Revisiting the Supply-Side Effects of Government Spending[*]

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Abstract

We revisit the macroeconomic effects of government consumption in the neoclassical growth model when agents face uninsured idiosyncratic investment risk. Under complete markets, a permanent increase in government consumption has no long-run effect on the interest rate and the capital-labor ratio, while it increases hours due to the negative wealth effect. These results are upset once we allow for incomplete markets. The same negative wealth effect now causes a reduction in risk taking and the demand for investment. This leads to a lower risk-free rate and, under certain conditions, also to a lower capital-labor ratio, and lower productivity.

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1 Introduction

Studying the impact of government spending on macroeconomic outcomes is one of the most celebrated policy exercises within the neoclassical growth model: it is important for understanding the business-cycle implications of fiscal policy, the macroeconomic effects of wars, and the cross-section of countries. Some classics include Hall (1980), Barro (1981, 1989), Aiyagari, Christiano and Eichenbaum (1992), Baxter and King (1993), Braun and McGrattan (1993), and McGrattan and Ohanian (1999, 2006).

These studies have all maintained the convenient assumption of complete markets, abstracting from the possibility that agents' saving and investment decisions, and hence their reaction to changes in fiscal policy, may crucially depend on the extent of risk sharing within the economy. This paper contributes towards filling this gap. It revisits the macroeconomic effects of government consumption within an incomplete-markets variant of the neoclassical growth model.

The key deviation we make from the standard paradigm is the introduction of uninsurable idiosyncratic risk in production and investment. All other ingredients of our model are the same as in the canonical neoclassical growth model: firms operate neoclassical constant-returns-to-scale technologies, households have standard CRRA/CEIS preferences, and markets are competitive.

The focus on idiosyncratic production/investment risk is motivated by two considerations. First, this friction is empirically relevant. This is obvious for less developed economies. But even in the United States, privately-owned firms account for nearly half of aggregate production and employment. Furthermore, the typical investor—the median rich household—holds a very undiversified portfolio, more than one half of which is allocated to private equity.[1] And second, as our paper shows, this friction upsets some key predictions of the standard neoclassical paradigm.

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[1] See Quadrini (2000), Gentry and Hubbard (2000), Carroll (2000), and Moskowitz and Vissing-Jørgensen (2002). Also note that idiosyncratic investment risks need not be limited to private entrepreneurs; they may also affect educational and occupational choices, or the investment decisions that CEO's make on behalf of public corporations. On this latter point, see Panousi and Papanikolaou (2008) for some supportive evidence.

In the standard paradigm, the steady-state values of the capital-labor ratio, productivity (output per work hour), the wage rate, and the interest rate, are all pinned down by the equality of the marginal product of capital with the discount rate in preferences. As a result, any change in the level of government consumption, even if it is permanent, has no effect on the long-run values of these variables.[2] On the other hand, because higher consumption for the government means lower net-of-taxes wealth for the households, a permanent increase in government consumption raises labor supply. It follows that employment and, by implication, output and investment increase. But the long-run levels of capital intensity and productivity unchanged.

The picture is quite different once we allow for incomplete markets. The same wealth effect that, in response to an increase in government consumption, stimulates labor supply in the standard paradigm, now also discourages investment. This is simply because risk taking, and hence investment, is sensitive to wealth. We thus find very different long-run effects. First, a permanent increase in government consumption necessarily reduces the risk-free interest rate. And second, unless the elasticity of intertemporal substitution is low enough, it also reduces the capital-labor ratio, productivity, and wages.

The effect on the risk-free rate is an implication of the precautionary motive: a higher level of consumption for the government implies a lower aggregate level of wealth for the households, which is possible in steady state only with a lower interest rate. If investment was risk-free, a lower interest rate would immediately translate to a higher capital-labor ratio. But this is not the case in our model precisely because market incompleteness introduces a wedge between the risk-free rate and the marginal product of capital—this wedge is simply the risk premium on investment. Furthermore, because of diminishing absolute risk aversion, this wedge is higher the lower the wealth of the households. It follows that the negative wealth effect of higher government consumption raises the risk premium on investment and can thereby lead to a reduction in the capital-labor ratio, despite the reduction in the interest rate. We show that a sufficient condition for this to be the case is that the elasticity of

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[2]This, of course, presumes that the change in government consumption is financed with lump-sum taxes. The efficiency or redistributive considerations behind optimal taxation are beyond the scope of this paper.

intertemporal substitution is sufficiently high relative to the income share of capital—a condition easily satisfied for plausible calibrations of the model.

Turning to employment and output, there are two opposing effects. On the one hand, as with complete markets, the negative wealth effect on labor supply contributes towards higher employment and output. On the other hand, unlike complete markets, the reduction in capital intensity, productivity, and wages contributes towards lower employment and output. Depending on the income and wage elasticities of labor supply, either of the two effects can dominate.

The deviation from the standard paradigm is significant, not only qualitatively, but also quantitatively. For our preferred parametrizations of the model, the following hold. First, the elasticity of intertemporal substitution is comfortably above the critical value that suffices for an increase in government consumption to *reduce* the long-run levels of the capital-labor ratio, productivity, and wages. Second, a 1% increase in government spending under incomplete markets has the same impact on capital intensity and labor productivity as a $0.5\% - 0.6\%$ increase in capital-income taxation under complete markets. Third, these effects mitigate, but do not fully offset, the wealth effect on labor supply. Finally, the welfare consequences are non-trivial: the welfare cost of a permanent 1% increase in government consumption is three times larger under incomplete markets than under complete markets.

The main contribution of the paper is thus to highlight how wealth effects on investment due to financial frictions can significantly modify the supply-side channel of fiscal policy. In our model, these wealth effects emerge from idiosyncratic risk along with diminishing absolute risk aversion; in other models, they could emerge from borrowing constraints. Also, such wealth effects are relevant for both neoclassical and Keynesian models. In this paper we follow the neoclassical tradition because this clarifies best our contribution: whereas wealth effects have been central to the neoclassical approach with regard to labor supply, they have been mute with regard to investment.

To the best of our knowledge, this paper is the first to study the macroeconomic effects of government consumption in an incomplete-markets version of the neoclassical growth

paradigm that allows for uninsurable investment risk. A related, but different, exercise is conducted in Heathcote (2005) and Challe and Ragot (2007). These papers study deviations from Ricardian equivalence in Bewley-type models like Aiyagari's (1994), where borrowing constraints limit the ability of agents to smooth consumption intertemporally. In our paper, instead, deviations from Ricardian equivalence are not an issue: our model allows households to freely trade a riskless bond, thus ensuring that the timing of taxes and the level of debt has no effect on allocations, and instead focuses on wealth effects on investment due to incomplete risk sharing.

The particular framework we employ in this paper is a continuous-time variant of the one introduced in Angeletos (2007). That paper showed how idiosyncratic capital-income risk can be accommodated within the neoclassical growth model without loss of tractability, studied the impact of this risk on aggregate saving, and contrasted it with the impact of labor-income risk in Bewley-type models (Aiyagari, 1994; Huggett, 1997; Krusell and Smith, 1998). Other papers that introduce idiosyncratic investment or entrepreneurial risk in the neoclassical growth model include Angeletos and Calvet (2005, 2006), Buera and Shin (2007), Caggeti and De Nardi (2006), Covas (2006), and Meh and Quadrini (2006).[3] The novelty of our paper is to study the implications for fiscal policy in such an environment.

Panousi (2008) studies the macroeconomic effects of capital taxation within a similar environment as ours. That paper shows that, when agents face idiosyncratic investment risk, an increase in capital taxation may paradoxically stimulate more investment in general equilibrium. This provides yet another example of how the introduction of idiosyncratic investment risk can upset some important results of the neoclassical growth model.

The rest of the paper is organized as follows. Section 2 introduces the basic model, which fixes labor supply so as to focus on the most novel results of the paper. Section 3 characterizes its equilibrium and Section 4 analyzes its steady state. Section 5 examines the steady-state effects of government consumption on the interest rate and capital accumulation. Section 6 considers three extensions that endogenize labor supply. Section 7 examines the dynamic

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[3]Related are also the earlier contributions by Leland (1968), Sandmo (1970), Obstfeld (1994), and Ace-moglu and Zilibotti (1997).

response of the economy to a permanent change in government consumption. Section 8 concludes. All the formal results are explained in the main text; but the complete proofs are delegated to the Online Appendix.

2 The basic model

Time is continuous, indexed by $t \in [0, \infty)$. There is a continuum of infinitely-lived households, indexed by i and distributed uniformly over $[0, 1]$. Each household is endowed with one unit of labor, which it supplies inelastically in a competitive labor market. Each household also owns and runs a firm, which employs labor in the competitive labor market but can only use the capital stock invested by the particular household.[4] Households cannot invest in other households' firms and cannot otherwise diversify away from the shocks hitting their firms, but can freely trade a riskless bond. Finally, all uncertainty is purely idiosyncratic, and hence all aggregates are deterministic.

2.1 Households, firms, and idiosyncratic risk

The financial wealth of household i, denoted by x_t^i, is the sum of its holdings in private capital, k_t^i, and the riskless bond, b_t^i:

$$x_t^i = k_t^i + b_t^i. \tag{1}$$

The evolution of x_t^i is given by the household budget:

$$dx_t^i = d\pi_t^i + [R_t b_t^i + \omega_t - T_t - c_t^i]dt, \tag{2}$$

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[4]One can think of a household as a couple, with the wife running the family business and the husband working in the competitive labor market (or vice versa). The key assumption, of course, is only that the value of the labor endowment of each household is pinned down by the competitive wage, and is not subject to idiosyncratic risk.

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where $d\pi_t^i$ is the household's capital income (i.e., the profits it enjoys from the private firm it owns), R_t is the interest rate on the riskless bond, ω_t is the wage rate, T_t is the lump-sum tax, and c_t^i is the household's consumption. Finally, the familiar no-Ponzi game condition is also imposed.

Whereas the sequences of prices and taxes are deterministic (due to the absence of aggregate risk), firm profits, and hence household capital income, are subject to undiversified idiosyncratic risk. In particular:

$$d\pi_t^i = [F(k_t^i, n_t^i) - \omega_t n_t^i - \delta k_t^i]dt + \sigma k_t^i dz_t^i. \tag{3}$$

Here, n_t^i is the amount of labor the firm hires in the competitive labor market, F is a constant-returns-to-scale neoclassical production function, and δ is the mean depreciation rate. Idiosyncratic risk is introduced through dz_t^i, a standard Wiener process that is i.i.d. across agents and across time. This can be interpreted either as a stochastic depreciation shock or as a stochastic productivity shock, the key element being that it generates risk in the return to capital. The scalar σ measures the amount of undiversified idiosyncratic risk and can be viewed as an index of market incompleteness, with higher σ corresponding to a lower degree of risk sharing (and $\sigma = 0$ corresponding to complete markets). Finally, without serious loss of generality, we assume a Cobb-Douglas specification for the technology: $F(k, n) = k^\alpha n^{1-\alpha}$, with $\alpha \in (0, 1)$.[5]

Turning to preferences, we assume an Epstein-Zin specification with constant elasticity of intertemporal substitution (CEIS) and constant relative risk aversion (CRRA). Given a consumption process, the utility process is defined by the solution to the following integral equation:

$$U_t = E_t \int_t^\infty z(c_s, U_s) \, ds, \tag{4}$$

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[5]The characterization of equilibrium and the proof of the existence of the steady state extend to any neoclassical production function; it is only the proof of the uniqueness of the steady state that uses the Cobb-Douglas specification.

where

$$z(c, U) \equiv \frac{\beta}{1 - 1/\theta} \left[\frac{c^{1-1/\theta}}{((1-\gamma)U)^{\frac{-1/\theta+\gamma}{1-\gamma}}} - (1-\gamma)U \right]. \tag{5}$$

Here, $\beta > 0$ is the discount rate, $\gamma > 0$ is the coefficient of relative risk aversion, and $\theta > 0$ is the elasticity of intertemporal substitution.[6]

Standard expected utility is nested with $\gamma = 1/\theta$. We find it useful to allow $\theta \neq 1/\gamma$ in order to clarify that the *qualitative* properties of the steady state depend crucially on the elasticity of intertemporal substitution rather than the coefficient of relative risk aversion (which in turn also guides our preferred parameterizations of the model). However, none of our results rely on allowing $\theta \neq 1/\gamma$. A reader who feels uncomfortable with the Epstein-Zin specification can therefore ignore it, assume instead standard expected utility, and simply replace γ with $1/\theta$ (or vice versa) in all the formulas that follow.

