Public-Private Partnerships for Liquidity Provision

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1 Summary

Extreme bouts of uncertainty and fear wreak havoc in financial markets and expose leveraged institutions to potentially devastating liquidity shocks. The natural antidote for fear is insurance, but this is nowhere to be found in a private sector in panic mode. In this context, the government must step to the plate by either replacing the missing insurance markets, or by supporting private insurance provision. In this note we develop a simple model to gauge the impact of different degrees of public-private partnerships in liquidity provision during extreme confidence crises.

We study the case of a bank with profitable long term opportunities but that can none-the-less be subject to a severe liquidity shock before then. Unless the bank prepares for this shock in advance by obtaining capital or insurance, it goes bankrupt when hit by the shock, an event that has significant social, and possibly, private costs.

There are two obstacles preventing the bank from using the private capital markets to protect itself from the liquidity shock: a coordination failure, whereby a single investor cannot ensure that the bank will survive but many of them together can, and secondly, Knightian uncertainty, which essentially means that investors believe expected losses from the liquidity shock will be greater than the "true" assessment. In this context, private sector solutions

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are simply too costly – extreme dilution in the case of capital raising and exorbitant premia in the case of insurance.

A government with sufficient resources and less prone to panic than private investors can solve the problem directly by purchasing shares at non-Knightian prices (i.e., substantially higher than market prices). However, this solution requires large initial outlays, creates a myriad of issues associated to large public ownership of banks, and exposes the government to substantial net-revenue volatility.

Thus, on one extreme the private sector is unable to solve the problem by itself, and on the other extreme the government-alone intervention is plagued by practical problems. This dilemma explains why the financial stabilization plan recently announced encourages public-private partnerships. In this note we evaluate and characterize different partnerships of this kind including the brand new convertible-equity plan.

All the plans we consider contain either a government insurance to new equity holders in the form of a minimum share price in the future, or some government contingent "loan" to banks until uncertainty subsides, or a combination of both. They all reduce government revenue's exposure relative to a pure-public equity injection.

The arrangements we consider extract all the surplus from bankers, in the sense of leaving them indifferent between having the policy implemented or not. The benefit of the policy for them is avoiding the liquidity crisis, the cost is dilution of a future which they see as brighter than the marginal investor. Existing shareholders are also diluted, but since they are affected by Knightian uncertainty, the elimination of the liquidity risk is extremely valuable to them and hence the stock price rises rather than falls with the dilution.

To illustrate these mechanisms, we construct an artificial scenario that resembles the situation of Citi before the conversion announcement of February 27th. Our scenario is conservative as it assumes that the expected long run value of the bank (C) is only one fourth of its historically maximum capitalization value, and that the liquidity injection required is five times the pre-plan capitalization value. Agents perceive this liquidity shock as happening with close to 90 percent probability, while in reality (in the model) it is half of that. The pre-plan price is calibrated to be $2 per share. In this scenario we show that offering a minimum share price guarantee to new equity holders of $2.7 can boost share prices to $6.7 and attract all the private capital that is needed to withstand the potential liquidity shock. The expected net fiscal cost of this intervention is about $1b and the standard deviation of this revenue is $2b.

We also discuss a super-guarantee scenario, which ensures the elimination of the coordination failure without any pledge of government resources aside from the guarantee. In this case, the guaranteed price is $5.8, which immediately boosts the share prices to $8.3 and allows the bank not only to fetch the capital to fight a potential liquidity shock, but also to obtain an upfront payment of $9.5b. This policy has an expected net *gain* for the government of $3.8b and a standard deviation of this revenue of $11b.

As a benchmark, the equivalent pure-public equity injection requires that shares' purchases take place at a prices of $7. The government's expected gain from this policy is $10b but the volatility rises to $33b, and it requires that the government hold about 60 percent of the shares. Finally, there is a convertible preferred shares intervention, which reduces revenue volatility to $21b without sacrificing the expected return of the equity injection.[1]

This framework can also be used to analyze public-private partnerships to remove toxic assets from banks. By extension of our previous results, the main lesson is again that guarantee type mechanisms are effective in reducing the government's exposure to these assets's risk.

