Inefficient Automation*

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Abstract

How should the government respond to automation? We study this question in a heterogeneous agent model that takes worker displacement seriously. We recognize that displaced workers face two frictions in practice: reallocation is slow and borrowing is limited. We first show that these frictions result in inefficient automation. Firms fail to internalize that displaced workers have a limited ability to smooth consumption while they reallocate. We then analyze a second best problem where the government can tax automation but lacks redistributive tools to fully overcome borrowing frictions. The equilibrium is (constrained) inefficient. The government finds it optimal to slow down automation on efficiency grounds, even when it has no preference for redistribution. Using a quantitative version of our model, we find that the optimal speed of automation is considerably lower than at the laissez-faire. The optimal policy improves aggregate efficiency and achieves welfare gains of 4%. Slowing down automation achieves important gains even when the government implements generous social insurance policies.

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1 Introduction

Automation technologies raise productivity but disrupt labor markets, displacing workers and lowering their earnings (Humlum, 2019; Acemoglu and Restrepo, 2022). The increasing adoption of automation has fueled an active debate about appropriate policy interventions (Lohr, 2022). Despite the growing public interest in this question, the literature has yet to produce optimal policy results that take into account the frictions that workers face in practice when they are displaced by automation.

The existing literature that justifies taxing automation assumes that worker reallocation is frictionless or absent altogether. First, recent work shows that a government that has a preference for redistribution should tax automation to mitigate its distributional consequences (see Guerreiro et al., 2017 and subsequent work by Costinot and Werning 2018; Korinek and Stiglitz 2020). This literature assumes that automation and labor reallocation are instrinsically efficient, and that the government is willing to sacrifice efficiency for equity. Second, an extensive literature finds that taxing capital in the long-run — and automation, by extension — might improve efficiency in economies with incomplete markets (Aiyagari, 1995; Conesa et al., 2009). This literature abstracts from worker displacement and labor reallocation.

In this paper, we take worker displacement seriously and study how a government should respond to automation. In particular, we recognize that workers face two important frictions when they reallocate or experience earnings losses. First, reallocation is slow: workers face barriers to mobility and may go through unemployment or retraining spells before finding a new job (Jacobson et al., 2005; Lee and Wolpin, 2006). Second, credit markets are imperfect: workers have a limited ability to borrow against future incomes (Jappelli and Pistaferri, 2017), especially when moving between jobs (Chetty, 2008).

We show that these frictions result in *inefficient* automation. A government should tax automation — even if it does not value equity — when it lacks redistributive instruments to fully alleviate borrowing frictions. The optimal policy *slows down* automation while workers reallocate but does not tax it in the long-run. Quantitatively, we find important welfare gains from slowing down automation.

We incorporate reallocation and borrowing frictions in a dynamic model with endogenous automation and heterogeneous agents. There is a continuum of occupations, and workers come in overlapping generations. Firms invest in automation to expand their productive capacity. Automated occupations become less labor intensive, which displaces workers but increases output as labor reallocates to non-automated occupations. Displaced workers face reallocation frictions: they receive random opportunites to move between occupations, experience a temporary period of unemployment or retraining when they do so (Alvarez and Shimer, 2011), and incur a productivity loss due to the specificity of their skills (Adão et al., 2020). Workers also face financial frictions: they are not insured against the risk that their occupation is automated and face borrowing constraints (Huggett, 1993; Aiyagari, 1994). This baseline model has the minimal elements needed to study our question. We enrich this model for our quantitative analysis.

We have two main theoretical results. Our first result shows that the interaction between slow reallocation and borrowing constraints results in *inefficient automation*. Displaced workers experience earnings losses when their occupation is automated, but expect their income to increase as they slowly reallocate and find a new job. This creates a motive for borrowing to smooth consumption during this transition. When borrowing and reallocation frictions are sufficiently severe, displaced workers are pushed against their borrowing constraints.[1] Their consumption profiles are steeper than those of unconstrained workers who price the firms' equity. There is a wedge between how workers value the returns to automation over time. Effectively, firms fail to internalize that displaced workers have a limited ability to smooth consumption. Private and social incentives to automate do not coincide.

Our second result characterizes optimal policy. In principle, the government could restore efficiency if it was able to fully relax borrowing constraints using redistributive transfers. This is unlikely in practice.[2] This motivates us to study second best interventions, where the government can tax automation and (potentially) implement active labor market interventions but is unable to fully alleviate

[1] This is consistent with the empirical evidence. Displaced workers borrow to smooth consumption when they are able to (Sullivan, 2008). Many workers are constrained and are either unable to borrow or forced to delever their existing debt (Braxton et al., 2020).

[2] Governments often do not have have access to such rich instruments, which is precisely what motivates the public finance literature (Piketty and Saez, 2013). Moreover, the taxes required to pay for the transfers could tighten constraints for other workers (Aiyagari and McGrattan, 1998) and carry large dead-weight losses (Guner et al., 2021), and the take-up of transfers could be low (Schochet et al., 2012). We allow for various forms of social insurance in our quantitative model.

the borrowing constraints of displaced workers by redistributing income.[3]

We find that the equilibrium is generically (constrained) inefficient, as defined by Geanakoplos and Polemarchakis (1985). Automation and reallocation choices impose *pecuniary externalities* on workers. Firms do not internalize that automation displaces workers and lowers their earnings, and workers do not internalize how their reallocation affects the wage of their peers. The optimal policy addreses these pecuniary externalities. This policy reduces the present discounted value of output (net of resource costs) compared to the laissez-faire, but increases welfare through two channels (Bhandari et al., 2021): it improves *aggregate efficiency* by changing the flows of aggregate consumption over time, and it improves *redistribution* by changing how consumption is allocated across workers.

We show that the government should tax automation on efficiency grounds — even when it has no preference for redistribution. In particular, the government should *slow down* automation while labor reallocation takes place but should not intervene in the long-run. The logic is as follows. Output gains from automation are *back-loaded*, since they materialize slowly as more workers reallocate. The government values future gains *less* than firms do. It recognizes that automated workers have steeper consumption profiles and are *effectively* more impatient than the average worker who prices the firms' equity. Slowing down automation lowers output but improves *aggregate efficiency* by flattening consumption profiles, raising consumption early on in the transition when displaced workers value it more.

We then suppose that the government can tax automation but cannot implement active labor market interventions. This is motivated by the fact that such interventions have mixed results (Card et al., 2018) or unintended effects (Crépon and van den Berg, 2016). The rationale for taxing automation is reinforced or dampened, depending on the duration of unemployment / retraining spells. If spells are short, workers rely excessively on reallocation as a source of insurance and the government taxes automation more. The opposite occurs if spells are long.

We conclude the paper with a quantitative exploration. Our goal is to evaluate the efficiency and welfare gains from slowing down automation, while allowing

[3] These instruments are already used in many countries. For example, US taxes vary by type of capital and in fact *favor* automation (Acemoglu et al., 2020). South Korea recently reduced its tax credits on investment in automated technologies, the canton of Geneva in Switzerland taxes automated cashiers, and Nevada imposed an excise tax on autonomous vehicles. See Kovacev (2020) for a detailed review of these cases.

for various redistributive instruments. Our theoretical analysis found that workers' consumption profiles are key for optimal policy. These profiles are determined by reallocation frictions and the ability of workers to smooth consumption. Thus, we enrich our baseline model to ensure it performs well along these dimensions. First, we introduce idyosincratic mobility shocks (Artuç et al., 2010), which leads to a dynamic discrete choice for reallocation and gross flows across occupations (Moscarini and Vella, 2008). Second, we add uninsured earnings risk (Floden and Lindé, 2001), which produces a realistic distribution of savings. We also allow for progressive income taxation (Heathcote et al., 2017) and unemployment benefits (Krueger et al., 2016) to account for existing insurance that helps workers.

The constrained efficient intervention slows down the speed of automation substantially compared to the laissez faire. A government that only values *efficiency* should tax automation so as to reduce its half-life by a factor of 2 at least. This policy achieves sizable welfare gains of about 4% in consumption equivalent terms. The gains are even larger (around 6%) for a utilitarian government that values redistribution since the policy improves not only efficiency but also equity.

We then consider two alternative calibrations and two alternative polices. First, our benchmark calibration is conservative regarding the average duration of unemployment spells (1 quarter for the average US worker). These spells could be longer for workers permanently displaced by automation. We thus increase their average duration, which steepens the consumption profiles of automated workers. The government finds it optimal to slow down automation even more. Second, we increase the amount of liquidity in the economy to a particularly high level (McKay et al., 2016). The welfare gains from slowing down automation decrease substantially (as expected) but remain high compared to the alternative policies we consider next. Finally, we allow the government to insure automated workers directly by giving them a generous lump-sum transfer of $10k — the maximum amount allowed by the Reemployment Trade Adjustment Assistance program (RTAA) for instance. The transfers achieve smaller welfare gains than those from slowing down automation, especially when the government does not value redistribution. Put it differently, these transfers are effective in improving equity but do little to alleviate borrowing constraints in the medium-run and address inefficient automation. Combining transfers and automation taxes achieves large welfare gains.

Our paper relates to several strands of the literature. We contribute to the literature on the labor market impact of automation (Acemoglu and Restrepo, 2018; Martinez, 2018; Humlum, 2019; Moll et al., 2021; Hémous and Olsen, 2022) by studying optimal policy in an economy with frictions and quantifying the gains from slowing down automation. Moreover, we show that taxing automation improves *both* efficiency and equity, while there is a trade-off in the efficient economies studied in the literature (Guerreiro et al., 2017; Costinot and Werning, 2018; Thuemmel, 2018; Korinek and Stiglitz, 2020). In this literature, taxing automation results in production inefficiency (Diamond and Mirrlees, 1971). Instead, the optimal policy preserves (or restores) production efficiency in our model. Finally, Costinot and Werning (2018) point to sufficient statistics for the optimal taxation of automation in static (efficient) economies. Our analysis uncovers empirical moments that determine how a government should slow down automation to improve efficiency.

The rationale we propose for taxing automation also complements a large literature on capital taxation due to equity considerations (Judd, 1985; Chamley, 1986), dynamic inefficiency (Diamond, 1965; Aguiar et al., 2021), or pecuniary externalities when markets are incomplete (Conesa et al., 2009; Dávila et al., 2012; Dávila and Korinek, 2018). Optimal policies in our model also address pecuniary externalities. However, these externalities are distinct from the type encountered in the incomplete markets literature. They rely neither on the presence of uninsured idiosyncratic risk, nor on endogenous borrowing constraints. In addition, the literature on pecuniary externalities has almost exclusively studied static (or two-period) models or long-run stationary equilibria. The *timing* of these externalities (i.e., how front- or back-loaded they are) plays no role in optimal policy. In contrast, the rationale for intervention that we propose applies during the *transition* to the long run, and the timing of externalities is central to optimal policy.

Methodologically, our quantitative model combines two state-of-the-art frameworks: (i) dynamic discrete choice models with mobility shocks (Artuç et al., 2010) used for studying the impact of technologies and trade; and (ii) heterogeneous-agent models (Huggett, 1993; Aiyagari, 1994) used for analyzing consumption and insurance. Our analysis also contributes to the public finance literature studying optimal taxation (Heathcote et al., 2017) and social insurance (Imrohoroglu et al., 1995; Golosov and Tsyvinski, 2006) in dynamic models with heterogeneous agents.

2 Model

Time is continuous and there is no aggregate uncertainty. Periods are indexed by $t \geq 0$. The economy consists of a continuum of workers, a continuum of occupations, and a final goods producer. In this section, we specify the technologies, preferences, reallocation frictions, and resource constraints of this economy. We will describe assets, incomes, and borrowing frictions in Section 4 when discussing the decentralized equilibrium.

2.1 Technology

Occupations use labor as an input. Final goods are produced by aggregating the output of all occupations.

Occupations. Occupations are indexed by $h \equiv [0,1]$. They use a decreasing returns to scale technology

$$y_t^h = F^h\left(\mu_t^h\right), \tag{2.1}$$

where μ_t^h denotes the flow of effective labor in a given occupation.

Technology adoption. At time $t = 0$, some occupations can be automated (e.g., routine-intensive occupations). We denote the share of automatable occupations by ϕ. The degree of automation in an occupation is α.[4] The occupations' outputs are

$$F^h(\mu) = \begin{cases} \hat{F}(\mu;\alpha) & \text{if automated} \\ F(\mu) & \text{otherwise} \end{cases}, \tag{2.2}$$

where $F(\cdot)$ and $\hat{F}(\cdot)$ are neoclassical technologies. By definition, $\hat{F}(\cdot;0) \equiv F(\cdot)$ absent automation. Automated occupations are less labor intensive than non-automated occupations, i.e., $\hat{F}_\mu(1;\alpha)$ decreases with α.[5] In other words, automation is a labor-displacing technology.[6] Automation can increase productivity and

[4] For now, automation is chosen once and for all. We introduce gradual investment later on. This allows us to clarify that the optimal policy is to *slow down* automation while labor reallocates.

[5] An increase in α decreases the marginal product of labor *within* automated occupations. However, the *aggregate* marginal product of labor can increase, as we show in Section C in the Supplementary Material. This is the case in our quantitative model.

[6] Some forms of automation might complement labor too. We focus on automation technologies

raise output directly, but it also comes at a cost. The technology has to be maintained: it requires some continued investment which diverts resources away from production. The technology $\hat{F}(\cdot;\alpha)$ implicitly captures these output gains and costs.[7] We will impose some regularity assumptions later on.

Final good. Aggregate output is produced by combining the output y^h of all occupations with a neoclassical technology

$$Y_t = G\left(\left\{y_t^h\right\}\right). \tag{2.3}$$

In the following, we suppose that these inputs are complements. Moreover, we impose some *symmetry* across occupations.

Assumption 1 (Symmetry)**.** *The technology of the final good producer $G(\cdot)$ is continuously differentiable, additively separable and symmetric in its arguments.*

This assumption together with the strict concavity of $G(\cdot)$ ensure that the economy behaves *as if* there were only automated ($h = A$) and non-automated ($h = N$) occupations. This allows us to define the aggregate production function

$$G^{\star}(\boldsymbol{\mu};\alpha) \equiv G\left(\left\{F^h\left(\mu^h\right)\right\}\right) \tag{2.4}$$

where $\boldsymbol{\mu} \equiv \left(\mu^A, \mu^N\right)$ are the flows of workers employed in each automated and non-automated occupations, with the degree of automation α being implicit in $\left\{F^h\left(\mu^h\right)\right\}$. The technology $G^{\star}(\cdot)$ is total production net of automation costs. We refer to it as *output* in the following. For illustration, we provide an example of $G^{\star}(\boldsymbol{\mu};\alpha)$ in Section C in the Supplementary Material. This example uses a task-based model (Acemoglu and Restrepo, 2018).

2.2 Workers

There are overlapping generations of workers who are born and die at a constant rate $\chi \in [0, +\infty)$. A worker is indexed by four idiosyncratic states: their initial

that displace labor, such as industrial robots, certain types of artificial intelligence, autonomous vehicles, automated cashiers, etc.

[7] For an example of $\hat{F}(\cdot;\alpha)$, see the production function (7.1) and footnote 31 in Section 7.

occupation of employment (h); their age (s); their productivity (ξ); and their employment status (e). In the following, we let $\mathbf{x} \equiv (h, s, \xi, e)$ denote the workers' idiosyncratic states and π denote the associated measure.

Preferences. Workers consume, and supply inelastically one unit of labor when employed. Workers' preferences are represented by

$$U_0 = \mathbb{E}_0 \left[\int \exp\left(-\rho t\right) u\left(c_t\right) dt \right] \tag{2.5}$$

for some discount rate $\rho > 0$ and some isoelastic utility $u\left(c\right) \equiv \frac{c^{1-\sigma}-1}{1-\sigma}$ with $\sigma > 0$.

Reallocation frictions. We assume that the process of labor reallocation is *slow* and *costly*. Reallocation is slow for three reasons. First, existing generations of workers are given the opportunity to reallocate to another occupation with intensity λ.[8] Second, workers who reallocate across occupations enter a temporary state of non-employment which they exit at rate $\kappa > 0$. This state can be interpreted either as involuntary unemployment due to search frictions or as temporary exit from the labor force during which workers retrain for their new occupation. Third, new generations of workers enter the labor market gradually at rate $\chi < \lambda$, at which point they can choose any occupation.[9] Finally, reallocation is costly for two reasons. First, workers do not produce while not employed. Second, they incur a permanent productivity loss $\theta \in (0, 1]$ after they have reallocated to a new occupation, i.e. $h'_t \neq h$. This productivity loss captures the workers' skill specificity, i.e. the lack of transferability of their skills across occupations. Thus, workers' productivity evolves as

$$\xi_t = \mathbf{1}_{\{e_t=1\}} \zeta_t \quad \text{with} \quad \zeta_t = \begin{cases} \lim_{\tau \uparrow t} \zeta_\tau & \text{if } h'_t\left(\mathbf{x}\right) = h \\ \left(1-\theta\right) \times \lim_{\tau \uparrow t} \zeta_\tau & \text{otherwise} \end{cases} \tag{2.6}$$

with $\zeta_t \equiv e_t \equiv 1$ at birth. The employment status switches to $e_t = 0$ upon real-

[8] That is, workers' mobility decision is purely time-dependent, which delivers tractable expressions. We allow for state-dependent mobility in our quantitative model (Section 7).

[9] We introduce overlapping generations because young cohorts account for a substantial share of labor reallocation across occupations (Adão et al., 2020). We assume that new generations enter at a sufficiently low rate for existing workers to reallocate in equilibrium.

location and reverts to $e_t = 1$ upon exiting unemployment. Labor is distributed uniformly across occupations initially ($t = 0$).

Occupational choices. Occupational decisions for existing or new generations consist of choosing the occupation with the highest value

$$\max_{h' \in \{A,N\}} V_t^{h'}\left(\mathbf{x}'\left(h';\mathbf{x}\right)\right) \tag{2.7}$$

where $V_t^h(\cdot)$ is the continuation value associated to automated and non-automated occupations. For existing generations, the state $\mathbf{x}'(h';\mathbf{x})$ captures the unemployment / retraining spells that displaced workers go through and the permanent productivity loss they experience. Newborns are subject to neither.

2.3 Resource Constraint

For each occupation h, the output is given by

$$y_t^h = F^h\left(\frac{1}{\phi^h}\int \mathbf{1}_{\{h(\mathbf{x})=h\}}\xi d\pi_t\right), \tag{2.8}$$

where $\phi^A \equiv \phi$ and $\phi^N \equiv 1-\phi$ denote the mass of automated and non-automated occupations. Finally, the aggregate resource constraint is

$$G^*\left(\left\{\mu_t^h\right\}\right) = \int c_t(\mathbf{x})d\pi_t \tag{2.9}$$

where $c_t(\mathbf{x})$ is the consumption of a worker with idiosyncratic state $\mathbf{x}$.

3 Efficient Allocation

We now characterize the set of efficient allocations. The planner faces two choices: how to reallocate labor and assign consumption after automation has taken place (ex post); and the degree of automation (ex ante).

In the following, we impose regularity assumptions to rule out corner solutions. These are needed for a meaningful discussion of automation and reallocation. First, we assume that the cost of automation is such that there is positive but partial automation at the first best.[10] Second, we suppose that the parameters governing reallocation costs ($1/\kappa$ and θ) are small enough that reallocation occurs.

Assumption 2 (Interior solutions)**.** *The direct effect of automation $G^{\star}(\mu, \mu'; \alpha)$ is concave in α and satisfies $\partial_\alpha G^{\star}(\mu, \mu'; \alpha)|_{\alpha=0} > 0$ and $\lim_{\alpha \to +\infty} \partial_\alpha G^{\star}(\mu, \mu'; \alpha) = -\infty$ for any $0 \leq \mu < 1$ and $\mu' > 1$. Finally, the average unemployment duration $(1/\kappa)$ and the productivity loss (θ) are sufficiently small so that*

$$(1-\theta) \int_0^{+\infty} (1 - \exp(-\kappa t)) Z_t^N dt \geq Z^A$$

where Z^A and $\{Z_t^N\}$, defined in Appendix A.1, are positive and independent of (θ, κ).

3.1 Efficient Labor Reallocation

We start with the efficient allocations of labor and consumption after automation has occurred. The planner solves

$$V^{FB}(\alpha; \boldsymbol{\eta}) \equiv \max_{\{\mathbf{c}_t, m_t, \hat{m}_t, \boldsymbol{\mu}_t\}_{t \geq 0}} \sum_h \phi^h \int_{-\infty}^{+\infty} \eta_s^h \int_0^{+\infty} \exp(-(\rho + \chi) t) u\left(c_{s,t}^h\right) dt ds \quad (3.1)$$

subject to the constraints (3.2)–(3.7) below and a symmetric initial distribution of labor $\left(\mu_0^A = \mu_0^N = 1\right)$. Here, $c_{s,t}^h$ denotes the consumption in period t of the generation born in period s and initially located in an occupation $h \in \{A, N\}$, and $\boldsymbol{\eta} \equiv \{\eta_s^h\}$ are the Pareto weights on workers based on their birth period and, for initial generations $s \leq 0$, their initial occupation too.

An allocation must satisfy the resource constraint

$$C_t = G^{\star}(\boldsymbol{\mu}_t; \alpha) \quad (3.2)$$

where

$$C_t \equiv \sum_h \phi^h \int_0^{+\infty} \chi \exp(-\chi s) c_{s,t}^h ds \quad (3.3)$$

is aggregate consumption. The laws of motion for *effective* labor supplies $\boldsymbol{\mu}_t \equiv \{\mu_t^A, \mu_t^N\}$ are

$$d\mu_t^A = -(\lambda m_t + \chi \hat{m}_t) \mu_t^A dt \quad \text{with} \quad \mu_0^A = 1 \quad (3.4)$$

[10] The first part of Assumption 2 is satisfied if the cost of automation $F(1) - \hat{F}(1; \alpha)$ is sufficiently convex, i.e. more and more resources are diverted as automation increases.

in automated occupations $h = A$, for some exit shares $m_t, \hat{m}_t \in [0,1]$, and

$$d\mu_t^N \equiv -\frac{\phi}{1-\phi} d\mu_t^A - d\tilde{\mu}_t - \theta d\hat{\mu}_t \quad \text{with} \quad \mu_0^N = 1 \tag{3.5}$$

in the non-automated occupations $h = N$, with

$$d\tilde{\mu}_t = \left(\lambda \frac{\phi}{1-\phi} m_t \mu_t^A - (\kappa + \chi)\, \tilde{\mu}_t\right) dt \quad \text{with} \quad \tilde{\mu}_0 = 0 \tag{3.6}$$

$$d\hat{\mu}_t = (\kappa \tilde{\mu}_t - \chi \hat{\mu}_t)\, dt \qquad \text{with} \quad \hat{\mu}_0 = 0 \tag{3.7}$$

for each $t \geq 0$. From (3.4), we see that labor reallocation happens through two margins: the reallocation of existing generations (at rate λ) or the arrival of new ones (at rate χ). The planner chooses the shares m_t and $\hat{m}_t$ of each of these workers to reallocate. From (3.6), we see that existing generations who reallocate enter a pool of unemployed which they leave gradually – either their unemployment spell ends (at rate κ) or they are replaced by a new generation (at rate χ). At that point, they become active in their new occupation (3.7) but incur a productivity loss ($\theta > 0$) and are replaced gradually (at rate χ). The effective labor supply in non-automated occupations evolves as in (3.5), capturing unemployment and productivity losses.

