Bertola (1991) for arguments along these lines. As noted in footnote 5 above, separation costs may be paid by the firing employer, by the hiring employer, or by the worker himself in reality, with little or no substantial effects.

$w(\tau)$ declines as $l(\tau)$ increases, and in what follows this will make it possible to solve explicitly for the firm's marginal employment cost $\phi(l(\tau), \varepsilon_g)$.

To find the wage schedule, we use the optimality condition (4) for open vacancies in the Nash-bargaining condition (11), to obtain

$$J^g(\tau) = J^u + \frac{\beta}{1-\beta} \frac{cv(\tau)}{\xi\theta^\nu} \qquad j^g(\tau) = \frac{\beta}{1-\beta} \frac{c\dot{v}(\tau)}{\xi\theta^\nu}. \qquad (15)$$

This can be inserted in (14) to obtain

$$w(\tau) = rJ^u + \frac{\beta c}{1-\beta} \frac{(r+\delta)v(\tau) - \dot{v}(\tau)}{\xi\theta^\nu}. \qquad (16)$$

Recalling that $C(\tau) = cv(\tau)$ in (7), solving for $\dot{v}(\tau)$, and inserting the resulting expression into equation (16), the terms in $v(\tau)$ cancel out to yield a relationship between the wage rate, employment, and marginal employment costs at a firm of age τ:

$$(1-\beta)w(\tau) = (1-\beta)rJ^u + \beta(\eta_g - \delta F) - \beta\phi(l(\tau), \varepsilon_g) - \beta\sigma l(\tau).$$

We now recognize that the joint evolution of $l(\tau)$ and $w_g(\tau)$ implies a relationship between wages and employment at each firm. Writing $w_g(\tau) = f(l(\tau))$, and noting that $\phi(l(\tau), \varepsilon_g) \equiv f(l(\tau)) + f'(l(\tau))l(\tau)$ by its definition as the marginal change in the firm's wage bill as employment increases, we obtain a functional condition which must be satisfied by the $f(\cdot)$ relationship between wages and employment in the model's equilibrium:

$$f(l) = (1-\beta)rJ^u + \beta(\eta_g - \delta F) - \beta f'(l)l - \beta\sigma l.$$

It is easy to verify that this first-order differential equation admits a linear solution which, in terms of the $l(\tau)$ and $w(\tau)$ schedules of interest, reads

$$w_g(\tau) = (1-\beta)rJ^u + \beta(\eta_g - \delta F) - \frac{\beta\sigma}{1+\beta} l(\tau) \qquad (17)$$

and is readily interpreted: the first term is a weighted average with Nash-bargain weights of the worker's outside option and the constant portion of the producer surplus from employment, namely the intercept of the marginal revenue function and the expected firing cost of the marginal worker; the second term reflects the fact that, inasmuch as $\sigma > 0$, the marginal producer surplus being bargained upon declines (linearly) as more and more workers are employed. As wages are linear in employment, we have $\phi(l, \eta_g) \equiv w_g + (\partial w_g/\partial l)l = w_g - (\beta\sigma/(1+\beta))l$. Using this and $C(\tau) = cv(\tau)$ in equation (7) yields

$$(r+\delta)\frac{cv(\tau)}{\xi\theta^\nu} = \eta_g - \delta F - w_g(\tau) - \frac{\sigma}{1+\beta} l(\tau) + \frac{c}{\xi\theta^\nu} \dot{v}(\tau). \qquad (7')$$

We are now ready to characterize a good firm's optimal vacancy policy. Differentiating equations (16) and (7') with respect to τ, eliminating $\dot{w}_g(\tau)$, and using (1) to eliminate $\dot{l}(\tau)$, we obtain a second-order differential equation in $v(\cdot)$:

$$\ddot{v}(\tau) - (r+\delta)\dot{v}(\tau) - \sigma\frac{1-\beta}{1+\beta}\frac{(\xi\theta^\nu)^2}{c} v(\tau) = 0,$$

whose solutions satisfying $\lim_{\tau \to \infty} v(\tau) = 0$ have the form:

$$v(\tau) = v(0)e^{-\lambda \tau}, \qquad \lambda \equiv \tfrac{1}{2}\left(\sqrt{(r+\delta)^2 + 4\frac{(\xi \theta^\nu)^2}{c}\sigma \frac{1-\beta}{1+\beta}} - (r+\delta) \right) > 0. \qquad (18)$$

Equation (18) will play a key role in the efficiency section below, where we show that the decentralized equilibrium generates values of λ that are inefficiently small for any given vacancy/unemployment ratio. Given $r + \delta$, which indexes the good firm's degree of impatience along the adjustment path, the rate at which a firm's posted vacancies and employment vary over time is an increasing function of $\sigma(1-\beta)/(1+\beta)$ and of $\xi \theta^\nu$, and a decreasing function of $c/\xi \theta^\nu$. To interpret these features, consider that the vacancy-posting policy determines the *growth rate* of employment. As the firm's expansion progresses (i.e. as τ increases), the marginal revenue from successful matches declines according to σ times employment growth; given the character of bargained wages, however, the firm retains only a fraction (smaller for larger β) of such marginal benefits. Such declining marginal benefits reduce incentives for the firm to post vacancies and, in turn, fewer posted vacancies translate into a decline in the rate of employment growth according to the matching probability intensity $\xi \theta^\nu$ in equation (1). As to the role of $c/\xi \theta^\nu$, note that firms' incentives to post vacancies disappear more quickly when the denominator is larger and posted vacancies are more easily filled; recalling that c indexes the extent to which the unit cost of vacancies increases with their number, a larger c gives incentives to spread out vacancy posting over time, hence is associated to smaller values of λ in (18).

