Earnings Uncertainty and Aggregate Wealth Accumulation

By Ricardo J. Caballero*

This paper argues that precautionary savings due to uninsurable earnings uncertainty are likely to be an important source of aggregate wealth accumulation. The stylized model presented in this paper can easily generate levels of wealth above 60 percent of the observed net wealth in the United States, net of conventional life-cycle savings. (JEL 023, 131)

The textbook version of Modigliani's life-cycle model indicates that, for the low real interest rates and degree of intertemporal substitution found in U.S. data, individuals' consumption paths should be approximately flat. If income paths are assumed to be flat until retirement, then consumption-smoothing generates a hump-shaped wealth model for individuals. In this case, both the amount *and* the motive for life-cycle savings are firmly rooted on the expected path of income.

This paper presents a simple overlapping-generations model in which not only the expected but also higher moments of the income path determine aggregate savings. Individual consumers accumulate precautionary savings to self-insure against potential earning misfortunes. When young, consumers accumulate more (on average) than they would were their income streams certain. This behavior yields a hump-shaped saving model even in the absence of a retirement motive. The model is highly stylized and is designed to isolate the long-run implications of earnings uncertainty for aggregate wealth accumulation. As such, the results obtained should be interpreted as measures of the potential *incremental* contribution of precautionary savings due to

earnings uncertainty, not as a description of U.S. wealth accumulation.

Borrowing parameter values from related research, I show that, within the context of the stylized model, precautionary savings due to earnings uncertainty alone can easily generate aggregate wealth levels above 60 percent of the observed *net* U.S. total stock of wealth. This conclusion is robust to the presence of lifetime uncertainty, regardless of whether annuity markets exist or not.

The evidence presented in this paper is just illustrative; however, the magnitude of the numbers found, together with related and consistent evidence presented by Jonathan Skinner (1988), brings about the need for more research on the behavior of consumers in the presence of earnings uncertainty and incomplete markets and its implications for aggregate wealth accumulation. Section I sets up the savings problem of an individual agent, deals with aggregation under certain horizons, and discusses empirical evidence shedding some light on the parameters determining the relative importance of precautionary savings due to earnings uncertainty. Section II extends the previous results to the case of uncertain horizons. It shows the interactions between precautionary savings and horizon uncertainty and shows the robustness of Section I's conclusions to the introduction of lifetime uncertainty and annuity markets. Section III summarizes the results, discusses some welfare implications, relates the findings to previous literature, and suggests possible extensions of the model.

*Department of Economics, Columbia University, New York, NY 10027. I am most grateful to Samuel Bentolila, Olivier Blanchard, Stanley Fischer, Glenn Hubbard, Anil Kashyap, Laurence Kotlikoff, Richard Lyons, Danny Quah, and three anonymous referees for their useful comments.

I. The Model: Certain Horizon

A. *Individual Behavior*

The behavior of an individual consumer in the presence of uninsurable labor-income (earnings) uncertainty has been extensively studied since the work of Hayne E. Leland (1968) and Agnar Sandmo (1970) (e.g., Jacques H. Drèze and Franco Modigliani, 1972; Bruce L. Miller, 1974, 1976; David S. Sibley, 1975; Jack Schechtman and Vera L. S. Escudero, 1977; David Levhari et al., 1980; Miles S. Kimball, 1987; Olivier J. Blanchard and N. Gregory Mankiw, 1988; Skinner, 1988; Stephen P. Zeldes, 1989; Caballero, 1990). The chief conclusion of this research agenda is that whenever preferences can be characterized by a separable utility function with convex marginal utility, the slope of the consumption path rises as the level of income uncertainty increases.[1]

In this paper, I make assumptions to isolate savings arising from precautionary reasons in the face of uncertain earnings from other sources of wealth accumulation. The base model is such that neither life-cycle nor cyclical savings exist. Consumption preferences of each individual i are characterized by a constant-absolute-risk-aversion utility function. Consumers live for T years. Throughout most of the paper, both the interest and discount rates are constant and equal to zero, while labor income is uncertain and uninsurable and follows a random walk.[2] Initial wealth is equal to zero, although this assumption is relaxed in Section II, when bequests are introduced into the model.

Formally, the problem can be stated as follows:

$$\max_{c_{t+i}} E_t \left[\sum_{i=0}^{T-t} -\frac{1}{\theta} e^{-\theta c_{t+i}} \right]$$

[1] In Caballero (1990), I study the implications of precautionary savings for the time-series behavior of consumption. Here, on the other hand, I look at the potential importance of precautionary savings in explaining long-run aggregate wealth accumulation.

[2] The consumer/worker is assumed to have an inelastic labor supply. The source of labor uncertainty may come from both wages and employment.

subject to

$$c_{t+i} = y_{t+i} + A_{t+i-1} - A_{t+i} \text{ for } 0 \le i \le T - t$$

$$c_T = y_T + A_{T-1}$$

$$A_0 = 0$$

$$y_t \equiv y_t^* + y_0$$

$$y_t^* = y_{t-1}^* + w_t \qquad w_t \text{ i.i.d.}$$

where E_t is the conditional (on information available at time t) expectations operator, θ is the coefficient of absolute risk aversion $(\theta > 0)$, c is consumption, y is labor income; w is labor-income innovation, A is nonhuman wealth, and i.i.d. indicates that the w_t are independently and identically distributed. The Appendix shows that, under these conditions, the consumption function of an individual i is

$$c_{it} = y_{it} + \frac{A_{it-1}}{T - t + 1} - \frac{\Gamma(T - t)}{2}$$

with

$$\Gamma = \frac{1}{\theta} E[e^{-\theta w}] > 0.$$

A simple example of Γ is obtained when w is $\mathcal{N}(0, \sigma^2)$:

$$(1) \qquad \Gamma = \frac{\theta}{2}\sigma^2.$$

The assumptions above grant not only a constant slope of the consumption path, Γ, but also a unitary marginal propensity to consume out of income. This determines that individual i's savings at age t, defined as $s_{it} \equiv y_{it} - c_{it}$, are a deterministic function of time, even though consumption and income are stochastic.

