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Excessive Dollar Debt: Financial Development and Underinsurance

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ABSTRACT

We propose that the limited financial development of emerging markets is a significant factor behind the large share of dollar-denominated external debt present in these markets. We show that when financial constraints affect borrowing and lending between domestic agents, agents undervalue insuring against an exchange rate depreciation. Since more of this insurance is present when external debt is denominated in domestic currency rather than in dollars, this result implies that domestic agents choose excessive dollar debt. We also show that limited financial development reduces the incentives for foreign lenders to enter emerging markets. The retarded entry reinforces the underinsurance problem.

ALTHOUGH OBSERVERS STILL DEBATE THE CAUSES underlying recent emerging markets' crises, one factor they agree on is that domestic firms' contracting of external debt in dollars as opposed to domestic currency creates balance sheet mismatches that lead to bankruptcies and dislocations.[1]

The evidence is that most contracts between foreign lenders and borrowers in emerging markets take the form of dollar debt (see Hausmann, Panizza, and Stein (2001)). However, although foreign lenders must eventually be repaid in dollars, in principle, there is no reason that these payments cannot be contingent on the exchange rate. For example, contingencies can be added explicitly by indexing debt contracts, or implicitly, by foreign lenders receiving domestic currency payments that they then convert into dollars. As a result, we are left asking why the choice of dollar debt is in the best interests of borrowers in emerging markets. On the one hand, the attraction to dollar debt is that dollar interest rates are lower than the domestic ones. On the other hand, dollar debt exposes firms to

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[1] Much of the analysis of the currency–balance sheet channel assumes that companies choose dollar-denominated debt. See, for example, Chang and Velasco (1999), Krugman (1999), Aghion, Bacchetta, and Banerjee (2001), and Caballero and Krishnamurthy (2001a).

a balance sheet mismatch. Is the low price of dollar debt worth the balance sheet risk for a firm in an emerging market? Do prices allocate the risk efficiently? Should a policy maker be concerned that companies underprice the risk of dollar debt and therefore take on too much of it? We address these questions in this paper.

Analysis of this issue has, for the most part, centered on the (harmful) incentives of the government.[2] In the context of sovereign debt, Calvo and Guidotti (1990) argue that once foreign lenders purchase domestic-currency-denominated debt, governments have an incentive to devalue and reduce the real value of their debt (see also Calvo (1996) and Allen and Gale (2000)). Foreign lenders rationally anticipate this and avoid purchasing domestic currency debt. First, these explanations seem most compelling for high inflation countries (Latin America), rather than the Asian countries where chronic inflation was not a problem. Moreover, as Calvo (2000) points out, it is hard to extend this argument to private sector debt if we are interested in the connection between debt choices and financial difficulties. The problem is that if balance sheet mismatches are indeed costly, firms will prefer to introduce contingencies into their liabilities to avoid them.

In our model, all agents are risk neutral but demand insurance because they face a risk of liquidation (or production interruptions) in bad states of the world, and they might need resources at times when the country faces international borrowing constraints. This demand arises from the observation, first made by Froot, Scharfstein, and Stein (1993), that the anticipation of borrowing constraints in a dynamic setting motivates firms to hedge. Since the exchange rate also depreciates in bad states of the world, borrowing in domestic currency as opposed to dollars provides more of this insurance.

We show that when financial constraints affect borrowing and lending between domestic agents, their valuation of this insurance is less than its social value. The undervaluation is because some agents who purchase the insurance will not need it. In this case, the agent will sell his excess resources to those who do need it. The financial constraint places a friction in this transaction. It limits the amount that agents who need insurance can pay to those who provide it, and places a wedge between the social valuation of insurance and the equilibrium return to providing this insurance. In a dynamic setting, agents undervalue insurance and take on too much dollar debt.

Our result differs from the sovereign debt literature cited above, because we show that domestic firms in financially underdeveloped economies will misvalue the insurance afforded by borrowing in domestic currency. The fault lies with financial constraints in the private sector rather than a misguided government. The result also explains why the dollar debt problem extends to the private sector's debt choices, and why the private sector might undervalue indexing their debt contracts.

[2] Constraints on domestic currency external borrowing may also have a domestic policy origin. Until recently, the Chilean tax code penalized external borrowing in domestic currency vis-à-vis dollar-denominated borrowing.

In the sovereign debt literature, lenders charge higher prices for lending in domestic currency because of the sovereign moral hazard. In our model, foreign lenders extend loans at actuarially fair prices. However, we show that the same mechanism responsible for underinsurance can also affect the supply decisions of foreign lenders. We allow foreign lenders to pay a fixed cost to enter domestic financial markets. In this case, they are able to value more of the collateral of domestic agents. We show that returns on entry are closely linked to the equilibrium return on providing insurance to domestic agents against bad states of the world. As a result, the distortion in the valuation of insurance by the domestic agents also lowers entry by the foreign lenders.

Although our explanation for dollar liabilities is also driven by an insurance mispricing mechanism, it is quite distinct from those that point out that fixed exchange rates offer free insurance and creates moral hazard that distorts investment choices (see, e.g., Dooley (1997)). In these models, fixing the exchange rate offers free insurance to firms that borrow in dollars and therefore encourages dollar borrowing.[3] In our model, on the other hand, it is not government misbehavior but financial underdevelopment that creates the private underinsurance problem. This result may explain why the dollar debt problem extends across emerging markets, regardless of exchange rate systems.[4,5]

In methodology, our paper relates to a growing literature on aggregate liquidity shortages (Diamond and Dybvig (1983), Allen and Gale (1994), Holmstrom and Tirole (1998, 2001), Krishnamurthy (2002), and Diamond and Rajan (2001)). Each of these papers studies different macroeconomic and asset price consequences of an aggregate liquidity shortage. The canonical model in this literature is Diamond and Dybvig, who study banking structure and the effects of runs on aggregate liquidity. Allen and Gale present a model in which aggregate liquidity shortages affect asset price volatility, and endogenize the links between market participation, aggregate liquidity, and asset prices.

Our modeling approach owes most to the Holmstrom and Tirole (1998, 2001) model of aggregate liquidity in the context of firms. Their papers motivate a role for the state in the creation of liquid assets when there are aggregate shocks. Our basic model economy relates to that in Caballero and Krishnamurthy (2001a), whose central departure from the literature is that they consider two forms of liquidity, one domestic and one international. In this sense, the paper also relates to the recent work by Diamond and Rajan (2001) in which bank's solvency constraints play a role similar to our domestic collateral in determining domestic asset prices. In Caballero and Krishnamurthy, there are two forms of liquidity,

[3] Distortions of private sector incentives due to free insurance is also behind the government-bailout-type models, such as Burnside, Eichenbaum, and Rebelo (2001).

[4] See, for example, the evidence of external dollar debt in fixed as well as flexible exchange rate systems in Hausmann et al. (2001).

[5] In most currency crises, governments run out of resources to bail out firms and the firms that borrow in dollars end up being badly hurt. Thus, we are left with the question as to how much free insurance a rational firm can expect the government to provide. The free insurance models require a government with deep pockets. Our explanation has the advantage of relying only on the resources of the private sector.

because foreign and domestic agents have different technologies to seize collateral on nonrepayment of loans. Aside from the different substantive issue that concerns us in this paper, the model of this paper builds the asymmetry between domestic and foreign lenders from their different valuation of nontradable goods rather than from an asymmetry in collateral valuation.

The paper proceeds as follows. Section I presents our model. Section II discusses the underinsurance result. Section III explores the connection between underinsurance and domestic financial development. This section also serves as a transition to Section IV, where we discuss external supply problems that may arise in this context. Section V concludes.

I. The Model

Our model has two sets of agents, domestic entrepreneurs/firms and foreign investors. There are three periods, which we define as $t = 0,1,2$. All agents are risk neutral and competitive. Domestic agents borrow from foreign investors, choose the contingency in their liabilities, and invest in production at date 0. Then at date 1, there are an idiosyncratic and an aggregate shock that determine the funds required to continue production. The agents' ability to cope with this shock depends on the value of their assets minus contracted liabilities. The question of currency denomination of liabilities turns on whether liabilities are sufficiently contingent to insure against this shock. At date 2, debts are fully repaid and all agents consume.

