Online Appendix for "School Admissions Reform in Chicago and England: Comparing Mechanisms by their Vulnerability to Manipulation"

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This online appendix includes proofs of all mathematical results in the main text and a documentation appendix for Table 1.

THEOREM 1: *Suppose each student has a complete rank ordering and $k > 1$. The old Chicago Public Schools mechanism (Chi^k) is at least as manipulable as any weakly stable mechanism.*

PROOF:

Fix a problem P and let φ be an arbitrary mechanism that is weakly stable. Suppose that Chi^k is not manipulable for problem P.

Claim 1: Any student assigned under $\text{Chi}^k(P)$ receives her top choice.

Proof. If not, since each student has a complete rank order list, $|I| > Q$, $k > 1$, there must be a student that is assigned to a school s he has not ranked first. Consider the highest composite score student i who is unassigned. Student i can rank school s first and will be assigned a seat there in the first round of Chi^k mechanism instead of some student who has not ranked school s first. That contradicts Chi^k is not manipulable for problem P.

Claim 2. The set of students who are assigned a seat under $\text{Chi}^k(P)$ is equal to the set of top Q composite score students.

Proof. If not, there is a school seat assigned to a student j who does not have a top Q score. Let student i be the highest scoring top Q student who is not assigned. Since student i has a complete rank order list, she can manipulate Chi^k by ranking student j's assignment as her top choice again contradicting Chi^k is not manipulable for problem P.

Since each of the top Q students is matched to her top choice in matching $\text{Chi}^k(P)$, all other students are unassigned.

Claim 3. In problem P, matching $\text{Chi}^k(P)$ is the unique weakly stable matching.

Proof. By Claims 1 and 2 it is possible to assign each one of the top Q students a seat at their top choice school under P and $\text{Chi}^k(P)$ picks that matching. Let $\mu \neq \text{Chi}^k(P)$. That means under μ there exists a top Q student i who is not

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assigned to her top choice s. Pick the highest composite score such student i. Since all higher score students are assigned to their top choices, either there is a vacant seat at her top choice s or it admitted a student with lower composite score. In either case the pair (i, s) strongly blocks matching μ. Hence $\text{Chi}^k(P)$ is the unique weakly stable matching under P.

We are now ready to complete the proof. By Claim 3, $\varphi(P) = \text{Chi}^k(P)$ and hence mechanism φ assigns all top Q students a seat at their top choices. None of the top Q students has an incentive to manipulate φ since each receives her top choice. Moreover no other student can manipulate φ because regardless of their stated preferences, $\varphi(P) = \text{Chi}^k(P)$ remains the unique weakly stable matching and hence φ picks the same matching for the manipulated economy. Hence, any other weakly stable mechanism is also not manipulable under P.

PROPOSITION 1: *Suppose there are at least k schools and let $k > 1$. The old Chicago mechanism* (Chi^k) *is more manipulable than truncated serial-dictatorship* (Sd^k) *CPS adopted in 2009.*

PROOF:

Chi^k is a special case of the Fpf^k mechanism where all schools are first preference first schools with an identical priority ranking. Similarly Sd^k is a special case of GS^k where all schools have an identical priority ranking. Therefore Chi^k being *as manipulable as* Sd^k directly follows from Proposition **??**. We complete the proof by giving an example where Chi^k is manipulable even though Sd^k is not.

There are three students and three schools each with one seat. The student preferences and the uniform school priorities are:

$$\begin{array}{ll} R_{i_1} : s_1, s_2, s_3, i_1 & \qquad \pi_{s_1} : i_1, i_2, i_3 \\ R_{i_2} : s_1, s_2, s_3, i_2 & \qquad \pi_{s_2} : i_1, i_2, i_3 \\ R_{i_3} : s_2, s_3, s_1, i_3 & \qquad \pi_{s_3} : i_1, i_2, i_3 \end{array}$$

The outcomes of Chi^2 and Sd^2 are:

$$\text{Chi}^2(R) = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & i_2 & s_2 \end{pmatrix} \quad \text{and} \quad \text{Sd}^2(R) = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}.$$

Since no student remains unmatched under Sd^2, strategy-proofness of Sd implies that no student can manipulate Sd^2 under profile R. In contrast

$$\text{Chi}^2(R_{-i_2}, R'_{i_2}) = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

where R'_{i_2} is any preference relation student i_2 ranks school s_2 as his first choice, and therefore

$$\text{Chi}^2_{i_2}(R_{-i_2}, R'_{i_2})\, P_{i_2}\, \text{Chi}^2_{i_2}(R)$$

implies that Chi^2 is vulnerable under profile R. Hence Chi^2 is more manipulable than Sd^2. It is straightforward to extend this example to show that Chi^k is more manipulable than Sd^k for $k > 2$.

PROPOSITION 2: *Let $\ell > k > 0$ and suppose there are at least ℓ schools. Then GS^k is more manipulable than GS^ℓ.*

PROOF:

Suppose there is a student i and preference $\hat{P}_i$ such that

$$GS^\ell_i(\hat{P}_i, P_{-i}) \;\; P_i \;\; GS^\ell_i(P). \tag{1}$$

For any student j, let P^ℓ_j be the truncation of P_j after the ℓ^{th} choice. This means that in P^ℓ_j any choice after the top ℓ in P_j are unacceptable, and choices among the top ℓ are ordered according to P_j. Observe that relation (1) implies that

$$GS_i(\hat{P}^\ell_i, P^\ell_{-i}) \;\; P_i \;\; GS_i(P^\ell). \tag{2}$$

Since GS is strategy-proof, relation (2) implies that student i does not receive one of her top ℓ choices from the GS mechanism under profile P^ℓ. Hence, $GS_i(P^\ell) = GS^\ell_i(P) = i$.

For $k < \ell$, there are two cases to consider.

Case 1: $GS^k_i(P) = i$.

Let $GS^\ell_i(\hat{P}_i, P_{-i}) = s$ and let $\tilde{P}_i$ be such that s is the only acceptable school.

Claim: $GS^k_i(\tilde{P}_i, P_{-i}) = s$.
Proof: First note that $GS^\ell_i(\tilde{P}_i, P_{-i}) = s$. Moreover, by definition

$$GS^\ell(\tilde{P}_i, P_{-i}) = GS(\tilde{P}_i, P^\ell_{-i}) \qquad \text{and} \qquad GS^k(\tilde{P}_i, P_{-i}) = GS(\tilde{P}_i, P^k_{-i}).$$

Gale and Sotomayor (1985) (see also Theorem 5.34 of Roth and Sotomayor 1990) implies that

$$GS_i(\tilde{P}_i, P^k_{-i}) \;\; R_i \;\; GS_i(\tilde{P}_i, P^\ell_{-i}).$$

Substituting the definitions,

$$GS^k_i(\tilde{P}_i, P_{-i}) \;\; R_i \;\; \underbrace{GS^\ell_i(\tilde{P}_i, P_{-i})}_{=s}.$$

Since c is the only acceptable school in $\tilde{P}_i$, the claim follows. ◇

Thus, in the first case, student i can manipulate GS^k:

$$\underbrace{GS_i^k(\tilde{P}_i, P_{-i})}_{=s} \;\; P_i \;\; \underbrace{GS_i^k(P)}_{=i}.$$

Case 2: $GS_i^k(P) \neq i$.