2 The model

There are three dates, 0, 1 and 2. In the last date ($t = 2$) the value of the bank can be H or L, with probabilities μ and $1-\mu$ respectively, with expected value $W = \mu H + (1-\mu) L$. Everybody understands these probabilities. There is also an intermediate liquidity shock, which may occur at $t = 1$. If this shock occurs, the bank requires a capitalization of K or goes bankrupt. This shock occurs with probability λ and bankers and the government understand this. However, current and potential investors are Knightian with respect to it and act as though λ were equal to $\lambda^K \geq \lambda$. The nature of the liquidity shock is such that bankers cannot wait until $t = 1$ to raise funds, and hence they must raise capital holdings at $t = 0$ to be prepared for the liquidity shock scenario. The liquidity and fundamental shocks are independent.

[1] Although as we recently learned from Citigroup's experience, it has the disadvantage that "the market" may not think of preferred shares as an efficient instrument to fight liquidity shortages.

Absent any policy, the price of each of the N shares outstanding is

$$p_0 = \left(1 - \lambda^K\right) \frac{W}{N}$$

because investors believe that the probability that the bank survives is only $1 - \lambda^K$. The value per share of the bank from the point of view of the banker (who is assumed to look after the long-term non-Knightian interest of the shareholders) is

$$V = (1 - \lambda) \frac{W}{N} \geq p_0.$$

We consider three policy scenarios. The first is a straightforward purchase of new ordinary shares by the government in exchange for K; the second is a purchase of convertible preferred shares and the third is the offer of share-price insurance to new investors. The second and third options attempt to limit the government's involvement by creating the conditions for private investors to be willing to supply capital. Two obstacles need to be overcome in order for private capital to enter:

- "Knightian discount": investors assess the future according to λ^K instead of λ, so they believe expected losses from the liquidity shock will be greater than the "true" assessment.

- Coordination failure: new investors will only receive positive returns if the bank survives. A single (small) investor might not be willing to invest if he expects to be the only one to do so, even if he would be willing to invest if he knew for sure that sufficient capital would be supplied to ensure the bank's survival.

All forms of interventions imply some sort of transfer between taxpayers, current investors and new investors. To make them comparable, they are all designed so that the banker's welfare after the intervention is still V, and hence they are indifferent between any of these policies or not policy at all (i.e., not diluting current shareholders and taking the risk that the bank is liquidated at $t = 1$).

2.1 Equity injection

Suppose *someone* injects K dollars into the bank in exchange for y new shares. This ensures that the bank survives even if there is a liquidity shock.

The value per share (for the banker) given this injection is

$$V' = \frac{W + (1-\lambda) K}{N + y}$$

If the policy is designed to extract all the surplus to the banker so that $V' = V$, then y must satisfy

$$\frac{y}{N} = \frac{\lambda}{1-\lambda} + \frac{K}{W} \tag{1}$$

Once the capital injection removes the liquidation risk, the bank simply becomes a risky asset with four possible states: $\{S, L\}$, $\{S, H\}$, $\{NS, L\}$, $\{NS, H\}$. The ex-post values of shares in each of these states are

$$\begin{aligned} p_{S,L} &= \frac{L}{N+y} \\ p_{S,H} &= \frac{H}{N+y} \\ p_{NS,L} &= \frac{K+L}{N+y} \\ p_{NS,H} &= \frac{K+H}{N+y} \end{aligned} \tag{2}$$

As of $t = 0$, these payoffs are assessed differently by Knightian investors and by non-Knightian bankers or the government. Intervention will make the share price rise

$$p'_o = \frac{W + \left(1 - \lambda^K\right) K}{N + y} > p_0 = \left(1 - \lambda^K\right) \frac{W}{N}$$

even though current shareholders are being diluted. The reason for this rise is that they are Knightian and hence exaggerate the value of now being protected against the liquidity shock.[2]

So far we have not specified who is providing the capital injection. If the government does it, then it will receive the risky payoffs in (2) in exchange for K up-front. Since the government is intervening directly, neither Knightian discount nor coordination are an issue.