2.2   Government

The government consumes output at the rate G_t. Government spending is deterministic, it is financed with lump-sum taxation, and it does not affect either the household's utility from private consumption or the production of the economy. The government budget constraint is given by:

$$dB_t^g = [R_t B_t^g + T_t - G_t]dt, \tag{6}$$

where B_t^g denotes the level of government assets (i.e., minus the level of government debt). Finally, a no-Ponzi game condition is imposed to rule out explosive debt accumulation.

2.3   Aggregates and equilibrium definition

The initial position of the economy is given by the cross-sectional distribution of (k_0^i, b_0^i). Households choose plans $\{c_t^i, n_t^i, k_t^i, b_t^i\}_{t \in [0,\infty)}$, contingent on the history of their idiosyncratic shocks, and given the price sequence and the government policy, so as to maximize their

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[6]To make sure that (4) defines a preference ordering over consumption lotteries, one must establish existence and uniqueness of the solution to the integral equation (4); see Duffie and Epstein (1992).

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lifetime utility. Idiosyncratic risk, however, washes out in the aggregate. We thus define an equilibrium as a deterministic sequence of prices $\{\omega_t, R_t\}_{t \in [0,\infty)}$, policies $\{G_t, T_t\}_{t \in [0,\infty)}$, and macroeconomic variables $\{C_t, K_t, Y_t\}_{t \in [0,\infty)}$, along with a collection of individual contingent plans $(\{c_t^i, n_t^i, k_t^i, b_t^i\}_{t \in [0,\infty)})_{i \in [0,1]}$, such that the following conditions hold: (i) given the sequences of prices and policies, the plans are optimal for the households; (ii) the labor market clears, $\int_t n_t^i = 1$, in all t; (iii) the bond market clears, $\int_i b_t^i + B_t^g = 0$, in all t; (iv) the government budget is satisfied in all t; and (v) the aggregates are consistent with individual behavior, $C_t = \int_i c_t^i$, $K_t = \int_i k_t^i$, and $Y_t = \int_i F(k_t^i, n_t^i)$, in all t. (Throughout, we let $\int_i$ denote the mean in the cross-section of the population.)

3   Equilibrium

In this section we characterize the equilibrium of the economy. We first solve for a household's optimal plan for given sequences of prices and policies. We then aggregate across households and derive the general-equilibrium dynamics.

3.1   Individual behavior

Since employment is chosen after the capital stock has been installed and the idiosyncratic shock has been observed, optimal employment maximizes profits state by state. By constant returns to scale, optimal firm employment and profits are linear in own capital:

$$n_t^i = \bar{n}(\omega_t)k_t^i \qquad \text{and} \qquad d\pi_t^i = \bar{r}(\omega_t)k_t^i dt + \sigma dz_t^i, \tag{7}$$

where

$$\bar{n}(\omega_t) \equiv \arg\max_n [F(1, n) - \omega_t n] \qquad \text{and} \qquad \bar{r}(\omega_t) \equiv \max_n [F(1, n) - \omega_t n] - \delta.$$

Here, $\bar{r}_t \equiv \bar{r}(\omega_t)$ is the household's expectation of the return to its capital prior to the realization of the idiosyncratic shock z_t^i, as well as the mean of the realized returns in the

cross-section of firms. Analogous interpretation applies to $\bar{n}_t \equiv \bar{n}(\omega_t)$.

The key result here is that households face risky, but linear, returns to their capital. To see how this translates to linearity of wealth in assets, let h_t denote the present discounted value of future labor income net of taxes, a.k.a. *human wealth*:

$$h_t = \int_t^\infty e^{-\int_t^s R_j dj}(\omega_s - T_s)ds. \tag{8}$$

Next, define *effective wealth* as the sum of financial and human wealth:

$$w_t^i \equiv x_t^i + h_t = k_t^i + b_t^i + h_t. \tag{9}$$

It follows that the evolution of effective wealth can be described by:

$$dw_t^i = [\bar{r}_t k_t^i + R_t(b_t^i + h_t) - c_t^i]dt + \sigma k_t^i dz_t^i. \tag{10}$$

The first term on the right-hand side of (10) measures the expected rate of growth in the household's effective wealth; the second term captures the impact of idiosyncratic risk.

The linearity of budgets, together with the homotheticity of preferences, ensures that, for given prices and policies, the household's consumption-saving problem reduces to a tractable homothetic problem as in Samuelson's and Merton's classic portfolio analysis. It then follows that the optimal policy rules are linear in wealth, as shown in the next proposition.

Proposition 1. *Let $\{\omega_t, R_t\}_{t\in[0,\infty)}$ and $\{G_t, T_t\}_{t\in[0,\infty)}$ be equilibrium price and policy sequences. Then, equilibrium consumption, investment and bond holdings for household i are given by*

$$c_t^i = m_t w_t^i, \quad k_t^i = \phi_t w_t^i, \quad and \quad b_t^i = (1 - \phi_t)w_t^i - h_t, \tag{11}$$

where ϕ_t, the fraction of effective wealth invested in capital, is given by

$$\phi_t = \frac{\bar{r}_t - R_t}{\gamma \sigma^2}, \tag{12}$$

while m_t, the marginal propensity to consume out of effective wealth, satisfies the recursion

$$\frac{\dot{m}_t}{m_t} = m_t + (\theta - 1)\rho_t - \theta\beta, \tag{13}$$

with $\rho_t \equiv \phi_t \bar{r}_t + (1 - \phi_t)R_t - \frac{1}{2}\gamma\phi_t^2\sigma^2$ denoting the risk-adjusted return to saving.

Condition (12) simply says that the fraction of wealth invested in the risky asset is increasing in the risk premium $\mu_t \equiv \bar{r}_t - R_t$ and decreasing in risk aversion γ and the amount of risk σ. Condition (13) is essentially the Euler condition: it describes the growth rate of the marginal propensity to consume as a function of the anticipated path of risk-adjusted returns to saving. Whether higher risk-adjusted returns increase or reduce the marginal propensity to consume depends on the elasticity of intertemporal substitution. To see this more clearly, note that in steady state this condition reduces to $m = \theta\beta - (\theta - 1)\rho$, so that higher ρ decreases m if and only if $\theta > 1$; that is, a higher risk-adjusted return to saving increases the fraction of savings out of effective wealth if and only if the EIS is higher than one. This is due to the familiar tension between the income and substitution effects implied by an increase in the rate of return.

3.2 General equilibrium

Because individual consumption, saving and investment are linear in individual wealth, aggregates at any point in time do not depend on the extent of wealth inequality at that time. As a result, the aggregate equilibrium dynamics can be described with a low-dimensional recursive system.

Define $f(K) \equiv F(K, 1) = K^\alpha$ as the production function in intensive form (output per work hour). From Proposition 1, the equilibrium ratio of capital to effective wealth and the equilibrium risk-adjusted return to savings are identical across agents and can be expressed as functions of the current capital stock and risk-free rate: $\phi_t = \phi(K_t, R_t)$ and $\rho = \rho(K_t, R_t)$,

where

$$\phi(K, R) \equiv \frac{1}{\gamma\sigma^2}\left(f'(K) - \delta - R\right) \quad \text{and} \quad \rho(K, R) \equiv R + \frac{1}{2\gamma\sigma^2}\left(f'(K) - \delta - R\right)^2.$$

Similarly, the wage is given by $\omega_t = \omega(K_t)$, where $\omega(K) \equiv f(K) - f'(K)K = (1 - \alpha)f(K)$. Using these facts, aggregating the policy rules of the agents, and imposing market clearing for the risk-free bond, we arrive at the following characterization of the general equilibrium of the economy.

Proposition 2. *In equilibrium, the aggregate dynamics satisfy the following ODE system*

$$\dot{K}_t = f(K_t) - \delta K_t - C_t - G_t \tag{14}$$

$$\frac{\dot{C}_t}{C_t} = \theta\left(\rho_t - \beta\right) + \tfrac{1}{2}\gamma\sigma^2\phi_t^2 \tag{15}$$

$$\dot{H}_t = R_t H_t - \omega_t + G_t \tag{16}$$

$$K_t = \frac{\phi_t}{1 - \phi_t} H_t \tag{17}$$

with $\omega_t = \omega(K_t)$, $\phi_t = \phi(K_t, R_t)$, *and* $\rho_t = \rho(K_t, R_t)$.

This system has a simple interpretation. Condition (14) is the resource constraint of the economy; it follows from aggregating budgets across all households and the government, imposing labor- and bond-market clearing, and using the linearity of individual firm employment to individual capital together with constant returns to scale, to get $Y_t = \int_i F(k_t^i, n_t^i) = F(\int_i k_t^i, \int_i n_t^i) = F(K_t, 1)$. Condition (15) is the aggregate Euler condition for the economy; it follows from aggregating consumption and wealth across agents, together with the optimality condition (13) for the marginal propensity to consume. Condition (16) expresses the evolution of the present value of aggregate net-of-taxes labor income in recursive form; it follows from the definition of human wealth combined with the intertemporal government budget, which imposes that the present value of taxes equals the present value of government consumption. Finally, condition (17) represents market-clearig

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in the bond market; more precisely, it follows from aggregating bond holdings and invest-ment across agents to get $B_t = (1 - \phi_t)W_t - H_t$ and $K_t = \phi_t W_t$, using the latter to replace W_t in the former, and imposing $B_t = 0$.

This system characterizes the equilibrium dynamics of the economy under both complete and incomplete markets. In particular, conditions (14), (16) and (17) are exactly the same under either market structure; the key differences between complete and incomplete markets rest in the Euler condition (15) and in the relation between the risk-adjusted return ρ_t, the risk-free rate R_t and the marginal product of capital $f'(K_t) - \delta$.

When $\sigma = 0$ (complete markets), arbitrage imposes that $R_t = f'(K_t) - \delta = \rho_t$ and the Euler condition reduces to its familiar complete-market version, $\frac{\dot{C}_t}{C_t} = \theta(R_t - \beta)$. When instead $\sigma > 0$ (incomplete markets), there are two important changes. First, the precaution-ary motive for saving introduces a positive drift in consumption growth, represented by the term $\frac{1}{2}\gamma\sigma^2\phi_t^2$ in the Euler condition (15). And second, the fact that investment is subject to undiversifiable idiosyncratic risk introduces a wedge between the risk-free rate and the marginal product of capital, so that $R_t < \rho_t < f'(K_t) - \delta$. It is worth noting here that the first effect is also shared by Aiyagari (1994) and other Bewley-type models that consider *labor-income* risk, whereas the second effect relies on the presence of *capital-income* risk.

