

On the Sign of the Investment–Uncertainty Relationship

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Understanding the effects of uncertainty over any decision variable has fascinated economists for a long time. Risk aversion and incomplete markets are likely to make the investment–uncertainty relationship negative (e.g., Roger Craine, 1989; Joseph Zeira, 1989). What happens in the absence of risk aversion and incomplete markets is, however, ambiguous.

Richard Hartman (1972) and Andrew B. Abel (1983, 1984, 1985) found that in the presence of (symmetric) convex costs of adjustment, mean-preserving increases in price uncertainty raise investment of a competitive firm as long as the profit function is convex in prices. On the other hand, the recent literature on irreversible investment (e.g., Robert S. Pindyck, 1988; Giuseppe Bertola, 1988) has shown that increases in uncertainty lower investment. All these results have been derived under either risk neutrality or complete markets.¹

Intuition suggests that the explanation for such a difference lies with the asymmetric nature of adjustment costs in the irreversible-investment case, as compared with the symmetry of the adjustment-cost mechanisms proposed by Abel and Hartman. Although this intuition is confirmed in this paper, asymmetric adjustment costs are

shown not to be sufficient to explain why the results differ. In fact, a more hidden but at least as important difference between these two literatures is that the former assumes perfect competition and constant returns to scale, whereas the latter assumes either imperfect competition or decreasing returns to scale (or both).²

The purpose of this paper is to highlight the role of the decreasing marginal return to capital assumption (due to either imperfect competition or decreasing returns to scale [or both]) in determining the effects of adjustment-cost asymmetries on the sign of the response of investment to changes in uncertainty (under risk neutrality). For this, the paper develops a simple model with a cost-of-adjustment mechanism general enough to consider both symmetric-convexity and irreversibility as special cases. One of the most important findings is the lack of robustness of the negative relationship between investment and uncertainty under asymmetric adjustment costs³ to changes in the degree of competition. In fact, when firms are nearly competitive, the conclusion of Hartman and of Abel holds no matter how asymmetric adjustment costs are. Studying adjustment-cost mechanisms has a central role in understanding the dynamics of investment and its business-cycle implications, but conclusive results about the sign of the instantaneous relationship between uncertainty and investment should not be

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¹The financial literature on investment has considered risk aversion through a premium in the discount rate determined by the CAPM, (capital asset pricing model), intertemporal CAPM, or consumption CAPM. However, often this discount rate is left unchanged when studying the response of investment to uncertainty changes (e.g., Pindyck, 1988 pp. 974–5), thereby omitting the effect of changes in uncertainty on investment due to risk aversion (and incomplete markets).

²In the typical version of the irreversible-investment problem, there is no cost of upward adjustments; thus, imperfect competition and (or) decreasing returns to scale are required to bound the size of the firm.

³In this paper, *asymmetric* adjustment cost refers to the case in which it is more expensive to adjust downward than upward. Certainly, the opposite case is a trivial extension of the case studied in this paper.

expected from the adjustment-costs literature alone.

Section I develops the minimum framework necessary to understand the main issues involved. For this, a simple partial-equilibrium two-period model in which adjustment costs are (weakly) convex and possibly asymmetric and in which managers (owners) are risk-neutral is derived. Section II specializes the previous model to perfect competition. The result of Hartman and of Abel is shown to be robust to asymmetries in the adjustment-costs function, including the irreversible investment case. Hence, investment and uncertainty are positively correlated even in the extreme case of irreversible investment, as long as the firm faces a very elastic demand curve (and returns to scale are nondecreasing). Section III confirms the fact that the combination of important degrees of imperfect competition and adjustment-costs asymmetry may reverse the positive correlation between uncertainty and investment. In obtaining this result, imperfect competition is not only necessary, but is also the paramount factor. Section IV summarizes the results and discusses the roles of increasing and decreasing returns to scale. The latter makes a negative investment-uncertainty relationship more likely, whereas increasing returns makes it less likely. Finally, the Appendix presents an infinite-horizon version of the perfect-competition-adjustment-costs model.