[2]Note that if $\lambda^K = \lambda$, then $p'_o = p_o$, even though the equilibrium with policy avoids the liquidity crisis. But since we imposed that the policy leaves bankers indifferent, the dilution cost exactly offsets the liquidity shock survival gain.

2.2 Equity insurance

The objective of this approach is to make sure that new private investors are willing to inject K in exchange for y shares, paying a price of $p = \frac{K}{y}$ per share. This can be done by the government offering a price guarantee, essentially granting new investors a put option at a prespecified price p^G. Let us assume that values are such that the option is exercised if and only if the value of the bank at $t = 2$ is L.[3] Assuming for a moment that coordination is not an issue, then the price that investors will be willing to pay for the new shares plus the put option is

$$p^{INS} = \mu \frac{H + \left(1 - \lambda^K\right) K}{N + y} + (1 - \mu)\, p^G$$

In the H state investors will obtain a pro-rate share of H plus any unused portion of the K they put in. In the L state the investors will simply exercise the option. For them to be willing to invest in the first place, p^G must be such that $p^{INS} = \frac{K}{y}$. Solving for p^G yields:

$$p^G = \frac{1}{1 - \mu} \left[\frac{K}{y} - \mu \frac{H + \left(1 - \lambda^K\right) K}{N + y} \right] \tag{3}$$

As in the simple equity injection, this intervention will immediately boost the share prices of the existing shares, even though these shares are not insured[4].

2.3 Super-guarantee

There is one caveat to the above analysis, which is that it solves the Knightian discount problem but not the coordination problem. Under the above policy there is a Nash equilibrium where each investor believes that they are the only one who is going to invest, therefore the bank will not survive the liquidity shock should it take place and therefore the (Knightian) value of the new

[3] This requires that $\frac{H-K}{N+y} > p^G > \frac{L}{N+y}$, where p^G is the result of the calculation in the text. Analogous calculations can be performed if these inequalities do not hold.

[4] The formula for p^G in (3), could yield a negative number. This would mean that no equity insurance is necessary to address the problem of "Knightian discount", although it may be necessary in order to eliminate coordination failure.

shares plus the put option is

$$p^{uncoordinated} = \left(1 - \lambda^K\right) \frac{W}{N} + \lambda^K p^{SG}$$

In order to make sure that investors are willing to enter regardless of the actions of other investors (i.e. implementing capitalization as a *unique* Nash equilibrium), the government needs to set p^{SG} so that $p^{uncoordinated} = \frac{K}{y}$ (a super-guarantee). This requires

$$p^{SG} = \frac{1}{\lambda^K} \left[\frac{K}{y} - \left(1 - \lambda^K\right) \frac{W}{N} \right] \tag{4}$$

Once this guarantee is in place, the bank will survive for sure and the coordination problem goes away.

Observe that the formula for p^{SG} in (4), could yield a negative number. This would mean that there is no coordination problem and private investors would be willing to enter even if they believe they will be the only ones to do so. Using (1), it is straightforward to show that if $\lambda^K = \lambda$ then (4) gives a negative number (assuming $W > K$, i.e. positive NPV even after the shock). Thus, although we may think of them as separate problems, both the coordination and the Knightian discount problems arise only if $\lambda^K > \lambda$.

Assuming again that the superguarantee will be exercised only in state L,[5] the price that the new shares plus options fetch is

$$p^{ins,unique} = \mu \frac{H + \left(1 - \lambda^K\right) K}{N + y} + (1 - \mu)\, p^{SG} > \frac{K}{y}$$

so the bank raises more funds than it needs by taking advantage of the more valuable options that the government is granting. This surplus could in principle be taxed away up-front.

