A Brief Note on Disinflation in a Menu Cost Model

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1   Introduction

This paper studies an economy's response in a menu cost model after an unanticipated credible monetary stabilization plan is announced. In particular, we are interested in describing the reaction of output, the frequency of price adjustment and price dispersion.

Motivated by Argentina's history in the years previous to the fixed-dollar-peg of the 1990's, it seems reasonable to believe that foward-looking agents anticipating future lower money growth rates and inflation would have altered their pricing behavior before the fixed-peg was actually in place.

The above could cast doubts on Alvarez, Beraja, Gonzalez-Rozada and Neumeyer (2012) (ABGN from here on) interpretation of the evidence on menu cost models by studying the argentinean economy during this exact period, since their theoretical results are derived for economies with constant money growth rates.

With this in mind, we will use ABGN's model to conduct the following experiment. Consider an economy that is first at its stationary equilibrium with a high constant money growth rate. A fully-credible stabilization plan is announced consisting of a gradual decrease in the growth rate of money for a fixed time interval T until it reaches a new lower constant rate.

This takes the economy out of its stationary equilibrium, making the price setting firms' value and policy functions time-dependent with feedback from the general equilibrium and results in the impossibility of deriving closed form expressions for the frequency of price adjustment and price dispersion as it is done in ABGN.

For the most part, this piece objective is to numerically compute the rational expectations equilibrium of the model resulting from the experiment described and compare the results to those in ABGN.

In Sections 2 and 3 we present the model and characterize some equilibrium implications. Section 4 describes the numerical algorithm. Section 5 exhibits the results.

2 The Model

The model closely resembles that of Golosov, Lucas (2007). However, we follow Alvarez, Beraja, Gonzalez-Rozada and Neumeyer (2012) in specifying the fixed cost of adjusting prices proportional to current profits and letting the idiosyncratic cost shocks be permanent. Additionally, we assume that products life is exponentially distributed.

As mentioned, we are interested in studying the transition of an economy that is initially at a steady state with high money growth rate to a new steady state with low money growth rate.

2.1 Household's problem

Aggregate consumption c_t is given by the Dixit-Stiglitz aggregator where differentiated goods are indexed by its price and marginal cost (P, Z)

$$c_t = \left[\int_{P,Z} C_t(P)^{\frac{\eta-1}{\eta}} \phi_t(dP, dZ) \right]^{\eta/(\eta-1)}$$

Lifetime utility depends on consumption, leisure l_t and real money balances $\frac{m_t}{p_t}$ as

$$\int_0^\infty e^{-rt} \left[\frac{c_t^{1-\gamma}}{1-\gamma} - \alpha l_t + log\left(\frac{m_t}{p_t}\right) \right] dt \tag{1}$$

where p_t is the aggregate price level.

$$p_t = \left[\int_{P,Z} P^{1-\eta} \phi_t(dP, dZ) \right]^{1/(1-\eta)}$$

Let $\{\mu^m(t), t \geq 0\}$ be a deterministic decreasing continuous time sequence of money growth rates with $lim_{t \to T} \mu^m(t) = \mu$ and m_0 the initial money supply, so that $m_t = m_0 e^{\int_0^t \mu^m(s)ds}$. Moreover, let $\{R(t), W(t), t \geq 0\}$ be the sequence of nominal interest rates and wages and B_0 initial nominal bond holdings.

We write the consumer's budget constraint as

$$B_0 + m_0 \geq \int_0^\infty Q_t \left[c_t p_t + (R_t - \mu_t^m) m_t - W_t l_t - \Pi_t \right] dt \tag{2}$$

where the discount factor $Q_t = e^{-\int_0^t R(s) ds}$ and Π_t is profit income coming from firms. The household's problem consists of choosing sequences $\{c(t), l(t), m(t), t \geq 0\}$ to maximize (1) subject to (2), taking $\{p(t), R(t), W(t), \Pi_t, \mu_t^m, t \geq 0\}$ as given.

2.2 Firm's problem

A firm with price P faces consumer demand $C_t(P)$, together with nominal wages W_t and a stochastic shock to its marginal cost Z_t.

Firms enter the period with a price level carried over from the past and have to decide a sequence of stopping times $\{T_i\}_0^\infty$ at which the adjustment cost is payed, and prices $\{P_i\}_0^\infty$ could be set.

Define $z_t = ln(Z_t)$ and the log-markup $x_t = ln(\frac{P_t}{Z_t W_t})$. The laws of motion are

$$dz_t = \mu^z dt + \sigma dB_t - z dN_t$$

Moreover, while no price adjustment is made and $dN_t = 0$

$$dx_t = -(R_t + \mu^z - r) dt - \sigma dB_t$$

Here, dB_t is standard-normal distributed and N_t is the counter of a Poisson process with constant arrival rate per unit of time ρ at which z restarts with $z = 0$ and the firm must set its initial markup. In other words, firms are assumed to die with instantaneous probability ρ and are replaced by new firms.