Proposition 1 (Efficient labor reallocation)**.** *The efficient reallocation of labor is characterized by two stopping times $\left(T_0^{FB}, T_1^{FB}\right)$. All workers from existing generations reallocate to non-automated occupations until T_0^{FB} whereas all new generations reallocate until T_1^{FB}. The stopping times satisfy the smooth pasting conditions*

$$\int_{T_0^{FB}}^{+\infty} \beta_t(\boldsymbol{\eta})\, u'(C_t)\, \Delta_t dt = 0 \quad \text{and} \quad \mathcal{Y}^N_{T_1^{FB}} = \mathcal{Y}^A_{T_1^{FB}} \tag{3.8}$$

where $\mathcal{Y}_t^h \equiv 1/\phi^h \partial_h G^\star(\boldsymbol{\mu}_t; \alpha)$ is the marginal product of labor, the effective discount factor $\beta_t(\boldsymbol{\eta})$ is defined in Appendix A.1, and

$$\Delta_t \equiv \underbrace{\exp(-\chi t)}_{OLG} \left\{ \underbrace{(1-\theta)}_{\text{Productivity cost}} \underbrace{(1 - \exp(-\kappa(t - T_0)))}_{\text{Unemployment spell}} \mathcal{Y}_t^N - \mathcal{Y}_t^A \right\} \tag{3.9}$$

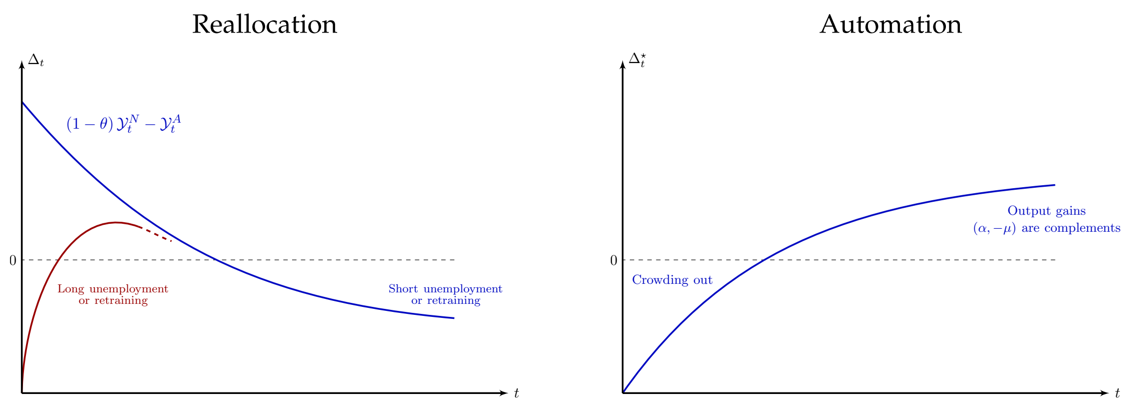
for all $t \geq T_0^{FB}$ is the response of output to labor reallocation.

Proof. See Appendix A.1. □

Automation drives a wedge between the marginal productivities of labor in automated and non-automated occupations. The planner reallocates workers between those to close this wedge and increase output over time. The planner displaces (existing) workers until the marginal benefit of doing so is zero. Reallocating workers helps close the wedge in marginal productivities $\left(\mathcal{Y}_t^N \geq \mathcal{Y}_t^A\right)$ but comes at the expense of a productivity loss $(\theta > 0)$ and the foregone production while unemployed $(1/\kappa > 0)$. The planner reallocates new workers until the marginal productivities are equalized, which takes time since new generations arrive slowly $(1/\chi > 0)$.

The benefits and costs of reallocating displaced workers are expressed in the output response Δ_t. The left panel of Figure 3.1 illustrates these flows for the case of no overlapping generations $(\chi \to 0)$. When unemployment / retraining spells are short $(1/\kappa \to 0)$, the flows Δ_t are *front-loaded:* they are initially positive and then gradually decline as more workers enter non-automated occupations. On the contrary, they are *back-loaded* when unemployment spells are sufficiently long because, at short horizons, displaced workers do not produce while unemployed.

Figure 3.1: Responses of output to reallocation and automation



To complete the characterization of the first best, labor allocations are

$$\mu_t^A = \exp\left(-\lambda \min\left\{t, T_0^{\text{FB}}\right\} - \chi t\right) \tag{3.10}$$

$$\mu_t^N = 1 + \frac{\phi}{1-\phi}(1-\theta)\left(1-\mu_t^A\right) + \frac{\phi}{1-\phi}\theta\left\{1 - \exp\left(-\chi t\right)\right\} \tag{3.11}$$

for all $t \in \left[0, T_1^{\text{FB}}\right)$ with no unemployment or retraining, or (A.10)–(A.13) in the general case. After T_1^{FB}, the planner allocates new generations across occupations so as to keep the marginal productivities equal across those.

3.2 Efficient Automation

We now turn to the efficient automation decision. The planner solves

$$\max_{\alpha \geq 0} V^{FB}(\alpha; \boldsymbol{\eta}) \tag{3.12}$$

Proposition 2 (Efficient automation)**.** *The degree of automation α^{FB} is unique and interior. A necessary and sufficient condition is*

$$\int_0^{+\infty} \beta_t(\boldsymbol{\eta})\, u'(C_t)\, \Delta_t^{\star} dt = 0 \tag{3.13}$$

where

$$\Delta_t^{\star} \equiv \frac{\partial}{\partial \alpha} G^{\star}(\boldsymbol{\mu}_t; \alpha) \tag{3.14}$$

for all $t \geq 0$ denotes the response of output to automation.

Proof. See Appendix A.2 □

Automation expands the production possibility frontier as labor reallocates between occupations, but it comes at a resource cost. The planner maximizes the present discounted value of the additional output that this reallocation allows, given reallocation frictions and the cost of automation. The time profile of these benefits and costs depends on the complementarity between automation and reallocation. We maintain the assumption below throughout. It ensures that the gains from automation are realized gradually, as workers reallocate to occupations where their marginal product is higher. It is satisfied in our quantitative model under standard functional forms. The right panel of Figure 3.1 shows that the returns on automation Δ_t^* are *back-loaded* in this case. Automation crowds out consumption early on but eventually increases output (and consumption) as labor reallocates.

Assumption 3 (Complementarity)**.** *Automation and labor reallocation are complements. That is,*

$$\hat{G}(\mu, \alpha) \equiv G^{\star}(\mu, 1 + \Theta(1 - \mu); \alpha)$$

has decreasing differences in (μ, α) for all $\mu \in (0, 1)$ and $\Theta \in (0, 1]$.

4 Decentralized Equilibrium

We now turn to the decentralized equilibrium. We first describe the problem of a representative firm which chooses automation and labor demands. We next describe the workers' problem, including the assets they trade, the frictions they face and their sources of incomes. Finally, we define a competitive equilibrium.

4.1 Firms

The representative firm chooses the degree of automation α and labor demands $\boldsymbol{\mu}$ to maximize the value of its equity

$$\max_{\alpha \geq 0} \hat{V}(\alpha) \quad \text{with} \quad \hat{V}(\alpha) \equiv \int_0^{+\infty} Q_t \hat{\Pi}_t(\alpha)\, dt \tag{4.1}$$

where $\{Q_t\}$ is the appropriate stochastic discount factor,[11] and

$$\hat{\Pi}_t(\alpha) \equiv \max_{\boldsymbol{\mu} \geq 0} G^\star(\boldsymbol{\mu}; \alpha) - \phi \mu^A w_t^A - (1-\phi)\, \mu^N w_t^N \tag{4.2}$$

are optimal profits given equilibrium wages $\{w_t^h\}$.

4.2 Workers

We now specify the assets that workers trade and the constraints they face.

Assets and states. Workers save in bonds available in zero net supply. We suppose that financial markets are incomplete: workers are unable to trade contingent securities against the risk that their occupation becomes automated.[12] We suppose that workers initially employed in automated occupations form a large household. This allows them to achieve full risk sharing against the risk of being allowed to

[11] As usual, equity is implicitly priced by workers who are marginally unconstrained.

[12] We rule out complete markets for two reasons: financial markets participations is limited in practice (Mankiw and Zeldes, 1991); and workers' equity holdings are typically not hedged against their employment risk (Poterba, 2003). The absence of contingent securities is precisely what motivates the literature on the regulation of automation. The equilibrium would be efficient if workers could trade contingent securities before occupations become automated.

move across occupations (at rate λ) or not.[13] Workers trade annuities against the risk of their death. Workers are now indexed by five idiosyncratic states: their holdings of bonds (a); their initial occupation of employment (h); their age (s); their productivity (ξ); and their employment status (e). We let $\mathbf{x} \equiv (a, h, s, \xi, e)$ denote the vector of states and π the associated measure.

Budget constraint. Worker's flow budget constraint is

$$da_t(\mathbf{x}) = \left[\mathcal{Y}_t^\star(\mathbf{x}) + (r_t + \chi)\, a_t(\mathbf{x}) - c_t(\mathbf{x})\right] dt \tag{4.3}$$

where $\mathcal{Y}_t^\star$ is total income consisting of labor income, profits and taxes, $r_t \geq 0$ is the return on savings, and c_t is consumption. The initial condition is $a_s(\mathbf{x}) = a_s^{\text{birth}}(\mathbf{x})$ at birth, where $a_s^{\text{birth}}(\mathbf{x})$ is the stock of inherited assets which we discuss below.

Borrowing frictions. Workers are subject to borrowing constraints

$$a_t(\mathbf{x}) \geq \underline{a} \tag{4.4}$$

where the borrowing limit is $\underline{a} \leq 0$. We focus on how borrowing frictions affect automated workers. For this reason, we abstract for now from borrowing frictions for new generations. We re-introduce them later in our quantitative model.

Income and occupational choice. Total income consists of effective labor income $\hat{\mathcal{Y}}_{s,t}^h$ and profits Π_t. That is,

$$\mathcal{Y}_t^\star(\mathbf{x}) = \hat{\mathcal{Y}}_{s,t}^h + \Pi_t \tag{4.5}$$

Profits are

$$\Pi_t \equiv G^*(\boldsymbol{\mu}_t; \alpha) - \int \mathbf{1}_{\{h(\mathbf{x})=h\}} \xi w_t^h d\pi_t \quad \text{with} \quad \mu_t^h = \frac{\int \mathbf{1}_{\{h(\mathbf{x})=h\}} \xi d\pi_t}{\phi^h} \tag{4.6}$$

For simplicity, we suppose that profits are claimed symmetrically — all our results carry through if we assume that automated workers claim no profits. Finally, workers still face the occupational choice (2.7).

[13] This assumption prevents an artificial dispersion in the distribution of assets and implies that a worker's reallocation history is irrelevant. It allows us to retain tractability and abstract from insurance considerations at this point. We relax this assumption in our quantitative model.

4.3 Equilibrium

The resource constraint is still given by (2.8)–(2.9) and wages are

$$w_t^h \equiv 1/\phi^h \partial_h G^\star (\boldsymbol{\mu}_t; \alpha) \tag{4.7}$$

All agents act competitively. We choose the price of the final good as *numéraire*. We define a competitive equilibrium below. We characterize the equilibrium in Appendix A.3.

Definition 1 (Competitive equilibrium)**.** A competitive equilibrium consists of a degree of automation α, and sequences for effective labor supplies $\{\mu_t^h\}$, consumption and savings functions $\{c_t(\mathbf{x}), a_t(\mathbf{x})\}$, interest rate, wages, profits and incomes $\{r_t, \{w_t^h\}, \Pi_t, \{\mathcal{Y}^\star(\mathbf{x})\}\}$, and distributions of states $\{\pi_t(\mathbf{x})\}$ such that: (i) automation and labor demands are consistent with the firm's optimization (4.1)–(4.2); (ii) consumption and savings are consistent with workers' optimization; (iii) interest rates ensure that the resource constraint is satisfied

$$\int c_t(\mathbf{x})\, d\pi_t = G^*(\boldsymbol{\mu}_t; \alpha); \tag{4.8}$$

(iv) wages, profits and incomes are given by (4.5)–(4.7); and (v) the distribution of states is consistent with workers' choices and the producitvity law of motion (2.6).

5 Inefficient Equilibrium

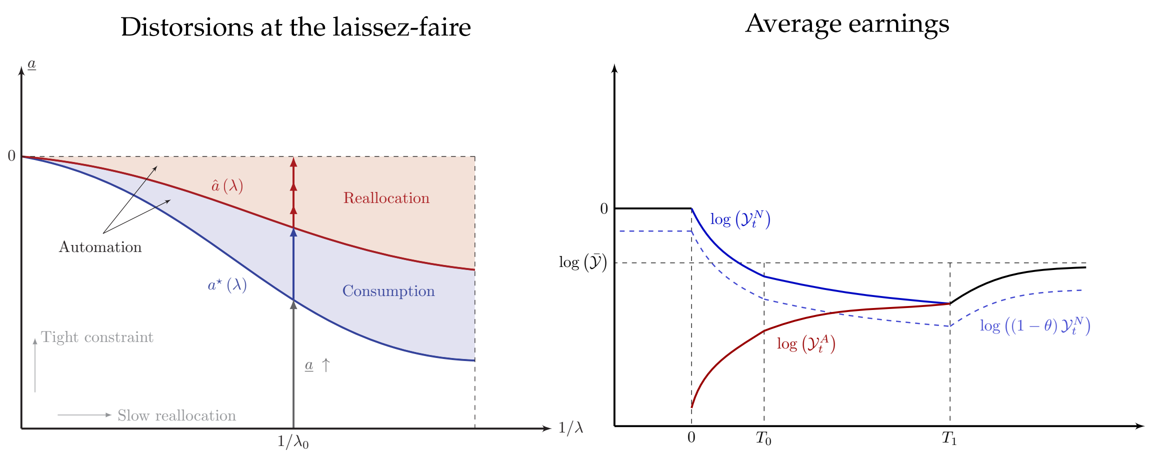
We now present the first main result of this paper: the equilibrium is inefficient when reallocation and borrowing frictions are sufficiently severe. Section 5.1 elaborates on why the equilibrium degree of automation is inefficient.

Proposition 3 (Failure of First Welfare Theorem)**.** *The laissez-faire equilibrium is inefficient if and only if reallocation frictions* (λ, κ, χ) *and borrowing frictions* $(\underline{a})$ *are such that* $a^\star(\lambda, \kappa, \chi) < \underline{a} \leq 0$ *for some threshold* $a^\star(\cdot)$ *defined in Appendix A.4. The threshold satisfies* $a^\star(\lambda, \kappa, \chi) < 0$*, i.e., inefficiency can occur, if and only if labor reallocation is slow* $1/\lambda > 0$ *or* $1/\kappa > 0$ *or* $1/\chi < +\infty$*.*

Proof. See Appendix A.4. □

The left panel of Figure 5.1 illustrates this result in the space of reallocation frictions $(1/\lambda)$ and borrowing frictions $(\underline{a})$. This space is partitioned in two main regions. The equilibrium is efficient as long as the frictions fall in the white region where $\underline{a} \leq a^{\star}(\cdot)$. This occurs when either reallocation is sufficiently fast or borrowing constraints are sufficiently loose. In constrast, the equilibrium is inefficient when the frictions fall into either one of the colored regions where $\underline{a} > a^{\star}(\cdot)$.[14]

Figure 5.1: Laissez-faire: distorsions and labor incomes



To understand this result, the right panel of Figure 5.1 depicts the paths of the average earnings for workers initially employed in each occupation, i.e., born in periods $s < 0$.[15] Their earnings are

$$\hat{\mathcal{Y}}_{s,t}^{h} = w_t^A + \overbrace{\left(1 - \exp\left(-\lambda \min\left\{t, T_0\right\}\right)\right)}^{\text{Mass of workers who reallocate}} \overbrace{\left[\Theta_t\left(\lambda, \kappa\right) \times \left(1-\theta\right) w_t^N - w_t^A\right]}^{\text{Self-insurance through reallocation}} \tag{5.1}$$

where $\Theta_t(\lambda, \kappa)$ captures the share of workers who exited unemployment or retraining (Appendix A.3). When reallocation is *slow*, automation decreases the income of workers displaced by automation. This decrease is not fully persistent though. Their income slowly rises after they reallocate and increases from w_t^A to $(1-\theta)w_t^N$ over time. Therefore, these workers wish to borrow while they slowly reallocate. The following remark states this insight.

[14] It should be noted that the threshold $a^{\star}(\lambda)$ is non-monotonic in its arguments. In particular, $\lim_{1/\lambda \to +\infty} a^{\star}(\lambda) = 0$ when existing workers never reallocate (as in Guerreiro et al., 2017).

[15] Workers born in $s \geq 0$ all earn the same wage $\hat{\mathcal{Y}}_{s,t}^{h} = w_t^N$ since they are allowed to choose their occupation and never reallocate subsequently. New workers never become borrowing constrained in this version of the model.

Remark 1. *Workers displaced by automation expect their income to partially recover as they slowly reallocate. This creates a motive for borrowing.*

When reallocation and borrowing frictions are sufficiently mild, workers are never borrowing constrained and the equilibrium is efficient, i.e., the white region in the left panel of Figure 5.1. As the frictions become more severe, borrowing constraints eventually bind $\underline{a} > a^\star$, i.e., the blue and red regions. In this case, consumption choices become distorted, and so do reallocation choices if the frictions are even more severe $\underline{a} > \hat{a}$. Proposition 9 in Appendix B formalizes this discussion. As we show in the next section, automation becomes inefficient whenever workers are borrowing constrained.

5.1 Why Is Automation Inefficient?

To understand why automation is inefficient, we compare the private and social incentives to automate[16]

$$\text{(LF)} \qquad \int_0^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c^N_{0,t}\right)}{u'\left(c^N_{0,0}\right)} \Delta^\star_t dt \ = 0 \tag{5.2}$$

$$\text{(FB)} \qquad \int_0^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c^A_{0,t}\right)}{u'\left(c^A_{0,0}\right)} \Delta^\star_t dt \ = 0 \tag{5.3}$$

where $\Delta^\star_t$ is the response of output to automation in (3.14), as depicted in Figure 3.1. Firms — just like the government — increase automation until the marginal returns $\Delta^\star_t$ are zero in present discounted value. The (intertemporal) *marginal rate of substitution* (MRS) that they internalize are potentially different, however. Absent borrowing constraints, all workers share the same MRS. In this case, the private and social incentives coincide, and the degree of automation is efficient. When automated workers become borrowing constrained, their consumption profiles are steeper than those of unconstrained workers who price the firms' equity. There is a wedge between how workers value the returns to automation over time.[17]

[16] To obtain (5.3), we use Proposition 2 and the envelope condition (A.42) in Appendix A.4. This expression holds for *any* weights η that the planner assigns to workers.

[17] This wedge would occur even if the sequence of interest rates was fixed (as in a small open economy) or in a model with an outside Ricardian household that invests in firm equity. Beyond

Effectively, firms fail to internalize that displaced workers have a limited ability to smooth consumption while they reallocate. Private and social incentives do not align and the degree of automation is inefficient.[18]

The mechanism described above operates only while workers reallocate. The reason is that they are not borrowing constrained in the long-run. In Section 6.5, we allow for gradual investment in automation (instead of once and for all) which clarifies that automation is inefficient only along the transition to the long-run.

Efficient special cases. The existing literature on the regulation of automation focuses on two efficient benchmarks that obtain as two limit cases in our model. Suppose first that labor reallocation is *instantaneous* $(1/\lambda \to 0, 1/\kappa \to 0$ and $1/\chi \to +\infty)$ as in Costinot and Werning (2018). In this case, the model is static $T_0, T_1 \to 0$. Workers initially employed in automated occupations reallocate immediately so as to ensure $(1-\theta)\, w_t^N = w_t^A$. Workers initially employed in non-automated occupations earn w_t^N instead. Automation has *distributional* effects but there is no motive for borrowing since income changes are fully permanent. As a result, borrowing frictions are irrelevant and the equilibrium is efficient. This explain why slow reallocation is *necessary* for inefficiency. Suppose instead that reallocation is slow but there are no borrowing frictions $(\underline{a} \to -\infty)$ as in Guerreiro et al. (2017).[19] Wages remain higher in non-automated occupations until the gap closes at $t = T_1$. Therefore, automation has distributional effects and creates a motive for borrowing, but there is no wedge between the MRS of automated and non-automated workers.

The mechanism in practice. Our mechanism relies on displaced workers becoming borrowing constrained while they slowly reallocate. Empirically, workers who

this wedge, automation is distorted for two additional reasons in general equilibrium: (i) the MRS of unconstrained non-automated workers (or the interest rate that firms face) changes; and (ii) so do wages (and hence) profits.

[18] It is worth noting that the economy can be *inefficient* while still achieving *production efficiency* (Diamond and Mirrlees, 1971). This is the case when borrowing and reallocation frictions are sufficiently severe to distort consumption choices, but not sufficiently so to distort the reallocation choices, i.e., the blue region in the left panel of Figure 5.1. In this case, the distorsion in automation simply affects the timing of the output and consumption stream $\{C_t\}$ and the economy moves *along* its production possibility frontier (as opposed to inside).

[19] In Guerreiro et al. (2017), reallocation takes place (entirely) through new generations $1/\chi < +\infty$. That is, existing generations are not allowed to move in their model. In our model, this corresponds to the case where workers never receive reallocation opportunities $1/\lambda \to +\infty$ or unemployment spells $1/\kappa$ are prohibitively long (Assumption 2).

loose their job indeed attempt to borrow to smooth consumption (Sullivan, 2008), but are often unable to do so or are even forced to delever their existing debt (Braxton et al., 2020). While we abstract from ex-ante heterogeneity across workers, our mechanism is more likely to be relevant when automation affects workers with small liquidity buffers. For example, industrial robots, automated cashiers, or autonomous vehicles would tend to displace low-to-middle income routine workers who are more likely to be hand-to-mouth. In contrast, artificial intelligence software for natural language processing tends to affect higher income skilled workers for which borrowing frictions are less severe. Finally, it is worth noting that our mechanism might, in theory, also apply to technological innovations that increase labor demand in certain occupations / sectors and cause labor to reallocate to those.[20] However, automation is distinct in that it displaces workers, lowers their earnings and forces them into unemployment. Borrowing frictions are more likely to bind as a result. As such, the mechanism is likely to apply more generally to other labor-displacing investments, such as when firms "off-shore" production.

6 Optimal Policy Interventions

We now discuss optimal policy. In Section 6.1, we state the general Ramsey problem, discuss policy instruments that implement a first best, and then consider a constrained Ramsey problem of a government that has a more restricted set of instruments. In Section 6.2, we show that the equilibrium is generically *constrained inefficient*. In Section 6.3, we show that the government should tax automation on efficiency grounds. Section 6.4 introduces equity concerns. Section 6.5 introduces gradual investment and shows that the optimal policy is to slown down automation. Finally, Section 6.6 allows for other forms of investment.