Claim 1: $\exists j \in I$ such that $GS_j^k(P) = j$ although $GS_j^\ell(P) \neq j$.

Proof: Suppose not. Then, since $GS_i^\ell(P) = i$ and $GS_i^k(P) \neq i$, there is a school that is assigned strictly more students under $GS^k(P)$ than $GS^\ell(P)$. This is a contradiction to Gale and Sotomayor (1985), which requires that each school is weakly worse off under GS^k (since profile P^k is a truncation of profile P^ℓ). ⋄

Pick any $j \in I$ such that $GS_j^k(P) = j$ although $GS_j^\ell(P) \neq j$. Let $GS_j^\ell(P) = s$ and let $\tilde{P}_j$ be such that s is the only acceptable school.

Claim 2: $GS_j^k(\tilde{P}_j, P_{-j}) = s$.

Proof: Since $GS_j^\ell(P) = s$, we have $GS_j^\ell(\tilde{P}_j, P_{-j}) = c$ as well. Moreover, by definition

$$GS^\ell(\tilde{P}_j, P_{-j}) = GS(\tilde{P}_j, P_{-j}^\ell) \quad \text{and} \quad GS^k(\tilde{P}_j, P_{-j}) = GS(\tilde{P}_j, P_{-j}^k).$$

Gale and Sotomayor (1985) implies that

$$GS_j(\tilde{P}_j, P_{-j}^k) \;\; R_j \;\; GS_j(\tilde{P}_j, P_{-j}^\ell).$$

Substituting the definitions,

$$GS_j^k(\tilde{P}_j, P_{-j}) \;\; R_j \;\; \underbrace{GS_j^\ell(\tilde{P}_j, P_{-j})}_{=s}.$$

Since s is the only acceptable school in $\tilde{P}_j$,

$$GS_j^k(\tilde{P}_j, P_{-j}) = s,$$

which establishes the claim. ⋄

Thus, for the second case, student j can manipulate GS^k:

$$\underbrace{GS_j^k(\tilde{P}_j, P_{-j})}_{=s} \;\; P_j \;\; \underbrace{GS_j^k(P)}_{=j}.$$

Finally, we describe a problem where GS^ℓ is not manipulable by any student, but GS^k is manipulable by some student for $\ell > k > 0$. Suppose there are two students, i_1 and i_2, and two schools, s_1 and s_2, each with one seat. The students have identical preferences which rank s_1 ahead of s_2 and both schools have identical priority rankings where student i_1 has higher priority than student i_2. Under GS^2, no student can manipulate because each one is assigned a school and GS is strategy-proof. In contrast, student i_2 is unassigned under GS^1, and he can benefit from ranking s_2 as his top choice. This example can be generalized to the case of GS^k and GS^ℓ for any $\ell > k > 0$. Since all schools have the same priority ranking in this example, it also proves that $\textsc{Sd}^k$ is more manipulable than $\textsc{Sd}^\ell$ for any $\ell > k > 0$. This completes the proof.[1]

PROPOSITION 3: *Suppose there are at least k schools where $k > 1$. Then* $\textsc{Fpf}^k$ *is more manipulable than GS^k.*

PROOF:

For any student j, let P_j^k be the truncation of P_j after the k^{th} choice. By definition,

$$\textsc{Fpf}^k(P) = \textsc{Fpf}(P^k) \qquad \text{and} \qquad GS^k(P) = GS(P^k).$$

Suppose that no student can manipulate $\textsc{Fpf}^k$. We will show that no student can manipulate GS^k either. Consider two cases:

Case 1: $\textsc{Fpf}^k(P) = \textsc{Fpf}(P^k)$ is stable under profile P.

Since $\textsc{Fpf}(P^k)$ is stable under P, it is stable under P^k as well. Moreover, $GS(P^k)$ is stable for P^k by definition. Since the set of unmatched students across stable matchings is the same (McVitie and Wilson 1970), for all students i,

$$GS_i(P^k) = i \quad \Leftrightarrow \quad \textsc{Fpf}_i(P^k) = i. \tag{3}$$

Pick some student i. If $GS_i^k(P^k) \neq i$, then student i receives one of her top k choices. This implies that i receives one of her top k choices under GS. Since GS is strategy-proof, student i cannot manipulate GS^k.

Suppose $GS_i^k(P^k) = i$ and student i can manipulate. We derive a contradiction. Since i can manipulate, there exists some school s and preference $\hat{P}_i$ such that

$$\underbrace{GS_i^k(\hat{P}_i, P_{-i}^k)}_{=s} \; P_i \; i.$$

Observe that s is not one of the top k choices of student i under P_i for otherwise student i could manipulate GS. Construct $\tilde{P}_i$ which lists s as the only acceptable school.

[1] It is also possible to provide an alternative, indirect proof of this result using the equilibrium interpretation of the definition of weakly more manipulable than together with the characterization of the set of Nash equilibria in the preference revelation game induced by GS^k in Theorem 6.5 of Haeringer and Klijn (2009).

Matching $GS^k(\hat{P}_i, P^k_{-i})$ remains stable under $(\tilde{P}_i, P^k_{-i})$ and therefore

$$GS^k_i(\tilde{P}_i, P^k_{-i}) = s.$$

Since $GS(P^k)$ is stable under P^k and $GS^k_i(P^k) = i$ by assumption, relation (3) implies

$$\text{FPF}_i(P^k) = i.$$

By Roth (1984), matching $\text{FPF}(P^k)$ is not stable under $(\tilde{P}_i, P^k_{-i})$ since student i remains single under $\text{FPF}(P^k)$ although not under stable matching $GS^k(\hat{P}_i, P^k_{-i})$. Since matching $\text{FPF}(P^k)$ is not stable under $(\tilde{P}_i, P^k_{-i})$, but it is stable for P^k, the only possible blocking pair of $\text{FPF}(P^k)$ in $(\tilde{P}_i, P^k_{-i})$ is (i, s). But since $\text{FPF}_i(P^k) = i$, this implies that (i, s) also blocks $\text{FPF}(P^k)$ under P^k, which is the desired contradiction. Thus, in case 1, no student can manipulate GS^k.