A. Technology and Preferences

Domestic agents are ex ante identical and have equal access to the same production technology. All production requires foreign (or dollar) goods and produces domestic (or baht) goods. Domestic agents have no dollars, so they must borrow from foreigners to finance all production. At date 0, a firm borrows b_0 dollars from a foreigner and creates capital of k at a cost of $c(k)$. Thus,

$$c(k) \leq b_0. \tag{1}$$

To generate an interior solution, we assume that the function $c(k)$ is convex and increasing. Once created, the capital is "baht." It generates domestic goods, and its value as collateral varies with the exchange rate.

We note in advance that our model is entirely real. Hence, any allusion to the exchange rate refers to the real exchange rate. We denote the exchange rate as e (the formal definition is below).

At date 2, if all goes well, capital generates Ak units of baht goods. However, as part of the normal churn of the economy, production may be interrupted at date 1 by an idiosyncratic shock. If this happens, the firm is required to import an additional unit of foreign goods per unit of capital to realize output of Ak baht. If a firm chooses not to do so, then its output falls to $a < A \equiv a + \Delta$ on the capital that is not salvaged. Thus, if a firm chooses to salvage a fraction $\theta \leq 1$ of its capital units, then its date 2 output is $(1-\theta)ak + \theta\, Ak$ and the firm imports θk units of goods.

Firms that are affected by this shock are *distressed* type, and those that do not are *intact* type.

Let $\omega \in \{l, h\}$ be the aggregate state of the world at date 1. In the h-aggregate state, no firm suffers from a liquidity shock. However, when the aggregate state is l, half of the firms need to reinvest. The shock is countrywide in the sense that it affects a positive measure of firms in the l-state, but it is idiosyncratic in that an individual firm has a probability of 0.5 of being affected by it in the l-state. The probability of the l-state is π, and that of the h-state is $1 - \pi$.

At date 2, the domestic entrepreneurs/firms repay the debts accumulated at date 0 and date 1 out of production proceeds. They consume the excess. Their preferences are

$$U^d = c^B + c^D \quad c^B, c^D \geq 0, \tag{2}$$

where c^B is consumption of baht goods, and c^D is consumption of dollar goods.

Unlike domestic agents, foreigners have preferences only over the consumption of dollar goods at date 2,

$$U^f = c^D. \tag{3}$$

Foreigners can lend dollars to domestic agents to finance production at both date 0 and date 1. We assume they have large endowments of dollars at each of these dates. We also assume that they have access to a storage technology for these endowments, providing a gross rate of return of one. These assumptions pin down the dollar risk-free interest rate at one.

B. Liability Denomination and Contingency

Domestic agents borrow at date 0 from foreigners, using contracts that are fully contingent on the aggregate state:

DEFINITION 1 (CONTINGENT LIABILITY CONTRACT): *For $\omega \in \{l, h\}$, a date 0 contingent liability contract between a domestic firm and foreign investor specifies date 2 repayments, f^ω dollars, and date 0 funding of b_0 dollars. Since foreign investors are risk neutral, competitive, and the dollar interest rate is one,*

$$b_0 = \pi f^l + (1 - \pi) f^h. \tag{4}$$

This definition only allows for aggregate contingencies in the liability structure of firms. Flexibility in specifying liabilities contingent on the type of firm (distressed/intact) could provide greater insurance. We assume that the identity of firms that experience the date 1 production shock in the l-state is private information of that firm, and is not observable by lenders. (Caballero and Krishnamurthy (2001b) examine this issue further.)

Although we define the repayments in units of dollars, this definition does not automatically mean that all debt is in dollars. The puzzling question in emerging markets is why firms take on so much noncontingent dollar debt from foreigners. Since the repayments in Definition 1 are contingent on the aggregate state, they are not the same as the repayments of a noncontingent dollar debt contract.

We prove that in equilibrium, $e^l > e^h = 1$. Consider how noncontingent debt contracts would be represented under Definition 1. A noncontingent dollar debt contract specifies dollar repayments of $f^h = f^l \equiv d_{dollar}$. A noncontingent baht debt contract has baht repayments of $b^h = b^l \equiv b_{baht}$. If we convert these to dollar equivalent repayments, we find that $f^h = b_{baht}$ and $f^l = b_{baht}/e^l$. Since $e^l > 1$, for the same dollar repayments in the high state (i.e., $b_{baht} = d_{dollar}$), the baht contract has lower repayments in the l-state (i.e., $b_{baht}/e^l < d_{dollar}$).

Thus, we look at the liability denomination question as a contingency question: How high are dollar contracted repayments in the h-state compared to the l-state?

C. Credit Constraints and Collateral

Liability choices matter because firms face credit constraints. We introduce credit constraints by requiring firms to post collateral to secure all financing.

We recall that foreigners do not value any baht goods because they do not consume these goods. Therefore, eventual repayments to foreigners can never be in the form of baht goods. We assume that domestic agents have an exogenously specified endowment of foreign goods arriving at date 2 given by w. Following the sovereign debt literature, we define w as international collateral.

Although all of w is collateral, we assume that not all of the output from production is collateral. That is, production returns $(1-\theta)ak + \theta Ak$. In a perfect capital market, firms can pledge all of this output to lenders. However, we assume that the reinvestment at date 1 is not observable and verifiable. Thus, courts cannot verify the extra output of $(A-a)\,k$ due to reinvestment and can only enforce repayments up to ak. For $\theta > 0$, $(1-\theta)ak + \theta Ak$ is clearly larger than ak. This limited collateral assumption is central to our results.

Assumption 1 (Collateral): *Lenders demand collateral against all loans. Each domestic firm has international collateral of w dollar goods and domestic collateral of ak baht goods. Thus, for each ω, the total debt capacity of a firm, measured in dollars, is*

$$f^\omega \leq w + \frac{ak}{e^\omega}. \tag{5}$$

The collateral value of the firm depends on the exchange rate. Since ak is baht collateral, as the exchange rate depreciates, the dollar value of this collateral falls. Our question is: Since collateral is worth less in the l-state, and since firms will need resources to finance their production shocks in this state, do firms match this collateral sensitivity by choosing the appropriate amount of contingency in their liabilities?

We note that Assumption 1 rules out the possibility of equilibrium default in our model. Lenders rationally anticipate the value of a borrower's collateral in each state of the world, and never demand repayments above this collateral value.

The assumption also implies that in equilibrium, foreign debt repayments of f^{ω} will never exceed w. This is because foreigners only value w for consumption, and if the country has contracted debt above w, it will have to default on this debt. Since foreign lenders rationally anticipate this and there is never default in the model, they never demand repayments above w.

Assumption 1 is all that is required to generate our results. However, we can better explain decisions and equilibrium if we consider a slight variation of this problem.

If a foreigner is repaid in baht goods at date 2, he will exchange the baht goods for dollar goods at the exchange rate of e. However, he does not need to wait until date 2. He could also exchange the claim on the date 2 baht goods for a claim on date 2 dollar goods at date 0. That is, the foreign lender is indifferent between waiting until date 2 to swap out of claims against domestic collateral and swapping out of them at date 0.

Rather than having foreigners lend against ak and then swap these goods at date 2 for some of the w dollar goods, we directly impose a constraint under which foreigners only lend against w. We impose a similar constraint, that domestic lending be only against the collateral of ak. These two assumptions are unnecessary for the workings of the model and our main results, but they do simplify the exposition.