Using the facts that

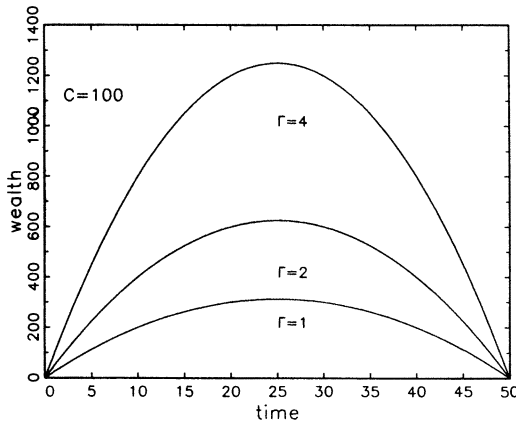$$s_{it} = \frac{\Gamma(T - t)}{2} - \frac{A_{it} - 1}{T - t + 1}.$$

$$A_{it} = A_{it-1} + s_{it}$$

FIGURE 1. THE PATHS OF WEALTH FOR
DIFFERENT VALUES OF Γ

and $A_0 \equiv 0$, it is possible to express the saving function of individual i (of age t) as

$$s_{it} = \frac{\Gamma(T+1-2t)}{2}.$$

Summing over time, total wealth accumulated at time t by individual i (born at time 0) is

$$A_{it} \equiv \sum_{j=1}^{t} s_{ij} = \frac{\Gamma(T-t)t}{2}.$$

Figure 1 shows the path of (nonhuman) wealth for different values of Γ; later I will argue that these values are realistic. It is apparent that the model here described generates a hump-shaped path of individual wealth. Precautionary savings due to earnings uncertainty have a role very similar to retirement in Modigliani and Richard Brumberg's (1954) life-cycle model. Contrary to their model, however, the precautionary motive does not depend on the expected path of income. For example, if, instead of a driftless random walk, income follows a random walk with positive drift, precautionary savings due to earnings uncertainty would not change, although total

wealth accumulation would fall due to a negative life-cycle component.[3]

The complete separation between life-cycle and precautionary savings is a result of the specific assumptions made. However, the principle is not: in general, savings for earnings-precautionary reasons do not change one-for-one with changes in the expected income path.

B. Aggregate Behavior

The purpose of this paper is to provide a simple model highlighting the potential role of precautionary motives due to earnings uncertainty in the determination of wealth accumulation. For this, I assume that there is no population growth and that the income process restarts with each consumer.[4] One drawback of these assumptions is that I have to concentrate on the factors determining aggregate wealth as opposed to (flow) aggregate saving, since the latter is zero in the steady state. The advantage, on the other hand, is that most of the results can be expressed in closed form.

Assuming that there are T cohorts of size T^{-1} (hence, population size is invariant to T) and that every cohort lives for T periods yields the following expression for aggregate steady-state wealth, W:

$$(2) \qquad W \equiv \frac{1}{T} \sum_{i=1}^{T} A_{it} = \frac{\Gamma(T^2-1)}{12}.$$

This expression reveals the potential role of precautionary motives for wealth accumulation. For example, if equation (1) is used as an approximation for Γ, doubling the coefficient of risk aversion or the variance of

---

[3]Certainly, if one lets earnings be a fixed constant after retirement, precautionary savings are reduced. This, however, is not a result of the reduction in expected income but of the reduction in earnings uncertainty after retirement. In a realistic situation, this is likely to be offset by health uncertainty.

[4]The latter assumption can be relaxed without changing the results in any important way. However, given the random-walk assumption for the income process, relaxing the memoryless assumption would imply that eventually some consumers would have negative earnings almost surely.

labor income doubles the steady-state level of aggregate wealth.[5] In partial equilibrium, the transition between steady states is smooth and lasts for T periods; this is described by

$$W^h = \frac{\Gamma^+ (T^2 - 1)}{12}$$

$$- \left(\frac{\Gamma^+ - \Gamma^-}{2T} \right) \sum_{k=h}^{T} (k - h)(T - k)$$

with W^h denoting aggregate wealth h periods after the new conditions were set, and with Γ^+ and Γ^- denoting the new and old Γ, respectively.

Before more meaningful examples can be presented, it is necessary to gather some information on the determinants of Γ. The next two subsections survey empirical evidence shedding some light on possible values of the coefficient of risk aversion and the degree of labor-income uncertainty found in U.S. data. These provide the basic elements needed to construct reference examples.

C. Coefficient of Risk Aversion

If one is willing to restrict the coefficient of risk aversion to be equal to the inverse of the coefficient of intertemporal substitution, it is possible to use the wealth of results found in the "Euler equation"-approach literature. Recent work in this area (e.g., Robert E. Hall, 1988) suggests that the intertemporal substitution parameter is unlikely to be larger than 0.3. This implies, under some standard preference restrictions, a coefficient of relative risk aversion above 3.[6]

---

[5]In this model, changes in the coefficient of risk aversion have first-order effects on the slope of the consumption path and hence on the level of aggregate wealth. This contrasts with previous models (e.g., Laurence Kotlikoff et al., 1987) where the coefficient of risk aversion is not of primary importance in the determination of savings.
[6]Lars P. Hansen and Kenneth J. Singleton (1983) present somewhat lower estimates of the coefficient of risk aversion, although 3 is well within their confidence intervals.