6.1 Ramsey Problem

We suppose at this point that the government has access to a set of taxes $\{\boldsymbol{\tau}_t\}$. This set includes a distorsionary tax on automation $\{\tau^{\alpha}\}$, and arbitrary taxes and

[20] Workers could become borrowing constrained due to anticipatory effects: they expect higher earnings in the booming sector. This type of anticipatory effect is likely to be quantitatively small (Poterba, 1988). The effect might actually be the opposite: borrowing constraints are relaxed as their future earnings increase (Jappelli, 1990).

transfers to redistribute income such as very rich lump sum tranfers $\{T_t^h\}$, non-linear income taxes $\{\mathcal{T}_t(\cdot)\}$, severance payments $\{\varsigma_t\}$, etc. For tractability and to obtain more compact expressions, we assume in the following that workers cannot borrow $\underline{a} \to 0$ and abstract from overlapping generations $1/\chi \to +\infty$. All the insights carry through in the general case with $1/\chi < +\infty$.

The government chooses these taxes to solve

$$\max \sum_h \phi^h \eta^h \int_0^{+\infty} \exp(-\rho t)\, u\left(c_t^h\right) dt \tag{6.1}$$

for a given set of Pareto weights $\boldsymbol{\eta}$, subject to the following implementability constraints. First, consumption and reallocation choices are consistent with workers' optimization, i.e., equations (A.21)–(A.25), (A.29) and (A.31) in Appendix A.3 augmented with taxes. Second, effective labor supplies are given by equations (A.10)–(A.13). Third, automation is consistent with firms' optimality condition (A.39) given taxes. Finally, wages and profits are given by (A.32)–(A.33) and earnings are given by (5.1).

6.1.1 Implementing a First Best

The inefficiency that we document operates when displaced workers become borrowing constrained. A government that has access to a sufficiently rich set of redistributive tools to fully undo borrowing frictions could, in theory, restore efficiency without taxing automation directly. To see this, consider the *wedge* between the optimality conditions for automation at the laissez-faire (5.3) and the first best (5.2)

$$\tau^\alpha \equiv \int_0^{+\infty} \exp(-\rho t) \left(\frac{u'\left(c_{0,t}^N\right)}{u'\left(c_{0,0}^N\right)} - \frac{u'\left(c_{0,t}^A\right)}{u'\left(c_{0,0}^A\right)} \right) \Delta_t^\star dt \tag{6.2}$$

This wedge corresponds to the linear tax on automation that would implement a particular first best. When the government can relax borrowing constraints by redistributing income directly, the MRS of automated and non-automated workers coincide: a first best can be implemented without taxing automation ($\tau^\alpha = 0$). Three interventions could in principle achieve this outcome.

First, *targeted* lump sum tranfers $\{T_t^h\}$ (indexed by worker and time) could implement any efficient allocation.[21] The literatures on optimal income taxation (Piketty and Saez, 2013) and the regulation of automation rule out such a rich set of transfers, in part because the informational requirements to implement them are too large. That said, some existing policies partially insure displaced workers, e.g., Reemployment Trade Adjustment Assistance program (RTAA) in the US. However, this type of programs have shown low take-up rates (Schochet et al., 2012) and often have unintended consequences (Crépon and van den Berg, 2016). We allow for realistic direct transfers in our quantitative model (Section 7).

Second, the government could undo workers' borrowing constraints via *symmetric* transfers $\{T_t\}$. Effectively, the government would borrow on behalf of workers in the short-term and repay its debt later on by taxing them. In practice, the future tax burden would tighten borrowing constraints (Aiyagari and McGrattan, 1998) and carry potentially large distorsions (Guner et al., 2021), limiting or entirely reversing the benefits of the transfers. The fiscal cost is also likely to be prohibitive. The payments need to be generous enough to ensure that no worker is constrained — a scenario that the literature on heterogeneous agents has not seriously considered. The size of transfers is further limited by the fact that future higher taxes could push the poorest workers into default.

Finally, a non-linear income tax $\mathcal{T}_t(\cdot)$ (Mirrlees, 1971; Atkinson and Stiglitz, 1976) or unemployment insurance could help relax borrowing constraints too. However, they would not implement a first best in practice, as they reduce labor supply and distort incentives to reallocate between occupations. In addition, non-linear income taxes are a particularly blunt tool to redistribute *across* occupations when incomes substantially vary *within* occupations due to idiosyncratic shocks. We allow for both non-linear income taxation and unemployment insurance in our quantitative model.

6.1.2 Constrained Ramsey Problem

We now assume that the government cannot *fully* alleviate borrowing frictions and implement a first best. We abstract from social insurance programs altogether at this point and re-introduce them later in our quantitative model. Instead, the government has access to a simple set of instruments that depend on calendar

[21] A version of the Second Welfare Theorem holds in our model (Proposition 10 in Appendix B).

time alone: a linear tax on automation τ^{α}, and active labor market interventions (Card et al., 2018) that tax or subsidize labor reallocation $\{\varsigma_t\}$.[22]

These instruments are already used in many economies and do not require the government to know which occupations are automated or which workers are displaced. For instance, US taxes vary by type of capital (e.g., equipment, software, structures) and industry (due to differential depreciation allowances), and seem to be favoring automation instead of taxing it (Acemoglu et al., 2020). Concrete policies discriminating against automation technologies (Kovacev, 2020) include: (i) South Korea's reduction in the automation tax credit aimed at protecting workers in high-tech manufacturing, (ii) the Swiss canton of Geneva's tax on retail stores installing automated cashiers, and (iii) Nevada's excise tax on transportation companies using autonomous vehicles that would displace human drivers.

The government effectively controls two choices with its instruments: the degree of automation α; and the reallocation of displaced workers T_0. All other choices must be consistent with workers' and firms' optimality. The government's constrained Ramsey problem reduces to the following primal problem.

Lemma 1 (Primal problem)**.** *The government's problem reduces to*

$$\max_{\{\alpha, T_0, \boldsymbol{\mu}_t, \mathbf{c}_t\}} \sum_h \phi^h \eta^h \int_0^{+\infty} \exp(-\rho t)\, u\left(c_t^h\right) dt$$

subject to the laws of motion for effective labor

$$\begin{aligned} \mu_t^A &= \exp\left(-\lambda \min\{t, T_0\}\right) \\ \mu_t^N &= 1 + \frac{\phi}{1-\phi}(1-\theta)\left(1-\mu_t^A\right) \end{aligned}$$

and the consumption allocations

$$c_t^h = \underbrace{1/\phi^h \partial_h G^\star(\boldsymbol{\mu}_t; \alpha)}_{\textit{Initial wage}} + \underbrace{\left(1-\exp\left(-\lambda \min\{t, T_0\}\right)\right)\Gamma_t^h}_{\textit{Reallocation gains}} + \underbrace{G^\star(\boldsymbol{\mu}_t; \alpha) - \sum_h \mu_t^h \partial_h G^\star(\boldsymbol{\mu}_t; \alpha)}_{\textit{Profits}},$$

[22] To abstract from income effects, we assume that the large family (Section 4.2) reimburses lump sum any reallocation taxes or subsidies it perceives. The latter can take the form of credits for retraining programs or unemployment insurance (when positive), or penalties such as imperfect vesting of retirement funds (when negative).

for each occupation $h \in \{A, N\}$, where reallocation gains are

$$\Gamma_t^A \equiv (1-\theta)\, 1/\phi^N \partial_N G^\star(\boldsymbol{\mu}_t; \alpha) - 1/\phi^A \partial_A G^\star(\boldsymbol{\mu}_t; \alpha)$$

in automated occupations and $\Gamma_t^N = 0$ in non-automated occupations, in the particular case without unemployment / retraining spells $(1/\kappa \to 0)$. The general case is similar but involves the laws of motion for effective labor (A.10)–(A.13) and reallocation gains (A.27)–(A.28) in Appendices A.1 and A.3.

6.2 Constrained Inefficiency

We now show that the government should intervene even when its instruments are limited. Formally, Appendix A.5 establishes that the laissez-faire is *generically* constrained inefficient in the sense of Geanakoplos and Polemarchakis (1985).

To see this, compare the private and social incentives to automate and reallocate

$$\begin{aligned}
\int_0^{+\infty} \exp(-\rho t) \frac{u'\left(c_t^N\right)}{u'\left(c_0^N\right)} \Delta_t^\star dt &= -\Phi^\star\left(\alpha^{SB}, T_0^{SB}; \boldsymbol{\eta}\right) \\
\underbrace{\int_{T_0^{SB}}^{+\infty} \exp(-\rho t) \frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)} \Delta_t dt}_{\text{laissez-faire}} &= \underbrace{-\Phi\left(\alpha^{SB}, T_0^{SB}; \boldsymbol{\eta}\right)}_{\text{pecuniary externalities}}
\end{aligned}$$

where the terms $\Phi^\star(\cdot)$ and $\Phi(\cdot)$ capture the *pecuniary externalities* that automation and reallocation impose on workers — which we define in Appendix A.5. The government takes into account that an increase in automation (α) reduces wages in automated occupations, but increases profits that benefit all workers (or *some* workers when profits are not claimed symmetrically).[23] Similarly, the government internalizes that an increase in reallocation (T_0) reduces wages in non-automated occupations, but lifts wages in automated occupations. Firms and workers do not internalize these effects. We show that these pecuniary externalities do not net out at the laissez-faire in presence of reallocation and borrowing frictions.

This finding echoes the constrained-inefficiency results in the incomplete markets literature (Lorenzoni, 2008; Dávila and Korinek, 2018). The nature of the in-

[23] Again, all our results carry through in the case where displaced workers do not claim profits. Assuming that profits are claimed symmetrically is conservative, if anything, since the increase in profits partly compensates for the decline in labor income experienced by displaced workers.

efficiency is different, however. Constrained inefficiency occurs in our economy despite the absence of uncertainty and incomplete markets, or endogenous borrowing constraints. Instead, it occurs when firms and workers make *technological choices*, and borrowing constraints distort the (shadow) prices that these agents face.[24] It is well-known that technological choices can result in inefficiencies by *themselves* (Acemoglu, 2009). However, our model is set up so that they are a source of inefficiency *only* when borrowing constraints bind.

6.3 Taxing Automation on Efficiency Grounds

We now present the second main set of results of this paper, which signs optimal policy interventions. We show that the government should tax automation on efficiency grounds — even when it does not have a preference for redistribution.

Pareto weights. Taxing automation has two effects. The first effect is *aggregate*: it generates an intertemporal substitution between current resources (the automation cost, or investments more generally) and future output. The importance of this effect for welfare depends on the distribution of marginal utilities over time — the intertemporal MRS. The second effect is *distributional*: some workers benefit from this intervention more than others through the pecuniary externalities we discussed above. The importance of this second effect depends on the distribution of marginal utilities and Pareto weights across workers. The two effects correspond to the *aggregate efficiency* and *redistribution* components of the decomposition in Bhandari et al. (2021).[25]

To highlight the new rationale for policy intervention that we propose, we initially abstract from equity altogether (the second effect). We suppose that the government intervenes *exclusively* to improve aggregate efficiency (the first effect). This is achieved by choosing Pareto weights $\boldsymbol{\eta}^{\text{effic}}$ so that the distributional effects net out.[26] In particular, these *efficiency* weights imply that the government values

[24] Labor reallocation — just like automation — is isomorphic to a technological choice in the Arrow-Debreu construct. Each worker owns a firm that chooses the type of labor services to provide.

[25] Bhandari et al. (2021) decompose the welfare effects of policy changes into gains in: (i) aggregate efficiency from changes in total resources, (ii) redistribution from changes in ex ante consumption shares, and (iii) insurance from changes in consumption risk. In our baseline model, taxing automation affects welfare via (i) and (ii) alone. In our quantitative model, (iii) is also present.

[26] See Appendices A.6–A.7 for details. The weights are inversely related to the workers' marginal

displaced workers *less* compared to a utilitarian government that values equity. We reintroduce equity considerations in Section 6.4.

6.3.1 With Active Labor Market Interventions

For now, we continue to allow for labor market interventions. The proposition shows that the government should *curb* (or *tax*) automation on efficiency grounds.

Proposition 4 (Second best). *Suppose that the government controls automation, as well as labor reallocation. Then, curbing automation is optimal.*

Proof. See Appendix A.6. □

To understand the result, compare the private and social incentives to automate[27]

$$\text{(LF)} \quad \int_0^{+\infty} \exp(-\rho t) \frac{u'\left(c_t^N\right)}{u'\left(c_0^N\right)} \Delta_t^\star dt = 0 \tag{6.3}$$

$$\text{(SB)} \quad \int_0^{+\infty} \exp(-\rho t) \left\{ \sum_h \phi^h \eta^{h,\text{effic}} \frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)} \right\} \Delta_t^\star dt = 0 \tag{6.4}$$

where $\Delta_t^\star$ is the response of output to automation and is given by (3.14). Both the firms and the government increase automation until the gains $\Delta_t^\star$ are zero in present discounted value. We have assumed that automation and reallocation are complements (Assumption 3). Therefore, the flows $\Delta_t^\star$ are *back-loaded* (as in the right panel of Figure 3.1). The firm initially incurs a resource cost when investing in automation ($\Delta_t^\star < 0$ for small t), and the gains are realized gradually as workers reallocate ($\Delta_t^\star > 0$ for large t). The government values future gains $\Delta_t^\star$ less than firms do. It recognizes that displaced workers have steeper consumption profiles than non-automated workers who price the firms' equity. In other words, firms are *effectively* too patient and (partly) overlook that the gains from automation take time to materialize. Formally, the left-hand-side of (6.4) is negative when evaluated at the laissez-faire. The optimal tax on automation improves *aggregate efficiency* by flattening consumption profiles — raising consumption early on in the transition

utilities. Absent borrowing constraints, they take the familiar form $1/\eta^{\text{effic},h} \propto u'\left(c_0^h\right)$.

[27] Note that the effective Pareto weights are $\eta^h \equiv \eta^{h,\text{effic}}/u'\left(c_0^h\right)$. This normalization is for convenience.

when displaced workers value it more — at the expense of decreasing the present discounted value of output (net of resource costs). The following remark summarizes these insights.

Remark 2. *Firms (partly) overlook that the gains from automation take time to materialize. The optimal tax on automation improves aggregate efficiency. It raises consumption early on in the transition, precisely when displaced workers are borrowing-constrained.*

6.3.2 Without Active Labor Market Interventions

In practice, ex post policies can be difficult to implement. Active labor market interventions often produce mixed results (Card et al., 2018), or have unintended consequences for untargeted workers (Crépon and van den Berg, 2016). For instance, this would be the case with *gross* flows between occupations, as in our quantitative model. For this reason, we now suppose that the government controls automation (ex ante) but is unable to control labor reallocation (ex post).

Proposition 5 (Second best — ex ante only)**.** *Suppose that the government only controls automation — but labor reallocation T_0 must be consistent with workers' optimization. This reinforces the government's desire to curb automation when unemployment / retraining spells are short ($1/\kappa \to 0$). On the contrary, this reduces the government's desire to curb automation when they are long ($1/\kappa > 1/\kappa^\star$ for some threshold $1/\kappa^\star > 0$)*

Proof. See Appendix A.7. □

Again, it is useful to inspect the social incentives to automate

$$\text{(SB)} \qquad \int_0^{+\infty} \exp\left(-\rho t\right) \sum_h \phi^h \eta^{h,\text{effic}} \frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)} \times \left\{\Delta_t^\star + \phi^A \lambda \exp\left(-\lambda T_0\left(\alpha^{\text{SB}}\right)\right) \mathbf{1}_{\left\{t > T_0\left(\alpha^{\text{SB}}\right)\right\}} T_0'\left(\alpha^{\text{SB}}\right) \Delta_t\right\} dt \; = 0, \tag{6.5}$$

and compare them to the private incentives (6.3). Here, Δ_t is the response of output to reallocation (3.9). Missing active labor market interventions provide an additional motive for policy intervention. The government internalizes the indirect effect of automation on output Δ_t due to the reallocation it induces $T_0'(\cdot) > 0$, in addition to the direct effect. Workers' reallocation at the laissez-faire satisfies

$$\text{(LF)} \qquad \int_{T_0^{\mathrm{LF}}}^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)} \Delta_t dt = 0 \tag{6.6}$$

Absent borrowing constraints, all workers share the same MRS. In this case, the indirect effect of automation Δ_t is no cause for intervention either, given (6.6). When borrowing constraints bind, the private and social incentives to automate differ due to *both* the direct effect $\Delta_t^\star$ and the indirect effect Δ_t. The government should curb automation based on the direct effect (Section 6.3.1). The sign of the indirect effect depends on the duration of unemployment / retraining spells.

When unemployment spells are short $1/\kappa \to 0$, the flows Δ_t are *front-loaded* (see Figure 3.1). Workers enjoy a higher wage after they reallocate $\Delta_0 > 0$, but their new wage declines gradually as more workers enter non-automated occupations $\lim_{t\to+\infty} \Delta_t < 0$. Constrained workers put an excessive weight on early, positive payoffs: binding constraints incentivize them to rely on mobility to self-insure. This indirect effect reinforces the government's desire to *curb* automation.

When unemployment spells are sufficiently long, the flows Δ_t are *back-loaded* instead. Workers' earnings decrease during unemployment $\Delta_0 < 0$, before they are paid the wage in their new occupation. Constrained workers put an excessive weight on early, negative payoffs: binding constraints limit their ability to use mobility to self-insure. The indirect effect dampens the government's desire to curb automation, and could in principle lead the government to *stimulate* automation.[28]

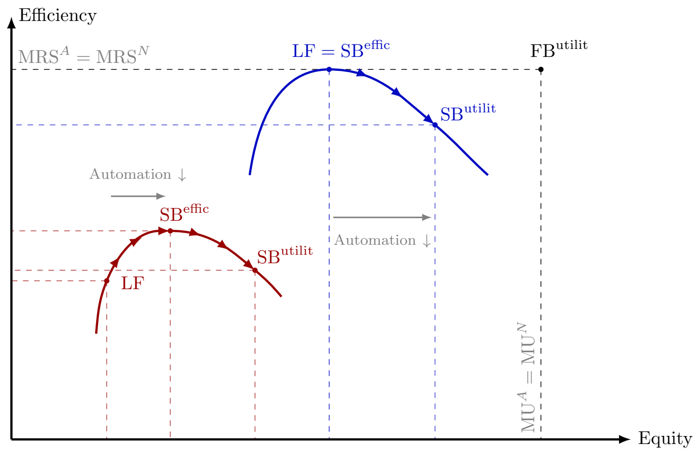
6.4 Equity Concerns

We now introduce equity concerns in our model. This allows us to clarify our contribution relative to the literature on the taxation of automation on equity grounds. Proposition 8 in Appendix A.8 shows that a government with a preference for redistribution curbs automation even if the economy is efficient. Figure 6.1 illustrates this result schematically and connects back to our previous results.

Automation has distributional effects: it reduces equity at the laissez-faire (LF

[28] The average duration of unemployment spells $1/\kappa$ is bounded above by Assumption 2. Therefore, the case where the government stimulates automation might not present itself. Quantitatively, the relevant case is the one where the government *taxes* automation even without active labor market interventions (Section 7).

Figure 6.1: Second best with efficiency ($\underline{a} \to -\infty$) and inefficiency ($\underline{a} \to 0$)



in the figure) relative to the first best of a utilitarian planner ($\mathrm{FB}^{\mathrm{utilit}}$). Displaced workers are worse off and their marginal utility is (persistently) higher than other workers' $\mathrm{MU}^A > \mathrm{MU}^N$. In an efficient economy (blue line in the figure), the intertemporal marginal rates of substitution of displaced workers coincide with the equilibrium interest rate faced by firms who automate $\mathrm{MRS}^A = \mathrm{MRS}^N$. The government does not intervene ($\mathrm{LF} = \mathrm{SB}^{\mathrm{effic}}$) unless it has a preference for redistribution ($\mathrm{SB}^{\mathrm{utilit}}$), in which case it taxes automation and sacrifices efficiency to improve equity. This is the canonical trade-off between equity and efficiency emphasized in the literature on the regulation of automation. In an inefficient economy, there is a wedge between the (intertemporal) marginal rate of substitutions of different workers $\mathrm{MRS}^A < \mathrm{MRS}^N$. Firms are effectively too patient: automation is inefficient. The government can improve *both* efficiency and equity by taxing automation, i.e., there is no trade-off.

6.5 Slowing Down Automation

An extensive literature argues that taxing capital might improve insurance (Conesa et al., 2009; Dávila et al., 2012) or prevent capital overaccumulation (Aiyagari, 1995) in economies with incomplete markets. These two rationales share two features: they rely on the presence on uninsured idiosyncratic risk and optimal policies affect investment in the *long-run*.

The rationale that we propose is conceptually distinct. First, we find that taxing automation is optimal even absent idiosyncratic uncertainty.[29] Second, our mechanism implies that the government should *slow down* automation only while labor reallocation takes place and workers are borrowing constrained, but has no reason to tax automation in the long-run. To clarify this last point, we extend our model and allow for gradual investments in automation. We assume that the law of motion of automation is $d\alpha_t = (x_t - \delta\alpha_t)\,dt$ for some depreciation rate δ and gross investment rate x_t, and that changes in automation are subject to a convex adjustment cost. The proposition below states the result.

Proposition 6 (No intervention in the long-run)**.** *A (utilitarian) government does not intervene in the long-run. That is* $\alpha_t^{LF}/\alpha_t^{FB} \to 1$ *as* $t \to +\infty$.

Proof. See Appendix A.9. □

The reason why the government should not intervene in the long-run is that workers have no incentive to borrow once labor reallocation is complete (Remark 1). That said, some workers could remain borrowing constrained in richer environments with uninsured income risk (as in our quantitative model). This creates a motive for policy intervention in the long-run too (see footnote 40).

6.6 The Direction of Investments

So far, firms could only invest in automation. Taxing it thus unequivocally reduces *total* investment. We now allow investments in a Hicks-neutral technology. We assume that aggregate output is

$$G^\star(\boldsymbol{\mu};\alpha,A) = A\hat{G}(\boldsymbol{\mu};\alpha) - \psi(\alpha) - \Phi(A)$$

and firms choose automation α and productivity A. Hicks-neutral investments do not cause worker displacement. The adjustment is instantaneous and workers are not borrowing constrained. Therefore, the optimal policy changes the *direction* of investments: taxing automation but subsidizing Hicks-neutral investments.

It is also worth noting that our analysis abstracts from other reasons why the government might want to subsidize investment, e.g., firm credit constraints, externalities, etc. Therefore, our results do not necessarily imply that automation

[29] Our result does not rely on the presence of overlapping generations either (Section 6.1), in contrast with Diamond (1965).

should be taxed *on net*. Rather, they suggest that automation should be taxed *relative to* other forms of investment, e.g., through lower investment subsidies as in South Korea.

7 Quantitative Model

In the remaining of this paper, we quantitatively evaluate the efficiency rationale for slowing down automation — even when allowing for various redistributive instruments. To this end, we enrich our baseline model along several dimensions that are important for a credible normative analysis. Section A in the Supplementary Material provides further details on the quantitative model.

