

# International Liquidity Management: Sterilization Policy in Illiquid Financial Markets

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## Abstract

During the booms that precede crises in emerging economies, policy makers often struggle to limit capital flows and their expansionary consequences. The main policy tool for this task is sterilization –essentially a swap of international reserves for public bonds. However, there is an extensive debate on the effectiveness of this policy, with many arguing that it may be counterproductive once the (over-) reaction of the private sector is considered. But what forces account for the private sector’s reaction remain largely unexplained. In this paper we provide a model to discuss these issues. We emphasize the *international liquidity management* aspect of sterilization over the traditional monetary one, a re-focus that seems warranted when the main concern is external crisis prevention. We first demonstrate that policies to smooth expansions in anticipation of downturns can be Pareto improving in economies where domestic financial markets are underdeveloped. We then discuss the implementation and effectiveness of this policy via sterilization. The greatest risk of policy arises in situations where policy is most needed – that is, when financial markets are illiquid. Our mechanism is akin to the “implicit bailout” problem, although the central bank acts non-selectively and only intervenes through open markets in our model. Illiquidity replaces corruption and ineptitude. In addition to an appreciation of the currency and the emergence of a quasi-fiscal deficit, the private sector’s reaction to sterilization may lead to an expansion rather than the desired contraction in aggregate demand or nontradeables investment and to a bias toward short term capital inflows. The main insights extend to international liquidity management issues more generally.

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# 1 Introduction

During the booms that invariably precede crises in emerging economies, policy makers often struggle to limit capital flows and their expansionary consequences. They primarily rely on tight monetary policy. In particular, they attempt to sterilize capital inflows through an open market sale of domestic bonds or increased reserve requirements.<sup>1</sup>

These sterilized interventions can be extremely large. During the early 1990s in Chile, for example, the exchange intervention meant that over three quarters of its large capital inflow — amounting to around seven percent of its GDP per year — went to international reserves accumulation at the central bank. The sterilization of this intervention increased the ratio of international reserves to monetary base from 3.5 around 1990 to over 6.0 by 1993. This pattern was repeated in many emerging economies during the early 1990's, when capital flows to the developing world surged. In fact most of the economies involved in the crises of the second half of the 1990's had heavily sterilized inflows. Illustratively, many of these countries' central banks exhausted their stock of treasury securities in the process, and had to resort to alternative sterilization mechanisms (see e.g. Glick (1998)).

While sterilization is a widely used tool, both policy makers and academics have warned that it comes along with a number of difficulties and risks. Building on the Mundell-Fleming logic, many argue that sterilization is, at best, ineffective. When capital markets are integrated and there is a simultaneous attempt to stabilize the exchange rate, the central bank has no control over the money supply because the private sector can undo an open market sale of bonds for money (Mundell (1962)). The policy literature, noting the increase in capital inflows that accompany sterilization, argues that since it is these flows that fuel what is perceived as an excessive expansion in aggregate demand and exchange rate overvaluation, sterilization is counterproductive (e.g. Calvo et al. 1993, Williamson 1995, Corbo and Hernandez 1996, Massad 1997).<sup>2</sup> Building on Sargent and Wallace's (1981) unpleasant monetary arithmetics, Calvo (1991) formally shows that, by raising domestic

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<sup>1</sup>E.g., Calvo et al. (1993) p. 146 write: "Sterilized intervention has been the most popular response to the present episode of capital inflows in Latin America." And so does the World Bank (1997), p. 181: "Sterilization was the most widely and intensively used policy response to the arrival of capital inflows among the countries in our sample." The sample included 22 emerging economies.

<sup>2</sup>In his statement on behalf of the Latin American Governors of the Fund at the joint World Bank - IMF annual meeting of the Board of Governors held in Hong Kong (1997), Massad writes: "The high rate of return on capital in a booming economy attracts large inflows of external resources. These inflows are further encouraged by the appreciation of the domestic currency, which is characteristic of economies experiencing rapid productivity growth. Capital flows stimulate domestic demand and could push up domestic interest rates *if the monetary authority safeguards domestic equilibrium*. This, in turn, could provide a further incentive for capital inflows. The probable outcome will be continued appreciation of the local currency, the resulting risk of widening the current account deficit, and the greater danger that these capital flows will be reversed, should some negative external shock occur." (page 4, our emphasis).

interest rates, the government increases its debt-service burden and creates a quasi-fiscal deficit that may jeopardize the very stabilization attempt that is supposedly being protected by the sterilization.

Uniformly, this debate has viewed the effects of sterilization as arising from changes in the composition of the government's liabilities (money versus bonds). However, as noted above, in a typical sterilized intervention the central bank also accumulates substantial international reserves as assets and some mix of domestic currency and bonds as liabilities, while the private sector's balance sheet changes in the opposite direction. We argue in this paper that the impact that sterilization has on the *asset* side of the government's balance sheet *and* on its counterpart in the private sector is central — and perhaps the chief factor — in understanding its consequences. That is, we emphasize the *international liquidity management* aspect of sterilization over its monetary implications.

We build this view on two salient features of emerging economies vis-à-vis developed ones: First, it is crisis-prevention rather than day-to-day fine-tuning that typically shapes their macroeconomic policy. And second, external crises are invariably associated with a country's shortage of *international* liquidity.<sup>3</sup> The sovereign debt literature echoes both of these points (see Eaton and Gersowitz (1981), Bulow and Rogoff (1989)). A country's debt capacity is tied to its aggregate international collateral (or liquidity), and therefore external crises occur when this external constraint binds. This paper studies the use of sterilization as a tool for international liquidity management. Since sterilization is effectively an intervention in domestic assets markets, an adequate treatment of this liquidity management issue requires us to be explicit not only about the presence of an aggregate external constraint, but also about the structure of domestic assets and their liquidity.

For a concrete example of the environment and policy issues addressed in this paper, consider the following: Suppose, that all domestic production in the economy requires only imported goods. In order to put up a building in downtown Bangkok, a Thai developer must import all the raw materials for the building. Suppose that this building is not acceptable collateral to a foreigner, so that loans to this developer, against the collateral of the building, will only be forthcoming from other domestics. Lacking any internationally liquid assets to exchange for the raw materials, it would appear that the real estate developer is in a dilemma. However, suppose that foreigners do accept, as internationally liquid assets, claims on export sector receivables. Then, construction can proceed as long as the real estate developer can find a domestic with export sector revenue who will accept the building

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<sup>3</sup>See Caballero and Krishnamurthy (1999) for a model of crises based on collateral shortages. For us, an internationally liquid asset is one that can be sold (or borrowed against) at a moment's notice to an international investor without suffering a steep discount. A shortage of international liquidity means that the quantity of these assets, net of any pre-existing external debt, is insufficient to meet all external financial needs.

as collateral. At the aggregate level, domestic investment — building construction — is constrained by the supply of internationally liquid assets. Will domestic agents adequately assess the international liquidity value and costs of the assets and liabilities they generate when making their investment decisions? Following Caballero and Krishnamurthy (1999), we show that the answer to this question is typically negative in an emerging market. That is, too much international liquidity is sacrificed in the real estate boom.

Now suppose that the economy is currently in an investment boom in which many loans are being made, there is much real estate development, and the economy is trading away its international liquidity. Anticipating a shortage of international liquidity in the future, the central bank tries to increase the international liquidity provisions of the economy by offering public bonds in exchange for internationally liquid assets during the boom. The sterilized intervention leaves the central bank with more international reserves while leaving the private sector with more public bonds. The direct effect of this transaction is just a reallocation of international liquidity from the private sector to the central bank. Does this financial reorganization have any real effects? In particular, does the reallocation curtail real estate investment today and have the desired effect of mitigating the future liquidity shortage? Can the action ever backfire, as practitioners warn, leading to a further loss of liquidity, and biasing capital inflows towards shorter maturities? We show in this paper that the answers to these questions depend on the explicit and implicit commitments of the government, and on the degree of development of domestic financial markets.

A central finding is that the outcomes of sterilization may hinge on the liquidity of secondary markets for government bonds. Sterilization can be very transaction intensive and the large volumes can test the liquidity of secondary markets. If the government bonds sold in sterilization are illiquid, there is a sort of “liquidity” mismatch that arises in the central bank’s balance sheet which can lead to an aggregate loss of liquidity and an expansion rather than a contraction in aggregate demand or nontradeables investment.

The basic mechanism behind backfiring is similar to a “bailout” problem, although it derives from financial market deficiencies rather than from moral hazard or government ineptitude. For example, suppose that the government acquires international reserves today and issues completely illiquid government bonds to the market. That is, the bonds are long term and have no secondary market, so that they must be held until maturity. Now suppose that the commitment of the government to supplying the reserves in the event of the crisis is shorter in term — e.g., the government is expected to supply the reserves over the next months and the bonds do not mature for a year. Then, by its action and commitment the government has increased its support of domestic asset prices during downturns and, contrary to its goal, effectively reduced the cost of capital for real estate builders.<sup>4</sup> Real

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<sup>4</sup>See Frankel (1997) for a discussion of different sterilization models from the perspective of a model where

estate builders take out loans at this low cost of capital to “arbitrage” the government’s short-term liquidity commitment. We show that this mismatch leads to increased external borrowing and a shift in the composition of borrowing toward short maturities.<sup>5,6,7</sup>

Unlike the international finance literature, we emphasize the international liquidity management aspect of sterilization over its monetary aspect. Changes in international reserves and public debt are central to our analysis. While not addressing sterilization, both Woodford (1990) and Holmstrom and Tirole (1998) present models in which policy has real effects through changes in public debt. Government policies are non-Ricardian because public debt provides the private sector with liquid assets that they are unable to create for themselves. Holmstrom and Tirole (1998) show that when there are aggregate shocks the private sector may have a shortage of liquid assets. Public bonds make up this shortfall and government policy has real effects. In our model, on the other hand, issuing public bonds does not increase private liquidity. In fact we shall assume that if the government issue a public bond, the private sector anticipates a tax liability which reduces domestic private liquidity one for one. As a result, policy along the lines of Woodford (1990) or Holmstrom and Tirole (1998) will be Ricardian in our model. At a more abstract level, nonetheless, our policies are related to theirs in that government policy acts through changing the liquidity of the (in our case, international) assets ultimately held by the private sector.

In section 2 we lay out our basic model. We show that when domestic financial markets are underdeveloped an externality arises whereby the private sector draws down its international liquidity too fast (over time) relative to the constrained efficient outcome. This sets the stage for the policy discussion.

In section 3 we introduce a government/central bank and describe its rights and commitments. We demonstrate conditions under which sterilization policy is effective and leads to Pareto improvements and those under which it is completely undone by the private sector. When domestic secondary markets are illiquid, the government action can backfire, leading to a net loss of international liquidity and a Pareto loss.

In section 4 we show that our perspective naturally accommodates two additional sources public bonds and corporate bonds are imperfect substitutes.

<sup>5</sup>With some relabeling, this mechanism can also be illustrated via a fixed exchange rate commitment. Suppose that the government has reserves to sustain a fixed exchange rate over the next year, but has issued bonds that expire later. Then capital inflows to purchase the bonds cannot take advantage of the fixed exchange rate unless the bonds can be sold in liquid secondary market in the next year. If this is not so, other shorter term assets will be created to take advantage of the government commitment. The domestic private sector finds that there is good demand for such assets and takes out loans (i.e. sells the asset) and increases real investment.

<sup>6</sup>The shift in composition of capital inflows during periods of intensive sterilization has been documented by, e.g., Montiel and Reinhart (1999) and Calvo, Leiderman, and Reinhart (1993).

<sup>7</sup>Dooley (1999) has also emphasized some of the insurance aspects of interventions.

of concern during sterilization episodes: the rise in the quasi-fiscal deficit and the shortening of the maturity of capital flows. We show that when domestic secondary markets are illiquid, the former symptom may arise *even* when the sterilization fails to raise corporate interest rates. The latter symptom, on the other hand, stems from domestic agents' undervaluation of the insurance aspect of long term debt, yet another manifestation of the externality highlighted in section 2.

Section 5 adds money and a lending channel. Within the context of this expanded model we supplement our results with the standard Mundell-Fleming insight that sterilization is more likely to succeed when the central bank has no commitment to supply international reserves at a pre-specified price. This advantage of a flexible exchange rate system is limited, in that the core issues discussed in the previous section remain, and that the additional success in sterilizing comes from implicit transfers rather than Pareto-improvements. Section 6 concludes and is followed by an appendix.

## 2 A Model of Crises and Illiquid Financial Markets

In this section we develop a simple open economy model where two forms of liquidity, domestic and international, are required to fulfill investment plans. International liquidity determines a firm's ability to borrow from international financiers. Domestic liquidity, on the other hand, determines a firm's ability to borrow from domestic financiers. A claim on a piece of land in Patagonia is a domestically liquid asset in Argentina, whereas the dollar reserves backing the currency board or export sector receivables are counted as internationally liquid assets. Shortages of either form of liquidity may lead to difficulties. We shall focus on and describe an external crisis as a situation where, primarily, there is an aggregate shortage of international liquidity. That is, the sum of the international liquidity of each firm is less than that required to fulfill all investment plans. In this case, domestic liquidity serves to allocate international liquidity to the highest value of use.