2.4 Capitalization through preferred shares

An alternative approach is for the government to inject capital in the form of preferred shares. Several variants are possible, depending on the interest rate they carry and on whether they can be redeemed or converted into ordinary

[5] Analogous calculations can be performed if parameters are such that the option will be exercised in other states.

shares, at whose request and at what conversion rate. We analyze one of these possibilities here.

The government gives the bank K and receives K preferred shares in return. At $t = 0$, the bank has the option to redeem them at a price of \$1 per share but only if it raises private equity capital[6]. At $t = 1$ the bank must pay a preferred dividend of rK and again has the option to redeem the preferred shares for \$1 per share. If the bank does not redeem the shares it has the option to convert them into x ordinary shares at any time or pay back K plus another preferred dividend rK at $t = 2$.

Suppose it is impossible to raise private capital and that the bank only considers converting at $t = 2$ (we show below that the bank never converts at $t = 1$). Then the value for the banker in each of the four scenarios is respectively

$$\begin{aligned} p_{S,L} &= \max\left\{\frac{L - rK}{N + x}, \frac{L - (1 + 2r)K}{N}\right\} \\ p_{S,H} &= \max\left\{\frac{H - rK}{N + x}, \frac{H - (1 + 2r)K}{N}\right\} \\ p_{NS,L} &= \max\left\{\frac{L + K - rK}{N + x}, \frac{L - 2rK}{N}\right\} \\ p_{NS,H} &= \max\left\{\frac{H + K - rK}{N + x}, \frac{H - 2rK}{N}\right\} \end{aligned} \tag{5}$$

The banker will value this policy proposal by weighting these possibilities using "true" probabilities.

In addition, the banker may instead attempt to raise K in private capital by issuing ordinary shares, either at $t = 0$ or $t = 1$ and use the proceeds to redeem the preferred shares. At $t = 0$ the number z of new shares that the banker would need to issue is given by

$$\begin{aligned} V^{Knightian} &= \frac{W + \left(1 - \lambda^K\right)K}{N + z} = \frac{K}{z} \\ &\Leftrightarrow \frac{z}{N} = \frac{K}{W - \lambda^K K} \end{aligned} \tag{6}$$

[6] The current (Feb 25, 2009) version of the US bank rescue plan envisions a 6 month period during which the bank may delay taking public capital in order to find private capital instead. The analysis here assumes, for expositional purposes, that the government injects K immediately but gives the bank the option to return it if it finds private capital.

$V^{Knightian}$ represents the investors' valuation per share, which is computed assuming that the bank will survive (because on-equilibrium the private sector supplies the capital and off-equilibrium the government does). This must be equal to the price per share $\frac{K}{z}$. The value for the banker of obtaining private capital in this way is

$$V' = \frac{W + (1 - \lambda) K}{N + z}$$

If r and x are sufficiently high, the banker will choose to look for private capital rather than be stuck with the preferred shares.

Overall, he will accept the scheme in the first place if $V' \geq V$. As before, extracting the surplus requires

$$\frac{z}{N} = \frac{\lambda}{1 - \lambda} + \frac{K}{W} \tag{7}$$

Combining equations (6) and (7), the banker will both accept the scheme and raise private capital as long as

$$\frac{\lambda}{1 - \lambda} + \frac{K}{W} \geq \frac{K}{W - \lambda^K K} \tag{8}$$

If condition (8) is met, the capitalization will be achieved entirely by the private sector. The only role of the temporary injection of public capital is to remove the bad equilibrium where no one is willing to invest. Notice that the above condition is exactly what is required for the insurance approach (which disregards the coordination problem) to be effective even with $p^G = 0$. Hence *if the only problem is one of coordination a policy of injecting preferred shares with a sufficiently onerous conversion factor and/or interest rate solves it at zero cost to the government.*

Suppose instead that the Knightian discount is severe and condition (8) is not met. What will be the consequence of the government's intervention? At $t = 1$, after paying the preferred dividend rK, the banker has three options:

1. To raise K in private capital and redeem the preferred shares
2. To convert them into x ordinary shares immediately
3. To wait until period 2 and then decide whether to convert or pay the preferred dividend rK

It is immediate that option 3 dominates option 2 since there is no penalty for waiting before deciding to accept the dilution.[7] The banker must then decide between options 1 and 3. By now the K shock either has or has not taken place so the Knightian uncertainty is removed and everyone assesses the probabilities of states H and L as μ and $1-\mu$.