Given that τ is distributed exponentially across good firms with parameter δ, (18) implies that open vacancies are distributed exponentially over τ with parameter $\delta + \lambda$. This and the Nash-bargaining outcome (15) yield an expression for the mean of a worker's gain upon being matched with a good job from unemployment:

$$\bar{J}^g - J^b = \frac{\beta}{1-\beta}\frac{c}{\xi \theta^\nu}(\delta + \lambda)\int_0^\infty v(\tau)e^{-(\delta + \lambda)\tau}d\tau = \frac{\beta}{1-\beta}\frac{c}{\xi \theta^\nu}\frac{\delta + \lambda}{\delta + 2\lambda}v(0).$$

In turn, this and (10) yield

$$w_b = z + \frac{\beta c}{1-\beta}\left(\frac{\delta + \lambda}{\delta + 2\lambda}\theta - \frac{\gamma}{\xi \theta^\nu}\right)v(0). \qquad (19)$$

We have from (12) and (15) evaluated at $\tau = 0$ that

$$rJ^b = w_b + \gamma \frac{\beta c}{1-\beta}\frac{v(0)}{\xi \theta^\nu}. \qquad (20)$$

Using this, the condition $J^b = J^u$, and (18) in (16), we finally obtain an expression for the good firms' wage schedule:

$$w_g(\tau) = w_b + \frac{\beta c}{1-\beta}\frac{v(0)}{\xi \theta^\nu}(\gamma + (r+\delta + \lambda)e^{-\lambda \tau}). \qquad (21)$$

The productivity parameters η_g and η_b do not appear in these equations, but do influence the equilibrium wage rates indirectly through firms' labour demand policies and the resulting vacancy/unemployment ratio. The wage schedules in (19) and (21) depend in standard fashion on the flow benefits from not being employed, z, on the cost of an open vacancy c, and on the labour market tightness index θ (see e.g. Pissarides (1987)).

In our framework, they also depend on $v(0)$, λ and τ, reflecting the different desirability of additional labour for firms with similar business conditions but different employment, and the resulting vacancy behaviour in (18).

To complete the characterization of the firms' problem, we use (18) in (1) and integrate to obtain an expression for firm-specific employment dynamics,

$$l(\tau) = l(0) + \frac{\xi\theta^\nu}{\lambda}(1 - e^{-\lambda\tau})v(0). \tag{22}$$

Two boundary conditions determine $v(0)$ and $l(0)$. As all employees of a bad firm are indifferent between employment and unemployment, a bad firm's wage rate is fixed by supply conditions and independent of its employment level. Hence, we have $\phi(l(0), \varepsilon_b) = w_b$ in equation (6), which gives us a first boundary condition:

$$\gamma v(0) = \sigma l(0)\frac{\xi\theta^\nu}{c} - \frac{\xi\theta^\nu}{c}(\tilde{\eta}_b - w_b),$$

where $\tilde{\eta}_b \equiv \eta_b(r + \gamma)F$. The second boundary condition is obtained by equating the expressions for $w_g(0)$ resulting from evaluation of (17) and (21) at $\tau = 0$, and recalling that $J^u = J^b$ as given by (20):

$$v(0)\left(\frac{r + \delta + \beta\gamma + \lambda}{1 - \beta}\right) = -\frac{\sigma}{1 + \beta}l(0)\frac{\xi\theta^\nu}{c} + \frac{\xi\theta^\nu}{c}(\tilde{\eta}_g - w_b),$$

where $\tilde{\eta}_g \equiv \eta_g - \delta F$. Inserting the bad-time wage expression (19) in these two conditions, we obtain a linear system which is readily solved for $v(0)$ and $l(0)$.

This completes the solution of a firm's optimal policy, conditional on a constant $\theta \equiv V/U = V/(1-L)$. We can then proceed to aggregate individual firms' policies in a steady-state situation where V and L are indeed constant and, in equilibrium, yield the θ value consistent with individual policies. As the arrival rates of negative and positive business conditions shocks are constant at δ and γ, the density of τ is exponential and the steady-state proportions of good and bad firms are $\gamma/(\gamma+\delta)$ and $\delta/(\gamma+\delta)$. Using these cross-sectional distributions along with the firm-level vacancy and employment paths in (17) and (20), we obtain

$$V = \frac{\gamma\delta}{\gamma+\delta}\int_0^\infty v(\tau)e^{-\delta\tau}d\tau = \frac{\gamma}{\gamma+\delta}\frac{\delta}{\delta+\lambda}v(0), \tag{23}$$

$$L = \frac{\delta\gamma}{\gamma+\delta}\int_0^\infty l(\tau)e^{-\delta\tau}d\tau + \frac{\delta}{\gamma+\delta}l(0) = l(0) + \frac{\gamma\xi\theta^\nu}{(\gamma+\delta)(\lambda+\delta)}v(0). \tag{24}$$

Recalling the definition $\theta \equiv V/(1-L)$, these equations and the solutions for $l(0)$ and $v(0)$ in terms of θ form a nonlinear condition which must be satisfied in a steady-state equilibrium with labour hoarding. We use a simple search routine to find θ, and to characterize the labour market's steady state in terms of the parameters.[12]

12. Not all parameterizations are consistent with $l(0) > 0$, of course: while it would be possible to derive equilibrium conditions for cases where the conditions in (5) are slack, such cases are uninteresting for our purposes. We have not performed an analytic study of the model's stability properties, but numerical experimentation indicates that there is only one stable equilibrium with positive vacancies and positive unemployment. Depending on parameters, $V/(1-L)$ can be upward or downward sloping as a function of θ, and besides the well-defined equilibrium of interest there may be an unstable equilibrium at $\theta = 0$ or a meaningless equilibrium with negative unemployment.