We set up a model to explain how these two forms of liquidity interact and arrive at our main result of this section: in a dynamic context, when domestic liquidity is low agents will undervalue holding international liquidity relative to the constrained efficient outcome. Domestic liquidity is low when domestic financial markets are illiquid and underdeveloped. Thus, we demonstrate the existence of Pareto improving policies in cases when domestic financial markets are underdeveloped. In the next section we introduce a government and study the effectiveness of sterilization in implementing Pareto gains.

While the incidence of crises in emerging economies certainly have to do with aggregate shocks, in the model that follows we suppress them in order to highlight the essence of our mechanism. We show that under certain conditions the decentralized economy will fully

anticipate a crisis, and still will not do enough about it.<sup>8</sup>

## 2.1 The Economic Environment, Assets, and Balance Sheets

### 2.1.1 Basic setup

*Time.* The world lasts three periods. Time is indexed as  $t = 0, 1, 2$ . Date 0 is the fully flexible period when agents design the productive structure, ownership structure, and portfolio allocations. Date 1 is the crisis period, when agents must shift resources from the future to cope with shocks in the present. Date 2 represents the unconstrained future, when the economy is (relatively) rich in resources.

*Agents and heterogeneity.* There are two types of agents: (i) a continuum of unit measure of domestic entrepreneurs-consumers (henceforth, domestics) with linear preferences over date 2 consumption of a single good, and (ii) foreign financiers (henceforth, foreigners) with large endowments at all dates and linear preferences with no discounting, thus the international gross interest rate is one.<sup>9</sup>

*Endowment, Production, Investment, and Liquidity.* We assume that domestic agents are endowed at date 0 with  $w$  units of an internationally liquid asset – e.g., the present value of export sector receivables – and access to a production technology.

On net, domestic agents must import materials from the rest of the world in order to produce. They do this by pledging their international liquidity to foreigners and taking on foreign debt of  $d_{0,f}$ . Production has a time-to-build aspect. Investments are made at dates 0 and 1, and output is realized at date 2. Let  $k$  denote the total amount of capital devoted to production at the beginning of date 1, inherited from date 0. Then creating capital of  $k$  requires a date 0 investment of  $c(k)$  units of imported goods.  $c(k)$  is assumed to be strictly increasing, convex and positive.

We capture the normal churn of the economy, with its implied domestic heterogeneity, with a simple Bernoulli process. At date 1, half of the firms are spared of further investment and go on to produce  $Rk$  units of goods at date 2. The rest experiences a productivity fall,  $\Delta \equiv R - r > 0$ , which can be offset by reinvesting a fraction  $\theta \leq 1$  of  $k$ , in units of the imported good, in order to realize output at date 2 of:

$$\tilde{R}(\theta)k = (r + \theta\Delta)k \leq Rk.$$

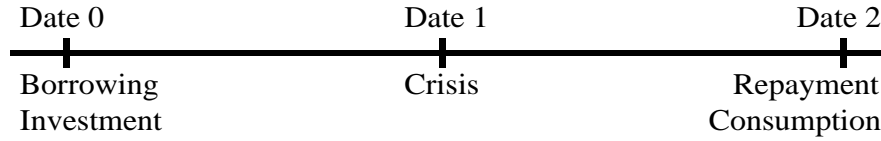
This time-to-build structure underlines a critical link between financing and production during a crisis. Firms in any economy have ongoing capital needs (working capital, etc.).

<sup>8</sup>See Caballero and Krishnamurthy (1999) for a similar model with aggregate shocks.

<sup>9</sup>The distinction between foreigners and domestics needs not be as stark as posited here. Domestic savers, for example, may be grouped with one or the other depending on the nature of the shocks faced by the economy. Our insights can be easily extended to more complex environments along this direction.

Starving firms of capital has the effect of shutting down production units in a potentially wasteful manner – to anticipate results, this is our crisis. The dual of this shutting down of production units is that the marginal value of capital during a crisis is very high.

The timeline is given below.



Date 0 Budget Constraint:  $d_{0,f} \leq w$   
 Investment Constraint:  $c(k) \leq d_{0,f}$

Production: Time-to-build

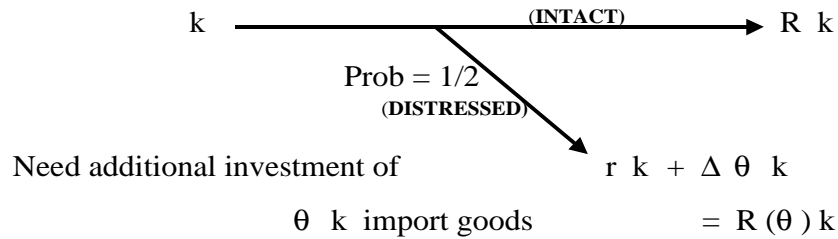


Figure 1: Time Line

In order to explore the financial problems that concern us, we make assumptions to rule out key insurance contracts, and to allow for a borrowing constraint that limits a distressed firm to take on debt with promised repayments of less than  $rk$ .<sup>10</sup> We shall return to these assumptions shortly.

**Assumption 1 (Non-observability of Production Shock)**

*The production shock at date 1 is idiosyncratic. The identity of firms receiving the shock is private information.*

**Assumption 2 (Domestic Borrowing Constraint)**

*A domestic lender can only be sure that a firm will produce  $rk$  units of goods at date 2. Any excess production based on physical reinvestment at date 1 is neither observable nor verifiable.*

There are two types of assets in this economy, those that are domestically liquid and those that are internationally liquid.  $rk$  is domestically liquid in that it can be pledged to

<sup>10</sup>Since the production shock is not observed, even intact firms that do not receive the shock have borrowing capacity of only  $rk$ . Of course, only the distressed firms need to raise funds, so the borrowing constraint on the intact firm never binds.



another domestic in order to take on debt. On the other hand, we assume that  $rk$  cannot be pledged to a foreign investor. Foreigners only lend to a firm against its internationally liquid assets of  $w$ . This means that the maximum amount of external debt the country can take on is  $w$ .<sup>11</sup> While much of this asymmetry can have a microeconomic origin, there are sovereign aspects that reinforce it. We return to this issue in the next section.

**Assumption 3 (Liquidity Bias)**

*Foreigners lend to domestic firms only against the backing of  $w$ . Domestic firms lend against both  $w$  and  $rk$ .*

*Financial structure and Balance Sheets.* Firms can raise finance at date 0 and date 1 from either domestic or foreign investors. We assume that all finance must be default free and fully secured debt – either by domestic liquidity in the case of domestic firms, or international liquidity in the case of foreign investors.

Date 0 decisions result in firms arriving at date 1 with installed capital of  $k$  and foreign debt of  $d_{0,f}$ . At date 1, a firm that receives a shock is distressed (S), while a firm that escapes the shock is intact (I). The balance sheet of a domestic firm has assets of  $rk$  units of domestic liquidity and  $w$  units of international liquidity, and foreign debt of  $d_{0,f}$ . The simplest way to think of the asymmetric treatment of collateral by foreigners and domestic firms, is to think of foreigners studying a balance sheet of the firm as perceiving only  $w$  as assets. On the other hand, a domestic firm sees both this quantity as well as  $rk$  as assets. Thus the debt constraint with respect to foreigners at date 0 is,

$$d_{0,f} \leq w.$$

At date 1, if a firm takes on additional debt with foreigners, the date 1 debt constraint is:

$$d_{0,f} + d_{1,f} \leq w.$$

**2.1.2 Discussion of main assumptions**

Let us pause at this juncture and discuss our main assumptions, starting from the two borrowing constraints.

We have assumed a friction that prevents a domestic entrepreneur from borrowing fully up to his output from another domestic. Investment at date 0 produces  $rk$  goods at date 2, and depending on the realization of shocks and reinvestment, an additional  $\Delta k$  of output.

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<sup>11</sup>The stark distinction between domestic firms and foreigners in their valuation of assets is only made for simplicity. In reality, many residents behave like our foreigners at time of distress (e.g., households may be behind capital flights), and many foreigners behave like our domestic firms (e.g., institutional specialists well informed and connected with the domestic establishment).

In a well functioning domestic financial market – i.e. if there were no frictions in borrowing from another domestic – both the  $rk$  and the  $\Delta k$  of output could be sold to obtain funds for investment. In assuming that only the output of  $rk$  is observable and verifiable, we have restricted claims sold to another domestic to  $rk$ . The modeling of this borrowing constraint is not unusual. Kiyotaki and Moore (1997) follow a similar avenue by assuming that output is not verifiable, but that physical collateral (land) is. Our  $rk$  is akin to their collateral. Holmstrom and Tirole (1998) develop a model with an ex-ante unobserved effort choice where a fraction of output has to be paid as a rent to the entrepreneur so that he exerts effort. While our model does not have ex-ante moral hazard, it shares the feature that a fraction of output (i.e.  $\Delta k$ ) can never be promised to a lender. Indeed, any model with borrowing constraints will share the feature that some fraction of future output is unpledgeable. As we will see, this gives rise to some standard features: the amount of investment at date 1 will depend on the liquidity of firms at date 1, firms will value holding liquidity from date 0 to date 1.

When we say that the the domestic financial market is underdeveloped we are referring to the fact that  $\Delta k$  cannot be borrowed against. In Caballero and Krishnamurthy (1999), we introduce a parameter,  $\lambda$ , that varies the amount of the extra  $\Delta k$  that can be borrowed against. Thus we arrive at comparative statics on the degree of development of domestic financial markets by relaxing the borrowing constraint. In this paper we simply assume that  $\lambda = 0$ , so that only  $rk$  can be borrowed against.

While the modeling of the domestic borrowing constraint is not unusual, the asymmetry in the borrowing constraint between domestics and foreigners is non-standard.  $rk$  cannot be borrowed against from foreigners, while it can from domestics. First of all, there is empirical support for this assumption. For example, Mexico, during the 94-95 crisis, used its oil revenues as collateral to back the liquidity package that it received. In many instances the bias is directly justified by mandates on foreign institutional investors (e.g. limits on real estate investments, see Blommestein, 1997).<sup>12,13</sup> Theoretically, the assumption is most similar to that of the that of the sovereign debt literature wherein the assumption is typically

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<sup>12</sup>Of course there are important exceptions (e.g. “too big to fail” utilities), but see, e.g., Kang and Stulz (1997) for systematic Japanese evidence showing that, for small firms, those that are export oriented are favored by foreign investors. See, e.g. Blommestein (1997), for a discussion of how real estate and other assets considered highly illiquid or exposed to exchange risk are generally avoided (sometimes by mandate) by foreign institutional investors. Interestingly, the very few exceptions to the sovereignty principle, by which the rating on debt issued by a country’s corporate sector is bounded from above by that country’s government debt rating, are for companies which belong to the export sector.

<sup>13</sup>The September 1998 report on the Asian crisis by the World Bank, describes firms that borrow in foreign currency as “predominantly large exporting firms with ties to foreign companies, and they have better adjusted to the crisis...” (box 4.3, page 62). The 1997 Industrial Survey in Thailand reflected that of those firms that borrow in foreign currency, 88 percent export, have an average of 818 employees, a debt-equity ratio of 3.12, a relatively high capacity utilization and optimism during the crisis, 70 and 37 percent

that only a fraction of exports or net exports is international collateral. On the other hand, the sovereign debt literature typically just imposes international collateral as an aggregate constraint. We take a microeconomic perspective by assuming that  $w$  is held by individual agents in the economy who can trade it among themselves and with foreigners.

Sensible as it may be, here we have simply posited assumption 3 regarding liquidity bias. In the next section we introduce a government and sovereign risk, thereby providing an alternative —and more explicitly modeled— grounding for the assumption.

The last assumption worth commenting on is the non-observability of the idiosyncratic production shock at date 1. Domestic agents are ex-ante identical at date 0. If the production shock was verifiable, domestic agents would write contracts amongst themselves to insure that they would be ex-post identical as well. There would be no heterogeneity at date 1 amongst domestics and frictions in the domestic market would have no economic effect. Assuming non-observability is necessary to study the impact of these frictions.

## 2.2 The Microeconomic Problem

Domestics have two sets of decisions. At date 1, given the date 0 choices of other firms (through prices) and the realization of the idiosyncratic shock, a domestic firm must decide how much to borrow (lend) and reinvest. At date 0, a firm must decide how much to invest and how much international liquidity to retain. We solve this problem by backward induction, starting from date 1.

*Date 1 problem.* Consider the problem of a distressed firm in raising funds to alleviate its production shock. A choice of  $\theta k$  will result in output at date 2 of  $\tilde{R}(\theta)k$  goods. In order to save a fraction  $\theta$  of the distressed unit, the firm must raise finance and reinvest  $\theta k$  import goods. It can do this in two ways. First, the firm may have some international liquidity at date 1 that it can use to borrow directly from foreigners. That is the firm can always raise directly,

$$d_{1,f} \leq w - d_{0,f}.$$

The latter quantity is always positive, and represents the minimum that a firm can raise at date 1. The rest must come from intact firms, which also have access to foreign investors since their capacity to borrow abroad at date 1 is also  $w - d_{0,f}$ .