Denote by $\{W_t^n, R_t^n, c_t^n\}$ the equilibrium values from time t to $t + n$ of nominal wages,

interest rates and aggregate consumption. The firm's value function at time t is

$$V_t(P_t, Z_t, W_t^n, R_t^n, c_t^n) = max_{\{T,P_T\}} \mathbb{E}_t \left[\int_t^T \frac{Q_s}{Q_t} e^{-\rho(s-t)} \Pi(P_t, Z_s, W_s, c_s) ds + \right.$$

$$\left. + \frac{Q_T}{Q_t} e^{-\rho(T-t)} (V_T(P_T, Z_T, W_T^n, R_T^n, c_T^n) - c\phi(Z_T, W_T, c_T)) \right]$$

where $\Pi(.)$ are the period return profits while the price is unchanged and c is the menu cost that we assume is proportional to the fricitionless profits $\phi(.)$ (the profits that would prevail if prices were continuously adjusted). These costs are payed in labor units.

3 Equilibrium characterization

In this section, we will describe some implications of the consumer's problem for this economy. Later on, we will use these and other properties of the shocks to rewrite the firm's problem in a more tractable fashion.

3.1 Household's optimality

The household's problem implies that we can write the demand for the differentiated products as

$$C_t(P_t) = c_t^{1-\gamma\eta} \alpha^{-\eta} \left(\frac{P_t}{W_t} \right)^{-\eta} \tag{3}$$

Furthermore, in equilibrium nominal wages and bond prices will have to satisfy

$$W_t = \alpha R_t m_t \tag{4}$$

$$Q_s = Q_t e^{-\int_t^s R_v dv} = Q_t e^{-r(s-t)} \frac{W_t}{W_s} \tag{5}$$

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Finally, we can obtain a differential equation characterizing the evolution of nominal interest rates $\{R(t), t \geq 0\}$ for the sequence of money supply growth rates $\{\mu^m(t), t \geq 0\}$ as

$$\frac{dR_t}{dt} = R_t(R_t - (\mu_t^m + r)) \tag{6}$$

Using market clearing of goods and the fact that income profits are $\Pi_t = p_t c_t - W_t l_t$, the budget constraint of the household becomes

$$\frac{B_0}{m_0} + 1 = \int_0^\infty e^{-\int_0^t (R_v - \mu_v^m)dv}(R_t - \mu_t^m)dt \tag{7}$$

In this way, the solution to (6) has to satisfy (7) and converge to the constant money growth rate steady state

$$lim_{t \to T} R_t = r + \mu$$

The above differential equation is a Riccatti equation with known solution given by

$$R_t = \frac{(r + \mu)}{e^{-\int_t^\infty (\mu_s^m + r)ds} + (r + \mu) \int_t^\infty e^{-\int_t^s (\mu_v^m + r)dv} ds} \tag{8}$$

If necessary, by adjusting $\frac{B_0}{m_0}$ we make sure that it satisfies (7) and hence R_t is the equilibrium nominal interest rate.

3.2 Firm's problem re-statement

Using (3) it is possible to find an expression for the period return and frictionless profits

$$\Pi(P_t, Z_s, W_s, c_s) = c_s^{1-\gamma\eta}\alpha^{-\eta}W_s \left(\frac{P_t}{W_s}\right)^{-\eta}\left(\frac{P_t}{W_s} - Z_s\right)$$

$$\phi(z_T, W_T, c_T) = c_T^{1-\gamma\eta}\alpha^{-\eta}W_T e^{Z_T(1-\eta)}\frac{(\eta - 1)^{\eta-1}}{\eta^\eta}$$

Using equilibrium conditions (4) and (5), define $W_t J_t(z_t, x_t, R_t^n, c_t^n) = V_t(P_t, Z_t, W_t^n, R_t^n, c_t^n)$ where,

$$J_t(x_t, z_t, R_t^n, c_t^n) = max_{\{T, x_T\}} \mathbb{E}_t \left[\int_t^T e^{-(r+\rho)(s-t)} \tilde{\Pi}(x_s, z_s, c_s) ds + \right.$$

$$+ \left. e^{-(r+\rho)(T-t)} (J_T(x_T, z_T, R_T^n, c_T^n) - c\tilde{\phi}(z_T, c_T)) \right]$$

$$\tilde{\Pi}(x_s, z_s, c_s) = \frac{\eta^\eta}{(\eta-1)^{\eta-1}} \tilde{\phi}(z_s, c_s) \left(e^{x_s(1-\eta)} - e^{-x_s \eta} \right)$$

$$\tilde{\phi}(z_t, c_t) = c_t^{1-\gamma\eta} \alpha^{-\eta} e^{z_t(1-\eta)} \frac{(\eta-1)^{\nu-1}}{\nu^\nu}$$