Case 2: $\text{FPF}(P^k)$ is not stable for profile P.

In this case, a student i along with a *first preference first school* s block $\text{FPF}(P^k)$: That is, there exists $j \in \text{FPF}_s(P^k)$ such that not only i has higher base priority than j at school s, but also $s \; P_i \; \text{FPF}_i(P^k)$.

Construct $\tilde{P}_i$ so that school s is the only acceptable school for student i. Since $j \in \text{FPF}_s(P^k)$ and student i has higher base priority than student j at school s, we must have $i \in \text{FPF}_s(\tilde{P}_i, P^k_{-i})$. But this means that

$$\underbrace{\text{FPF}_i(\tilde{P}_i, P^k_{-i})}_{=s} \;\; P_i \;\; \text{FPF}_i(P^k),$$

contradicting the assumption that no student can manipulate FPF at P^k.

Finally we give an example where FPF^2 is manipulable but GS^2 is not. It is straightforward to extend the example for any $k > 2$. There are three students and three *first preference first* schools each with one seat. Since all schools are first preference first, FPF mechanism reduces to the special case of the Boston mechanism in this example. The student preferences and the (uniform) school priorities are:

$$\begin{array}{ll} R_{i_1} : s_1, s_2, s_3, i_1 & \pi_{s_1} : i_1, i_2, i_3 \\ R_{i_2} : s_1, s_2, s_3, i_2 & \pi_{s_2} : i_1, i_2, i_3 \\ R_{i_3} : s_2, s_3, s_1, i_3 & \pi_{s_3} : i_1, i_2, i_3 \end{array}$$

The outcomes of FPF^2 and GS^2 are:

$$\text{FPF}^2(R) = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & i_2 & s_2 \end{pmatrix} \quad \text{and} \quad GS^2(R) = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}.$$

Since no student remains unmatched under GS^2, strategy-proofness of GS implies that no student can manipulate GS^2 under profile R. In contrast

$$\text{FPF}^2(R_{-i_2}, R'_{i_2}) = \begin{pmatrix} i_1 & i_2 & i_3 \\ s_1 & s_2 & s_3 \end{pmatrix}$$

where R'_{i_2} is any preference relation student i_2 ranks school s_2 as his first choice, and therefore

$$\text{FPF}^2_{i_2}(R_{-i_2}, R'_{i_2})\, P_{i_2}\, \text{FPF}^2_{i_2}(R)$$

implies that FPF^2 is vulnerable under profile R. Hence FPF^2 is more manipulable than GS^2.

LEMMA 1: *Fix a set of agents $I' \subset J \cup C$. Let φ, ψ be two stable mechanisms such that, for any preference profile P, and any agent $i \in I'$,*

$$\varphi_i(P)\ R_i\ \psi_i(P).$$

Then mechanism ψ is as strongly manipulable as mechanism φ for members of I'.

PROOF:

Let $I' \subset J \cup C$ and mechanisms φ, ψ be as in the statement of the Lemma. Let preference profile P, agent $i \in I'$, and preference relation $\hat{P}_i$ be such that

$$\varphi_i(\hat{P}_i, P_{-i})\ \ P_i\ \ \varphi_i(P). \tag{4}$$

We want to show that there exists a preference relation $\tilde{P}_i$ such that

$$\psi_i(\tilde{P}_i, P_{-i})\ \ P_i\ \ \psi_i(P).$$

By assumption

$$\varphi_i(P)\ \ R_i\ \ \psi_i(P). \tag{5}$$

Let the preference relation $\tilde{P}_i$ be such that only agents in $\varphi_i(\hat{P}_i, P_{-i})$ are acceptable to agent i under $\tilde{P}_i$. Since matching $\varphi(\hat{P}_i, P_{-i})$ is stable under profile $(\hat{P}_i, P_{-i})$, it is also stable under profile $(\tilde{P}_i, P_{-i})$. Moreover by Roth (1984), agent i is matched with the same number of agents on the other side of the market at any stable matching under any given preference profile, and in particular under profile $(\tilde{P}_i, P_{-i})$. Therefore, since only agents in $\varphi_i(\hat{P}_i, P_{-i})$ are acceptable to agent i under $\tilde{P}_i$, stability of matching $\varphi(\hat{P}_i, P_{-i})$ under $(\tilde{P}_i, P_{-i})$ implies

$$\psi_i(\tilde{P}_i, P_{-i}) = \varphi_i(\hat{P}_i, P_{-i}). \tag{6}$$

Hence, by (4), (5), and (6), we have

$$\underbrace{\psi_i(\tilde{P}_i, P_{-i})}_{=\varphi_i(\hat{P}_i, P_{-i})} \; P_i \; \varphi_i(P) \; R_i \; \psi_i(P),$$

which shows that agent i can manipulate mechanism ψ by reporting $\tilde{P}_i$. This completes the proof.

PROPOSITION 4: *$GS^{\mathcal{J}}$ is strongly more manipulable than $GS^{\mathcal{C}}$ for colleges.*

PROOF:

Given any problem, the college-optimal stable matching is weakly preferred to student-optimal stable matching by any college (Gale and Shapley 1962). Therefore, Lemma **??** implies $GS^{\mathcal{J}}$ is as strongly manipulable as $GS^{\mathcal{C}}$ for colleges.

Next, we give a problem where $GS^{\mathcal{C}}$ is not manipulable by any college, while some college can manipulate $GS^{\mathcal{J}}$. Suppose there are two students, j_1 and j_2, and two colleges, c_1 and c_2, each with one seat. The student and college preferences are

$$\begin{array}{ll} R_{j_1} : c_1, c_2, j_1 & \quad R_{c_1} : \{j_2\}, \{j_1\}, \emptyset \\ R_{j_2} : c_2, c_1, j_2 & \quad R_{c_2} : \{j_1\}, \{j_2\}, \emptyset. \end{array}$$

The outcomes of $GS^{\mathcal{C}}$ and $GS^{\mathcal{J}}$ are:

$$GS^{\mathcal{C}}(R) = \begin{pmatrix} j_1 & j_2 \\ c_2 & c_1 \end{pmatrix} \quad \text{and} \quad GS^{\mathcal{J}}(R) = \begin{pmatrix} j_1 & j_2 \\ c_1 & c_2 \end{pmatrix}.$$

Since each college obtains its top choice under $GS^{\mathcal{C}}$, no college can manipulate. However, if college c_1 declares that only j_2 is acceptable, it can manipulate $GS^{\mathcal{J}}$. This completes the proof.

