RICARDO J. CABALLERO

Near-Rationality, Heterogeneity, and Aggregate Consumption

IT IS WELL KNOWN that when applied to nondurables consumption, the simplest form of the Permanent Income (PIH) model (Hall 1978) does not survive formal hypothesis testing. Simply put, there is more serial correlation on aggregate consumption than is implied by the PIH model. Alternatively, in income space, there is excess smoothness to income innovations and excess sensitivity to lagged income (Deaton 1987; Campbell and Deaton 1989).[1] These findings have led to a large number of potential explanations. To name a few, over the last decade attention has focused on models that shorten the effective horizon of consumers either through liquidity constraints (Flavin 1981; Hayashi 1982; Campbell and Mankiw 1989) or life cycle aspects (Clarida 1991; Galí 1991), models that emphasize potential correlations between income shocks and changes in the intertemporal marginal rate of substitution of consumption (Christiano 1987; Caballero 1990; Heaton 1991), models that challenge the "maintained" statistical properties about the aggregate income process (Diebold and Rudebusch 1991) or consumption process (Christiano, Eichenbaum, and Marshall 1991; Heaton 1991; Ermini 1991), and models where individual agents have superior information relative to the econometrician (West 1988; Campbell and Deaton 1989; Quah 1990).

In this paper I offer another potential explanation of the departure between the path of actual consumption and PIH consumption: that *heterogeneous* agents do not

update their individual nondurables consumption patterns continuously.[2] However, contrary to the case of durable goods where such behavior can be justified by the presence of large transaction costs (Lam 1991; Grossman and Laroque 1990; Bar-Ilan and Blinder 1992; Bertola and Caballero 1990; Caballero 1993; Eberly 1993; and Beaulieu 1993), nondurable consumption patterns are unlikely to be affected by such costs, at least in significant amounts. This imposes an additional constraint on the *infrequent actions* explanation for aggregate nondurables puzzles: the cost of such inaction must be small in utility terms. Thus, I explore whether *microeconomic* near-rationality in the Akerlof-Yellen (1985) sense has the potential to generate aggregate consumption dynamics similar to those observed in actual U.S. data.[3]

The empirical section proceeds in two steps: It first estimates a nonrepresentative agent version of an Akerlof-Yellen–type model, without imposing the constraint that individual consumers' utility losses be small; and asks whether such a model can account for the short-run behavior of aggregate consumption. It then returns to the initial motivation of microeconomic consumption policies, and asks whether the utility losses implied by the estimates are indeed small. The answer to these two questions turns out to be affirmative. First, the model simultaneously explains the observed excess smoothness of consumption to wealth innovations and the excess sensitivity of consumption to lagged income changes. It also explains conditional asymmetries found in the data: in good times, consumers respond more promptly to positive than to negative wealth stocks, while the opposite is true in bad times. And second, the estimated dollar-equivalent utility cost of the near-rational microeconomic strategy is only about 0.03γ percent of consumption per year, where γ is the coefficient of relative risk aversion.

Section 1 presents the microeconomic model and its connection with aggregate outcomes. The results are presented in section 2, and section 3 computes the implied microeconomic utility loss. Final remarks are provided in section 4.

1. THE MODEL

There is a large number of consumers, approximated by a continuum, and indexed by $i \in [0, 1]$. I assume that if individuals updated their consumption patterns continuously, their level of consumption would be proportional to their wealth. I call this level of consumption the Permanent Income Level (PIH).[4] Thus,

---

2. For the reader wondering why we need another explanation of the rejections of the basic PIH, I cite Lucas' argument for the nonredundancy of his (1989) monetary nonneutrality paper: "Because to paraphrase Tolstoy's observation about happy and unhappy families, complete market economies are all alike, but each incomplete market economy is incomplete in its own individual way."

3. See Cochrane (1989) for a nice demonstration that the observed aggregate departures from PIH involve only second-order losses for a *representative* agent. In contrast to his paper, I assume here that there is no perfect insurance of idiosyncratic risk. It is also important to note that I am not looking for the microeconomic policy that minimizes individual losses given the aggregate path of consumption, but just assessing the potential relevance of a particular, but compelling, form of near-rationality.

4. A simple (but not the only) model that delivers this is the certainty-equivalent model with the discount rate equal to the interest rate.

$$c_i^*(t) = \lambda_i + h_i(t) , \tag{1}$$

where c_i^* is the logarithm of PIH consumption, λ_i is the logarithm of the marginal propensity to consume out of wealth, and h_i is the logarithm of wealth. All of these refer to individual i at time t.

I assume that the logarithm of actual consumption by (near-rational) individual i, $c_i(t)$, on the other hand, remains constant most of the time and is reset only when $z_i(t) \equiv c_i(t) - c_i^*(t)$ reaches a lower trigger point L or an upper trigger point U.[5] To simplify the exposition, I assumed that $L = -U$ and that when either of the trigger points is reached, $z_i(t)$ is brought back to zero.[6,7]

From the definition of z_i as the log-departure (percentage deviation) between actual and PIH consumption, it is possible to express the rate of growth of individual i's consumption (equal to zero, except at a measure zero set of points in time when it is infinite) as follows:

$$dc_i(t) = dc_i^*(t) + dz_i(t) .$$

Multiplying each side of this equation by $\alpha_i(t)$, the share of individual i's consumption in aggregate consumption, integrating over i, and rearranging, yields

$$dC(t) = dC^*(t) + \int_0^1 dz_i(t) \, di + \int_0^1 (\alpha_i(t) - \alpha_i^*(t)) \, dc_i^*(t) \, di$$

$$+ \int_0^1 (\alpha_i(t) - 1) dz_i(t) \, di ,$$

where capital letters denote aggregates (for example, dC represents the rate of growth of aggregate consumption), and α_i^* is the share of individual i's PIH consumption in PIH aggregate consumption. Assuming that the rates of growth of individuals' PIH consumption are orthogonal to the difference between their shares in actual and PIH consumption, and that the changes in individuals' disequilibria are also independent of their shares in actual consumption, we obtain a simpler expression for aggregate consumption growth:

---

5. Actual consumption being constant between adjustments is just a convenient simplification. It is trivial to extend the model to the case where—when not adjusted discretely—consumption grows at a positive and constant rate, or even at a stochastic rate—as long as this growth rate does not exactly match the (stochastic) rate of growth of PIH consumption.