Irwin Friend and Marshal E. Blume (1975), on the other hand, obtain an independent measure of the degree of risk aversion using cross-sectional data on households' asset holdings. Their comprehensive analysis of the data, leads them to the conclusion that the coefficient of relative risk aversion of U.S. asset-holders exceeds 2.

D. Human Wealth Uncertainty

Measuring human wealth uncertainty is difficult, since it requires assessing the average uninsurable risk faced by individuals. Benjamin Eden and Ariel Pakes (1981) and Kotlikoff and Pakes (1988) have suggested doing this by measuring the variance of consumption changes. Unfortunately, using readily available aggregate data averages out much of the microeconomic uncertainty. Microeconomic consumption data, on the other hand, do not match the required theoretical consumption measure (in adequate length) and are subject to substantial noise.

An alternative procedure is to look directly at earnings data. Uncertainty in the annuity value of wealth is then obtained from the estimated stochastic process of earnings. Again, aggregate measures of income uncertainty are easily attainable; however, they are not likely to provide a good proxy for the uncertainty faced by individuals unless idiosyncratic risk is fully insurable. On the other hand, disaggregate-data studies usually involve short time-series observations. The latter fact impedes a good understanding of the degree of persistence of income shocks, a crucial issue in establishing the links between income and human wealth uncertainty. In spite of this shortcoming, disaggregate data seem more appropriate for addressing the earnings-uncertainty issue.

Thomas E. MaCurdy (1982) designs a random sample from the Michigan Panel Study of Income and Dynamics (PSID) for the years 1968–1977 (annual). His preferred specifications for wages and earnings, respectively, are

$$\Delta x_t = e_t - 0.484 e_{t-1} - 0.066 e_{t-2}$$

$$\Delta z_t = v_t - 0.411 v_{t-1} - 0.106 v_{t-2}$$

with $\sigma_e^2 = 0.061$ and $\sigma_v^2 = 0.054$, where x and z are the logarithms of wages and earnings, respectively.

Strictly speaking, these estimates do not fit the theoretical model, since the equations are in logarithms and include moving-average coefficients. It is still possible, nonetheless, to calculate the variance of the rate of change of the annuity value of human wealth, although it will no longer be equal to the variance of income innovations. An additional problem with the moving-average terms is that, when combined with the finite-horizon assumption, they imply a nonconstant variance of the annuity value of human wealth.[7]

Given these caveats, if $T = 50$ the estimates of the standard deviation of (proportional) changes in the annuity value of human wealth calculated from the process of wages varies from 0.123 at birth to 0.247 one period before death. If earnings are used instead of wages, it varies from 0.115 at birth to 0.232 one period before death.

Hall and Frederic Mishkin (1982) measure income uncertainty through the residual of a regression of PSID data on demographic and life-cycle variables. Their procedure yields estimates of human wealth uncertainty of the same order of magnitude as those in MaCurdy's work.

These estimates have to be taken with caution. First of all, they represent the uncertainty as measured by the econometrician, which is not necessarily the same as the uncertainty faced by individuals, who are likely to have more information about their future earnings. Furthermore, even if this is not the case, the uninsurable component of labor-income risk may be smaller than measured earnings uncertainty; for example, insurance within the family (Kotlikoff and Avia Spivak, 1981) may reduce nondiversifiable risk. Hence, on one hand, the above estimates represent an upper

bound for the uninsurable component of labor income. Yet, on the other hand, some of MaCurdy's procedures bias downward the estimate of total labor-income uncertainty faced by individuals. For instance, only white males between the ages of 25 and 46 who have been continuously married to the same spouse are considered. All these characteristics tend to smooth large and unexpected changes in income. Also, to avoid outliers, observations with large changes in wages or earnings are excluded. Finally, a dummy for each year is used, ruling out aggregate uncertainty, perhaps the most uninsurable of all the risks faced by an individual. Hall and Mishkin's procedure is subject to the same issues, as they remove all demographic and life-cycle components from their residuals.

Given these considerations, it seems sensible to postulate—although with a great deal of incertitude—that labor-income uncertainty (as measured by the standard deviation of the percentage change in the annuity value of human wealth) is, on average, a number on the order of 10 percent, and perhaps larger than this. The next subsection combines these estimates with the estimates on the coefficient of risk aversion presented above, to speculate on the possible importance of precautionary motives for U.S. wealth accumulation.

E. Some Relevant Examples

As a reference case, it is convenient to assume that labor-income innovations are normally distributed. Columns 2 and 4 in Table 1 present values of average Γ's for coefficients of relative risk aversion and earnings uncertainty in the neighborhood of those discussed above. The contribution of precautionary motives to the slope of an

---

[7]Certainly the moving-average coefficients also have implications for short-run fluctuations in savings. This, however, is of second-order importance for this paper; hence, it will be disregarded. I also disregard the potential implications of the conditional heteroscedasticity embodied in a logarithm equation (Caballero, 1990).