Finally note that condition (17) can be solved for R_t as a function of the contempora-neous (K_t, H_t), so that the equilibrium dynamics of the economy reduce to a simple three-dimensional ODE system in (C_t, K_t, H_t). Indeed, the equilibrium dynamics can be approxi-mated with a simple shooting algorithm, similar to the one applied to the complete-markets neoclassical growth model. For any historically given K_0, guess some initial values (C_0, H_0) and use conditions $(14) - (16)$ to compute the entire path of (C_t, K_t, H_t) for $t \in [0, T]$, for some large T; then iterate on the initial guess till (C_T, K_T, H_T) is close enough to its steady-state value.[7] In the special case of a unit EIS ($\theta = 1$), we have that $m_t = \beta$ and hence $C_t = \beta(K_t + H_t)$ for all t. One can then drop the Euler condition from the dynamic system and analyze the equilibrium dynamics with a simple phase diagram in the (K, H) space,

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[7]This presumes that a turnpike theorem holds true in our model; we expect this to be the case at least for σ small enough, by continuity to the complete-markets case.

much alike in a textbook exposition of the neoclassical growth model. Either way, this is a significant gain in tractability relative to other incomplete-markets models, where the entire wealth distribution—an infinite dimensional object—is usually a relevant state variable for aggregate equilibrium dynamics. As in Angeletos (2007), the key is that individual policy rules are linear in individual wealth, so that aggregate dynamics are invariant to the wealth distribution.

4   Steady State

We henceforth parameterize government spending as a fraction g of aggregate output to study the steady state of the economy, that is, the fixed point of the dynamic system in Proposition 2. We now show that this can be characterized as the solution to a system of two equations in K and R.

First, note that the growth rate of consumption must be zero in steady state. Setting $\frac{\dot{C}_t}{C_t} = 0$ into the Euler condition (15) gives:

$$\rho = \beta - \frac{1}{2}\frac{\gamma}{\theta}\sigma^2\phi^2. \tag{18}$$

Using then the facts that $\rho = R + \frac{1}{2\gamma\sigma^2}\left(f'(K) - \delta - R\right)^2$ and $\phi = \frac{1}{\gamma\sigma^2}\left(f'(K) - \delta - R\right)$, we can write the above as follows:

$$f'(K) - \delta - R = \sqrt{\frac{2\theta\gamma\sigma^2\left(\beta - R\right)}{\theta + 1}}. \tag{19}$$

This condition gives the combinations of K and R that are consistent with stationarity of aggregate consumption (equivalently, with stationarity of aggregate wealth).

Second, note that the growth rate of human wealth from (16) must also be zero in steady state. From this we get that:

$$H = \frac{\omega - G}{R},$$

which simply states that human wealth must equal the present value of wages net of taxes.

Substituting this into the bond-market clearing condition (17), and using $\omega = (1 - \alpha)f(K)$ and $G = gf(K)$, we get the following:

$$K = \frac{\phi(K, R)}{1 - \phi(K, R)} \frac{(1 - \alpha - g)f(K)}{R}. \tag{20}$$

This condition gives the combinations of K and R that are consistent with stationarity of human wealth and bond-market clearing.

In any steady state, the capital stock and the risk-free rate must jointly solve equations (19) and (20). In the Appendix we further show that a solution to this system exists and is unique. We thus reach the following result.

Proposition 3. *The steady state exists and is unique. The steady-state levels of the capital stock and the risk-free rate are given by the solution to the system of equations (19) and (20).*

To understand the determination of the steady state of our model and its relation to its complete-markets counterpart, note first that condition (18) imposes $\rho < \beta$. That is, the risk-adjusted return to saving must be *lower* than the discount rate. In particular, ρ must be low enough just to offset the precautionary motive for saving. If the risk-adjusted return were higher than this critical level, consumption (and wealth) would increase over time without bound, which would be a contradiction of steady state. Conversely, if the risk-adjusted return were lower than this level, consumption (and wealth) would shrink to zero, which would once again be a contradiction of steady state. Combining this with the fact that $R < \rho$, we infer that the risk-free rate is also lower than the discount rate: $R < \beta$. At the same time, because $\rho < f'(K) - \delta$, it is unclear whether the marginal product of capital is higher or lower than the discount rate. Using these observations, along with the fact that the complete-markets steady state features $f'(K) - \delta = R = \beta$, we conclude that incomplete markets necessarily reduce the risk-free rate but can have an ambiguous effect on the capital stock. In simple words, the precautionary motive guarantees that the interest rate is lower under incomplete markets than under complete markets, but this does not necessarily translate to a higher capital stock, because investment risk introduces a wedge
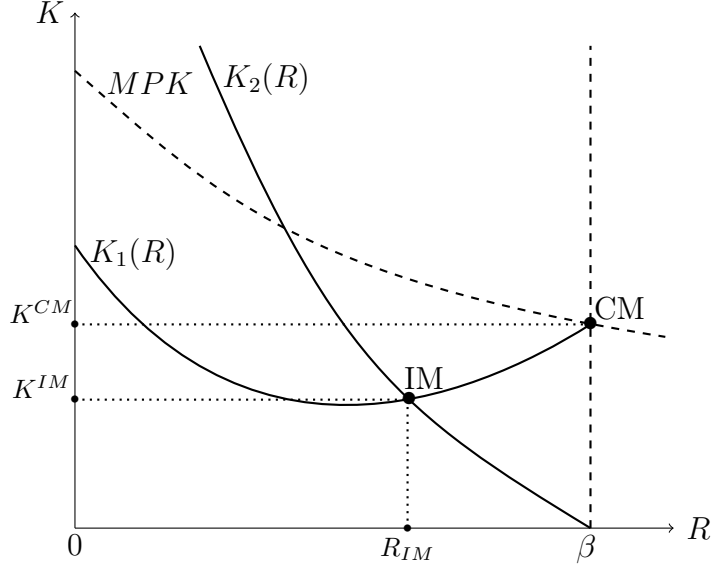
Figure 1. A graphical representation of the steady state.

between the marginal product of capital and the interest rate.

A graphical representation of the steady state helps appreciate further this tension between the precautionary motive and the risk premium in our model (and will also facilitate the comparative statics of the steady state). Let $K_1(R)$ and $K_2(R)$ denote the functions defined by solving, respectively, equations (19) and (20) for K as functions of R. We discuss the properties of these functions in what follows and illustrate them in Figure 1.[8]

Consider first the curve $K_1(R)$. When $\sigma = 0$, condition (19) reduces to $f'(K) - \delta = R$. The complete-markets counterpart of $K_1(R)$ is therefore given by a standard curve for the marginal product of capital, represented by curve MPK in Figure 1. The positive risk premium introduced on investment when $\sigma > 0$ implies that curve $K_1(R)$ lies uniformly below curve MPK. Indeed, the distance between the two curves measures the risk premium, as captured by the right-hand side of (19). Clearly, the latter is decreasing in R: the higher the risk-free rate, the lower the risk premium in steady state. To understand the intuition behind this property, take for a moment the interest rate to be exogenously given. Then, an increase

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[8]For a formal derivation of all the properties discussed here, see Lemma 1 in the Appendix.

in R would lead to an increase in the steady-state level of wealth. Because of diminishing absolute risk aversion, the increase in wealth would stimulate capital accumulation. However, because of diminishing returns to capital accumulation, the ratio of capital to wealth, i.e. the fraction ϕ, would fall. But then the risk premium, which is given by $\frac{1}{2}\gamma\phi^2\sigma^2$, would also fall. And because wealth explodes as $R \to \beta$, while K remains bounded, it follows that the risk premium must vanish as $R \to \beta$. These observations explain why the distance between the two curves indeed falls monotonically with R, and vanishes as $R \to \beta$.

To recap, there are two important economic effects behind curve $K_1(R)$. On the one hand, a higher R raises the opportunity cost of capital. This effect, which is present under both complete and incomplete markets, tends to discourage investment. On the other hand, a higher R is possible in steady state under incomplete markets only if aggregate wealth is higher in that steady state. This wealth effect, which is present only under incomplete markets, tends to encourage investment. Moreover, from condition (19) it is immediate that $K_1(R)$ is U-shaped, as illustrated in Figure 1. Therefore, the opportunity-cost effect must be dominating for low R, while the wealth effect must be dominating for high R.[9]

Let's now turn to the curve $K_2(R)$. The complete-markets counterpart of $K_2(R)$ is the vertical line at $R = \beta$: as $\sigma \to 0$, $K_2(R)$ converges to this vertical line, whereas for any $\sigma > 0$, $K_2(R)$ lies to the left of this vertical line. In Lemma 1 in the Appendix we show that $K_2(R)$ is monotonically decreasing in R, with $K_2(R) \to +\infty$ as $R \to 0$ and $K_2(R) \to 0$ as $R \to \beta$.[10] The intuition for the monotonicity of $K_2(R)$ is simple. For given K, and hence given ω, an increase in R reduces both H and $\phi(K, R)$, and thereby necessarily reduces the right-hand side of (20). But then, for (20) to hold with the lower R, it must be that K also falls, which explains why $K_2(R)$ is decreasing.

Since $K_1(R)$ and $K_2(R)$ are continuous in R, and using their limiting properties from above, it is then clear that the two curves intersect at least once at some $R \in (0, \beta)$. But, as already mentioned, we further show in the Appendix that this intersection is in fact unique.

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[9]We verify these properties in Lemma 1 in the Appendix, where we further show that the relative strength of these two effects is such that $\partial K_1/\partial R > 0$ if and only if $\theta > \phi/(1 - \phi)$. This property will also turn out to be important in the next section, where we analyze the steady-state effects of government spending.

[10]Simulations suggest that the curve is also convex, but we have not been able to prove this.

The incomplete-markets steady state of our model is thus represented by point IM in Figure 1, while its complete-markets counterpart is represented by point CM. For the particular economy we have considered in this figure, the steady-state capital stock is lower under incomplete markets than under complete markets. However, the opposite could also be true. Clearly, a sufficient condition for the steady-state capital stock to be lower than under complete markets is that the two curves intersect on the upward portion of $K_1(R)$, or that the wealth effect on investment due to risk aversion dominates the usual opportunity cost effect. The following proposition identifies a condition that is both necessary and sufficient for the capital stock to be lower than under complete markets.

Proposition 4. *The steady-state level of capital is lower under incomplete markets ($\sigma > 0$) than under complete markets ($\sigma = 0$) if and only if $\theta > \frac{\phi}{2-\phi}$.*

This result, which was first reported in Angeletos (2007), highlights how augmenting the neoclassical growth model for idiosyncratic capital-income risk can lead to lower aggregate saving, and thereby to lower aggregate output and consumption than under complete markets. This result stands in contrast to Aiyagari (1994), which documents how labor-income risk raises aggregate saving.[11] We refer the interested reader to that earlier work for a more extensive discussion and quantification of this result. In the remainder of our paper, we focus on the effects of government spending, which is our main question of interest.

5 The long-run effects of government consumption

In this section we study how the steady state changes when the rate of government consumption increases. The analysis will make clear that the different impact that government spending has in our model as compared to the standard paradigm originates precisely from the wealth effects that idiosyncratic risk introduces in the demand for investment.

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[11]In Aiyagari (1994), a precautionary motive implies $R < \beta$, but the absence of investment risk maintains $R = f'(K) - \delta$, from which it is immediate that the capital stock is necessarily higher under incomplete markets.
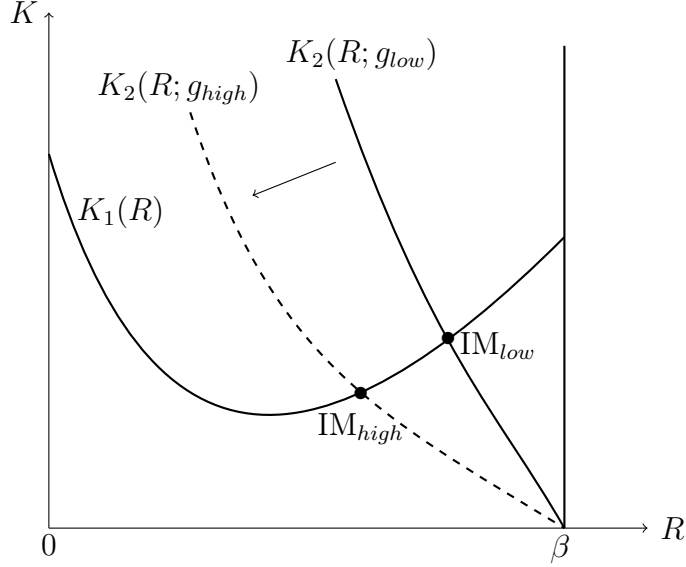
Figure 2. The steady-state effects of government consumption.

