# Online Appendix to Trade Theory with Numbers: Quantifying the Consequences of Globalization 

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#### Abstract

This appendix provides the derivation of theoretical results used in Sections 3.1, 3.4, 4.1, 4.2, 4.3, and 5.1 as well as additional information about the treatment of trade imbalances, import tariffs, and data used in our quantitative analysis.


## 1 Price Index in Section 3.1

The goal of this section is to establish Equation (14) in Section 3.1:

$$
\begin{equation*}
P_{i j}=\tau_{i j} c_{i}^{p} \times\left(\left(\frac{E_{j}}{c_{i j}^{x}}\right)^{\frac{\delta}{1-\sigma}} \frac{\tau_{i j} c_{i}^{p}}{P_{j}}\right)^{\eta} \times\left(\frac{R_{i}}{c_{i}^{e}}\right)^{\frac{\delta}{1-\sigma}} \times \xi_{i j} \tag{14}
\end{equation*}
$$

We do so for the case of a model of monopolistic competition with firm-level heterogeneity similar to the one considered in Arkolakis et al. (2008). We then heuristically explain how a similar expression for $P_{i j}$ arises in Krugman (1980), Eaton and Kortum (2002), and Bernard et al. (2003).

The basic environment is a strict generalization of Arkolakis et al. (2008) in which: (i) entry, exporting, and production activities may use both primary factors of production and intermediate goods and (ii) fixed costs of exporting may be partially paid both in the origin and destination countries. Compared to the original model of monopolistic competition with firm-level heterogeneity by Melitz (2003), this basic environment is more general in that it allows an arbitrary number of asymmetric countries, intermediate goods, and fixed exporting costs that may be paid in both countries, but less general in that it restrictions the distribution of firm-level productivity to be Pareto, as discussed below.

Specifically, there is a continuum of goods indexed by $\omega \in \Omega$. We denote by $\Omega_{j}$ the set of goods available for purchase in country $j$. In line with the assumptions of Sections 3.1 and 3.4, we assume that both final goods and intermediate goods are aggregated via the same Constant Elasticity of Substitution (CES) function:

$$
\begin{aligned}
C_{j} & =\left(\int_{\Omega_{j}} c_{j}(\omega)^{(\sigma-1) / \sigma} d \omega\right)^{\sigma /(\sigma-1)} \\
I_{j} & =\left(\int_{\Omega_{j}} i_{j}(\omega)^{(\sigma-1) / \sigma} d \omega\right)^{\sigma /(\sigma-1)}
\end{aligned}
$$

where $\sigma>1$ and $c_{j}(\omega)$ and $i_{j}(\omega)$ are the quantity of good $\omega$ demanded in country $j$ for final consumption and production, respectively. The associated price index for both final and intermediate goods in country $j$ is given by

$$
P_{j}=\left(\sum_{j} P_{i j}^{1-\sigma}\right)^{1 /(1-\sigma)}
$$

where

$$
P_{i j}=\left(\int_{\Omega_{i j}}\left(p_{j}(\omega)\right)^{1-\sigma} d \omega\right)^{1 /(1-\sigma)}
$$

with $\Omega_{i j}$ the set of goods sold by firms from country $i$ in country $j$.
Productivity levels $\phi$ are independently drawn across monopolistically competitive firms in each country from a Pareto distribution, $G$, with dispersion parameter $\theta>\sigma-1$ and lower bound $b$, which we set to 1 :

$$
\begin{equation*}
G(\phi)=1-\phi^{-\theta} \text { for all } \phi \geq 1 \tag{1}
\end{equation*}
$$

In order to get a productivity draw, firms must pay a fixed entry $\operatorname{cost} c_{i}^{e} f_{i}^{e}$. In order to sell in country $j$, firms from country must then pay a fixed exporting costs $c_{i j}^{x} f_{i j}$. Once fixed entry costs and fixed exporting costs have been paid, the constant cost of producing and delivering one unit of good $\omega$ from country $i$ in country $j$ is given by $c_{i}^{p} \tau_{i j} / \phi_{i}(\omega)$, where $\phi_{i}(\omega)$ denotes the productivity of the firm producing good $\omega$ in country $i$ and $\tau_{i j} \geq 1$ denotes the iceberg trade costs between country $i$ and country $j$. The endogenous costshifters $c_{i}^{e}, c_{i j}^{x}$, and $c_{i}^{p}$ are functions of the price of primary factors of production-typically labor-and the price of intermediate goods. In order to establish Equation (14), however, we do not need to take a stand on what those functions are.