5. ON CROSS-SECTIONAL REALISM

Before turning to the aggregate steady-state implications of search frictions in the next section, it is useful to briefly discuss our model's characterization of cross-sectionally heterogeneous firm behaviour. This section aims at highlighting the microeconomic mechanism behind aggregate outcomes on the one hand and, on the other, at arguing that our stylized model does shed light on several empirical features of real-life labour markets.

To illustrate the insights discussed in what follows, we select parameters that yield reasonable values for unemployment and aggregate vacancy posting. Our model's constant-returns matching technology is similar to that estimated by Blanchard and Diamond (1989), and an unemployment elasticity $v = -0.4$ is roughly consistent with their findings on U.S. data. To select reasonable baseline values for parameters other than those pertaining to the matching technology, we consider a hypothetical first-best, frictionless, full-employment equilibrium and normalize parameters so that aggregate output would be unity in that case. As long as $\eta_b \leq z$, all workers are employed by high-productivity firms in full-flexibility equilibrium; for our purposes, η_b should be large enough to yield labour hoarding in equilibrium: we choose $\eta_b = 0.9z$. There are $\gamma/(\gamma + \delta)$ good firms in steady state, hence each of them should employ $(\gamma + \delta)/\gamma$ units of labour and, for full employment to be efficient, labour's marginal revenue product should be at least as large as the income-equivalent flow from not working, z. Taking this condition to hold with equality, we let the baseline parameters satisfy the relationship:[13]

$$\eta_g = z + \sigma \frac{\gamma + \delta}{\gamma}. \tag{25}$$

The total output from firms in the labour market can be measured integrating revenues across all firms, i.e. adding up quantities at market prices.[14] For full-employment output to equal unity, we then impose that

$$1 = \frac{\gamma}{\gamma + \delta} \int_0^{(\gamma + \delta)/\gamma} (\eta_g - \sigma l) dl,$$

to imply that $\eta_g = 1 + \frac{1}{2}\sigma(\gamma + \delta)/\gamma$, and complete our set of full-flexibility technological parameters setting $\sigma = 0.25$ and $\gamma = \delta = 0.15$. Taken at face value, these probability intensities mean that our firms experience business-conditions shocks of the size we consider about every six and a half years on average.

As to parameters which would have no role in the frictionless baseline case, we set $r = 0$, so that vacancy-posting policies maximize steady-state undiscounted profits; and $c = z$, so that the marginal cost of open vacancies coincides with the income-equivalent flow enjoyed by unemployed workers and, by (25), with labour's full-employment marginal revenue product. The parameters β and F represent labour market institutions in our model: we shall analyze in some detail the implications of a wide range of F values for the "neutral" $\beta = v$ value, and briefly discuss the implications of higher or lower β values.

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13. We should of course stress that our parameterization is only meant to yield suggestive results. As a referee points out, it is somewhat arbitrary to assume that, at the margin, workers would be indifferent between employment and unemployment in a frictionless equilibrium. Some experimentation suggests that the results we discuss below are qualitatively unaffected by this: in particular, the positive association between welfare and unemployment across firing-cost values is robust to relaxation of the equality in (25) in either direction.

14. Integration constants may harmlessly be set to zero: since the number of firms is exogenously given, fixed costs are irrelevant in the equilibrium we describe.

The solid lines in Figure 1 depict the dynamic path of employment, vacancy posting, marginal profitability and wages at a firm that has remained in the good state for τ years. The dashed lines represent the corresponding values at firms in the bad state.

The first panel shows the path of vacancies. The first feature to notice is that these remain finite throughout, which implies that the path of hiring is also smooth and that employment is accumulated slowly over time—in sharp contrast with the character of labour shedding, which in our model is fully accomplished in the period when business conditions turn bad. This follows from the asymmetric character of the hiring and firing technologies, which has been noted before with reference to aggregate time-series evidence (see e.g. Pissarides (1985)). In the context of our model, which focuses on cross-sectional firm-level phenomena, the prediction is in accordance with the empirical evidence on microeconomic job creation and destruction as presented by Davis and Haltiwanger (1992). They show that the cross-sectional distribution of job destruction is highly concentrated, with about 80% of destruction accounted for by establishments that shrink by more than 20% over the span of a year.

Davis and Haltiwanger (1992) also show that job creation falls substantially with age and size of the plant. Young (1 year old) establishments have a creation rate about four times larger than that of much older firms (15 years and older), while the same ratio for destruction is about 2. This is in principle consistent with the second feature of panel (a), which is that the path of vacancy posting is decreasing with age. Unfortunately, this is only very indirect evidence in favour of the implications of the model since our concept of age is not one of plant age—for which data are plenty—but one of "expansion age". Size and, especially, marginal profitability (see below) are concepts closer to our definition of "age". Davis and Haltiwanger (1992) show that the creation pattern described above still holds when plants are sorted by size, but this is much less dramatic (when compared to the pattern of destruction) than when sorted by plant age.[15]

Panel (b) depicts the path of employment, which is a monotonic transformation of the integral of the previous panel. Thus, the concavity of the employment path is the result of the decreasing path of vacancy posting at the firm level. The new information in this panel is that a firm that just turns good has a fairly sizable labour force. As will be apparent in the next set of panels, this is the result of labour hoarding by the firm while in the bad state. Firms realize that search frictions make fast hiring very costly, thus keep levels of employment that are inefficiently high from a static point of view while in the bad state.