5.1 Characterization

To study the long-run effects of an increase in the level of government consumption, we again use a graphical representation of the steady state, namely Figure 2, which is a variant of Figure 1.

Let the initial level of government spending be $g = g_{low}$ and suppose that the corresponding steady state is given by point IM_{low} in Figure 2. Subsequently, let government spending increase to $g = g_{high} > g_{low}$. Note that condition (19) does not depend on g, and hence an increase in government consumption does not affect the $K_1(R)$ curve. Rather, it is condition (20), and the $K_2(R)$ curve, that depend on g. In particular, because a higher g means lower net-of-taxes labor income, and hence a lower H in steady state for any given R, an increase in government consumption causes the $K_2(R)$ curve to shift leftwards, as illustrated in Figure 2. This leftward shift is a manifestation of the negative wealth effect of higher lump-sum taxes on investment.

The new steady state is then represented by point IM_{high}. Clearly, the leftward shift in the $K_2(R)$ curve leads unambiguously to a decrease in R. The impact on K, on the other

hand, is ambiguous. This is because, as explained in the previous section, a reduction in R entails two opposing effects on the demand for investment: the familiar opportunity-cost channel tends to encourage investment, while the novel wealth channel of our model tends to discourage investment. As evident from the figure, if the two curves intersect on the upward portion of the $K_1(R)$ curve, that is, in the portion where the wealth effect dominates, then the increase in g leads to a reduction in K. In the Appendix we show that the intersection occurs in the upward portion of the $K_1(R)$ curve if and only if $\theta > \frac{\phi}{1-\phi}$. Finally, it is easy to check that $\phi < \alpha$.[12] We thus reach the the following result.

Proposition 5. *In steady state, an increase in government consumption (g) necessarily decreases the risk-free rate (R), while it locally decreases the capital-labor ratio (K/N), labor productivity (Y/N), the wage rate (ω), and the saving rate $(s \equiv \delta K/Y)$ if and only if $\theta > \frac{\phi}{1-\phi}$. A sufficient condition for the latter is that $\theta > \frac{\alpha}{1-\alpha}$.*

This is the key theoretical result of our paper. It establishes that, as long as the EIS is sufficiently high relative to the income share of capital, a permanent increase in the rate of government consumption has a negative long-run effect on both the interest rate and the capital intensity of the economy.

It is important to appreciate how this result deviates from the standard neoclassical paradigm. With complete markets, in steady state the interest rate is equal to the discount rate $(R = \beta)$, and the capital-labor ratio is determined by the equality of the marginal product of capital to the discount rate $(f'(K/N) - \delta = \beta)$. It follows that, in the long run, government consumption has no effect on either R or K/N, Y/N, ω, and s. This is true even when labor supply, N, is endogenous.[13] The only difference is that, with endogenous labor supply, N changes with g. In particular, when labor supply is fixed, the increase in government consumption simply leads to a one-to-one decrease in private consumption. When instead labor supply is elastic, the increase in government consumption has a negative

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[12]To see this, note from condition (20) that $\frac{\phi}{1-\phi} = \frac{RK}{(1-\alpha-g)f(K)}$. Combining this with the fact that $R < f'(K) - \delta < \alpha f(K)/K$, we get $\frac{\phi}{1-\phi} < \frac{\alpha}{1-\alpha-g} < \frac{\alpha}{1-\alpha}$ and hence $\phi < \alpha$.

[13]We endogenize labor in Section 6.

wealth effect, inducing agents to work more. The capital stock then increases one-to-one with labor supply, so as to keep the the capital-labor ratio and the interest rate invariant with g.

In our model, instead, government consumption has non-trivial long-run effects on both the interest rate and the capital intensity of the economy. Building on the earlier discussions, we can now summarize the key mechanism in our model as follows. Because households face consumption risk, they have a precautionary motive to save. Because preferences exhibit diminishing absolute risk aversion, this motive is stronger when the level of wealth is lower. It follows that, by reducing household wealth, higher government spending stimulates precautionary saving. But then, the risk-free rate at which aggregate saving can be stationary has to be lower, which explains why the risk-free rate R falls with g. At the same time, because of diminishing absolute risk aversion, the reduction in wealth tends to discourage the demand for investment. Provided that the positive effect of the lower opportunity cost of investment is not strong enough to offset this negative wealth effect, the capital-labor ratio K/N also falls with g.

5.2   Calibration and numerical simulation

For empirically plausible calibrations of the model, the critical condition $\theta > \frac{\phi}{1-\phi}$ appears to be satisfied quite easily. For example, take the interest rate to be $R = 4\%$ and labor income to be 65% of GDP (as in US data). This implies that H is about 16 times GDP. With a capital-output ratio of 4 (again as in US data), this translates to an H of about 4 times K. Since in steady state $\frac{\phi}{1-\phi} = \frac{K}{H}$, this exercise gives a calibrated value for $\frac{\phi}{1-\phi}$ of about 0.25. This critical value is lower than most of the recent empirical estimates of the elasticity of intertemporal substitution, which are in most cases above 0.5 and often even above 1.[14] Hence, a negative long-run effect of government consumption on aggregate saving and productivity appears to be the most likely scenario.

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[14] See, for example, Vissing-Jørgensen and Attanasio (2003), Mulligan (2002), and Gruber (2005). See also Guvenen (2006) and Angeletos (2007) for related discussions on the parametrization of the EIS.

In the remainder of this section, we make a first pass at the potential quantitative importance of our results within the context of our baseline model. In the next section we then turn to an enriched version of the model that allows for endogenous labor supply, as well as a certain type of agent heterogeneity.

The economy is fully parameterized by $(\alpha, \beta, \gamma, \delta, \theta, \sigma, g)$, where α is the income share of capital, β is the discount rate, γ is the coefficient of relative risk aversion, δ is the (mean) depreciation rate, θ is the elasticity of intertemporal substitution, σ is the standard deviation of the rate of return on private investment, and g is the share of government consumption in aggregate output.

In our baseline parametrization, we take $\alpha = 0.36$, $\beta = 0.042$, and $\delta = 0.08$; these values are standard in the literature. For risk aversion, we take $\gamma = 5$, a value commonly used in the macro-finance literature to help generate plausible risk premia. For the elasticity of intertemporal substitution, we take $\theta = 1$, a value consistent with recent micro and macro estimates.[15] For the share of government, our baseline value is $g = 25\%$ (as in the United States) and a higher alternative is $g = 40\%$ (as in some European countries).

What remains is σ. Unfortunately, there is no direct measure of the rate-of-return risk faced by the "typical" investor in the US economy. However, there are various indications that investment risks are significant. For instance, the probability that a privately held firm survives five years after entry is less than 40%. Furthermore, even conditional on survival, the risks faced by entrepreneurs and private investors appear to be very large: as Moskowitz and Vissing-Jørgensen (2002) document, not only is there a dramatic cross-sectional variation in the returns to private equity, but also the volatility of the book value of a (value-weighted) index of private firms is twice as large as that of the index of public firms—one more indication that private equity is more risky than public equity. Note then that the standard deviation of annual returns is about 15% per annum for the entire pool of public firms; it is over 50% for a single public firm (which gives a measure of firm-specific risk); and it is about 40% for a portfolio of the smallest public firms (which are likely to be

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[15]See the references in footnote 14.

similar to large private firms).

Given this suggestive evidence, and lacking any better alternative, we let $\sigma = 30\%$ for our baseline parameterization and consider $\sigma = 20\%$ and $\sigma = 40\%$ for sensitivity analysis. Although these numbers are somewhat arbitrary, it is reassuring that the volatility of individual consumption generated by our model is comparable to its empirical counterpart. For instance, using the Consumer Expenditure Survey (CEX), Malloy, Moskowitz and Vissing-Jørgensen (2006) estimate the standard deviation of consumption growth to be about 8% for stockholders (and about 3% for non-stockholders). Similarly, using data that include consumption of luxury goods, Aït-Sahalia, Parker and Yogo (2001) get estimates between 6% and 15%. In our simulations, on the other hand, the standard deviation of individual consumption growth is less than 5% per annum (along the steady state).

Putting aside these qualifications about the parametrization of σ, we now examine the quantitative effects of government consumption on the steady state of the economy. Table 1 reports the per-cent reduction in the steady-state values of the capital-labor ratio (K/N), labor productivity (Y/N), and the saving rate (s), relative to what their values would have been if g were 0.[16] Complete markets are indicated by CM and incomplete markets by IM.

	K/N		Y/N		s		τ^k_{equiv}
	CM	IM	CM	IM	CM	IM	CM
baseline	**0**	**−10.02**	**0**	**−3.73**	**0**	**−1.14**	**17**
$\sigma = 40\%$	0	−12.18	0	−4.57	0	−1.21	20
$\sigma = 20\%$	0	−6.78	0	−2.5	0	−0.88	12
$g = 40\%$	0	−17.82	0	−6.82	0	−2.05	28

Table 1. The steady-state effects of government consumption .

In our baseline parametrization, the capital-labor ratio is about 10% lower when $g = 25\%$ than when $g = 0$. Similarly, productivity is about 4% lower and the saving rate is about 1 percentage point lower. These are significant effects. They are larger (in absolute value) than the steady-state effects of precautionary saving reported in Aiyagari (1994). They are

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[16]Here, since labor supply is exogenously fixed, the changes in K and Y coincide with those in K/N and Y/N; this is not the case in the extensions with endogenous labor supply in the next section.

equivalent to what would be the steady-state effects of a marginal tax on capital income equal to 17% in the complete-markets case. (The tax rate on capital income that would generate the same effects under complete markets is given in the last column of the table, as τ_{equiv}^k.)

Not surprisingly, the effects are smaller if σ is lower (third row) or if γ is lower (not reported), because then risk matters less. On the other hand, the effects are larger when $g = 40\%$ (final row): productivity is almost 18% lower, the saving rate is 2 percentage points lower, and the tax on capital income that would have generated the same effects under complete markets is 28%.

Table 2 turns from level to marginal effects: it reports the change in K/N, Y/N, and s as we increase government spending by 1 percent, either from 25% to 26%, or from 40% to 41%. In the first case, productivity falls by 0.19%; in the second, by 0.26%. This is equivalent to what would have been, under complete markets, the effect of increasing the tax rate on capital income by about 0.75 percentage points in the first case, and about 0.8 percentage points in the second case.

	K/N		Y/N		τ_{equiv}^k
	CM	IM	CM	IM	CM
$g = 25\% \rightarrow 26\%$	0	-0.52	0	-0.19	0.75
$g = 40\% \rightarrow 41\%$	0	-0.71	0	-0.26	0.8

Table 2. Long-run effects of a permanent 1% increase in government consumption.

6   Endogenous labor

In this section we endogenize labor supply in the economy. We consider three alternative specifications that achieve this goal without compromising the tractability of the model.

6.1 GHH preferences

One easy way to accommodate endogenous labor supply in the model is to assume preferences that rule out income effects on labor supply, as in Greenwood, Hercowitz and Huffman (1998). In particular, suppose that preferences are given by $U_0 = \mathbb{E}_0 \int_0^\infty e^{-\beta t} u(c_t, l_t) dt$, with

$$u(c_t, l_t) = \tfrac{1}{1-\gamma} [c_t + v(l_t)]^{1-\gamma}, \tag{21}$$

where l_t denotes leisure and v is a strictly concave, strictly increasing function.[17] The analysis can then proceed as in the benchmark model, with labor supply in period t given by $N_t = 1 - l(\omega_t)$, where $l(\omega) \equiv \arg\max_l \{v(l) - \omega l\}$.