I. Basic Framework

Each firm is in place for two periods⁴ and faces an isoelastic demand function:

$$(1) \quad P_t = Q_t^{(1-\psi)/\psi} Z_t$$

where ψ ($\psi \geq 1$) is a markup coefficient that takes the value of 1 under perfect competition, P and Q are respectively the price and quantity of the good sold, and Z is a stochastic term described by a lognormal

⁴See the Appendix for a multiperiod version of the perfect-competition version of the model.

random-walk process:⁵

$$Z_t = Z_{t-1} \exp \varepsilon_t$$

with ε distributed normally with mean $-\sigma^2/2$ and variance σ^2 .

Technology is described by a homogeneous Cobb-Douglas production function:

$$Q = (AL^\alpha K^{1-\alpha})^\gamma$$

with A a scale parameter, L labor, K capital, α the labor share, and γ a returns-to-scale parameter.

Under these conditions the profit function, $\Pi(K, Z)$, is equal to

$$\Pi(K_t, Z_t) = h Z_t^\eta K_t^\mu$$

where

$$h = (1 - \alpha\gamma/\psi) A^{(\gamma/\psi)/(1-\alpha\gamma/\psi)}$$

$$\times \left(\frac{\alpha\gamma}{\psi w} \right)^{(\alpha\gamma/\psi)/(1-\alpha\gamma/\psi)}$$

$$\eta \equiv \frac{1}{1 - \alpha\gamma/\psi} > 1$$

$$\mu \equiv \frac{(1 - \alpha)\gamma/\psi}{1 - \alpha\gamma/\psi} \leq 1$$

and w is the (constant) wage paid to labor.⁶

Letting $C(I)$ denote the cost of changing the stock of capital by I units and assuming (without loss of generality) neither depreci-

⁵Assuming a stationary process instead of a (log) random walk does not change the conclusions.

⁶Note that the unambiguous convexity of $\Pi(\cdot, \cdot)$ with respect to Z depends on the fact that, given Q , the relation between P and Z depends on neither technology nor preference parameters. If, for example, (1) were replaced by $Q = P^{\psi/(1-\psi)} Z$, the profit function would no longer be convex with respect to Z . However, the specification in (1) seems more appropriate since it permits analysis of perfect competition as the limit case when $\psi = 1$, without altering the variance of the fundamental source of uncertainty, Z . In fact, when $\psi = 1$, (1) corresponds to the specification of Hartman and of Abel.

ation nor discounting yields the following two-period optimization problem for a single firm:

$$(2) \quad V_1(K_0, Z_1) = \max_{I_1} \Pi(K_1, Z_1) - C(I_1) + E_1[V_2(K_1, Z_2)]$$

subject to $K_1 = K_0 + I_1$

where V_i represents the value function at time i .

The first-order condition of this problem is

$$(3) \quad \Pi_{K_1}(K_0 + I_1, Z_1) - C_I(I_1) + E_1[V_{2K_1}(K_0 + I_1, Z_2)] = 0.$$

Finally, the second-period (terminal) value function is just

$$(4) \quad V_2(K_1, Z_2) = \max_{I_2} \Pi(K_1 + I_2, Z_2) - C(I_2).$$

The remainder of this section presents a general investment-cost function, while the rest of the paper discusses the role of competition, returns to scale, and the shape of the investment-cost function in determining the investment-uncertainty relationship.

The cost of changing the stock of capital by I units, denoted by $C(I)$, includes both direct and adjustment costs:

$$C(I) = I + [I > 0]\gamma_1 I^\beta + [I < 0]\gamma_2 |I|^\beta$$

where $\beta \geq 1$, γ_1 and γ_2 are two nonnegative parameters, and the price of capital has been set equal to 1.⁷

This parameterization of $C(I)$ is quite general. For example, except for the addition of I to reflect the direct cost of capital, the symmetric adjustment-cost case used by Abel (1983) is achieved when $\gamma_1 = \gamma_2 > 0$ and $\beta > 1$, and the irreversible-investment

case of Pindyck (1988) and Bertola (1988) corresponds to the case in which $\gamma_1 = 0$, $\gamma_2 = \infty$, and $\beta = 1$.

II. Perfect Competition

In order to isolate the role of competition, I will postpone issues of returns to scale until Section IV. For now, the technology is assumed to exhibit homogeneity of degree one with respect to capital and labor ($\gamma = 1$).