Panel (c) displays the path of marginal profitability for a good firm, and its counterpart at a bad one as a reference value. Marginal revenues of a good firm decline over time as a result of our assumption of decreasing marginal profits, and of its interaction with slow but monotonic employment growth. The convexity of this curve results from decreasing vacancy posting and the implied slower and slower employment growth, which in turn reflect decreasing marginal profitability of additional workers.

Panel (d) presents the wages. Two features deserve comment: the wage in bad times is higher than marginal profitability, and workers share the high return of young, highly profitable firms. The empirical evidence on the behaviour of wages across firms of different ages and sizes is mixed for the predictions of our model. If age is interpreted as chronological age of a plant—which we do not view as the most appropriate interpretation—the evidence is that wages are mildly increasing (after controlling for size and other characteristics) rather than decreasing with respect to age. The positive correlation between wages

15. This is consistent with stationarity of the size distribution if destruction does not fall as much with size, which is the case in U.S. data.

(a) Vacancy Posting at 'Age' τ

(b) Employment at 'Age' τ

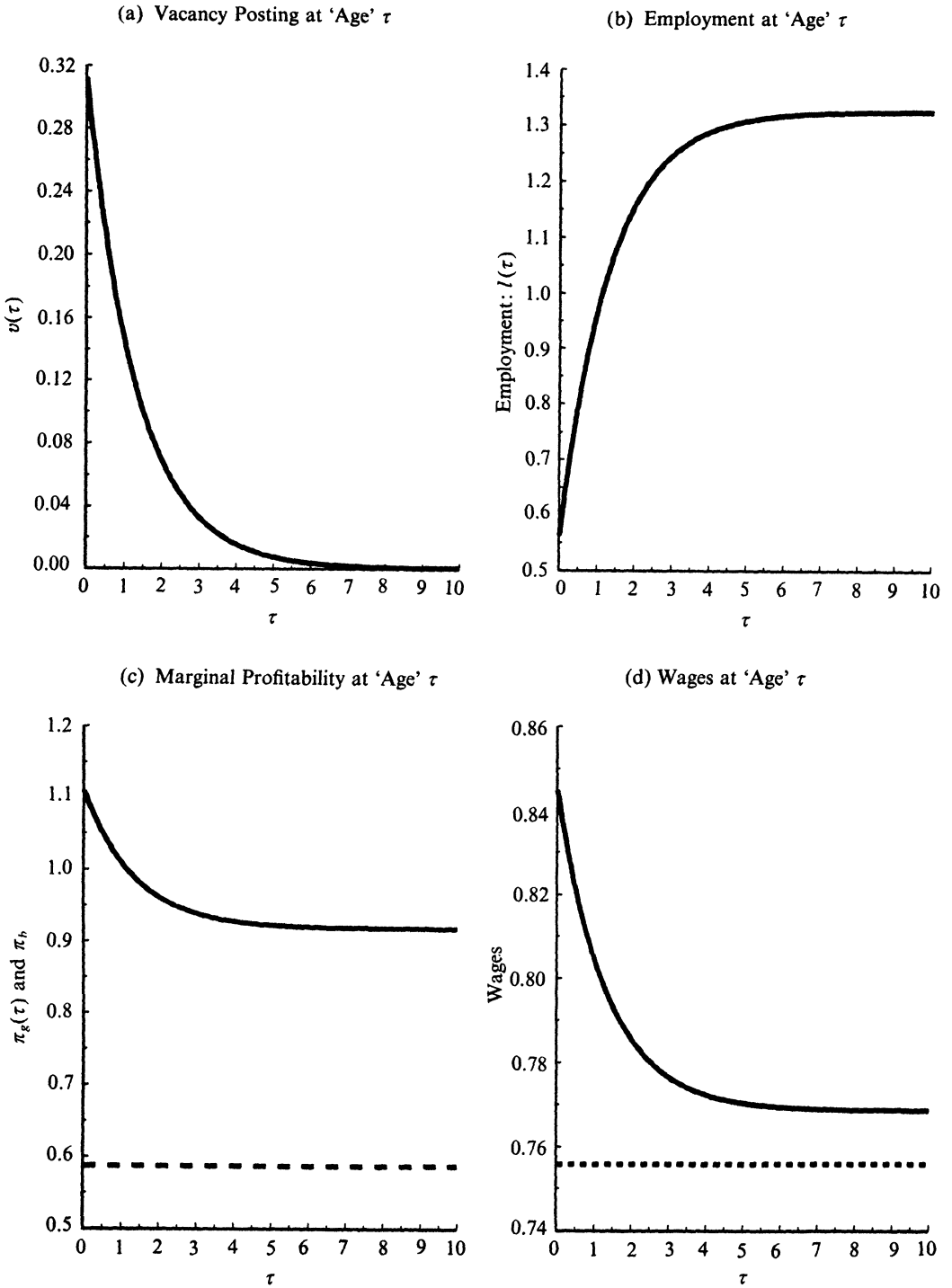(c) Marginal Profitability at 'Age' τ

(d) Wages at 'Age' τ

FIGURE 1

and the establishment size is even stronger; see e.g. Layard, Nickell and Jackman (1991) for Europe, and Davis and Haltiwanger (1991) for U.S. manufacturing establishments. The evidence is less negative if employment size is associated with a high labour/capital ratio; Krueger and Summers (1988) find that wages are decreasing with respect to this ratio.