A distressed firm can use its domestic liquidity to access the international liquidity of the intact firms. In equilibrium, the latter discount the domestic liquidity at a rate of  $L_1 \geq 1$  in providing international liquidity.  $L_1$  is the date 1 interest rate. It is not an interest rate that is driven by expectations of default or currency depreciation. Rather, it is driven by respectively. The same statistics for firms that do not borrow in foreign currency (75 percent of firms) are: 46 percent, 139 employees, 2.36, 61 percent and 19 percent. See Dollar and Hallward-Dreimeier (1998).

liquidity considerations.  $L_1$  is strictly greater than one only when the country is “illiquid,” in the sense that its aggregate availability of international liquidity is less than its needs (see below).<sup>14</sup> For this reason, we refer to  $L_1 - 1$  as the international liquidity premium.<sup>15</sup>

Through this “credit chain” —which represents the domestic financial markets in our framework— the distressed firm is able to aggregate the international liquidity of the economy and pledge this to foreigners to raise resources for date 1 reinvestment. Let  $d_{1,f}$  and  $d_{1,d}$  represent the foreign and domestic debt contracted by a distressed firm at date 1, respectively. We can then write the problem of a distressed firm as,

$$\begin{aligned}
 (P1) \quad V_s &\equiv \max_{\theta, d_{1,f}, d_{1,d}} w + \tilde{R}(\theta)k - d_{0,f} - d_{1,f} - d_{1,d} \\
 s.t. \quad (i) & \quad d_{1,f} + d_{0,f} \leq w \\
 (ii) & \quad \frac{d_{1,d}}{L_1} + d_{1,f} + d_{0,f} \leq w + \frac{rk}{L_1} \\
 (iii) & \quad \theta k = d_{1,f} + \frac{d_{1,d}}{L_1} \\
 (iv) & \quad \theta \leq 1.
 \end{aligned}$$

Constraints (i) and (ii) are balance sheet constraints (net marketable assets greater than liabilities), while constraint (iii) reflects that new investment must be fully paid with the resources received by the firm at date 1 in taking on debts of  $d_{1,f}$  and  $d_{1,d}$ . Constraint (iv) is purely technological.

An intact firm at date 1 has only one decision: how much finance will it extend to the distressed firm. Suppose that the firm accepts claims at date 1 of  $x_{1,d}$  (face value of date 2 goods) in return for making a date 1 contribution of  $x_{1,d}/L_1$  which is financed with new external debt  $d_{1,f}^s$ . Then,

$$\begin{aligned}
 (P2) \quad V_i &\equiv \max_{x_{1,d}} w + Rk + x_{1,d} - d_{0,f} - d_{1,f}^s \\
 s.t. & \quad d_{0,f} + d_{1,f}^s \leq w \\
 & \quad \frac{x_{1,d}}{L_1} \leq d_{1,f}^s
 \end{aligned}$$

*Date 0 problem.* At date 0, a firm looking forward to date 1 can expect to find itself as either distressed or intact. Thus the decision at date 0 is,

$$\begin{aligned}
 (P3) \quad \max_{k, d_{0,f}} & \quad (V_s + V_i)/2 \\
 s.t. & \quad d_{0,f} \leq w \\
 & \quad c(k) = d_{0,f}.
 \end{aligned}$$

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<sup>14</sup>Allen and Gale (1994) also develop a liquidity based model of asset prices, in which prices may be based on “cash in the market” rather than fundamentals. Similarly, in our model, when the country is illiquid interest rates are partly driven by the supply of international liquidity.

<sup>15</sup>While it can be represented as a shift in the interest parity condition, it is important to realize that it is domestic suppliers of international liquidity, rather than foreigners, who earn the liquidity premium.

### 2.3 Equilibrium and Crises

*Equilibrium.* Market clearing in the domestic debt market at date 1 (capital letters denote aggregate quantities) requires that the aggregate amount of domestic debt taken on by distressed firms is fully funded by intact firms:

$$\begin{aligned} D_{1,d} &= \frac{1}{2}d_{1,d} \\ X_{1,d} &= \frac{1}{2}x_{1,d}. \end{aligned}$$

Therefore, market clearing,

$$D_{1,d} = X_{1,d}, \tag{1}$$

determines the gross interest rate,  $L_1$ .

An *equilibrium* of this economy consists of date 0 and date 1 decisions,  $(k, d_{0,f})$  and  $(\theta, d_{1,f}, d_{1,d}, x_{1,d})$ , respectively, and prices  $L_1$ .<sup>16</sup> Decisions are solutions to the firms' problems (P1), (P2), and (P3) given prices. At these prices, the market clearing condition (1) holds.

Let us now study equilibrium in more detail. Starting from date 1, consider financing and investment choices of the distressed firm given  $(k, d_{0,f})$ . First, if  $\Delta \geq 1$ , then the distressed firm would choose to save as many of its production units as it can. It may borrow up to its international debt capacity,

$$d_{1,f} = w - d_{0,f}. \tag{2}$$

If the amount raised from international investors,  $w - d_{0,f}$ , is less than the funds needed for restructuring,  $k$ , the firm will have to access the domestic debt market to make up the shortfall. It will choose to do this as long as  $\Delta \geq L_1$ , or the return on restructuring exceeds the interest rate. If the firm borrows fully up to its domestic debt capacity, it will issue debt totalling,

$$d_{1,d} = rk, \tag{3}$$

and raise funds with which to pay for imported goods of  $\frac{rk}{L_1}$ . As long as the sum of  $\frac{rk}{L_1}$  and the right hand side of (2) is more than the borrowing need, the firm is unconstrained in its reinvestment at date 1 and all production units will be saved. In this case, the firm will borrow less than its domestic debt capacity (and perhaps less than the international debt capacity).

Intact firms can tender at most their excess international debt capacity of  $w - d_{0,f}$  in return for purchasing domestic debt. They will choose to do this as long as the domestic interest rate exceeds the international rate of one,  $L_1 \geq 1$ .

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<sup>16</sup>Where we have used the fact that at equilibrium prices,  $\frac{x_{1,d}}{L_1} = d_{1,f}^s$ .

Assume for a moment that  $\Delta \geq L_1 \geq 1$  so that distressed firms borrow as much as they can, and intact firms lend as much as they can. Then, in total the economy can import  $w - d_{0,f}$  goods, which is directed to the distressed firms. A necessary condition for all production units to be saved is that,

$$\frac{k}{2} \leq w - d_{0,f}. \quad (4)$$

We shall refer to this constraint as the *international liquidity constraint*. When neither (3) nor (4) binds, all production units are saved. Since there is excess supply of funds from intact firms relative to domestic demand for funds, there is no international liquidity premium, and  $L_1$  is equal to the international interest rate (one).

The other extreme case is when both (3) and (4) bind. Equilibrium in the domestic debt market requires that,

$$\frac{rk}{L_1} = w - d_{0,f}.$$

Since (3) binds, distressed firms borrow fully up to their debt capacity. As (4) binds, intact firms purchase this debt with all of their excess funds. Solving for  $L_1$ , yields

$$L_1 = \frac{rk}{w - d_{0,f}} > 1. \quad (5)$$

That is, in this case the international liquidity premium is positive.  $L_1$  is above the (gross) international interest rate in order to clear the domestic market for scarce international liquidity. One half times the numerator in (5) corresponds to the transferable domestic resources owned by distressed firms. The international liquidity premium is positive when these resources are greater than one half times the denominator, which corresponds to the excess international liquidity owned by intact firms.

Define the *index of domestic illiquidity* as the difference between the marginal profit of saving a distressed production unit and the domestic interest rate of  $L_1$ . When (4) binds, this is simply,

$$s_d = \Delta - L_1,$$

Equilibrium at date 1 can place the economy in one of four regions, classified according to which of the two (domestic and international) liquidity constraints are binding. These regions are summarized in the appendix.<sup>17</sup> In the main text we focus on a crisis scenario,

<sup>17</sup>Depending on the date 0 choices of  $k$  and  $w - d_{0,f}$  (see the appendix) any of four regions are possible. Date 0 decisions then pin down these choices and tell us which region will prevail at date 1. A property of the model is that, depending on parameter values, any of the four regions are equilibrium outcomes of the model. This is largely due to the fact that model is one of certainty - agents at date 0 know exactly which of the four regions will prevail at date 1. In Caballero and Krishnamurthy (1999), the model includes aggregate uncertainty (essentially in  $w$ ), so that the occurrence of any particular region is a surprise and at date 0 agents face a date 1 world where these regions are blended.

where both liquidity constraints are binding. At the aggregate level, the economy is liquidity constrained with respect to foreigners; at the individual level, firms are liquidity constrained with respect to other domestics since they are selling all of their domestic liquidity in aggregation; real investment is constrained; domestic spreads are positive; and the interest rate of  $L_1$  is above the international interest rate. This is the most interesting configuration for the prevention-policy questions we intend to address in the main section of the paper.

**Technical Assumption 1 (Conditions for Crisis)**

*Assume that:*

$$1 \leq r < \Delta$$

$$c'^{-1} \left( \frac{\Delta + R}{1 + \Delta} \right) + c \left( c'^{-1} \left( \frac{\Delta + R}{1 + \Delta} \right) \right) < w < c'^{-1} \left( \frac{1 + R}{2\Delta} \right) + c \left( c'^{-1} \left( \frac{1 + R}{2\Delta} \right) \right).$$

The assumption guarantees that in equilibrium  $\Delta > L_1 > 1$  and  $\theta < 1$ . This ensures focus on a case where both (3) and (4) bind.

**Proposition 1 (Crisis Region)**

*Under assumption 1, date 0 decisions of  $k$  and  $d_{0,f}$  are such that both the international constraint and the domestic constraint are binding. The international liquidity premium and the domestic illiquidity index are positive, and some projects are downsized.  $L_1 > 1, s_d > 0, \theta < 1$ .*

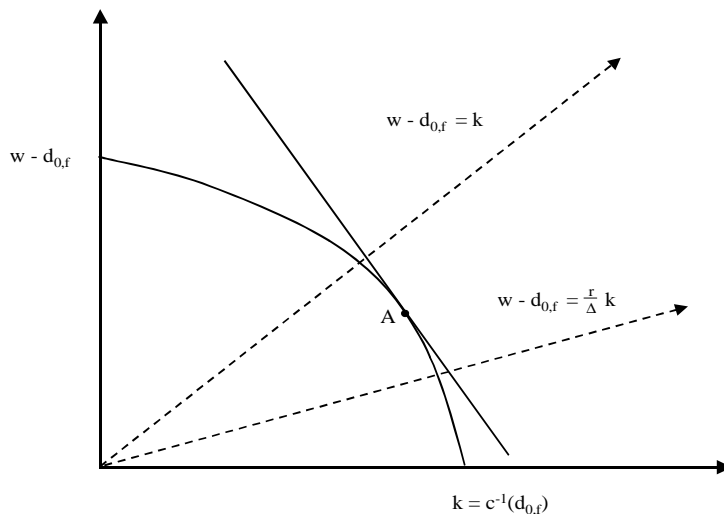


Figure 2: (P3)

Graphically, the solution to (P3) is represented in figure 2. The inner curve represents the budget set. This is the set of points of  $(w - d_{0,f}, k)$  such that the date 0 budget constraint

is satisfied with equality. The tangent line reflects the tradeoff between holding domestic and international liquidity. Its slope is given as  $-\frac{R+r\frac{\Delta}{L_1}}{L_1+\Delta}$ , so that higher values of  $L_1$  make the line flatter.

In order to arrive at the crisis region, point A needs to lie between the two rays. This is the area where,  $k > w - d_{0,f}$ , but  $\frac{r}{\Delta}k < w - d_{0,f}$ .

## 2.4 Constrained Inefficiency

When the economy is in the crisis-region it is international liquidity that presents the dominant bottleneck. Domestic liquidity, on the other hand, determines the allocation of surplus during these crises. We show next that when there is insufficient domestic liquidity, the market surplus allocation will not correspond to that of the central planner. In particular, the central planner will desire to allocate all the surplus to the liquidity suppliers. This will generally not happen when domestic liquidity is low as those in need have insufficient commitment to compensate the international liquidity providers adequately.

**Proposition 2 (Underprovision of International Liquidity)** *When both liquidity constraints are binding at date 1, the decentralized equilibrium is constrained inefficient. A central planner can effect a Pareto improvement by forcing the private sector to decrease  $(k, d_{0,f})$ , thereby increasing the international liquidity of the economy. The welfare gain from this intervention rises with the index of domestic illiquidity,  $s_d$ .*

Proof: First let us rewrite (P3), substituting in the value function from (P1) and (P2). A date 0 choice of  $(k, d_{0,f})$  result in date 2 resources (net of any contracted debt) of,

$$(w - d_{0,f})\Delta + \frac{rk}{L_1}\Delta$$

if the firm is distressed. This is because  $(w - d_{0,f})$  is directly pledged to foreigners, and the proceeds invested at the project return of  $\Delta$ . The  $rk$  of domestic liquidity is sold at the interest rate of  $L_1$ , and the proceeds invested at  $\Delta$ . If the firm is intact, date 2 resources are,

$$(w - d_{0,f})L_1 + Rk.$$

Thus the date 0 program is,

$$\begin{aligned} (P4) \quad \max_{k, d_{0,f}} \quad & (R + r\frac{\Delta}{L_1})k + (\Delta + L_1)(w - d_{0,f}) \\ \text{s.t.} \quad & w \geq d_{0,f} \\ & c(k) = d_{0,f} \end{aligned}$$

Consider the program for a central planner who directly chooses  $(k, d_{0,f})$  to maximize the equally weighted sum of utilities of agents in this economy. To do this we simply



substitute the expression for  $L_1$ , (5), into the objective of (P4), arriving at an expression that is free of prices. The program for a central planner is,

$$(P5) \quad \max_{K, D_{0,f}} \quad (R+r)K + 2\Delta(W - D_{0,f})$$

$$s.t. \quad w \geq d_{0,f}$$

$$c(K) = D_{0,f}$$

The solutions to (P5) are the constrained efficient decisions of the economy. The only difference between the programs (P4) and (P5) is in the objective. Subtracting the objective in (P5) from that of (P4) we arrive at,

$$s_d \left( \frac{r}{L_1} K - (W - D_{0,f}) \right).$$

At a given equilibrium, this term must be zero. But it is apparent that individuals and the central planner value a marginal unit of international liquidity and domestic liquidity quite differently. Moreover, this misvaluation is directly proportional to  $s_d$ , the domestic illiquidity index.