7.1 Firms

Production. Occupations produce intermediate goods with technology

$$y_t^h = F\left(\mu_t^h; \alpha_t^h\right) = A^h \left(\varphi^h \alpha_t^h + \mu_t^h\right)^{1-\eta} \tag{7.1}$$

for some elasticity $\eta \in (0,1)$ and productivities $A^h, \varphi^h > 0$.[30] We set $\varphi^A \geq 0$ in automated occupations and $\varphi^N \equiv 0$ in non-automated occupations. The aggregate technology has a constant elasticity of substitution

$$G\left(\left\{y_t^h\right\}\right) \equiv \left(\sum_h \phi^h \left(y_t^h\right)^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}} \tag{7.2}$$

for some elasticity $\nu < 1$. Automated occupations rent the stock of automation on spot markets (Guerreiro et al., 2017) at rate $\{r_t^\star\}$ from a mutual fund.

Investment. A competitive mutual fund invests workers' savings in government's bonds and automation. The law of motion of automation is

$$d\alpha_t = (x_t - \delta\alpha_t)\, dt, \tag{7.3}$$

[30] That is, labor and automation are perfect substitutes within occupations as in Acemoglu and Restrepo (2018) (their tasks correspond to our occupations).

where δ is the rate of depreciation, and x_t is the investment rate. Investment is subject to quadratic adjustment cost $\Omega(x_t; \alpha_t) = \omega (x_t/\alpha_t - \delta)^2 \alpha_t$.[31] In particular, the effective price of investment x_t falls as automation α_t increases.[32] The government taxes automation linearly at rate $\{\tau_t^x\}$ and rebates the revenue to the mutual fund.

7.2 Workers

There are still overlapping generations of workers that are are replaced at rate χ. A worker is indexed by five states: their asset holdings (a); their occupation of employment (h); their employment status (e); a variable that indicates whether they ever switched occupations (ξ); and the mean-reverting component of their productivity (z). We let $\mathbf{x} \equiv (a, h, e, \xi, z)$ be the workers' states and π its measure.

Assets and constraints. Workers invest in the mutual fund with return $\{r_t\}$. In addition, they have access to annuities which allows them to self-insure against survival risk. Financial markets are otherwise incomplete: workers cannot trade contingent securities against the risk that their occupation becomes automated, against the risk that they are not able to relocate, against unemployment risk, or against idiosyncratic productivity risk. Workers now face the budget constraint

$$da_t(\mathbf{x}) = \left[\mathcal{Y}_t^{\text{net}}(\mathbf{x}) + (r_t + \chi) a_t(\mathbf{x}) - c_t(\mathbf{x})\right] dt \tag{7.4}$$

where $\mathcal{Y}_t^{\text{net}}(\mathbf{x})$ denotes net income and r_t is the return on the mutual fund. Workers still face the borrowing constraint (4.4). They hold $a^{\text{birth}}(\mathbf{x}) = 0$ assets at birth.

Occupational choice. Workers choose their first occupation of employment at birth. They supply labor and are given the opportunity to move between occupations with intensity λ. Moreover, workers are subject to linearly additive taste shocks when choosing between occupations. These taste shocks are independent over time and distributed according to an Extreme Value Type-I distribution with mean 0 and variance $\gamma > 0$, as is standard in the literature (Artuç et al., 2010). In partic-

[31] This specification provides a micro-foundation for the cost of automation in our baseline model. The production function net of investment is $\hat{F}(\mu; \alpha) \equiv A(\varphi^A \alpha + \mu)^{1-\eta} - \delta\alpha$ at the steady state.

[32] This captures the price decline of automation technologies over time (Graetz and Michaels, 2018).

ular, workers choose a *non-automated* occupation with hazard

$$\mathcal{S}_t(\mathbf{x}) = \frac{(1-\phi)\exp\left(\frac{V_t^N(\mathbf{x}'(N;\mathbf{x}))}{\gamma}\right)}{\sum_{h'}\phi^{h'}\exp\left(\frac{V_t^{h'}(\mathbf{x}'(h';\mathbf{x}))}{\gamma}\right)}, \tag{7.5}$$

where $V_t^h(\cdot)$ denotes the continuation value associated to automated $(h = A)$ and non-automated $(h = N)$ occupations, and the parameter γ governs the elasticity of labor supply. Workers who reallocate go through unemployment / retraining spells which they exit at rate κ, and experience a productivity loss θ.

Income. Employed workers $(e = E)$ earn a gross labor income

$$\mathcal{Y}_t^{\text{labor}}(\mathbf{x}) = \xi \exp(z)\, w_t^h, \tag{7.6}$$

with the productivity consisting of a permanent component (ξ) and a mean-reverting component (z). The permanent component captures the productivity loss (2.6) that workers incur when reallocating. The mean-reverting component evolves as

$$dz_t = -\rho_z z_t dt + \sigma_z dW_t \tag{7.7}$$

with persistence $\rho_z^{-1} > 0$ and volatility $\sigma_z > 0$. Following Krueger et al. (2016), we suppose that unemployed workers $(e = U)$ receive benefits that are proportional to the gross labor income they would have earned if they had remained employed in their previous occupation. The replacement rate is $b \in [0, 1]$, and we assume that these earnings take the form of home production (Alvarez and Shimer, 2011).[33] We suppose that workers claim profits in proportion to their idiosyncratic (mean-reverting) productivity, as in Auclert et al. (2018).[34] Workers net income is

$$\mathcal{Y}_t^{\text{net}}(\mathbf{x}) = \mathcal{T}_t\left(\mathcal{Y}_t^{\text{labor}}(\mathbf{x}) + \exp(z)\,\Pi_t\right)$$

[33] This last assumption is mostly innocuous. Its only purpose is to avoid introducing an additional motive for distorsionary taxation to finance unemployment insurance.

[34] This assumption implies that workers claim labor and profit income in proportion to their idiosyncratic (mean-reverting) productivity. It is the most neutral possible, as it ensures that the government has no incentives to tax (or subsidize) automation to reduce workers' income risk.

where $\mathcal{T}_t(y) = y - \psi_0 y^{1-\psi_1}$ captures progressive taxation (Heathcote et al., 2017).

7.3 Policy and Equilibrium

The government's flow budget constraint is

$$dB_t = (T_t + r_t B_t - G_t)\, dt \tag{7.8}$$

where B_t is the government's asset holdings, T_t is total tax revenues and G_t is government spending. The resource constraint is now

$$\int a_t(\mathbf{x})\, d\pi_t = -B_t \tag{7.9}$$

The wages are still given by (4.7). The rental rate of automation adjusts so that the firm's demand for automation α_t^A equals the supply α_t from the mutual fund. We normalize the final good price to 1. A competitive equilibrium is defined as before.

8 Quantitative Evaluation

We now use the model to evaluate the importance of our mechanism and perform policy experiments. Section 8.1 discusses the calibration. Section 8.2 describes the laissez-faire transition. Section 8.3 discusses policy interventions. Finally, Supplementary Material B provides details about our numerical implementation.

8.1 Calibration

We parameterize the model using a mix of external and internal calibration. We interpret our initial stationary equilibrium (before automation) as the year 1970. Table 8.1 shows the parameterization.

External calibration. External parameters are set to standard values in the literature. The initial labor share $1-\eta$ is 0.64 based on BLS data. We pick $\varphi^A = 0.3/0.7$ so that 30% of activities within automated occupations effectively become automated (McKinsey, 2017). The depreciation rate δ is 10%, as in Graetz and Michaels (2018). The elasticity of substitution across occupations ν is 0.75, in between the values in Buera and Kaboski (2009) and Buera et al. (2011).[35] The inverse elasticity of intertemporal substitution σ is 2. We set the replacement rate χ to obtain an average active life of 50 years. We pick the unemployment exit hazard parameter κ to match the average unemployment duration in the U.S., as measured by Alvarez and Shimer (2011). The productivity loss θ when moving between occupations is set to match the earnings losses in Kambourov and Manovskii (2009). As in Auclert et al. (2018), we rule out borrowing $\underline{a} = 0$. We use the annual income process estimated by Floden and Lindé (2001) using PSID data and choose the mean reversion ρ_z and volatility σ_z in our continuous time model accordingly. The replacement rate when unemployed b is 0.4, following Ganong et al. (2020). Government spending relative to consumption G_t/C_t is 50% at the initial steady state. The progressivity of the tax schedule ψ_1 is 0.181, as in Heathcote et al. (2017). We choose the intercept of the tax schedule ψ_0 so that the government can finance $G_t/C_t = 0.5$ at the initial steady state. Finally, the ratio of liquidity to GDP $-B_t/Y_t$ is 0.75 at the initial and final steady states, which lies between the values used by Kaplan et al. (2018) and McKay et al. (2016). During the transition, we let the supply of liquidity converge exponentially to its long run level with a half-life of roughly 15 years, following Guerrieri and Lorenzoni (2017).

Internal calibration. We calibrate seven parameters internally: the discount rate (ρ); the mobility hazard (λ); the Fréchet parameter (γ); the occupations' productivities $\left(A^h\right)$; the adjustment cost for automation (ω); and the share of automated occupations (ϕ). We pick these to jointly match seven moments. The discount rate targets an annualized real interest rate of 4 percent. We adjust the mobility hazard to match an occupational mobility rate of 10% per year at the initial steady state, which corresponds to the U.S. level in 1970 in Kambourov and Manovskii (2008). The Fréchet parameter targets an elasticity of labor supply of 2 for the stock of workers (i.e., all generations) following Hsieh et al. (2019).[36] The occupations' productivity $\left\{A^h\right\}$ are such that output is 1 and wages are identical across occu-

[35] We interpret automated occupations as routine-intensive ones which are well represented in manufacturing. Accordingly, we set the elasticity of substitution between automated and non-automated occupations to that between manufacturing and other sectors. The structural change literature strongly suggests that the these occupations are gross complements.

[36] We compute this elasticity in our model by simulating a 10% wage increase in one of the occupations and leaving the other one unchanged.

Table 8.1: Calibration

Parameter	Description	Calibration	Target / Source
Workers			
ρ	Discount rate	0.102	4% real interest rate
σ	EIS (inverse)	2	-
χ	Death rate	$1/50$	Average working life of 50 years
$\underline{a}$	Borrowing limit	0	Auclert et al. (2018)
Technology			
A^A, A^N	Productivities	$(0.938, 1.157)$	Initial output (1)
$1-\eta$	Initial labor share	0.64	1970 labor share (BLS)
δ	Depreciation rate	0.1	Graetz and Michaels (2018)
ϕ	Fraction of automated occupations	0.546	Routine occs. share in 1970
φ^A	Automation productivity	0.43	Fraction of automated activities
ω	Adjustment cost	4	Half-life of automation
ν	Elasticity of subst. across occs.	0.75	(Buera and Kaboski, 2009; Buera et al., 2011)
Mobility frictions			
λ	Mobility hazard	0.312	Occupational mobility rate in 1970
$1/\kappa$	Average unemployment duration	$1/3.2$	Alvarez and Shimer (2011)
θ	Productivity loss from relocation	0.18	Kambourov and Manovskii (2009)
γ	Fréchet parameter	0.052	Elasticity of labor supply
Government			
ψ_0	Tax intercept	0.35	BEA
ψ_1	Tax elasticity	0.181	Heathcote et al. (2017)
$-B/Y$	Liquidity / GDP	0.75	Liquid assets / GDP (Kaplan et al., 2018)
Income process			
ρ_z	Mean reversion	0.0228	Floden and Lindé (2001)
σ_z	Volatility	0.1025	Floden and Lindé (2001)
b	Replacement rate	0.4	Ganong et al. (2020)

pations at the initial stationary equilibrium. The mass of automated occupations ϕ targets an employment share of 56% in routine occupations in 1970 (Bharadwaj and Dvorkin, 2019). Finally, we choose the investment adjustment cost ω so that automation converges to its long-run level with a half-life of 20 years.[37]

8.2 Automation, Reallocation and Inequality

We start by simulating the transition of our economy to its long-run steady state with automation. The economy is initially at its steady state without automation $\alpha = 0$ and no investment takes place $\left(\varphi^A = 0\right)$. In period $t = 0$, automation becomes possible ($\varphi^A > 0$). The initial steady state with $\alpha = 0$ is now unstable. We consider a small increase in automation $\alpha_0 > 0$ which initiates the convergence to the new long-run (stable) steady state with automation. We choose the initial stock α_0 to be $1/10$ of its long-run level.[38]

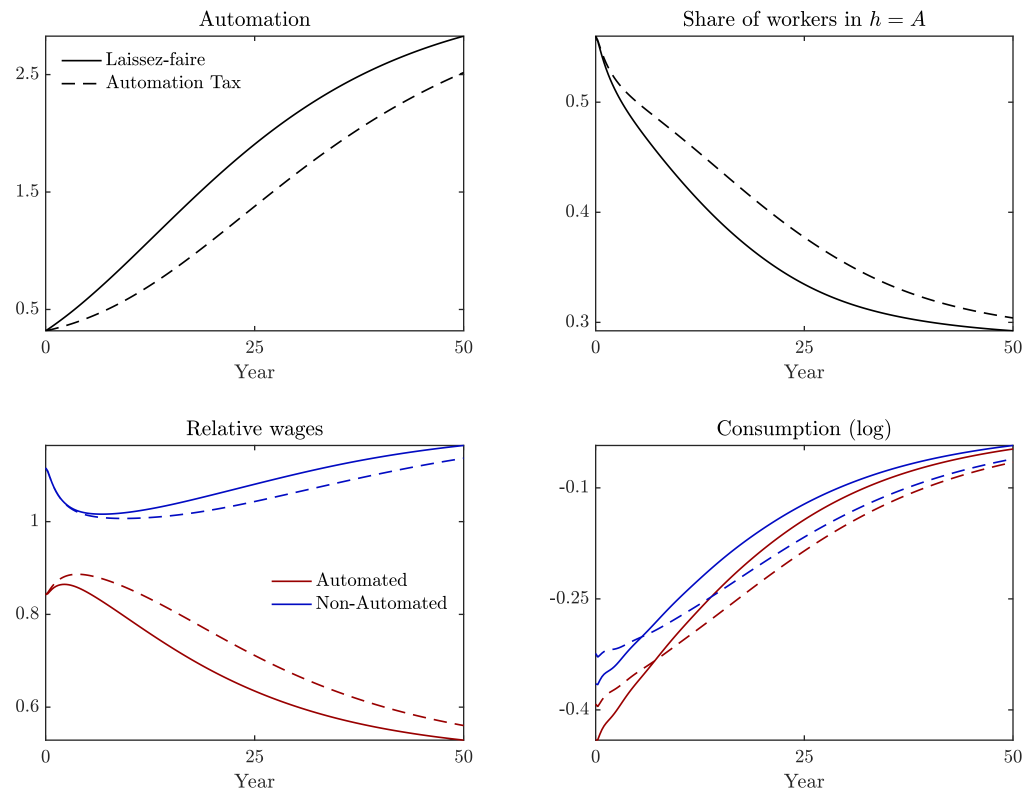
Figure 8.1 illustrates the transition at the laissez-faire (solid lines). Automation converges to its steady state with a half-life of 20 years. The rise in automation displaces workers and reallocates labor away from automated occupations. Despite this reallocation, wages decline gradually in automated occupations (red line) but increase in non-automated occupations (blue line) since the two occupations are gross complements. Inequality rises substantially even at short horizons: the relative wage in automated occupations is about 35% lower after 15 years compared to its steady state level. Finally, automated workers consume less and have steeper consumption profiles — their MRS is lower — as they are more likely to become borrowing constrained.[39] The same figure illustrates the effect of slowing down automation (dashed lines). The sequence of distortionary taxes on automation $\{\tau_t^x\}$ that we feed in are such that the half-life of automation increases to roughly

[37] This is a typical convergence rate in neoclassical growth models. This exercise is rather conservative. The empirical half-life of automation is about 10 years (Acemoglu and Restrepo, 2020). A slower convergence rate dampens our mechanism by limiting the consequences of automation early on during the transition.

[38] For comparison, the stock of automation in 1990 was roughly $1/5$ of its level in 2020 (Acemoglu and Restrepo, 2020). If anything, choosing a lower share dampens our mechanism by limiting the impact of automation early on during the transition.

[39] The share of hand-to-mouth workers is roughly 18% at the initial steady state, which is somewhat lower than the estimates found in the literature on heterogeneous agent models (Kaplan et al., 2018). A lower share of hand-to-mouth workers is conservative with respect to our mechanism since it reduces the share of displaced workers who become borrowing constrained.

Figure 8.1: Allocations



Notes: Solid curves correspond to the laissez-faire anddashed curves to the equilibrium with the automation tax. Red curves denote workers initially in automated occupations and blue curves in non-automated ones. Relative wages are wages normalized by their initial steady state level.

25 years. As expected, labor reallocation slows down and so does the fall in wages and consumption in automated occupations. Finally, consumption profiles become flatter and the wedge between MRSs closes faster, as the share of automated workers who are constrained is much less persistent.

8.3 Second Best Policies and Welfare

We now solve for the optimal policy and quantify welfare gains. The government maximizes

$$\mathcal{W}(\boldsymbol{\eta}) \equiv \int_{-\infty}^{+\infty} \int \eta_t(\mathbf{x})\, V_t^{\text{birth}}(\mathbf{x})\, d\pi_t(\mathbf{x})\, dt \tag{8.1}$$

where $V_t^{\text{birth}}(\mathbf{x})$ is the value of a worker born in period t that draws a state $\mathbf{x}$, and $\eta_t(\mathbf{x})$ are Pareto weights. The government maximizes this objective by choosing taxes on investment $\{\tau_t^x\}$ and rebating the proceedings to the mutual fund.

As in our tractable model, we work with the primal problem. Solving for the exact sequence of $\{\alpha_t\}$ is computationally challenging and beyond the scope of this paper. Instead, we restrict our attention to simpler (or arguably more realistic) parametric perturbations of this sequence. Details are provided in Supplementary Material A.3 and B.[40] For each of these perturbations, we compute the transition dynamics and evaluate welfare (8.1). We then find the second best sequence of automation $\{\alpha_t^{\text{SB}}\}$ and calculate welfare gains $\Delta\mathcal{W}$ (in consumption equivalent terms) relative to the laissez-faire. We repeat this exercise using *efficiency* and *utilitarian* weights $\eta_t(\mathbf{x})$ (Supplementary Material A.3). Efficiency weights are chosen so that the government has no preference for redistribution, whereas utilitarian weights introduce a motive for redistribution.

Table 8.2 reports our findings. In our benchmark calibration, the government finds it optimal to slow down automation substantially *on efficiency grounds alone*. The optimal half-life is about 47 years — more than double the half-life at the laissez faire — and this policy achieves sizable welfare gains of roughly 4%. The gains are even larger with utilitarian weights (roughly 6%) since slowing down automation improves not only efficiency but also equity. The optimal speed of automation turns out to be similar in both cases because the additional incentives to redistribute are small when automation takes place sufficiently slowly. As anticipated in Remark 2, we find in our simulations that these welfare gains are achieved by flattening consumption profiles and raising consumption early on during the transition when displaced workers value it more.

Alternative calibrations. We consider two alternative calibrations of our model.[41] The goal is to explore the sensitivity of results to two important features that affect workers' consumption profiles. Our first alternative calibration focuses on the average duration of unemployment / retraining spells upon reallocation. We increase their average duration $(1/\kappa)$ to 2 years with the idea that workers dis-

[40] In particular, we do not constrain automation to converge to its laissez-faire level in the long-run. The reason is that our quantitative model also features uninsured idiosyncratic risk which introduces an additional motive for intervention. It is well known that a long-run tax (or subsidy) on *capital* can be optimal when markets are incomplete (Section 6.5). However, we find that long-run interventions produce modest improvements in the government's objective (8.1).

[41] For each of these two alternative calibrations, we re-calibrate the rest of the parameters to match the same moments as in our benchmark.

Table 8.2: Welfare Gains $\Delta\mathcal{W}$ from Second Best Interventions

		Alternative calibrations		Alternative policies	
	Benchmark	Long unempl.	High liquid.	Transfers	Joint
Efficiency	3.8%	3.5%	0.6%	0.3%	3.9%
Utilititarian	5.9%	5.8%	2.3%	3.0%	8.7%

Note: 'Benchmark' corresponds to the gains from optimal automation taxes under the calibration described in Section 8.1. 'Long unempl.' and 'High liquid.' denote alternative calibrations with $1/\kappa = 2$ and $-B/Y = 1.4$. 'Transfers' corresponds to the gains from an alternative policy that transfers $10k to automated workers at time $t = 0$ financed with government debt. 'Joint' combines both optimal automation taxes and targeted transfers. 'Efficiency' and 'Utilitarian' compute the gains and optimal automation taxes using the two Pareto weights (Supplementary Material A.3).

placed by automation could take more than the 3 months needed by the typical U.S. worker to exit unemployment (or retrain).[42] Our benchmark calibration with shorter unemployment / retraining can also be interpreted as one where the government adopts active labor market interventions that facilitate retraining. We find that the optimal half-life of automation increases to 52 years with longer unemployment / retraining as a larger share of displaced workers become borrowing constrained during their long reallocation spells. The welfare gains from slowing down automation are comparable to those obtained with shorter unemployment / retraining. This suggests that slowing down automation is optimal even when the government facilitates retraining.

Our second alternative calibration focuses on the degree of liquidity in the economy. We increase the ratio of liquidity to GDP $(-B/Y)$ from our benchmark 0.75 to 1.4 (as in McKay et al., 2016). This level of liquidity is several times larger than effective liquid asset holdings by the average US household (Kaplan et al., 2018), which substantially alleviates borrowing constraints. We find much smaller welfare gains, especially under efficiency weights as anticipated in Section 4.3.

Targeted transfers. Government transfers that target automated workers could in principle be an effective tool to respond to automation. In particular, we showed

[42] We suppose that unemployment benefits last for the entire duration of the reallocation spells. This policy is similar to Trade Readjustment Allowances which extend benefits to workers negatively affected by foreign imports while they retrain.

in Section 6.1.1 that a government could implement a first best allocation without taxing automation if the transfers fully alleviate the borrowing constraints. We allow for realistic targeted transfers in the following, and compare the welfare gains that they produce to those from the optimal tax on automation. Specifically, at time $t = 0$, the government gives a transfer of $10k to workers initially employed in automated occupations. The transfers are financed via an increase in debt. These transfers are rather generous: they correspond to the maximum amount allowed by the Reemployment Trade Adjustment Assistance program (RTAA).[43]

The fourth column in the table shows that the targeted transfers alone produce only small improvements in aggregate efficiency. The welfare gains are only 0.3% under the efficiency weights — much smaller than the 3.8% gains from the automation tax (first column). On the contrary, the gains are substantially larger with utilitarian weights (3.0%) yet still about half of those obtained with the optimal automation tax (5.9%). Together, these results imply that targeted transfers of this magnitude are an effective tool for redistributing towards automated workers but do little to alleviate borrowing constraints in the medium-run and address inefficient automation. Finally, we combine the optimal automation tax with targeted transfers which delivers substantially higher welfare gains when the government is utilitarian.