6. These symmetry assumptions are harmless for the purpose of this paper; see Caballero (1992). In the empirical and utility-loss computation sections, however, I center the inaction interval around the constant that makes the sample averages of aggregate PIH and actual consumption equal, which is a weak (long-run average) budget constraint.

7. Of course, the specific form of this near-rational microeconomic rule need not be taken literally. Having fixed barriers is merely a mathematical simplification of the idea that as consumers get further away from their PIH consumption levels, on average, they are more likely to update their actual consumption level. See Caballero and Engel (1992) for a discussion of this point.

$$dC(t) = dC^*(t) + dZ(t) \ ,$$

where, after exchanging derivatives and integrals,

$$dZ(t) \equiv d \int_0^1 z_i(t) \ di \ .$$

Thus, dZ represents the change in the average departure of (the log of) actual and PIH consumption across all individuals. Letting $f(z, t)$ represent the *cross-sectional* density of z_is at time t permits us to write dZ as

$$dZ(t) = d \int_L^U zf(z, t) \ dz \ ,$$

or

$$dZ(t) = \int_L^U zdf(z, t) \ dz \ . \tag{2}$$

This says that the dynamic difference between the *aggregate* rate of consumption growth and its PIH counterpart can be described in terms of the changes in the cross-sectional density of the z_is. Alternatively, one can describe the path of aggregate consumption directly through the gross flows of microeconomic units upgrading and downgrading their consumption patterns:

$$dC(t) = P(t) - M(t) \ , \tag{3}$$

where $P(t)$ and $M(t)$ are the consumption upgrading and downgrading flows, respectively. The connection between equations (2) and (3) arises from the fact that the evolution of $P(t)$ and $M(t)$ is closely related to the evolution of the cross-sectional density of the z_is. In order to describe this connection more fully, one needs to make the properties of the driving processes explicit. For this, let each individual's PIH consumption be described by the process:

$$dc_i^*(t) = \theta dt + \sigma dW_i(t) \ , \tag{4}$$

where W_i is a standard Brownian motion such that $E[dW_i(t)dW_j(t)] = (\sigma_A^2/\sigma^2)dt$ for $\{j \neq i; j \in [0, 1]\}$. The parameters θ, σ_A^2, and σ^2 are the aggregate drift, and aggregate and total (the sum of aggregate and idiosyncratic) variances, respectively.

Since Brownian motions are continuous processes, the upgrading flow in a time-interval dt, starting at t, $P(t)$, must be a function of the number of consumers in the "neighborhood" of the lower trigger barrier, $-U$, at time t. No unit is "at" $-U$ since this is a trigger point, thus the leading term defining the neighborhood is the first

(right) derivative of the density at $-U$, $f_z(-U, t)^+$. How deep is the neighborhood (how many units are "close" to $-U$) and how many of these units reach the trigger point in the time-interval dt is determined by the quadratic variation of Brownian motion (the "shakeout"), $(\sigma^2/2)dt$; the larger is σ the deeper is the neighborhood, and about half of these units will move in the direction of the barrier in a small time interval. The upgrading flow is then obtained by multiplying the number of upgrading consumers by the size of their adjustment, U. This yields

$$P(t) = U \frac{\sigma^2}{2} f_z(-U, t)^+ dt .$$

A similar derivation shows that

$$M(t) = -U \frac{\sigma^2}{2} f_z(U, t)^- dt .$$

Thus, the actual rate of growth of aggregate nondurables consumption is[8]

$$dC(t) = U \frac{\sigma^2}{2} \{f_z(-U, t)^+ + f_z(U, t)^-\} dt , \tag{5}$$

which can be compared with the equation describing the aggregate rate of growth under the PIH, obtained from integrating equation (4):

$$dC^*(t) = \theta dt + \sigma_A dW_A(t) , \tag{6}$$

with $W_A(t)$ a standard Brownian motion.

Equations (5) and (6) show that the rates of growth of actual and PIH consumption—dC and dC^*, respectively—are described by very different mechanisms. The latter results from aggregating the infinitesimal changes of all units in the economy, while the former corresponds to the sum of large changes in the consumption patterns of an infinitesimal fraction of the population. The key elements to determine in equation (5) are the derivatives of the cross-sectional density at its boundaries, $f_z(-U, t)^+$ and $f_z(U, t)^-$. I postpone the formal description of these terms until the appendix. In what follows I provide an informal discussion of the behavior of such derivatives, which I use to summarize the main empirical implications of the model.

The Mechanism

In order to clarify the mechanism underlying the basic results, let me use a formally implausible but useful example.[9] Imagine that the economy has not had an

---

8. See Propositions 2 and 3 in Caballero (1993) for a formal derivation of a similar equation in the context of durable goods.

9. It is formally implausible in the sense that the path described cannot be generated by a Brownian motion. It corresponds, instead, to a momentary change in the drift of the PIH consumption process.
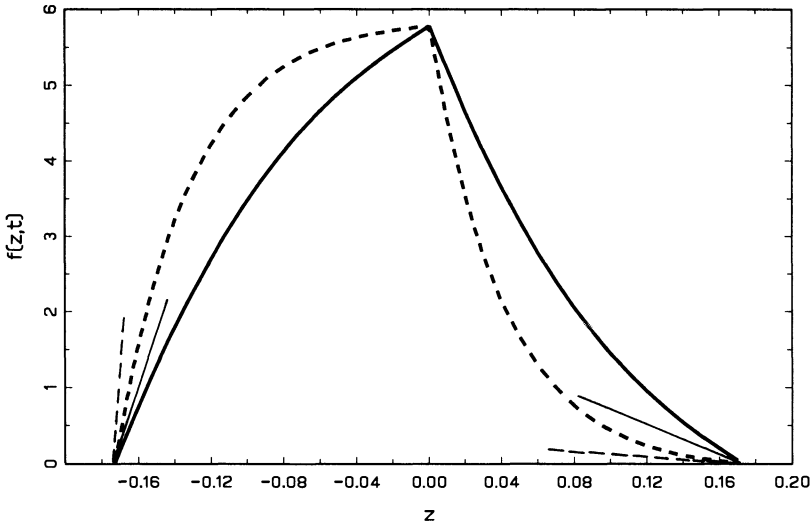
Fig. 1. Cross-sectional Densities

aggregate surprise for a long time, so $f(z, t)$ has converged to a density like that depicted by the solid curve in Figure 1, where the skewness is due to the presence of a positive drift in consumers' wealth ($\theta > 0$).[10] In this steady state $dZ(t) = 0$, $dC(t) = dC^*(t) = \theta dt$, and $f_z(-U)^+ > |f_z(U)^-|$ (solid tangents), which says that the positive steady-state rate of consumption growth is supported by a larger fraction of consumers upgrading their consumption patterns than by consumers downgrading theirs.[11]