Given our CES assumption, the total demand for a good $\omega$ in country $j$ with price $p_{j}(\omega)$ is equal to

$$
q_{j}(\omega)=p_{j}(\omega)^{-\sigma} P_{j}^{\sigma-1} E_{j},
$$

where $E_{j} \equiv P_{j}\left(C_{j}+I_{j}\right)$ is total expenditure in country $j$. Under monopolistic competition, prices are a constant markup over marginal costs,

$$
\begin{equation*}
p_{j}(\omega)=\frac{\sigma}{\sigma-1} \frac{c_{i}^{p} \tau_{i j}}{\phi_{i}(\omega)} \text { for all } \omega \in \Omega_{i j} \tag{2}
\end{equation*}
$$

From here onwards we drop the index for goods $\omega$ and keep track of goods by their origin $i$ and productivity $\phi$.

For each origin country $i$, there exists a productivity cut-off $\phi_{i j}^{*}$ such that firms from country $i$ sell in market $j$ if and only if $\phi \geq \phi_{i j}^{*}$. Letting $x_{i j}(\phi) \equiv P_{j}^{\sigma-1} E_{j}\left(\frac{\sigma}{\sigma-1} \frac{c_{i}^{p} \tau_{i j}}{\phi}\right)^{1-\sigma}$ denote total sales by a firm with productivity $\phi$ from country $i$ in country $j$, the cut-off productivities $\phi_{i j}^{*}$ are determined by equating profits to zero $x_{i j}\left(\phi_{i j}^{*}\right) / \sigma=c_{i j}^{x} f_{i j}$. Letting
$\widetilde{\sigma} \equiv\left(\frac{\sigma}{\sigma-1}\right)^{1-\sigma}$, this can be rearranged as

$$
\begin{equation*}
\phi_{i j}^{*}=\left(\frac{\sigma}{\widetilde{\sigma}} \frac{c_{i j}^{x} f_{i j}}{E_{j}}\right)^{1 /(\sigma-1)} \frac{c_{i}^{p} \tau_{i j}}{P_{j}} \tag{3}
\end{equation*}
$$

Now let $M_{i}$ denotes the measure of firms (operating and non-operating) in country $i$, i.e. the measure of firms that have paid the fixed entry costs, $c_{i}^{e} f_{i}^{e}$. Using the previous notation and Equation (2), we can rewrite the origin-and-destination-specific price index, $P_{i j}$, as

$$
\begin{equation*}
P_{i j}=\left(M_{i} \widetilde{\sigma} \int_{\phi_{i j}^{*}}^{\infty}\left(\frac{c_{i}^{p} \tau_{i j}}{\phi_{i}(\omega)}\right)^{1-\sigma} d G_{i}(\phi)\right)^{\frac{1}{1-\sigma}} \tag{4}
\end{equation*}
$$

Under the assumption that firm-level productivity is Pareto distributed (Equation 1), we get

$$
P_{i j}=\left(\frac{M_{i} \theta \widetilde{\sigma}}{\theta-(\sigma-1)}\left(c_{i}^{p} \tau_{i j}\right)^{1-\sigma}\left(\phi_{i j}^{*}\right)^{\sigma-\theta-1}\right)^{\frac{1}{1-\sigma}}
$$

Together with Equation (3), this implies

$$
P_{i j}=\left(\frac{M_{i} \theta \widetilde{\sigma}}{\theta-(\sigma-1)}\left(c_{i}^{p} \tau_{i j}\right)^{1-\sigma}\left(\left(\frac{\sigma}{\widetilde{\sigma}} \frac{c_{i j}^{x} f_{i j}}{E_{j}}\right)^{1 /(\sigma-1)} \frac{c_{i}^{p} \tau_{i j}}{P_{j}}\right)^{\sigma-\theta-1}\right)^{\frac{1}{1-\sigma}} .
$$