As mentioned above, however, in our model firms and plants undergo repeated product cycles, and the "age" index τ is better interpreted as an index of productivity rather than a measure of the plant's chronological age. From this perspective, the wage evidence is more favourable to our model's cross-sectional implications. Measures of *marginal* profitability are difficult to obtain, of course, but there is evidence that workers do appropriate part of firms profitability (see e.g. Krueger and Summers (1988), and Layard, Nickell and Jackman (1991)), Blanchflower and Oswald (1988) show that 40% of the workers interviewed in the 1982 Workplace Industrial Relations Survey for the U.K. indicated that the profitability/productivity of their firm was one of the main factors determining their wage demands. This holds for union as well as non-union workers, and is the most commonly cited factor (even more than the cost of living). A more indirect piece of evidence can be obtained from the attempt by Layard *et al.* to identify the importance of internal (to the firm) factors in wage negotiations. They concluded that these are not important in the Scandinavian countries where bargaining is mostly centralized but it reaches values close to 30% for countries like the U.S. and Japan.

In summary, our stylized model does appear capable of replicating and rationalizing a few important aspects of labour flows at the microeconomic level and, with some qualifications, of cross-sectional wage differentials. We turn next to the aggregate and welfare implications of the model.

6. ON EFFICIENCY

In this section we address issues of aggregate efficiency in the context of the previous sections' technological and matching structure. For simplicity, we discuss efficiency in terms of steady-state, undiscounted optimization, whose decentralized counterpart is obtained by setting $r = 0$ in the expressions obtained above: as in Hosios (1990), the simplification is inconsequential for the issues we address. Like the market's equilibrium, the relevant social optimization problem has no closed-form solution. Still, inspection of the first-order conditions for optimal aggregate allocations affords insights into various sources of inefficiency.

The efficiency properties of the market we consider are most easily interpreted if decreasing marginal-revenue schedules of "firms" (or production units) reflect decreasing-returns production of homogeneous output.[16] Integrating the expressions given above for marginal productivity and marginal vacancy-posting costs, each unit's direct contributions in terms of production and vacancy-posting costs are quadratic in $l(\cdot)$ and $v(\cdot)$, respectively. Since employment in a good firm results in (possibly costly) firing with probability δ, the steady-state mobility cost per unit of employment is δF for each of the $l(\tau) - l(0)$ workers who become redundant upon realization of a negative productivity shock. Thus,

16. Alternatively, the model could be interpreted in terms of monopolistically competitive pricing of differentiated goods: as shown in Dixit and Stiglitz (1977), monopoly power is not necessarily associated with inefficient outcomes if it reflects socially valuable product differentiation.

a "good" unit of age τ contributes a production-equivalent flow

$$\eta_g l(\tau) - \frac{\sigma}{2} l(\tau)^2 - \frac{c}{2} v(\tau)^2 - \delta F(l(\tau) - l(0))$$

to the social objective function. The flow benefit from operation of a production unit experiencing bad productivity conditions, which posts no vacancies and, like a unit which has just received a positive productivity shock, employs $l(0)$ units of labour:

$$\eta_b l(0) - \frac{\sigma}{2} l(0)^2.$$

In steady state, a fraction $\delta/(\gamma + \delta)$ of the unitary mass of firms is experiencing bad business conditions. The remaining $\gamma/(\gamma + \delta)$ firms enjoy good productivity, but a mass of size $\delta\gamma/(\gamma + \delta)$ experiences a negative shock at every point in time, and reduces its employment from its current level to $l(0)$. Across all "good" firms and across all firms which are experiencing a transition from the good to the bad state, the date of the last positive business conditions shock (hence their "age" τ) is distributed exponentially, with parameter δ. Aggregating across firms, and adding the output-equivalent flow z from workers who are not currently working, we obtain an expression for steady-state production net of mobility and vacancy costs:

$$Y = \frac{\gamma}{\gamma + \delta} \left(\int_0^\infty \delta e^{-\delta\tau} \left((\eta_g - \delta F) l(\tau) - \frac{\sigma}{2} l(\tau)^2 - \frac{c}{2} v(\tau)^2 \right) d\tau \right) \tag{26}$$

$$+ \frac{\delta}{\delta + \gamma} \left((\eta_b + \gamma F) l(0) - \frac{\sigma}{2} l(0)^2 \right) + z U.$$

We shall constrain the social labour-allocation problem by the matching technology introduced above. Hence, we have

$$\dot{l}(\tau) - \xi \left(\frac{V}{U} \right)^v v(\tau) = 0, \qquad \forall \tau \in [0, \infty), \tag{27}$$

to which dynamic constraint we associate a schedule of Hamiltonian shadow prices $\{A^*(\cdot)\}$. Like its counterpart $A(\tau)$ in decentralized equilibrium, $A^*(\cdot)$ measures the marginal contribution to output of employment at a firm of age τ.

Given the matching technology in equation (27), the social planner must devote effort to the matching process or "post vacancies", like the business firms of the previous sections. Quite unlike decentralized decision makers, however, the social planner is aware that each unit's vacancy-posting and employment policies are relevant to the *aggregate* matching probability. This can be formalized by associating shadow prices μ and ρ to the definitional relationships between firm-level employment and vacancy schedules to their aggregate counterparts, and including the expressions

$$\mu \left(V - \frac{\gamma}{\gamma + \delta} \int_0^\infty \delta e^{-\delta\tau} v(\tau) d\tau \right), \tag{28}$$

$$\rho \left(\left(1 - \int_0^\infty \delta e^{-\delta\tau} l(\tau) d\tau - \frac{\delta}{\delta + \gamma} l(0) \right) - U \right) \tag{29}$$

in the social planner's Lagrangian objective. The shadow price μ in (28) indexes the social

cost of aggregate vacancies in terms of their negative effect on the matching process in
(27). More aggregate vacancies slow down matching and, as defined, μ measures the social
value of a *smaller* V which, for given $\{v(\cdot)\}$, would make it easier to match workers with
vacancy-posting firms. Symmetrically, ρ is the shadow price of aggregate unemployment
in terms of the matching process. Since more unemployment yields speedier matching in
(27), a higher U is socially good in this respect, and ρ measures the positive effect that
higher unemployment would have on allocation of labour to firms which are posting
vacancies.