TABLE 1—STEADY-STATE RATIO OF WEALTH TO CONSUMPTION, WITH CERTAIN LIFETIME

σ/Y	$\theta C = 3$		$\theta C = 4$	
	Γ	W/C	Γ	W/C
0.05	0.38	0.78	0.50	1.04
0.10	1.50	3.12	2.00	4.17
0.15	3.38	7.03	4.50	9.37

individual's consumption path is large. For example, it does not seem unreasonable to assume that U.S. data are likely to be well characterized by coefficients of relative risk and degree of labor-income uncertainty around 3 and 10 percent, respectively. In this case, the contribution of precautionary motives to the slope of the consumption path of an individual is 1.5 percent of average consumption per year.[8]

The ratio of steady-state wealth to aggregate average consumption, W/C, can be obtained from equations (1) and (2) and the fact that average consumption (C) is equal to average income (Y):

$$\frac{W}{C} = \frac{(T^2-1)(\theta C)(\sigma/Y)^2}{24}.$$

Columns 3 and 5 of Table 1 show that, with reasonable values of the coefficient of relative risk aversion (θC) and the standard deviation of the rate of growth of labor income (σ/Y), it is possible to impute sizable amounts of wealth to precautionary savings due to earnings uncertainty.

According to estimates of the Board of Governors of the Federal Reserve System (1990), the ratio of private wealth to private consumption in the United States is around 5.2. The estimates of Table 1 are of the same order of magnitude. For example, if the coefficient of relative risk aversion is equal to 3, $T = 50$, and labor-income uncertainty is 10 percent per year, then W/C is around 3. On the other hand, if labor-income uncertainty is 15 percent per year, then W/C is around 7. Furthermore, in a previous paper (Caballero, 1990), I show that if the distribution of labor-income innovations is asymmetric, in the sense that very bad low-probability events (without a positive counterpart) exist (unemployment?), Γ is substantially larger than in the normal-distribution case. This type of phe-

nomenon magnifies all the numbers presented in most of the tables of the paper. Of course one could also imagine situations in which this asymmetry is reversed and the size of Γ is reduced.[9] In general, most of the results presented in this paper are scaled by Γ (i.e., by $1/\theta$ times the expected value of $e^{-\theta w}$); thus, assessing the impact of changes in the distributional assumptions about labor-income innovations is trivial.

F. Interest Rates and Discounting

Interest and discount rates are assumed to be equal to zero throughout the paper. These assumptions are momentarily relaxed in this subsection in order to study their impact on the main conclusions of the paper. Still, in order to isolate the saving motive studied here from conventional intertemporal-substitution arguments, I maintain the assumption of equal interest and discount rates; thus, in the absence of precautionary motives, the aggregate stock of wealth is zero.

Under the random-walk assumption for labor income, the interest rate has no implications for the value of Γ.[10] However, it does have an effect on the amount the consumer needs to save today in order to build up his defenses against possible future income misfortunes: the larger the interest rate, the less the amount of current saving that is required to finance future consumption. The new consumption function (see the Appendix) for individual i reveals this:

$$c_{it} = y_{it} + \frac{1-\alpha}{\alpha(1-\alpha^{T-t})} A_{it-1}$$

$$- \alpha \left[\frac{1}{1-\alpha} - \frac{(T-t+1)\alpha^{T-t}}{1-\alpha^{T-t+1}} \right] \Gamma$$

---

[8]Very large values of Γ have to be interpreted with caution. Given that the exponential utility function does not preclude negative consumption, it is possible that very large values of Γ may overestimate the potential wealth accumulation of the young.

[9]As is the case with the lognormal distribution, for example. See Caballero (1990) for an extensive discussion of the conditional heteroscedasticity-precautionary savings issues arising in this case.

[10]Since the annuity value of an income innovation remains unchanged and equal to the income innovation.

where α is equal to $1/(1 + \text{real interest rate})$. Trivial algebra shows that the term preceding Γ in the new consumption function is smaller (in absolute value) than $(T - t)/2$, thus lowering the amount of precautionary savings at each point in time. As for the aggregate, analytical expressions are obtainable but are unrevealing.

What is important, however, is that the ratio of wealth in an economy with positive interest rate to aggregate wealth when the interest rate is zero is not far from one for reasonable interest rates: 0.97 and 0.89 for annual interest rates of 2 and 5 percent, respectively. Thus, the simplifying assumptions on interest rates are not likely to weaken the main results of the paper.

The model is partial-equilibrium (or small open economy) in nature. Instead of determining the level of wealth, one could use it to determine the level of the interest rate, given the level of wealth; if the aggregate technology has decreasing returns to capital, precautionary savings due to earnings uncertainty lower the equilibrium interest rate. In this case, an interesting reinforcing effect arises from the fact that lower interest rates increase precautionary savings, reducing interest rates even further.

In the next section, the zero-interest-rate assumption is reinstated, but lifetime becomes uncertain. The issue studied there is whether the presence of bequests may alter the potential importance of precautionary savings due to the uninsurability of labor-income uncertainty for aggregate wealth accumulation.[11]

---

[11] This paper discusses the case of involuntary bequests. It is also possible to provide strong arguments in favor of a precautionary-motives explanation for wealth accumulation in the context of an altruistic model. In this framework, the steady-state equilibrium interest rate (r) is lower than that in the case in which precautionary motives are not taken into account. As long as there are decreasing returns to capital, this implies a larger steady-state capital stock. In terms of the model shown in this paper, a steady state would require $r = \delta - \theta^2\sigma^2/2$ (where δ is the discount rate). For example, assuming that labor-income uncertainty is 10 percent and that the coefficient of relative risk aversion is equal to 3 would imply an equilibrium interest rate 4.5-percent lower than the equilibrium rate of an economy with no precautionary savings.

II. Lifetime Uncertainty

In this section, I introduce horizon uncertainty and show that the interaction of this with precautionary savings due to earnings uncertainty results in even larger wealth accumulation than when the horizon is certain.[12] Again, the model described below is such that longevity risk alone is not a source of additional savings.