Moreover, perfect competition is taken only as an expository device to illustrate the consequences of a highly elastic demand. Indeed, Pindyck (1990) provides compelling arguments against mean-preserving changes in price-uncertainty experiments when competition is strictly perfect and investment is fully irreversible.

When competition is perfect $\mu = 1$; hence, the profit function becomes linear with respect to the stock of capital. This yields a simple first-order condition at time 2 [see eq. (4)]:

$$hZ_2^\eta - [I_2 > 0](1 + \gamma_1 \beta I_2^{\beta-1}) - [I_2 < 0](1 - \gamma_2 \beta |I_2|^{\beta-1}) = 0.$$

Thus, I_2 is determined by

$$I_2 = \begin{cases} \left[\frac{hZ_2^\eta - 1}{\gamma_1 \beta} \right]^{1/(\beta-1)} & \text{for } I_2 > 0 \text{ or } hZ_2^\eta \geq 1 \\ - \left[\frac{1 - hZ_2^\eta}{\gamma_2 \beta} \right]^{1/(\beta-1)} & \text{for } I_2 < 0 \text{ or } hZ_2^\eta < 1. \end{cases}$$

The most important feature of this solution is that it does not depend on K_1 ; hence, the value function at time 2 is only linearly linked to K_1 through the profit function. It is easy to show that in this case $V_{2K_1} = hZ_2^\eta$.⁸

⁸The fact that $V_{2K_1} = hZ_2^\eta$ can be easily proved by noticing that I_2 does not depend on the stock of capital at time 1; therefore, $V_{2K_1} = \Pi_{K_1}(K_2, Z_2)$. Given that $\Pi(K_2, Z_2) = hZ_2^\eta K_2$ and $K_2 = K_1 + I_2$, then $V_{2K_1} = hZ_2^\eta$.

⁷Relaxing this assumption is trivial.

Plugging the expressions for $V_{2K_1}(\cdot, \cdot)$, $\Pi_K(\cdot, \cdot)$, and $C_I(\cdot)$ into equation (3) provides the first-order condition for the perfectly competitive, constant-returns-to-scale firm (at time 1):

$$hZ_1^\eta(1 + e^{[\eta(\eta-1)/2]\sigma^2}) - [I_1 > 0](1 + \gamma_1\beta I_1^{\beta-1}) - [I_1 < 0](1 - \gamma_2\beta |I_1|^{\beta-1}) = 0$$

which yields the following investment function at time 1:

$$I_1 = \begin{cases} \left[\frac{hZ_1^\eta(1 + e^{[\eta(\eta-1)/2]\sigma^2}) - 1}{\gamma_1\beta} \right]^{1/(\beta-1)} & \text{for } I_1 > 0 \text{ or } hZ_1^\eta(1 + e^{[\eta(\eta-1)/2]\sigma^2}) \geq 1 \\ - \left[\frac{1 - hZ_1^\eta(1 + e^{[\eta(\eta-1)/2]\sigma^2})}{\gamma_2\beta} \right]^{1/(\beta-1)} & \text{for } I_1 < 0 \text{ or } hZ_1^\eta(1 + e^{[\eta(\eta-1)/2]\sigma^2}) < 1. \end{cases}$$

Again, investment at time 1 does not depend on either past or future capital stocks. This lack of “intertemporal links” does not depend on the two-period assumption. In fact, this insight also holds for the n -period model (see Appendix) and is crucial in determining the irrelevance of the shape (besides convexity) of the investment-cost function, under partial equilibrium and risk neutrality, vis-à-vis the response of investment to changes in the level of uncertainty.