ASSUMPTION 1A* (FOREIGN LENDING): *All foreign lending takes the form of liability contracts that are fully secured by the foreign good collateral of w. Lending is default free, so that*

$$f^{\omega} \leq w. \tag{6}$$

ASSUMPTION 1B* (DOMESTIC LENDING): *Domestic firms can lend to each other at either date 0 or date 1. All domestic lending is fully secured by baht revenues. However, the domestic financial market is underdeveloped, so that agents can only use ak of the date 2 baht revenues to secure financing from another domestic agent,*

$$f^{D,\omega} \leq ak. \tag{7}$$

It turns out that since domestic agents are identical at date 0, there is no reason for domestic agents to borrow or lend from each other against the ak at date 0. At date 0, $f^{D,\omega}$ is always zero. This is convenient, because it allows us to restrict our focus to the currency denomination of foreign liabilities. Domestic lending can occur at date 1, but at this time, uncertainty is resolved and the contingency issue is moot.

D. Decisions and the Credit Chain

We solve the decision problem of a firm by backward induction. At date 0, the firm borrows b_0 funds to create capital of k. At date 1, there are two possible states of the world. In the h-state, there are no shocks, and all firms continue to produce Ak. Entrepreneurs repay f^h and consume

$$V^h = Ak + w - f^h. \tag{8}$$

In the l-state, firms divide into distressed and intact groups. A distressed firm raises funds to alleviate its production shock. A choice of θk will result in output at date 2 of $(1-\theta)ak+\theta Ak$ goods. To salvage a fraction θ of distressed capital, the firm must borrow and invest θk imported goods.

The firm can do this in two ways. First, it can go to foreigners to raise additional funds. That is, the firm can always directly raise

$$b_1 = f_1^l \leq w - f^l \tag{9}$$

Second, the distressed firms can turn to intact firms for loans.

There is an asymmetry between domestic and foreign agents. Unlike foreigners, domestic agents value the ak of baht output. Thus, the distressed firm can access foreign funds indirectly by borrowing from the intact firms, who in turn use their international collateral to borrow from foreign agents. Since the exchange rate is e^l, the distressed firm can borrow a maximum amount of ak/e^l dollars from intact firms. This credit chain represents the domestic financial market in our framework. By using this chain, the distressed firm can aggregate the resources of the economy and pledge this to foreigners, thus raising resources for date 1 reinvestment.

The decision problem of a distressed firm is

$$\begin{array}{llll} (PI) & V_s^l & \equiv \max_{\theta, f_1^l, f_1^D} & w + ak + \theta k(A-a) - f^l - f_1^l - f_1^D \\ & s.t. & (i) & f_1^l \leq w - f^l \\ & & (ii) & f_1^D \leq ak \\ & & (iii) & \theta k = f_1^l + \frac{f_1^D}{e^l} \\ & & (iv) & 0 \leq \theta \leq 1. \end{array} \tag{10}$$

Constraints (i) and (ii) are the international and domestic collateral constraints. Constraint (iii) is that investment must be financed by the resources raised from the debt issues of f_1^l and f_1^D. Constraint (iv) is purely technological.

An intact firm at date 1 decides how much it will lend to the distressed firm. If the intact firm lends x_1^D/e_1 dollars at date 1 against collateralized baht goods of x_1^D at date 2, then

$$\begin{array}{llll} (P2) & V_i^l & \equiv \max_{x_1^D} & w + Ak + x_1^D - \frac{x_1^D}{e^l} - f^l \\ & s.t. & & \frac{x_1^D}{e^l} \leq w - f^l. \end{array} \tag{11}$$

The constraint is that the intact firm can, at most, lend $w - f^l$ dollars to the distressed firm.

Date 0 problem. At date 0, a firm maximizes its expected profits over the events of being either distressed or intact, and in either the low or the high state. Thus, the decision at date 0 is

$$\begin{array}{lll} (P3) & \max_{k,b_0,f^w} & (1-\pi)V^h + \pi(V_s^l + V_i^l)/2 \\ & & f^h, f^l \leq w \\ & s.t. & b_0 = \pi f^l + (1-\pi)f^h \\ & & c(k) \leq b_0. \end{array} \tag{12}$$

E. Equilibrium and Exchange Rates

An equilibrium of this economy consists of date 0 and date 1 decisions, (k, b_0, f^ω) and $(\theta, f_1^\omega, f_1^D, x_1^D)$, respectively, and prices e^ω. Decisions are solutions to the firms' problems (P1), (P2), and (P3) given prices of e^ω. At these prices, the financial market clears.

The only equilibrium price is the exchange rate. From the preferences of domestic agents, the following must hold true.

LEMMA 1. *Let c^B and c^D denote the equilibrium consumption of any intact entrepreneur in the domestic economy at date 2:*

- *If c^B, $c^D > 0$, then $e = 1$.*
- *If $c^B > 0$, but $c^D = 0$, then $e \geq 1$.*

The case of $c^B = 0$ and $c^D \geq 0$ can never occur in our model, since production always generates at least some baht and domestic agents must consume this baht.

The exchange rate is one as long as the solution is at an interior where domestic agents consume both baht as well as dollar goods. However, if $c^D = 0$, the economy runs out of dollar goods and the exchange rate depreciates further to reflect this scarcity.

Since this is precisely the case we are interested in, we construct an equilibrium in which this happens at date 1 in the low state, that is, where $e^l > e^h = 1$.

Equilibrium in the l-state. What pins down the exchange rate when the economy runs out of dollar goods? Intact firms have $w - f^l$ of dollar goods that they sell to distressed firms to use in production. Distressed firms pay for these dollars by selling f_1^D of baht to intact firms. The exchange rate is the price in this trade

$$\frac{1}{2} f_1^D = \frac{1}{2}(w - f^l)e^l. \tag{13}$$

This exchange rate is really a date 2 forward exchange rate. Since the international interest rate is one and interest parity must hold, the date 1 exchange rate is the date 2 exchange rate divided by the gross domestic interest rate. The model has a free parameter in that we need not pin down the domestic interest rate. By choosing this interest rate to be equal to one, we can call e^ω the date 1 exchange rate as well.

A distressed firm that borrows against its international collateral to salvage its capital generates $A - a$ units of baht goods at date 2 per unit of foreign debt.

We let $\Delta \equiv A - a$ be the baht return to salvaging one unit of capital. Since the international interest rate is one, as long as $\Delta \geq 1$, the distressed firm chooses to borrow as much as it can against its international collateral ($b_1 = w - f^l$).

If the amount raised from foreign investors, $w - f^l$, is less than the funds needed for salvaging all of its capital, k, then the firm will have to access the domestic financial market to make up the shortfall. It can sell up to ak date 2 baht to another domestic agent at the exchange rate of e^l. It will choose to do this as long as the baht return on restructuring exceeds the exchange rate ($\Delta \geq e^l$). The maximum amount of funds raised is

$$\frac{f_1^D}{e^l} \leq \frac{ak}{e^l}. \tag{14}$$

As long as the sum of ak/e^l and $w - f^l$ is more than the borrowing need of k, the firm is unconstrained in its reinvestment at date 1, and all production units will be salvaged. In this case, the firm will borrow less than ak/e^l in domestic financial markets.

Intact firms will lend dollars to distressed firms as long as the exchange rate weakly exceeds one ($e^l \geq 1$). The most that intact firms can lend is their excess international collateral of $w - f^l$.

If we assume that $\Delta \geq e^l \geq 1$, then distressed firms will borrow as much as they can and intact firms lend as much as possible. In total, the economy imports $w - f^l$ goods, which are all lent to the distressed firms. A necessary condition for all production units to be salvaged is that $\frac{k}{2} \leq w - f^l$. For a given equilibrium $\theta \leq 1$, we refer to the constraint

$$\theta \frac{k}{2} \leq w - f^l \tag{15}$$

as the international collateral constraint.

As long as the international collateral constraint does not bind, both c^B and c^D are positive. From Lemma 1, we note that this will mean that $e^l = 1$. However, if the constraint does bind, the economy will have sold all of its dollar goods, and from Lemma 1, we see that the exchange rate exceeds one.