A. Absence of Annuity Markets

The framework is similar to that in the previous section, except for the fact that there is a constant per-period survival probability p between periods 1 and T. The maximum age (T), however, is still known with certainty. The formal problem of the individual is

$$\max_{c_{t+i}} E_t \left[\sum_{i=0}^{T-t} -p^i \frac{1}{\theta} e^{-\theta c_{t+i}} \right]$$

subject to

$$c_{t+i} = y_{t+i} + A_{t+i-1} - A_{t+1} \text{ for } 0 \leq i \leq T-t$$

$$c_T = y_T + A_{T-1}$$

$$A_0 \geq 0$$

$$y_t \equiv y_t^* + y_0$$

$$y_t^* = y_{t-1}^* + w_t \qquad w_t \text{ i.i.d.}$$

---

[12] The formal treatment of the role of uncertain horizons on the consumption/savings profiles starts with the work of Menahem E. Yaari (1965). His work was followed by many studies (e.g., Levhari and Mirman, 1977; James B. Davies, 1981; Eytan Sheshinski and Yoram Weiss, 1981; Andrew B. Abel, 1983, 1985; R. Glenn Hubbard, 1984; Zvi Eckstein et al., 1985; Hubbard and Kenneth L. Judd, 1987; Kotlikoff et al., 1987). Some of the main issues addressed by this literature are: the effect of changes in the degree of lifetime uncertainty on the level of current consumption; the possibility of generating slow dissaving of the elderly; involuntariness and endogeneity of bequests; the effects of social security systems on welfare and wealth accumulation; imperfections in annuity markets; alternatives to the existence of complete annuity markets; and the effects of social security on welfare and aggregate savings when borrowing constraints are present.

It is particularly important to notice that, in the absence of annuity markets, the budget constraint remains unchanged. This implies that the variance of the annuity value of human wealth also remains unchanged. In fact, the new consumption function is similar to that of the previous section, except for the fact that there is an additional (negative) term in the slope of the consumption path to take into account the increase in discounting of future-periods utility (see Appendix). The new slope, Γ^*, can be written as follows:

$$(3) \qquad \Gamma^* = \Gamma + \frac{\log p}{\theta}.$$

Equation (3) illustrates that, under the type of preferences assumed in this paper, uninsurable earnings uncertainty and lifetime uncertainty do not interact in determining the slope of the consumption path. The interaction between both types of uncertainty comes only through the risk of longevity, which raises the "overaccumulation" of the young. It is important to notice, however, that contrary to most of the literature on uncertain horizons, risk of longevity does not have any effect on savings *unless* there is income uncertainty.

Aggregating over time yields the same saving equation as before, with the exception that now bequests have to be introduced in the definition of wealth. Assuming initially that bequests are received at birth ($A_{i0} \geq 0$), the saving function can be written as follows:

$$s_{it} = \frac{\Gamma^*(T+1-2t)}{2} - \frac{A_{i0}}{T-t+1}.$$

As before, summing over time yields the wealth of individual i at time t (born at time 0):

$$A_{it} = \frac{\Gamma^*(T-t)t}{2} + A_{i0}\left(1 - \frac{t}{T}\right).$$

Normalizing by $(1-p^T)/(1-p)$, so that the population size is independent of p and T, implies that at each point in time there are $p^{i-1}(1-p)/(1-p^T)$ individuals of age i. Hence, in a steady state, aggregate wealth is defined as:

$$W \equiv \left(\frac{1-p}{1-p^T}\right)\sum_{t=1}^{T} p^{t-1}A_{it}.$$

Using the fact that in equilibrium bequests must equal the initial wealth of the just born,

$$A_{i0}\left(\frac{1-p}{1-p^T}\right) = (1-p)W.$$

It is then possible to determine the steady-state level of wealth:

$$W = \Gamma^*\left(\frac{T\{(T-1)(1-p^T)(1-p)-2p[1+(T-1)p^T-Tp^T]\}}{2(1-p)^3[1-p^T+Tp^{T-1}(2p-1)]}\right).$$

Keeping expected lifetime constant (i.e., increasing T when p is lowered), Table 2 shows that the presence of lifetime uncertainty does not preclude precautionary motives (the only source of wealth accumulation in this model) from delivering large amounts of steady-state wealth. For example, if $p = 0.995$, $\theta C = 3$, and $\sigma/Y = 0.1$, the wealth:consumption ratio is around 4.9. If $p = 0.990$, on the other hand, this ratio rises to 9.2. Several effects account for the change in steady-state wealth when p falls. First, the effective discounting of individual wealth rises, reducing the slope of the con-

TABLE 2—STEADY-STATE RATIO OF WEALTH TO CONSUMPTION, KEEPING EXPECTED LIFETIME CONSTANT

	$p = 0.995, T = 57$		$p = 0.990, T = 69$	
σ/Y	$\theta C = 3$	$\theta C = 4$	$\theta C = 3$	$\theta C = 4$
0.05	0.76	1.37	0.32	1.96
0.10	4.88	6.86	9.17	13.80
0.15	11.70	16.00	23.90	33.40

sumption path and, as a result, wealth accumulation. This implies that the threshold model, in which there are no precautionary savings due to income uncertainty, yields a negative steady-state stock of wealth. Second, a larger fraction of individuals die before age T, increasing bequests (see discussion below) and, therefore, wealth in the hands of youngsters. Third, the demographic structure changes in the direction of younger consumers. This, combined with the second effect, tends to raise steady-state wealth. Fourth, in order to keep expected lifetime constant, T must rise, increasing the longevity risk. Individuals must save for the additional lifetime income uncertainty arising from the possible "extra" years alive; this is the main factor responsible for the increase in the steady-state stock of wealth.