The first order condition of (P5) gives,

$$c'(K) = \frac{R+r}{2\Delta},$$

while that of (P4) yields,

$$c'(k) = \frac{R+r\frac{\Delta}{L_1}}{\Delta + L_1}.$$

Graphically, we can represent the solutions to (P4) and (P5) in figure 3.

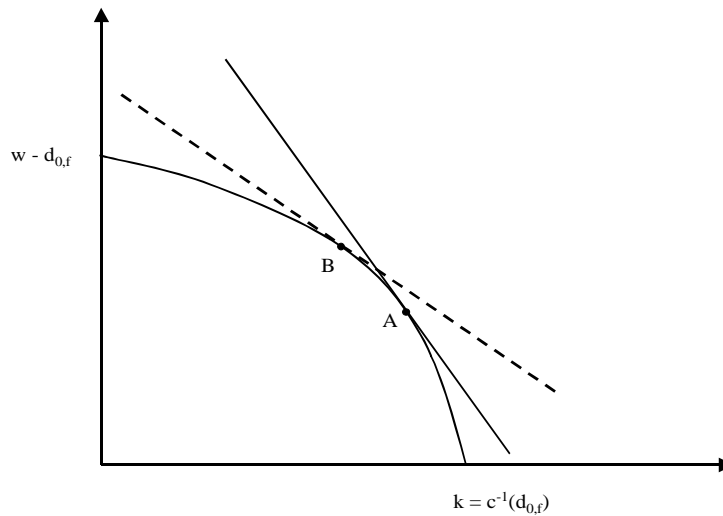


Figure 3: (P4) and (P5)

The two tangent lines reflect the tradeoff between retaining liquidity and making physical investments. The steeper of the two (the solid line) represents the objective in (P4), the decentralized case. On the other hand, the dashed line is the objective in the central planner's problem. From the point of view of retaining liquidity, the central planner is only concerned with protecting international liquidity. This is because international liquidity is all that can be used to attract foreign investment. However at the microeconomic (decentralized) level, both domestic and international liquidity can be used to secure financing. This is the basic tension between the individual and the central planner's problem.

The solutions to the programs are points A and B in figure 3. It is clear that the central planner will prefer to retain more international liquidity than individual firms when  $s_d > 0$ .<sup>18</sup> In other words, a central planner must find a mechanism to induce the private sector to reduce  $k$ .

In sum, financial constraints create a wedge between internal and external returns. When  $rk$  is small, distressed firms are borrowing constrained. This creates a spread between the internal rate of return for such a firm ( $\Delta$ ) and the rate of return that can be credibly promised to outside domestic financiers ( $L_1$ ). The domestic spread of  $\Delta - L_1$  is just the dual of an inefficient domestic capital allocation process. Indeed if firms could pledge all of  $\Delta$  to a financier the borrowing constraint would never bind and  $s_d$  would be equal to zero. In a dynamic setting, the domestic spread leads to an undervaluation of international liquidity. Since at the date 1 crisis there is a shortage of international liquidity, abating it requires an ex-ante decision to carry an extra unit of international liquidity into date 1. However, doing so provides the holder less than the full social return on this liquidity – in fact, the shortfall is proportional to the domestic spread. Anticipation of this reduces the private sector's incentive to accumulate and provision international liquidity.

### 3 The Government, Asset Liquidity, and Sterilization Policy

Having set the stage for policy intervention, we can now consider whether sterilization policy is likely to succeed in offsetting the expansionary consequences of capital inflows.

In a sterilization, the central bank sells public bonds to domestics in order to reabsorb the monetary expansion brought about by capital inflows. A capital inflow plus sterilization leaves the central bank with more international reserves as assets, and more domestic government bonds as liabilities. Thus, in reduced form, the central bank sells bonds for international reserves. In our language, this corresponds to a swap of domestic liquidity for

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<sup>18</sup>In Caballero and Krishnamurthy (1999) we show that when all date 2 output can be pledged to another domestic – that is, both  $rk$  as well as  $\Delta k$ , – then  $s_d = 0$  and there is no externality conducive to overborrowing and overinvestment. Hence intervention is only justified when domestic financial markets are undeveloped.

some of the international liquidity held by the private sector (which needs to borrow abroad to buy the public bonds).

Widespread as it may be, sterilization is perceived as a “risky” strategy, hampered by the possible overreaction of the private sector. But the mechanisms behind this “risk” are not well understood, let alone modeled. In this section we offer a methodic analysis of sterilization in an economy with underdeveloped financial markets for private and public instruments. We shall begin by noting conditions under which the private sector *completely undoes* the central bank’s action. In cases where this does not occur, we demonstrate conditions under which the policy can be Pareto improving, and those under which it results in a Pareto *loss*. Before doing so, however, we must introduce the (consolidated) government, its instruments and constraints, and the implications of its action for asset liquidity.

### 3.1 Preliminaries

#### 3.1.1 Public Bonds

We consolidate the central bank and the treasury. The minimum number of ingredients we need in order to address our policy question is one public financial instrument and a tax to finance any quasi-fiscal deficit that the sterilization policy may generate.<sup>19</sup> We start with this minimum and enrich the set of public financial instruments to include money in section 4, when we discuss exchange rate systems.

We formally describe public bonds in the next assumption, and justify it at the end of the preliminaries-section, when we discuss the key assumptions of this section.

#### Assumption 4 (Public Bonds)

*At date 0, the government issues public bonds with face value  $B$ :*

- *(Long Maturity) These bonds mature at date 2, but can be bought and sold in secondary markets at date 1.*
- *(Illiquidity) A sale at date 1 of one unit of a date 2 government bond suffers a real cost of  $0 \leq \alpha < 1$ . Selling  $X$  units of bonds only recovers  $\frac{X(1-\alpha)}{L_1}$  units of international liquidity.*

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<sup>19</sup>Importantly, taxes are always paid in units of liquidity (either domestic or international). That is, suppose that a firm has  $rk$  units of domestic liquidity at date 1, and that the government levies a tax of  $T$  on the firm. Then, after the transaction the firm will be left with  $rk - T$  units of domestic liquidity, and the government will be left with  $T$  units of domestic liquidity. Thus, if the firm has a tax liability of  $T$  to the government, this simply reduces its liquidity with respect to other firms. That is, unlike Woodford(1990) and Holmstrom and Tirole (1998), our government cannot *create* liquidity. The simplest way to think of taxes is that they alter the balance sheet of a firm by introducing an additional liability. This then affects the firm’s ability to raise finance from other agents.

### 3.1.2 Sovereign Risk and Short Horizons

Aside from having the ability to issue public bonds and finance interest payments with taxes, the government can also disrupt the chain of payments connecting the domestic private sector and foreign investors, thereby selectively defaulting on foreigners.

#### **Assumption 5 (Sovereign Risk)**

*The government suspends convertibility at date 1 and imposes two restrictions on exchange:*

- *Foreigners are prevented from transporting any date 2 goods from within the economy across the border.*
- *Any international liquidity holdings of the government/central bank are released to the private sector only on presentation of an invoice for imported goods.*

*Short Horizons.* At date 1 the government will suspend convertibility and repudiate any date 2 claims directly owed to foreigners. Foreigners holding claims directly on export sector revenues ( $w$ ) are unaffected by suspension, since they can seize these receivables directly. However a foreigner holding a claim on a domestic firm or a domestic government bond has only one choice. He must sell these claims to a domestic agent who has some international liquidity and exit. That is domestic claims are internationally liquid only to the extent that they can be exchanged for the private sector's international liquidity at date 1.

#### **Lemma 1 (Foreigner's Short Horizon)**

*Foreigners have short horizons with respect to domestic claims. Any domestic claim acquired at date 0 will always be sold at date 1. Claims on international liquidity are free of this short horizon.*

### 3.1.3 Discussion of assumptions

It is widely understood that absent the monetary channel, if domestic assets are perfect substitutes for international assets (or foreign reserves) sterilization must fail (see e.g. Mundell (1962)). The reasoning is often applied to fixed exchange rate systems where capital flows in to arbitrage any interest differential between domestic and foreign assets, rendering monetary policy useless. Our model must also contend with this logic, though for our purpose the argument is best restated in terms of international liquidity. Since in sterilization the central bank removes international liquidity from the private sector in exchange for domestic bonds, if foreigners view these bonds as perfect substitutes for the subtracted liquidity, sterilization must fail.

It is easy to see in this context that if public bonds were one period, the reserves that the government acquires in the sterilization would exactly repay the public bonds and the date

0 sterilization transaction would be fully undone at date 1. Since the government reverses its own action very quickly, policy must fail. Essentially the one period bonds are viewed as perfect substitutes for the reserves they replace. It is worth pointing out that we do not require the extreme assumption made above, but just that a fraction of the new government liabilities matures after some potential external crisis.

Long maturity of public bonds is not a sufficient condition for imperfect substitutability, because there is still a secondary market open and the government will inject its reserves into this market at date 1. There are two types of domestic assets, corporate assets and government bonds. Since foreigners cannot repatriate payments at date 2, any assets held until date 2 are effectively defaulted on to foreigners. As a result, foreigners must sell all assets at date 1, and limit their date 0 holdings to reflect this fact. In principle, if the government commits to inject all of the foreign reserves gained during sterilization back into the market at date 1, then a foreign investor would be willing to buy a domestic bond since he knows he can sell these at date 1 for the government's reserves.<sup>20</sup> Requiring that the public reserves are released only on presentation of an invoice for imported goods implies that the foreign investor cannot, directly, repatriate the return from selling the domestic bond at date 1. Taken together, the two assumptions on sovereign risk imply that domestic assets are imperfect substitutes for the foreign assets removed from the private sector during sterilizations, and this allows some space for the policy to be non-Ricardian. Finally, the sovereign risk assumption also provides a deeper foundation for our earlier assumption on liquidity bias.

Our sovereign risk assumption is similar to that made in the widely accepted and empirically supported sovereign debt literature (see e.g. Obstfeld and Rogoff (1998) ch. 6). The main difference is that, since decentralization is not their main concern, the sovereign debt literature simply posits a maximum repayment for a country. In our setup domestic financial markets are modeled explicitly and hence we have to be careful about how private actions may affect the maximum repayment and perhaps undo government actions. Both aspects of our assumption, suspension of convertibility and selective default/rescheduling have been observed in recent years (e.g. Malaysia for the former, and Ecuador, Korea and Mexico for the latter).

Finally, we will be interested in studying the role of illiquidity in the secondary public bond market. Our modeling of illiquidity – transactions costs incurred for early liquidation – is not unusual. As we will see below, illiquidity can exacerbate the impact of the government's maturity mismatch on the private sectors' asset liquidity. On one hand, illiquidity is

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<sup>20</sup>Imagine the following scenario: the government issues 1 bond for \$1 at date 0. The foreigner buys this bond at date 0 for \$1. He sells this at date 1, knowing that the government will inject the \$1 back into the market, and this will provide the liquidity for his exit from the country.

not an assumption insofar as we explore scenarios with and without it. On the other, in an environment where domestic financial markets are underdeveloped, public bonds are often—albeit less than the private sector’s instruments— illiquid as well.

With these preliminaries behind us, let us turn to the outcomes of policy.

### 3.2 Sterilization Policy

**Definition:**

*Sterilization is an open market sale at date 0 of government bonds with face value  $B$  for  $\frac{B}{L_0}$  units of international liquidity.*

Suppose that sterilization takes place. To purchase these bonds the private sector borrows against its international liquidity of  $w$  and acquires bonds. Let  $L_0$  denote the gross interest rate on this bond, so that the government collects in total  $\frac{B}{L_0}$  international reserves. At date 1, the government offers these reserves to purchase domestic assets, receiving  $L_1 \frac{B}{L_0}$  units of domestic assets in return. Then the budget constraint of the government is given by:

$$T + L_1 \frac{B}{L_0} = B, \tag{6}$$

where any shortfall is made up with a lump-sum tax on firms (in units of domestic liquidity).