Now assume that this economy is followed by a sequence of positive and constant aggregate shocks: $\sigma_A dW_A(r) = \omega(dt) > 0$. This causes an immediate jump of PIH consumption growth to the new rate $dC^*(t) = \theta dt + \omega$. The rate of growth of actual consumption, on the other hand, only picks up slowly as more and more units approach the barrier that triggers upward changes, and fewer approach the down-grading barrier. In terms of equation (5), the slopes of the cross-sectional density at the boundaries—indexing the number of consumers altering their consumption patterns—change slowly over time. In this process—while the slopes change suffi-ciently to match the PIH rate of consumption growth—part of the "force" of the new driving force is absorbed by the shift in the cross-sectional density (and slopes at the boundaries), which induces *excess smoothness* of aggregate consumption to wealth innovations.

The other prominent fact about consumption, *excess sensitivity*, is best under-

---

10. If the no-action microeconomic policy is to let consumption grow at the rate θ' instead of zero, the relevant drift for the density is $(\theta - \theta')$.

11. Note that $f_z(-U)^+ = |f_z(U)^-| + 2\theta/(U\sigma^2)$.

stood by terminating the expansion;[12] in this case dC^* falls immediately back to θdt, while dC returns to its old level more slowly as the "abnormally" large (small) number of units close to the upgrading (downgrading) barrier introduces inertia. The return of the slopes of the cross-sectional density at the boundaries back to those of the solid line in Figure 1 illustrates this. That is, excess sensitivity results from the slow use of the "force" absorbed (stored) by the cross-sectional density during the expansion.

The same example can be used for the case in which there is an initial contraction, showing that excess smoothness and excess sensitivity occur in both directions.

Further Implications

In addition to excess smoothness and sensitivity features, the model has more subtle implications arising from the rich dynamics generated by the endogenous evolution of the cross-sectional density. The magnitude and timing of the response of consumption to wealth innovations depend on the shape of the cross-sectional density at each point in time, which depends on the stochastic environment faced by consumers and on the path of aggregate shocks in particular.

For example, if the economy has been experiencing a sequence of positive shocks, most consumers are likely to be grouped on the upgrading half of their state space.[13] This translates into a cross-sectional density with shape as depicted by the solid curve in Figure 2, where the value of $Z(t)$, denoted by Z_1 in the figure, is very low. At this point, a further reduction in $Z(t)$ is very difficult since the distribution to the left of the target/return point approaches a uniform distribution, which is easily shown to be invariant to positive aggregate shocks (see Caplin and Spulber 1987). This limit uniform distribution has the property that the fraction of consumers upgrading their consumption patterns after a positive (continuous) aggregate shock ΔH—which leads to a change in PIH consumption equal to ΔH—is equal to $\Delta H / U$, and since the size of their change is U, the product of these two quantities is approximately ΔH, precisely the PIH response. This limit is never literally reached; however it suggests that when $Z(t)$ is low, consumption—satisfying $C(t) = C^*(t) + Z(t)$— is unlikely to exhibit much excess smoothness with respect to a new *positive* wealth shock. Conversely, actual consumption should respond very little to a *negative* change in wealth, since most of this would be absorbed by the increase in $Z(t)$, owing to the change in the shape of the cross-sectional density. Exactly the opposite happens if the economy has been experiencing a sequence of negative wealth shocks, so that the initial cross-sectional density looks like the dashed curve in Figure 2 (with mean Z_2).

Figure 3 illustrates the response of consumption to changes in PIH consumption (due to changes in wealth) for different histories of aggregate shocks. The 45° line

---

12. For a clear distinction between the excess smoothness and sensitivity findings, see Campbell and Deaton (1989).

13. See the discussion in terms of the slopes at the trigger barriers in the previous section. Also see Caballero (1993) for an extensive discussion of the relation between the shape of the cross-section distribution and aggregate dynamics in a two-sided (S, s) model similar to that presented here.
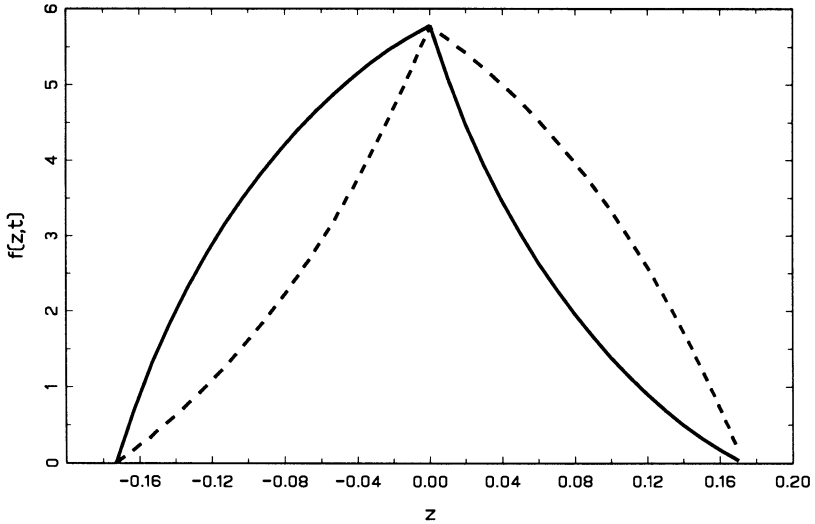
Fig. 2. Cross-sectional Densities

depicts the PIH responses, while the dashed and solid lines portray the responses as indicated by a near-rational model simulated with the parameters found in the empirical section and shown in Table 1 below. The solid line corresponds to a case in which consumption has been growing very fast (4 percent per quarter) for some time. The increasing slope of this curve shows that in good times there is more excess smoothness to negative than to positive changes. Exactly the opposite occurs in a case in which consumption has been declining for a long time at the rate of 4 percent per quarter. This is illustrated by the short-dashed line. Finally, the long-dashed line represents an intermediate case where the responses are fairly symmetric.

2. RESULTS

The model presented until now has the potential to account for the short-run behavior of aggregate consumption. The purpose of this section is to find out whether the correlations it generates are actually close to those present in U.S. data and to estimate the basic parameters of the model. The latter will be used in the next section to compute the implicit utility loss for near-rational agents.