This can be rewritten as

$$J_t(x, z, R_t^n, c_t^n) = max \left\{ J_t^a(z, R_t^n, c_t^n), J_t^i(x, z, R_t^n, c_t^n) \right\}$$

$$J_t^a(z, R_t^n, c_t^n) = -c\tilde{\phi}(z, c_t) + max_{\{T, x'\}} \left\{ \int_0^T \mathbb{E} \left[\tilde{\Pi}(x_s, z_s, c_{t+s}) | x_0 = x', z_0 = z \right] ds + \right.$$

$$\left. + e^{-(r+\rho)T} \mathbb{E} \left[J_{t+T}(x_T, z_T, R_{t+T}^n, c_{t+T}^n) | x_0 = x', z_0 = z \right] \right\}$$

$$J_t^i(x, z, R_t^n, c_t^n) = max_{\{T\}} \left\{ \int_0^T \mathbb{E} \left[\tilde{\Pi}(x_s, z_s, c_{t+s}) | x_0 = x, z_0 = z \right] ds + \right.$$

$$\left. + e^{-(r+\rho)T} \mathbb{E} \left[J_{t+T}(x_T, z_T, R_{t+T}^n, c_T^n) | x_0 = x, z_0 = z \right] \right\}$$

Given the homogeneity in z_t and c_t of the value function and the fact that z_t has independent increments allow us to write the normalized function $v_t(x, R_t^n, c_t^n) = \frac{J_t(x, z, R_t^n, c_t^n)}{\tilde{\phi}(z_t, c_t)}$

as

$$v_t(x, R_t^n, c_t^n) = max\left\{v_t^a(R_t^n, c_t^n), v_t^i(x, R_t^n, c_t^n)\right\}$$

$$v_t^a(R_t^n, c_t^n) = max_{\{T,x'\}}\left\{\left[\int_0^T \mathbb{E}\left[\frac{\eta^\eta}{(\eta-1)^{\eta-1}}\left(\frac{c_{t+s}}{c_t}\right)^{1-\gamma\eta}e^{(z_s-z_0)(1-\eta)}\left(e^{x_s(1-\eta)}-e^{-x_s\eta}\right)|x_0=x'\right]ds + \right.\right.$$

$$\left.+e^{-(r+\rho)T}\mathbb{E}\left[\left(\frac{c_{t+T}}{c_t}\right)^{1-\gamma\eta}e^{(z_T-z_0)(1-\eta)}v_{t+T}(x_T, R_{t+T}^n, c_{t+T}^n)|x_0=x'\right]\right\}-c$$

$$v_t^i(x, R_t^n, c_t^n) = max_{\{T\}}\left\{\left[\int_0^T \mathbb{E}\left[\frac{\eta^\eta}{(\eta-1)^{\eta-1}}\left(\frac{c_{t+s}}{c_t}\right)^{1-\gamma\eta}e^{(z_s-z_0)(1-\eta)}\left(e^{x_s(1-\eta)}-e^{-x_s\eta}\right)|x_0=x\right]ds + \right.\right.$$

$$\left.+e^{-(r+\rho)T}\mathbb{E}\left[\left(\frac{c_{t+T}}{c_t}\right)^{1-\gamma\eta}e^{(z_T-z_0)(1-\eta)}v_{t+T}(x_T, R_{t+T}^n, c_{t+T}^n)|x_0=x\right]\right\}$$

Finally, using the laws of motion for z_t, x_t we obtain simplified expressions for the value of adjusting prices/markups and inaction

$$v_t^a(R_t^n, c_t^n) = -c + max_{\{T,x'\}}\left\{\left[\int_0^T \int_{-\infty}^{+\infty} f(\varepsilon, s, c_{t+s}, c_t)g(x', \varepsilon, s, R_{t+s})d\Phi(\varepsilon)ds + \right.\right. \tag{9}$$

$$+e^{-(r+\rho)T}\frac{(\eta-1)^{\eta-1}}{\eta^\eta}\int_{-\infty}^{+\infty}f(\varepsilon, T, c_{t+T}, c_t)v_{t+T}(x'-(R_{t+s}+\mu^z-r)T-\sigma\sqrt{T}\varepsilon, R_{t+T}^n, c_{t+T}^n)d\Phi(\varepsilon)\right\}$$

$$v_t^i(x, R_t^n, c_t^n) = max_{\{T\}}\left\{\left[\int_0^T \int_{-\infty}^{+\infty} f(\varepsilon, s, c_{t+s}, c_t)g(x, \varepsilon, s, R_{t+s})d\Phi(\varepsilon)ds + \right.\right. \tag{10}$$