Kotlikoff and Lawrence H. Summers (1981) show that a large amount of U.S. wealth accumulation must be explained in terms of intertemporal transfers. In the model presented here, the only structural reason for wealth formation is the overaccumulation by youngsters to protect themselves against future misfortunes; however, it is still the case that one can determine the ratio of *involuntary* transfers to the stock of wealth (Abel, 1985). Using the equation

$$\frac{B}{W} = 1 - p^T - \frac{1+(T-1)p^T - Tp^{T-1}}{T(1-p)}$$

(see the Appendix for the derivation) reveals that, if $p = 0.995$, approximately 13 percent of steady-state wealth can be imputed to bequests, while if $p = 0.990$, this number rises to 28 percent.

Total wealth and the bequest:wealth ratio are not invariant to the timing of bequests. The later bequests are received, the more people borrow against future bequests, reducing capital accumulation and the relative contribution of bequests to wealth formation. The Appendix shows that, if bequests are received at time t^* instead of at time 1, total steady-state wealth, W^*, is equal to

$$W^* = p^{t^*-1}W$$

TABLE 3—STEADY-STATE RATIO OF WEALTH TO CONSUMPTION, KEEPING EXPECTED LIFETIME CONSTANT AND WITH $t^* = T/2$

σ/Y	$p = 0.995, T = 57$		$p = 0.990, T = 69$	
	$\theta C = 3$	$\theta C = 4$	$\theta C = 3$	$\theta C = 4$
0.05	0.66	1.19	0.23	1.40
0.10	4.25	5.97	6.55	9.93
0.15	10.20	13.90	17.10	23.90

and the bequests:wealth ratio is

$$\frac{B^*}{W^*} = p^{t^*-1}\left(1 - p^{T-t^*+1}\right)$$

$$-\frac{1+(T-1)p^T - Tp^{T-1}}{T(1-p)}.$$

Table 3 is the analogue of Table 2, except that now $t^* = T/2$. Not surprisingly, if borrowing against future bequests is allowed for—or if the precautionary savings motive due to income uncertainty is strong enough to offset the needs for borrowing against future bequests—wealth accumulation is lower than for the case in which $t^* = 1$. Nonetheless, it is still true that sizable amounts of wealth accumulation can be generated just out of earnings-precautionary motives. In fact, the wealth:consumption ratio is larger than in the certain-horizon case for most of the parameter combinations presented.

The negative effect of late bequests on saving behavior by the young is taken into account in these calculations. Once this is done, the net importance of intergenerational transfers on the steady-state level of wealth accumulation disappears, in spite of the large observed bequests flows.

The results in this section depend on the lack of fair annuity markets. Although annuity markets exist in the United States, they do not seem to be "fair." Moreover, these markets are seldom used. Why this is so is beyond the scope of this paper (see Eckstein et al., 1985; Kotlikoff et al., 1987; Benjamin M. Friedman and Mark J. War-

shawsky, 1990). Nonetheless, the next sub-section assesses the effect of a complete annuity market on the potential importance of precautionary savings for wealth accumulation.

B. Complete Annuity Markets

In the selfish world described in this paper, fair annuity markets rule out bequests.[13] Thus, initial wealth is zero as in the certain-horizon problem. The other important difference with respect to the model presented in the previous subsection is that now the gross return on savings is p^{-1}; therefore, the lifetime budget constraint becomes

$$\sum_{j=1}^{T} p^{j-1}(c_{ij} - y_{ij}) = 0.$$

Table 4 shows the steady-state wealth:consumption ratio (see Appendix). For each individual consumer, the case of annuity markets resembles the certain-horizon problem with positive interest rate (and equal discount rate) but with more periods to save for due to the longevity risk.[14] From the aggregate point of view, the difference between these two cases relies on the demographic structure. These two effects determine that wealth accumulation is above that of the certain-horizon case. Not surprisingly, however, for most parameters the stock of steady-state wealth of the incomplete-annuity-markets case is larger than when annuity markets are complete. Most importantly, however, the presence of annuity markets does not invalidate the fact that sizable amounts of wealth due to precautionary measures against earnings uncer-

TABLE 4—STEADY-STATE RATIO OF WEALTH TO CONSUMPTION, WITH COMPLETE ANNUITY MARKETS

σ/Y	$p = 0.995, T = 57$		$p = 0.990, T = 69$	
	$\theta C = 3$	$\theta C = 4$	$\theta C = 3$	$\theta C = 4$
0.05	1.11	1.48	1.75	2.33
0.10	4.43	5.91	7.00	9.33
0.15	9.97	13.30	15.70	21.00

tainty are consistent with feasible preferences and uncertainty levels.

III. Conclusions

Many sources of saving and dissaving are discussed in the literature (e.g., health reasons, voluntary-bequests motives, nonlinear income profiles); hence, trying to assess the exact contribution of any of them to actual wealth accumulation is difficult and far too ambitious. Here the objective is more modest, and it consists of comparing the wealth that would be accumulated due to earnings-uncertainty precautionary motives with the *observed net wealth* in the United States. It is shown that reasonable parameters are consistent with levels of wealth originating from precautionary attitudes toward earnings uncertainty, in excess of 60 percent of the observed net U.S. wealth. These results are consistent with Skinner's (1988) independent finding on the potential importance of precautionary savings. He shows that about 56 percent of an individual's life-cycle wealth could be explained by the cautious behavior of young consumers.

It is important to realize that my ruling out of voluntary bequests is not responsible for the main conclusions of this paper. Of course the particular numbers and even the general framework would change if the selfishness assumption is relaxed; however, precautionary savings due to income uncertainty works its way toward wealth accumulation through its impact on the slope of the consumption path, and this occurs regardless of the nature of bequests.