This section is subdivided into three parts. The first part characterizes the PIH part of the model, C^*. The second part focuses on the dynamic part of the model, on Z, and provides estimates of the inaction index, U, and of the amount of microeconomic level uncertainty, σ. The final part of this section presents consumption facts and discusses the consistency of these facts with the model's implications.

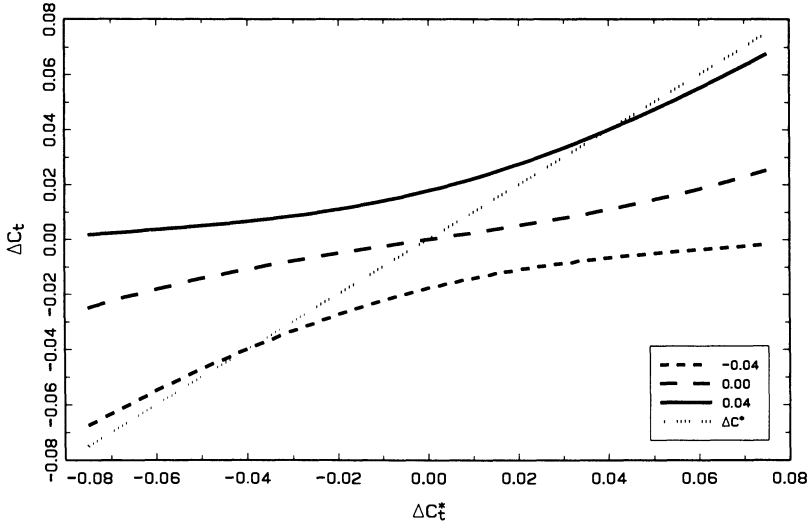I choose U.S. nondurable goods consumption in 1982 dollars (GCN82) divided

FIG. 3. Endogenous Smoothness

by population (GPOP) as the basic consumption series to be explained. The income measure corresponds to U.S. disposable income (GYD82) in 1982 dollars, divided by population. I also use U.S. total consumption expenditures in 1982 dollars (GC82) to construct the savings rate, which is an input in the forecasting equation used to construct a measure of PIH consumption. All series are quarterly from 1955:1 to 1990:1 and were obtained from CITIBASE (keywords in parenthesis).

The PIH Consumption Process

Aggregating the first difference version of (1), yields

$$dC^*(t) = \int_0^1 \alpha_i^*(t) dh_i(t) \, di \, ,$$

or

$$dC^*(t) = dH(t) \, , \tag{8}$$

where dH is the rate of growth of aggregate wealth.

Using Campbell and Deaton's (1989, p. 360) log-linearized representation of the consumption process as implied by the simplest PIH model, I construct an empirical counterpart to equation (8):

$$\Delta C_{t+1}^* \approx \left(\frac{r}{r - \mu} \right) e^{Y_t - C_t^*} \sum_{i=1}^{\infty} (1 + \rho)^{-i} (E_{t+1} - E_t) \Delta Y_{t+i} \, , \tag{9}$$

where Δ is the quarterly change operator, r is the interest rate, ρ is the discount rate, Y_t is the logarithm of per capita disposable income, μ is the unconditional expected value in the change of Y_t, and E_x represents the expectation condition on the information available at time x.

In order to construct an estimate of the revision of the expectation term in equation (9), I follow Campbell (1987) and Campbell and Deaton (1989), and estimate a VAR for the joint income growth–savings rate process.[14] The savings rate, s_t, is constructed by subtracting total per capita consumption expenditures (scaled to make the savings rate stationary)[15] from per capita disposable income, and dividing by the latter.

The estimated first order VAR (removing constants) is

$$\begin{pmatrix} \Delta Y_t \\ s_t \end{pmatrix} = \begin{pmatrix} 0.069 & -0.0173 \\ -0.096 & 0.846 \end{pmatrix} \begin{pmatrix} \Delta Y_{t-1} \\ s_{t-1} \end{pmatrix} + \begin{pmatrix} u_{1t} \\ u_{2t} \end{pmatrix} .$$

Denoting the matrix of coefficients by $\mathbf{A}$, the vector of estimated residuals by $\mathbf{u_t}$, the identity matrix by $\mathbf{I}$, and letting $e_1 \equiv [1, 0]$, we have

$$\sum_{i=1}^{\infty} (1 + \rho)^{-i} (E_{t+1} - E_t)\Delta Y_{t+i} = e_1 (\mathbf{I} - \mathbf{A}/(1 + \rho))^{-1} \mathbf{u_t} .$$

Replacing this expression in equation (9), assuming $r = \rho = 0.06$ and imposing the constraint that average PIH consumption be equal to average disposable income, yields an estimate for the path of the rate of growth of PIH consumption. Since the consumption measure to be explained includes only nondurables, a subcategory of total consumption expenditures, I adjust for secular changes in expenditure allocation by renormalizing the average rate of growth of PIH consumption to match the average rate of growth of nondurables consumption: $\overline{dC^*} = \overline{dC}$. The quarterly standard deviation of the PIH consumption series so constructed is 0.014, which is about 85 percent larger than the standard deviation of the rate of growth of actual consumption (0.0075). The estimated PIH series, dC^*, is the aggregate shock which drives the key stochastic partial differential equation tracing the path of the cross-section distribution of disequilibria. This partial differential equation is described in the next paragraph.

Dynamics

The next step is to estimate equation (5). The key ingredients of this equation are U, σ, and the path of the slopes of the cross-sectional density at the boundaries. The latter is the most difficult and time-consuming part of the problem, since it requires

14. Including the savings rates in the VAR is a very efficient way to include information about future changes in income not captured by a univariate income process (see Campbell 1987).

15. See Campbell (1987). The scaling coefficient is 1.0986.

TABLE 1

ESTIMATES

U	σ	$R^2_{\Delta Z}$	$R^2_{\Delta C}$
5.7	7.4	0.71	0.47
(1.2)	(1.6)	—	—

NOTES Standard errors are in parentheses. Entries are in percents, except for the R^2s Sample: 55 1–90 1 (but the first ten observations are not included in the sum of squared residuals, to reduce the impact of initial conditions)

one to track down the path of the cross-sectional density; this amounts to solving the following stochastic partial differential equation:

$$df(z,\ t)\ =\ f_z(z,\ t)\ dC^*(t)\ +\ \frac{\sigma^2}{2}\ f_{zz}(z,\ t)\ dt\ ,$$

subject to the boundary conditions: $f(-U,\ t) = f(U,\ t) = 0, f(0,\ t)^+ = f(0,\ t)^-$ and $f_z(0,\ t)^+ - f_z(0,\ t)^- = f_z(U,\ t)^- - f_z(-U,\ t)^+$, for each combination of parameters, U and σ. The estimation procedure minimizes the sum of squared differences between actual consumption growth and the outcome of equation (5) by searching over combinations of U and σ. The appendix describes this procedure in detail.