THEOREM 2: *Let φ be an arbitrary stable mechanism. Then*

a) φ is as strongly manipulable as $GS^{\mathcal{C}}$ for colleges,

b) $GS^{\mathcal{J}}$ is as strongly manipulable as φ for colleges, and

c) $GS^{\mathcal{C}}$ is as strongly manipulable as φ for students.

PROOF:

Let φ be any stable mechanism and P be any preference profile. Then

a) $GS^{\mathcal{C}}_c(P) \; R_c \; \varphi(P)$ for any $c \in C$,

b) $\varphi_c(P)\ R_c\ GS_c^{\mathcal{J}}(P)$ for any $c \in C$, and

c) $\varphi_j(P)\ R_j\ GS_j^{\mathcal{C}}(P)$ for any $j \in J$

by Gale and Shapley (1962). Therefore Lemma **??** implies the desired result.

PROPOSITION 5: *The discriminatory auction is intensely and strongly more manipulable than the uniform-price auction.*

PROOF:

Let δ denote the discriminatory auction and Υ denote the uniform-price auction. Fix $\epsilon > 0$ and a bidder i. Let t_{-i} be the type profile of all other bidders. The type of each bidder is the vector of his valuations. Given t_{-i}, order the $k(|I|-1)$ valuations of all bidders in $I \setminus \{i\}$ from highest to lowest. Let b_1 be the highest valuation, b_2 be the next highest valuation, and so on. That is, $b_1 \geq b_2 \geq \cdots \geq b_{k(|I|-1)} > 0$.

Let $t_i = (v_i^1, \ldots, v_i^k)$ be the type of bidder i. We will consider two cases. For the first case bidder i will not be able to manipulate the uniform-price auction. For the second case he potentially can but whenever that happens he will have an at least as profitable deviation under the discriminatory auction.

Case 1: $v_i^1 < b_k$. For this case bidder i's highest valuation is less than b_k. Therefore if he reports his true values under the uniform-price auction, he will not receive any object and will not make any payment. Hence $u_i\big(\Upsilon(t)\big) = 0$. In order to have a profitable manipulation, bidder i will need to receive an object. However, since $v_i^1 < b_k$, that will require bidder i to pay a unit price that is higher than his highest valuation. Hence $u_i\big(\Upsilon(t_i', t_{-i})\big) - u_i\big(\Upsilon(t)\big) \leq 0$ for any $t_i' \in T_i$, showing there exists no profitable manipulation of the uniform-price auction for Case 1.

Case 2: $v_i^1 \geq b_k$. Let bidder i receive m units under the uniform price auction when he reports his true type $t_i = (v_i^1, \ldots, v_i^k)$. That means $v_i^m \geq b_{k-m+1}$ and the market clearing-price for profile t is

$$p^* = \begin{cases} \max\{v_i^{m+1}, b_{k-m+1}\} & \text{if } m < k \\ b_{k-m+1} & \text{if } m = k \end{cases}$$

which in turn implies

$$u_i\big(\Upsilon(t)\big) = (v_i^1 + \cdots + v_i^m) - mp^* \geq 0. \tag{7}$$

Let the potential manipulation $\hat{t}_i = (\hat{v}_i^1, \ldots, \hat{v}_i^k)$ be such that bidder i receives n units under $\Upsilon(\hat{t}_i, t_{-i})$. Then the market-clearing price for profile $(\hat{t}_i, t_{-i})$ is

$$\hat{p} = \begin{cases} \max\{\hat{v}_i^{n+1}, b_{k-n+1}\} & \text{if } n < k \\ b_{k-n+1} & \text{if } n = k \end{cases}$$

and hence

$$u_i\big(\Upsilon(\hat{t}_i, t_{-i})\big) = (v_i^1 + \cdots + v_i^n) - n\hat{p}.$$

Observe that,

$$\hat{p} \geq b_{k-n+1}. \tag{8}$$

Suppose

$$u_i\big(\Upsilon(\hat{t}_i, t_{-i})\big) - u_i\big(\Upsilon(t)\big) = (v_i^1 + \cdots + v_i^n - n\hat{p}) - (v_i^1 + \cdots + v_i^m - mp^*) > 0$$

and thus bidder i can manipulate the uniform-price auction at profile t. We will construct $\tilde{t}_i \in T_i$ such that

$$u_i\big(\delta(\tilde{t}_i, t_{-i})\big) - u_i\big(\delta(t)\big) > u_i\big(\Upsilon(\hat{t}_i, t_{-i})\big) - u_i\big(\Upsilon(t)\big) - \epsilon.$$

First observe that $u_i\big(\delta(t)\big) = 0$, since bidder i pays her reported valuation for each unit she wins under the discriminatory auction. Let $\tilde{t}_i = (\tilde{v}_i^1 \ldots \tilde{v}_i^k)$ be such that

$$\tilde{v}_i^\ell = \begin{cases} b_{k-n+1} + \frac{\epsilon}{2n} & \text{if } \ell \leq n \\ 0.5 b_{k-n+1} & \text{if } \ell > n \end{cases}$$

Given t_{-i}, bidder i wins n units and pays $b_{k-n+1} + \frac{\epsilon}{2n}$ for each unit upon reporting $\tilde{t}_i$. Therefore inequalities 7 and 8 imply

$$\begin{aligned} u_i\big(\delta(\tilde{t}_i, t_{-i})\big) - u_i\big(\delta(t)\big) &= \big(v_i^1 + \cdots + v_i^n - n(b_{k-n+1} + \frac{\epsilon}{2n})\big) - 0 \\ &= (v_i^1 + \cdots + v_i^n - nb_{k-n+1}) - \frac{\epsilon}{2} \\ &> (v_i^1 + \cdots + v_i^n - n\hat{p}) - (v_i^1 + \cdots + v_i^m - mp^*) - \epsilon \\ &= u_i\big(\Upsilon(\hat{t}_i, t_{-i})\big) - u_i\big(\Upsilon(t)\big) - \epsilon \end{aligned}$$

showing that bidder i has an at least as profitable manipulation, subject to an upper bound of ϵ deviation, under the discriminatory auction for Case 2.

This covers all cases, so to complete the proof, we describe an example where some bidders can manipulate δ, but not Υ. Suppose that all bidders other than bidder 1 have the same value $\bar{v}$ for all of the units. Bidder 1's value for the first unit is strictly greater than $\bar{v}$, while her value for each of the remaining units is strictly less than $\bar{v}$. Under the uniform price auction, when bidders are truthful, every bidder wins one unit. Bidder 1 cannot manipulate to win more units because she would have to pay $\bar{v}$ for the additional units. She does not want to manipulate to win fewer units because she obtains strictly positive utility by reporting the truth and she cannot manipulate to change the price she pays. No other bidder would find it strictly profitable to manipulate because each would still have to pay at least $\bar{v}$ for that unit, and none can change the price paid. Hence, no bidder can manipulate the uniform-price auction. Under the discriminatory price auction, when each bidder reports truthfully, every bidder wins one unit. However, bidder 1 would prefer to under-report her valuation for the first unit to pay less for it. Hence, for this example, bidder 1 can manipulate δ, but not Υ.