9 Conclusion

We presented two novel results in economies where workers displaced by automation face reallocation and borrowing frictions. First, automation is inefficient when these frictions are sufficiently severe. Firms fail to internalize that workers displaced by automation have a limited ability to smooth consumption while they reallocate. Second, absent redistributive tools that fully alleviate borrowing frictions, the government should slow down automation while displaced workers reallocate but not tax it in the long-run. The optimal policy improves aggregate efficiency, raising consumption early on in the transition precisely when displaced workers value it more. Quantitatively, we found that slowing down automation achieves substantial efficiency and welfare gains, even when the government can implement generous transfers to displaced workers.

[43] Average earnings are $65k at the initial steady state.

To derive sharp results and clarify the mechanisms at play, our model necessarily abstracted from many features. Some of these are worth discussing now. Tax-codes often subsidize capital and R&D expenditures on the grounds that firms face credit constraints or that there are externalities involved — features that our analysis has ignored. Thus, our results do not necessarily imply that automation technologies ought to be taxed *on net*, as is the case for automated cashiers in the Swiss canton of Geneva or automonous vehicles used by transportation companies in Nevada. Instead, they imply that subsidies on investment in automation should be lowered *temporarily* while the economy adjusts and displaced workers reallocate, which is similar to the lower tax credits for automation in South Korea.

Our quantitative model points to two directions for future work. First, we found that the optimal policy is crucially determined by how steep the consumption profiles of workers displaced by automation are. It would be interesting to measure these profiles and compare them to the estimates for the average US worker used in our quantitative exercises. For instance, the profiles could be steeper if automated workers are unemployed for longer while they reallocate. Second, the quantitative model is rich enough to tackle other optimal policy questions where the dynamics of labor reallocation and asset markets imperfections are relevant, such as how governments should manage declining regions or the economy's adjustment to international trade.

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Online Appendix for: Inefficient Automation

This online appendix contains the proofs and derivations of all theoretical results for the article “Inefficient Automation.” The end of this appendix contains additional results referenced in the main article.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded “A.” or “B." refer to the main article.

A Proofs and Derivations

A.1 Proof of Proposition 1

The proof consists of two steps. In the first step, we decompose the problem (3.1)–(3.7) into a dynamic problem and a sequence of statics ones. In the second step, we characterize the efficient allocation of labor across occupations and the associated level of consumption.

Objective. The planner's problem is equivalent to

$$\max_{\{C_t,Q_t,m_t,\hat{m}_t,\mu_t\}_{t\geq 0}} \int_0^{+\infty} \exp\left(-\rho t\right) U_t\left(C_t\right) dt \tag{A.1}$$
$$\text{s.t. } (3.2)-(3.7)$$

with the felicity function

$$\exp\left(-\rho t\right) U_t\left(C_t\right) \equiv \max_{\left\{c_s^h\right\}} \chi \sum_h \phi^h \int_0^{+\infty} \eta_{t-s}^h \exp\left(-\left(\rho+\chi\right)s\right) u\left(c_s^h\right) ds \tag{A.2}$$
$$\text{s.t. } C_t = \chi \sum_h \phi^h \int_0^{+\infty} \exp\left(-\chi s\right) c_s^h ds$$

Here, $c_s^h \equiv c_{t-s,t}^h$ denotes the consumption in period t of the generation born in period $t-s$ and initially located in occupation h. Solving the static problem,

$$c_s^h = \frac{\left(\eta_{t-s}^h \exp\left(-\rho s\right)\right)^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp\left(-\chi\tau\right)\left(\eta_{t-\tau}^h \exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}} d\tau} C_t \tag{A.3}$$

and the felicity function $U_t\left(\cdot\right)$ is given by

$$U_t\left(C_t\right) \equiv \beta_t\left(\boldsymbol{\eta}\right) u\left(C_t\right) \tag{A.4}$$

$$\beta_t\left(\boldsymbol{\eta}\right) \equiv \chi \int_0^{+\infty} \bar{\eta}_{t-s} \exp\left(-\rho\left(s-t\right)-\chi s\right) \times \left(\frac{\left(\bar{\eta}_{t-s}\exp\left(-\rho s\right)\right)^{\frac{1}{\sigma}}}{\chi \int_0^{+\infty} \exp\left(-\chi\tau\right)\left(\bar{\eta}_{t-\tau}\exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}} d\tau}\right)^{1-\sigma} ds. \tag{A.5}$$

where $(\bar{\eta}_{t-s})^{\frac{1}{\sigma}} \equiv \sum_h \phi^h \left(\eta^h_{t-s}\right)^{\frac{1}{\sigma}}$ is the average Pareto weight within a generation.

Labor allocation. Fix some period $T \geq 0$. Consider the planner's decision to reallocate workers employed in automated occupations, i.e. the choice of $\{m_t\}$ and $\{\hat{m}_t\}$. Using a standard variational argument, it is optimal to reallocate all members of existing generations $(m_t = 1)$ if and only if the present discounted value of the marginal labor productivities is higher in non-automated occupations

$$\int_T^{+\infty} \exp\left(-\rho\left(t-T\right)\right) U_t'\left(C_t\right) \Delta_t dt > 0, \tag{A.6}$$

where

$$\Delta_t \equiv \exp\left(-\chi t\right)\left(\left(1-\theta\right)\left(1-\exp\left(-\kappa\left(t-T\right)\right)\right)\mathcal{Y}_t^N - \mathcal{Y}_t^A\right) \tag{A.7}$$

captures the marginal increase in output from reallocating an additional worker. This term reflects the difference in marginal productivities across occupations $\mathcal{Y}_t^h \equiv 1/\phi^h \partial_h G^\star\left(\boldsymbol{\mu}_t;\alpha\right)$, the permanent productivity loss θ, the average duration of unemployment spells $1/\kappa$, and the share $\exp\left(-\chi t\right)$ of the marginal workers that survive. The planner reallocates none of these workers $(m_t = 0)$ if and only if the inequality (A.6) is reversed. Similarly, the planner reallocates all members of entering generations $(\hat{m}_t = 1)$ if and only if

$$\int_T^{+\infty} \exp\left(-\rho\left(t-T\right)\right) U_t'\left(C_t\right) \exp\left(-\chi\left(t-T\right)\right)\left[\mathcal{Y}_t^N - \mathcal{Y}_t^A\right] dt > 0, \tag{A.8}$$

and reallocates none of them $(\hat{m}_t = 0)$ if and only if the inequality is reversed. The planner chooses an interior solution $\hat{m}_t \in (0,1)$ otherwise. By Assumptions 1–2, there exists some $T_0^{\text{FB}} > 0$ such that the planner reallocates all members of existing generations $(m_t = 1)$ for all $t \in \left[0, T_0^{\text{FB}}\right)$. In period $T = T_0^{\text{FB}}$, the left-hand side of (A.6) is zero. Inspecting (A.6) and (A.8), the planner continues to reallocate entering generations since they are subject to neither a productivity cost nor unemployment. That is, there exists some T_1^{FB} with $0 < T_0^{\text{FB}} < T_1^{\text{FB}}$ such that the planner reallocates all members of new generations $(\hat{m}_t = 1)$ for all $t \in \left[0, T_1^{\text{FB}}\right)$. Furthermore, $T_1^{\text{FB}} < +\infty$ since the technologies $F(\cdot)$ and $\hat{F}(\cdot)$ satisfy Inada conditions. From $t = T_1^{\text{FB}}$ onward, the left-hand side of (A.8) holds with equality and the planner chooses $\hat{m}_t \in (0,1)$ to ensure that the marginal productivities are equalized $\mathcal{Y}_t^N = \mathcal{Y}_t^A$ for all $t \in \left[T_1^{\text{FB}}, +\infty\right)$. The planner does not reallocate existing generations ($m_t = 0$) for all $t \geq T_0^{\text{FB}}$. Summing up,

$$m_t = \begin{cases} 1 & \text{if } t \in \left[0, T_0^{\text{FB}}\right) \\ 0 & \text{if } t \in \left[T_0^{\text{FB}}, +\infty\right) \end{cases} \quad \text{and} \quad \hat{m}_t = \begin{cases} 1 & \text{if } t \in \left[0, T_1^{\text{FB}}\right) \\ \in (0,1) & \text{if } t \in \left[T_1^{\text{FB}}, +\infty\right) \end{cases} \tag{A.9}$$

with $\{\hat{m}_t\}$ chosen for $t \geq T_1^{\text{FB}}$ such that the effective labor supplies in the two occupations remain constant over time. The two stopping times satisfy (3.8) in the text. Solving the differential equation (3.4) and evaluating using (A.9) gives

$$\mu_t^A = \exp\left(-\lambda \min\left\{t, T_0\right\} - \chi t\right) \tag{A.10}$$

for all $t \in [0, T_1)$, evaluated at $T_0 = T_0^{\text{FB}}$ and $T_1 = T_1^{\text{FB}}$. Solving (3.5)–(3.7) gives

$$\mu_t^N \equiv 1 + \frac{\phi}{1-\phi}\left(1 - \mu_t^A\right) - \tilde{\mu}_t - \theta\hat{\mu}_t \tag{A.11}$$

and

$$\tilde{\mu}_t = \frac{\lambda}{\lambda-\kappa}\frac{\phi}{1-\phi}\exp\left(-\left(\kappa+\chi\right)t\right)\left(1 - \exp\left(-\left(\lambda-\kappa\right)\min\left\{t, T_0\right\}\right)\right) \tag{A.12}$$

$$\begin{aligned} \hat{\mu}_t = &\frac{\lambda}{\lambda-\kappa}\frac{\phi}{1-\phi}\exp\left(-\chi t\right) \times \\ &\qquad \Big[\left(1 - \exp\left(-\kappa \min\left\{t, T_0\right\}\right)\right) - \frac{\kappa}{\lambda}\left(1 - \exp\left(-\lambda \min\left\{t, T_0\right\}\right)\right) \\ &\qquad + \left(1 - \exp\left(-\left(\lambda-\kappa\right)T_0\right)\right)\left[\exp\left(-\kappa T_0\right) - \exp\left(-\kappa \max\left\{t, T_0\right\}\right)\right]\Big] \end{aligned} \tag{A.13}$$

for all $t \in [0, T_1)$, evaluated at $T_0 = T_0^{\text{FB}}$ and $T_1 = T_1^{\text{FB}}$. For $t \geq T_1^{\text{FB}}$, the effective labor supplies $\boldsymbol{\mu}_t$ adjust so that the marginal productivities are equalized across occupations $\mathcal{Y}_t^N = \mathcal{Y}_t^A$. The expression (3.11) in the text is obtained by taking the limit with no unemployment ($1/\kappa \to 0$). Finally, consumption is given by aggregate output $C_t = G^\star\left(\boldsymbol{\mu}_t; \alpha\right)$.

Assumption 2. We have supposed so far that that the average unemployment duration and the productivity loss are sufficiently small that labor mobility takes place at the first best, i.e. $T_0^{\text{FB}} > 0$. This occurs whenever the productivity cost associated to reallocation is sufficiently small

$$\theta \leq 1 - \frac{\int_0^{+\infty} \exp\left(-\left(\rho+\chi\right)t\right) U_t'\left(\tilde{C}_t\right) \tilde{\mathcal{Y}}_t^A dt}{\int_0^{+\infty} \left(1-\exp\left(-\kappa t\right)\right) \exp\left(-\left(\rho+\chi\right)t\right) U_t'\left(\tilde{C}_t\right) \tilde{\mathcal{Y}}_t^N dt} \tag{A.14}$$

where the terms on the right-hand side are defined as above, but evaluated with an alternative technology and a counterfactual sequence of (effective) labor supplies and consumption. These labor supplies are still given by (A.10)–(A.13) but are now evaluated at $T_0 \equiv 0$ and T_1 given by (3.8). Consumption is still given by output $C_t = G^\star\left(\tilde{\boldsymbol{\mu}}_t;\alpha\right)$. We evaluate automation at some automation level $\bar{\alpha} > 0$ such that $\partial_\alpha G^\star\left(\mu,\mu';\bar{\alpha}\right) > 0$ when reallocation has not taken place yet $\mu = \mu' = 1$.[44,45] By definition, the sequences of consumption and the marginal productivities in (A.14) are not indexed by any of the mobility parameters (θ,κ) since existing generations do not reallocate at this allocation. Therefore, the restriction (A.14) effectively puts an upper bound (jointly) on the average unemployment duration $1/\kappa$ and the productivity loss θ. The coefficients $\left\{Z^A, Z_t^N\right\}$ in Assumption 2 can be read from the numerator and denominator in (A.14).

A.2 Proof of Proposition 2

We first consider a perturbation of the planner's ex post problem as the level of automation changes and we derive an envelope condition. We then state the optimality condition for the planner's ex ante problem.

Envelope. By Proposition 1, the planner's ex post problem (3.1)–(3.7) can be equivalently formulated as

$$V^{\mathrm{FB}}\left(\alpha;\boldsymbol{\eta}\right) = \max_{\{T_0,T_1\}} \int_0^{+\infty} \exp\left(-\rho t\right) U_t\left(C_t\right) dt \tag{A.15}$$

subject to the resource constraint $C_t = G^\star\left(\boldsymbol{\mu}_t;\alpha\right)$, the effective labor supplies given by (A.10)–(A.13) and the restriction $0 < T_0 < T_1 < +\infty$. Note that the problem is differentiable in α by Assumption 1 and is Lipschitz continuous in $\{T_0, T_1\}$. There-

[44] Such a threshold $\bar{\alpha} > 0$ exists by Assumption 1 and the rest of Assumption 2.

[45] Whenever automation satisfies $\alpha \geq \bar{\alpha}$, which is the case at the first best (Proposition 2), the right-hand side in (A.14) remains larger than θ so labor reallocation still takes place.

fore, the following envelope condition applies

$$\frac{\partial}{\partial \alpha} V^{\text{FB}}(\alpha; \boldsymbol{\eta}) = \int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t) \times \underbrace{\frac{\partial}{\partial \alpha} G^{\star}(\boldsymbol{\mu}_t; \alpha)}_{\equiv \Psi_t(\alpha)} dt = 0, \tag{A.16}$$

where consumption $\{C_t\}$, the labor supplies $\{\boldsymbol{\mu}_t\}$ and the terms $\{\tilde{\mu}_t, \hat{\mu}_t\}$ are those characterized in Appendix A.1 when evaluated at α.

Optimality. The solution to the planner's ex ante problem (3.12) is unique and interior. We first show that the solution is interior. First, note that $\alpha^{\text{FB}} > \bar{\alpha}$ where $\bar{\alpha}$ is the exogeneous level of automation implicit in (A.14), i.e. Assumption 2. The reason is that $\Psi_t(\alpha) > 0$ for all $\alpha \in [0, \bar{\alpha}]$ and all $t > 0$. This follows by Assumptions 1–2 and the fact that $\mu_t < 1$ and $\Theta_t(1 - \mu_t) > 1$ since (at least) some members of existing generations reallocate. Therefore,

$$\int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t) \times \Psi_t(\alpha)\, dt > 0 \tag{A.17}$$

for all $\alpha \in [0, \bar{\alpha}]$, so $\alpha^{\text{FB}} > \bar{\alpha}$. Furthermore, $\alpha^{\text{FB}} < 1$ since

$$\lim_{\alpha \to 1} \int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t) \times \Psi_t(\alpha)\, dt = -\infty \tag{A.18}$$

by Assumption 2. Therefore, the solution is interior. Uniqueness follows from the concavity of the value (A.15) in α. To see that, consider two automation levels (α_0, α_1) and let $\mu_t(\alpha)$ and $\mu_t'(\alpha)$ denote the associated effective labor supplies in $h \in \{A, N\}$ at the first best. Now, consider a convex combination $\hat{\alpha} \equiv c\alpha_0 + (1-c)\alpha_1$ of the automation levels, for some $c \in (0, 1)$. Note that the effective labor supplies $\hat{\mu}_t^{(\prime)} = c\mu_t^{(\prime)}(\alpha_0) + (1-c)\mu_t^{(\prime)}(\alpha_1)$ are feasible under the laws of motions (3.4)–(3.7). Therefore,

$$\begin{aligned} G^{\star}\left(\mu_t(\hat{\alpha}), \mu_t'(\hat{\alpha}); \hat{\alpha}\right) \geq & G^{\star}\left(\hat{\mu}_t, \hat{\mu}_t'; \hat{\alpha}\right) \\ \geq & cG^{\star}\left(\mu_t(\alpha_0), \mu_t'(\alpha_0); \alpha_0\right) \\ & + (1-c)\, G^{\star}\left(\mu_t(\alpha_1), \mu_t'(\alpha_1); \alpha_1\right) \end{aligned} \tag{A.19}$$

for all periods $t \geq 0$. The second inequality follows by concavity of the aggregate technology with respect to labor supplies and α (Assumption 2).[46] The concavity of the value (A.15) then follows immediately by concavity of the felicity function (A.4). Therefore, a necessary and sufficient condition for an optimum is

$$\frac{\partial}{\partial \alpha} V^{\mathrm{FB}}\left(\alpha^{\mathrm{FB}}; \boldsymbol{\eta}\right) = 0 \tag{A.20}$$

since $V^{\mathrm{FB}}(\cdot; \boldsymbol{\eta})$ is differentiable everywhere. The result follows immediately from (A.16) and (A.20).

A.3 Characterization of Equilibrium

We now characterize the competitive equilibrium in our baseline model (Section 4). We omit distorsionary and lump sum taxes (Section 6) to save on notation.

Ex post. We start by characterizing the equilibrium conditional on an automation level α. The presence of borrowing frictions implies that some workers are potentially borrowing constrained. As a result, the decentralized equilibrium is characterized by four stopping times: the times until which existing and new generations reallocate to non-automated occupations (T_0, T_1); and the times between which workers initially employed in automated occupations are borrowing constrained (S_0, S_1). We start by characterizing the latter, before turning to the former and solving for equilibrium prices.

1. **Consumption-savings.** The time at which workers initially employed in automated occupations become borrowing constrained (S_0) is such that workers deplete their savings[47]

$$\left(\hat{c}^{A}_{0,S_0}\right)^{-\sigma} = \exp\left(\int_0^{S_0} r_t dt - \rho S_0\right)\left(\hat{\mathcal{Y}}^{A}_{0,S_0} + \Pi_{S_0} + \hat{r}_{S_0}\underline{a}\right)^{-\sigma} \tag{A.21}$$

[46] The aggregate technology $G^{\star}$ inherits the concavity of neoclassical technologies used by the final good producer (G) and in each occupation $(F, \hat{F})$.

[47] We have $S_0 = S_1 \to +\infty$ when these workers never become borrowing constrained, since all workers effectively become hand-to-mouth as the economy converges to its new stationary equilibrium. Without loss of generality, we can also set $S_0 = S_1 \equiv 0$. For notational convenience, we choose to do so in the following.

given the budget restriction

$$\hat{c}^A_{0,S_0} = \frac{\int_0^{S_0} \exp\left(-\int_0^t \hat{r}_s ds\right)\left(\hat{\mathcal{Y}}^A_{0,t} + \Pi_t\right) dt + a^A_0 - \exp\left(-\int_0^{S_0} \hat{r}_s ds\right) \underline{a}}{\int_0^{S_0} \exp\left(-\int_0^t \hat{r}_s ds\right) \exp\left(\frac{1}{\sigma}\left(\int_0^t r_s dt - \rho t\right)\right) dt} \tag{A.22}$$

where $\hat{\mathcal{Y}}^A_{0,t}$ is labor income, Π_t are profits, $\hat{r}_t \equiv r_t + \chi$ is the effective return on bonds and $a^A_0 \equiv 0$.[48] The time at which these workers stop being borrowing constrained (S_1) is the one where their savings flow equals the change in their borrowing constraint[49]

$$\hat{c}^A_{S_1,+\infty} = \hat{\mathcal{Y}}^A_{0,S_1} + \Pi_{S_1} + \hat{r}_{s_1}\underline{a} \tag{A.23}$$

with $\hat{c}^h_{t,t'}$ defined by analogy with (A.22), $a^A_{S_1} \equiv \underline{a}$ and the last term in the denominator being zero. The same workers are unconstrained for all $t \geq S_1$. That is, the consumption of automated workers is given by $c^A_t = \exp\left(\frac{1}{\sigma}\left(\int_0^t \hat{r}_s ds - \rho s\right)\right)\hat{c}^A_{0,S_0}$ before the borrowing constraint binds $t \in [0, S_0)$, $c^A_t = \hat{\mathcal{Y}}^A_t + \Pi_t + \hat{r}_t\underline{a}$ when the borrowing constraint binds $t \in [S_0, S_1)$ and

$$c^A_t = \exp\left(\frac{1}{\sigma}\left(\int_{S_1}^t r_s ds - \rho\,(t - S_1)\right)\right)\hat{c}^A_{S_1,+\infty} \tag{A.24}$$

afterwards. In turn, workers initially employed in non-automated occupations and members of generations born at $s > 0$ are unconstrained for all $t \geq 0$. Their consumption is given by

$$c^N_t = \exp\left(\frac{1}{\sigma}\left(\int_0^t r_s ds - \rho t\right)\right)\hat{c}^N_{0,+\infty} \tag{A.25}$$

Finally, aggregate consumption is given by

$$\begin{aligned} C_t =& \phi \exp(-\chi t)\, c^A_t + (1-\phi)\exp(-\chi t)\, c^N_t \\ &+ \chi \int_0^t \exp(-\chi(t-s))\, c^{\text{new}}_{s,t} ds, \end{aligned} \tag{A.26}$$

[48] Lump sum taxes on annuities $T_t(\mathbf{x}) \equiv \chi a_t(\mathbf{x})$ are implicit in the expressions above.

[49] In theory, workers could be constrained over multiple, separate intervals of time. We rule this case out since it does not occur for the parametrizations of interest. This explains why (A.23) implicitly assumes that workers are unconstrained for all periods $t \geq S_1$.

where $c^{\text{new}}_{s,t}$ is the consumption of new generations born at s which is similar to (A.25).