The results are presented in Table 1. The first column shows that, on average, individual agents let their consumption patterns depart from the level of consumption indicated by the PIH by up to 6 percent before updating their patterns. The second column shows that the standard deviation of total uncertainty faced by individuals (relevant for nondurables purchases) is about 8 percent per year. These estimates of uncertainty do not seem out of the range of microeconomic evidence (see Hall and Mishkin 1982; MaCurdy 1982). The last two columns illustrate the fit of the model. It is apparent that the model explains a large fraction of the dynamic behavior of the discrepancy between actual and PIH consumption, ΔZ, and of actual consumption growth.[16] The properties of the estimated consumption path are compared to those of the path of actual and PIH aggregate consumption in the next section.

Facts and Findings

Table 2 presents the basic (linear) facts (column 1) and the corresponding implications of the model (column 2). The coefficients $\beta_{\Delta H}$ show the average response of actual and estimated consumption growth to (unexpected) changes in wealth. These

16. The likelihood is relatively flat along a ray where both U and σ rise. This makes the particular estimates of these parameters fairly unrobust to changes in the value assumed for interest and discount rates. If one eliminates this identification problem by fixing the estimate of uncertainty at the level obtained for $\rho = r = 0.06$, however, the main conclusions are robust to changes in ρ and r. The index of inaction, U, is essentially unchanged for large values of these parameters, and rises to only 7 percent when $\rho = r = 0.04$. The aggregate and welfare implications of these estimates are indistinguishable.
Given the identification problem mentioned above, the standard errors reported in table 1 are suspect. For this reason, I prefer to take the conservative view that the results presented show that one can calibrate the model to explain most aggregate consumption facts, rather than as precise estimates.

TABLE 2

BASIC FACTS AND RESULTS

	ΔC	$\widehat{\Delta C}$
$\beta_{\Delta H}$	0.387	0.565
	(0.034)	(0.030)
$\beta_{\Delta Y(-1)}$	0.157	0.207
	(0.068)	(0.082)

NOTES: Standard errors are in parentheses. The coefficients $\beta_{\Delta H}$ and $\beta_{\Delta Y(-1)}$ are obtained in separate regressions (these include constants). Sample: 58:3–90:1.

TABLE 3

ASYMMETRIES

	ΔC	$\widehat{\Delta C}$
β_{flex}	0.49	0.64
	(0.06)	(0.05)
β_{rigi}	0.34	0.53
	(0.04)	(0.04)

NOTES: Standard errors in parentheses. All regressions include a constant. Sample: 58:2–90:1.

are obtained from simple OLS regressions of the rate of growth of consumption on the rate of wealth growth. Both coefficients are significantly below 1, which is the PIH response. This yields a measure of the *excess smoothness* of consumption to wealth changes.[17] The coefficients $\beta_{\Delta Y(-1)}$ highlight the other well-known fact about consumption: its *excess sensitivity* to lagged (therefore anticipated) income changes. Actual consumption growth is positively correlated with lagged, disposable income growth, and the same is true for the estimated model.[18]

Table 3 illustrates some of the aspects of the nonlinearities described in section 1. For this, I have split wealth surprises into two groups according to the slopes in Figure 3: The first one, denoted *flex,* contains the observations for which the response of consumption to wealth shocks is likely to be large: positive innovations in wealth given that the beginning of period value of Z is below its mean (given that previous times were very favorable), and negative changes in wealth given that the beginning of period value of Z is above its mean (given that previous times were very unfavorable). The second one, denoted *rigi,* contains the opposite extreme; that is, the observations for which the response of consumption to wealth changes is likely to be low: positive innovations in wealth given that the beginning of period value of Z is above its mean (given that previous times were very unfavorable), and negative changes in wealth given that the beginning of period value of Z is below

17. This is a somewhat stronger concept of excess smoothness than the one used in the literature, where it is used to denote the fact that the variance of actual changes in consumption is less than the variance of changes in PIH consumption. Here, the ratio of actual to PIH consumption growth variances is 0.65. This is larger than the number obtained by Campbell and Deaton (1989), who used labor income instead of disposable income to construct PIH consumption. The qualitative result is the same, however.

18. The estimate of the same coefficient for PIH consumption growth is -0.01 and nonsignificant. Adding a drift term to the consumption pattern, whereby average consumption growth is fully accounted for by planned consumption growth, leaves the results of Table 2 virtually unchanged.

its mean (given that previous times were very favorable). This yields the basic equation:

$$\Delta C_{t+1} = (\beta_{flex}1[flex] + \beta_{rigi}1[rigi])\Delta H_{t+1} , \tag{11}$$

where

$$1[flex] \equiv 1[(\Delta H(t + 1) > \overline{\Delta H} \mid Z(t) < \bar{Z}) \text{or} (\Delta H(t + 1) < \overline{\Delta H} \mid Z(t) > \bar{Z})] ,$$

$$1[rigi] \equiv 1[(\Delta H(t + 1) > \overline{\Delta H} \mid Z(t) > \bar{Z}) \text{or} (\Delta H(t + 1) < \overline{\Delta H} \mid Z(t) < \bar{Z})] ,$$

An equation similar to (11) is used for the estimated consumption path.[19]

According to the discussion above, we would expect to observe less excess smoothness—that is, a larger β—during *flex* than during *rigi* periods. Table 3 shows that this is indeed the case.

Having shown that the basic implications of the model are consistent with the data, it remains to show that the *microeconomic* utility loss of not updating consumption continuously is small. I turn to this in the next section, although the task should not be difficult since the inaction range estimated/calibrated in the previous section is already small.