$$+e^{-(r+\rho)T}\frac{(\eta-1)^{\eta-1}}{\eta^\eta}\int_{-\infty}^{+\infty}f(\varepsilon, T, c_{t+T}, c_t)v_{t+T}(x-(R_{t+s}+\mu^z-r)T-\sigma\sqrt{T}\varepsilon, R_{t+T}^n, c_{t+T}^n)d\Phi(\varepsilon)\right\}$$

where $\Phi(\varepsilon)$ is the c.d.f. of a standard normal and

$$f(\varepsilon, s, c_{t+s}, c_t) = \frac{\eta^\eta}{(\eta-1)^{\eta-1}}\left(\frac{c_{t+s}}{c_t}\right)^{1-\epsilon\eta}e^{(\mu^z s+\sigma\sqrt{s}\varepsilon)(1-\eta)}$$

$$g(x, \varepsilon, s, R_{t+s}) = e^{(x-(R_{t+s}+\mu^z-r)s-\sigma\sqrt{s}\varepsilon)(1-\eta)}-e^{-(x-(R_{t+s}+\mu^z-r)s-\sigma\sqrt{s}\varepsilon)\eta}$$

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4 Numerical computation of equilibrium

In order to compute the rational expectations equilibrium of the model we do a discrete-time, discrete-state approximation of the value functions in (9), (10)

$$
\tilde{v}_t^a(R_t^n, c_t^n) = -c + max_{\{x'\}} \left\{ \frac{\eta^\eta}{(\eta-1)^{\eta-1}} \left(e^{x'(1-\eta)} - e^{-x'\eta} \right) \Delta + \right. \tag{11}
$$

$$
\left. +e^{-(r+\rho-\mu^z(1-\eta))\Delta} \left(\frac{c_{t+\Delta}}{c_t} \right)^{1-\gamma\eta} \sum_{\varepsilon=\underline{\varepsilon}}^{\bar{\varepsilon}} \Phi(\varepsilon)e^{\sigma\sqrt{\Delta}\varepsilon(1-\eta)} v_{t+\Delta}(x' - (R_{t+\Delta} + \mu^z - r)\Delta - \sigma\sqrt{\Delta}\varepsilon, R_{t+\Delta}^n, c_{t+\Delta}^n) \right\}
$$

$$
\tilde{v}_t^i(x, R_t^n, c_t^n) = \frac{\eta^\eta}{(\eta-1)^{\eta-1}} \left(e^{x(1-\eta)} - e^{-x\eta} \right) \Delta + \tag{12}
$$

$$
+ e^{-(r+\rho-\mu^z(1-\eta))\Delta} \left(\frac{c_{t+\Delta}}{c_t} \right)^{1-\gamma\eta} \sum_{\varepsilon=\underline{\varepsilon}}^{\bar{\varepsilon}} \Phi(\varepsilon)e^{\sigma\sqrt{\Delta}\varepsilon(1-\eta)} v_{t+\Delta}(x - (R_{t+\Delta} + \mu^z - r)\Delta - \sigma\sqrt{\Delta}\varepsilon, R_{t+\Delta}^n, c_{t+\Delta}^n)
$$

The law of motion for the log-markup if no adjustment is made between t and $t+\Delta$, nor the product dies with probability $e^{-\rho\Delta}$ is

$$
x_{t+\Delta} = x_t - (R_t + \mu^z - r)\Delta - \sigma\sqrt{\Delta}\varepsilon
$$

The numerical algorithm is as follows:

1. Pose a sequence for $\{\mu_t\}_0^n$ where μ_0 and μ_n are the constant values for money growth rates at the initial and new stationary equilibriums.

2. Use (7) and (8) to solve for the sequence of equilibrium interest rates $\{R_t\}_0^n$

3. Start with a guess for a vector c_0^n and a fixed convergence length n for which $v_n(x, R_n^n, c_n^n) = v(x, r + \mu_n, c_n)$, where c_n is the constant value at the new stationary equilibrium of aggregate consumption.

4. Use (11) and (12) to iterate backwards on the value function and calculate the sequence of policy functions $\{\tilde{g}_t(x, c_t^n)\}_0^n$ (This will be a sequence of inaction thresholds and return points).

5. Sample from the initial joint stationary distribution of (x, z) (when money growth is high at μ_0). Simulate a path for shocks $\{z_t\}_0^n$ using dz. Calculate $\{x_t\}_0^n$ per using $\{\tilde{g}_t(x, c_t^n)\}_0^n$ and dx.

6. Repeat step 5. multiple times to get the simulated distribution of log-relative prices $z_t + x_t$ at each point in time for a given c_0^n.