A topic for future research is the potential welfare implication of earnings insur-

---

[13]Note that the level of wealth attained in Kotlikoff et al.'s (1987) case with collected pooling between children and parents can be seen as a combination of the results in this and the previous subsection. Furthermore, the complete-annuity-markets case corresponds to the limit case of the pooling equilibrium when the number of family members is large.

[14]Note that in this case $\Gamma^* = \Gamma$, since the annuity interest cancels with the discounting term due to lifetime uncertainty.

ance mechanisms within the context of a general-equilibrium model. Not surprisingly, the implications of the introduction of a labor-income insurance mechanism resemble the role of social security in a model with retirement and uncertain lifetime (Sheshinski and Weiss, 1981; Abel, 1983; Hubbard, 1984; Kotlikoff et al., 1987). An extended version of this paper (Caballero, 1988) shows that, for the case in which the aggregate technology is linear, a reduction of labor-income uncertainty benefits not only current, but also future generations even in a completely selfish world, as long as the coefficient of risk aversion is not too small. Early generations benefit unambiguously since they receive not only the direct utility gain of a more certain environment, but also increase their lifetime consumption by eating part of the capital stock. Future generations also perceive the direct utility gain but reduce their lifetime consumption since there is a lower capital stock. The utility gain turns out to outperform the consumption-level effect. Generalizations of the technology used for these simple experiments seem to be an important next step.

Also, tax policy may have insurance components (Robert Barsky et al., 1986) that could largely affect welfare and wealth accumulation. Quantifying the extent of the insurance component of different taxes is another interesting issue for future research.

To summarize, precautionary savings are likely to be an important source of wealth accumulation in the United States. However, the motives for these savings may also be a major source of welfare reduction.

APPENDIX

A. Consumption Function (Certain Horizon)

The derivations of this appendix are based in the approach proposed in Caballero (1990).[15] The Euler equation of the opti-

---

[15]See Kotlikoff and Pakes (1988) for a somewhat similar approach.

mization problem [equation (1)] is

$$(A1) \qquad e^{-\theta c_t} = E_t[e^{-\theta c_{t+1}}].$$

It is easily verifiable (by substitution) that the process

$$c_{t+i+1} = \frac{1}{\theta} E_t[e^{-\theta w}] + c_{t+i} + w_{t+i+1}$$

satisfies (A1). Replacing the stochastic process of consumption and income in the intertemporal budget constraint yields the consumption function shown in Section I of the paper.

B. Consumption Function (Uncertain Horizon)

The derivation is identical to the previous section except for the fact that the Euler equation is now

$$(A2) \qquad e^{-\theta c_t} = pE_t[e^{-\theta c_{t+1}}].$$

As before, it is easy to verify that the process

$$c_{t+i+1} = \frac{1}{\theta} E_t[e^{-\theta w}] + \frac{\log p}{\theta} + c_{t+i} + w_{t+i+1}$$

satisfies (A2).

Given the absence of annuity markets, the budget constraint is identical to the perfect-horizon case. Once more, substituting the consumption and income processes in the budget constraint yields the consumption and saving functions under horizon uncertainty.

C. Ratio of Bequest to Total Wealth

Bequests-originated wealth can be written as

$$\frac{B_h}{W_h} = \left\{ \sum_{i=1}^{T} p^{i-1}\left(1 - \frac{i}{T}\right) - 1[h>1] \sum_{i=1}^{h-1} p^{i-1} \right\}$$

for h, the time at which bequests are expected to be received. The first term corresponds to the weighted sum of the propor-

tion of bequests held by individuals of age i, had bequests been received at time 1, whereas the second term corresponds to borrowing against the bequest to be received at time h. Simple geometric-sum formulas lead to the equations shown in the text.

D. Annuity Markets and Interest Rates

As said in the text, the only difference from the case without annuity markets is that now wealth holding receives a gross return equal to p^{-1}. Using the same steps as before yields the following consumption function:

$$c_{it} = y_{it} + \frac{1-p}{1-p^{T-t}} p^{-1} A_{it-1}$$

$$- p \left[\frac{1}{1-p} - \frac{(T-t+1)p^{T-t}}{1-p^{T-t+1}} \right] \Gamma.$$

Hence,

$$s_{it} = \left(p^{-1} - 1 \right) A_{it-1} - \frac{1-p}{1-p^{T-t}} p^{-1} A_{it-1}$$

$$+ p \left[\frac{1}{1-p} - \frac{(T-t+1)p^{T-t}}{1-p^{T-t+1}} \right] \Gamma.$$

The results shown in Table 4 correspond just to the weighted sum of saving across age and individuals. Replacing p by α above yields the microeconomic equations for the case with positive interest rates and a certain horizon. The aggregation is different from the uncertain-horizon case, however, since the age distribution of the population differs.