2. **Labor reallocation.** Labor income $\hat{\mathcal{Y}}^h_{s,t}$ in period t for a generation born in s and initially located in occupation h is

$$\hat{\mathcal{Y}}^h_{s,t} = w^A_t + (1 - \exp(-\lambda \min\{t, T_0\})) \left(\Theta_t(\lambda,\kappa)(1-\theta) w^N_t - w^A_t\right) \quad \text{(A.27)}$$

if $h = A, s < 0$ and $\hat{\mathcal{Y}}^h_{s,t} = w^N_t$ otherwise, where

$$\Theta_t(\lambda,\kappa) \equiv \frac{1-\phi}{\phi} \frac{\hat{\mu}_t \exp(\chi t)}{1 - \exp(-\lambda \min\{t, T_0\})} \quad \text{(A.28)}$$

is the share of workers who exited their unemployment spell, with $\{\hat{\mu}_t\}$ given by (A.13) evaluated at the equilibrium stopping times.In any period $t = T$, workers initially employed in automated occupations, i.e., $h = A$, $s < 0$, decide as a large household whether to reallocate to non-automated occupations or not. It is never optimal to postpone mobility. Thus, these workers effectively choose a stopping time T_0. When making this choice, they internalize the effect of this stopping time on labor income, taking prices as given, i.e., the direct effect of T_0 in (A.27)–(A.28) as well as the impulse response of $\{\hat{\mu}_t\}$. Therefore, the optimal stopping time satisfies

$$\int_{T_0}^{+\infty} \exp(-\rho t)\, u'\left(\hat{c}^A_t\right) \Delta_t dt = 0 \quad \text{(A.29)}$$

where $\Delta_t \equiv \exp(-\chi t)\left\{(1-\theta)(1-\exp(-\kappa(t-T_0))) w^N_t - w^A_t\right\}$ captures the marginal increase in labor incomes when the large family reallocates additional workers.[50] This condition becomes

$$\int_{T_0}^{+\infty} \exp\left(-\int_{T_0}^{t} r_\tau d\tau\right) \Delta_t dt = 0 \quad \text{(A.30)}$$

in the case where existing workers are unconstrained after they stop reallocating $t \geq T_0$.

The second stopping time T_1 is such that wages are equalized across auto-

[50] A worker who reallocates between occupations internalizes the risk that she will die through her discount factor $\exp(-(\rho+\chi)t)$, not through the flows Δ_t — contrary to the planner.

mated and non-automated occupations

$$w^A_{T_1} = w^N_{T_1} \tag{A.31}$$

Fixing a sequence of interest rates $\{r_t\}$, the conditions (A.21)–(A.23), (A.27)–(A.28) and (A.30)–(A.31) pin down the equilibrium stopping times (T_0, T_1) and (S_0, S_1).[51] Effective labor supplies $\boldsymbol{\mu}_t$ are given by (A.10)–(A.13) evaluated at the stopping times (T_0, T_1). Aggregate consumption is given by (A.24)–(A.26).

3. **Equilibrium prices.** Equilibrium wages and profits are

$$w^h_t = 1/\phi^h \partial_h G^\star(\boldsymbol{\mu}_t; \alpha) \;\; \forall h \tag{A.32}$$

$$\Pi_t \equiv Y_t - \phi \mu^A_t w^A_t - (1-\phi)\,\mu^N_t w^N_t \tag{A.33}$$

where $Y_t \equiv G^\star(\boldsymbol{\mu}_t; \alpha)$ is equilibrium output. Finally, the interest rate that ensures that $C_t = Y_t$ at equilibrium is

$$r_t = \rho + \frac{\sigma}{Y_t}\left(\phi w^A_t \partial_t \mu^A_t + (1-\phi)\, w^N_t \partial_t \mu^N_t\right) \tag{A.34}$$

when the borrowing constraint does not bind $t \in [0, S_0) \cup [S_1, +\infty)$. The expression for the interest rate when the borrowing constraint binds $t \in [S_0, S_1)$ involves additional terms, so we omit it for concision since we do not use it in the following. Finally,

$$\partial_t \mu^A_t = \left(\mathbf{1}_{\{t<T_1\}}\chi + \mathbf{1}_{\{t<T_0\}}\lambda\right)\mu^A_t \tag{A.35}$$

$$\begin{aligned}\partial_t \mu^N_t = & -\frac{\phi}{1-\phi}\partial_t \mu^A_t - \left(\lambda \frac{\phi}{1-\phi}\mathbf{1}_{\{t<T_0\}}\mu^A_t - (\kappa+\chi)\,\tilde{\mu}_t\right) \\ & + (\theta-1)\,(\kappa\tilde{\mu}_t - \chi\hat{\mu}_t)\end{aligned} \tag{A.36}$$

using (3.4)–(3.7) and the definition of the stopping times.

Ex ante. We now characterize the equilibrium choice of automation. A necessary

[51] We can actually show that (T_0, T_1) and (S_0, S_1) are unique, given $\{r_t\}$.

condition for an interior optimum is[52,53]

$$\int_0^{+\infty} \exp\left(-\int_0^t r_s ds\right) \frac{\partial}{\partial \alpha} \Pi_t(\alpha)\, dt = 0 \tag{A.37}$$

Furthermore, the following envelope condition applies

$$\frac{d}{d\alpha} \Pi_t(\alpha) = \frac{\partial}{\partial \alpha} G^\star(\boldsymbol{\mu}_t; \alpha) \tag{A.38}$$

Therefore,

$$\int_0^{+\infty} \exp\left(-\int_0^t r_s ds\right) \frac{\partial}{\partial \alpha} G^\star(\boldsymbol{\mu}_t; \alpha) = 0 \tag{A.39}$$

This condition is both necessary and sufficient, by Assumption 2.

A.4 Proof of Proposition 3

The result states that laissez-faire equilibrium is inefficient if and only if the borrowing frictions are sufficiently severe $\underline{a} > a^\star$ for some $a^\star \leq 0$. For our purpose, it is sufficient to show that the laissez-faire either satisfies all the restrictions that characterizes first best allocations (Section 3) or violates *at least* one of those. At this point, we do not elaborate on the nature of the inefficiency. Throughout, we define the aggregate and individual allocation $\{\bar{X}_t\}$ with $\bar{X}_t \equiv \left(\left\{\bar{c}^h_{s,t}, \bar{a}^h_{s,t}\right\}, \left\{\bar{\mathcal{Y}}^h_t\right\}, \bar{Y}_t\right)$ to be the one that occurs in the laissez-faire equilibrium without borrowing frictions ($\underline{a} \to -\infty$). We let $(\bar{T}_0, \bar{T}_1)$ denote the associated stopping times. Prices are defined similarly. To economize on notation, the dependence on the reallocation parameters $(\lambda, \kappa, \theta, \chi)$ is implicit when there is no ambiguity. We show sufficiency first, then necessity.

Sufficiency. Define the threshold $a^\star \equiv \inf_t \bar{a}^A_{s,t}$ for existing generations $s < 0$. Then, the laissez-faire allocation coincides with $\{\bar{X}_t\}$ whenever $\underline{a} \leq a^\star$.[54] It suffices to show that $\{\bar{X}_t\}$ is efficient — i.e. there exist some weights $\{\eta^h_s\}$ that implement

[52] The static profit function (4.2) is differentiable in the level of automation by Assumption 1.

[53] As usual,wWe suppose that equity is priced by marginally unconstrained workers. By no arbitrage with bonds, the return on equity (pre-annuities) is $\{r_t\}$.

[54] All other workers, i.e., any occupation h and generation $s \geq 0$, hold at least as much bonds as those initially employed in automated occupations.

this allocation as a first best. When workers are unconstrained,

$$\bar{c}^h_{s,\tau}/\bar{c}^h_{s,t} = \exp\left(\frac{1}{\sigma}\left(\int_t^\tau r_k dk - \rho\left(\tau - t\right)\right)\right) \quad \text{for all } (h,s) \text{ and } t,\tau \geq s \tag{A.40}$$

This quantity does not depend on the initial occupation of employment (h) nor the birth date (s). Therefore, there exists a set of weights $\{\eta^h_s\}$ and coefficients $\{b_t\}$ such that $c^h_{s,t} = b_t\left(\eta^h_{t-s}\exp\left(-\rho s\right)\right)^{\frac{1}{\sigma}} C_t$ for all initial occupations (h), generations (s) and periods $t \geq s$. The sequence $\{b_t\}$ is chosen to satisfy the definition of aggregate consumption (3.2). As a result, the equilibrium consumption allocation coincides with its first best counterpart (A.3) when the planner uses the weights $\{\eta^h_s\}$. It remains to show that the equilibrium stopping times $(\bar{T}_0, \bar{T}_1)$ also coincide with their first best counterparts. When workers are unconstrained, the first stopping time is characterized by (A.29). Then,

$$\int_{T_0}^{+\infty} \exp\left(-\rho t\right) u'\left(\bar{c}^h_{s,t}\right) \Delta_t dt = 0 \quad \text{for all } (h,s) \tag{A.41}$$

using the workers' optimality conditions (A.30) and (A.40). Furthermore, the following envelope condition applies

$$U'_t\left(C_t\right) = \eta^h_s \exp\left(-\rho s\right) u'\left(\bar{c}^h_{s,t}\right) \quad \text{for all } (h,s) \text{ and } t \geq s \tag{A.42}$$

using the planner's intratemporal problem (A.2). It follows that the first stopping time $(\bar{T}_0)$ coincides with its first best counterpart (3.8), using (A.41)–(A.42). Finally, so does the second stopping time $(\bar{T}_1)$ since effective labor supplies still evolve as (A.10)–(A.13) in both cases. To complete the proof of sufficiency, note that $-\infty < a^\star \leq 0$. In the limit where reallocation is fast $1/\lambda, 1/\kappa \to 0$ and $1/\chi \to +\infty$, we have $\bar{\mathcal{Y}}^A_t = \bar{\mathcal{Y}}^N_t$ for all $t \geq 0$ by Proposition 1. Therefore no borrowing takes place and $a^\star \to 0$ in this limit.

Necessity. Define $a^\star$ as above. Let $\underline{a} > a^\star$. Then, there exist some periods $0 \leq t < \tau$ such that

$$c^A_{s,\tau}/c^A_{s,t} > \exp\left(\frac{1}{\sigma}\left(\int_t^\tau r_k dk - \rho\left(\tau - t\right)\right)\right) \quad \text{for all } s < 0 \tag{A.43}$$

at the laissez-faire for workers initially employed in automated occupations. In contrast, the relation above holds with equality for workers initially employed in non-automated occupations since they are unconstrained at equilibrium. It follows that there exist occupation h, generations $s<0$ and s' and periods $s' \leq t < \tau$, such that $c^A_{s,\tau}/c^A_{s,t} \neq c^h_{s',\tau}/c^h_{s',t}$. Therefore, the equilibrium allocation does not satisfy the first best restriction (A.3). We conclude that this equilibrium is inefficient.

A.5 Constrained Inefficiency

Proposition 7 (Constrained inefficiency)**.** *Fix the production function $G^\star$. Suppose that the* laissez-faire *is constrained efficient for some Pareto weights $\boldsymbol{\eta}$. Then, there exists a perturbation of the production function $G^{\star\prime} = \mathcal{G}\left(G^\star, \epsilon\right)$ (with $\mathcal{G}\left(G^\star, \epsilon\right) \to G^\star$ uniformly as $\epsilon \to 0$) and a threshold $\bar{\epsilon} > 0$ such that the second best and* laissez-faire *for this alternative economy do not coincide for all $0 < \epsilon \leq \bar{\epsilon}$.*

The government's optimality conditions to reallocate and automate are

$$\int_{T_0^{\text{SB}}}^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)} \Delta_t dt = \Phi\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right) \tag{A.44}$$

and

$$\int_{0}^{+\infty} \exp\left(-\rho t\right) \frac{u'\left(c_t^N\right)}{u'\left(c_0^N\right)} \Delta_t^\star dt = \Phi^\star\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right) \tag{A.45}$$

respectively. The terms on the left-hand side of (A.44)–(A.45) correspond to the private incentives to automate and reallocate, respectively. The terms on the right-hand capture pecuniary externalities that affect workers through wages and profits — which firms and workers do not internalize. These pecuniary externalities are given by[55]

$$\Phi\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right) \equiv \int_{T_0^{\text{LF}}}^{+\infty} \exp\left(-\rho t\right) \hat{\Phi}_t\left(\cdot\right) dt \tag{A.46}$$

$$\Phi^\star\left(\alpha^{\text{SB}}, T_0^{\text{SB}}; \boldsymbol{\eta}\right) \equiv \int_{0}^{+\infty} \exp\left(-\rho t\right) \hat{\Phi}_t^\star\left(\cdot\right) dt \tag{A.47}$$

[55] The expressions below are obtained by rearranging the optimality conditions from the government's problem (Lemma 1). The derivation of these expressions uses the fact that the stopping time T_0 is chosen optimally, i.e., an envelope condition applies.

where[56]

$$\hat{\Phi}_t(\cdot) \equiv -\frac{\exp\left(\lambda T_0^{\mathrm{SB}}\right)}{\lambda}\frac{1}{\phi^A\eta^A}\left\{\phi^N\eta^N\frac{u'\left(c_t^N\right)}{u'\left(c_0^N\right)}\left[\hat{w}_t^N - \sum_h \phi^h\mu_t^h\hat{w}_t^h\right]\right.$$
$$+\phi^A\eta^A\frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)}\left[\exp\left(-\lambda T_0^{\mathrm{SB}}\right)\hat{w}_t^A + (1-\theta)\frac{\phi^N}{\phi^A}\hat{\mu}_t\left(T_0^{\mathrm{SB}}\right)\hat{w}_t^N\right.$$
$$\left.\left.-\sum_h \phi^h\mu_t^h\hat{w}_t^h\right]\right\} \tag{A.48}$$

$$\hat{\Phi}_t^\star(\cdot) \equiv -\frac{1}{\phi^N\eta^N}\left\{\phi^N\eta^N\frac{u'\left(c_t^N\right)}{u'\left(c_0^N\right)}\left[\hat{w}_t^{N,\star} - \sum_h \phi^h\mu_t^h\hat{w}_t^{h,\star}\right]\right.$$
$$+\phi^A\eta^A\frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)}\left[\Delta_t^\star + \exp\left(-\lambda T_0^{\mathrm{SB}}\right)\hat{w}_t^{A,\star} + (1-\theta)\frac{\phi^N}{\phi^A}\hat{\mu}_t\left(T_0^{\mathrm{SB}}\right)\hat{w}_t^{N,\star}\right.$$
$$\left.\left.-\sum_h \phi^h\mu_t^h\hat{w}_t^{h,\star}\right]\right\} \tag{A.49}$$

for the reallocation and automation decisions, respectively.[57] In turn, $\hat{\mu}_t\left(T_0^{\mathrm{SB}}\right)$ denotes the mass of workers (A.13) who have reallocated and completed their unemployment spell, while the sequences $\left\{\hat{w}_t^h\right\}$ and $\left\{\hat{w}_t^{h,\star}\right\}$ denote the perturbation of wages $w_t^h \equiv \partial_h G\left(\mu_t, \Theta_t\left(1-\mu_t\right); \alpha\right)$ with respect to a change in T_0 and α.[58]

The equilibrium is *constrained efficient* if and only if

$$\Phi\left(\alpha^{\mathrm{LF}}, T_0^{\mathrm{LF}}; \boldsymbol{\eta}\right) = \Phi^\star\left(\alpha^{\mathrm{LF}}, T_0^{\mathrm{LF}}; \boldsymbol{\eta}\right) = 0 \tag{A.50}$$

for *some* weights $\boldsymbol{\eta}$. We now show that if these conditions hold, there is a small perturbation of the production function such that the second best and laissez-faire

[56] The effective Pareto weights are $\eta^h/u'\left(c_0^h\right)$ (see footnote 27).

[57] The last term in each of the brackets in (A.48)–(A.49) corresponds to the change in profits. This is obtained using the definition of profits $\Pi_t = G^\star(\cdot) - \phi w_t^A\mu_t - (1-\phi)\, w_t^N\Theta_t\left(1-\mu_t\right)$ and equilibrium wages $w_t^h = 1/\phi^h G_h^\star(\cdot)$.

[58] Effective labor supplies $\left\{\mu_t^A, \mu_t^N\right\}$ are effectively indexed by T_0, as is apparent from (A.10)–(A.13). These quantities are evaluated at the degree of automation α^{SB} and the stopping time T_0^{SB}.

do not coincide. In particular, consider the perturbed production function

$$\mathcal{G}\left(G^{\star},\epsilon\right)=G^{\star}+\epsilon g\left(\boldsymbol{\mu};\alpha\right) \tag{A.51}$$

where g is any function that satisfies

$$g\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)=0 \tag{A.52}$$

for all $t\geq 0$,

$$\int_{T_0^{\mathrm{LF}}}^{+\infty}\exp\left(-\rho t\right)\frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)}\partial_h g\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)dt \quad =0 \tag{A.53}$$

for each occupation $h\in\{A,N\}$, and

$$\int_{0}^{+\infty}\exp\left(-\rho t\right)\frac{u'\left(c_t^N\right)}{u'\left(c_0^N\right)}\partial_\alpha g\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)dt \quad =0 \tag{A.54}$$

along the initial equilibrium. For instance,

$$g\left(\boldsymbol{\mu}_t;\alpha\right)\equiv\left\{\mu_t^{A,\mathrm{LF}}+\varrho\mu_t^{N,\mathrm{LF}}\right\}\left(\alpha^{\mathrm{LF}}-\alpha\right) \tag{A.55}$$

satisfies (A.52)–(A.54) when choosing $\varrho<0$ appropriately.

Then, the allocation $\left(\boldsymbol{\mu}_t^{\mathrm{LF}};\alpha^{\mathrm{LF}}\right)$ still satisfies all equilibrium conditions — workers' reallocation (A.29), firms' automation (5.2), and the resource constraint (4.8) — after a variation $\varepsilon>0$. That is, the laissez-faire is unchanged. It follows that the pecuniary externality that concerns reallocation (A.48) still nets out $\Phi\left(\alpha^{\mathrm{LF}},T_0^{\mathrm{LF}};\boldsymbol{\eta}\right)=0$. The reason is that this pecuniary externality only involves terms in $D^2_{\boldsymbol{\mu}}G^{\star}$, while the perturbation (A.55) is linear in $\boldsymbol{\mu}$ and cannot affect these terms.

Now, note that $\partial_\alpha g\left(\boldsymbol{\mu}_t;\alpha\right)$ is *increasing* over time when evaluated at the laissez-faire, so that $\partial_\alpha g\left(\boldsymbol{\mu}_0;\alpha\right)<0$ and $\lim_{t\to+\infty}\partial_\alpha g\left(\boldsymbol{\mu}_t;\alpha\right)>0$.[59] Furthermore, note that the sequence of relative marginal utilities $\left\{u'\left(c_t^A\right)/u'\left(c_t^N\right)\right\}$ is *decreasing* over time given (5.1). Put it differently, automated workers put a relatively higher weight on

[59] The first property follows from the definition (A.55) and the law of motions (A.10)–(A.13) for labor $\{\boldsymbol{\mu}_t\}$. The second and third properties follow immediately from (A.54). Note that the sequence of labor allocation — and hence $\partial_\alpha g\left(\boldsymbol{\mu}_t;\alpha\right)$ — could be non-monotonic after the second stopping time T_1 when there are overlapping generations — which we abstract from here (Section 6.1).

earlier flows compared to non-automated workers. It follows that

$$\int_0^{+\infty} \exp\left(-\rho t\right) \phi^A \eta^A \frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)} \partial_\alpha g\left(\boldsymbol{\mu}_t^{\mathrm{LF}}; \alpha^{\mathrm{LF}}\right) dt \quad < 0 \tag{A.56}$$

given (A.54). Thus, we constructed a variation $\mathcal{G}\left(G^\star, \epsilon\right)$ such that the pecuniary externality $\Phi^\star\left(\alpha^{\mathrm{SB}}, T_0^{\mathrm{SB}}; \boldsymbol{\eta}\right) \neq 0$. That is, the second best and the laissez-faire do not coincide after the perturbation $\varepsilon > 0$. Finally, $G^{\star,\prime}\left(G^\star, \epsilon\right) \to G^\star$ uniformly as $\epsilon \to 0$ given (A.55), as claimed.

A.6 Proof of Proposition 4

We first derive the optimality conditions associated to the problem (6.1). We then sign the wedge at the laissez-faire.

The equilibrium degree of automation α^{LF} satisfies

$$\int_0^{+\infty} \exp\left(-\rho t\right) \times \frac{u'\left(c_t^N\right)}{u'\left(c_0^N\right)} \Delta_t^\star dt = 0, \tag{A.57}$$

where $\Delta_t^\star$ is defined by (3.14) and denotes the response of aggregate output to automation. In turn, the second-best level of automation $\alpha^{\mathrm{SB}}\left(\boldsymbol{\eta}\right)$ satisfies

$$\int_0^{+\infty} \exp\left(-\rho t\right) \times \sum_h \phi^h \eta^h \frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)} \left(\Delta_t^\star + \Phi_t^{h,\star}\right) dt = 0 \tag{A.58}$$

where $\left\{\Phi_t^{h,\star}\right\}$ capture distributional effects between workers employed in different occupations, with $\sum_h \phi^h \Phi_t^{h,\star} \equiv 0$ for all periods t. By assumption, the weights $\boldsymbol{\eta} \equiv \boldsymbol{\eta}^{\mathrm{effic}}$ ensure that the distributional terms net out for the automation choice. Therefore,

$$\int_0^{+\infty} \exp\left(-\rho t\right) \times \sum_h \phi^h \eta^h \frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)} \Delta_t^\star dt = 0 \tag{A.59}$$

In the following, we let $\lambda_t^h \equiv u'\left(c_t^h\right) / u'\left(c_0^h\right)$. The sequence $\left\{\lambda_t^A\right\}$ is more *front-loaded* than $\left\{\lambda_t^N\right\}$ since labor incomes (and thus consumption) satisfy $\hat{\mathcal{Y}}_t^A \leq \hat{\mathcal{Y}}_t^N$ and the two converge eventually (Appendix A.3). Now, note that the sequence

$\{\Delta_t^\star\}$ is itself *back-loaded*. To see that, define the sequence $\{\Theta_t\}$ such that

$$\Theta_t\left(1-\mu_t^A\right)\equiv\mu_t^N \tag{A.60}$$

By Assumption 3,

$$\partial_\alpha G^\star\left(\mu_\tau^A,\Theta_t\left(1-\mu_\tau^A\right);\alpha\right)>\partial_\alpha G^\star\left(\mu_t^A,\Theta_t\left(1-\mu_t^A\right);\alpha\right)\equiv\Delta_t^\star, \tag{A.61}$$

for all $\tau\geq t$. . Furthermore,

$$\Delta_\tau^\star\equiv\partial_\alpha G^\star\left(\mu_\tau^A,\Theta_\tau\left(1-\mu_\tau^A\right);\alpha\right)>\partial_\alpha G^\star\left(\mu_\tau^A,\Theta_t\left(1-\mu_\tau^A\right);\alpha\right) \tag{A.62}$$

since $\Theta_\tau>\Theta_t$ as more workers exit unemployment after they reallocate, and $G^\star$ has increasing differences in (Θ,α) since the original production function G defined in (2.3) is neoclassical. It follows from (A.61)–(A.62) that the sequence $\{\Delta_t^\star\}$ is indeed backloaded. Finally, the left-hand side of (A.59) is negative at α^{LF} since the government's values relatively less flows which are more distant in the future. Therefore, it is optimal to *curb* automation.