The private sector’s balance sheet is altered by this transaction. Suppose that a firm purchases  $b$  units of bonds. At date 1, the firm will have an extra  $b - T$  units of domestic liquidity, while its international liquidity would have shrunk by  $\frac{b}{L_0}$  (or  $d_{0,f}$  has risen). Thus, if the firm is intact, its date 2 output is,

$$V_i = (w - d_{0,f})L_1 + Rk + b - T,$$

while if distressed, it is

$$V_s = (w - d_{0,f})\Delta + \Delta \frac{rk + (1 - \alpha)(b - T)}{L_1}.$$

But at date 0, the firm has a choice as to how much of these bonds to purchase at the interest rate  $L_0$ . Thus its date 0 problem can be written as,

$$\begin{aligned} (P6) \quad \max_{k, d_{0,f}, b} \quad & (w - d_{0,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha) \frac{\Delta}{L_1}) \\ \text{s.t.} \quad & d_{0,f} \leq w \\ & c(k) + \frac{b}{L_0} = d_{0,f}. \end{aligned}$$

Since the government purchases domestic assets at date 1, the market clearing condition gives us that,

$$L_1 = \frac{rK + (1 - \alpha)(B - T)}{W - D_{0,f} + 2 \frac{B}{L_0}} \tag{7}$$

### 3.3 Sterilization with Liquid Public Debt Markets

Let us start our study of the effectiveness of sterilization in a context where private asset markets are underdeveloped but secondary public debt markets are *domestically* liquid ( $\alpha = 0$ ).

#### **Lemma 2 (Date 0 Interest Rates)**

*If the private sector holds government bonds in equilibrium, the date 0 interest rate must satisfy,*

$$L_0 \geq L_1$$

This is easy to verify. In order for domestics to hold government bonds, they must be compensated for losing their international liquidity. Taking on an extra unit of debt costs  $\Delta + L_1$ . However purchasing one bond yields an extra  $L_0 \frac{\Delta + L_1}{L_1}$ . This gives us the inequality in the lemma.

Sterilization is naturally associated with a capital inflow, as either foreigners or domestic investors will attempt to buy the high yield government bonds. Foreigners have short horizons, however. Thus, if foreigners see a scenario where they can purchase these bonds at date 0 and sell them back to the domestic private sector at date 1, they will step in to purchase the bonds. They will do this only to the extent that the private sector has some international liquidity to offer at date 1. Thus the international liquidity of the public bonds is closely tied to the international liquidity of the private sector. The other potential buyer of the bonds is the domestic private sector itself. Since this sector does not have short horizons, it can always take advantage of the high return on the bonds by borrowing abroad (a capital inflow) to purchase the public bonds. However, once again, the capacity of the private sector to do so is limited by its international liquidity. In both cases it is the liquidity of the private sector that determines the outcome of sterilization.

#### **3.3.1 Liquidity Conservation**

#### **Lemma 3 (Liquidity Bias and Aggregate International Illiquidity)**

*If the government sterilizes so that the private sector is internationally illiquid,  $d_{0,f} = w$ , then foreigners will hold no domestic claims and restrict their holdings to international claims.*

Proof: see appendix.

This is the date 0 effect of future suspension of convertibility and market illiquidity. If the domestic private sector has no international liquidity to offer a foreigner when he sells domestic claims at date 1, then the foreigner's date 1 liquidity bias extends back to date 0. The foreigner anticipates that there will be a suspension of convertibility at date 1 and

that there will be no buyers of his domestic claims. This means he restricts his holdings of domestic claims at date 0. The foreigner only hold direct claims on international liquidity that are free of suspension of convertibility.

**Proposition 3 (Liquidity Conservation)**

Take the case where  $d_{0,f} < w$ . Let  $(k', d'_{0,f}, L'_1)$  be equilibrium choices and prices to (P6) and (7) when  $B = 0$  and let  $(k, d_{0,f}, b, L_1)$  be equilibrium choices and prices to (P6) and (7) when  $B > 0$  and  $L_0 = L_1$ . Then,

$$\begin{aligned} L_1 &= L'_1 \\ k &= k' \\ d_{0,f} &= d'_{0,f} + \frac{B}{L_0} \end{aligned}$$

Proof: see appendix.

The proposition is fairly intuitive. When  $d_{0,f} < w$ , foreigners are willing to hold the bonds at date 0. Moreover since the private sector is not internationally liquidity constrained, it can always borrow from foreigners to purchase the bonds that the government offers. Thus the gain in reserves of the government are offset one-for-one with a loss in international liquidity of the private sector —either at date 0 if the private sector purchases the bonds, or at date 1 if foreigners purchase the bonds— and the sterilization is completely undone.

**3.3.2 Pareto Improving Policy**

**Proposition 4 (Liquidity Contraction)** *There exists a sterilization policy of  $B > 0$  such that the resulting equilibrium of  $(k, d_{0,f}, b, L_1)$  is Pareto superior to  $(k', d'_{0,f}, L'_1)$ , as long as, in the resulting equilibrium,  $d_{0,f} = w$ . Optimal intervention raises the date 0 interest rate relative to date 1,  $L_0 > L_1$ , while lowering the date 1 interest rate relative to the case of no-intervention,  $L_1 < L'_1$ .*

Proof: As we showed in proposition 2, the decentralized equilibrium is constrained inefficient. Optimal policy should move the economy from point A to point B on figure 3. That is to say, optimal policy should reduce date 0 investment so that  $k < k'$ .

Suppose that the central bank offers  $B$  bonds at  $L_0$ . The program for a firm choosing to purchase  $b$  bonds is,

$$\begin{aligned} \max_{k, d_{0,f}, b} & (w - d_{0,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + \frac{\Delta}{L_1}) \\ \text{s.t.} & d_{0,f} \leq w \\ & c(k) + \frac{b}{L_0} = d_{0,f}. \end{aligned}$$



From the previous proposition, we know that sterilization is ineffective if  $d_{0,f} < w$ . Taking the other case ( $d_{0,f} = w$ ), we can rewrite the program as,

$$\begin{aligned} \max_{k,b} \quad & k(R + r\frac{\Delta}{L_1}) + (b - T)(1 + \frac{\Delta}{L_1}) \\ \text{s.t.} \quad & c(k) + \frac{b}{L_0} = w. \end{aligned}$$

The first order conditions for this program yield,

$$L_0 c'(k) = \frac{R + r\frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}}$$

Now optimal policy will be such that  $k < k'$ . For this to be the case, we must have that  $c'(k) < c'(k')$ .  $L_0$  satisfies,

$$\frac{1}{L_0} \frac{R + r\frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}} < c'(k') = \frac{R + r\frac{\Delta}{L'_1}}{\Delta + L'_1}$$

Rewriting this yields

$$L_0 > L_1 \frac{R + r\frac{\Delta}{L_1}}{R + r\frac{\Delta}{L'_1}} \frac{\Delta + L'_1}{\Delta + L_1}. \quad (8)$$

Consider the market clearing condition for  $L_1$ .

$$L_1 = L_0 \frac{rK + B - T}{2B}$$

which we can rewrite as,

$$L_1 \frac{B}{L_0} + L_1 \frac{B}{L_0} = rK + B - T.$$

Substituting in the government's budget constraint and rewriting,<sup>21</sup>

$$L_1 = L_0 \frac{rK}{B}$$

Whereas,

$$L'_1 = \frac{rK'}{W - D'_{0,f}}.$$

If  $k < k'$  then  $\frac{B}{L_0} = W - c(K) > W - D'_{0,f}$ . Therefore,  $L_1 < L'_1$  and  $B > 0$ . Combining this with (8), we can conclude that  $L_0 > L_1$ .<sup>22</sup> ■

<sup>21</sup>The budget constraint for the government is that,

$$T + L_1 \frac{1}{L_0} B = B.$$

When  $L_0 = L_1$ , the budget balances without having to raise taxes. When,  $L_0 > L_1$ , it must be that  $T > 0$  to pay the interest on the government debt.

<sup>22</sup>We also need to make sure that  $\Delta > L_1$  after the intervention so that we are still in the region where both liquidity constraints are binding.

$$rK < \frac{\Delta}{L_0} B = \Delta(W - c(K))$$

Since  $K < K'$ , if the condition is satisfied at the decentralized solution, it must also be satisfied at the central planner's solution.

When the central bank offers bonds at a high interest rate ( $L_0 > L_1$ ), there is clearly a capital inflow to purchase the bonds. However, when the private sector is internationally illiquid ( $d_{0,f} = w$ ), foreigners are unwilling to purchase the bonds since they recognize that there will be no buyers for the bonds at date 1. Hence the only buyers of the bonds are domestics. As long as domestics have reached their international debt capacity, government bonds cannot be purchased purely by borrowing abroad and saving. Expenditure must be reduced as well, which is why  $k$  falls. This is the heart of intervention, by issuing domestic bonds, the government crowds out private investment, while at the same time acquiring all of the international liquidity of the private sector. It can then transfer this back to the private sector at date 1. The government smooths shocks between date 0 and date 1. It removes liquidity from the private sector at date 0 by raising interest rates, then it injects this liquidity back into the markets at date 1, thereby lowering the cost of liquidity.

The main obstacle faced by the government in its attempt to reduce aggregate demand (investment in  $k$ ) at date 0 is the reaction of the private sector. In a successful sterilization  $L_1$  falls both below  $L_0$  and below  $L'_1$ . Remember that the private sector's incentives for international liquidity provision are linked to  $L_1$ . When  $L_1$  is low the private sector has very little incentive to do so. Thus when the government intervenes, it actually reduces the private sector's incentives for international liquidity provisioning. So much so that the private sector chooses to sell all of its liquidity provisions and hold domestic government bonds returning  $L_0 > L_1$ . As long as  $d_{0,f} = w$  the job of liquidity provisioning is left fully in the hands of the central bank.

The private sector anticipates a “bailout” and takes action. If the central bank is committed to supplying liquidity at date 1—in practice, via support of an exchange rate, or via a government guarantee—the private sector will try to take full advantage of this support. First, the private sector will have no incentive to hedge (independently hold its own international liquidity provisions.) Second, the private sector will attempt to undo the liquidity provisioning of the government. They will borrow abroad paying the opportunity cost of  $L_1$  and invest locally at the higher return of  $L_0$ . When  $d_{0,f} < w$  this action by the private sector fully offsets the liquidity provisioning by the central bank. Effectively, the private sector appropriates any international reserves held in the central bank. When  $d_{0,f} = w$  the private sector reaches its international debt capacity and it must reduce domestic expenditure in order to purchase government bonds.

### 3.4 Sterilization with Illiquid Public Debt Markets: Policy “Mistakes”

Is liquidity provisioning by the central bank a policy mistake? The response of the private sector, when viewed in isolation, would certainly suggest so. However, to appropriately judge the policy one must view the private sector response in conjunction with the central

bank action at both date 0 and date 1.

When sterilization is successful, the private sector is not the marginal international liquidity provider. The central bank takes over this job. Indeed, since  $L_1$  falls, the private sector has little incentive to do so. Is it possible to arrive at scenario where both,  $L_1$  falls so that the private sector has little incentive to liquidity provision, and the private sector remains the marginal liquidity provider? If so, the private sector would free-ride by cutting its liquidity provisioning (relative to the case of no-intervention) and on net the economy would lose international liquidity. In this section we show that this scenario is a very real possibility when domestic markets are illiquid. We shall demonstrate conditions under which the capital inflow accompanying sterilization leads not just to a purchase of government bonds but also to increased lending to the domestic private sector.

At date 1, government bonds are exchanged for international liquidity by both domestic distressed firms and any potential foreign investors. Distressed firms sell in order to receive funds for investment, while foreign holders sell in order to exit the market. We now reintroduce a friction in this transaction ( $\alpha > 0$ ).

The illiquid secondary market makes it harder to liquidate the two period government bond and exchange it for international liquidity. Transactions costs must be paid, there are search costs involved in making the exchange, and potentially even rents must be paid to market makers. This will further raise the required return for holding bonds at date 0, and will increase the domestic illiquidity index,  $s_d$ , at date 1.

### 3.4.1 Backfiring Policy

Suppose that the government sells bonds at date 0 in an attempt to sterilize, but does not intervene sufficiently so that the private sector is still internationally liquid at date 0.  $B$  government bonds are sold at interest rate of  $L_0$ . Consider the program for a firm choosing to hold  $b$  bonds at date 0,

$$\begin{aligned}
 (P7) \quad \max_{k, d_{0,f}, b} \quad & (w - d_{0,f})(\Delta + L) + k(R + r\frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha)\frac{\Delta}{L_1}) \\
 \text{s.t.} \quad & d_{0,f} \leq w \\
 & c(k) + \frac{b}{L_0} = d_{0,f}.
 \end{aligned}$$

#### Lemma 4 (Government Interest Rates)

*The date 0 interest rate on government bonds will always exceed the date 1 interest rate.  $L_0 > L_1$ .*

This lemma is easy to verify. Purchasing one government bond costs  $\frac{1}{L_0}$  units of international liquidity. At date 1, if the firm is distressed the bond can be sold for  $\frac{1-\alpha}{L_1}$  and the proceeds invested to yield output at date 2 of  $\Delta\frac{1-\alpha}{L_1}$ . If the firm is not distressed, the bond

is held till maturity to return one.<sup>23</sup> The opportunity cost of using one unit of international liquidity is to use it at date 1. If the firm is distressed, this yields  $\Delta$  at date 2, while if the firm is intact lending yields  $L_1$  at date 2. Thus firms purchase bonds as long as,

$$\Delta \frac{1 - \alpha}{L_1} + 1 \geq \frac{1}{L_0}(\Delta + L_1).$$

Rewriting,

$$L_0 \geq L_1 + \frac{\alpha \Delta L_0}{\Delta + L_1} > L_1.$$

Since government bonds are illiquid at date 1, they must pay a high interest rate in order to be held. Corporate borrowing rates on the other hand are determined by  $L_1$ , not  $L_0$ . If a firm was to sell some its domestic liquidity at date 0 and borrow (say, from another domestic), it would pay the interest rate of  $L_1$ . This is because another domestic would value the lost international liquidity at the opportunity cost – the return to lending at date 1, which is  $L_1$ .