REFERENCES

Abel, Andrew B., "Bequests and Social Security with Uncertain Lifetimes," mimeo, Harvard University, November 1983.
_____, "Precautionary Savings and Accidental Bequest," American Economic Review, September 1985, 75, 777–91.
Barsky, Robert B., Mankiw, N. Gregory and

Zeldes, Steve P., "Ricardian Consumers and Keynesian Properties," American Economic Review, September 1986, 76, 676–91.
Blanchard, Olivier J. and Mankiw, N. Gregory, "Consumption: Beyond Certainty Equivalence," American Economic Review, May 1988 (Papers and Proceedings), 78, 173–7.
Caballero, Ricardo J., "A Non Life Cycle Model of Wealth Accumulation," Discussion Paper No. 401, Columbia University, August 1988.
_____, "Consumption and Precautionary Savings: Empirical Implications," Journal of Monetary Economics, January 1990, 25, 113–36.
Davies, James B., "Uncertain Lifetime, Consumption, and Dissaving in Retirement," Journal of Political Economy, June 1981, 89, 561–78.
Drèze, Jacques H. and Modigliani, Franco, "Consumption Decisions Under Uncertainty," Journal of Economic Theory, December 1972, 5, 308–35.
Eckstein, Zvi, Eichenbaum, Martin S. and Peled, Dan, "The Distribution of Wealth and Welfare in the Presence of Incomplete Annuity Markets," Quarterly Journal of Economics, August 1985, 100, 789–806.
Eden, Benjamin and Pakes, Ariel, "On Measuring the Variance-Age Profile of Lifetime Earnings," Review of Economic Studies, July 1981, 48, 385–94.
Friedman, Benjamin M. and Warshawsky, Mark J., "The Cost of Annuities: Implications for Saving Behavior and Bequests," Quarterly Journal of Economics, February 1990, 105, 135–54.
Friend, Irwin and Blume, Marshal E., "The Demand for Risky Assets," American Economic Review, December 1975, 65, 900–22.
Hall, Robert E., "Intertemporal Substitution," Journal of Political Economy, April 1988, 96, 339–57.
_____ and Mishkin, Frederic S., "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households," Econometrica, March 1982, 50, 461–81.
Hansen, Lars Peter and Singleton, Kenneth J.,

"Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns," *Journal of Political Economy*, April 1983, *91*, 249–65.

Hubbard, R. Glenn, "Precautionary Savings Revisited: Social Security, Individual Welfare, and the Capital Stock," NBER (Cambridge, MA) Working Paper No. 1430, August 1984.

_____ and Judd, Kenneth L., "Social Security and Individual Welfare: Precautionary Saving, Borrowing Constraints, and the Payroll Tax," *American Economic Review*, September 1987, *77*, 630–46.

Kimball, Miles S., "Precautionary Saving in the Small and the Large," mimeo, University of Michigan, November 1987.

Kotlikoff, Laurence J. and Pakes, Ariel, "Looking for the News in the Noise-Additional Stochastic Implications of Optimal Consumption Choice," *Annales d'Economie et de Statistique*, January 1988, *9*, 29–46.

_____, Shoven, John B., and Spivak, Avia, "Annuity Markets, Savings, and the Capital Stock," in Z. Bodie, John B. Shoven, and David A. Wise, eds., *Issues in Pension Economics*, Chicago: University of Chicago Press, 1987, pp. 211–36.

_____ and Spivak, Avia, "The Family as an Incomplete Annuities Market," *Journal of Political Economy*, April 1981, *89*, 372–91.

_____ and Summers, Lawrence H., "The Contribution of Intergenerational Transfers in Aggregate Capital Accumulation," *Journal of Political Economy*, August 1981, *90*, 706–32.

Leland, Hayne E., "Savings and Uncertainty: The Precautionary Demand for Savings," *Quarterly Journal of Economics*, August 1968, *82*, 465–73.

Levhari, David and Mirman, Leonard J., "Savings and Consumption with an Uncertain Horizon," *Journal of Political Economy*, April 1977, *85*, 265–81.

_____, _____ and Zilcha, Itzhak, "Capital Accumulation Under Uncertainty," *International Economic Review*, October 1980, *21*, 661–71.

MaCurdy, Thomas E., "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis," *Journal of Econometrics*, January 1982, *18*, 83–114.

Miller, Bruce L., "Optimal Consumption with Stochastic Income Stream," *Econometrica*, March 1974, *42*, 253–66.

_____, "The Effect on Optimal Consumption of Increased Uncertainty in Labor Income in the Multiperiod Case," *Journal of Economic Theory*, March 1976, *13*, 154–67.

Modigliani, Franco and Brumberg, Richard, "Utility Analysis and the Consumption Function: An Interpretation of Cross-Section Data," in K. K. Kurihara, ed., *Post-Keynesian Economics*, New Brunswick, NJ: Rutgers University Press, 1954, pp. 338–436.

Sandmo, Agnar, "The Effect of Uncertainty on Savings Decisions," *Review of Economic Studies*, July 1970, *37*, 353–60.

Schechtman, Jack and Escudero, Vera L. S., "Some Results on 'An Income Fluctuation Problem'," *Journal of Economic Theory*, December 1977, *16*, 151–66.

Sheshinski, Eytan and Weiss, Yoram, "Uncertainty and Optimal Social Security Systems," *Quarterly Journal of Economics*, May 1981, *96*, 189–206.

Sibley, David S., "Permanent and Transitory Income Effects in a Model of Optimal Consumption with Wage Income Uncertainty," *Journal of Economic Theory*, August 1975, *11*, 68–82.

Skinner, Jonathan, "Risky Income, Life Cycle Consumption, and Precautionary Savings," *Journal of Monetary Economics*, September 1988, *22*, 237–55.

Yaari, Menahem E., "Uncertain Lifetime, Life Insurance, and the Theory of the Consumer," *Review of Economic Studies*, April 1965, *32*, 137–50.

Zeldes, Stephen P., "Optimal Consumption with Stochastic Income," *Quarterly Journal of Economics*, May 1989, *104*, 275–98.

Board of Governors of the Federal Reserve System (Flow of Funds Section, Division of Quarterly Statistics), *Balance Sheets of the U.S. Economy 1945–1989*, Washington, DC: Board of Governers of the Federal Reserve System, 1990.