This specification highlights an important difference between complete and incomplete markets with regard to the employment impact of fiscal shocks. Under incomplete markets, an increase in government spending can have a *negative* general-equilibrium effect on aggregate employment. This is never possible with complete markets, but it is possible with incomplete markets when an increase in g reduces the capital-labor ratio, and thereby the wage rate, which in turn discourages labor supply. Indeed, with GHH preferences, $\theta > \frac{\phi}{1-\phi}$ suffices for both K/N and N to fall with g in both the short run and the long run.

Although it is unlikely that wealth effects on labor supply are zero in the long run, they may well be very weak in the short run. In light of our results, one may then expect that after a positive shock to government consumption both employment and investment could drop on impact under incomplete markets. Indeed, an interesting extension would be to consider a preference specification that allows for weak short-run but strong long-run wealth effects on labor supply, as in Jaimovich and Rebelo (2006).

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[17]To allow for $\theta \neq 1/\gamma$, we let $U_t = \mathbb{E}_t \int_t^\infty z(c_\tau + v(l_\tau), U_\tau) d\tau$, with the function z defined as in condition (5).

6.2 KPR preferences

A second tractable way to accommodate endogenous labor supply is to assume that agents have homothetic preferences over consumption and leisure, as in King, Plosser, and Rebelo (1988). The specification assumed in that paper is $U_0 = \mathbb{E}_0 \int e^{-\beta t} u(c_t, l_t) dt$, with

$$u(c_t, l_t) = \frac{\left(c_t^{1-\psi} l_t^{\psi}\right)^{1-\gamma}}{1-\gamma}, \tag{22}$$

where l_t denotes leisure and $\psi \in (0,1)$ is a scalar. This specification imposes expected utility $(\theta = \gamma)$. To allow for $\theta \neq 1/\gamma$, we let $U_t = \mathbb{E}_t \int_t^{\infty} z(c_\tau^{\psi} l_\tau^{1-\psi}, U_\tau) d\tau$, with z defined as in (5).

The benefit of this specification is that it is standard in the literature (making our results comparable to previously reported results), while it also comes with zero cost in tractability.[18] The homotheticity of the household's optimization problem is then preserved and the equilibrium analysis proceeds in a similar fashion as in the benchmark model.[19] The only essential novelty is that aggregate employment is now given by $N_t = 1 - L(\omega_t, C_t)$, where

$$L(\omega_t, C_t) = \frac{\psi}{1-\psi} \frac{C_t}{\omega_t}.$$

The neoclassical effect of wealth on labor supply is then captured by the negative relationship between N_t and C_t (for given ω_t).

For the quantitative version of this economy, we take $\psi = 0.75$. This value, which is in line with King, Plosser, and Rebelo (1988) and Christiano and Eichenbaum (1992), ensures that the steady-state fraction of available time worked approximately matches US data. The rest of the parameters are as in the baseline specification of the benchmark model.

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[18]For convenience, we allow agents to trade leisure with one another, so that an individual agent can possibly consume more leisure than her own endowment of time.

[19]The proofs are available upon request.

6.3 Hand-to-mouth workers

A third approach is to split the population into two groups. The first group consists of the households that have been modeled in the benchmark model; we will call this group the "investors". The second group consists of households that supply labor but do not hold any assets, and simply consume their entire labor income at each point in time; we will call this group the "hand-to-mouth workers". Their labor supply is given by

$$N_t^{htm} = \omega_t^{\epsilon_\omega}(C_t^{htm})^{\epsilon_c}, \tag{23}$$

where C_t^{htm} denotes the consumption of these agents, $\epsilon_\omega > 0$ parameterizes the wage elasticity of labor supply, and $\epsilon_c > 0$ parameterizes the wealth elasticity.[20]

This approach could be justified on its own merit. In the United States, a significant fraction of the population holds no assets, has limited ability to borrow, and sees its consumption tracking its income almost one-to-one. This fact calls for a richer model of heterogeneity than our benchmark model. But is unclear what the "right" model for these households is. Our specification with hand-to-mouth workers is a crude way of capturing this form of heterogeneity in the model, while preserving tractability.

A side benefit of this approach is that it also gives freedom in parameterizing the wage and wealth elasticities of labor supply. Whereas the KPR preference specification imposes $\epsilon_\omega = -\epsilon_c = 1$, the specification introduced above permits us to pick much lower elasticities, consistent with micro evidence. The point is not to argue which parametrization of the labor-supply elasticities is more appropriate for quantitative exercises within the neoclassical growth model; this is the subject of a long debate in the literature, to which we have nothing to add. The point here is rather to cover a broader spectrum of empirically plausible quantitative results.

For the quantitative version of this economy, we thus take $\epsilon_\omega = 0.25$ and $\epsilon_c = -0.25$, which are in the middle of most micro estimates.[21] What then remains is the fraction of

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[20]Preferences that give rise to this labor supply are $u_t = c_t^{\zeta_c} - n_t^{\zeta_n}$, for appropriate ζ_c, ζ_n.

[21]See, for example, Hausman (1981), MaCurdy (1981), and Blundell and MaCurdy (1999).

aggregate income absorbed by hand-to-mouth workers. As mentioned above, a significant fraction of the US population holds no assets. For example, using data from both the PSID and the SCF, Guvenen (2006) reports that the lower 80% of the wealth distribution owns only 12% of aggregate wealth and accounts for about 70% of aggregate consumption. Since some households may be able to smooth consumption even when their net worth is zero, 70% is likely to be an upper bound for the fraction of aggregate consumption accounted for by hand-to-mouth agents. We thus opt to calibrate the economy so that hand-to-mouth agents account for 50% of aggregate consumption. This is also the value of the relevant parameter that one would estimate if the model were to match US aggregate consumption data—we can deduce this from Campbell and Mankiw (1989).[22]

6.4 The long-run effects of government consumption with endogenous labor

Our main theoretical result (Proposition 5) continues to hold in all of the above variants of the benchmark model: in steady state, a higher rate g of government consumption necessarily reduces the interest rate R; and it also reduces the capital-labor ratio K/N, labor productivity Y/N, and the wage rate ω if and only if the elasticity of intertemporal substitution θ is higher than $\frac{\phi}{1-\phi}$.[23]

What is not clear anymore is the effect of g on K and Y, because now N is not fixed. On the one hand, the reduction in wealth stimulates labor supply, thus contributing to an increase in N. This is the familiar neoclassical effect of government spending on labor supply. On the other hand, as long as $\theta > \frac{\phi}{1-\phi}$, the reduction in capital intensity depresses real wages, contributing towards a reduction in N. This is the novel general-equilibrium effect due to

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[22]Note that the specification of aggregate consumption considered in Campbell and Mankiw coincides with the one implied by our model. Therefore, if one were to run their regression on data generated by our model, one would correctly identify the fraction of aggregate consumption accounted for by hand-to-mouth workers in our model. This implies that it is indeed appropriate to calibrate our model's relevant parameter to Campbell and Mankiw's estimate.

[23]This is true as long as the steady state is unique, which seems to be the case but has not been proved as in the benchmark model. Also, in the variant with hand-to-mouth agents, we have to be cautious to interpret ϕ as the ratio of private equity to effective wealth for the investor population alone.

incomplete markets. The overall effect of government spending on aggregate employment is therefore ambiguous under incomplete markets, whereas it is unambiguously positive under complete markets.

Other things equal, we expect the negative general-equilibrium effect to dominate, thus leading to a reduction in long-run employment after a permanent increase in government spending, if the wage elasticity of labor supply is sufficiently high relative to its income elasticity. This is clear in the GHH specification, where the wealth effect is zero. It can also be verified for the case of hand-to-mouth workers, where we have freedom in choosing these elasticities, but not in the case of KPR preferences, where both elasticities are restricted to equal one.

Given these theoretical ambiguities, we now seek to get a sense of empirically plausible quantitative effects. As already discussed, the GHH case (zero wealth effects on labor supply) is merely of pedagogical value. We thus focus on the parameterized versions of the other two cases, the economy with KPR (homothetic) preferences and the economy with hand-to-mouth workers.

Table 3 then presents the marginal effects on the steady-state levels of the capital-labor ratio, productivity, employment, and output for each of these two economies, as g increases from 25% to 26%, or from 40% to 41%.[24] The case of KPR preferences is indicated by KPR, while the case with hand-to-mouth workers is indicated by HTM. In either case, complete markets are indicated by CM and incomplete markets by IM.

		K/N		Y/N		N		Y		τ^k_{equiv}
		CM	IM	CM	IM	CM	IM	CM	IM	CM
$g = 25\% \to 26\%$	KPR	0	-0.33	0	-0.12	1.4	1.27	1.4	1.15	0.52
	HTM	0	-0.3	0	-0.11	0.38	0.38	0.38	0.27	0.46
$g = 40\% \to 41\%$	KPR	0	-0.52	0	-0.19	1.76	1.53	1.76	1.34	0.68
	HTM	0	-0.36	0	-0.13	0.57	0.57	0.57	0.44	0.48

Table 3. Long-run effects with endogenous labor.

Regardless of specification, the marginal effects of higher government spending on capital

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[24]We henceforth focus on marginal rather than level effects just to economize on space.

intensity K/N and labor productivity Y/N are negative under incomplete markets (and are stronger the higher is g), whereas they are zero under complete markets. As for aggregate employment N, the wealth effect of higher g turns out to dominate the effect of lower wages under incomplete markets, so that N increases with higher g under either complete or incomplete markets. However, the employment stimulus is weaker under incomplete markets, especially in the economy with hand-to-mouth workers. The same is true for aggregate output: it increases under either incomplete or complete markets, but less so under incomplete markets. Finally, the incomplete-markets effects are on average equivalent to what would have been the effect of increasing the tax rate on capital income by about 0.55% under complete markets.

7 Dynamic responses

The results so far indicate that the long-run effects of government consumption can be significantly affected by incomplete risk sharing. We now examine how incomplete risk sharing affects the entire impulse response of the economy to a fiscal shock.[25]

Starting from the steady state with $g = 25\%$, we hit the economy with a permanent 1% increase in government spending and trace its transition to the new steady state (the one with $g = 26\%$). We conduct this experiment for both the economy with KPR preferences and the economy with hand-to-mouth workers, each parameterized as in the previous section; in either case, the transitional dynamics reduce to a simple system of two first-order ODE's in (K_t, H_t) when $\theta = 1$.[26]

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[25]Note that the purpose of the quantitative exercises conducted here, and throughout the paper, is not to assess the ability of the model to match the data. Rather, the purpose is to detect the potential quantitative significance of the particular deviation we took from the standard neoclassical growth model.

[26]Throughout, we focus on permanent shocks. Clearly, transitory shocks have no impact in the long run. As for their short-run impact, the difference between complete and incomplete markets is much smaller than in the case of permanent shocks. This is simply because transitory shocks have very weak wealth effects on investment as long as agents can freely borrow and lend over time, which is the case in our model. We expect the difference between complete and incomplete markets to be larger once borrowing constraints are added to the model, for then investment will be sensitive to changes in current disposable income even if there is no change in present-value wealth.