Policy can backfire because sterilization leads to a fall in  $L_1$ . When the government sterilizes and holds international liquidity provisions, it commits to supplying these to the market at date 1. If the bonds that the government issues are longer term and illiquid, these liquidity provisions go toward supporting domestic private sector assets. The corporate borrowing rate,  $L_1$ , falls and corporates find it attractive to borrow and increase investment at date 0.

To see why, consider an extreme case of illiquidity where the government bonds have no secondary market and must be held till maturity ( $\alpha = 1$ ). Market clearing at date 1 is as follows: Distressed firms sell  $rK - T$  total of domestic liquidity at the date 1 interest rate of  $L_1$ , the  $B$  government bonds have no secondary market and must be held until maturity. Since one-half of firms are distressed, in total, firms raise for investment

$$\frac{rK - T}{2L_1}.$$

Now at the other side of the market are intact firms and the government. The government has international liquidity of  $\frac{B}{L_0}$ , while intact firms have in aggregate,  $\frac{W - D_{0,f}}{2}$ . Thus market clearing is,

$$\frac{rK - T}{2L_1} = \frac{W - D_{0,f}}{2} + \frac{B}{L_0}. \quad (9)$$

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<sup>23</sup>It is also easy to check that foreigners will never purchase government bonds. The cost  $\alpha$  is only suffered if claims are liquidated at date 1. A foreigner has short horizons with respect to domestic claims. A domestic has short horizons only if his firm is distressed. With probability one-half, the domestic holds the claim to maturity and does not liquidate. Thus domestics have longer horizons and bear less of the illiquidity cost. Which means that as long as domestic are holding government bonds, it must be that foreigners shy away from holding them.

Rewriting this expression, yields

$$\frac{rK - T}{L_1} + \left( \frac{B}{L_1} - \frac{B}{L_0} \right) = W + \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}.$$

The loss to the government of intervening by issuing bonds at date 0 at the high rate of  $L_0$  and purchasing bonds at date 1 at the lower rate of  $L_1$  must be made up in taxes. Thus, rewrite this expression by substituting in,  $T = B - \frac{L_1}{L_0}B$ , as,

$$\frac{rK}{L_1} = W + \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}. \quad (10)$$

Now imagine that the private sector exactly offsets the government's reserve accumulation. That is to say, suppose that for every unit that  $\frac{B}{L_0}$  rises, the private sector takes on an extra unit of date 0 debt. In this case, the term in the parentheses on the right hand side would be unchanged by sterilization. However if this is the case, and  $B > 0$ , it must be that  $L_1$  falls, since the supply of liquidity to purchase private sector assets has risen by  $\frac{B}{L_1}$ . This is why sterilization backfires. The government supports private sector assets and hence lowers corporate borrowing costs.

The case where the government bonds are fully liquid highlights the key role played by illiquid markets in the previous conclusion. Suppose that  $\alpha = 0$ , so that government bonds can be sold at date 1 without any friction. In this case, distressed firms sell  $B$  bonds at the price of  $L_1$  at date 1. Thus append  $\frac{B}{L_1}$  to the left hand side of (10) to arrive at,

$$\frac{rK}{L_1} + \frac{B}{L_1} = W - D_{0,f} - \left( \frac{B}{L_0} - D_{0,f} \right) + \frac{B}{L_1}.$$

The government's reserves go towards purchasing back the bonds that it issued at date 0. Sterilization does not bring additional support for corporate assets, and hence  $L_1$  is unaffected.

Let us now state this result more formally.

**Proposition 5 (International Liquidity Loss)**

*Consider the case where  $\alpha > 0$ . Let  $(k', d'_{0,f}, L'_1)$  be equilibrium choices and prices to (P7) and (9) when  $B = 0$  and let  $(k, d_{0,f}, b, L_0, L_1)$  be equilibrium choices and prices to (P7) and (9) when  $B > 0$  but  $d_{0,f} < w$ . Then, we have that,*

$$\begin{aligned} L_1 &< L'_1 \\ k &> k' \\ d_{0,f} &> d'_{0,f} + \frac{B}{L_0} \end{aligned}$$

Proof: see appendix.

An equivalent way to think about this result is in terms of capital inflows. When the government sterilizes and commits to providing liquidity at date 1, foreigners come in to purchase short term assets that will be supported by the government. If these assets are the government bonds that are issued in sterilization, then the sterilization is offset. However, when the secondary market for government bonds is illiquid, foreigners will shy away from holding these bonds and will instead demand other short term assets. The corporate sector sees this as a borrowing opportunity on good terms and increases investment. Thus a symptom of counterproductive sterilization is that corporate borrowing rates fall relative to government borrowing rates. In other words, as  $B$  rises, the spread between the interest rate on government bonds and that on corporate lending,  $L_0 - L_1$  rises.<sup>24</sup>

Backfiring is a policy mistake, but one that is not easy to avoid in the environment of emerging markets. It occurs because the government does not sterilize enough—the private sector is left with some international debt capacity that it borrows against on favorable terms— *and* the domestic instruments that the government issues are illiquid. How much sterilization is enough? In the model, the government has to sell enough bonds so that the private sector reaches its international debt capacity ( $d_{0,f} = w$ ). That is the government takes all of the private sector’s liquidity. This is clearly an abstraction. Our supply curve of international funds is kinked at  $d_{0,f} = w$ . A more gradual slope would cause interest rates to rise for smaller interventions. Indeed the key point is that in unsuccessful sterilizations, corporate borrowing rates fall not rise, both absolutely and relative to government rates. Nevertheless, during the booms when international capital markets are all too willing to lend to emerging economies, the amount of required sterilization is much higher and as such the possibility that policy may backfire is very real. The second factor behind the failure of sterilization, that of illiquid secondary markets, is more structural and seems unavoidable in emerging markets.

### 3.5 Optimal Policy with Illiquid Secondary Markets

When there are illiquid secondary markets for government bonds, optimal policy must take this into account. Are there always Pareto improving policies? We answer this question next.

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<sup>24</sup>We have made the assumption that the secondary market for corporate assets is liquid, while the government asset market is illiquid. The assumption is clearly unrealistic, but our results depend on absolute liquidity of the government market rather than the relative liquidity of the two markets. The reason is that, *ceteris paribus*, when the government market is illiquid, sterilization leads to a liquidity mismatch in the government’s balance sheet. This means reserves will support private sector assets and lower the absolute cost of funds for the private sector - even if this cost of funds before and after policy was above the government bond interest rate. It is clearest to understand the backfiring result as sterilization causing a fall in the spread between corporate assets and government bonds.

**Proposition 6 (Sterilization with Illiquid Markets)** *There exists a sterilization policy of  $B > 0$  such that the resulting equilibrium of  $(k, d_{0,f}, b, L_0, L_1)$  is Pareto superior to  $(k', d'_{0,f}, L'_1)$ , as long as,*

- (i) *Domestic markets are sufficiently liquid (i.e small  $\alpha$ ), and*
- (ii) *In the resulting equilibrium,  $d_{0,f} = w$ , so that the private sector is internationally illiquid.*

*Optimal policy requires that, in equilibrium, the government raises date 0 interest rates,  $L_0 > L_1$ , and the date 1 interest rate falls relative to no-intervention,  $L_1 < L'_1$ .*

See the appendix for the proof. The result can be understood by studying figure 4. The figure traces out welfare for the decentralized equilibrium as a function of  $k$  ( $U^{PRIV}$ ), and welfare for the equilibrium assuming that the central planner intervenes via sterilization, and forces the economy to incur the secondary market illiquidity costs ( $U^{CP}$ ).

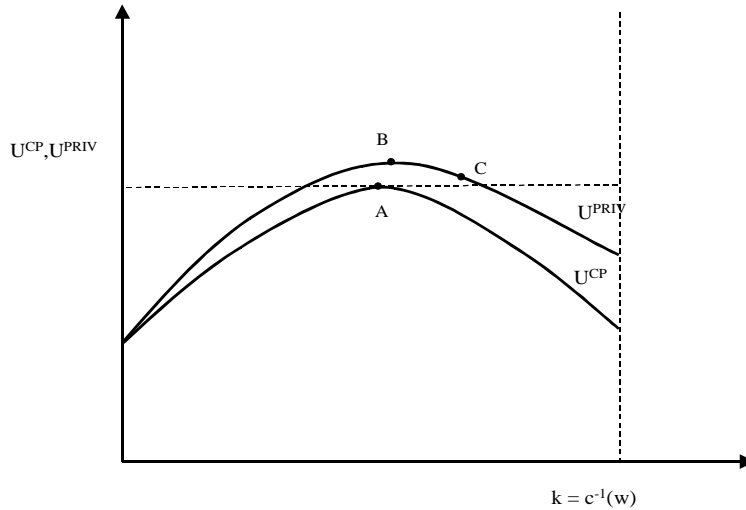


Figure 4:  $U^{CP}$  and  $U^{PRIV}$

As in proposition 2, it is clear that the central planner values international liquidity higher than the private sector, and domestic liquidity less. Thus the benefit of intervention is that it moves the private sector away from a sub-optimal choice. That is,  $K^{CP} < K'$ . However intervention has a cost, since firms must sell their bonds into an illiquid market. Thus, it is clear that intervention always lowers the welfare function when  $\alpha > 0$ .

Intervention is beneficial as long  $U^{CP}(K^{CP}) > U^{PRIV}(K^{PRIV})$ . But this depends on the size of the externality versus the cost of intervention. The private sector always chooses an inefficient point - point C on the figure. The cost of intervention is that it lowers welfare

to points on the lower curve. Thus if the cost is sufficiently high, point A lies below point C and intervention is not beneficial. On the other hand, in the extreme case where  $\alpha = 0$ ,  $U^{CP}$  rises to  $U^{PRIV}$ , so that point B is chosen by the central planner. It is always the case that intervention is beneficial in this case.

## 4 Further Costs of Failed Sterilizations: Quasi-Fiscal Deficits and Excessive Short Term Capital Inflows

In addition to fueling capital flows, sterilization often appreciates the currency, generates quasi-fiscal deficits, and biases capital flows toward short term debt (e.g. Reinhart and Reinhart (1998), Montiel and Reinhart (1997), and Calvo et al. (1993)). While each of these regularities may just be the equilibrium counterpart of the higher domestic interest rates associated with a successful sterilization, policymakers are often concerned that they are too large or too severe to correspond to the simple counterpart of a successful policy. We have already argued that failed sterilization leads to an increase in capital flows and aggregate demand. In this section, we show that the backfiring result we described in section 3 also produces the quasi-fiscal deficit and the shortening in the maturity of capital flows. Thus, these regularities may also correspond to symptoms of failed sterilization.

### 4.1 Quasi-Fiscal Deficits

Returning to the simple single-public financial instrument described in section 3, consider the budget constraint of the government. This equates revenues, comprised of taxes and the proceeds from selling the international reserves acquired during the sterilization episode, to the face value of the bonds issued:

$$T + L_1 \frac{B}{L_0} = B.$$

There are many ways in which this constraint can lead to an increase in the fiscal deficit. While when  $L_0 = L_1$ , the budget balances without having to raise taxes, when  $L_0 > L_1$  it must be that  $T > 0$  to pay the high interest on the government debt. While this situation arises with a successful sterilization (hence the high interest rates), it also does when there is an illiquid secondary market for bonds.<sup>25</sup> In lemma 4, we showed that when  $\alpha > 0$ , it

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<sup>25</sup>Quantitatively, it is seldom the case that the quasi-fiscal costs associated with sterilization are much larger than a quarter of a percent of GDP - a small number when compared with other factors involved in crises (see e.g. Kletzer and Spiegel (1998) who compute these costs for several Latin American and Asian economies and conclude that they are not only small, but also draw little reaction from governments and central banks). It is for this reason that we think of these costs as yet another dimension of a failed sterilization, rather than as the primary source of concern.



must be that  $L_0 > L_1$ . Hence, attempting to sterilize in an illiquid bond market can have large detrimental effects on the quasi-fiscal deficit, *even* without the rewards of a higher domestic corporate borrowing rates and a slowdown in aggregate demand.

Needless to say, this deterioration in the fiscal situation can be worsened if the reserves of international liquidity are not targeted back to the private sector in an efficient fashion at date 1. This will occur if the government receives a return less than  $L_1$  on its international reserves.

## 4.2 Excessive Short Term Capital Flows

The observed shortening in the maturity of capital flows following sterilization is particularly interesting from our point of view. The step in arriving at this result from our model is in defining short and long term debt in terms of their insurance features. One can think of long term debt as short term debt plus rescheduling insurance. A simple extension of the model presented in section 2 shows that agents undervalue the insurance component of long term debt as long as  $L_1 < \Delta$ , which is the case when domestic financial markets are underdeveloped. The result is akin to our previous result on the undervaluation of international liquidity.