PROPOSITION 6: *The Generalized First Price Auction is intensely and strongly more manipulable than the Generalized Second Price Auction.*

PROOF:

Given a type profile t, let $\textsc{Gsp}(t)$ denote the outcome of GSP auction and $\textsc{Gfp}(t)$ denote the outcome of GFP auction. Fix $\epsilon > 0$ and a bidder i. Let t_{-i} be the type profile of all other bidders. Recall that the type of each bidder is his valuation per click. Given t_{-i}, order the $|I|-1$ valuations of all bidders in $I \setminus \{i\}$ from highest to lowest. Let b_1 be the highest valuation, b_2 be the next highest valuation, and so on. That is, $b_1 \geq b_2 \geq \cdots \geq b_{|I|-1} > 0$.

Let $t_i = v_i$ be the type of bidder i. We will consider two cases with four sub-cases for the second case. For all cases except Case 2d, bidder i will not be able to manipulate the GSP auction. For Case 2d, he potentially can but whenever that happens he will have an at least as profitable deviation under the GFP auction.

Case 1: $v_i \leq b_k$.

In this case $u_i\big(\textsc{Gsp}(t)\big) = 0$ either because bidder i does not receive a slot, or because she receives a slot at 0 utility.[2] Let $t'_i = v'_i$ be a potential manipulation. For this manipulation to be profitable, bidder i shall receive a slot. Let this slot be slot ℓ. Then

$$b_{\ell-1} \geq v'_i \geq b_\ell \geq b_k \geq v_i$$

and therefore,

$$u_i\big(\textsc{Gsp}(t'_i, t_{-i})\big) = \alpha_\ell v_i - \alpha_\ell b_\ell = \alpha_\ell(v_i - b_\ell) \leq 0.$$

Hence bidder i does not have a profitable manipulation of GSP for Case 1.

Case 2: $v_i > b_k$.

Let bidder i receive slot m under GSP when he reveals his type truthfully. Then $b_{m-1} \geq v_i \geq b_m$ and

$$u_i\big(\textsc{Gsp}(t)\big) = \alpha_m v_i - \alpha_m b_m \geq 0. \tag{9}$$

Let $t'_i = v'_i$ be a potential manipulation and suppose bidder i receives slot ℓ under $t'_i = v'_i$. This implies $v'_i \geq b_\ell$. We have four sub-cases to consider.

Case 2a: $v'_i > b_{m-1}$.

For this case, $\ell \leq m-1$ and hence $b_\ell \geq b_{m-1} \geq v_i$. Therefore

$$u_i\big(\textsc{Gsp}(t'_i, t_{-i})\big) = \alpha_\ell v_i - \alpha_\ell b_\ell = \alpha_\ell(v_i - b_\ell) \leq 0$$

[2]The latter can happen only if $v_i = b_k$.

and thus, bidder i does not have a profitable manipulation of GSP for Case 2a.

Case 2b: $v'_i = b_{m-1}$.

For this case there is a tie and bidder i either receives slot $m-1$ at a cost of $\alpha_{m-1}b_{m-1}$ or slot m at a cost of $\alpha_m b_m$. If the former happens,

$$u_i\big(\text{GSP}(t'_i, t_{-i})\big) = \alpha_{m-1}v_i - \alpha_{m-1}b_{m-1} = \alpha_{m-1}(v_i - b_{m-1}) \leq 0.$$

If the latter happens,

$$u_i\big(\text{GSP}(t'_i, t_{-i})\big) = \alpha_m v_i - \alpha_m b_m = u_i\big(\text{GSP}(t)\big).$$

In either case, bidder i does not have a profitable manipulation of GSP.

Case 2c: Either $b_{m-1} > v'_i > b_m$ or $v'_i = b_m$ and bidder i receives slot m with tie-breaker.

In this case bidder i receives slot m at a cost of $\alpha_m b_m$. Therefore,

$$u_i\big(\text{GSP}(t'_i, t_{-i})\big) = \alpha_m v_i - \alpha_m b_m = u_i\big(\text{GSP}(t)\big),$$

and hence bidder i does not have a profitable manipulation of GSP.

Case 2d: $v'_i \leq b_m$ and bidder i receives a slot ℓ with $\ell > m$.

In this case

$$v_i \geq b_m \geq b_\ell \tag{10}$$

and

$$u_i\big(\text{GSP}(t'_i, t_{-i})\big) = \alpha_\ell v_i - \alpha_\ell b_\ell = \alpha_\ell(v_i - b_\ell) \geq 0. \tag{11}$$

Suppose $u_i\big(\text{GSP}(t'_i, t_{-i})\big) > u_i\big(\text{GSP}(t)\big)$ so that bidder i can manipulate GSP at profile t. We will construct $\tilde{t}_i \in T_i$ such that,

$$u_i\big(\text{GFP}(\tilde{t}_i, t_{-i})\big) - u_i\big(\text{GFP}(t)\big) > u_i\big(\text{GSP}(t'_i, t_{-i})\big) - u_i\big(\text{GSP}(t)\big) - \epsilon.$$

First observe that,

$$u_i\big(\text{GFP}(t)\big) = 0. \tag{12}$$

Let $\tilde{t}_i = \tilde{v}_i = b_\ell + \frac{\epsilon}{2\alpha_\ell}$. Given t_{-i}, bidder i either wins slot ℓ at a cost of $\alpha_\ell\big(b_\ell + \frac{\epsilon}{2\alpha_\ell}\big)$ or a better slot n (with $\alpha_n > \alpha_\ell$) at a cost of $\alpha_n\big(b_\ell + \frac{\epsilon}{2\alpha_n}\big)$. If the former happens,

$$u_i\big(\text{GFP}(\tilde{t}_i, t_{-i})\big) = \alpha_\ell v_i - \alpha_\ell\big(b_\ell + \frac{\epsilon}{2\alpha_\ell}\big) = \alpha_\ell(v_i - b_\ell) - \frac{\epsilon}{2}$$

and if the latter happens,

$$u_i\big(\text{GFP}(\tilde{t}_i, t_{-i})\big) = \alpha_n v_i - \alpha_n\big(b_\ell + \frac{\epsilon}{2\alpha_\ell}\big) = \alpha_n(v_i - b_\ell) - \frac{\alpha_n}{\alpha_\ell}\frac{\epsilon}{2} > \alpha_\ell(v_i - b_\ell) - \frac{\epsilon}{2}$$

where the last inequality holds by inequality 10 and $\alpha_n > \alpha_\ell$. Therefore,

$$u_i\big(\text{GFP}(\tilde{t}_i, t_{-i})\big) \geq \alpha_\ell(v_i - b_\ell) - \frac{\epsilon}{2}. \tag{13}$$