The results are presented in Figures 3 and 4. Time in years is on the horizontal axis, while deviations of the macro variables from their respective initial values are on the vertical axis. The interest rate and the investment rate are in simple differences, the rest of the variables are in log differences. The solid lines indicate incomplete markets, the dashed lines indicate complete markets.
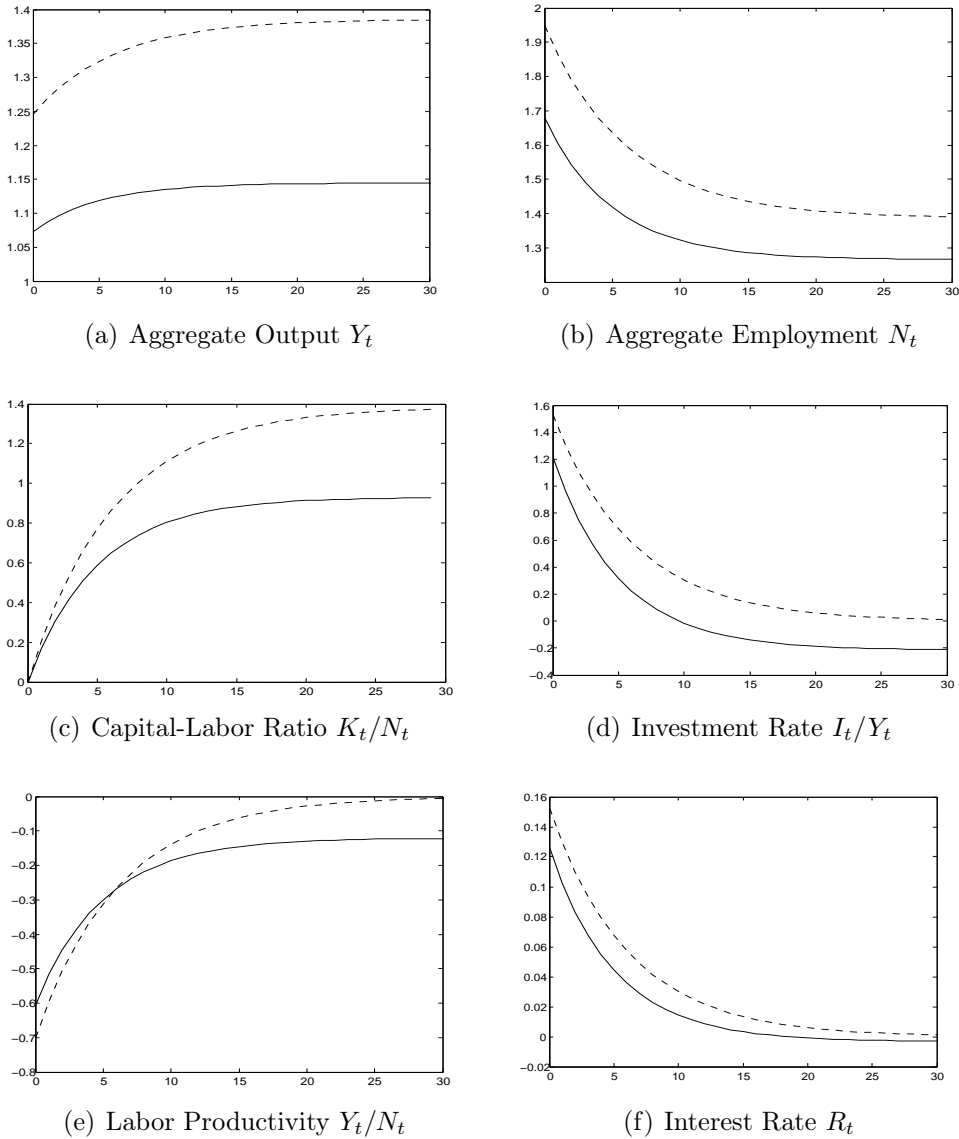

(a) Aggregate Output Y_t

(b) Aggregate Employment N_t

(c) Capital-Labor Ratio K_t/N_t

(d) Investment Rate I_t/Y_t

(e) Labor Productivity Y_t/N_t

(f) Interest Rate R_t

Figure 3. Dynamic responses to a permanent shock with KPR preferences.

(g) Aggregate Output Y_t

(h) Aggregate Employment N_t

(i) Capital-Labor Ratio K_t/N_t

(j) Investment Rate I_t/Y_t

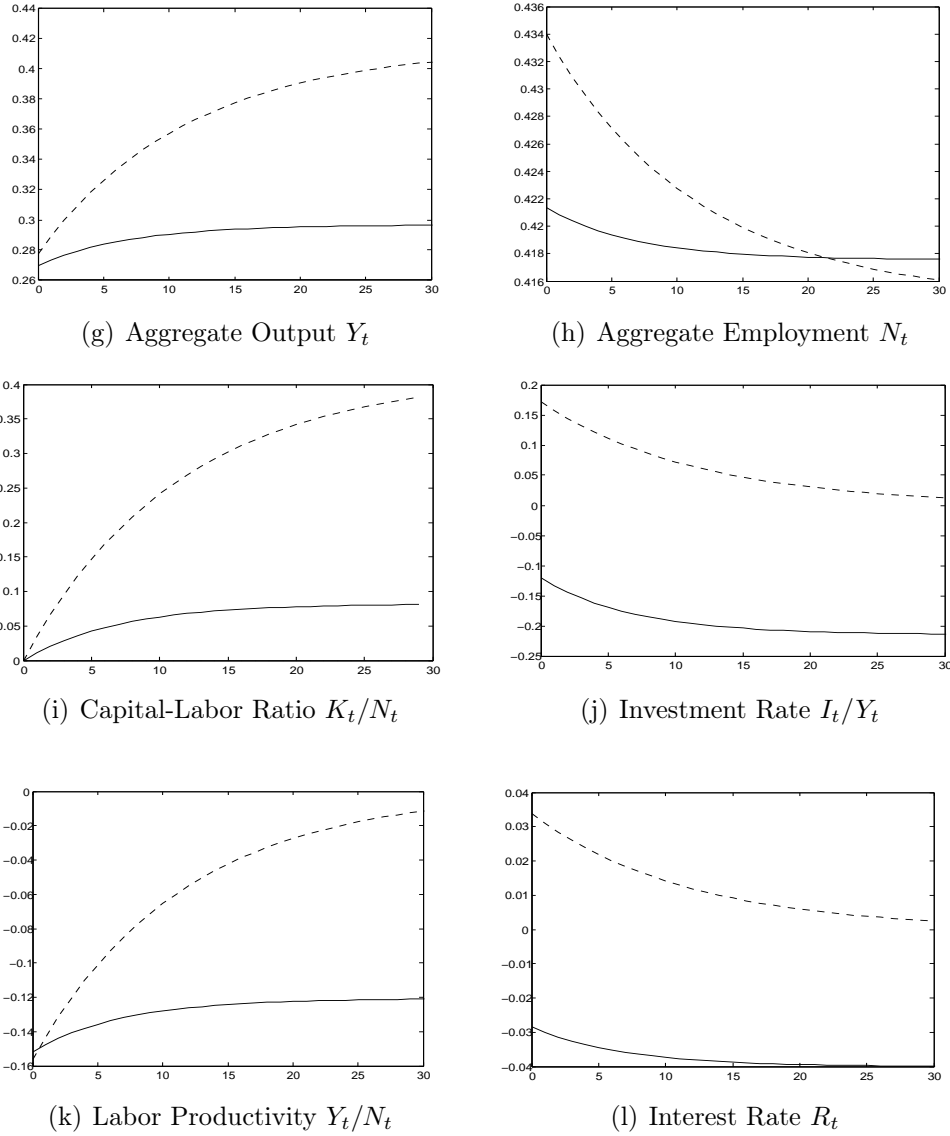(k) Labor Productivity Y_t/N_t

(l) Interest Rate R_t

Figure 4. Dynamic responses to a permanent shock with hand-to-mouth agents.

As evident in these figures, the quantitative effects of a permanent fiscal shock can be quite different between complete and incomplete markets. The overall picture that emerges is that the employment and output stimulus of a permanent increase in government spending is weaker under incomplete markets than under complete markets. And whereas we already knew this for the long-run response of the economy, now we see that the same is true for its short-run response.

This picture holds for both the economy with KPR preferences and the one with hand-to-mouth workers. But there are also some interesting differences between the two. The mitigating effect of incomplete markets on the employment and output stimulus of government spending is much stronger in the economy with hand-to-mouth workers. As a result, whereas the short-run effects of higher government spending on the investment rate and the interest are positive under complete markets in both economies, and whereas these effects remain positive under incomplete markets in the economy with KPR preferences, they turn negative under incomplete markets in the economy with hand-to-mouth workers.

To understand this result, consider for a moment the benchmark model, where there are no hand-to-mouth workers and labor supply is completely inelastic. Under complete markets, a permanent change in government spending would be absorbed one-to-one in private consumption, leaving investment and interest rates completely unaffected in both the short- and the long-run. Under incomplete markets, instead, investment and the interest rate would fall on impact, as well as in the long run. Allowing labor supply to increase in response to the fiscal shock ensures that investment and the interest rate jump upwards under complete markets. However, as long as the response of labor supply is weak enough, the response of investment and the interest rate can remain negative under incomplete markets.

As a final point of interest, we calculate the welfare cost, in terms of consumption equivalent, associated with a permanent 1% increase in government spending. Under complete markets, welfare drops by 0.2%, whereas under incomplete markets it drops by 0.6%. In other words, the welfare cost of an increase in government spending is three times higher under incomplete markets than under complete markets.[27]

To recap, the quantitative results indicate that a modest level of idiosyncratic investment risk can have a non-trivial impact on previously reported quantitative evaluations of fiscal policy. Note in particular that our quantitative economy with KPR preferences is directly comparable to two classics in the related literature, Aiyagari, Christiano and Eichenbaum (1992) and Baxter and King (1993). Therefore, further investigating the macroeconomic

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[27]Here we have assumed that government consumption has no welfare benefit, but this should not be taken literally: nothing changes if G_t enters separably in the utility of agents.

effects of fiscal shocks in richer quantitative models with financial frictions appears to be a promising direction for future research.

8   Conclusion

This paper revisited the macroeconomic effects of government consumption within a tractable incomplete-markets variant of the neoclassical growth model. Because private investment is subject to uninsurable idiosyncratic risk and because risk-taking is sensitive to wealth, the aggregate level of investment depends on the aggregate level of net-of-taxes household wealth for any given prices. It follows that an increase in government spending can crowd-out private investment simply by reducing household net worth. As a result, market incompleteness can seriously upset the supply-side effects of fiscal shocks: an increase in government consumption, even if financed with lump-sum taxation, tends to reduce capital intensity, labor productivity, and wages in both the short-run and the long-run. For plausible parameterizations of the model, these results appear to have not only qualitative, but also quantitative content.

These results might, or might not, be bad news for the ability of the neoclassical paradigm to explain the available evidence regarding the macroeconomic effects of fiscal shocks.[28] However, the goal of this paper was not to study whether our model could match the data. Rather, the goal was to identify an important mechanism through which incomplete markets modify the response of the economy to fiscal shocks: wealth effects on investment.

In our model, these wealth effects originated from uninsured idiosyncratic investment risk combined with diminishing absolute risk aversion. Borrowing constraints could lead to similar sensitivity of investment to wealth (or cash flow).[29] Also, this mechanism need not depend on whether prices are flexible (as in the neoclassical paradigm) or sticky (as in the

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[28]Whether the evidence is consistent with the neoclassical paradigm is still debatable. For example, using structural VARs with different identification assumptions, Ramey and Shapiro (1997) and Ramey (2006) find that private consumption falls in response to a positive shock to government consumption, as predicted by the neoclassical paradigm, while Blanchard and Perotti (2002) and Perotti (2007) find the opposite result.

[29]On this point, see Challe and Ragot (2007).

Keynesian paradigm). The key insights of this paper are thus clearly more general than the specific model we employed—but the quantitative importance of these insights within richer models of the macroeconomy is, of course, a widely open question.