7. Calculate the updated value of the sequence of aggregate consumption by

$$c_t = \alpha^{-\frac{\eta}{\gamma}} \left(\frac{W_t}{P_t}\right)^{\frac{1}{\gamma}}$$

8. Repeat from step 3. until convergence of c_0^n.

5 Results

In this section we present the equilibrium behavior of nominal interest rates, output/consumption, frequency of price changes and log-price dispersion in the transition from a high money growth steady state to a low money growth one. Even when interest rates are not of our immediate incumbency, they comprise expectations about future money growth rates. Hence, it is the most relevant general equilibrium feedback variable affecting the pricing behavior of firms in this economy and accounting for the difference between the results during the transition between steady states studied in this piece and the steady states themselves studied in ABGN. For details on model calibration the reader should refer to ABGN.

Figure 1 depicts the experiment. We feed the model with the actual M1 growth rate for Argentina between 1990 and 1994 which determines the model equilibrium nominal interest rate assuming that the future path of money growth rates is credible and perfectly known by the agents in this economy. In anticipation of lower money growth and inflation, the interest rate drops on impact around 50 percent and after 2 years is approximately at its new steady state value. We choose 1990 as our initial departure point since the first quarter of this

year coincides with the government announcement of a sharp contraction in money growth, followed by the Convertibility Plan in 1991.

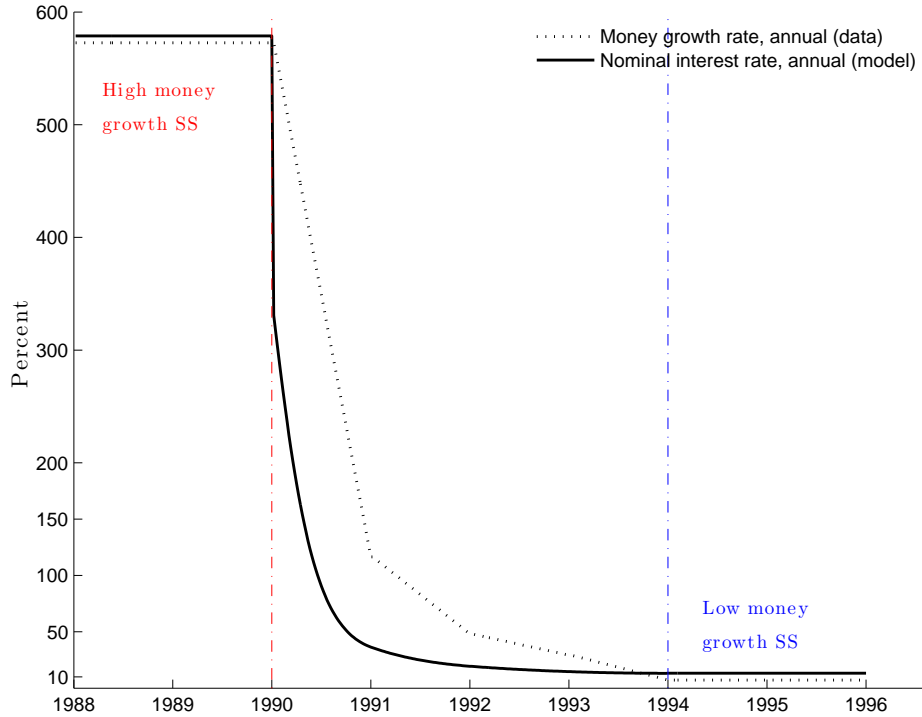Figure 1: Money growth and nominal interest rate

Figure 2 shows the consumption response expressed as percent changes from the initial steady state with high money growth. On impact, consumption drops around 2 percent. This relative small change given the magnitude of the shock results from the fact that all firms decrease their prices as a response to the large 50 percent decrease in nominal wages/increase in markups, which themselves are a consequence of the decrease in the nominal interest rate and hence real money balances.

Nonetheless, since we are starting with prices distributed according to the stationary distribution with high money growth, only the firms with markups that were already at its static optimum will decrease prices by the full nominal wage decrease. All the rest will decrease prices by a lower amount. In turn, the aggregate price level decreases less than the nominal wage and so consumption is lower on impact.

After the initial period, consumption increases above its old steady state level since money growth is still high and nominal interest rates are slowly decreasing which implies nominal wages are increasing. The large magnitude of this rebound results from the frequency of price adjustments being particularly small right after the initial impact since all firms had previously adjusted their prices.