We are ready to finalize Case 2d. Relations 9, 11, 12, and 13 imply

$$\underbrace{u_i\big(\text{GFP}(\tilde{t}_i, t_{-i})\big)}_{\geq \alpha_\ell(v_i - b_\ell) - \frac{\epsilon}{2}} - \underbrace{u_i\big(\text{GFP}(t)\big)}_{=0} > \underbrace{u_i\big(\text{GSP}(t'_i, t_{-i})\big)}_{=\alpha_\ell(v_i - b_\ell)} - \underbrace{u_i\big(\text{GSP}(t)\big)}_{=\alpha_m v_i - \alpha_m b_m \geq 0} - \epsilon$$

showing that bidder i has an at least profitable manipulation, subject to an upper bound of ϵ deviation, under GFP auction for Case 2d.

This covers all cases, so to complete the proof, we describe an example where some bidder can manipulate GFP, but no bidder can manipulate the GSP. Suppose that $v_1 > v_2 = \ldots = v_S = v_{S+1} > v_{S+2} > \ldots > v_N$. Under the GSP, when all bidders are truthful, the highest value bidder's payoff is $\alpha_1(v_1 - v_2) > 0$. She cannot change her payoff unless she reports a bid of v_2 or lower. If she reports her value to be v_2, she obtains a zero payoff. If she reports her value to be less than v_2, she does not win a slot and obtains a zero payoff. Hence, she cannot manipulate. Any bidder with value equal to v_2 who obtains a slot cannot manipulate. Reporting a value greater than v_1 will give the first slot, but this is not profitable. Reporting a value between v_1 and v_2 does not change her payoff. Reporting a value below v_2 prevents her from obtain a slot. Finally, no bidder with value less than v_2 can manipulate because the only way to change the outcome is to report a value greater than or equal to v_2, which is unprofitable. Hence, with this value distribution, no bidders can manipulate the GSP. In the GFP, if every bidder reports the truth, the outcome is the same as the GSP, but each bidder obtains a zero payoff. If bidder 1 reports a value less than v_1, but greater than v_2, she wins the first slot, but pays a lower price than had she reported the truth. Hence, bidder 1 can manipulate the GFP, but not GSP.

Parag Pathak and Tayfun Sonmez (2012): "School Admissions Reform in Chicago and England: Comparing Mechanisms by their Vulnerability to Manipulation" appendix

Table 1. School Admissions Reforms: Documentation Web Appendix

Allocation System	Year	From	To	Manipulable (More or Less?)	Source	References
Boston Public Schools (K, 6, 9)	2005	Boston	GS	Less	A,B,E	(1) Abdulkadiroglu, Atila and Tayfun Sonmez. 2003. "School Choice: A Mechanism Design Approach." *American Economic Review*, 101(1): 399-410. (2) Abdulkadiroglu, Atila, Parag A. Pathak, Alvin Roth and Tayfun Sonmez. 2005. "The Boston Public Schools Match." *American Economic Review,* Papers and Proceedings, 96: 368-371. (3) Abdulkadiroglu, Atila, Parag A. Pathak, Alvin E. Roth, and Tayfun Sonmez. 2006. "Changing the Boston Mechanism: Strategy-proofness as Equal Access." NBER Working Paper 11965. (4) Cook, Gareth. 2003. "School Assignment Flaws Detailed: Two economists study problem, offer relief." Boston Globe, September 12. (5) BPS. 2002-2010. "Introducing the Boston Public Schools."
Chicago Selective High Schools	2009	Boston[4]	SD[4]	Less	A,B,C	(1) Rossi, Rosalind. 2009. "8th Graders' Shot at Elite High Schools Better." Chicago Sun-Times, November 12. (2) CPS, 2009. "Post Consent Decree Assignment Plan." Office of Academic Enhancement, November 11. (3) Chicago Public Schools. 2009. "New Admissions Process: Frequently Asked Questions." (describes the advice for re-ranking schools).
	2010	SD[4]	SD[6]	Less	A,B,C	(1) CPS. 2010. "Guidelines for Magnet and Selective Enrollment Admissions for the 2011-2012 School Year." November 29. (2) Joseph, Abigayil and Katie Ellis, 2010. "Refinements to 2011-2012 Selective Enrollment and Magnet School Admission Policy." November 4. (3) CPS, 2011. "Application to Selective Enrollment High Schools." Available at www.cpsoae.org, Last accessed December 28, 2011.
Ghana - Secondary Schools	2007	GS[3]	GS[4]	Less	E	(1) Ajayi, Kehinde. 2011. "A Welfare Analysis of School Choice Reforms in Ghana." Working paper, Boston University. Available at: https://www2.bc.edu/~sonmezt/Ajayi, Last accessed December 28, 2011.
	2008	GS[4]	GS[6]	Less	E	(1) Ajayi, Kehinde. 2011. "A Welfare Analysis of School Choice Reforms in Ghana." Working paper, Boston University. Available at: https://www2.bc.edu/~sonmezt/Ajayi, Last accessed December 28, 2011. (2). GhanaWeb, 2010. "2010 Computerized School Selection and Placement for SHS Released." General News, Aug. 27. Available at: http://www.ghanaweb.com/GhanaHomePage/NewsArchive/artikel.php?ID=189231#, Last accessed December 28, 2011.
Denver Public Schools	2012	Boston[2]	GS[5]	Less	A,B	(1) Denver Public Schools, 2011-2012 Enrollment Guide. Available at: http://www.dpsk12.org/pdf/2011-2012%20Enrollment%20Guide%20English.pdf, Last accessed May 25, 2012. (2) Denver Public Schools, 2012-2013 Enrollment Guide. Available at: http://soco.dpsk12.org/, Last accessed May 25, 2012.
Seattle Public Schools	1999	Boston	GS	Less	A,B,C,E,F	(1) Denn, Rebeka. 1999."New Amendment in System Stresses Actual Preference." Seattle Post-Intelligencer, October 5. (2) Parents Involved in Community Schools v. Seattle School District, No. 1 United States District Court, Western District of Washington (Seattle), U.S. Court of Appeals for the Ninth Circuit, No. 01-35450. (3) MacGregor, Chris. 2007. "The REAL Truth About Seattle Public Schools Kindergarten Assignments." February 22. Available at: http://www.cybermato.com/projects/school-assignments/, Last accessed December 28, 2011. (4) Calvo, Naomi Ann. 2007. "How Parents Choose Schools: A Mixed-Methods Study of Public School Choice in Seattle." Ph.D. in Public Policy Dissertation, Graduate School of Arts and Sciences, Harvard University.
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England						
Bath and North East Somerset	2007*	FPF[3]	GS[3]	Less	A, D	(1) Bath and North East Somerset, 2006. "Council Meeting: Motion to Investigate Amendment of School Admissions Policy from 'First Preference First' to 'Equal Preferences'." July 13. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Bedford and Bedfordshire	2007*	FPF[3]	GS[3]	Less	A,D	(1) Bedforshire County Council, 2007. "Schools 1st: News and Information from Bedfordshire Children's Services." February, Issue 2. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Blackburn with Darwen	2007*	FPF[3]	GS[3]	Less	A,D	(1) Blackburn with Darwen Borough Council, 2012. "Secondary School Admissions for September 2012." Page 11 (description of equal preferences). (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Blackpool	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.