The asymmetry of adjustment costs has nothing to do with the sign of the response of investment to increases in uncertainty. Whether investment is positive or negative depends on the sign of the numerator, and this does not include the adjustment-cost parameters. An increase in uncertainty raises investment (or reduces disinvestment) for any (finite) level of adjustment costs. The asymmetry determines only that investment and disinvestment have different speeds of adjustment. This is fully consistent with Hartman’s (1972) and Abel’s (1983,

1984, 1985) conclusion for the symmetric case. Notice that this is true even for the case in which β is very close to 1, γ_1 is slightly greater than 0, and γ_2 is ∞ , that is, when investment is irreversible and there are almost no costs (besides the price itself) of adjusting the capital stock upward.⁹

In sum, the fact that asymmetric costs imply a larger disequilibrium (as compared to the frictionless capital stock) when demand realizations are low, does *not* affect the conclusion that, under constant returns to scale and perfect competition (as well as risk neutrality and partial equilibrium), increases in uncertainty raise investment. This is just a reflection of the fact that, under perfect competition, how much is invested today affects profits tomorrow, but not the level of investment tomorrow. Therefore, any increase in the expected marginal profitability of capital, including the one caused by an increase in price uncertainty, raises investment today.

The next section relaxes the perfect-competition assumption to show that the interaction between decreasing marginal profitability of capital, resulting from imperfect competition (or decreasing returns to scale), and asymmetric costs can generate a negative investment–uncertainty relationship. In determining this relationship, imperfect competition not only is necessary but also plays a central role.

III. Imperfect Competition

When competition is imperfect there is, in general, no closed-form solution for the investment function (even in the simple two-period context).¹⁰ The exceptions correspond to extreme assumptions about $(\gamma_1, \gamma_2, \beta)$: no adjustment costs $(0, 0, \beta)$, irreversible investment with no (adjustment) cost of increasing the stock of capital $(0, \infty, 1)$, capacity constraints with no cost of

⁹It is well known that, under perfect competition and constant returns to scale, the size of the firm is indeterminate; therefore, some convexity in (upward) adjustment costs ($\gamma_1 > 0, \beta > 1$) is required in order to bound (positive) investment.

¹⁰At least, there is none known to me.

scrapping capital $(\infty, 0, 1)$, and predetermined capital (∞, ∞, β) . Here, I solve numerically several examples of the general $(\gamma_1, \gamma_2, \beta)$ case.

To streamline the notation, it is convenient to assume (without loss of generality) that $Z_1 \equiv 1$. It is also simpler (again, without loss of generality) to assume that there is no initial capital, with the result that I_1 is always positive. With these assumptions, I_1 and I_2 are determined by the following two equations:

$$\mu h I_1^{\mu-1} - \gamma_1 \beta I_1^{\beta-1} + \beta E_1([I > 0] \gamma_1 I_2^{\beta-1} - [I < 0] \gamma_2 |I_2|^{\beta-1}) = 0$$

$$\mu h Z_2^\eta (I_1 + I_2)^{\mu-1} - 1 - \beta \{ [I > 0] \gamma_1 I_2^{\beta-1} - [I < 0] \gamma_2 |I_2|^{\beta-1} \} = 0$$

corresponding to the first-order conditions in periods 1 and 2, respectively.

The problem is even further simplified by replacing the assumption of a lognormal distribution for Z with a simple symmetric Bernoulli distribution.¹¹ Figure 1 illustrates the independent (of asymmetries) role of imperfect competition in generating the negative investment-uncertainty relationship.¹² In this figure, adjustment costs are entirely symmetric; however the Jensen's inequality argument of Hartman (1972) and Abel (1983, 1984, 1985) becomes less significant as competition becomes more imper-

fect (ψ gets larger).¹³ The figure shows that as ψ increases (i.e., as the elasticity of demand is reduced), investment responds less and less to changes in the level of uncertainty.¹⁴ In fact, when $\psi > 1.6$, the lines characterizing the investment-uncertainty trade-off in Figure 1 are almost horizontal, indicating practically no effects of changes in the level of uncertainty on investment decisions. Again, it should be remembered that this has been achieved under perfectly symmetric adjustment costs.

There are two channels for the dampening of the result of Hartman and of Abel under imperfect competition (and symmetric adjustment costs). First, as the elasticity of demand is reduced, the convexity of the marginal profitability of capital with respect to price uncertainty, η , is reduced (given the stock of capital).¹⁵ This can be seen more clearly by using perfect competition as a benchmark. Recall that under perfect competition $P_t = Z_t$; hence, an increase in Z_t raises revenues both directly (through $Q\Delta P$) and indirectly through the increase in optimal output (given the stock of capital). The latter effect is responsible for the convexity of the profit function with respect to Z_t . When the elasticity of demand is less than infinite, however, the indirect effect is less important, as the firm's desire to increase output is less than that under perfect competition since doing it brings the price of its goods down, lowering the direct effect of a positive change in Z . Thus, as the elasticity of demand falls, the profit function becomes less convex with respect to Z_t . Second, as the markup (ψ) rises, the marginal profitability of capital (Π_K) decreases more with a given increase in capital (i.e., $\Pi_{KK\psi} < 0$).¹⁶ This, again, dampens the response of investment to an increase in