Suppose that only a fraction  $1 - \psi$ , where  $0 < \psi < 1$ , of  $w$  is directly pledgeable to foreigners at date 1.<sup>26</sup> Debt that is taken on against this  $1 - \psi$  of international liquidity will be viewed as short term debt. Now suppose that the rest,  $\psi w$ , can be seized by foreigners at date 2, however doing so requires payment of a monitoring cost of  $0 < \epsilon < (\Delta - 1)$  at date 2. A domestic firm has two choices. (A) It can take on one period debt up to the limit of  $(1 - \psi)w$  at the international interest rate of one. Then at date 1, if it needs the funds, it can roll this over and take on additional debt of  $\psi w$ . However, the interest rate on this additional debt will obviously be above one to compensate the foreign lenders for bearing the monitoring cost. (B) It can take on long term debt against the full  $w$ , in which case the foreign lenders will *always* pay the monitoring cost to seize the additional  $\psi w$ . Thus, domestics face an upward sloping term structure of borrowing.

With option A, only firms that are distressed at date 1 will take on the additional debt and draw down  $\psi w$ . With option B, on the other hand, all firms will have pledged their extra-collateral ex-ante, and the intact firms will sell the corresponding international funds at date 1 to the distressed firms. The latter option is clearly socially preferable, since the social value of an extra unit of liquidity is  $\Delta - 1 > \epsilon$ . The problem, as before, is that the return to intact firms is only  $L_1 - 1$ , which could well be below  $\epsilon$  if domestic financial markets are illiquid. If this last inequality holds, the equilibrium is one where no firm values

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<sup>26</sup>Diamond (1991) develops a model of debt maturity structure based on liquidity risk. The sketch of our model is related but the maturity structures depends on aggregate liquidity risk.

the insurance service of long term debt and is willing to pay the high interest. This comes at great cost in terms of the aggregate supply of international liquidity since a central planner would prefer that firms take on the long term debt.

Since a failed sterilization lowers  $L_1$ , the incentives for short versus long term debt are adversely affected by it. Thus the capital inflow-composition problem is worsened when there is backfiring.<sup>27</sup>

## 5 Intermediation, Exchange Rate Systems, and the Mundell-Fleming Mechanism

Up to now, we have removed from our analysis any source of ex-ante heterogeneity among domestics and made no distinction between domestic assets issued by the central bank (money and bonds, in particular). In this section we go back to the basic model in section 2 but relax these two assumptions with the goal of connecting our framework to the traditional Mundell-Fleming based discussion.

Introducing ex-ante heterogeneity gives a role to domestic asset markets at date 0, when sterilization takes place. Money is an asset with unique transaction services in these financial markets. In particular, it facilitates bank lending. While none of our basic conclusions are modified by these extensions, we find that indeed—and for more or less the standard reasons—a flexible exchange rate system facilitates the success of sterilization. A monetary contraction is indeed useful in limiting agents date 0 borrowing even before the private sector runs out of international liquidity. However, in our lending-channel example, this additional power is gained at the cost of losing the Pareto nature of the intervention, as it involves a dynamic transfer from banks/savers to corporations.

### 5.1 Money and the Banking System

We introduce a minimalist structure to capture the special role of money in facilitating domestic financial transactions at date 0. Rather than starting with homogeneous domestic agents, we take the international wealth away from the corporate sector and allocate it to a blended banking/savers sector (henceforth, referred to as bankers). Bankers, like entrepreneurs, have linear preferences over date 2 consumption but have no investment opportunities aside from lending to the corporate sector.

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<sup>27</sup>Since a successful sterilization also lowers  $L_1$ , the reader may wonder whether the shortening of capital flows is also an implication of a successful sterilization. The answer to this question is that it will not. Since government bonds yield  $L_0$  which is high, firms will find it profitable to borrow as much as they can at date 0, which will include both short and (expensive) long-term debt. That is, the high  $L_0$ , replaces  $L_1$  in the preceding cost-benefit tradeoff.

In order to lend to the corporate sector at date 0, bankers require domestic collateral from corporations and need to hoard  $\mu < 1$  units of domestic money between dates 0 and 1 per unit of debt's face value. They are born with  $M^0$  units of high powered money. This means that the maximum amount of loans that the banking sector can make (to the corporate sector) is also  $\frac{M^0}{\mu}$ .<sup>28</sup>

The central bank fully backs up  $M^0$  with international reserves, which are exchange for money at date 1. Thus, the nominal exchange rate at date 1,  $E_1$ , is equal to one regardless of the exchange rate system prevailing at date 0. Banks are not needed at date 1, so bankers can participate directly in the financial markets without holding any money.

Taken together, these assumptions create a transmission mechanism for domestic monetary policy via the "lending channel."<sup>29</sup> A central bank that contracts or expands  $M^0/\mu$  can affect the amount of lending from bankers to the corporate sector. Our assumptions are designed to isolate the impact of this mechanism on the date 0 problem of taming the boom, as it is apparent that at date 1 the Central Bank will attempt to ensure perfect domestic aggregation of international collateral.

## 5.2 Monetary Policy in a Flexible Exchange Rate System

We remove bonds for now and fix  $M^0$ , so monetary policy takes the form of a tightening in reserve requirements,  $\mu$ . None of our main conclusions is affected by reintroducing bonds and implementing the monetary contraction via open market operations instead (see below).

At date 0, bankers lend as much as they can to firms as long as  $L_0 > L_1$ . That is, they supply:

$$X_0^s = \min \left[ \frac{M^0}{\mu}, W \right]. \quad (11)$$

Let  $X_0^{dh}$  denote the potential demand for loans, defined as the demand that would arise if  $L_0 = L_1$ . Then the amount lent,  $X_0$ , is:

$$X_0 = \min[X_0^s, X_0^{dh}] \quad (12)$$

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<sup>28</sup>These steps can be disentangled more finely. For example, separating bankers from savers, the story goes as follows: Banks face a reserve requirement on taking deposits from savers. Let  $\mu$  be the reserve requirement. Then, given  $M^0$  banks can only take in deposits of  $\frac{M^0}{\mu}$ . We additionally assume that banks cannot raise funds from savers in any other way than by taking deposits. This means that the maximum amount of loans that the banking sector can make (to the corporate sector) is also  $\frac{M^0}{\mu}$ . Last we assume that the  $rK$  of domestic liquidity that firms create is only tradeable among the corporate sector and banks. That is loans to a firm with some domestic liquidity can only be made by other firms (for example, via asset sales, trade credit, or mergers), or from savers through the banking sector.

<sup>29</sup>For discussion and evidence of the lending channel in the U.S., see for example, Kashyap, Stein and Wilcox (1993).

We focus on the case where the first terms in square brackets in both (11) and (12) bind.<sup>30</sup> Thus:

$$X_0 = \frac{M^0}{\mu}.$$

In equilibrium, corporate investment is effectively determined by credit availability:

$$c(K) = \frac{M^0}{\mu},$$

while the rest of bankers' funds is supplied at date 1 :

$$X_1 = W - X_0.$$

It is apparent that by controlling  $\mu$  the central bank can manipulate the timing of investment and hence aggregate liquidity provisions. It is here that exchange rate flexibility plays a role, as it limits the private sector's mechanisms to undo the monetary squeeze.

It is important to notice, nonetheless, that unlike the policies we discussed in the previous section, this mechanism relies on transfers as opposed to Pareto changes. In fact, since in this section we have allocated all the wealth to a sector which has no real investment margin (the bankers), the decentralized equilibrium is constrained efficient and  $\mu$  should be set to a very low value. Reducing the fragility of the corporate sector by raising  $\mu$  comes primarily at the expense of curtailing financial investment opportunities to bankers.

Furthermore, as in Mundell-Fleming, the counterpart of monetary control is that the exchange rate at date 0,  $E_0$ , is determined by the market.  $E_0$  must be such that banks are indifferent between selling their stock of money at date 0 and collecting the return for the funds that otherwise would have been involved in a loan package, and the return on the loan itself:

$$L_1 \left( \frac{\mu}{E_0} + 1 \right) = L_0 + L_1 \mu.$$

Solving for the exchange rate, yields:

$$E_0 = \frac{L_1 \mu}{L_1 \mu + s^0} < 1,$$

where  $s^0 \equiv L_0 - L_1$ . While in our simplified framework such appreciation has no impact on real allocations, in reality it may be in itself a source of concern. For example, if bankers had a choice on whether to consume an imported or a domestic good at date 0, some of their international liquidity would be diverted to the early consumption of foreign goods.

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<sup>30</sup>If  $X_0^{dh}$  binds instead,  $L_0 = L_1$ . This case is uninteresting for us since monetary policy, of the type discussed here, has obviously no effect on lending.

### 5.3 Monetary Policy in a Fixed Exchange Rate system

Suppose now that the exchange rate is fixed at one at date 0 as well, and the central bank stands ready to swap international reserves for domestic money at the private sector's will.

There are two basic scenarios to consider in this fixed exchange rate system. In the first one, foreigners do not value domestic money as collateral.<sup>31</sup> It is apparent that in this case bankers will offset any monetary contraction by selling their  $X_1$  to the central bank in exchange for domestic money. They will do so for as long as  $L_0 > L_1$  and  $X_1 > 0$ . Thus, as in Mundell-Flemming, monetary policy is futile. This changes once  $X_1 = 0$  for then the bankers are constrained in the same sense as firms were in section 3 in the scenario where sterilization worked. The “holy trinity” of open economy macroeconomics establishes that only two out of the following three are possible: effectiveness of monetary policy, control of the exchange rate, and free capital mobility. It is the endogenous canceling of the latter that gives back its powers to monetary policy.<sup>32</sup>

The other limit case to consider is when high-power money is part of international collateral. In that case, neither  $X_0$  nor  $X_1$  can be affected by monetary policy, thus monetary policy is useless. This case highlights an important aspect of the policy considerations we have stressed throughout the paper: In order to be successful in preventing an external crisis, the policymaker needs to be able to “hide” some of the private sector's *international* liquidity at date 0. This, it will not be able to do if it attempts it by selling highly internationally liquid instruments to its private sector.

## 6 Final Remarks

A central consideration of macroeconomic policy in emerging economies is *external-crisis* prevention. In practice, the main macroeconomic tool utilized for such purpose is the sterilization of capital inflows. However, existing models are not particularly well suited to study this policy. On one hand, the sovereign debt literature identifies the aggregate international constraint as limiting external debt repayments, and therefore links external crises to this aggregate constraint. But as it is designed to answer a different question, this literature suppresses domestic financial markets and is therefore unable to study outcomes of sterilization policy. On the other hand, the Mundell-Fleming framework directly addresses sterilization policy. However, as it essentially ignores all aspects of the external financial constraint and instead emphasizes the monetary aspect of sterilization, it is best suited

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<sup>31</sup>Recall that bankers need to hold the money at date 0 in order to make the loans. Hence, this assumption does not mean that foreigners wouldn't accept money as a method of payment at date 0, but that money in banks' hands does not count as collateral.

<sup>32</sup>Reisen (1993) argues that the “holy trinity” does not apply when the central bank uses reserve requirements rather than open market operations. We do not find support for such claim in our model.

to analyze policy in economies where international liquidity management is not a central issue – a situation that seems more apt for developed than developing economies, especially in the neighborhood of external crises. Moreover, neither approach is explicit about the degree of development of domestic financial markets and on how this affects the outcome of sterilization, a policy measure that relies heavily on these markets.

The framework proposed here roots both the need for policy as well as its outcomes in the illiquidity or underdevelopment of a country’s financial markets. The central bank is well aware of the external financial constraint and the consequences of reaching it abruptly. By sterilizing current capital inflows in an attempt to reduce aggregate demand, the central bank not only changes its own and the private sector’s portfolio significantly, but it also invites the latter to reassess risks and respond to such measure. It is this response which represents the biggest obstacle to the success of policy. How risks change with the intervention and how much the private sector does about it depend on the availability of assets, their liquidity, and the intervention rules of the central bank. All of these factors are central to our model.

When sterilization is successful, the private sector is not the marginal international liquidity provider as the central bank takes over this job. Indeed, since the domestic cost of capital at times of crises ( $L_1$ ) falls, the private sector has little incentive to do so. On the other hand, when sterilization backfires  $L_1$  also falls, but in this case the private sector remains the marginal liquidity provider. This outcome hinges on the liquidity of the public debt market. One of the central features of the swap of bonds for international reserves involved in a sterilization is the maturity mismatch that arises in the central bank’s balance sheet — international reserves (very short term) as assets and bonds (longer term) as liabilities. This maturity mismatch, while largely irrelevant if the public debt market remains liquid during crises, creates an “implicit insurance” to private claims if not, and hence it effectively lowers the cost of capital to the private sector. This observation is relevant since it is not uncommon to observe a decline in the spread between the private cost of capital and the public bonds yield during sterilizations.<sup>33</sup>

The framework we have proposed and the issues we have analyzed raise at least as many questions as they answer. In particular, since our model only has a corporate sector and a central bank, international liquidity management is limited to studying central bank

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<sup>33</sup>The case of Colombia during the early 90s —one of the “classic” examples of steep sterilization in Latin America— hints at the presence of these mechanisms. While the interest rate on public bonds rose sharply, the spread between private lending rates and public bonds’ rates declined significantly (from around seven percent previous to the 1991 sterilization to virtually zero during it), and the stock market soared. On the expenditure side, while investment fell sharply, consumption only experienced a mild decline and construction —as well as loans to it— kept rising. Of course, these facts must be interpreted with caution since many other variables and policies were taking place at the time.

asset and liability management policies – and its effects on the corporate sector. Liquidity management, in practice, has more layers. Argentina has considered liquidity requirements in the banking sector. Central banks often respond to inflows by increasing domestic reserve requirements. Until recently, Chile required the private sector to hold liquidity against short term external financing. Each of these actions results in liquidity provisioning at a more decentralized level. When is liquidity provisioning by the banking system more or less effective than that of the central bank? In assessing the international liquidity of a country, should we equally weight the holdings of the central bank and those of the domestic banking system? These are important questions that require, among other things, enriching the framework to include a domestic banking sector. This remains something we are working on.