Using the assumption that firm-level productivity is Pareto distributed (Equation 1),one can also check that aggregate profits are a constant share, $\frac{\sigma-1}{\sigma \theta}$, of aggregate revenues, $R_{i}$. Thus free entry implies that $c_{i}^{e} f_{i}^{e} M_{i}=\frac{\sigma-1}{\sigma \theta} R_{i}$. Combining the two previous expressions, we obtain

$$
\begin{equation*}
P_{i j}=c_{i}^{p} \tau_{i j}\left(\left(\frac{E_{j}}{c_{i j}^{x}}\right)^{1 /(1-\sigma)} \frac{c_{i}^{p} \tau_{i j}}{P_{j}}\right)^{\eta}\left(\frac{R_{i}}{c_{i}^{e}}\right)^{1 /(1-\sigma)} \xi_{i j} \tag{5}
\end{equation*}
$$

where $\eta \equiv \frac{\theta}{\sigma-1}-1$ and $\xi_{i j} \equiv\left(\frac{\tilde{\sigma}}{\eta \sigma}\right)^{1 /(1-\sigma)}\left(\frac{\sigma f_{i j}}{\tilde{\sigma}}\right)^{\eta /(\sigma-1)}\left(f_{i}^{e}\right)^{1 /(\sigma-1)}$. Using the convention $\delta=1$ under monopolistic competition, this establishes Equation (14) under monopolistic competition with firm-level heterogeneity.

The case of monopolistic competition without firm-level heterogeneity, as in Krugman (1980), can be dealt with in a similar manner. In the absence of firm-level heterogeneity,

Equation (4) simply becomes

$$
P_{i j}=\left(M_{i} \widetilde{\sigma}\left(\frac{c_{i}^{p} \tau_{i j}}{\phi_{i}}\right)^{1-\sigma}\right)^{\frac{1}{1-\sigma}}
$$

where $\phi_{i}$ is the productivity of a representative firm in country $i$. In this environment, aggregate profits are still a constant share, now given by $\frac{1}{\sigma}$, of aggregate revenues. Thus free entry implies $c_{i}^{e} f_{i}^{e} M_{i}=\frac{1}{\sigma} R_{i}$. Combining the two previous expressions, we obtain

$$
P_{i j}=c_{i}^{p} \tau_{i j}\left(\frac{R_{i}}{c_{i}^{e}}\right)^{1 /(1-\sigma)} \xi_{i j}
$$

where $\xi_{i j} \equiv\left(\frac{\sigma f_{i}^{e}}{\tilde{\sigma}}\right)^{1 /(\sigma-1)} \frac{1}{\phi_{i}}$. Using the convention $\eta=0$ in the absence of firm-level heterogeneity, this establishes Equation (14) under monopolistic competition without firmlevel heterogeneity.

The case of perfect competition, as in Eaton and Kortum (2002), and Bertrand competition, as in Bernard et al. (2003), is simpler in that the measure of goods potentially available from each country, $M_{i}$, is exogenously given. Thus the entry term, $\left(\frac{R_{i}}{c_{i}^{e}}\right)^{1 /(1-\sigma)}$, in Equation (5) drops out. Similarly, the measure of goods sold in each market no longer depends on whether revenues are large enough to cover fixed exporting costs, which explains why the term $\left(\frac{E_{j}}{c_{i j}^{x}}\right)^{1 /(1-\sigma)}$ drops out of Equation (5) as well. Using the convention $\delta=0$ under perfect and Bertrand competition, one can therefore again establish Equation (14) under these alternative market structures provided that productivity levels across goods are drawn from Fréchet distributions; see Eaton and Kortum (2002) and Bernard et al. (2003) for details.