Bolton	2007*	FPF[3]	GS[3]	Less	A,D	(1) Fogg, Val. 2007. "Admissions Arrangements to Primary and Secondary Schools September 2008." March 27. Report to Children's Services. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Bradford	2007*	FPF[3]	GS[3]	Less	A,D	(1) School Adjudicator. 2006. "Objection to the admission arrangements, eight Bradford schools." August 4, Office of Schools Adjudicator. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Brighton and Hove	2007	Boston[3]	GS[3]	Less	A,C,D,E	(1) Burgess, Simon, Ellen Greaves, Anna Vignoles, and Deborah Wilson. 2009. "What Parents Want: School preferences and school choice." Centre for Market and Public Organisation, Working Paper No. 09/222. (2) Brighton & Hove City Council. 2008. "New secondary admissions system agreed." Secondary Admissions Review. (3) Chiles, Andy. 2008. "Hundreds of pupils lose schools lottery." The Argus, March 3. (4) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Calderdale	2006	FPF[3]	GS[3]	Less	A,C	(1) School Adjudicator. 2005. "Objection to Admissions Arrangements of Twelve Calderdale Schools." July 7, Office of Schools Adjudicator. (2) Coldron, John. 2005. "Consultation on co-ordinated admission arrangements in Calderdale." Final Report, March. (3) Calderdale. 2012. "Admission to Secondary School 2012." Admissions Booklet, page 3.
Cornwall	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Cumbria	2007*	FPF[3]	GS[3]	Less	D	(1) Cumbria County Council. 2011. "Transfer to secondary school in Cumbria for September 2012." Admissions Booklet, page 8. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Darlington	2007*	FPF[3]	GS[3]	Less	D	(1) Darlington Local Authority. 2011. "Co-ordinated Admissions Scheme for Schools in Darlington." page 2. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Derby	2005*	FPF[4]	GS[4]	Less	A,D	(1) School Adjudicator. 2005. "Objection to Admissions Arrangement by Derby City Council." May 27. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Devon	2006*	FPF[3]	GS[3]	Less	A,D	(1) Devon City Council. 2007. "Consultation on Arrangements for Admission in 2008." Meeting MInutes, page 1. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Durham	2007	FPF[3]	GS[3]	Less	A,D	(1) Durham City Council. 2008. "Report of David Williams, Children and Young People's Services." (2) School Adjudicator. 2008. "Objection to Admissions Arrangement in Durham Comprehensive School." (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Ealing	2006*	FPF[6]	GS[6]	Less	A,D	(1) Ealing City Council. 2006. "School Admission Forum." Meeting Minutes from January 16. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
East Sussex	2007	Boston[3]	GS[3]	Less	A,D	(1) Evans, Geoff. 2007. "Admissions to primary schools in 2008/09." November 26 (Provides guide to how the equal preference system operates). (2) Evans, Geoff. 2007. "School Admissions Code." Meeting Minutes to Senior Management Team, February 6. (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Gateshead	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Halton	2007*	FPF[3]	GS[3]	Less	A,D	(1) Halton LA Executive Board. 2007. "School Admission Arrangments 2008/9." March 29th (page 28 describes formerly used first preference first arrangements). (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
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Hartlepool	2007	FPF[3]	GS[3]	Less	A,D	(1) Hartlepool Local Authority. 2011. "Coordinated Admission Scheme - Secondary Schools 2011-2012." (Section 5.1 describes Equal ranking scheme). (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Isle of Wright	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.

Kent	2007	Boston[3]	GS[4]	Less	A,D	(1) Young, Jenny. 2006. "Kent Admission Forum: An explanation of first preference first and equal preference criteria from the Office of Schools Adjudicator." May 2. (2) Turner, Rosalind and Sarah Hohler. 2010. "Secondary School Admissions in Kent." January 15. Agenda Item B5. Available at: http://democracy.kent.gov.uk/Published/ C00000535/M00003099/AI00011084/$ItemB5SecondaryAdmissionsPaper nal.docA.ps.pdf (3) Office of Schools Adjudicator. 2006. "Adjudicator publishes decisions on objections to admissions arrangements of 29 secondary schools in Kent." July 4, M2 Presswire. (4) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.)
Kingston upon Thames	2007*	FPF[3]	GS[4]	Less	A	(1) Royal Borough of Kingston upon Thames. 2004. "Executive Committee Minutes." March 16. Section 147 describes FP-3 arrangements in 2004. (2) Royal Borough of Kingston upon Thames. 2010. "In-Year Admission to Secondary School (Application form)." Learning and Children's Services. (Allows for four choices)
Knowsley	2007*	FPF[3]	GS[3]	Less	A,D	(1) Knowsley Council. 2011. "Knowsley Secondary Education Admissions: Information for Parents 2011/2012." Available at: http://www.knowsley.gov.uk/schooladmisisons (describes equal preferences). (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Lancashire	2007*	FPF[3]	GS[3]	Less	A,D	(1) Lancashire County Council. 2006. "Minutes of the South Area Admission Forum held in Cabinet Room C." September 27. Describes plan in the event that equal preferences were introduced. (2) Lancashire County Council. 2011. "Secondary School Admissions in Lancashire: Information for Parents 2012/2013." Available at: http://www.lancashire.gov.uk/e-admissions/ (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Lincolnshire	2007*	FPF[3]	GS[3]	Less	A,D	(1) Lincolnshire County Council. 2009. "Going to School in Lincolnshire 2009-2010." Page 5. Available at: http://www.lincolnshire.gov.uk/schooladmissions. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Luton	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Manchester	2007*	FPF[3]	GS[3]	Less	A,D	(1) Manchester City Council. 2011. "2012/2013 School Admissions Determiend Scheme and Policy." Page 5 describes current equal preference scheme. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Merton	2006	FPF[6]	GS[6]	Less	A,D	(1) Philpot, Richard. 2004. "The Need to Move to Equal Preference Based Arrangements for Admission to Community and Voluntary Controlled Secondary Schools for Entry Into School Between September 2005 and August 2006." Document explains advantages of equal preference (Appendix A). (2) Merton Council. 2011. "Starting Secondary School 2012: Transfer from Primary to Secondary School for Admission in September 2012." Available at: http://www.merton.gov.uk/admissions. (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
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North Lincolnshire	2007*	FPF[3]	GS[3]	Less	A,D	(1) North Lincolnshire Council. 2011. "Academic Year 2012-2013: Determined admission arrangements for secondary schools within North Lincolninshire." April. Page 4 describes equal preference system. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
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Oldham	2007*	FPF[3]	GS[3]	Less	A,D	(1) Baldwin, Ruth. 2007. "Primary Education: A Guide for Parents 2007-2008." Page 5 describes first preference first system. (2) Oldham Council. 2011. "Applying for Primary School 2012/13." Page 2 describes equal preference system. (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Peterborough	2007*	FPF[3]	GS[3]	Less	A,D	(1) Peterborough School Admissions Forum. 2007. "Annual report on School Admissions in 2007-2008." Page 1 describes move to equal preference system. (2) Peterborough City Council. 2010. "Determination of the Co-Ordinated Scheme for Secondary Schools." Page 1 describes equal preference system. (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.