Along the transition to a new higher consumption steady state there are two opposing forces. Nominal wages are increasing since money growth is always positive and the change in the nominal interest rate is small in comparison. However, nominal interest rates and expected inflation are decreasing which lowers the necessity and so the frequency of price adjustments as can be seen in Figure ??. The observed non-monotonicity in consumption response is a reflection of the above.
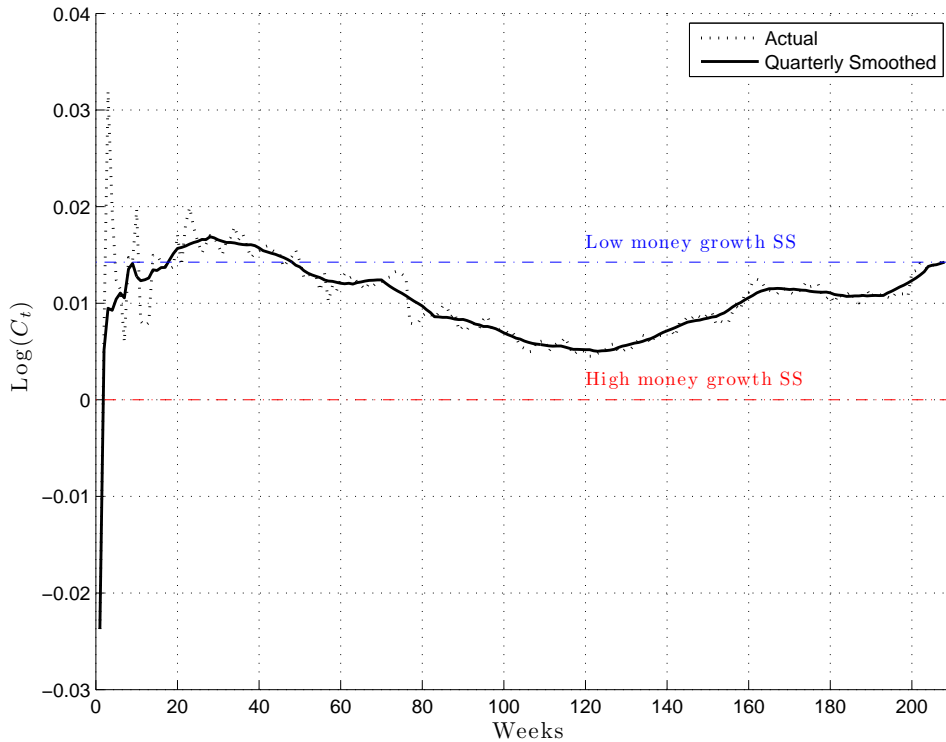
Figure 2: Consumption

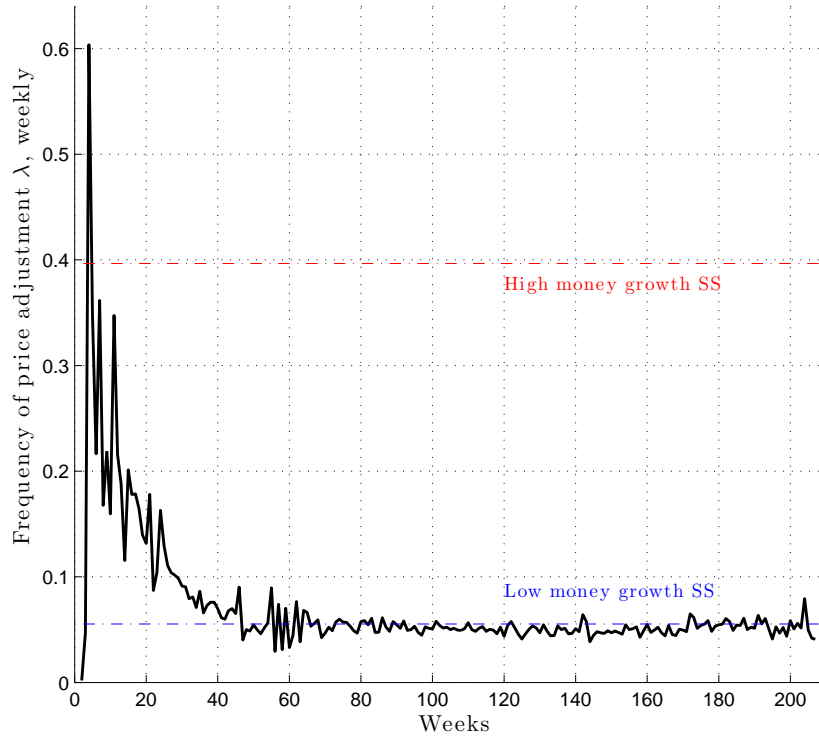Figure 3: Frequency of price adjustment λ