An important aspect left outside our analysis is the optimal financing of government expenditures. In this paper, we assumed that the increase in government spending is financed with lump-sum taxation, only because we wished to isolate wealth effects from the distortionary and redistributive effects of taxation. Suppose, however, that the government has access to two tax instruments, a lump-sun tax and a proportional income tax.[30] Clearly, with complete markets (and no inequality) it would be optimal to finance any exogenous increase in government spending with only lump-sum taxes. With incomplete markets, however, it is likely that an increase in government spending is financed with a mixture of both instruments: while using only the lump-sum tax would disproportionately affect the utility of poor agents, using both instruments permits the government to trade off less efficiency for more equality. Further exploring these issues, and the nature of optimal taxation for the class of economies we have studied here, is left for future research.

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[30]As in Werning (2007), this might be a good proxy for more general non-linear tax schemes.

References

[1] Acemoglu, D., Zilibotti, F., 1997. Was Prometheus Unbound by Chance? Risk, Diversification, and Growth. Journal of Political Economy 105, 709-751.

[2] Aït-Sahalia, Y., Parker, J. A., Yogo, M., 2001. Luxury Goods and the Equity Premium. NBER Working Paper 8417.

[3] Aiyagari, S. R., 1994. Uninsured Idiosyncratic Risk and Aggregate Saving. Quarterly Journal of Economics 109, 659-684.

[4] Aiyagari, S. R., Christiano, L. J., Eichenbaum, M., 1992. The Output, Employment, and Interest Rate Effects of Government Consumption. Journal of Monetary Economics 30, 73-86.

[5] Angeletos, G.-M., 2007. Uninsured Idiosyncratic Investment Risk and Aggregate Saving. Review of Economic Dynamics 10, 1-30.

[6] Angeletos, G.-M., Calvet, L.-E., 2005. Incomplete-Market Dynamics in a Neoclassical Production Economy. Journal of Mathematical Economics 41, 407-438.

[7] Angeletos, G.-M., Calvet, L.-E., 2006. Idiosyncratic Production Risk, Growth, and the Business. Journal of Monetary Economics 53, 1095-1115.

[8] Barro, R. J., 1981. Output Effects of Government Purchases. Journal of Political Economy 89, 1086-1121.

[9] Barro, R. J., 1989. The Ricardian Approach to Budget Deficits. Journal of Economic Perspectives 3, 37-54.

[10] Baxter, M., King R. G., 1993. Fiscal Policy in General Equilibrium. American Economic Review 83, 315-334.

[11] Blanchard, O., Perotti, R., 1999. An Empirical Characterization of the Dynamic Effects of Changes in Government Spending and Taxes on Output. NBER Working Paper 7269.

[12] Blundell, R., MaCurdy T., 1999. Labour Supply: A Review of Alternative Approaches. In: Ashenfelter, O., Card, D. (Eds.), Handbook of Labor Economics. 3.

[13] Braun, A. R., McGrattan E. R., 1993. The Macroeconomics of War and Peace. NBER Macroeconomics Annual 1993.

[14] Buera, F., Shin, Y., 2007. Financial Frictions and the Persistence of History: a Quantitative Exploration. Northwestern University mimeo.

[15] Cagetti, M., De Nardi M., 2006. Entrepreneurship, Frictions, and Wealth. Journal of Political Economy 114, 835-870.

[16] Campbell, J., Mankiw, N. G., 1989. Permanent Income, Current Income, and Consumption. NBER Macroeconomics Annual 1989.

[17] Carroll, C., 2000. Portfolios of the Rich. NBER Working Paper 430.

[18] Challe, E., Ragot, X., 2007. The Dynamic Effects of Fiscal Shocks in a Liquidity-Constrained Economy. University of Paris-Dauphine/Paris School of Economics mimeo.

[19] Christiano, L. J., Eichenbaum, M., 1992. Current Real-Business-Cycle Theories and Aggregate Labor Market Fluctuations. American Economic Review 82, 430-450.

[20] Covas, F., 2006. Uninsured Idiosyncratic Production Risk with Borrowing Constraints. Journal of Economic Dynamics and Control 30, 2167-2190.

[21] Duffie, D., Epstein L. G., 1992. Stochastic Differential Utility. Econometrica 60, 353-394.

[22] Gentry, W. M., Hubbard, R. G., 2000. Entrepreneurship and Household Saving. NBER Working Paper 7894.

[23] Greenwood, J., Hercowitz, Z., Huffman, G. W., 1988. Investment, Capacity Utilization and the Real Business Cycle. American Economic Review 78, 402-417.

[24] Gruber, J., 2005. A Tax-Based Estimate of the Elasticity of Intertemporal Substitution. NBER Working Paper 11945.

[25] Guvenen, F., 2006. Reconciling Conflicting Evidence on the Elasticity of Intertemporal Substitution: A Macroeconomic Perspective. Journal of Monetary Economics 53, 1451-1472.

[26] Hall, R., 1980. Stabilization Policy and Capital Formation. American Economic Review 70, 156-63.

[27] Hausman, J., 1981. Labor Supply: How taxes affect Economic Behavior. In Aaron, H., Pechman, J. (Eds.), Tax and the Economy. Brookings Institute, Washington DC.

[28] Heathcote, J., 2005. Fiscal Policy with Heterogeneous Agents and Incomplete Markets. Review of Economic Studies 72, 161-188.

[29] Huggett, M., 1997. The One-Sector Growth Model With Idiosyncratic Shocks. Journal of Monetary Economics 39, 385-403.

[30] Jaimovich, N., Rebelo S., 2006. Can News About the Future Drive the Business Cycle?. American Economic Review, forthcoming.

[31] King, Robert G., Plosser, C., Rebelo, S., 1988. Production, Growth and Business Cycles: I. the Basic Neoclassical Model. Journal of Monetary Economics 21, 195-232.

[32] Krusell, P., Smith, A. A., 1998. Income and Wealth Heterogeneity in the Macroeconomy. Journal of Political Economy 106, 867-896.

[33] Leland, H., 1968. Saving and Uncertainty The Precautionary Demand for Saving. Quarterly Journal of Economics 82, 465-473.

[34] MaCurdy, T., 1981. An Empirical Model of Labor Supply in a Life-Cycle Setting. Journal of Political Economy 86, 1059-1085.

[35] Malloy, C. J., Moskowitz, T. J., Vissing-Jørgensen A., 2006. Long Run Stockholder Consumption Risk and Asset Returns. Mimeo.

[36] McGrattan, E., Ohanian L. E., 1999. The Macroeconomic Effects of Big Fiscal Shocks: The Case of World War II. Federal Reserve Bank of Minneapolis Working Paper 599.

[37] McGrattan, E., Ohanian, L. E., 2006. Does Neoclassical Theory Account for the Effects of Big Fiscal Shocks: Evidence from World War II. Federal Reserve Bank of Minneapolis Research Department Staff Report 315.

[38] Meh, C. A., Quadrini, V., 2006. Endogenous Market Incompleteness with Investment Risks. Journal of Economic Dynamics and Control 30, 2143-2165.

[39] Moskowitz, T. J., Vissing-Jørgensen, A., 2002. The Returns to Entrepreneurial Investment: A Private Equity Premium Puzzle?. American Economic Review 92, 745-778.

[40] Mulligan, C. B., 2002. Capital, Interest, and Aggregate Intertemporal Substitution. NBER Working Paper 9373.

[41] Obstfeld, M., 1994. Risk-Taking, Global Diversification, and Growth. American Economic Review 84, 1310-1329.

[42] Panousi, V., 2008. Capital Taxation with Entrepreneurial Risk. Federal Reserve Board mimeo.

[43] Panousi, V., Papanikolaou, D., 2008. Investment, Idiosyncratic Risk, and Ownership. Federal Reserve Board/Northwestern University mimeo.

[44] Perotti, R., 2007. In search of the transmission mechanism of fiscal policy. NBER Macroeconomics Annual 2007.

[45] Quadrini, V., 2000. Entrepreneurship, Saving, and Social Mobility. Review of Economic Dynamics 3, 1-40.

[46] Ramey, V. A., 2006. Identifying government spending shocks: It's all in the timing. UCSD mimeo.

[47] Ramey, V. A., Shapiro, M. D., 1997. Costly capital reallocation and the effects of government spending. NBER Working Paper 6283.

[48] Sandmo, A., 1970. The Effect of Uncertainty on Saving Decisions. Review of Economic Studies 37, 353-360.

[49] Vissing-Jørgensen, A., Attanasio, O., 2003. Stock-Market Participation, Intertemporal Substitution, and Risk Aversion. American Economic Review 93, 383-391.

[50] Werning, I., 2007. Optimal Fiscal Policy with Redistribution. Quarterly Journal of Economics 122, 925-967.

Appendix: Proofs

Proof of Proposition 1 (individual policy rules). Let $J(w,t)$ denote the value function for the household's problem. The value function depends on time t because of discounting as well as because the price sequence $\{\omega_t, R_t\}_{t \in [0,\infty)}$ need not be stationary. However, the value function does not depend on i, because households have identical preferences, they have access to the same technology, and they face the same sequence of prices and the same stochastic process for idiosyncratic risk. The Bellman equation that characterizes the value function is given by:

$$
0 = \max_{m,\phi} \left\{ z(mw, J(w,t)) + \frac{\partial J}{\partial t}(w,t) + \frac{\partial J}{\partial w}(w,t)[\phi\bar{r}_t + (1-\phi)R_t - m]w \right.
$$
$$
\left. + \frac{1}{2}\frac{\partial^2 J}{\partial w^2}(w,t)\phi^2 w^2 \sigma^2 \right\}. \tag{24}
$$

The first term of the Bellman equation (24) captures utility from current-period consumption; the second term takes care of discounting and the non-stationarity in prices; the third term captures the impact of the mean growth in wealth; and the last term (Itô's term) captures the impact of risk.

Because of the CRRA/CEIS specification of preferences, an educated guess is that there exists a deterministic process B_t such that:

$$
J(w,t) = B_t \frac{w^{1-\gamma}}{1-\gamma}. \tag{25}
$$

Because of the homogeneity of J in w, the Bellman equation then reduces to the following:

$$
0 = \max_{m,\phi} \left\{ z(m, J(1,t)) + \frac{\partial J}{\partial t}(1,t) + \frac{\partial J}{\partial w}(1,t)[\phi\bar{r}_t + (1-\phi)R_t - m] + \frac{1}{2}\frac{\partial^2 J}{\partial w^2}(1,t)\phi^2\sigma^2 \right\}. \tag{26}
$$

It follows that the optimal m and ϕ are independent of w. Using (5) and (25), the above becomes

$$
0 = \max_{m,\phi} \left\{ \frac{\beta}{1-1/\theta}[B_t^{\frac{1/\theta-1}{1-\gamma}} m^{1-1/\theta} - 1] + \frac{\dot{B}_t/B_t}{1-\gamma} + [\phi\bar{r}_t + (1-\phi)R_t - m] - \frac{1}{2}\gamma\phi^2\sigma^2 \right\}. \tag{27}
$$

The first order conditions for ϕ and m give the following:

$$\phi_t = \frac{\bar{r}_t - R_t}{\gamma \sigma^2}, \tag{28}$$

$$m_t = \beta^\theta B_t^{\frac{1-\theta}{1-\gamma}}. \tag{29}$$

Substituting this into (27), using the definition of $\hat{\rho}_t$, and rearranging, we get

$$0 = \frac{\beta^\theta B_t^{\frac{1-\theta}{1-\gamma}} - \theta\beta}{\theta - 1} + \frac{\dot{B}_t/B_t}{1 - \gamma} + \hat{\rho}_t.$$

This ODE, together with the relevant transversality condition, determines the process for B_t. Using (29), this is equivalent to the following:

$$\frac{\dot{m}_t}{m_t} = m_t + (\theta - 1)\hat{\rho}_t - \theta\beta,$$

which is the Euler condition (13).