Taking into account these Lagrangian terms, and collecting terms under the integral
signs, we obtain an expression for steady-state social production net of mobility and
vacancy costs,

$$\tilde{Y} = \frac{\gamma}{\gamma + \delta} \left(\int_0^\infty \delta e^{-\delta \tau} \left(\eta_g^* l(\tau) - \frac{\sigma}{2} l(\tau)^2 - \frac{c}{2} v(\tau)^2 + \mu v(\tau) \right) d\tau \right) \tag{31}$$

$$+ \frac{\delta}{\delta + \gamma} \left(\eta_b^* l(0) - \frac{\sigma}{2} l(0)^2 \right) + z$$

where the "social revenue" parameters

$$\eta_g^* \equiv \eta_g - \rho - \delta F - z, \qquad \eta_b^* \equiv \eta_b - \rho + \gamma F - z \tag{30}$$

differ from their private counterparts in intuitive ways. Note in particular that each unit
of employment has social cost z in terms of foregone leisure, and an additional cost ρ in
that higher employment makes matching more difficult. In the bad-firm social production
function, the term γF accounts for resources saved when a worker is "hoarded" in a bad
firm: the mobility cost F does not need to be paid when (with probability intensity γ) the
firm experiences a positive productivity shock.

To proceed, we characterize maximization of (31) subject to (27), conditional on
aggregate vacancies V, unemployment U, and the relevant shadow prices μ and ρ. The
socially optimal $l^*(\cdot)$ and $v^*(\cdot)$ schedules must satisfy the Hamiltonian conditions

$$A^*(\tau) = (cv^*(\tau) + \mu) \left(\xi \left(\frac{V}{U} \right)^\nu \right)^{-1}, \tag{32}$$

$$\eta_g^* - \sigma l^*(\tau) + \dot{A}^*(\tau) = \delta A^*(\tau). \tag{33}$$

These are readily comparable to their counterpart in decentralized equilibrium (equations
(4) and (2) respectively), and have a straightforward economic interpretation. In steady
state, the social marginal cost of vacancies posted by a firm of "age" τ is $\mu + cv^*(\tau)$;
condition (33) imposes that this be equal to the social value of those same vacancies, i.e., to
the matching probability intensity times $A^*(\tau)$. The optimality condition for decentralized
vacancy-posting activity is similar but, crucially, sets $\mu = 0$, neglecting the aggregate conges-
tion effect of individual vacancy posting activities. The social shadow value $A^*(\cdot)$ obeys
the asset-valuation equation (33) and the transversality condition $\lim_{\tau \to \infty} A^*(\tau) e^{-\delta \tau} = 0$.
These too have a direct counterpart in decentralized equilibrium, but the social "dividend"
paid by each production unit features η_g^* as defined in (30) instead of $\tilde{\eta}_g - w_g(\tau)$: we see
from the definition $\tilde{\eta}_g \equiv \eta_g - \delta F$ that the decentralized equilibrium not only bases vacancy
posting on the market wage schedule $w_g(\cdot)$, but also sets $\rho = 0$, neglecting the beneficial
effect of (aggregate) unemployment on the matching process.

Combining (32) and (33) yields

$$\frac{\delta c}{\xi(V/U)^v}\, v^*(\tau) + \frac{\delta \mu}{\xi(V/U)^v} = \eta_g^* - \rho - \sigma l^*(\tau) + \frac{c}{\xi(V/U)^v}\, \dot{v}^*(\tau), \qquad (34)$$

which can be differentiated with respect to τ to obtain

$$\frac{\delta c}{\xi(V/U)^v}\, \dot{v}^*(\tau) = -\sigma \dot{l}^*(\tau) + \frac{c}{\xi(V/U)^v}\, \ddot{v}^*(\tau). \qquad (35)$$

Using (27) to substitute out $\dot{l}^*(t)$, and rearranging, we obtain an ordinary differential equation,

$$\ddot{v}^*(\tau) - \delta \dot{v}^*(\tau) - \frac{\sigma}{c}\left(\xi\left(\frac{V}{U}\right)^v\right)^2 v^*(\tau) = 0,$$

which does not feature the Nash–sharing parameter β but is otherwise identical to its market-equilibrium counterpart encountered in Section 4. Accordingly, we write

$$v^*(\tau) = v^*(0)e^{-\lambda^* \tau}, \qquad \lambda^* \equiv \tfrac{1}{2}\left(\sqrt{\delta^2 + 4(\xi(V/U)^v)^2\sigma/c} - \delta\right) > 0, \qquad (36)$$

and note that, for given V/U, vacancy posting declines more rapidly in the social plan than in market equilibrium. This finding is easily interpreted: the social planner aims at maximizing total producer's surplus rather than the excess of production over wages; while Nash–bargained wages decline with a firm's good-time age in market equilibrium, the social leisure flow is the same for workers employed by firms of all ages.