Figure 1 represents the market clearing for the case in which the international collateral constraint of (15) binds (for $\theta < 1$). The supply of dollars from intact firms is elastic at the international interest rate of one, up to $\frac{1}{2}(w - f^l)$. At this point, the economy has no more international collateral and so the supply of dollars turns vertical. The figure represents equilibria at points A and B. The points are distinguished by whether the distressed firms are credit constrained or not. Given $w - f^l$, in both equilibria θ is the same and less than one. However, in the case where (14) does not bind, the exchange rate is equal to Δ (this case is represented by the dashed upper line for demand corresponding to point B). In the other case (the downward sloping solid curve corresponding to point A), the exchange rate is

$$1 < e^l = \frac{ak}{w - f^l} < \Delta. \tag{16}$$

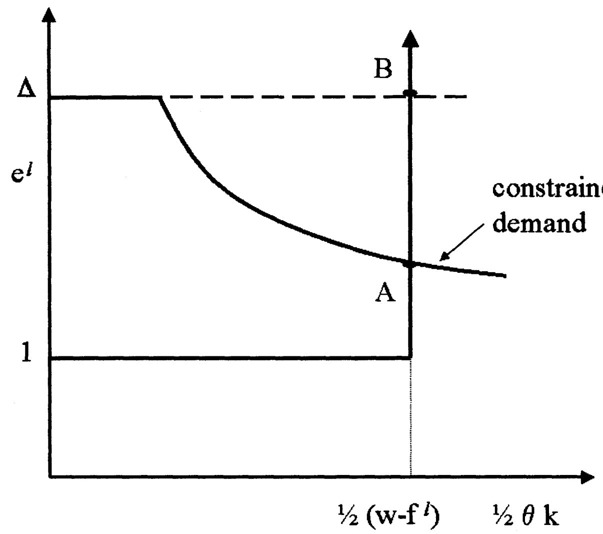

Figure 1. Market clearing in the l-state.

Since we are interested in equilibria in which the exchange rate is depreciated in the l-state, we assume that the international collateral constraint of (15) binds in this state. We are interested in the distinction in outcomes between the cases where (14) does and does not bind.

LEMMA 2. (EXCHANGE RATES)

- *In the h-state, the international collateral constraint does not bind. Therefore,* $e^h = 1$.
- *In the l-state, the international collateral constraint binds. Therefore,*

$$e^l = \min\left[\Delta, \frac{ak}{w - f^l}\right] > 1. \tag{17}$$

If (14) *binds, then* $e^l < \Delta$.

We must also make assumptions such that date 0 investment in capital is sufficiently profitable and gives an interior solution ($k < c^{-1}(w)$). We provide the assumptions on primitives required to generate these equilibria in the Appendix.

II. Underinsurance: Excessive Dollar Debt

Firms contract to make contingent debt repayments in dollars of f^h and f^l. There are two states of the world, and noncontingent dollar and baht debt have linearly independent repayments. Thus, spanning results apply and the contingent repayments of (f^h, f^l) can be implemented by contracting in a mixture of dollar and baht debt. There is excessive dollar debt when a central planner would choose a lower fraction of dollar debt than would private agents.

DEFINITION 2: *If f^h and f^l are debt repayment choices in the competitive decentralized equilibrium, and F^h and F^l are debt repayment choices of a central planner, then the economy has* **excessive dollar debt** *if*

$$\frac{f_h}{f_l} < \frac{F_h}{F_l}. \tag{18}$$

A. Competitive Equilibrium versus Planner's Choice

To arrive at the program for a firm at date 0, we substitute the value functions from (P1) and (P2) into (P3). Firms solve their decision problem, given exchange rates of $e^h = 1$ and $e^l > 1$ (as in Lemma 2).

If a firm chooses (k, f^h, f^l), it will make date 2 profits (net of any contracted debt) in the h-state of $V^h = Ak + w - f^h$ (equation (8)). In the l-state, if the firm is distressed, the date 2 profits are

$$V_s^l = (w - f^l)\Delta + \frac{ak}{e^l}\Delta. \tag{19}$$

$(w - f^l)$ is pledged to foreigners and the proceeds are invested at the project return of Δ. The ak of domestic collateral is sold at the exchange rate of e^l and the proceeds are invested at Δ. If the firm is intact, date 2 resources are

$$V_i^l = (w - f^l)e^l + Ak. \tag{20}$$

Combining these results, the date 0 program is

$$(P4) \quad \max_{k,f^h,f^l} \quad (1-\pi)(Ak + w - f^h) + \pi\tfrac{1}{2}((A + a\tfrac{\Delta}{e^l})k + (\Delta + e^l)(w - f^l))$$
$$s.t. \quad f^h, f^l \leq w$$
$$c(k) \leq \pi f^l + (1-\pi)f^h. \tag{21}$$

In both h and l states, the firm can increase its liabilities up to a maximum of w. The benefit of increasing f^h by one dollar is that the firm raises $1 - \pi$ dollars at date 0. This dollar is used to increase capital by $(1 - \pi)/(c'(k)$. The cost of doing so is that there is one dollar less in the h-state, which reduces date 2 consumption by one dollar. The ratio of benefit (in units of increased capital) to cost of increasing f^h is:

$$\frac{(1-\pi)/c'(k)}{1-\pi} = \frac{1}{c'(k)}. \tag{22}$$

Now we consider the same exercise in the l-state. Increasing f^l by one dollar raises π dollars at date 0. This dollar is used to increase capital by $\pi/c'(k)$. However, since in the l-state firms have to finance their production shock, the cost differs from that in the h-state. From the objective in (P4), we see that the cost is $\pi(\Delta+e^l)/2$. Thus, the same benefit-to-cost ratio in the l-state is:

$$\frac{\pi/c'(k)}{\pi(\Delta+e^l)}=\frac{1}{c'(k)(\Delta+e^l)/2}. \tag{23}$$

Comparing these last two expressions confirms our intuition that since firms will need resources to finance their production shock in the l-state, it is costlier to have more liabilities in this state ($(\Delta+e^l)/2>1$).

However, this cost is lower than that which a central planner would compute. In the l-state, a dollar certainly returns Δ when used in production. Since $e^l<\Delta$, the cost term for firms is strictly less than the planners ($(\Delta+e^l)/2<\Delta$). As a result, the planner will have firms choose to contract less liabilities in the l-state than in the competitive equilibrium outcome.

We confirm this intuition more formally by constructing the program of a central planner who maximizes an equally weighted sum of the utilities of the domestic agents, subject to the domestic and international collateral constraints.

Suppose the central planner makes a date 0 choice of (K, F^h, F^l) (capital letters denote the central planner's aggregate quantities). At date 2, in the high state, all firms earn profits of $V^h=AK+w-F^h$. In the low state, a distressed firm's profits are $(w-F^l)\Delta+(aK/e^l)\Delta$ (equation (19)). For the planning problem, we construct an objective that is free of prices. Substituting the market clearing condition of $e^l=aK/(w-F^l)$ into this profit expression, we obtain:

$$V_s^l=2(w-F^l)\Delta. \tag{24}$$

Similarly, an intact firm's profits are $(w-F^l)e^l+AK$ (equation (20)). After substituting in the market clearing condition, this becomes:

$$V_i^l=(a+A)K. \tag{25}$$

The efficient debt choices in this economy are given by the solution to

$$\begin{aligned}(P5)\quad \max_{K,F^h,F^l}\quad & (1-\pi)(AK+w-F^h)+\pi\tfrac{1}{2}((A+a)K+2\Delta(w-F^l))\\ s.t.\quad & F^h,F^l\le w\\ & c(K)\le\pi F^l+(1-\pi)F^h. \end{aligned}\tag{26}$$

We now compare the benefits/costs of increasing liabilities. Starting with the h-state, since the objectives corresponding to the h-state are the same across ($P4$) and ($P5$), the benefit/cost computation for both the planner and firms in increasing f^h is the same.

LEMMA 3. *In both (P4) and (P5),* $F^h=f^h=w$.

Proof: See the Appendix.