3. NEAR-RATIONALITY?

The (present value) utility cost of following the near-rational policy for an individual that has current departure $z(0)$ is equal to (the subindex i is suppressed for simplicity):

$$\int_0^\infty \int_{T-U}^{T+U} (\mathcal{U}(e^{c*(t)}) - \mathcal{U}(e^{c*(t)+z(t)})) \, e^{-\rho t} g(z, t) \, dz \, dt , \tag{12}$$

where T is the return point,[20] chosen to set the first order-term of the departure equal to zero,[21] $\mathcal{U}(.)$ is the consumer's instantaneous utility function, ρ is the discount rate, and $g(z, t)$ is the density of z at time t, conditional on $z = z(0)$ at $t = 0$.

I follow Cochrane (1989) and divide the integrand in equation (12) by the corresponding marginal utility evaluated at the PIH consumption level. This transforms the utility value expression into a dollar-equivalent expression. Dividing the integrand again, now by PIH consumption, one transforms expression (12) into a weighted (by the discount factor) average of flow costs expressed in term of percent-

19. The *flex* periods are: 57:4, 58:1, 58:4, 60:1, 60:3, 60:4, 61:1, 61:3, 62:1, 62:3, 63:1, 63:2, 63:4, 64:4, 66:4, 67:1, 67:3, 67:4, 68:3, 69:1, 70:2, 70:3, 70:4, 71:2, 71:3, 72:1, 72:2, 72:3, 72:4, 73:1, 74:3, 76:1, 76:3, 76:4, 77:1, 77:3, 77:4, 78:2, 78:4, 79:3, 81:1, 81:2, 81:3, 81:4, 82:1, 82:2, 82:3, 84:1, 84:3, 85:3, 86:1, 86:3, 87:3, 88:1, 89:3.

20. Which was set to zero before for expository simplicity.

21. This plays the role of the feasibility constraint in Cochrane's (1989) analysis.

age of PIH consumption sacrificed. Finally, multiplying by ρ one gets the annuity value of this present-value cost. I denote this expression by $F(z(0))$.

$$F(z(0)) \equiv \rho \int_0^\infty \int_{T-U}^{T+U} \frac{(\mathcal{U}(e^{c^*(t)}) - \mathcal{U}(e^{c^*(t)+z(t)}))}{\mathcal{U}'(e^{c^*(t)})e^{c^*(t)}} \, e^{-\rho t} g(z, t) \, dz \, dt \ . \quad (13)$$

Preserving the first two terms of the Taylor expansion of $\mathcal{U}(e^{c^*(t)+z(t)})$ around $z(t) = 0$, and taking the average of $F(z)$ over the ergodic density of z, $g(z)$, yields an expression for the average yearly cost—in terms of percentages of PIH consumption —due to the near-rational policy (see the appendix):

$$G \equiv \int_{T-U}^{T+U} F(z)g(z) \, dz = \int_{T-U}^{T+U} \left((1 - e^z) + \frac{\gamma}{2} (e^z - 1)^2 \right) g(z) \, dz \ ,$$

where γ is the coefficient of relative risk aversion. And since T is chosen so as to satisfy the budget constraint (weak form), $\int (e^z - 1)g(z)dz = 0$, G reduces to

$$G = \frac{\gamma}{2} \int_{T-U}^{T+U} (e^z - 1)^2 \, g(z) \, dz \ . \quad (14)$$

Evaluating this expression at the parameters found in the empirical section of the paper yields

$$G = 0.0003\gamma \ ,$$

an extremely small number for reasonable values of γ. This ties the argument of the paper: Not only a nonrepresentative agent/discontinuous action microeconomic policy can generate aggregate dynamics consistent with the behavior of U.S. nondurables consumption, but also the degree of inaction required to do so imposes very small costs on individual consumers.[22]

4. FINAL REMARKS

The model presented in the paper provides a structural interpretation of the main features of aggregate consumption. Excess smoothness and sensitivity arise naturally from the endogenous evolution of the cross-sectional density of individuals' short-run deviations from the PIH. The endogenous nature of the cross-sectional

---

22. Which obviously implies that if one is to justify the model in terms of adjustment costs rather near-rationality, these costs can be made quite small. The small size of the welfare loss required to generate significant implications for aggregate dynamics is consistent with Dixit's (1991) argument that small adjustment costs can lead to substantial inaction.

distribution also determines that the aggregate departure from the PIH varies over the business cycle, enriching the characterization of postwar U.S. data.

Within the framework discussed in the paper, heterogeneity plays a key role. Idiosyncratic shocks do not wash away because microeconomic consumption policies are nonlinear. In this context, information about the cross-section distribution of consumers' departures (the z_is) helps explaining the path of aggregate consumption. In the absence of microeconomic data, however, one needs to make a "guess" on the path of the cross-sectional distribution. This defines a "distribution" extraction problem, which is what I have done when estimating the model. The estimates suggest that *on average* consumers keep their consumption levels within 6 percent of their PIH consumption level, and that they face uncertainty about the driving forces of their PIH nondurables consumption of about 8 percent per year.

Despite occasional microeconomic departures from PIH consumption as large as 6 percent (when z is close to the barriers), the implied average cost of the *microeconomic* policy is extremely small: about 0.03γ percent of PIH consumption. In short, *near-rational* microeconomic consumers—in the Akerlof-Yellen (1985) sense—generate aggregate dynamics consistent with U.S. postwar nondurables consumption data.

APPENDIX A: DERIVATION OF EQUATION (5)

This section of the appendix begins with the observation that the diffusion-forward Kolmogorov equation associated to the controlled Brownian motion z_i, with driving process described by equation (4), is

$$dh(z, t) = \frac{\sigma^2}{2} h_{zz}(z, t) \, dt + \theta h_z(z, t) \, dt , \tag{A1}$$

subject to the initial condition:

$$h(z, 0) = \bar{h}(z) ,$$

and the boundary conditions:

$$h(-U, t) = h(U, t) = 0 ,$$

$$h(0, t)^+ = h(0, t)^-$$

and

$$h_z(0, t)^+ - h_z(0, t)^- = h_z(U, t)^- - h_z(-U, t)^+ ,$$

where $h(z, t)$ is the probability density of z_i at time t, conditional on the information available at time zero. The first two boundary conditions reflect the fact that $-U$ and

U are trigger points, the third one is a condition of continuity at zero. The last one is a conservation law which is the continuous state space equivalent of a simple discrete state space (regulated random walk) statement: the density at the return point is equal to (weighted by appropriate probabilities) the density in the contiguous states *plus* the density near the trigger points.