Consider first the value of choosing option 1. Let the number of shares the banker must offer in order to raise K from the private sector be y_S or y_{NS} depending on whether the shock has or has not occurred. y_S and y_{NS} satisfy[8]

$$\begin{aligned}\frac{W-rK}{N+y_S} &= \frac{K}{y_S} \Leftrightarrow \frac{y_S}{N} = \frac{K}{W-(1+r)K} \\ \frac{W+K-rK}{N+y_{NS}} &= \frac{K}{y_{NS}} \Leftrightarrow \frac{y_{NS}}{N} = \frac{K}{W-rK}\end{aligned}$$

The value for the banker if it decides to raise private capital is

$$\begin{aligned}V_{S,cap} &= \frac{W}{N+y_S} = \frac{W-(1+r)K}{N} \\ V_{NS,cap} &= \frac{W+K}{N+y_{NS}} = \frac{W-rK}{N}\end{aligned} \tag{9}$$

The interpretation of this formula is that private capital does not change the bank's net worth but simply allows it to pay back $(1+r)K$ at $t=1$ instead of $(1+2r)K$ at $t=2$.

Consider now the value of choosing option 3. Using (5), it is given by

$$\begin{aligned}V_{S,wait} &= \mu \max\left\{\frac{H-rK}{N+x}, \frac{H-(1+2r)K}{N}\right\} \\ &\quad + (1-\mu)\max\left\{\frac{L-rK}{N+x}, \frac{L-(1+2r)K}{N}\right\} \\ V_{NS,wait} &= \mu \max\left\{\frac{H+K-rK}{N+x}, \frac{H-2rK}{N}\right\} \\ &\quad + (1-\mu)\max\left\{\frac{L+K-rK}{N+x}, \frac{L-2rK}{N}\right\}\end{aligned}$$

[7]In reality the interest rate accrues continuously so if the option value of waiting to decide is sufficiently small the bank could decide to accept dilution sooner than the deadline.

[8]This assumes that $W > K$ so the NPV of the bank is still positive. When this is not the case, the bank would never choose to look for private capital.

Assume that x and r are such that the banker always prefers to convert in the L state and to pay the preferred dividend in the H state.[9] Then

$$\begin{aligned} V_{S,wait} &= \mu\left[\frac{H-(1+2r)\,K}{N}\right]+(1-\mu)\,\frac{L-rK}{N+x} \\ V_{NS,wait} &= \mu\left[\frac{H-2rK}{N}\right]+(1-\mu)\,\frac{L+K-rK}{N+x} \end{aligned} \tag{10}$$

The decision whether or not to seek private capital at $t=1$ will depend on the comparison of the value in (9) and those in (10). In general both higher x (dilution by the government) and higher r make the option of seeking private capital more attractive. In this example, x is a comparably more powerful inducement in the state where the shock has not occurred because it threatens to dilute a more valuable bank. This conclusion could change if we were in a region of the parameter/policy space where the decision to accept dilution depends on whether the liquidity shock has occurred and not just on H versus L.

Finally, there exists parameters such that it is possible to make x and r such that the banker wants to raise private capital in *any* state at $t=1$ even if, due to the Knightian discount, he does not wish to do so at $t=0$. In this case the government intervention is temporary - it only lasts until the Knightian uncertainty is resolved. After that, since there is no longer any disagreement between bankers and investors, they will find a price at which both are willing to go ahead with the capitalization.