Since there is no chance of a liquidity shock in the h-state, there is no reason to leave a slack in the debt repayment. Optimality requires firms to borrow as much as possible against w in this state and use the proceeds to increase K at date 0.

In the l-state, we confirm that the choices over liability diverge:

LEMMA 4. *If $\Delta > e^l$, then $F^l < f^l$, or debt repayments are set too high in the l-state in the decentralized equilibrium.*

Proof: See the Appendix.

In the objective in ($P4$), f^l is multiplied by $(\Delta + e^l)/2$. In the objective in ($P5$), f^l is multiplied by Δ. When $e^l < \Delta$, the latter is bigger and we arrive at the lemma.

PROPOSITION 1 (EXCESSIVE DOLLAR DEBT): *Suppose that the international collateral constraint of (15) binds in the l-state. If the domestic collateral constraint of* equation (14) *binds, so that $e^l < \Delta$, then firms contract excessive dollar debt. If (14) does not bind so that $e^l = \Delta$, then debt choices are efficient.*

This proposition follows from Lemma 2, $e^l < \Delta$ only if the domestic collateral constraint of (14) binds.

B. The Externality

The planner's choice differs from the competitive equilibrium because of an externality that arises when there are domestic collateral constraints.

The market price of a dollar in the l-state at date 1 is given by e^l. The marginal value of this dollar in production is Δ. The difference between these two valuations is responsible for the underinsurance result.

When (14) binds, the demand for dollars is depressed because the firms in need of dollars are credit constrained. This depressed demand distorts the market price of a dollar relative to its social value. If (14) does not bind, then the distressed firms bid up the price of dollars towards their marginal product of Δ and there is no distortion.

The distorted price affects the quantity of insurance purchased. The insurance decision is a date 0 decision to save one dollar into the l-state. If the firm turns out to be distressed, it uses this dollar in production to return Δ at date 2. However, if the firm is intact, the distorted price comes into play: The firm must sell the dollar at the price of $e^l < \Delta$ and fetches less than the social marginal product of Δ. Ex ante, this translates into underinsurance and the excessive dollar debt result.[6]

[6] We note that there is another factor that reinforces the underinsurance in our model. When $e^l < \Delta$, distressed firms sell their domestic collateral at $(ak)/e^l$ as opposed to $(ak)/\Delta$. We can show that this is an overvaluation, relative to the planner, of the collateral created by k investment. As a result, firms overborrow and overinvest at date 0. Although, we do not focus on this aspect of underinsurance because our interest in this paper is in understanding how liability choices (as opposed to asset/investment choices) are affected by financial development, we can show that when there are more than two aggregate states, the latter effect leads to overborrowing but does not affect insuring against the l-state.

III. Financial Development and Underinsurance

The main result of the previous section is that the excessive share of dollar debt in the liabilities of firms in emerging markets arises because credit constraints affect borrowing/lending relationships among domestic agents. We now consider an economy with a mix of firms that face no credit constraints in their domestic borrowing and the constrained ones of the previous section. We model financial development as increasing in the fraction of firms that are not credit constrained.

We simplify the analysis by ruling out domestic insurance markets contingent on aggregate shocks, which will naturally arise when firms are *ex-ante* heterogeneous. The results in this section are robust to relaxing this simplification.

A. Constrained and Unconstrained Firms

Assume that a fraction λ of the domestic firms face no constraints on domestic borrowing. For these firms, the date 1 domestic collateral constraint is

$$f_1^D \leq ((1-\theta)a + \theta A)k. \tag{27}$$

For $\theta > 0$, it is clear that $((1-\theta)a+\theta A)k > ak$. Thus, these firms are less credit constrained than those of (14). In fact, we show in the Appendix that since these firms are able to pledge all of their baht output as collateral, the domestic collateral constraint of (27) will never bind for them.

LEMMA 5. *(27) will never bind for unconstrained firms.*

Proof: See the Appendix.

Next, we consider the date 0 program for these firms. As before in the h state, $V^h = Ak+w-f^h$. In the l state, if the firm is intact, it makes profits of $V_i^l = Ak + (w-f^l)e^l$. If the firm is distressed, it makes profits of $V_s^l = ak + (\Delta - e^l)k + (w-f^l)e^l$, because the firm is able to salvage all of its capital units by borrowing k dollars at the exchange rate of e^l, and generating Δ baht at date 2.

Combining these expressions yields the date 0 program of an unconstrained firm:

$$\begin{array}{lrl} (P6) \quad \max_{k,f^h,f^l} & & (1-\pi)(Ak+w-f^h) + \pi((A-\frac{e^l}{2})k + e^l(w-f^l)) \\ & s.t. & f^h, f^l \leq w \\ & & c(k) = \pi f^l + (1-\pi)f^h. \end{array} \tag{28}$$

PROPOSITION 2 (FINANCIAL DEVELOPMENT AND EFFICIENCY): *If $\lambda = 1$ and the international collateral constraint binds in the l-state, then $e^l = \Delta$, $e^h = 1$, and*

$$\frac{f^h}{f^l} = \frac{F^H}{F^L}. \tag{29}$$

Proof: Suppose in contradiction that $\Delta > e^l$. Then the distressed firm will choose to borrow the maximum amount and salvage production units. We use Lemma 5 and note that (27) will never bind. As a result, distressed firms will issue debt and salvage all of their production units ($\theta = 1$). Thus $f_1^D/e^l = \frac{k}{2}$. However, since (15) binds, $\theta\frac{k}{2} = w - f^l$ for $\theta < 1$. This implies that

$$\frac{f_1^D}{e^l} > w - f^l, \tag{30}$$

which violates market clearing. There is excess demand for dollars at date 1. As a result, it must be that $\Delta = e^l$.

We substitute $e^l = \Delta$ into the program for a firm at date 0 in (P6).

$$\begin{aligned} \max_{k,f^h,f^l} \quad & (1-\pi)(Ak + w - f^h) + \pi\left(\tfrac{A+a}{2}k + \Delta(w - f^l)\right) \\ s.t. \quad & f^h, f^l \leq w \\ & c(k) \leq \pi f^l + (1-\pi) f^h. \end{aligned} \tag{31}$$

This program is identical to that of (P5). Hence, if $\lambda = 1$, the economy makes efficient debt choices.

This proposition clarifies the main result of the previous section: Since collateral is limited to ak in Assumption 1, firms are constrained in their domestic borrowing. This causes the distortion in prices and results in underinsurance. When all of the baht output of firms can be pledged as collateral, market prices reflect the social marginal product and insurance decisions are chosen optimally.

We conclude by showing that for the intermediate cases of $\lambda < 1$, the debt choices are monotone in λ. As financial development rises and more firms are unconstrained, the debt choices feature more insurance.

PROPOSITION 3 (FINANCIAL DEVELOPMENT AND UNDERINSURANCE): *Consider two economies indexed by λ and λ', where $\lambda > \lambda'$, and in both economies the international collateral constraint binds in the l-state. Then, for both constrained and unconstrained firms,*

$$\left.\frac{f^h}{f^l}\right|_{\lambda} \geq \left.\frac{f^h}{f^l}\right|_{\lambda'}. \tag{32}$$

Proof: See the Appendix.

IV. Limited Foreign Insurance: Further Costs of Domestic Financial Underdevelopment

The general principle behind our result is that credit constraints leads to constrained demand for funds. Those in need of funds are not credible in transferring the surplus created by these funds to the lenders. In a dynamic context, the latter find that the business of lending to firms with bad collateral is not profitable, and so they transfer their resources elsewhere. We apply this principle to explain the limited entry of specialist foreign lenders into domestic markets (i.e., credit line facilities, foreign banks).

So far, we have modeled foreigners as passive lenders who make no profits and willingly lend in either currency. This characterization of foreigners (and many domestic savers) is hardly realistic. This section extends the model to study the effects of financial development on foreign lending decisions. We introduce an active margin whereby foreign lenders may choose to pay a fixed cost and specialize in lending to the domestic market.