¹¹The interesting symmetric Bernoulli case is that in which the positive shock leads to positive investment in period 2 and a negative shock leads to disinvestment. The numerical problem is then trivial, as it consists of three equations (the expected value equation [first equation] and the two equations for the second period: one for the good realization and one for the bad realization) and three unknowns (investment in the first period and investment in the second period for the good and bad realizations). Certainly, generalizing this to any discrete-state space is trivial.

¹²All the curves (in all diagrams) are normalized by their respective level of investment under certainty.

¹³Notice that the level of adjustment costs is not important, since investment is normalized by the level of investment under certainty.

¹⁴Remember that $\psi = 1$ corresponds to the perfect competition case, whereas $\psi > 1$ represents imperfect competition (or decreasing returns to scale).

¹⁵Remember that η is decreasing in ψ .

¹⁶Remember that under perfect competition $\Pi_{KK} = 0$.

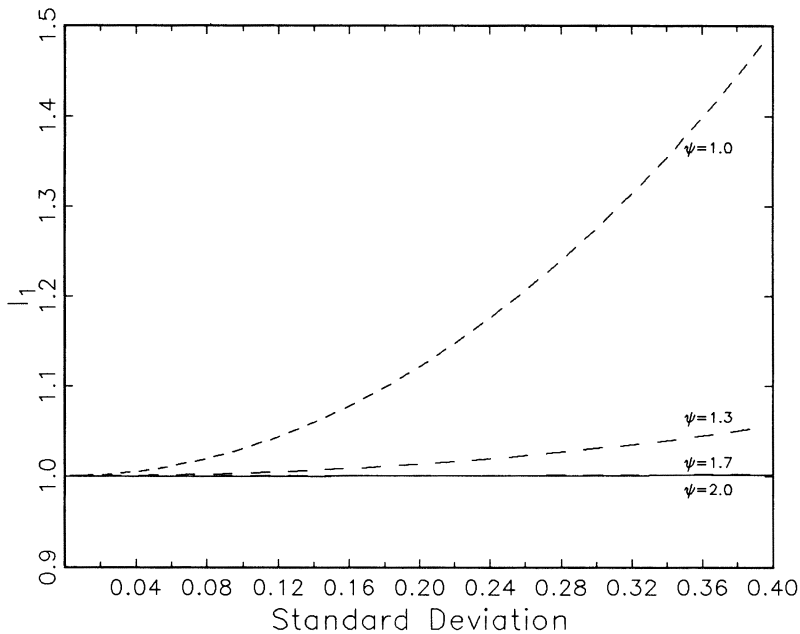


FIGURE 1. INVESTMENT AS A FUNCTION OF UNCERTAINTY FOR DIFFERENT VALUES OF ψ

price (demand) uncertainty, as the initial lift of the second period's expected marginal profitability of capital (due to Jensen's inequality) is curtailed by an increase in investment today. Combined, these two effects demonstrate that the result of Hartman and of Abel loses its strength under imperfect competition even when adjustment costs are symmetric.

Figure 2 shows the effects of asymmetric adjustment costs on the investment–uncertainty relationship, once competition is imperfect. The markup coefficient is 1.67 (this corresponds to an elasticity of demand equal to 2.5), and the cost of downward adjustments goes from being equal to the cost of upward adjustments of the capital stock (solid line), to being 50 times as expensive as the latter (short dashes). It is apparent that, given the presence of a significant degree of competition imperfection, the investment–uncertainty relationship becomes more negative as the adjustment-cost asymmetry gets larger.