3 Example

Let us consider a bank whose expected long run value is only one-fourth of its historically maximum capitalization value, and that it may require a liquidity injection that is five times its current capitalization value in order to be prepared for an extreme liquidity event. Investors perceive the liquidity shock as happening with close to 87% probability but the true probability of the liquidity shock is half the perceived one. The scenario is extremely negative and is designed to capture in very broad terms the situation of Citigroup. It is calibrated to match the recent value of Citi's shares before policy intervention (which we round at \$2). Table 1 contains the set of parameters we use.

[9] Analogous calculations can be performed in the other cases

Table 1: Parameter values

H	125
L	25
μ	.5
K	50
λ	0.43
λ^K	0.87
N	5

The expected value of the bank absent a liquidity shock is $W = 75$ (market capitalization for Citi was about \$270 billion at its peak) but there is substantial volatility around this expected value. Current market capitalization is $N \times p_0$=\$10b and the potential liquidity shock is K=\$50b. Table 2 illustrates the results of different forms of intervention

Table 2: Results of intervention

Pre-intervention share price (p0)	2.00
Banker's value per share (1-λ)W/N	8.50
Post-intervention share price (p0')	6.72 *
Price of new issue (K/y)	6.99 *
Dilution (y/(N+y))	59% *
Guaranteed price (pG)	2.68
Super-guaranteed price (pSG)	5.75
Price of shares-plus-superguarantees	8.32
Tax on superguarantee	9.54
Conversion price (K/x)	6.99
Preferred dividend rate (r)	36% **

	State						
	S,L	S,H	NS,L	NS,H			
Ex-post value per share *	2.06	10.28	6.17	14.40			
	Government revenue				Expected	Range	std
Equity injection	-35.28	23.58	-5.85	53.02	10.83	88.30	32.85
Insurance	-4.43	0.00	0.00	0.00	-0.96	4.43	1.83
Superguarantee	-16.91	9.54	9.54	9.54	3.81	26.45	10.90
Preferred shares	-27.90	30.96	17.94	17.94	10.83	58.87	21.01

* These figures apply to intervention via equity, insurance or superguarantee

The banker is only willing to accept up to 59% dilution in exchange for \$50b, which requires that investors pay \$6.99 per share. If the government provides the \$50b, it makes an expected profit of \$10.83b (since the government agrees with the banker's valuation of \$8.50 per share but it only pays \$6.99); the cost is the exposure to substantial fundamental volatility (standard deviation of \$32.85b).

If the government wishes to avoid this risk exposure, insurance is another option. Condition (8) does not hold, so even assuming away coordination failure, there is a need to offset the Knightian discount. A guaranteed price of \$2.68 per share suffices for this. This guarantee has an expected cost of \$0.96b with a standard deviation of \$1.83b.

In order to eliminate the possibility of coordination failure, the government needs to offer a super-guarantee of \$5.75 per share. The value of shares-plus-superguarantees is \$8.32 per share, which is higher than \$6.99, so the government can tax away that difference to obtain upfront revenue of \$9.54b. The overall expected net revenue of the superguarantee scheme is \$3.81b with a standard deviation of \$10.90b. Under any of these interventions the share price would jump to \$6.72 per share because the intervention ensures the survival of the bank.

If instead the government chooses to invest via preferred shares it has many possible combinations of x and r that it can use.[10] For example it could use the same conversion rate of \$6.99 per share that it would use if it injected equity and a preferred dividend rate of 36%.[11] Under these values, the bank would redeem the preferred shares at $t = 1$ if and only if there is no shock, giving the government a net revenue of rK=\$17.94b. If there is a shock, the bank would always end up converting the preferred shares into ordinary shares rather than paying another rK in preferred dividends. This would give the government a net profit of \$30.96b if the state turns out to be H and a net loss of \$27.90b if the state turns out to be L. Overall, this generates the same expected revenue of \$10.83b for the government as using ordinary shares but lower volatility because it is not fully exposed to fundamental risk.

[10]Subject always to leaving the banker indifferent between intervention and nonintervention.

[11]This is a rate per "period" and the length of a period is unspecified by the model. It should roughly correspond to how long the government estimates that it would take for the liquidity shock to either happen or not happen.