A. Foreign Specialists

We return to the model of Section III, with $0<\lambda<1$. We divide foreign lenders into two classes, specialists and nonspecialists. The specialists value baht goods as do domestic agents:

$$U^S = c^D + c^B. \tag{33}$$

Nonspecialists are exactly like the foreign lenders of the previous sections. They value only dollar goods, that is, $U = c^D$.

Unlike the nonspecialist, a specialist can invest in loans backed by domestic baht collateral. This modification captures the idea that specializing in the domestic market enables a foreign lender to receive higher returns on lending to domestic agents. We assume that all lenders (both specialists and nonspecialists) have a date 0 endowment of w^f dollars. There is a continuum of measure α of these specialists, α will shortly be endogenized by positing a cost of specializing.

B. Specialist Lending as Insurance

DEFINITION 3 (SPECIALIST LENDING CONTRACT): *A contract between a foreign specialist and a domestic firm specifies repayments of (f_S^h, f_S^l) and initial loan of b_0.*

$$b_0 q = (1-\pi) f_S^h + \pi f_S^l, \quad q \geq e^l. \tag{34}$$

The collateral constraints for this lending contract are

$$f_S^\omega \leq Ak + w \tag{35}$$

if the firm is unconstrained (in domestic markets), and

$$f_S^\omega \leq ak + w \tag{36}$$

if the firm is constrained.

In this definition, we have accounted for specialist lending against baht collateral by expanding the collateral constraint to include the baht output.

The required return of the specialist lender is $q \geq e^l$ (in (34)). This is because the specialist has a high return investment opportunity in the l-state. He can lend one dollar-good and receive $e^l > 1$ baht-goods in return at date 2. If the specialist converts all of his wealth into date 1 dollars in the l-state (e.g., by investing with a risk-neutral nonspecialist), he will earn the return of e^l on his w^f. Thus, the specialist lender must receive at least this return on date 0 lending.

Consider the problem of a constrained firm. This firm chooses to borrow from both specialists and nonspecialists at date 0. We modify (P4) to reflect this:

$$
\begin{aligned}
(P7) \quad \max_{k,f^h,f^l,f_S^h,f_S^l} \quad & (1-\pi)(Ak + w - f^h - f_S^h) + \\
& \pi\left(\frac{A + a\frac{\Delta}{e^l}}{2}k - f_S^l\frac{\Delta + e^l}{2e^l} + \frac{\Delta + e^l}{2}(w - f^l)\right) \\
s.t. \quad & f^h, f^l \leq w \qquad\qquad (37) \\
& 0 \leq f_S^h, f_S^l \leq ak \\
& c(k) \leq \pi f^l + (1-\pi)f^h + \tfrac{1}{q}(\pi f_S^l + (1-\pi)f_S^h).
\end{aligned}
$$

As in the previous sections, we shorten the collateral constraint for specialist loans (the second constraint) to being constrained only by ak. This is without loss of generality for the same reason as in previous sections.

The following lemma describes the insurance features of specialist lending.

Lemma 6. (Lending Contract as Insurance)
Consider an economy in which $e^l < \Delta$ and $0 < \alpha < \varepsilon$, where ε is positive but small. In this case, $f_S^h > 0$ and $f_S^l = 0$. In addition, the return to specialists is $q = (\Delta + e^l)/2$.

Proof: See the Appendix.

In the economy without specialists, the baht collateral of firms in the h-state is never borrowed against. Since specialists value that collateral, their advantage vis-a-vis nonspecialists is lending against the h-state collateral. This results in $f_S^h > 0$.

Since specialists are limited, they charge the premium of $q \geq e^l$ on their lending. Since nonspecialists lend at the international interest rate of one and $f^l < w$, a firm prefers to increase borrowing from a nonspecialist before it borrows from a specialist. This is why firms choose not to borrow against the l-state from nonspecialists ($f_S^l = 0$). Specialists provide more contingency and insurance than nonspecialists. In this sense, their lending is more domestic currency denominated.

Proposition 4 (Specialist Lending and Insurance): *Consider two economies indexed by α and α', where $\alpha > \alpha'$, $e^l < q < \Delta$ in both economies. Then*

$$
\left.\frac{\sum_{j\in\{const,unconst\}}\lambda_j\left(f_j^h + f_{S,j}^h\right)}{\sum_{j\in\{const,unconst\}}\lambda_j\left(f_j^l + f_{S,j}^l\right)}\right|_{\alpha} \geq \left.\frac{\sum_{j\in\{const,unconst\}}\lambda_j\left(f_j^h + f_{S,j}^h\right)}{\sum_{j\in\{const,unconst\}}\lambda_j\left(f_j^l + f_{S,j}^l\right)}\right|_{\alpha'}. \tag{38}
$$

Proof: See the Appendix.

As the mass of specialists increases, constrained firms raise their borrowing against the h-state. Moreover, these proceeds are used both in increasing k and insuring against the l-state. This can happen directly by receiving dollars in the l-state from specialists, or indirectly by reducing f^l from the nonspecialists. There is an indeterminacy in which of the lenders provides the l-state insurance. However, in total, the liability structure of the firms provide more insurance as the mass of specialists rises.[7]

C. Equilibrium, Entry, and Financial Development

We endogenize α and describe how financial development affects α and lending premia. Assume that specializing costs C. As a result, entry yields expected utility of $V^e = (w^f - C)q$ and nonentry yields utility of $V^{ne} = w^f$. The free entry condition is that α lenders choose to specialize such that, in equilibrium,

$$V^e = V^{ne}. \tag{39}$$

The return to specialists of q is a function of both the equilibrium amount of entry and the exogenous level of financial development (λ). In the Appendix, we prove the following comparative statics.

Lemma 7. *(q, α, and λ)*
As more specialists enter the market, q falls:

$$\frac{\partial q}{d\alpha} < 0.$$

As more firms in the economy are unconstrained, q rises:

$$\frac{\partial q}{d\lambda} > 0.$$

The first result is that as more specialists enter the market, the return to the marginal entrant falls, and q falls. We use previous results to establish the second comparative static. In Lemma 7 we noted that q was equal to $(\Delta + e^l)/2$ for small α. In the Appendix, we show that this positive relation between q and e^l holds more generally for any α. In Section III, we show that e^l was increasing in λ. As a result, q is also increasing in λ. In a more developed financial market, firms have more domestic collateral and the return on lending to these firms rises.

[7] We note that the proposition applies only to the case in which $q > e^l$. This is because when α is large, $q = e^l$ and unconstrained firms will also be borrowing from specialists in equilibrium. Since these firms are unconstrained, they are indifferent between borrowing against h-state collateral or l-state collateral. As a result, the liability structure is indeterminate, and we are unable to make statements about the insurance features of the liabilities. If we assume that they always borrow against h-state collateral, then the proposition continues to hold. In terms of welfare, increasing α is always beneficial, since it leads to more insurance between the h- and l-states.

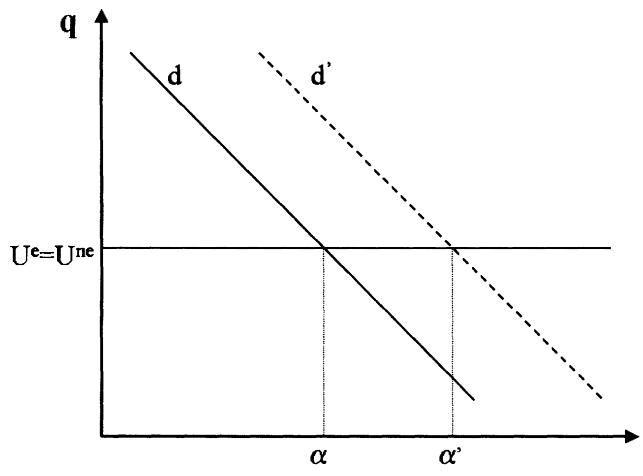

Figure 2. Entry by foreign specialists.