Adjustment costs deform the relationship between realizations of the shock (innova-

tions), ε_2 , and the stock of capital at time 2 (when compared with the costless adjustment case). For example, if capital is predetermined (unchangeable both upward and downward), there is no link whatsoever between the realization of the shock and the stock of capital in the second period. In the irreversible-investment case, on the other hand, the capital stock and shocks are only linked for “good” realizations of the latter (i.e., for realizations in which the capital in place is less than the desired stock of capital). In general, for the asymmetric case, the stock of capital responds more to “good” than to “bad” realizations (i.e., realizations in which the capital in place is larger than the desired stock of capital). When competition is imperfect, the determination of what is a “good” and a “bad” shock is endogenous. It depends on how much is invested in the first period. The less the firm invests in the first period, the more likely it is to get a good shock (i.e., one in response to which the lowest adjustment cost is paid). Certainly, the cost of this strategy is less output today. When uncertainty is larger, “very

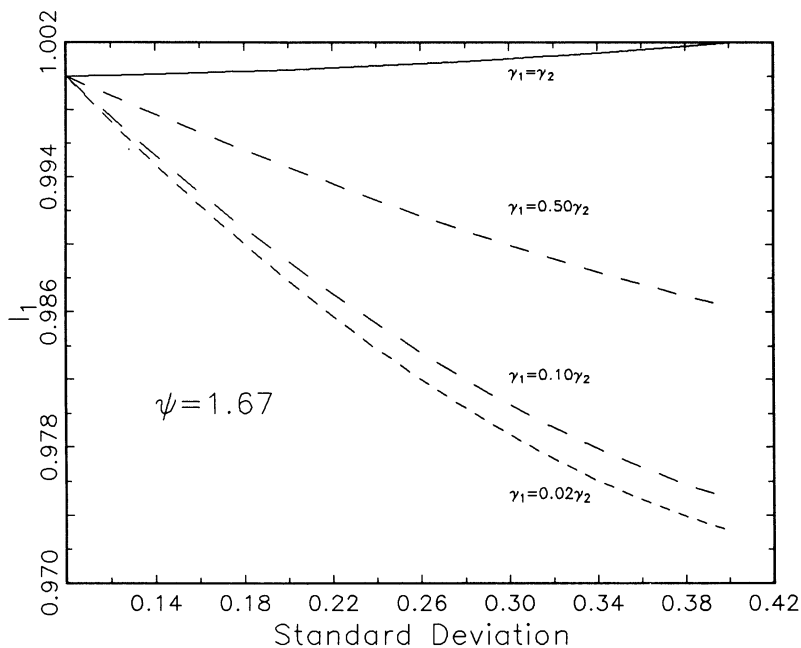


FIGURE 2. INVESTMENT AS A FUNCTION OF UNCERTAINTY FOR DIFFERENT DEGREES OF ASYMMETRY IN ADJUSTMENT COSTS ONCE COMPETITION IS IMPERFECT

good” and “very bad” news become more likely; but the larger the asymmetry of adjustment costs, the more expensive are the latter relative to the former. Thus, it is optimal to buy more protection, in the form of less initial investment, as the asymmetry and degree of uncertainty rise.

It is also worth noticing that the support of I_1 is larger in Figure 1 than in Figure 2, suggesting that imperfect competition (or decreasing returns to scale) not only is a necessary condition for asymmetric adjustment costs to affect the investment–uncertainty relationship, but also plays a central role.

IV. Conclusion

This paper has demonstrated that the presence of asymmetric adjustment costs is not sufficient to render a negative relationship between investment and mean-preserving changes in uncertainty. Some nonnegligible degree of imperfect competition is also required. In fact, the result of Hartman

(1972) and Abel (1983, 1984, 1985) (positive relationship between investment and uncertainty) for symmetric and convex adjustment costs under perfect competition fully carries over to the case of asymmetric adjustment costs. Furthermore, under very competitive conditions, the asymmetry of adjustment costs has little to do with the sign of the investment–uncertainty relationship. Today’s investment decisions depend almost exclusively on the price of capital (today and in the future)¹⁷ and the expected marginal profitability of capital. In this case, the marginal profitability of capital is only tenuously related to the level of capital; hence, the convexity of marginal profitability of capital with respect to prices is the dominant factor in determining the sign of the investment–uncertainty relationship. A simple Jensen’s inequality argument shows that the latter is positive.

¹⁷Assumed to be constant in this paper.