We find that the social planning solution differs from market equilibrium in two interrelated respects. The first one is quite standard and well understood (see, e.g., Hosios (1990)). In decentralized equilibrium individual firms' vacancy-posting policies (and the resulting employment path) disregard their own effect on the aggregate labour market's tightness, as indexed by μ and ρ in the social objective function. Also, individual firms are charged market wages (rather than the social opportunity cost of labour) for "use" of each unit of labour they employ. In models with only one or two types of jobs, these two effects cancel each other out if the market wage, as determined by β, happens to be just enough higher than z so as to internalize to every firm's vacancy posting policy the *social* cost in terms of matching efficiency, as determined by v, of higher employment and lower unemployment. (Such models may also feature active search decision on the part of workers, who are completely passive in the model we study. Under constant-returns matching, the same sharing rule will ensure that both workers' and firms' decisions are socially optimal.)

In our framework, however, there is a second and more complicated dimension to the discrepancy between private and socially optimal vacancy-posting policies: individual firms' productivity is cross-sectionally different not only in levels, but also in dynamic terms. To achieve social efficiency, the market wage should not only internalize to individual firms' optimization problems the overall desirability of unemployment (and undesirability of vacancies); it should also enhance the incentive to post vacancies for those firms where additional employment is more highly desirable, namely "young" good firms in our stylized framework. This cross-sectional perspective is novel to our framework of analysis. Formally, the speed λ at which vacancy-posting declines over a typical firm's product cycle would coincide with its socially optimal counterpart if $\beta = 0$. However, $\beta = 0$ would of course *not* address the overall distortion in general: it would appropriately distribute across firms the intensity of search but, except in the degenerate case where $v = 0$, it would not yield the correct level of job-market tightness.
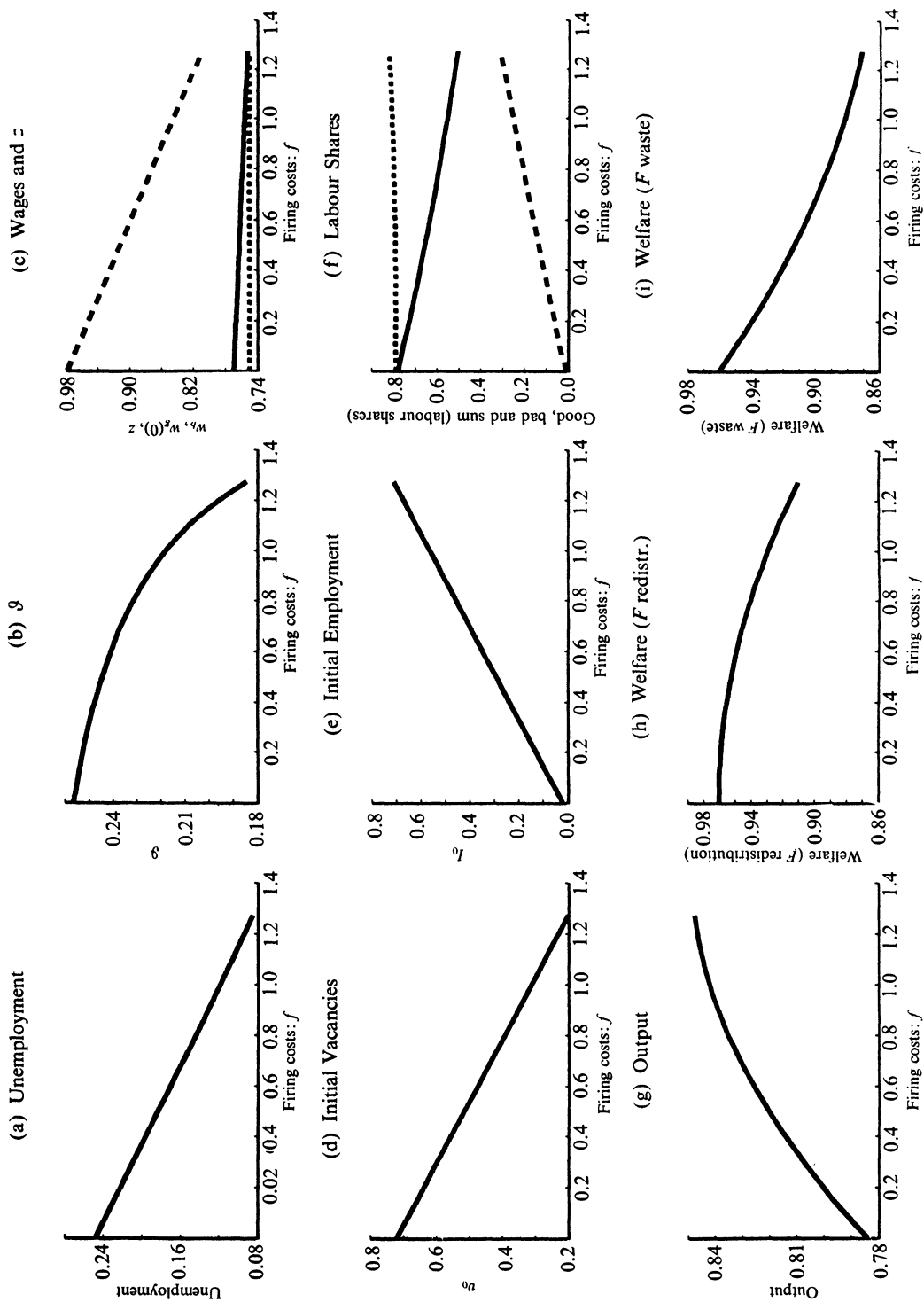
FIGURE 2

While we have derived the result in a specific and admittedly very stylized framework, the Nash bargaining device under the assumption of continuous renegotiation (or, non-enforceable long-term contracts) appears inadequate to internalize social incentives in models more general than the one we propose. Given that the dynamic outlook of different firms or production sites is heterogeneous, bargaining should at the very least take place over state-contingent wage *schedules* rather than on wages at every point in time. In our framework, the slope of the marginal productivity schedule and current employment levels index the relevant heterogeneity, but the point appears relevant to any model where firms face heterogeneous dynamic choices. A social planner would need to use as many instruments as there are active margins, and altering the Nash bargaining parameter β is not enough in the presence of cross-sectional dynamic heterogeneity.