If $f(z, 0) = \bar{h}(z)$ and there are no aggregate shocks, a direct application of the Glivenko-Cantelli theorem determines that (A1) and its boundary conditions also describe the path of the cross-sectional density $f(z, t)$. Although this step is not directly applicable here because there are aggregate shocks ($\sigma_A > 0$), Proposition 1 in Caballero (1993) shows that similar argument holds *conditional* on the realization of aggregate shocks. In this case the boundary conditions remain unchanged, but the partial differential equation (A1) is replaced by the *stochastic* partial differential equation:

$$df(z, t) = \frac{\sigma^2}{2} f_{zz}(z, t)\, dt + f_z(z, t)\, dC^*(t) \ . \tag{A2}$$

LEMMA A1: *Let* f(z, t) *denote the cross-sectional density at time* t, *satisfying the boundary conditions described above for* h(z, t), *and evolving according to* (A2), *then:*

$$\int_{-U}^{U} f_z(z, t)\, dz = 0 \ , \tag{a}$$

$$\int_{-U}^{U} f_{zz}(z, t)\, dz = 0 \ , \tag{b}$$

$$\int_{-U}^{U} z f_z(z, t)\, dz = -1 \ , \tag{c}$$

$$\int_{-U}^{U} z f_{zz}(z, t)\, dz = U\{f_z(-U, t)^+ + f_z(U, t)^-\} \ . \tag{d}$$

PROOF: Parts (a) and (b) are proved by integrating (A2) with respect to z between $-U$ and U, noticing that the integral of the left-hand side is zero for all t and that the diffusion term in dC^* cannot be offset by any other term in the equation. Parts (c) and (d) follow directly from integration by parts, and using the boundary conditions and parts (a) and (b) of this lemma. Q.E.D.

It is now straightforward to obtain equation (5). For this note that

$$dC(t) = dC^*(t) + dZ(t) = dC^*(t) + \int_{-U}^{U} z\, df(z, t)\, dz \ .$$

Replacing (A2) in the last expression, yields:

$$dC(t) = dC^*(t) + \frac{\sigma^2}{2} dt \int_{-U}^{U} z f_{zz}(z, t) \, dz + dC^*(t) \int_{-U}^{U} z f(z, t) \, dz . \qquad (A3)$$

Equation (5) is obtained by using Lemmas (1c) and (1d) in (A3):

$$dC(t) = U \frac{\sigma^2}{2} \{ f_z(-U, t)^+ + f_z(U, t)^- \} \, dt .$$

APPENDIX B: ESTIMATION OF EQUATION (5)

The difficulty of estimating equation (5) is due to the presence of the slopes $f_z(-U, t)^+$ and $f_z(U, t)^-$. The value of these slopes at t, however, depends not only on the realization of aggregate and idiosyncratic shocks, but also on the shape of the cross-sectional density *inside* the interval $(-U, U)$ in previous periods. In other words, in order to characterize the boundaries of the cross-sectional density over time, one needs to track down the path of the entire density. This is the strategy followed in the paper.

For each pair (U, σ), and the realization of the aggregate path $\{C_t^*\}_{t \geq 0}$, equation (A2) determines a path of a simulated cross sectional density, $f(z, t)$, where $f(z, 0)$ is taken as given and equal to the corresponding "steady-state" density (defined as the density that solves (A1) with $dh(z, t) = 0$). In order to reduce the impact of the initial distribution, the first ten observations are excluded from the sum of squared residuals. The estimation procedure consists in searching over U and σ until finding the pair (U, σ) that minimizes the sum of squared departures between the rate of growth of actual and PIH consumption.

The realization of $\{C^*\}_{t \geq 0}$ is not observed (estimated) continuously, but only at quarterly frequency. Instead of solving an extremely cumbersome filtering problem, I take the change in C^* during a quarter to be homogeneously distributed within the quarter. In this case the Fourier representation of the density at time t is (see Caballero 1993)

$$f(z, t) = g(z; \theta_t) + \sum_{n>0} e^{-\sigma^2/2[(\pi^2 n^2/U^2) + (\theta_t^2/\sigma^4)]} A_n(t) w_n(z, t) ,$$

where the time unit is a quarter, $\theta_t \equiv \Delta C_t^*$, $g(z; \theta_t)$ is the "steady state" density achieved if θ_t remains constant forever:

$$g(z; \theta_t) = \frac{1}{U} \begin{cases} \dfrac{e^{-\eta_t z} - e^{\eta_t U}}{1 - e^{\eta_t U}} & \text{if} \quad -U \leq z \leq 0 \\[3mm] \dfrac{e^{-\eta_t z} - e^{-\eta_t U}}{1 - e^{-\eta_t U}} & \text{if} \quad 0 < z \leq U , \end{cases}$$

with $\eta_t \equiv 2\theta_t/\sigma^2$,

$$w_n(z, t) = \sin\left(\frac{n\pi}{U}z\right) \cdot \begin{cases} e^{-(\eta_t/2)z} & \text{if } -U \leq z \leq 0 \\ e^{-(\eta_t/2)(z-U)}(-1)^{n+1} & \text{if } 0 < z \leq U, \end{cases}$$

and, finally,

$$A_n(t) = \frac{2}{U(1 + e^{\eta_t U})} \int_{-U}^{U} e^{\eta_t z} w_n(z, t) f(z, t - 1) \, dz.$$

APPENDIX C: DERIVATION OF G

A simple application of the Law of Iterated Expectations determines that

$$G \equiv \int_{T-U}^{T+U} F(z) g(z) \, dz$$

$$= \rho \int_0^\infty \int_{T-U}^{T+U} \frac{(\mathcal{U}(e^{c^*(t)}) - \mathcal{U}(e^{c^*(t)+z(t)}))}{\mathcal{U}'(e^{c^*(t)})e^{c^*(t)}} e^{-\rho t} g(z) \, dz \, dt. \tag{A4}$$

Keeping the first two terms of a Taylor expansion of $\mathcal{U}(e^{c^*(t)+z(t)})$ around $z(t) = 0$, yields

$$\frac{(\mathcal{U}(e^{c^*(t)}) - \mathcal{U}(e^{c^*(t)+z(t)}))}{\mathcal{U}'(e^{c^*(t)})e^{c^*(t)}} = \left((1 - e^z) + \frac{\gamma}{2}(e^z - 1)^2\right) \equiv Y(z). \tag{A5}$$

Replacing (A5) in (A4) and rearranging, yields

$$G = \rho \int_0^\infty e^{-\rho t} \, dt \int_{T-U}^{T+U} Y(z) g(z) \, dz.$$

Equation (14) in the paper follows immediately from this expression after imposing the feasibility constraint, $\int (1 - e^z) g(z) dz = 0$.