Conversely, when competition is imperfect, the marginal profitability of capital is significantly affected by the level of capital. An increase in investment today makes it more likely that the firm will find its second-period capital "too large" relative to the desired capital stock. When adjustment costs are asymmetric, having "too much" capital is worse than having "too little" of it, since increasing the stock of capital is cheaper than decreasing it. If this effect is sufficiently strong (i.e., the asymmetry of adjustment costs is large and the negative dependence of the marginal profitability of capital on the level of capital is strong), the investment-uncertainty relationship becomes negative. The irreversible-investment arguments analyzed in the literature typically correspond to this case.

Most of the analysis of this paper maintains the assumption of constant returns to scale. Relaxing this assumption, however, does not convey any additional difficulty. Convexity of the profit function with respect to prices in this case depends on the value of γ/ψ instead of just $1/\psi$. An increase in γ operates exactly like an equivalent reduction in the markup coefficient and vice versa. Hence, decreasing returns to scale makes a negative uncertainty-investment relationship more likely, whereas increasing returns offsets imperfect competition, bringing the uncertainty-investment relationship closer to the result of Hartman and of Abel.¹⁸

Overall, the results of this paper suggest that the relationship between changes in price uncertainty and capital investment under risk neutrality is not robust. Studying different adjustment-cost mechanisms is extremely important in determining the dynamics of investment and its business-cycle implications; however, it is very likely that it will be necessary to turn back to risk aver-

sion, incomplete markets, and lack of diversification to obtain a sturdier negative relationship between investment and uncertainty. Craine (1989) and Zeira (1989) have taken important steps along these lines.

APPENDIX

Consider the following discounted infinite-horizon version of the optimization problem presented in the paper for the perfect-competition case:

$$(A1) \quad V(K_{t-1}, Z_t) = \max_{I_t} \{ hZ_t^\eta K_t - I_t \\ - [I_t > 0] \gamma_1 I_t^\beta \\ - [I_t < 0] \gamma_2 |I_t|^\beta \\ + \delta E_t [V(K_t, Z_{t+1})] \}$$

subject to

$$K_t = \lambda K_{t-1} + I_t$$

$$\lim_{T \rightarrow \infty} \delta^T E_t [V(K_{T-1}, Z_T)] = 0$$

$$K_{t-1} \text{ given.}$$

The parameters δ and λ correspond to the discount factor and 1 minus the depreciation rate, respectively, both being less than 1. Also assume that $Z_t = Z_{t-1} W_t$, where W_t is any strictly positive independently and identically distributed random variable with mean 1 and log-standard deviation σ .

From the insights given by the two-periods problem, it seems reasonable to guess a value function of the form

$$(A2) \quad V(K_{t-1}, Z_t) = A(Z_t) + chZ_t^\eta K_{t-1}$$

where c is a constant and $A(\cdot)$ is a continuous function, both to be found.

After substituting (A1) into (A2) and obtaining the first-order conditions, it is possible to write investment as a function of the

¹⁸It is also easy to show that if there are costs of waiting to invest, in the sense that there are some advantages (besides the traditional convex-adjustment-costs arguments) of planning and investing with time, the investment-uncertainty relationship may become positive even when adjustment costs are asymmetric, returns to scale are constant or decreasing, and competition is very imperfect.

unknown parameter c :

$$I_t = \begin{cases} \left[\frac{hZ_t^\eta(1 + c\lambda\delta L(\eta, \sigma)) - 1}{\gamma_1\beta} \right]^{1/(\beta-1)} & \text{for } I_t > 0 \\ - \left[\frac{1 - hZ_t^\eta(1 + c\lambda\delta L(\eta, \sigma))}{\gamma_2\beta} \right]^{1/(\beta-1)} & \text{for } I_t < 0 \end{cases}$$

where $L(\eta, \sigma) = E_t[W_{t+1}^\eta]$, an increasing function of σ (Jensen's inequality). Plugging this back into the Bellman equation makes it possible to find c :

$$c = \frac{\lambda}{1 - \lambda\delta L(\eta, \sigma)}.$$

This substitution also leads to a functional equation for $A(\cdot)$:

$$(A3) \quad A(Z_t) = \delta E_t[A(Z_{t+1})] + G(Z_t)$$

where

$$\begin{aligned} G(x) \equiv & - \left[x \geq \{h[1 + c\lambda L(\eta, \sigma)]\}^{-1/\eta} \right] \\ & \times \left[\frac{hx^\eta[1 + c\lambda\delta L(\eta, \sigma)] - 1}{\gamma_1\beta} \right]^{1/(\beta-1)} \\ & \times [1 - c\delta L(\eta, \sigma)hx^\eta] \\ & - \left[x \geq \{h[1 + c\lambda L(\eta, \sigma)]\}^{-1/\eta} \right] \\ & \times \gamma_1 \left[\frac{hx^\eta[1 + c\lambda\delta L(\eta, \sigma)] - 1}{\gamma_1\beta} \right]^{\beta/(\beta-1)} \\ & + \left[x < \{h[1 + c\lambda L(\eta, \sigma)]\}^{-1/\eta} \right] \\ & \times \left[\frac{1 - hx^\eta[1 + c\lambda\delta L(\eta, \sigma)]}{\gamma_2\beta} \right]^{1/(\beta-1)} \\ & \times [1 - c\delta L(\eta, \sigma)hx^\eta] \\ & - \left[x < \{h[1 + c\lambda L(\eta, \sigma)]\}^{-1/\eta} \right] \\ & \times \gamma_2 \left[\frac{1 - hx^\eta[1 + c\lambda\delta L(\eta, \sigma)]}{\gamma_2\beta} \right]^{\beta/(\beta-1)}. \end{aligned}$$

Making the simplifying assumption that W_t has bounded support makes the function $G(x)$ a continuous bounded function. This, together with the Markov structure of the transition and the fact that $\delta < 1$, determines that $A(Z_t)$ exists and is unique (David Blackwell, 1965).¹⁹

As long as $\lambda\delta L(\eta, \sigma) < 1$, the instantaneous reward function is concave in the control I_t ; therefore, the investment function is also unique. Furthermore,

$$\begin{aligned} \frac{\partial I_t}{\partial \sigma} &= \{\beta(\beta - 1)([I > 0]\gamma_1 + [I < 0]\gamma_2) \\ &\quad \times (1 - \lambda\delta L(\eta, \sigma))^2\}^{-1} \\ &\quad \times |I_t|^{-1/\beta} hZ_t^\eta \delta \lambda^2 L_\sigma(\eta, 0) \\ &> 0 \end{aligned}$$

confirming the fact that the result of Hartman (1972) and Abel (1983, 1984, 1985) extends to the asymmetric-adjustment-cost case.

¹⁹Note that the function $A(\cdot)$ does not need to be unique to guarantee a unique investment function.

REFERENCES

- Abel, Andrew B., "Optimal Investment Under Uncertainty," *American Economic Review*, March 1983, 73, 228-33.
- _____, "The Effects of Uncertainty on Investment and the Expected Long-Run Capital Stock," *Journal of Economic Dynamics and Control*, February 1984, 7, 39-54.
- _____, "A Stochastic Model of Investment, Marginal Q and the Market Value of the Firm," *International Economic Review*, June 1985, 26, 305-22.
- Bertola, Giuseppe, "Adjustment Costs and Dynamic Factor Demands: Investment and Employment Under Uncertainty," Ph.D. Dissertation (Ch. 2), MIT, June 1988.
- Blackwell, David, "Discounted Dynamic Programming," *Annals of Mathematical*

- Statistics*, February 1965, 36, 226–35.
- Craine, Roger**, “Risky Business: The Allocation of Capital,” *Journal of Monetary Economics*, March 1989, 23, 201–18.
- Hartman, Richard**, “The Effects of Price and Cost Uncertainty on Investment,” *Journal of Economic Theory*, October 1972, 5, 258–66.
- Pindyck, Robert S.**, “Irreversible Investment, Capacity Choice, and the Value of the Firm,” *American Economic Review*, December 1988, 78, 969–85.
- , “A Note on Competitive Investment Under Uncertainty,” mimeo, MIT, June 1990.
- Zeira, Joseph**, “Cost Uncertainty and the Rate of Investment,” Hebrew University Working Paper No. 206, February 1989.