## 2 Autarky Equilibrium in Section 3.4

In order to derive the formula for the gains from trade in Section 3.4 (Equation 29) we have argued that in the autarky equilibrium, we must have

$$
\begin{equation*}
e_{j, k}^{A} / v_{j}^{A}=\sum_{l=1}^{S} \beta_{j, l} a_{j, k l} \tag{6}
\end{equation*}
$$

where $e_{j, k}^{A}$ denotes the share of expenditure on varieties from sector $k$ in country $j ; v_{j}^{A}$ denotes the ratio of total income to total revenues in country $j ; \beta_{j, l}$ denotes the share of
expenditure on final goods going to varieties from sector $l$; and $a_{j, k l}$ are now the elements of the Leontief inverse $\left(\boldsymbol{I} d-\boldsymbol{A}_{j}\right)^{-1}$, with $\boldsymbol{A}_{j} \equiv\left\{\alpha_{j, k s}\right\}^{\prime}$ and $\alpha_{j, k s}$ the share of intermediate goods from sector $k$ used in sector $s$ for production, entry, and exporting activities:

$$
c_{i, s}^{p}=c_{i, s}^{e}=c_{i i, s}^{x} \equiv Y_{i}^{1-\alpha_{i, s}} \prod_{k=1}^{S} P_{i, k}^{\alpha_{i, k s}} .
$$

We now establish Equation (6) formally.
In the autarky equilibrium, the total expenditure on varieties from sector $s$ must remain equal on total spending for final consumption plus total spending on intermediate goods by firms from other sectors:

$$
e_{j, s}^{A} E_{j}^{A}=\beta_{j, s} Y_{j}^{A}+\sum_{k=1}^{S} \alpha_{j, s k} r_{j, k}^{A} R_{j}^{A}
$$

But in the autarky equilibrium, revenue must be equal to expenditure sector-by-sector so that $e_{j, k}^{A}=r_{j, k}^{A}$ and $E_{j}^{A}=R_{j}^{A}$. Accordingly, we can rearrange the previous expression as

$$
e_{j, s}^{A} / v_{j}^{A}=\beta_{j, s}+\sum_{k=1}^{S} \alpha_{j, s k}\left(e_{j, k}^{A} / v_{j}^{A}\right)
$$

where we have used the definition of $v_{j}^{A} \equiv Y_{i}^{A} / R_{j}^{A}$. Equation (6) directly derives from the previous expression and the definition of $a_{j, k l}$ as a typical element of the Leontief inverse $\left(I d-A_{j}\right)^{-1}$.

## 3 Counterfactuals in Sections 4.1, 4.2, 4.3, and 5.1

The goal of this section is to derive the equilibrium conditions used for the counterfactual analysis of Sections 4.1, 4.2, 4.3, and 5.1. To do so, we consider an environment with multiple sectors, tradable intermediate goods, and trade imbalances. In line with the analysis of Section 4, we assume that import tariffs are imposed before markups (if any) and that fixed exporting costs are paid in the exporting country (this is explained in detail Section 5 of this Appendix). We then study a general counterfactual change in import tariffs and trade imbalances and discuss how these results are used to generate the quantitative results of Sections 4.1, 4.2, 4.3, and 5.1. Throughout our analysis we use world GDP as our numeraire so that $\sum_{i=1}^{n} Y_{i}=1$ in all equilibria.

Assumptions. In line with Section 3.4, we assume that production, entry, and exporting activities use primary factors of production and intermediate goods in the same propor-
tion: $c_{i, s}^{p}=c_{i, s}^{e}=c_{i i, s}^{x}=c_{i, s}$, where

$$
\begin{equation*}
c_{i, s} \equiv Y_{i}^{1-\alpha_{i, s}} \prod_{k=1}^{S} P_{i, k}^{\alpha_{i, k s}} . \tag{7}
\end{equation*}
$$

The assumption that fixed exporting costs are paid in the exporting country further implies that $c_{i j, s}^{x}=c_{i, s}$. In the presence of tariffs (imposed before markups), total variable trade costs are given by $\phi_{i j, s}=\tau_{i j, s}\left(1+t_{i j, s}\right)$, where $\tau_{i j, s}$ is the iceberg trade cost between country $i$ and country $j$ in sector $s$ and $t_{i j, s}$ is the ad-valorem import tariff. Under these assumptions, Equation (26) in the text simplifies into

$$
P_{i j, s}=\phi_{i j, s} c_{i, s}\left(\left(\frac{E_{j, s}}{c_{i, s}}\right)^{\frac{\delta_{s}}{1-\sigma_{s}}} \frac{\phi_{i j, s} c_{i, s}}{P_{j, s}}\right)^{\eta_{s}}\left(\frac{R_{i, s}}{c_{i, s}}\right)^{\frac{\delta_{s}}{1-\sigma_{s}}} \xi_{i j, s} .
$$

Since we allow for trade imbalances, we do not impose $E_{j}=Y_{j}$ and write $E_{j, s} / c_{i, s}$ instead of $\frac{e_{j, s}}{v_{j}} \frac{Y_{j}}{c_{i, s}}$. Similarly, we write $R_{i, s}$ rather than $\frac{r_{i, s}}{v_{i}} Y_{i}$.