Similarly, while sterilization —and international liquidity management in general, including external public debt management— may be the tool of choice in the short run, long term solutions to the problems we have highlighted are not cyclical but structural in nature. Our framework not only illustrates the second best options and policy problems, but also points at domestic financial underdevelopment as the primitive source of concern. It is important when thinking about second best solutions to also ask whether they will have any long run effects on the primitive problem. Taxing capital flows, for example, while obviously appropriate from the second best point of view, and even useful as a companion to sterilization, loses appeal once one thinks in terms of the medium and long term development of financial markets. Flexible exchange rates may have an advantage over fixed - for Mundell-Fleming reasons - but they may have long run detrimental effects on financial markets.

Regardless of the specific answers to these concerns, it appears to us that there is an increasing realization that a modern debate on issues such as the advantages and drawbacks of dollarization, capital flows taxation, liquidity requirements, and so on, ought to consider the asset markets aspects of the problem, and that the structure we have proposed here is a useful tool for such a task. We are currently exploring some of these structural problems in ongoing work.

## A Appendix

### A.1 Regions

Taking as given date 0 decisions of  $k$  and  $d_{0,f}$ , there are four possible regions that the economy can be in at date 1.

- Region I occurs when both the international liquidity constraint (4) and the domestic liquidity constraint (3) are slack. In this case, both the international liquidity premium and the domestic illiquidity index are zero and there is full project completion.  $L_1 = 1, s_d = 0, \theta = 1$ .
- Region II occurs when the international constraint is slack but the domestic constraint is binding. The international liquidity premium remains at zero, however the domestic illiquidity index is positive as the economy fails to aggregate all of its resources resulting in some projects being downsized.  $L_1 = 1, s_d > 0, \theta < 1$ .
- Region III occurs when the international constraint is binding but the domestic constraint is slack. The economy aggregates all of its resources, however this is insufficient in the aggregate and some projects are downsized.  $L_1 > 1, s_d = 0, \theta < 1$ .
- Region IV occurs when both the international constraint and the domestic constraint are binding. The international liquidity premium and the domestic illiquidity index are positives, and some projects are downsized.  $L_1 > 1, s_d > 0, \theta < 1$ .

### A.2 Proof of Lemma 3

Lemma 3 stated that under the assumption of suspension of convertibility and if  $d_{0,f} = w$ , then foreigners will not hold any domestic claims at date 0. The proof is as follows.

Suppose that the private sector holds  $B$  government bonds,  $rK$  domestic claims, and has  $w - d_{0,f} = 0$ . Suppose foreigners hold  $B_f$  government bonds. We show that  $B_f = 0$  in equilibrium.

When  $w - d_{0,f} = 0$ , the government controls all of the international liquidity in the economy. If the government suspends convertibility at date 1, it only releases reserves when shown an invoice for an imported good to fulfill investment plans. The government from soale of  $B + B_f$  bonds at date 0 has  $\frac{B+B_f}{L_0}$  of reserves.

Case I: Suppose that the foreigner goes to the central bank with  $B_f$  bonds and an invoice for  $B_f/L_1$  import goods. Then the government releases  $B_f/L_1$  reserves to the foreigner that it uses to pay the cost of the import good. The foreigner must sell this to the domestic distressed firm for  $B_f$  domestic claims. However, these claims have no value to foreigners



since they will surely be repudiated at date 2. Thus,  $B_f = 0$ .<sup>34</sup>

Case II: Suppose the foreigner goes to a domestic investor and offers  $B_f$  bonds for international liquidity. Since  $d_{0,f} = w$ , domestics have none, and can only get international liquidity by tendering bonds to the central bank. Suppose a domestic tenders  $B$  bonds to the central bank, shows import goods of  $B/L_1$  and then receives  $B/L_1$  reserves. However these reserves will exactly pay for the import goods, hence the domestic will be left with no liquidity to offer foreigners.

Finally let us show that only distressed firms will sell bonds to the central bank for its international reserves.

Case III: Either distressed or intact firms can tender  $rK + B - T$  to the government for  $\frac{B+B_f}{L_0}$  of reserves. If intact firms tender, they onsell the imported goods to distressed firms in exchange for some of  $rK + B - T$  of distressed firms. Suppose an intact firm tenders one domestic claim, it receives  $1/L_1$  import goods, which it sells to the distressed firm for  $L_1$  domestic claims. Thus it is indifferent between tendering and not. Assume that it does not.

Distressed firms receive all imported goods totalling,

$$\frac{rK + B - T}{L_1}$$

From (7), this is exactly equal to  $\frac{B+B_f}{L_0}$  which is all of the government's reserves. Thus when only distressed firms tender, this is an equilibrium.

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<sup>34</sup>We have made two unrealistic assumptions here. First, we have said that foreigners cannot take the imported goods and liquidate them outside the country for international liquidity. If there is any liquidation cost in this transaction, it is easy to see that foreigners would prefer not to hold domestic claims, since they must always bear this cost, while domestics never bear the cost. The more interesting case is that of over-invoicing. During periods of capital-controls domestic firms routinely over-invoice their imported goods. That is they claim higher prices than actual ones for their goods, thereby getting their hands on more valuable international reserves. Suppose that a firm can get away with over-invoicing by a multiple of  $M > 1$ . A firm that tenders one unit of domestic claim receives  $\frac{1}{ML_1}$  import goods, and  $\frac{M-1}{ML_1}$  international reserves. The import goods can be sold to a distressed firm, for  $\frac{1}{M}$  domestic claims. The international reserves can now be sold to a foreigner to redeem some of the foreigner's bonds. Foreigners selling  $B_f$  bonds can receive at maximum  $\frac{M-1}{M} \frac{B+B_f}{L_0}$  reserves. Thus let,

$$L_{1,f} = \frac{B_f}{\frac{M-1}{M} P_B (B + B_f)}$$

This is the discount that foreigners sell their bonds at. In equilibrium, foreigners will hold only enough bonds so that  $L_{1,f} = L_1$ . This means that a fraction  $\frac{M-1}{M}$  of international reserves can be promised away to foreigners by the private sector at date 0. Over-invoicing creates a leak in the system.

### A.3 Proof of Proposition 3

Proposition 3 stated conditions under which the private sector completely undoes the central banks sterilization operation.

Take prices first. Let us rewrite (7) to give,

$$L_1(W - D_{0,f}) + 2L_1 \frac{B}{L_0} = rK + B - T.$$

Now suppose that  $k = k'$  and  $d_{0,f} = d'_{0,f} + \frac{B}{L_0}$ , then,

$$L_1(W - D'_{0,f}) + L_1 \frac{B}{L_0} = rK' + B - T.$$

However, given the government's budget constraint, (6), we can conclude that  $L_1$  does not depend on  $B$ .

Fixing prices at  $L_1 = L'_1$ , let us consider the firm's optimization problem and verify that  $k = k'$  and  $d_{0,f} = d'_{0,f} + \frac{B}{L_0}$  is a solution. Since,

$$w - d_{0,f} > 0,$$

it must also be that,

$$w - d'_{0,f} > \frac{B}{L_0} > 0.$$

Additionally, since,

$$c(k) + \frac{B}{L_0} = d_{0,f}$$

then substituting,

$$c(k') + \frac{B}{L_0} = d'_{0,f} + \frac{B}{L_0}.$$

Thus the solutions satisfy the budget constraints. Since the objective of the program is linear, they also satisfy the F.O.C's. The last part of the proposition,  $L_0 = L_1$ , follows from domestic arbitrage. ■

### A.4 Proof of Proposition 5

The statement is that when domestic secondary markets are illiquid and  $d_{0,f} < w$ , sterilization can backfire.

The proof is by showing that there is no other possible equilibrium. Suppose there was an equilibrium with  $k \leq k'$  and  $L_1 \geq L'_1$ . The private sector purchases the bonds through increased borrowing and reduced expenditure. However if this is the case then from market clearing we have a contradiction,

$$L_1 = \frac{rK + (1 - \alpha)(B - T)}{W - D_{0,f} + 2\frac{B}{L_0}} < \frac{rK'}{W - D'_{0,f}} = L'_1$$

Consider an equilibrium with  $L_1 \leq L'_1$  and  $K \leq K'$  - where at least one of the inequalities is strict. From the firm's first order conditions,

$$c'(k) = \frac{R + r \frac{\Delta}{L_1}}{1 + \frac{\Delta}{L_1}} < \frac{R + r \frac{\Delta}{L'_1}}{1 + \frac{\Delta}{L'_1}} = c'(k').$$

Since  $c(k)$  is strictly convex, this is a contradiction. The case with  $L_1 \geq L'_1$  and  $K \geq K'$ , can be ruled out by the same logic. Hence the only equilibrium is,  $L_1 < L'_1$ ,  $k > k'$ , and  $d_{0,f} > d'_{0,f} + \frac{B}{L_0}$ . ■

## A.5 Proof of Propostion 6

Proposition 6 derived optimal policy when secondary markets were illiquid.

At date 1, the private sector sells the government bonds in return for the international liquidity. When  $\alpha > 0$ , this transaction suffers a real cost. Thus intervention can be costly because it requires the private sector to sell more bonds into an illiquid market. Policy can result in a Pareto improvement as long as this cost is not too high. In the extreme case when  $\alpha = 0$ , policy always leads to Pareto improvement.

First let us define  $U^{PRIV}$  as,

$$(P8) \quad \max_{k, d_{0,f}} \quad U^{PRIV} \equiv (w - d_{0,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1})$$

$$s.t. \quad d_{0,f} \leq w$$

$$c(k) = d_{0,f}.$$

The first order condition for this program is,

$$c'(k') = \frac{R + r \frac{\Delta}{L_1}}{\Delta + L_1}$$

The optimal choice is denoted  $k'$  to refer to the no-intervention point. The market clearing condition remains that of (5).

Suppose that a central bank offered  $B$  bonds for sale at the interest rate of  $L_0$ , but bearing the illiquidity cost of  $\alpha$ . This program must be altered as follows:

$$\max_{k, d_{0,f}, b} \quad (w - d_{0,f})(\Delta + L_1) + k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha) \frac{\Delta}{L_1})$$

$$s.t. \quad d_{0,f} \leq w$$

$$c(k) + \frac{b}{L_0} = d_{0,f}.$$

Since we require that the central bank sell enough bonds so that  $d_{0,f} = w$ , we can rewrite as:

$$\max_{k, b} \quad k(R + r \frac{\Delta}{L_1}) + (b - T)(1 + (1 - \alpha) \frac{\Delta}{L_1})$$

$$s.t. \quad c(k) + \frac{b}{L_0} = w.$$

The first order condition for this program is,

$$c'(k) = \frac{1}{L_0} \frac{R + r \frac{\Delta}{L_1}}{1 + (1 - \alpha) \frac{\Delta}{L_1}}$$

and market clearing gives,

$$L_1 = \frac{rK + (1 - \alpha)(B - T)}{2 \frac{B}{L_0}}$$

Let us now simplify this expression using the fact that the government's budget constraint is  $T + B \frac{L_1}{L_0} = B$ . Then, this can be rewritten as,

$$L_1 = \frac{1}{1 + \alpha} \frac{rK}{\frac{B}{L_0}}. \quad (13)$$

Substituting this back into the objective in (P8) allows us to generate the welfare function for the central planner assuming that it intervenes sufficiently so that  $d_{0,f} = w$ . This expression is given by,

$$U^{CP}(K) \equiv K \left( R + \frac{r}{1 + \alpha} \right) + (W - c(K)) 2\Delta.$$

The central bank can choose  $B$  to arrive at any point on this function. Thus, assuming intervention, the central bank will choose to implement,

$$C'(K^{CP}) = \frac{R + \frac{1}{1 + \alpha} r}{2\Delta}.$$

We can subtract the objective in  $U^{CP}$  with that in (P8) to see how the private sector's choice differs from that of a central planner.

$$U^{CP} - U^{PRIV} = (W - c(K)) s_d - \frac{1}{1 + \alpha} \frac{rK}{L_1} (s_d + \alpha\Delta) \quad (14)$$

As in proposition 2, it is clear that the central planner values international liquidity higher than the private sector, and domestic liquidity less. Thus the benefit of intervention is that it moves the private sector away from a sub-optimal choice. That is,  $K^{CP} < K'$ . However intervention has a cost, since firms must sell their bonds into an illiquid market. To compute this cost let us substitute market clear condition for  $L_1$ , (13), into (14):

$$U^{CP} - U^{PRIV} = -\alpha \frac{1}{1 + \alpha} rK.$$

Thus, it is clear that intervention always lowers the welfare function when  $\alpha > 0$ . Intervention is beneficial as long  $U^{CP}(K^{CP}) > U^{PRIV}(K^{PRIV})$ . For  $\alpha$  sufficiently small, this inequality will hold. ■

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