Proof of Proposition 2 (equilibrium dynamics). Since aggregate labor demand is $\int_i n_t^i = \bar{n}(\omega_t)K_t$ and aggregate labor supply is 1, the labor market clears if and only if $\bar{n}(\omega_t)K_t = 1$. It follows that the equilibrium wage satisfies $\omega_t = F_L(K_t, 1)$ and, similarly, the equilibrium mean return to capital satisfies $\bar{r}_t = F_K(K_t, 1) - \delta$. The bond market, on the other hand, clears if and only if $0 = (1 - \phi_t)W_t + H_t$. Combining this with $K_t = \phi_t W_t$ gives condition (17).

Combining the intertemporal government budget with the definition of human wealth, we get

$$H_t = h_t = \int_t^\infty e^{-\int_t^s R_j dj} (\omega_s - G_s) ds. \tag{30}$$

Expressing this in recursive form gives condition (16).

Let $\bar{\rho}_t \equiv \phi_t \bar{r}_t + (1 - \phi_t)R_t$ denote the *mean* return to total saving. Aggregating the household budgets gives $\dot{W}_t = \bar{\rho}_t W_t - C_t$. Combining this with (16) and with $K_t + H_t = W_t$,

we get that $\dot{K}_t = \dot{W}_t - \dot{H}_t = (\bar{\rho}_t W_t - C_t) - (R_t H_t - \omega_t + G_t)$. Using $\bar{\rho}_t W_t = \bar{r}_t \phi_t W_t + R_t (1 - \phi_t) W_t = \bar{r}_t K_t + R_t H_t$, we get $\dot{K}_t = \bar{r}_t K_t + \omega_t - C_t - G_t$. Together with the fact, in equilibrium, $\bar{r}_t K_t + \omega_t = F(K_t, 1) - \delta K_t$, this gives condition (14), the resource constraint.

Finally, using $C_t = m_t W_t$, and therefore $\dot{C}_t / C_t = \dot{m}_t / m_t + \dot{W}_t / W_t$ together with $\dot{W}_t = \bar{\rho}_t W_t - C_t = (\bar{\rho}_t - m_t) W_t$ and (13), gives condition (15), the aggregate Euler condition.

Proof of Proposition 3 (steady state). First, we derive the two equations characterizing the steady state K and R. In steady state, the Euler condition gives

$$0 = \theta(\bar{\rho} - \beta) - (\theta - 1) \tfrac{1}{2} \gamma \sigma^2 \phi^2,$$

where
$$\bar{\rho} = R + \frac{[f'(K) - \delta - R]^2}{\gamma \sigma^2} \quad \text{and} \quad \phi = \frac{f'(K) - \delta - R}{\gamma \sigma^2}.$$

Combining and solving for $f'(K)$ gives condition (19). Condition (20), on the other hand, follows directly from (16) and (17).

Next, we prove existence and uniqueness of the steady state. Let $\mu(R)$ and $\phi(R)$ denote, respectively, the risk premium and the fraction of effective wealth held in capital, when K is given by (19):

$$\mu(R) \equiv \sqrt{\frac{2\theta \gamma \sigma^2}{1 + \theta}(\beta - R)} \quad \text{and} \quad \phi(R) \equiv \sqrt{\frac{2\theta}{\gamma \sigma^2 (1 + \theta)}(\beta - R)}.$$

Note that $\mu'(R) < 0$ and $\phi'(R) < 0$. Next, let $K(R)$ denote the solution to (19), or equivalently

$$K(R) = \left[\frac{\mu(R) + \delta + R}{\alpha}\right]^{\frac{1}{\alpha - 1}}. \tag{31}$$

Finally, let
$$D(R; g) \equiv (1 - \alpha - g) \frac{K(R)^{\alpha - 1}}{R} - \frac{1 - \phi(R)}{\phi(R)}. \tag{32}$$

Note that we have used $\omega = (1 - \alpha) Y$, $G = gY$, and $Y = f(K) = K^\alpha$, where $\alpha > 0, g \geq 0$, and $\alpha + g < 1$. To establish existence and uniqueness of the steady state , it suffices to show

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that there exists a unique R that solves $D(R; g) = 0$.

Fix g henceforth, and consider the limits of D as $R \to 0^+$ and $R \to \beta^-$. Note that $\mu(0) = \left(\frac{2\theta\gamma\sigma^2}{1+\theta}\beta\right)^{1/2}$ is finite and hence both $\phi(0)$ and $K(0)$ are finite. It follows that

$$\lim_{R \to 0^+} D(R; g) = (1 - \alpha - g)K(0)^{\alpha-1} \lim_{R \to 0^+} \frac{1}{R} - \frac{1}{\phi(0)} + 1 = +\infty.$$

Furthermore, $\mu(\beta) = 0$, implying $\phi(\beta) = 0$ and $K(\beta) = K_{compl} \equiv (f')^{-1}(\beta)$ is finite. It follows that

$$\lim_{R \to \beta^-} D(R; g) = (1 - \alpha - g)K(\beta)^{\alpha-1}\frac{1}{\beta} - \lim_{R \to \beta^-} \frac{1}{\phi(R)} + 1 = -\infty.$$

These properties, together with the continuity of $D(R)$ in R, ensure the existence of an $R \in (0, \beta)$ such that $D(R) = 0$.

If we now show that $D(R; g)$ is strictly decreasing in R, then we also have uniqueness. To show this, note that, from (32),

$$\frac{\partial D}{\partial R} = (1 - \alpha - g)\frac{K(R)^{\alpha-1}}{R^2}\left[(\alpha - 1)R\frac{K'(R)}{K(R)} - 1\right] + \frac{\phi'(R)}{\phi(R)^2}. \tag{33}$$

Now note that

$$K^{\alpha-1} = \frac{f'(K)}{\alpha}, \quad \frac{K'}{K} = \frac{1}{\alpha - 1}\frac{\mu' + 1}{f'(K)}, \quad \text{and} \quad \frac{\phi'}{\phi^2} = \frac{\gamma\sigma^2\mu'}{\mu^2},$$

where we suppress the dependence of K, μ, and ϕ on R for notational simplicity. It follows that

$$\begin{aligned}\frac{\partial D}{\partial R} &= \frac{1 - \alpha - g}{\alpha}\frac{f'(K)}{R^2}\left[R\frac{\mu' + 1}{f'(K)} - 1\right] + \frac{\gamma\sigma^2\mu'}{\mu^2} = \\ &= \frac{1 - \alpha - g}{\alpha}\frac{R\mu' + R - f'(K)}{R^2} + \frac{\gamma\sigma^2\mu'}{\mu^2}.\end{aligned}$$

Since $\mu'(R) < 0$ and $R < f'(K(R))$ for all $R \in (0, \beta)$, we have that $\partial D/\partial R < 0$ for all

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$R \in (0, \beta)$, which completes the argument.

Proof of Proposition 4 (incomplete vs complete markets). Since $\dot{W}_t = \bar{\rho}_t W_t - C_t = (\bar{\rho}_t - m_t) W_t$, wealth stationarity requires $\bar{\rho} = m$. Combining this with the Euler equation (15) in steady state, we get

$$\frac{\theta + 1}{2} \phi(f'(K) - \delta - R) - \theta(\beta - R) = 0.$$

From this, and for steady-state capital to be lower than under complete markets, that is, for $f'(K) - \delta > \beta$, it has to be the case that

$$\frac{\theta + 1}{2} \phi(\beta - R) - \theta(\beta - R) < 0,$$

which, since $\beta - R > 0$, gives $\theta > \phi/(2 - \phi)$.

Lemma 1 (properties of curves $K1$ and $K2$). *Let $K_1(R)$ and $K_2(R)$, $R \in (0, \beta)$, be the functions defined by solving, respectively, conditions (19) and (20) for K as a function of R, and let MPK denote the inverse of the marginal-product-of-capital function.*

(i) For any $\sigma > 0$, $K_1(R)$ satisfies the following properties: $0 < K_1(R) < MPK(R)$ for all R; $MPK(R) - K_1(R)$ is decreasing in R and vanishes as $R \to \beta$; and, finally, $\partial K_1/\partial R > 0$ if and only if $\theta > \frac{\phi}{1-\phi}$, which in turn is true if and only if R is sufficiently high.

(ii) For any $\sigma > 0$, $\partial K_2/\partial R < 0$ always; $K_2(R) \to +\infty$ as $R \to 0$, $K_2(R) \to 0$ as $R \to \beta$.

(iii) For any $R \in (0, \beta)$, as $\sigma \to 0$, $K_1(R) \to MPK(R)$ and $K_2(R) \to \beta$. (That is, the $K_1(R)$ curve converges to the $MPK(R)$ curve, while the $K_2(R)$ curve converges to the vertical line at $R = \beta$.)

Proof of Lemma 1. **(i)** From (19) it is clear that $f'(K) - \delta = R$ under complete markets, whereas $f'(K) - \delta = R + \mu(R) > R$ under incomplete markets. Hence, for any given R, steady-state K under incomplete markets is lower than under complete markets, which means

that $K_1(R)$ lies below $MPK(R)$ for every R. Since $\mu(R) = \sqrt{2\theta\gamma\sigma^2(\beta - R)/(1 + \theta)}$, it is clear that $\mu(R)$, i.e. the distance between $K_1(R)$ and $MPK(R)$, is decreasing in R, and that it tends to zero as $R \to \beta$. Finally, recall that (19) is equivalent to

$$\theta(\bar{\rho} - \beta) - (\theta - 1)\gamma\phi^2\sigma^2 = 0,$$

where $\bar{\rho} = \phi(f'(K) - \delta) + (1 - \phi)R$, and $\phi = (f'(K) - \delta - R)/\gamma\sigma^2$. Applying the implicit function theorem, we get

$$\left.\frac{\partial K}{\partial R}\right|_{(19)} = \frac{\phi - \theta(1 - \phi)}{\phi(\theta + 1)}\frac{1}{f''},$$

which proves that

$$\frac{\partial K_1}{\partial R} < 0 \Leftrightarrow \theta < \frac{\phi}{1 - \phi}.$$

Since ϕ, and therefore $\phi/(1 - \phi)$, is decreasing in R, it follows that θ is less likely to be lower than the threshold when R is high enough. So for R sufficiently high, $\partial K_1/\partial R > 0$. (ii) Using the fact that $\phi = (f'(K) - \delta - R)/\gamma\sigma^2$, it is easy to see that $K_2(R) \to +\infty$ as $R \to 0$. Moreover, since both sides of (20) are zero as $(R, K) \to (\beta, 0)$, we have that $K_2(R) \to 0$ for $R \to \beta$. From (20), using $\phi = (f'(K) - \delta - R)/\gamma\sigma^2$, $f'(K) = \alpha K^{\alpha-1}$, and $\omega - G = (1 - \alpha - g)K^\alpha$, we get

$$\left.\frac{\partial K}{\partial R}\right|_{(20)} = -\frac{(1 - \alpha)\frac{K^{\alpha-1}}{R^2} + \frac{\gamma\sigma^2}{\mu^2}}{\frac{(1-\alpha)^2}{R}K^{\alpha-2} + \frac{\gamma\sigma^2}{\mu^2}(1 - \alpha)\frac{f'}{K}} < 0,$$

which proves that $\partial K_2/\partial R < 0$ always. (ii) Follows from the above.

Proof of Proposition 5 (steady-state impact of government spending). From (32), we have that $\partial D/\partial g < 0$. Together with the property that $\partial D/\partial R < 0$, this implies that the steady-state R necessarily decreases with g. The impact of g on the steady-state K then follows from the fact that $K_1(R)$, defined by (19), does not depend on g and is increasing in R if and only if θ is higher than $\phi/(1 - \phi)$.