Figure 2 represents the equilibrium for the entry decision.[8] The solid line d is the demand for foreign specialist funds as a function of q. The line d' represents demand in an economy which is more financially developed (i.e., λ is higher). Inspection of the figure leads to Proposition 5.

PROPOSITION 5 (FINANCIAL DEVELOPMENT AND SPECIALIST ENTRY): *Financial development increases the entry of specialists into the domestic lending market (α is increasing in λ).*

D. Financial Development and Lending Premia

The limited foreign entry described in the previous section can feed back into the price charged by foreign specialists through a thin-market externality. Suppose foreign lenders prefer to lend in a market in which there are already other foreign lenders. Allen and Gale (1994) provide a microeconomic model for this phenomena. We defer to their results, and take a reduced form approach to this issue by positing complementarity in foreign entry decisions. Suppose that C is a function of the amount of entry:

$$C(\alpha) = \begin{cases} \bar{C} & \text{if } \alpha \leq \hat{\alpha} \\ \underline{C} & \text{if } \alpha > \hat{\alpha}. \end{cases}$$

[8] Foreigners require that,

$$\frac{\Delta + e^l}{2} = q = \frac{1}{1 - \frac{C}{w^f}}$$

in order to enter. As a result, α is such that the equilibrium exchange rate is $e^l = 2\frac{1}{1-\frac{C}{w^f}} - \Delta$.

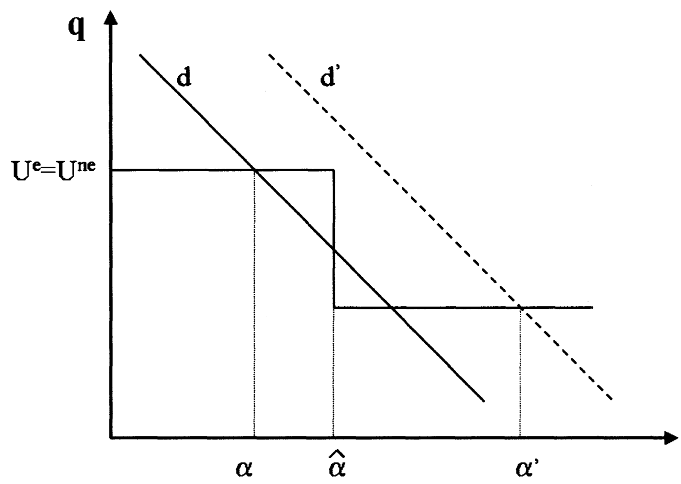

Figure 3. Complementary entry decisions.

Figure 3 illustrates the effect of complementarity. At the low level of financial development corresponding to d, there is the possibility that foreign specialists anticipate limited entry by others and stay out of the market. However, if the market is sufficiently developed (d'), this possibility disappears, since specialists expect others to enter and therefore enter themselves. Comparing across these two cases, we see that the lending premia fall as λ rises.

V. Conclusion

We began the paper with the following question: The large share of external debt in emerging markets that is dollar denominated has played a central role in most recent crises. However, since this is a private decision, why do firms expose themselves to the risks of dollar debt?

We answer this question by showing that the choice over liability denomination was equivalent to a choice over how much insurance to purchase against states of the world when international collateral is scarce. The central result of our analysis is that when domestic financial markets are underdeveloped, the private valuation of this insurance will be distorted relative to a planner's valuation. The distortion leads to underinsurance.

If there is a drop in returns to providing insurance, then the supply of this insurance by foreign specialists also falls. Countries with limited financial development also have fewer foreign credit lines and foreign lending in domestic currency. This situation is exacerbated by complementarities in the lending decisions of foreign specialists.

The primitive result in our analysis is one of underinsurance. Denominating external liabilities in dollars is just one manifestation of this underinsurance.

Other forms of underinsurance include the limited availability of external credit lines and the large amount of short-term external debt in emerging markets. We conjecture that the manifestation of underinsurance in a particular country depends on institutional factors, chief among which is the exchange rate system. We are currently investigating this issue.

Finally, the dollar-debt problem we have discussed is due to an externality. Our investigations into the role of the government to correct this externality have proven fruitful. We are able to rationalize some canonical government policies such as capital inflow taxation or liquidity requirements as Pareto improving in some situations, although not without drawbacks (Caballero and Krishnamurthy (2001b)). We are also able to show that well-designed monetary and international reserve management policies can be effective (Caballero and Krishnamurthy (2002)). Without a theory for why governments may undertake these policies, it has not been possible to study the relative merits of these policies. We are using our framework to analyze these issues currently.

Appendix

Proof of Lemma 3: We form the Lagrangian for (P5),

$$\begin{aligned}\mathcal{L}^* =&(1-\pi)(AK+W-F^h)+\pi\frac{1}{2}((A+a)K+2\Delta(W-F^l))\\ &-\lambda(c(K)-\pi F^l-(1-\pi)F^h)-\mu_h(F^h-W)-\mu_l(F^l-W).\end{aligned} \tag{A1}$$

First,

$$\frac{\partial\mathcal{L}^*}{\partial K}=(1-\pi)A+\pi\frac{A+a}{2}-\lambda c'(K)=0. \tag{A2}$$

Likewise, if $\mu_h=0$,

$$\frac{\partial\mathcal{L}^*}{\partial F^h}=-(1-\pi)+\lambda(1-\pi)=\frac{1-\pi}{c'(K)}\left((1-\pi)A+\pi\frac{A+a}{2}-c'(K)\right)>0. \tag{A3}$$

We substitute in λ from above and note that the project is sufficiently profitable at date 0 to arrive at the inequality. Since $\partial\mathcal{L}^*/\partial F^h>0$, it must be that $F^h=W$. The same proof applies for (P4).

Proof of Lemma 4: We note that,

$$F^l=\frac{c(K)-(1-\pi)w}{\pi} \tag{A4}$$

$$f^l=\frac{c(k)-(1-\pi)w}{\pi}. \tag{A5}$$

From the FOC,

$$c'(K) = \frac{1}{\Delta}\left((1-\pi)A + \frac{\pi}{2}(A+a)\right) \tag{A6}$$

$$c'(k) = \frac{2}{e^l + \Delta}\left((1-\pi)A + \frac{\pi}{2}(A + a\frac{\Delta}{e^l})\right). \tag{A7}$$

If $e^l < \Delta$ then $c'(k) > c'(K)$ and $c(k) > c(K)$. From the first set of equations, this also means that $f^l > F^l$.

Proof of Lemma 5: If the unconstrained firms salvage θ units of capital by issuing debt that raises θk dollars, then the maximum amount of dollars they can raise is,

$$\frac{(1-\theta)a + \theta A}{e^l}k > \theta\frac{A}{e^l}k. \tag{A8}$$

Since

$$A = a + \Delta > \Delta \geq e^l, \tag{A9}$$

we conclude that

$$\frac{(1-\theta)a + \theta A}{e^l}k > \theta k. \tag{A10}$$

Thus, given any θ, unconstrained firms will always be able to obtain the funds required to restructure all of their capital units.

Proof of Proposition 3: From (P6), the FOC for an unconstrained firm is (the "hat" denotes choices for unconstrained firms),

$$(1-\pi)A + \pi\left(A - \frac{e^l}{2}\right) = e^l c'(\hat{k}). \tag{A11}$$

Note that $\hat{k}$ is strictly decreasing in e^l. Also, from the budget constraint

$$c(\hat{k}) = \pi w + (1-\pi)\hat{f}^l \tag{A12}$$

$\hat{f}^l$ is also strictly decreasing in e^l. From the same program for constrained firms, the FOC is

$$(1-\pi)A + \pi\left(\frac{A}{2} - a\frac{\Delta}{2e^l}\right) = \frac{\Delta + e^l}{2}c'(k). \tag{A13}$$

Again we conclude that f^l and k are strictly decreasing in e^l.