The explicit expression for $g(z)$ is obtained from the solution to the Kolmogorov equation (A1) (with its boundary conditions) when $dh(z, t) = 0$, and it corresponds to $g(z; \theta)$.

LITERATURE CITED

Akerlof, George A., and Janet L. Yellen. "Can Small Deviations from Rationality Make Significant Differences to Economic Equilibria?" *American Economic Review* 75 (September 1985), 708–21.

Bar-Ilan, Avner, and Alan S. Blinder. "Consumer Durables: Evidence on the Optimality of Doing Nothing." *Journal of Money, Credit, and Banking* 24 (May 1992), 253–72.

Beaulieu, Joseph. "Utilizing Cross-sectional Evidence in Modeling Aggregate Time Series: Consumer Durables with Fixed Costs of Adjustment." Federal Reserve Board Working Paper no. 93-13, (1993).

Bertola, Giuseppe, and Ricardo J. Caballero. "Kinked Adjustment Costs and Aggregate Dynamics" in *NBER Macroeconomics Annual 1990,* edited by Olivier J. Blanchard and Stanley Fischer. Cambridge: MIT Press, 1990.

Caballero, Ricardo J. "Consumption Puzzles and Precautionary Savings." *Journal of Monetary Economics* 25 (January 1990), 113–36.

———. "A Fallacy of Composition." *American Economic Review* 82 (December 1992), 1279–92.

———. "Durable Goods: An Explanation for Their Slow Adjustment." *Journal of Political Economy* 101 (April 1993), 351–84.

Caballero, Ricardo J., and Eduardo M.R.A. Engel. "Beyond the Partial Adjustment Model." *American Economic Review, Papers and Proceedings* 82 (May 1992), 360–64.

Campbell, John Y. "Does Saving Anticipate Declining Labor Income? An Alternative Test of the Permanent Income Hypothesis." *Econometrica* 55 (1987), 1249–73.

Campbell, John Y., and Angus S. Deaton. "Why Is Consumption So Smooth?" *Review of Economic Studies* 56 (July 1989), 357–74.

Campbell, John Y., and N. Gregory Mankiw, "Consumption, Income, and Interest Rates: Reinterpreting the Time Series Evidence." In *NBER Macroeconomics Annual 1989,* edited by Olivier J. Blanchard and Stanley Fischer. Cambridge: MIT Press, 1989.

Christiano, Lawrence J. "Why Is Consumption Less Volatile than Income." *Federal Reserve Bank of Minneapolis Quarterly Review* 11 (Fall 1987), 2–20.

Christiano, Lawrence J., Martin S. Eichenbaum, and David A. Marshall. "The Permanent Income Hypothesis Revisited." *Econometrica* 59 (March 1991), 397–423.

Clarida, Richard H. "Aggregate Stochastic Implications of the Life Cycle Hypothesis." *Quarterly Journal of Economics* 106 (August 1991), 851–68.

Cochrane, John H. "The Sensitivity of Tests of the Intertemporal Allocation of Consumption to Near-Rational Alternatives." *American Economic Review* 79 (June 1989), 319–37.

Deaton, Angus S. "Life-Cycle Models of Consumption: Is the Evidence Consistent with the Theory?" In *Advances in Econometrics: Fifth World Congress,* vol. 2, edited by Truman F. Bewley. New York: Cambridge University Press, 1987.

Diebold, Francis X., and Glenn D. Rudebusch. "Is Consumption Too Smooth? Long Memory and the Deaton Paradox." *Review of Economics and Statistics* 73 (February 1991), 1–17.

Dixit, Avinash, "Analytical Approximations in Models of Hysteresis." *Review of Economic Studies* 58 (January 1991), 141–51.

Eberly, Janice. "Adjustment of Consumers' Durable Stocks: Evidence from Automobile Purchases." Wharton mimeo, 1993.

Ermini, Luigi. "Effects of Transitory Consumption and Temporal Aggregation on Consumption." Mimeo, University of Hawaii-Manoa, 1991.

Flavin, Marjorie A. "The Adjustment of Consumption to Changing Expectations about Future Income." *Journal of Political Economy* 89 (October 1981), 974–1009.

Galí, Jordi. "Budget Constraints and Time-Series Evidence on Consumption." *American Economic Review* 81 (December 1991), 1238–53.

Grossman, Sanford J., and Guy Laroque. "Asset Pricing and Optimal Portfolio Choice in the

Presence of Illiquid Durable Consumption Goods." *Econometrica* 58 (January 1990), 25–51.

Hall, Robert E. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence." *Journal of Political Economy* 86 (December 1978), 971–87.

Hall, Robert E., and Frederick S. Mishkin. "The Sensitivity of Consumption to Transitory Income: Estimates from Panel Data on Households." *Econometrica* 50 (March 1982), 461–81.

Hansen, Lars P., and Kenneth J. Singleton. "Stochastic Consumption, Risk Aversion, and the Temporal Behavior of Asset Returns." *Journal of Political Economy* 91 (April 1983), 249–65.

Hayashi, Fumio. "The Permanent Income Hypothesis: Estimation and Testing by Instrumental Variables." *Journal of Political Economy* 90 (October 1982), 895–916.

Heaton, John C. "The Interaction between Time-Nonseparable Preferences and Time Aggregation." MIT Sloan School of Management Working Paper 3376-92-EFA, December 1991.

Lam, Sam-Pok. "Permanent Income, Liquidity, and Adjustment of Automobile Stocks: Evidence from Panel Data." *Quarterly Journal of Economics* 106 (1991), 203–30.

Lucas, Robert E., Jr. "The Effects of Monetary Shocks When Prices Are Set in Advance." Mimeo, University of Chicago, November 1989.

MaCurdy, Thomas E. "The Use of Time Series Processes to Model the Error Structure of Earnings in a Longitudinal Data Analysis." *Journal of Econometrics* 18 (January 1982), 83–114.

Quah, Danny. "Permanent and Transitory Movements in Labor Income: An Explanation for 'Excess Smoothness' in Consumption." *Journal of Political Economy* 98 (June 1990), 449–75.

West, Kenneth D. "The Insensitivity of Consumption to News about Income." *Journal of Monetary Economics* 21 (January 1988), 17–33.