Figure 4 presents the dispersion in log-prices. There appears to be a monotonic decrease as inflation converges to the new lower steady state. The magnitudes are rather small which comes as no surprise given the results in ABGN when comparing steady states with high and low inflation.
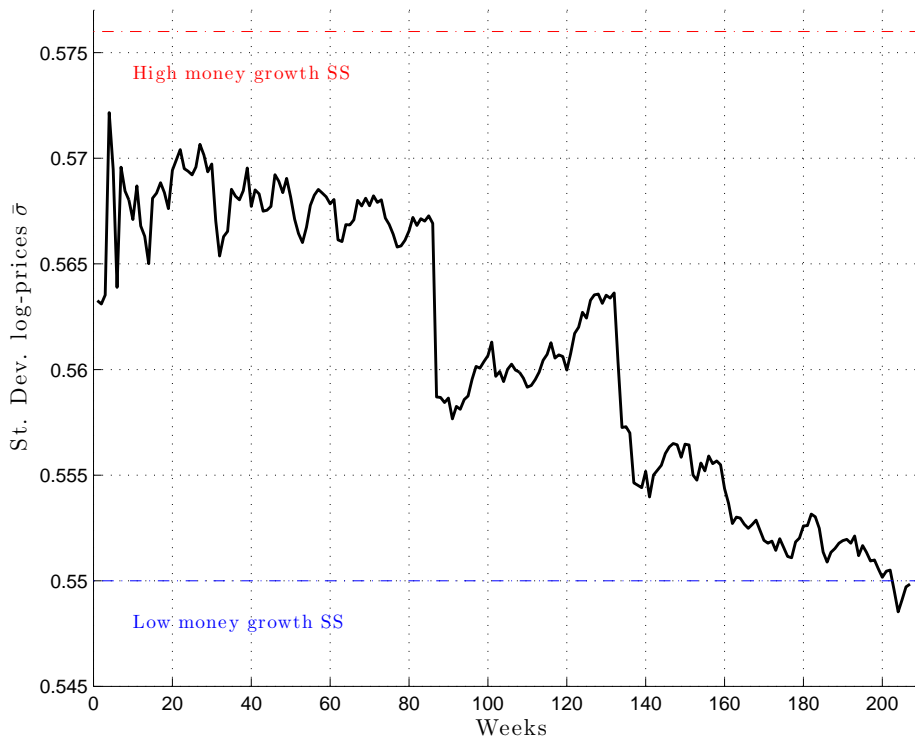
Figure 4: Dispersion of relative prices

Figure 5 illustrates the relationship between the frequency of price adjustments, λ, and the model implied expected inflation rate as defined in ABGN with a 6 months ahead prediction window. In steady state, it would be equivalent to look at money growth rates directly. Nevertheless, it is key given the nature of our experiment to consider expectations about the future inflation rate as it is the most relevant variable affecting pricing behavior. Identically to ABGN we fit a NLLS regression to obtain the elasticity of λ with respect to expected inflation at high levels (η) and the semi-elasticity at low levels (ϵ).

The elasticity η is estimated at 0.73, above the theoretical 2/3 derived in ABGN for steady states with small menu costs and high inflation relative to the cost shock volatility. Furthermore, the semi-elasticity ϵ is 3% i.e. the percentage change in λ when the expected inflation rate goes from 0 to 1 percent.