We know that if $e^l = \Delta$, the private sector debt choices coincide with the efficient choices. If we take the other case where $e^l < \Delta$, the market clearing condition in the l-state is

$$\lambda\hat{k} + \frac{ak}{e^l}(1-\lambda) = (1+\lambda)w - f^l(1-\lambda) - 2\hat{f}^l\lambda. \tag{A14}$$

We want to prove that f^h/f^l (for both constrained and unconstrained firms) is weakly increasing in λ. This is obviously true when $e^l = \Delta$. In the other case, we construct a proof by contradiction. Suppose not, then for two economies in which $\lambda > \lambda'$, we have that $e^l < e^{l'}$. However if this is true, then $\hat{f}^l > \hat{f}^{l'}$, $\hat{k} > \hat{k}'$, $f^l > f^{l\prime}$, $k > k'$. From the market clearing condition, $e^l > e^{l'}$, which is a contradiction.

Proof of Lemma 6: For a fixed k, we consider the amount of money raised at date 0 by increasing f_s^ω in (P7), and compare this to the cost in terms of the objective. Increasing f_S^h raises $(1-\pi)\frac{1}{q}$ more resources at date 0 and incurs costs of $(1-\pi)$ in the objective. This gives a gross borrowing cost of q. Increasing f^h raises $(1-\pi)$ more resources at date 0 and costs $(1-\pi)$. Increasing f_S^l raises $\pi\frac{1}{q}$ more resources at date 0, but costs $\pi[(e^l+\Delta)/2e^l]$. The gross borrowing cost is $(q/e^l)[(\Delta+e^l)/2]$.

We compare each of these to the cost/benefit ratio of increasing f^l. This raises π resources at date 0 and costs $\pi[(\Delta+e^l)/2]$ in the objective for a gross borrowing cost of $(\Delta+e^l)/2$. Since

$$\frac{\Delta+e^l}{2} < \frac{q}{e^l}\frac{\Delta+e^l}{2},$$

the constrained firm will always choose $f_S^l = 0$. Borrowing from the specialist in the l-state is dominated by borrowing from the nonspecialist.

However, since specialists lend in equilibrium, $f_S^h > 0$. For small values of α, the return to specialist lending is determined by the value of dollars to firms in the l-state. Thus, $q = (\Delta+e^l)/2 > e^l$. Also, at this price, unconstrained firms choose not to borrow from specialists.

Proof of Proposition 4: We first write the program for an unconstrained firm:

$$\begin{aligned}
\max_{k,f^h,f^l,f_S^h,f_S^l} \quad & (1-\pi)(Ak+w-f^h-f_S^h)+ \\
& \pi\Big(\Big(A-\tfrac{e^l}{2}\Big)k - f_S^l + e^l(w-f^l)\Big) \\
s.t. \quad & f^h, f^l \le w \\
& 0 \le f_S^h, f_S^l \le Ak \\
& c(k) \le \pi f^l + (1-\pi)f^h + \tfrac{1}{q}\left(\pi f_S^l + (1-\pi)f_S^h\right).
\end{aligned} \tag{A15}$$

In the region where $q > e^l$, the unconstrained firms will not borrow from specialists. Thus, all lending by specialists must go to constrained firms.

To show that ratio of h-state to l-state liabilities rises with α, we need to show that $(1-\lambda)f^l + \lambda\hat{f}^l$ falls. This is because we know from Lemma 6 that for the constrained firms, f_S^h will rise with α, while $f_S^l = 0$.

We show the result in two steps. First we show that e^l falls as α increases, and second, we show that this implies that $(1-\lambda)f^l + \lambda\hat{f}^l$ falls.

Consider an increase in the mass of specialists of $d\alpha$. Since this increase goes toward constrained firms altering their date 0 investment and borrowing against the l-state (from nonspecialists),

$$(w^f - C)d\alpha = (1-\lambda)c'(k)dk - (1-\lambda)\pi df^l > 0. \tag{A16}$$

The market clearing condition in the l-state is

$$\lambda\hat{k}+\frac{ak}{e^l}(1-\lambda)=(1+\lambda)w-f^l(1-\lambda)-2\hat{f}^l\lambda. \tag{A17}$$

Thus,

$$\frac{(1-\lambda)ak}{e^{l2}}de^l=\frac{(1-\lambda)a}{e^{l2}}dk+\lambda d\hat{k}+(1-\lambda)df^l+2\lambda d\hat{f}^l. \tag{A18}$$

If $de^l\geq 0$, then from the FOCs we know that $dk\leq 0, d\hat{k}\leq 0, d\hat{f}^l\leq 0$. However, given (A16), $df^l<0$. From (A18), $de^l<0$, which is a contradiction. Thus, $de^l<0$ for $d\alpha>0$.

Given that $de^l<0$, from the FOCs we know that $dk>0, d\hat{k}>0, d\hat{f}^l>0$. From (A18), $(1-\lambda)df^l+\lambda d\hat{f}^l<0$.

Proof of Lemma 7: We first note that q is proportional to e^l. This is because in the region that specialists only lend to constrained firms, q begins at $(\Delta+e^l)/2$ and decreases linearly. When there are sufficient specialists, specialists also lend to unconstrained firms, resulting in $q=e^l$.

The proof follows the same logic as that of Proposition 4. First, since specialists lend to both constrained and unconstrained firms, we note that

$$(w^f-C)d\alpha=(1-\lambda)c'(k)dk+\lambda c'(\hat{k})d\hat{k}-(1-\lambda)\pi df^l-\lambda\pi d\hat{f}^l>0. \tag{A19}$$

If $de^l\geq 0$ for $d\alpha>0$, then from the FOCs we know that $dk\leq 0, d\hat{k}\leq 0, d\hat{f}^l\leq 0$. However, given (A19), $df^l<0$. From (A18) this means that $de^l<0$, which is a contradiction. Thus, $de^l<0$ for $d\alpha>0$.

For the λ comparative static, after a little algebra we can show that

$$\frac{(1-\lambda)ak}{e^{l2}}de^l=\frac{(1-\lambda)a}{e^{l2}}dk+\lambda d\hat{k}+(1-\lambda)df^l+2\lambda d\hat{f}^l+(1-\lambda)ak(\hat{k}-2(w-\hat{f}^l))d\lambda. \tag{A20}$$

The last term on the right-hand side (RHS) is positive for $d\lambda>0$, because on net, unconstrained firms are borrowers of dollars in the market at date 1. Thus, we only need to show that the first term is nonnegative. The proof is by contradiction. Suppose $de^l\leq 0$ for $d\lambda>0$. Then the first term on the RHS is positive. However from the FOCs we know that if $de^l\leq 0$, then $dk\leq 0, d\hat{k}\leq 0, d\hat{f}^l\leq 0$, and $df^l<0$. From (A20) this means that $de^l<0$, which is a contradiction. Thus, $de^l>0$ for $d\lambda>0$.

Parameter assumptions We examine the technical assumptions on parameters that we have used. First, we require that $w=F^h$ in ($P5$), or that the return to investing domestically exceeds that of investing abroad:

$$(1-\pi)A+\pi\frac{A+a}{2}\geq c'(w). \tag{A21}$$

Second, we require that the solution features some insurance against the l-state, so that $F^l < w$:

$$c'(w)\Delta \geq (1-\pi)A + \pi\frac{A+a}{2}. \tag{A22}$$

Finally, we require that equilibrium has $1 < e^l < \Delta$. The FOC for the program in (P4) is,

$$c'(k)\frac{\Delta + e^l}{2} = (1-\pi)A + \pi\frac{1}{2}\left(A + a\frac{\Delta}{e^l}\right). \tag{A23}$$

We denote the solution to this equation as $k(e)$. Then the largest value of k is attained when $e = 1$, and the smallest value when $e = \Delta$. Using this knowledge as well as the market clearing condition leads to:

$$\frac{\pi a k(1)}{w - c(k(1))} < \Delta \tag{A24}$$

$$\frac{\pi a k(\Delta)}{w - c(k(\Delta))} > 1. \tag{A25}$$

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