A.7 Proof of Proposition 5

We proceed as in Appendix A.6. The equilibrium stopping time T_0^{LF} satisfies

$$\int_{T_0^{\mathrm{LF}}}^{+\infty}\exp\left(-\rho t\right)\times\frac{u'\left(c_t^A\right)}{u'\left(c_0^A\right)}\Delta_t=0, \tag{A.63}$$

where Δ_t is defined by (3.9) and denotes the response of aggregate output to labor reallocation. In turn, the second-best level of automation $\alpha^{\mathrm{SB}}(\boldsymbol{\eta})$ satisfies

$$\int_0^{+\infty}\exp\left(-\rho t\right)\sum_h\phi^h\eta^h\frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)}\times \\ \left\{\Delta_t^\star+\phi^A\lambda\mathbf{1}_{\left\{t>T_0\left(\alpha^{SB}\right)\right\}}\exp\left(-\lambda T_0\left(\alpha^{\mathrm{SB}}\right)\right)T_0'\left(\alpha^{\mathrm{SB}}\right)\Delta_t+\hat{\Phi}_t^h\right\}dt=0, \tag{A.64}$$

where $T_0'(\cdot)>0$ denotes the response of reallocation at the laissez-faire and $\{\hat{\Phi}_t^h\}$ capture distributional effects between workers employed in different occupations, with $\sum_h \phi^h \hat{\Phi}_t^h \equiv 0$ for all periods t. By assumption, the weights are $\boldsymbol{\eta} \equiv \boldsymbol{\eta}^{\text{effic}}$ so as to ensure that the distributional terms net out. Therefore,

$$\int_0^{+\infty} \exp\left(-\rho t\right) \sum_h \phi^h \eta^h \frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)} \times \\ \left\{\Delta_t^\star + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{SB}\right)\right\}} \exp\left(-\lambda T_0\left(\alpha^{\text{SB}}\right)\right) T_0'\left(\alpha^{\text{SB}}\right) \Delta_t\right\} dt \ = 0, \tag{A.65}$$

Again, let $\lambda_t^h \equiv u'\left(c_t^h\right)/u'\left(c_0^h\right)$. The sequence $\left\{\lambda_t^A\right\}$ is still more *front-loaded* than $\left\{\lambda_t^N\right\}$. For the reasons outlined in Section 6.3.2, the sequence $\{\Delta_t\}$ can itself be front- or back-loaded depending on the average duration of unemployment / retraining spells. When this reallocation is fast, i.e. $1/\kappa$ small, the sequence $\{\Delta_t\}$ is *front-loaded*.[60] In this case, the term involving $\{\Delta_t\}$ in (A.65) is negative at α_0^{LF} since the government values futureflows more than automated workers. This reinforces the government's desire to *curb* automation. When this reallocation is slow, i.e. $1/\kappa > 1/\kappa^\star$ large, the sequence $\{\Delta_t\}$ is *back-loaded*.[61] Therefore, the term involving $\{\Delta_t\}$ in (A.65) is positive at α_0^{LF}. This reduces the government's desire to curb automation. In theory, this case might not present itself. The reason is that workers might decide not to reallocate altogether if the average duration of unemployment is too long (Assumption 2). In this case, we set $1/\kappa^\star \equiv +\infty$.

A.8 Second best with equity concerns

Proposition 8 (Second best with equity concerns)**.** *Consider the special case of our model with no borrowing frictions — so that the laissez-faire is efficient. Suppose that the government is utilitarian. The government should curb automation whether it can control labor reallocation or not.*

We focus on the case with ex-ante interventions only to streamline the exposition. The proof is very similar in the case where the government intervenes ex ante

[60] The sequence is initially positive as $(1-\theta)\, w_t^N > w_t^A$ at the equilibrium stopping time (Appendix A.3 and the left panel of Figure 3.1). It declines over time as wages converge, and eventually becomes negative.

[61] In the limit with infinitely long unemployment spells, $1/\kappa \to +\infty$, the sequence is entirely back-loaded since workers are unemployed for a long-time. The sequence thus increases over time. However, $(1-\theta)\, w_t^N < w_t^A$ when workers exit unemployment (Appendix A.3 and the left panel of Figure 3.1), so workers would choose not to reallocate in the first place.

(automation) and ex post (labor reallocation). Suppose that there are no borrowing frictions ($\underline{a} \to -\infty$). Then, the decentralized equilibrium is efficient (Proposition 3). As a result, the Negishi weights $\eta^h = 1/u'\left(c_t^h\right)$ support this allocation as a first best (Section 3). This allocation is necessarily second best as well. Abstracting again from overlapping generations ($\chi \to 0$), the equilibrium level of automation α^{LF} satisfies

$$\int_0^{+\infty} \exp\left(-\rho t\right) \sum_h \phi^h \frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)} \times \\ \left\{\Delta_t^\star + \Phi_t^{h,\star} + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{\text{LF}}\right)\right\}} \exp\left(-\lambda T_0\left(\alpha^{\text{SB}}\right)\right) T_0'\left(\alpha^{\text{LF}}\right)\left(\Delta_t + \Phi_t^h\right)\right\} = 0 \tag{A.66}$$

where Δ_t and $\Delta_t^\star$ are defined by (3.9) and (3.14) evaluated at the relevant allocation, and Φ_t^h and $\Phi_t^{h,\star}$ are distributional effects associated to more automation and more reallocation, respectively. By definition, these distributional pecuniary effects satisfy $\sum_h \phi^h \Phi_t^h = \sum_h \phi^h \Phi_t^{h,\star} = 0$ for all periods $t \geq 0$. Now, note that[62]

$$\frac{u'\left(c_t^h\right)}{u'\left(c_0^h\right)} = \frac{U_t'\left(C_t^{\text{LF}}\right)}{U_t'\left(C_0^{\text{LF}}\right)} \tag{A.67}$$

using (A.42), where C_t^{LF} denotes aggregate output at the laissez-faire. Therefore, the pecuniary effects net out

$$\int_0^{+\infty} \exp\left(-\rho t\right) \frac{U_t'\left(C_t^{\text{LF}}\right)}{U_t'\left(C_0^{\text{LF}}\right)} \times \\ \left\{\Delta_t^\star + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{\text{LF}}\right)\right\}} \exp\left(-\lambda T_0\left(\alpha^{\text{SB}}\right)\right) T_0'\left(\alpha^{\text{LF}}\right) \Delta_t\right\} = 0 \tag{A.68}$$

Now, consider the second best problem for a utilitarian goverment. The second best degree of automation with equity concerns $\alpha^{\text{SB}}\left(\bar{\boldsymbol{\eta}}\right)$ then satisfies (A.66), except that current marginal utilities are not divided by $u'\left(c_0^h\right)$. For the second best level

[62] Again, see footnote 27.

of automation to remain unchanged with the utilitarian weights, it has to be that

$$\int_0^{+\infty} \exp\left(-\rho t\right) \frac{U_t'\left(C_t^{\mathrm{LF}}\right)}{U_t'\left(C_0^{\mathrm{LF}}\right)} \sum_h \phi^h u'\left(c_0^h\right) \times \\ \left\{\Delta_t^\star + \Phi_t^{h,\star} + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{\mathrm{LF}}\right)\right\}} \exp\left(-\lambda T_0\left(\alpha^{\mathrm{LF}}\right)\right) T_0'\left(\alpha^{\mathrm{LF}}\right)\left(\Delta_t + \Phi_t^h\right)\right\} = 0 \tag{A.69}$$

Equivalently,

$$\int_0^{+\infty} \exp\left(-\rho t\right) \frac{U_t'\left(C_t^{\mathrm{LF}}\right)}{U_t'\left(C_0^{\mathrm{LF}}\right)} \sum_h \phi^h u'\left(c_0^h\right) \times \\ \left\{\Phi_t^{h,\star} + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{\mathrm{LF}}\right)\right\}} \exp\left(-\lambda T_0\left(\alpha^{\mathrm{LF}}\right)\right) T_0'\left(\alpha^{\mathrm{LF}}\right) \Phi_t^h\right\} = 0, \tag{A.70}$$

using (A.68). Furthermore, note that $u'\left(c_0^A\right) > u'\left(c_0^N\right)$ since automated workers are worse off. In addition, note that

$$\Phi_t^{\star,A} + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{\mathrm{LF}}\right)\right\}} \exp\left(-\lambda T_0\left(\alpha^{\mathrm{LF}}\right)\right) T_0'\left(\alpha^{\mathrm{LF}}\right) \Phi_t^A < 0 \tag{A.71}$$

$$\Phi_t^{\star,N} + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{\mathrm{LF}}\right)\right\}} \exp\left(-\lambda T_0\left(\alpha^{\mathrm{LF}}\right)\right) T_0'\left(\alpha^{\mathrm{LF}}\right) \Phi_t^N > 0 \tag{A.72}$$

since these terms capture the distributional effects of automation in general equilibrium. As automation increases, workers initially employed in these occupations are worse off. They reallocate more as a result of this change, but still earn no more than those initially employed in automated occupations (Appendix A.3). Putting this together,

$$\int_0^{+\infty} \exp\left(-\rho t\right) \frac{U_t'\left(C_t^{\mathrm{LF}}\right)}{U_t'\left(C_0^{\mathrm{LF}}\right)} \sum_h \phi^h u'\left(c_0^h\right) \times \tag{A.73}$$

$$\left\{\Phi_t^{h,\star} + \phi^A \lambda \mathbf{1}_{\left\{t > T_0\left(\alpha^{\mathrm{LF}}\right)\right\}} T_0'\left(\alpha^{\mathrm{LF}}\right) \Phi_t^h\right\} < 0, \tag{A.74}$$

since the left-hand side of (A.73) puts a higher weight on negative payoffs. Therefore, automation is *excessive*, regardless of the average duration of unemployment / retraining spells.

A.9 Proof of Proposition 6

We assume that output (net of investment costs) is

$$Y_t = G^* (\boldsymbol{\mu}_t; \alpha_t) - x_t \alpha_t - \Omega (x_t / \alpha_t) \alpha_t, \tag{A.75}$$

where x_t is the gross investment rate in automation and $\Omega(\cdot)$ is a convex function. The law of motion of automation is $d\alpha_t = (x_t - \delta \alpha_t) dt$ for depreciation rate $\delta > 0$.

We next show that the laissez-faire allocation converges to its *first best* counterpart in the long-run. It follows that it also converges to its *second best* counterpart, regardless of whether the government has commitment or not.[63]

We now guess and verify that the laissez-faire converges to the first best with utilitarian weights $\eta_s^h \propto \exp(-\rho s)$. It suffices to verify that the equilibrium sequence of interest rates $r_t \to \rho$ as $t \to +\infty$. The reason is twofold. First, other aggregates allocations are continuous in $\{r_t\}$ (Section A.3) so that the guess that $\{\alpha_t, x_t, \boldsymbol{\mu}_t, \Pi_t\}$ converge to their first best steady state counterparts is verified too — this part is very similar to the proof of Proposition 10 in Appendix B so we omit it. Second, individual allocations $\mathbf{c}$ are symmetric across workers $c_{s,t}^h = C_t$ both at the laissez-faire and the first best with weights $\boldsymbol{\eta}$.[64] As a result, individual allocations necessarily concide too in the long-run.

To show that $r_t \to \rho$ as $t \to +\infty$, note that all workers are (marginally) unconstrained at equilibrium except for the surviving mass $\exp(-\chi t)$ of workers born in $s < 0$ and initially employed in automated occupations $h = A$. Furthermore, note that all these other workers earn the same income

$$\hat{\mathcal{Y}}_t^{\text{unconstr}} \equiv \frac{1}{1 - \phi \exp(-\chi t)} \left\{ G^\star (\cdot) - x_t \alpha_t - \omega x_t^2 \alpha_t - \phi \exp(-\chi t) \hat{\mathcal{Y}}_t^{\text{constr}} \right\} \tag{A.76}$$

where $\hat{\mathcal{Y}}_t^{\text{constr}} < +\infty$ is the income of constrained workers. Therefore, the income of unconstrained workers converges to the long-run aggregate consumption at the first best $\hat{\mathcal{Y}}_t^{\text{unconstr}} \to C^{\text{FB}}$ as $t \to +\infty$, using the fact that all other aggregates converge to their first best counterpart and the aggregate resource constraint. It

[63] The reason is that workers are hand-to-mouth $\underline{a} \to 0$ so that all the state variables in the government's problem are under its control.

[64] At the laissez-faire, workers are hand-to-mouth so that $c_{s,t}^h = \hat{\mathcal{Y}}_{s,t}^h$ and $\hat{\mathcal{Y}}_{s,t}^h / \hat{\mathcal{Y}}_{s',t}^{h'} \to 1$ as $t \to +\infty$ for all occupations (h, h') and generations (s, s'), using (5.1) and the fact that wages are equalized in the long-run. At the first best, symmetry follows directly from (A.3).

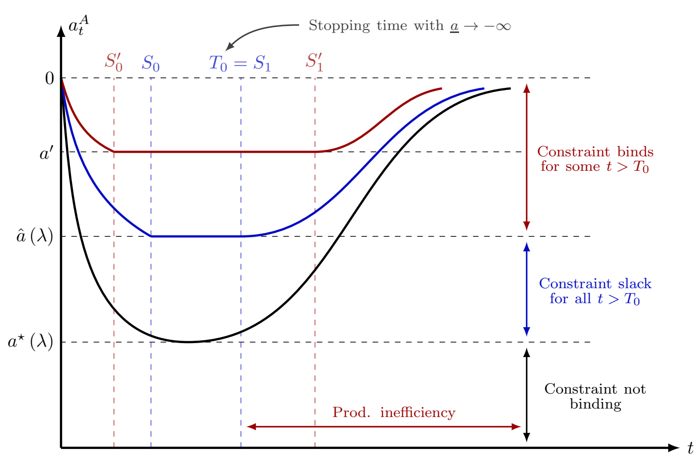
follows that individual consumption $c_t^{\text{unconstr}} \to C^{\text{FB}}$ by market clearing. Thus, the interest rate converges to the discount rate $r_t \to \rho$, using (A.21).

B Additional Results

Proposition 9 (Distorsions in PE)**.** *Fix prices and profits at the level that prevails in an efficient economy without borrowing constraints $\underline{a} \to -\infty$. Then, the consumption choices are distorted if and only if $\underline{a} > a^\star(\lambda, \kappa, \theta, \chi)$ where $a^\star(\cdot)$ is defined in Proposition 3. Furthermore, the labor supply choices are distorted if and only if $\underline{a} > \hat{a}(\lambda, \kappa, \theta, \chi)$ for some threshold $\hat{a}(\cdot) \geq a^\star(\cdot)$.*

Proof. We have already shown that consumption choices are distorted if and only if $\underline{a} > a^\star(\lambda, \kappa, \theta)$ as part of Proposition 3. We now show that labor supply choices are distorted if and only if $\underline{a} > \hat{a}(\lambda, \kappa, \theta, \chi)$ for some threshold $\hat{a}(\cdot) \geq a^\star(\cdot)$. Figure B.1 depicts the dynamics of assets and these thresholds graphically. Throughout, we denote by $\bar{T}_0$ the stopping time that prevails at the efficient equilibrium with no borrowing constraints ($\underline{a} \to -\infty$). All prices are understood to be the ones at this equilibrium.

Figure B.1: Assets



Sufficiency. We proceed in three steps. First, we show that labor supplies are distorted only if borrowing constraints bind at $t = \bar{T}_0$, where $\bar{T}_0$ is the stopping time in the economy with no borrowing frictions. Second, we show that these constraints bind at $t = \bar{T}_0$ if $\underline{a} = 0$. Third, we show that there is some $\hat{a}$ with

$a^\star \leq \hat{a} \leq 0$ such that the constraints bind at $t = \bar{T}_0$ if $\hat{a} < \underline{a} \leq 0$. Finally, we show that the constraints do *not* bind at $t = \bar{T}_0$ if $\underline{a} \leq \hat{a}$. The desired result follows immediately.

Step 1. We show that labor supply choices are distorted only if borrowing bind at equilibrium at $t = \bar{T}_0$. To see this, note that the reallocation decision (A.30) when unconstrained is purely forward-looking. In particular, they are not indexed by workers' asset holdings, and whether they were constrained in any period $t < \bar{T}_0$. Therefore, the labor supply choices are distorted only if borrowing constraints bind in period $t = \bar{T}_0$ (see Appendix A.3 for when this is the case).

Step 2. We now show that the borrowing constraints are binding in period $t = \bar{T}_0$ if $\underline{a} = 0$. To derive a contradiction, suppose that this is not the case. Then, all workers are hold no assets since none of them can save. Furthermore, their Euler equations hold with equality since they are unconstrained. Therefore,

$$\frac{\hat{\mathcal{Y}}^A_{s,t} + \Pi_t}{\hat{\mathcal{Y}}^A_{s,\bar{T}_0} + \Pi_{\bar{T}_0}} = \frac{\hat{\mathcal{Y}}^N_{s,t} + \Pi_t}{\hat{\mathcal{Y}}^N_{s,\bar{T}_0} + \Pi_{\bar{T}_0}} \tag{B.1}$$

for all $s < 0$ and $t \geq \bar{T}_0$ since preferences are isoelastic. However, this restriction cannot hold since $\hat{\mathcal{Y}}^A_t$ increases over time while $\hat{\mathcal{Y}}^N_t$ decreases, using labor incomes (A.27)–(A.28). This leads to the desired contradiction.

Step 3. By continuity of the equilibrium with respect to $\underline{a}$, there exists some $\hat{a}$ with $a^\star \leq \hat{a} \leq 0$ such that the borrowing constraints are binding at equilibrium in period $t = \bar{T}_0$ if $\hat{a} < \underline{a} \leq 0$. This threshold satisfies

$$\hat{a} = \begin{cases} a' & \text{if } a' \leq 0 \\ -\infty & \text{otherwise} \end{cases}, \tag{B.2}$$

where a' ensures that workers initially employed in automated occupations do not want to save or dissave in period $t = \bar{T}_0$

$$\frac{\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^t \hat{r}_s ds\right)\left(\hat{\mathcal{Y}}^A_t + \Pi_t\right) dt + a'}{\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^t \hat{r}_s ds\right) \exp\left(\frac{1}{\sigma}\left(\int_{\bar{T}_0}^t r_s ds - \rho\left(t - \bar{T}_0\right)\right)\right) dt} = \hat{\mathcal{Y}}^A_{\bar{T}_0} + \Pi_{\bar{T}_0} + \hat{r}_{\bar{T}_0} a', \tag{B.3}$$

where incomes and prices are those at the equilibrium with no borrowing frictions.

Step 4. It remains to show that the borrowing constraints are *not* binding at equilibrium in period $t = \bar{T}_0$ if $\underline{a} \leq \hat{a}$. Consider first the case where $\hat{a} = a' \leq 0$. Now, consider a decrease in the borrowing constraint from a' to $\underline{a} \leq a'$. Then, the borrowing constraint does not bind in period $t = \bar{T}_0$ when

$$\hat{r}_{\bar{T}_0}^{-1} \geq \int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^{t} \hat{r}_s ds\right) \exp\left(\frac{1}{\sigma}\left(\int_{\bar{T}_0}^{t} r_s ds - \rho\left(t - \bar{T}_0\right)\right)\right) dt \tag{B.4}$$

as total income exceeds consumption when $\underline{a} \leq a'$. This condition holds since otherwise $a' > 0$ using (B.3), given that the sequence $\{\bar{\mathcal{Y}}_t^0 + \bar{\Pi}_t\}$ is increasing at the original equilibrium, and $r_t \geq \rho$ for all t if the economy without borrowing constraints grows over time. Now, consider the second case where $\hat{a} = -\infty$, i.e., borrowing constraints always bind in period $t = \bar{T}_0$. Then, $\underline{a} \leq \hat{a}$ is never satisfied so the statement still holds.

Necessity. Let $\underline{a} > \hat{a}$ where the threshold is given by (B.2). Then, automated workers are constrained at $t = \bar{T}_0$. We now show that reallocation and automation decisions are distorted in this case. Fix the continuation sequences $\{\bar{r}_t\}_{t \geq \bar{T}_0}$ and $\{\bar{w}_t^h\}_{t \geq \bar{T}_0}$ at the frictionless equilibrium. By definition of the stopping time $\bar{T}_0$,

$$\int_{\bar{T}_0}^{+\infty} \exp\left(-\int_{\bar{T}_0}^{t} \bar{r}_\tau d\tau\right) \bar{\Delta}_t dt = 0 \tag{B.5}$$

using (A.30), where Δ_t was defined in Appendix A.3. However, these workers are hand-to-mouth over a choice-specific interval $t \in [\bar{T}_0, S_1)$. Note that the optimality condition (A.29) is *generically* not satisfied when (B.5) holds.[65] □

Proposition 10 (Second Welfare Theorem)**.** *A first best allocation supported by some Pareto weights* $\boldsymbol{\eta}$ *can be decentralized with lump sum transfers*

$$\tau_{s,t}^h = \frac{\left(\eta_s^h \exp\left(-\rho\left(t-s\right)\right)\right)^{\frac{1}{\sigma}}}{\chi \sum_h \phi^h \int_0^{+\infty} \exp\left(-\chi\tau\right)\left(\eta_{t-\tau}^h \exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}} d\tau} C_t - \left\{\hat{\mathcal{Y}}_{s,t}^h + C_t - \sum_k \phi^k \mu_t^k \mathcal{Y}_t^k\right\}$$

for each initial occupation h, all ages $s \leq t$ and calendar time t, where the quantities on the right-hand side are given by Proposition 1 and (5.1).

[65] If it happens to be satisfied, there is a small perturbation of $1/\kappa$ that ensures it does not.

Proof. We first show that the first best allocation $\{X_t\}$ associated to the weights $\boldsymbol{\eta}$ is part of an equilibrium (ex post), given the transfers

$$\tau_{s,t}^{h}=\frac{\left(\eta_{s}^{h}\exp\left(-\rho\left(t-s\right)\right)\right)^{\frac{1}{\sigma}}}{\chi\sum_{h}\phi^{h}\int_{0}^{+\infty}\exp\left(-\chi\tau\right)\left(\eta_{t-\tau}^{h}\exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}}d\tau}C_{t}-\left\{\hat{\mathcal{Y}}_{s,t}^{h}+C_{t}-\sum_{k}\phi^{k}\mu_{t}^{k}\mathcal{Y}_{t}^{k}\right\}\tag{B.6}$$

and the level of automation α^{FB}. Then, we show that the equilibrium level of automation is α^{FB} (ex ante) when anticipating $\{X_t\}$.

Ex post. Fix automation at α^{FB} and the transfers (B.6). We guess and verify that the sequence of interest rate $\{r_t\}$, wages $\{w_t^h\}$, profits $\{\Pi_t\}$ are part of an equilibrium,

$$\exp\left(-\int_{0}^{t}r_{s}ds\right)=\exp\left(-\rho t\right)\frac{U_{t}'\left(C_{t}\right)}{U_{0}'\left(C_{0}\right)}\quad\text{for all } t\geq 0,\tag{B.7}$$

and

$$w_{t}^{h}=\mathcal{Y}_{t}^{h}\quad\text{for each } h\in\{A,N\}\text{ and all } t\geq 0,\tag{B.8}$$

and

$$\Pi_{t}\equiv C_{t}-\phi\mu_{t}^{A}\mathcal{Y}_{t}^{A}-\left(1-\phi\right)\mu_{t}^{N}\mathcal{Y}_{t}^{N},\tag{B.9}$$

and that the associated allocation is first best. The right-hand side of (B.7)–(B.9) correspond to the first best. It suffices to show that the planner's allocations of consumption (A.3) and labor (A.10)–(A.13) are consistent with workers' optimality given these prices. By construction, the other equilibrium conditions are satisfied: labor markets clear given wages (B.8) and the resource constraint (4.8) is satisfied.