The integrals in equation (31) are all available in closed form for given $v(0)$ and $l(0)$, but the socially optimal levels of U and V can only be computed numerically. It is not particularly interesting to perform such exercises, however. Rather, we briefly discuss several comparative-static results for market allocations in response to changes in the size F of institutional and technological labour mobility costs, which is arguably not only a likely policy instrument, but also particularly relevant to the interaction between allocative shocks, inefficient matching, and mobility costs.

Figure 2 plots the reaction of several endogenous variables to changes in F. Quite intuitively, higher firing costs yield less labour mobility and a lower unemployment rate in panel a and a lower steady-state vacancies/unemployment ratio in panel b, as good firms post fewer vacancies initially (panel d). The latter effect needs not dominate, however, and for different parameter configuration θ rises with F rather than fall: lower unemployment implies that firms find it more difficult to fill their vacancies, and therefore approach their optimal employment level more slowly, further tightening the labour market as more vacancies are kept open along the adjustment path. Higher and higher firing costs imply an *increasing* labour share in output, plotted in panel f as the upper dashed line: the increase in the lower dashed line, which plots the wage bill paid by bad firms, more than compensates the decline of good firms' wage bill, displayed as a solid line. The labour share is only very loosely related to workers' welfare, of course. On the one hand, the total size of output and the absolute value of the wage bill are also affected by the size of F (and we comment on this next); on the other, the income-equivalent flow from unemployment, z, should be taken into account as well.[17]

The most interesting results are those displayed in panels g, h, and i. For our baseline parameters, output is an increasing function of mobility costs. Two effects are at work when higher and higher firing costs are considered: lower unemployment would increase output if the proportion of those employed at good and bad firms could be kept constant but, for a given unemployment rate, higher firing costs imply more labour hoarding and a less efficient allocation of labour. In the figure, the first effect is dominant, though the second one could possibly come to dominate (and yield a decreasing relationship between output and F) if the unemployment rate becomes so low as to make labour reallocation very expensive.[18]

17. Interestingly, labour shares differ only very slightly when dramatically different β values are considered. As it turns out, the income share effects of workers' bargaining power at the individual, marginal-match level largely wash out when their equilibrium effect on unemployment is taken into account.

18. We have also experimented with comparative statics on β, the Nash bargaining strength of workers. Unemployment is higher for larger β values, hence the beneficial, unemployment-reducing effects of higher F dominate their negative effect on the efficiency of labour allocation over a wider range of F values. Stretching the interpretation beyond our model's stylized representation of labour-market interactions, and noting that "job security" legislation might well be concerned with output maximization, this result might rationalize the fact that legal restrictions on labour shedding are more stringent in highly unionized sectors and countries.

Panels h and i display the effect of larger F on two measures of (aggregate) welfare, or consumable output. In both panels, $z(1-L)$ is added to output to account for the output-equivalent flow enjoyed by the unemployed, and the resources absorbed by firms' search activities are subtracted from the resulting measure of gross output; in panel i firing costs are considered lost to society as well, as is appropriate if F reflects actual resource expenditure at the aggregate level rather than (third-party) redistribution or explicit constraints on labour-shedding policies (this measure of welfare corresponds to Y). In all cases, consumable resources are a decreasing function of the mobility cost F, largely because of the decline in the unemployment rate and in the output-equivalent flow associated with it.

7. CONCLUSION

The model we propose in this paper can be used to study positive and normative aspects of steady-state equilibrium in a market where firms of endogenous size experience idiosyncratic shocks and undergo a costly search process to hire their workers. Within the confines of our model, we have highlighted the interactions between job-security provisions and sectoral shocks in determining the natural rate of unemployment, allocation of labour, and the extent of labour hoarding. We have also found that, when productivity and search costs are dynamically heterogeneous across firms, decentralized wage bargains imply important cross-sectional inefficiencies, which may overshadow the static search inefficiencies that simpler models focus on.

At the empirical level, our model is potentially capable of interpreting the variable intensity of labour re-allocation across different sectors, countries, and periods. Like other models of matching and bargaining, ours takes as given the size and frequency of idiosyncratic shocks to demand and technology. In our framework, however, these interact with the economy's institutional characteristics in determining the *realized* (as opposed to *desired*) intensity of labour re-allocation, and this should make it possible to use information on (e.g.) the extent of job security and labour's bargaining strength when interpreting the size of flows into and out of unemployment. While the model we propose is much too stylized for empirical applications, its implications are in qualitative accord with several pieces of cross-sectional evidence. Many extensions appear feasible and should not significantly alter the qualitative results emphasized above. In realistic situations, labour flows out of employment and into unemployment for reasons unrelated to firms' business conditions. Currently employed workers retire, or quit because of changes in their own (rather than their employers') conditions; and jobs are sought be new entrants to the labour force as well as by job losers. It would be conceptually simple, but analytically tedious, to account for such flows in our model allowing for exogenous quits and for balancing inflows into unemployment in a steady-state situation.

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