Plymouth	2007*	FPF[3]	GS[3]	Less	A,D	(1) Gorton, Jayne. 2009. "Local Authority Report 2009 to the Schools Adjudicator from Plymouth City Council." Page 5 and 6 describe new equal preference system and old first preference first system. (2) Plymouth City Countil. 2011. "The Next Step Parents Guide: Secondary admissions September 2012." Page 1 describes how equal preference system works. (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
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Portsmouth	2007*	FPF[3]	GS[3]	Less	D	(1) Portsmouth City Council. 2010. "Co-ordinated Scheme - Secondary Admissions 2011/12." Page 2 describes equal preference scheme with example. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Richmond	2005	FPF[6]	GS[6]	Less	D	(1) London Borough of Richmond upon Thames. 2011. "Primary School Admissions 2011/2012." Education and Children's Services Overview and Scrutiny Committee, June 13. Page 48 describes equal preference system. PAge 49 describes previous first preference first system. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Sefton primary	2007	Boston[3]	GS[3]	Less	A,D	(1) Sefton Schools Forum. 2007. "Meeting Minutes." September 24. Agenda Item No. 9 describes Sefton has now adopted equal preference scheme. (2) Sefton Council. 2011. "Determined Admission Arrangements and Criteria for Sefton Community Secondary Schools 2012-2013." Page 25. (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Sefton secondary	2007	FPF[3]	GS[3]	Less	A,D	(1) Sefton Schools Forum. 2007. "Meeting Minutes." September 24. Agenda Item No. 9 describes Sefton has now adopted equal preference scheme. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Slough	2006*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Somerset	2007*	FPF[3]	GS[3]	Less	A,D	(1) Somerset City Council. 2007. "2008-09 School Admission Arrangements Item 7 Appendix A." March 28. Describes replacing first preference first with equal preference. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
South Gloucestershire	2007*	FPF[3]	GS[3]	Less	A,D	(1) South Glouchestershire Admission Forum. 2008. "Annual Report Academic Year 2007-2008." Page 4 describes previous first preference first and equal preference. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
South Tyneside	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Southhampton	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Stockton	2007*	FPF[3]	GS[3]	Less	A, D	(1) Stockton City Council. 2008. "Major Change: School Admission Arrangements for September 2008." Page 4. (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Stoke-on-Trent	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Suffolk	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Sunderland	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Surrey	2007	FPF[3]	GS[3]	Less	A,D	(1) "Shake up for policy on getting places at school." 27 January 2006. Newsquest. (2) Get Hampshre (S&B Media) 2006. "Survey flags up key changes for schools." January 26. Available at: http://www.gethampshire.co.uk/ (3) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
	2010	GS[3]	GS[6]	Less	A	(1) Admissions Forum. 2011. "Admission arrangement for Surrey County Council's Community and Voluntary Controlled Schools September 2012."
Sutton	2006	FPF[6]	GS[6]	Less	A,D	(1) Wilson School. 2011. "Admissions: Frequently Asked Questions." (2) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Swindon	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Tameside	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.

Telford and Wrekin	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Torbay	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Warrington	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Warwickshire	2007*	FPF[7]	GS[7]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Wilgan	2007*	FPF[3]	GS[3]	Less	D	(1) Coldron, John, et. al., "Secondary School Admissions." Sheffield Hallam University and National Centre for Social Research, Research Report DCSF-RR020.
Wales						
Wrexham County Borough	2011	FPF[3]	GS[3]	Less	A	(1) Wrexham County Borough. 2010. "Report CLAO/31/10 Review of School Admissions Policy." December 14. (2) Wrexham County Borough. "School Admission Arrangements 2012/2013 - Annual Consultation on Admission Arrangements, Including Admission Numbers." Available at: http://www.wrexham.gov.uk.

Notes. * For changes in the 2007 code, an asterisk indicates that we assume that the number of choices allowed has not changed. A - Documentation from schools (brochures) or official policy minutes; B - Direct communication with school officials; C - Documentation from press clippings; D - Coldron report; E - Other academic papers; F - Other online materials. In some cases, we do not know the exact year the mechanism changed, the years correspond to the last possible year.

Other choice plans referenced in text

Cambridge	(1) Cambridge Public Schools. 2003. "Schools at a Glance." (2) School Committee. 2001. "Approval of Modifications to the Cambridge Public Schools Choice Plan." December 18. Contains description of entire choice plan.
Charlotte-Mecklensburg	(1) Hastings, Justine, Thomas J. Kane, and Douglas O. Staiger. 2006. "Preferences and Heterogeneous Treatment Effects in a Public School Choice Lottery." NBER Working paper, No. 12145.
Denver	(1) Denver Public Schools. 2003. "Denver Public School Choice Guide." Department of Planning and Research.
Miami-Dade	Cited in other papers (Ergin-Sonmez, Abdulkadiroglu-Che-Yasuda). Personnel communication based on visit to Miami in September 2007.
Providence	(1) Provide Public Schools. 2009. "Parent Handbook 2009-2010." Available at: http://www.providenceschools.org, Page 24.
Tampa- St. Petersburg Fl	(1) Tobin, Thomas C. 2003. "Want first choice now? Many say no." St. Petersburg Times, August 21. (2) "School Choice Q&A." St. Petersburg Times, September 14, 2003.