Focusing on the consumption allocation first, we now show that: (i) workers can afford these allocations with a balanced budget given wages (B.8), profits (B.9) and the reallocation choice implicit in (A.10)–(A.13); and (ii) these consumption allocations ensure that Euler equations hold with equality given the interest rate (B.7). Pre-transfer labor incomes $\hat{\mathcal{Y}}_{s,t}^{h}$ are given by (A.27)–(A.28). By construction, transfers (B.6) ensure that the first best consumption allocations (A.3) are affordable

$$\frac{\left(\eta_{s}^{h}\exp\left(-\rho\left(t-s\right)\right)\right)^{\frac{1}{\sigma}}}{\chi\sum_{h}\phi^{h}\int_{0}^{+\infty}\exp\left(-\chi\tau\right)\left(\eta_{t-\tau}^{h}\exp\left(-\rho\tau\right)\right)^{\frac{1}{\sigma}}d\tau}C_{t}=\hat{\mathcal{Y}}_{s,t}^{h}+\Pi_{t}+\tau_{s,t}^{h}\tag{B.10}$$

since wages $\{w_t^h\}$ and profits $\{\Pi_t\}$ are given by (B.8)–(B.9) and equilibirum output satisfies $Y_t = C_t$. We still have to show that the consumption allocations (B.10) are optimal. Consider the planner's intratemporal problem (A.2). At the optimum of this problem, consumption allocations satisfy

$$\eta_{t-s}^h \exp(-\rho s)\, u'\left(c_{t-s,t}^h\right) = \exp(-\rho t)\, U_t'(C_t) \tag{B.11}$$

It follows that workers' Euler equations hold with equality, by definition of the sequence of interest rates (B.7) and using restriction (B.11).

Turning to the labor allocation, we now show that the effective labor supplies coincide with the first best ones (A.10)–(A.13) given the sequence of wages (B.8), profits (B.9) and interest rate (B.7). Occupational choices are still characterized by two stopping times $\left(T_0^{LF}, T_1^{LF}\right)$. The first stopping time $\left(T_0^{LF}\right)$ satisfies

$$\begin{aligned}\int_{T_0^{LF}}^{+\infty} \exp\left(-\int_{T_0^{LF}}^{t} r_\tau d\tau\right) \exp(-\chi t) \times \\ \left((1-\theta)\left(1-\exp\left(-\kappa\left(t-T_0^{LF}\right)\right)\right) w_t^N - w_t^A\right) dt = 0\end{aligned} \tag{B.12}$$

since transfers (B.6) ensure that workers are unconstrained, and using (A.30). It follows that the equilibrium stopping time coincides with the first best $T_0^{LF} = T_0^{\text{FB}}$, using the definition of the first best stopping time (3.8), wages (B.8) and the MRS (B.7). The proof for the second stopping time $\left(T_1^{LF}\right)$ is very similar, so we omit it.

Ex ante. Finally, we show that the first best automation α^{FB} solves the firm's problem (4.1) when it anticipates the equilibrium sequence $\{X_t\}$. Using (A.39), the interest rates (B.7) and the fact that reallocation is unchanged if automation is α^{FB},

$$\int_0^{+\infty} \exp(-\rho t)\, U_t'(C_t) \frac{\partial}{\partial \alpha} G^\star\left(\boldsymbol{\mu}_t; \alpha^{FB}\right) = 0 \tag{B.13}$$

with $C_t = G^\star\left(\boldsymbol{\mu}_t; \alpha^{FB}\right)$, so the degree of automation is efficient $\alpha^{LF} = \alpha^{\text{FB}}$. □

Lemma 2. *Suppose that either: there are no reallocation frictions $1/\lambda, 1/\kappa, \theta \to 0$; or there are no borrowing frictions $\underline{a} \to -\infty$. Then, $\tau_{s,t}^h = \mathbf{0}$ implements a first best.*

Proof. Consider first the case without reallocation frictions $1/\lambda, 1/\kappa, \theta \to 0$. Then, marginal productivities (and incomes) are equalized across occupations $\mathcal{Y}_t^A = \mathcal{Y}_t^N$. Fix the weights $\eta_t^A = \eta_t^N \propto \exp(\rho t)$. Using (B.6), $\left\{\tau_{s,t}^h\right\} = \mathbf{0}$ with these weights.

Now, consider the case without borrowing frictions $\underline{a} \to -\infty$ so that the equilibrium is efficient. Fix a set of weights $\left\{\eta_t^h\right\}$ such that

$$\int \exp\left(-\left(\rho+\chi\right)\tau\right) U_\tau'\left(C_\tau\right) \tau_{t,\tau}^h d\tau = 0 \tag{B.14}$$

using (B.6). By Proposition 10, the efficient allocation associated to these weights can be implemented with transfer $\left\{\tau_{s,t}^h\right\}$ that have a present discounted value of zero for each worker. Thus, $\left\{\tau_{s,t}^h\right\} = \mathbf{0}$ also implements this allocation since workers are unconstrained. $\square$

Supplementary Material for: Inefficient Automation

This supplementary material **is not intended for publication**. It first describes in more detail the quantitative model in the article “Inefficient Automation” and its numerical implementation. It then discusses issues related to the aggregate marginal product of labor in a task-based model.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded “A.” or “B." refer to the main article.

A Quantitative Model

In this appendix, we describe our quantitative model in more detail and discuss the approach used to solve the model numerically. Section A.1 provides a recursive formulation of the workers' problem. Section A.2 states and characterizes the solution to the occupations' problem. Section A.3 discusses the second best.

A.1 Workers' Problem

We discretize time into periods of constant length $\Delta \equiv 1/N > 0$, and solve the workers' problem in discrete time.[1] The workers' problem can be formulated recursively

$$V_t^h\left(a, e, \xi, z\right) = \max_{c, a'} u\left(c\right)\Delta + \exp\left(-\left(\rho + \chi\right)\Delta\right) V_{t+\Delta}^{h,\star}\left(a', e, \xi, z\right) \tag{A.1}$$

$$\text{s.t. } a' = \left(\mathcal{Y}_t\left(\mathbf{x}\right) - c\right)\Delta + \frac{1}{1 - \chi\Delta}\left(1 + r_t \Delta\right) a$$

$$a' \geq 0$$

for employed workers $(e = E)$ and unemployed workers $(e = U)$. The continuation value $V^\star$ before workers observe the mean-reverting component of their income is given by

$$V_t^{h,\star}\left(a, e, \xi, z\right) = \int \hat{V}_t^h\left(a, e, \xi, z'\right) P\left(dz', z\right), \tag{A.2}$$

where $\hat{V}_t\left(\cdot\right)$ is the continuation value associated to the discrete occupational choice. The continuation value for employed workers $(e = E)$ associated to this discrete

[1] Alternatively, we could have formulated the workers' problem in continuous time and solved the associated partial differential equation using standard finite difference methods. However, (semi-)implicit schemes are non-linear in our setting due to the discrete occupational choice. This requires iterating on (A.1)–(A.5) to compute policy functions which limits the efficiency of these schemes. We found that explicit schemes were unstable unless we use a particularly small time step Δ which again proves relatively inefficient. Formulating and solving the workers' problem in discrete time proves to be relatively fast.

chocie problem is[2]

$$\hat{V}_t^h(a,e,\xi,z) = (1-\lambda\Delta)\, V_t^h(a,e,\xi,z) + \lambda\Delta\gamma \log\left(\sum_{h'} \phi^{h'} \exp\left(\frac{V_t^{h'}\left(a, e'\left(h',\mathbf{x}\right),\xi,z\right)}{\gamma}\right)\right) \tag{A.3}$$

with $e'(\cdot) = E$ if $h' = h$ and $e'(\cdot) = U$ otherwise. The associated mobility hazard across occupations is

$$\mathcal{S}_t\left(h';\mathbf{x}\right) = \frac{\phi^{h'} \exp\left(\frac{V_t^{h'}(\mathbf{x}'(h';\mathbf{x}))}{\gamma}\right)}{\sum_{h''} \phi^{h''} \exp\left(\frac{V_t^{h''}(\mathbf{x}'(h'';\mathbf{x}))}{\gamma}\right)} \tag{A.4}$$

In turn, the continuation value for unemployed workers $(e = U)$ is

$$\hat{V}^h(a,e,\xi,z) = (1-\kappa\Delta)\, V^h(a,e,\xi,z) + \kappa\Delta V^h\left(a,1,\xi'\left(h',\mathbf{x}\right),z\right) \tag{A.5}$$

where $\mathcal{S}(\cdot)$ is the mobility hazard, and $\xi'(\cdot) = (1-\theta)\,\xi$ when the reallocation spell is complete. New generations who enter the labor market draw a random productivity z from its stationary distribution and then choose their occupation with a hazard similar to the employed workers'. The only difference is that they experience neither an unemployment spell nor a productivity loss. Worker's labor income is

$$\mathcal{Y}_t(\mathbf{x}) = \begin{cases} \xi \exp(z)\, w_t^h & \text{if } e = E \\ b\mathcal{Y}_t^{h'}(a,E,\xi,z) & \text{otherwise} \end{cases}, \tag{A.6}$$

with $h' \neq h$ denoting the previous occupation of employment. The permanent component of workers' income (ξ) is reduced by a factor $(1-\theta)$ whenever a worker who exits unemployment chooses to enter her new occupation. Finally, the mean-reverting component income (z) evolves as

$$z' = (1 + (\rho_z - 1)\,\Delta)\, z + \sigma_z\sqrt{\Delta}W' \quad \text{with} \quad W' \sim \text{i.i.d.}\mathcal{N}(0,1) \tag{A.7}$$

[2] See Artuç et al. (2010) for the derivation.

A.2 Firms' Problem

We solve the mutual fund's and the firm's problem in continuous time. The mutual fund invests in automation subject to convex adjustment costs and rents its stock to the firm. The mutual fund's problem can be formulated recursively

$$r_t W_t(\alpha) = \max_{\{x,\alpha'\}} r_t^{\star}\alpha - (1+\tau_t^x)\, x - \omega\left(\frac{x}{\alpha} - \delta\right)^2 \alpha + (x - \delta\alpha)\, W_t'(\alpha) + \frac{\partial}{\partial t} W_t(\alpha) \tag{A.8}$$

$$\text{s.t. } x \geq 0$$

where α is the stock of automation, x is gross investment, i.e. $d\alpha_t = (x_t - \delta\alpha_t)\, dt$, $r_t^{\star}$ is the rental rate of automation, and τ_t^x is a potential distorsionary tax on investment. The optimal supply of automation satisfies

$$(r_t + \delta)\left((1+\tau_t^x) + 2\omega\left(x_t^{\star} - \delta\right)\right) = \left\{r_t^{\star} + \omega\left[\left(x_t^{\star}\right)^2 - \delta^2\right]\right\} + \partial_t \tau_t^x + 2\omega\partial_t x_t^{\star}, \tag{A.9}$$

with $x_t^{\star} \equiv x_t/\alpha_t$, together with the law of motion

$$d\alpha_t = \left(x_t^{\star} - \delta\right)\alpha_t dt, \tag{A.10}$$

the initial condition $\alpha_0 = 0$ and a standard transversality condition. In turn, the firm's problem is

$$\max_{\left\{\alpha_t^h, \mu_t^h\right\}} G^{\star}\left(\left\{\alpha_t^h, \mu_t^h\right\}\right) - \phi^A r_t^{\star}\alpha_t^A - \sum_h \phi^h w_t^h \mu_t^h \quad \text{s.t.} \quad \alpha_t^N = 0$$

where α_t^h and μ_t^h denote the the amount of automation and labor services that the firm rents in *each* automated ($h = A$) or non-automated ($h = N$) occupations, and

$$G^{\star}\left(\left\{\alpha_t^g, \mu_t^g\right\}\right) = \left(\sum_g \phi^g \left\{A^g\left(c\alpha_t^g + \mu_t^g\right)^{(1-\eta)}\right\}^{\frac{\nu-1}{\nu}}\right)^{\frac{\nu}{\nu-1}}$$

is the aggregate production function. The equilibrium rental rate is $r_t^\star \equiv c w_t^A$, with the wages given by

$$w_t^h = (1-\eta)\frac{1}{c\alpha_t^h + \mu_t^h}\frac{\left\{A^h\left(c\alpha_t^h + \mu_t^h\right)^{(1-\eta)}\right\}^{\frac{\nu-1}{\nu}}}{\sum_g \phi^g\left\{A^g\left(c\alpha_t^g + \mu_t^g\right)^{(1-\eta)}\right\}^{\frac{\nu-1}{\nu}}} G^\star\left(\left\{\alpha_t^g, \mu_t^g\right\}\right)$$

for each $h \in \{A, N\}$. Finally, market clearing for inputs requires that the firm rents the stock of automation supplied by the mutual fund

$$\alpha_t^A = \alpha_t/\phi \quad \text{and} \quad \tilde{\alpha}_t^N = 0,$$

and that the firm hires the (effective) labor supplied in each occupation

$$\mu_t^h = \frac{1}{\phi^h}\int \mathbf{1}_{\{e=1, h'=h\}}\xi d\pi_t$$

for each $h \in \{A, N\}$.

A.3 Second Best

In this appendix, we state the second best problem we consider in our numerical exercise and discuss our choice of Pareto weights.

Objective. The government's objective is

$$\begin{aligned}\mathcal{W} \equiv &\chi\int_{-\infty}^{0}\int \eta_s(\mathbf{x})\exp\left((\rho+\chi)s\right)V_0^{\text{old}}(\mathbf{x})\,\pi_{s,0}^{\text{old}}(d\mathbf{x})\,ds\\ &+\chi\int_0^{+\infty}\eta_s V_s^{\text{new}} ds,\end{aligned} \tag{A.11}$$

for some Pareto weights $\boldsymbol{\eta}$. The first and second terms capture the contributions of existing ($s < 0$) and new generations ($s \geq 0$), respectively. Following Calvo and Obstfeld (1988), these (continuation) values are evaluated at birth.[3] The value $\exp\left((\rho+\chi)s\right)V_0^{\text{old}}$ is the continuation utility of existing generations over periods

[3] This explains the presence of the additional discounting $\exp\left((\rho+\chi)s\right)$ for existing generation $s < 0$.

$t \geq 0$. The measure $\pi_{s,0}^{\text{old}}$ is the distribution of idiosyncratic states in period $t = 0$ for existing generations born in $s < 0$ (conditional on survival). In turn, the value

$$V_t^{\text{new}} \equiv \int \gamma \log \left(\sum_h \phi^h \exp \left(\frac{V_t^h (0, 1, 0, z)}{\gamma} \right) \right) P^\star (dz) \tag{A.12}$$

is the continuation utility for new generations born in period $t = s \geq 0$, which reflects their occupational choice.[4] Here, $P^\star$ denotes the ergodic distribution of the income process $z'|z \sim P(z)$, i.e., the distribution of productivities at birth.

Pareto weights. We choose two sets of weights: weights that capture the *efficiency* motive for policy intervention, and utilitary weights. We now describe these efficiency weights. Our approach is similar to the one we adopted in our tractable model (Section 5.3). The weights that the government puts on a given worker are inversely related to this worker's marginal utility at birth (evaluated at the laissez-faire transition). This ensures that the government has no incentive to redistribute resources (at birth) to improve equity. In particular, the government weights constrained workers (with a higher marginal utilitary) *less* compared to a utilitarian government. We also assume the the government discounts generations at rate ρ over time, which ensures that the planner does not discriminate across generations at the first best — which is evident from equation (A.3) in the Online Appendix. Therefore, the weights assigned to old generations satisfy

$$\eta_s (\mathbf{x}) = \exp(-\rho s) \times 1/\partial_a V_0^{\text{old,LF}} (\mathbf{x}), \tag{A.13}$$

where $1/\partial_a V_0^{\text{old,LF}} (\mathbf{x})$ is the marginal utility of financial wealth at the laissez-faire. In turn, the weights assigned to new generations satisfy

$$\exp(-\rho s) / \eta_s (z) = \sum_h \mathcal{S}_s^h (0, 1, 0, z) \, \partial_a V_t^{h,\text{LF}} (a, 1, 0, z) \Big|_{a=0} \tag{A.14}$$

for all $s \geq 0$ since new generations start with no financial assets $a = 0$ and have not reallocated yet $\xi = 0$.

Summarizing, the government's objective becomes

[4] Members of a new generation are born with no assets $a = 0$, are employed $e = 1$, and have not incurred the productivity cost associated to switching occupations $\xi = 0$.

$$
\begin{aligned}
\mathcal{W} \equiv & \int \frac{V_0\left(\mathbf{x}\right)}{\partial_a V_0^{\text{old,LF}}\left(\mathbf{x}\right)} \pi_0\left(d\mathbf{x}\right) ds \\
& + \chi \int_0^{+\infty} \exp\left(-\rho s\right) \frac{V_s^{\text{new}}}{\int \sum_h \mathcal{S}_s^h\left(0,1,0,z'\right) \partial_a V_s^{h,\text{LF}}\left(a,1,0,z'\right)\Big|_{a=0} P^\star\left(dz'\right)} ds,
\end{aligned}
\tag{A.15}
$$

where

$$
\pi_0\left(d\mathbf{x}\right) \equiv \int_{-\infty}^0 \chi \exp\left(\chi s\right) \pi_{s,0}^{\text{old}}\left(d\mathbf{x}\right) ds \tag{A.16}
$$

is the unconditional (initial) distribution of idiosyncratic states. When solving for the constrained efficient steady state, we maximize the contribution of generations $s \to +\infty$ to the objective (A.15), i.e., $\lim_{s\to+\infty} V_s^{\text{new}}$.

Policy tools and implementability. The government maximizes the objective (A.11) by choosing an appropriate sequence of distortionary taxes on investment $\{\tau_t^x\}$ and rebating the proceedings back to the mutual fund or the workers. The implementability constraints consist of workers' reallocation and consumption choices.

B Numerical Implementation

We now describe how we solve for the stationary equilibrium and the transition.

Workers' problem. We solve the problem worker's (A.1) using the standard endogenous grid method (Carroll, 2006). In theory, this problem could be non-convex since it involves a discrete choice across occupations. However, we find that this is not the case in our calibration. The variance of the taste shocks γ is sufficiently large that the value function remains concave. We use Young (2010)'s non-stochastic simulation method to iterate on the distribution. Finally, we discretize the income process on a 7-point grid using the method of Rouwenhorst (1995).

Firm's problem. The firm's optimal choice of investment and automation is characterized by the non-linear system of differential equations (A.9)–(A.10). We solve this system using a standard shooting algorithm. Fixing an initial value for investment x_0, we iterate the system forward. We then adjust this initial value until automation converges to its long-run level.

Policy. For numerical reasons, we restrict our attention to simple perturbations of $\{\alpha_t\}$ from the sequence that prevails at the laissez-faire. We do so by repeatedly feeding sequences of taxes $\{\tau_t^x\}$ in the mutual fund's problem (A.8).[5] These taxes

$$\tau_t^x = \exp\left(-\beta t\right)\hat{\tau} + \bar{\tau} \tag{B.1}$$

consist of a persistent component $\hat{\tau}$ and a permanent one $\bar{\tau}$. The taxes converge to monotonically to their permanent level. The persistent component allows to *slow down* automation early on during the transition. In turn, the permanent component controls the long-run level of automation. It is well-known that a long-run tax (or subsidy) on *capital* can be optimal when markets are incomplete — it can improve insurance and / or prevent dynamic inefficiency (Section 5.4). We choose a subsidy $\bar{\tau} = -39.9\%$ so that the economy converges to its constrained efficient steady state. We set the mean-reversion speed β so that the half-life of τ_t^x is the same as the one of automation at the laissez-faire (20 years). Finally, we optimize over $\hat{\tau}$ on a fine grid to find the second best intervention. The Pareto weights (Section A.3) are evaluated at the allocation with the permanent subsidy $\bar{\tau}$ (but no persistent tax $\hat{\tau}$). This ensures that $\bar{\tau}$ is the optimal long-run policy.

C A Task-Based Example

We introduced a general technology $G^\star$ in our benchmark model (Section 2). For illustration, we provide here an explicit example that uses the task-based model of Acemoglu and Restrepo (2018).[6] There are two types of occupations: automatable A (share ϕ) and non-automatable N (share $1-\phi$). Labor and automation (or capital) are perfect substitutes within automatable occupations. The aggregate

[5] The differential equation (A.9) can become stiff when prices are sufficiently persistent. We thus evaluate prices at the *laissez-faire* to avoid stability issues. Re-optimizing for a given sequence of taxes $\{\tau_t^x\}$ yields a new sequence $\{\alpha_t\}$ which was feasible in the original government's problem.

[6] The functional form is similar to the one we adopt in our quantitative model (Section 6). To streamline the exposition, we assume here that the elasticity of substitution between occupations is 1, whereas occupations are gross substitutes in our quantitative model.

production function (excluding the cost of automation) is

$$
\begin{aligned}
G\left(\mu^{A},\mu^{N};\alpha\right) &= \exp\left(\int_{0}^{\phi}\log\left(\varphi\alpha+\mu^{A}\right)+\int_{1-\phi}^{1}\log\left(\mu^{N}\right)\right) \\
&= \left(\varphi\alpha+\mu^{A}\right)^{\phi}\left(\mu^{N}\right)^{1-\phi}
\end{aligned}
\tag{C.1}
$$

where $\alpha>0$ is the degree of automation (the stock of robots), μ^{A} and μ^{N} are labor demands in automatable and non-automatable occupations, and φ is the productivity of automation. We allow automatable occupations to be partially automated, e.g., because automation is chosen once and for all in our benchmark model or due to adjustment costs in our quantitative model, which is the only difference with Acemoglu and Restrepo (2018).

We show below that an increase in automation α decreases the marginal productivity of labor (MPL) *within* automatable occupations, while potentially raising the *aggregate* MPL. Consider an increase in the degree of automation α. Then,

$$
\frac{d}{d\alpha}\log\left(\mathrm{MPL^{A}}\right)=(\phi-1)\,\frac{1}{\varphi\alpha+\mu^{A}}\leq 0
$$

since $\phi,\varphi\in[0,1]$. In turn,

$$
\frac{d}{d\alpha}\log\left(\mathrm{MPL^{N}}\right)=\phi\frac{1}{\varphi\alpha+\mu^{A}}>0
$$

since $\partial_{\alpha}F\left(\cdot\right)>0$. That is, the MPL declines in automatable occupations but increases in non-automatable occupations. The marginal productivity of labor at the aggregate level, i.e., workers' average wage rate,

$$
\mathrm{MPL}\equiv\frac{\phi\mu^{A}}{\phi\mu^{A}+(1-\phi)\,\mu^{N}}\mathrm{MPL^{A}}+\frac{(1-\phi)\,\mu^{N}}{\phi\mu^{A}+(1-\phi)\,\mu^{N}}\mathrm{MPL^{N}}
$$

can increase or decrease, depending on $\left(\mu^{A},\mu^{N},\phi\right)$.

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