Figure 5: Frequency of price adjustment λ and expected inflation
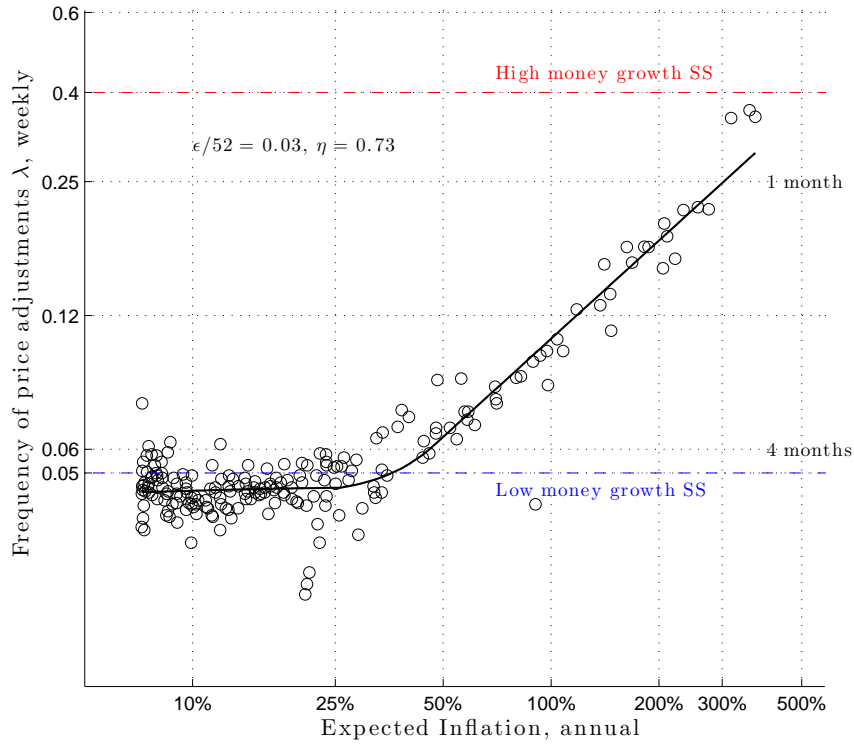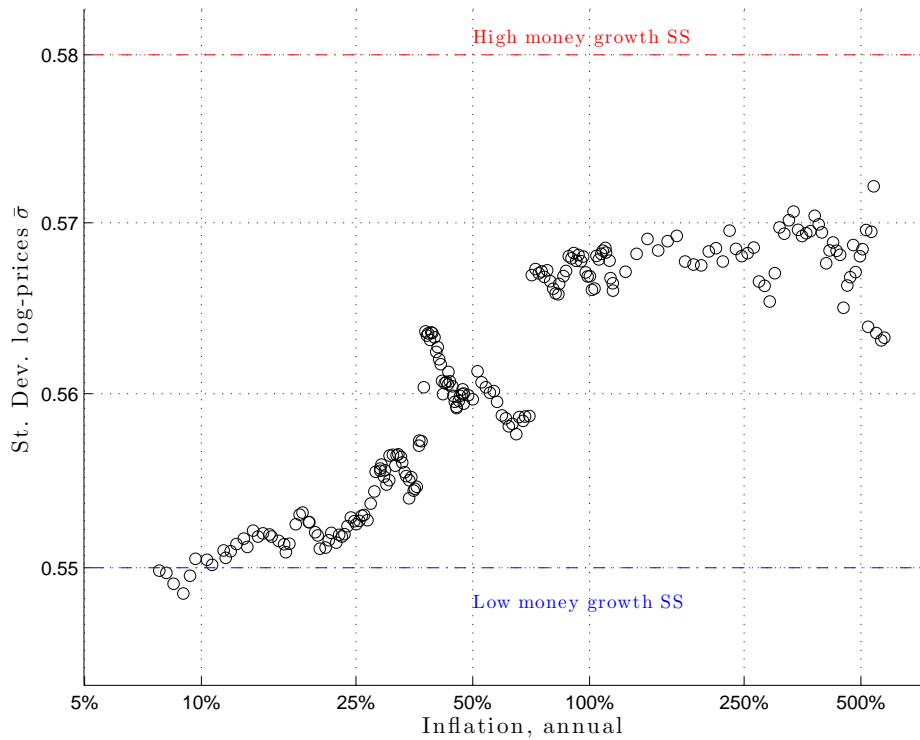

Figure 6: Dispersion of relative prices and inflation

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In Figure 6 the standard deviation of log-prices $\bar{\sigma}$ is plotted against inflation. The elasticity is approximately zero at all inflation levels.

6 Final Remarks

We set out to study the behavior of consumption, the frequency of price adjustment and price dispersion along the transition of an economy from a high money growth rate steady state to a with low money growth one after this stabilization plan is announced.

After numerically solving for the rational expectations equilibrium we conclude the model's predictions are in line with the observed argentinean experience in the period studied in ABGN.

The frequency of price changes elasticity at high inflation is in the ballpark of the estimates in ABGN, which vary from 0.51 to 0.68 depending on the methodology. The elasticity of log-price dispersion is approximately zero, somewhat lower than the estimated in ABGN, although the inflation variation in the experiment in this paper is smaller as well.

In sum, ABGN's interpretation of Argentina's evidence on menu cost models by considering its steady state predictions, appear to be robust to conducting the analysis in a non-stationary economy during a disinflation process.

Comments and questions:

1) I haven't done so here, but it looks like in response to a one time unanticipated positive shock to the money supply, the nominal interest rates would go up or stay unchanged. Is this so? What do you think about this?

2) There is a non-monotonicity involved in the response of output to a one time unanticipated positive shock to the money supply coming from the fact that the frequency of price changes and the nominal wage both increase. It looks like there is a size of the shock that maximizes the output response. Is this so? This size must be different depending on the

initial inflation rate. What about the state of the economy? Say output is below steady state. Does it require a larger size money shock to have the same impact on output than if output is at its steady state level?

3) I looked around a little bit and I have not seen any menu cost type model in an open economy. Do you think it could be interesting to do so and be able to talk about, say, real exchange rates? I'm thinking there are a number of papers who have addressed the issue of stabilization plans in small open economies (Calvo, Vegh and company..) which is related to what I've done here. Also, maybe as a new take or alternative to the so called New Open Economy Macroeconomics?