Let $D_{j}$ denote the net trade deficit in country $j$. In line with Section 5.1, we model trade deficits as a lump-sum transfer from the rest of the world to country $j$. We consider two alternative assumptions about the nature of trade deficits: $D_{j}=\varkappa_{j} Y_{j}$ and $D_{j}=$ $\varkappa_{j} \sum_{i=1}^{n} Y_{i}$, where $\varkappa_{j}>0$ is treated as an exogenous structural parameter. Given our choice of numeraire, $\sum_{i=1}^{n} Y_{i}=1$, we can express these two assumptions more compactly as

$$
D_{j}=\varkappa_{j} Y_{j}^{\mu}
$$

where $\mu$ is a binary variable equal to 1 if trade deficits, $D_{j}$, are assumed to be a constant share of domestic GDP, $Y_{j}$, and 0 if they are assumed to be a constant share of world GDP. Trade Equilibrium. Since $P_{j, s}^{1-\sigma_{s}}=\sum_{i} P_{i j, s}^{1-\sigma_{s}}$, we must have

$$
\begin{equation*}
P_{j, s}=\left(\sum_{i}\left(\phi_{i j, s} c_{i, s}\right)^{\left(1-\sigma_{s}\right)\left(1+\eta_{s}\right)}\left(\frac{E_{j, s}}{c_{i, s}}\right)^{\delta_{s} \eta_{s}}\left(\frac{R_{i, s}}{c_{i, s}}\right)^{\delta_{s}}\left(\xi_{i j, s}\right)^{1-\sigma_{s}}\right)^{\frac{1}{\left(1-\sigma_{s}\right)\left(1+\eta_{s}\right)}} . \tag{8}
\end{equation*}
$$

Under the assumption that varieties are aggregated in a CES fashion within each sector, we know that the share of expenditure, $\lambda_{i j, s}$, on varieties from sector $s$ produced in country $i$ and sold in country $j$ satisfies $\lambda_{i j, s}=P_{i j, s}^{1-\sigma_{s}} / P_{j, s}^{1-\sigma_{s}}$. Thus Equation (8) implies

$$
\begin{equation*}
\lambda_{i j, s}=\frac{\left(\phi_{i j, s} c_{i, s}\right)^{-\varepsilon_{s}} c_{i, s}^{-\delta_{s} \eta_{s}}\left(\frac{R_{i, s}}{c_{i, s}}\right)^{\delta_{s}} \chi_{i j, s}}{\sum_{l}\left(\phi_{l j, s} c_{l, s}\right)^{-\varepsilon_{s}} c_{l, s}^{-\delta_{s} \eta_{s}}\left(\frac{R_{l, s}}{c_{l, s}}\right)^{\delta_{s}} \chi_{l j, s}} . \tag{9}
\end{equation*}
$$

where the trade elasticity $\varepsilon_{s}=\left(\sigma_{s}-1\right)\left(1+\eta_{s}\right)$ and $\chi_{i j, s} \equiv \xi_{i j, s}^{1-\sigma_{s}}$.
In the presence of tariffs and trade imbalances, total expenditure on varieties from sector $s$ in country $j$ can be decomposed as follows,

$$
E_{j, s}=\beta_{j, s}\left(Y_{j}+D_{j}+T_{j}\right)+\sum_{k=1}^{S} \alpha_{j, s k} R_{j, k}
$$

where $T_{j}=\sum_{i=1}^{n} \sum_{s=1}^{S} \frac{t_{i j, s}}{1+t_{i j, s}} X_{i j, s}$ are tariff revenues. By definition, bilateral trade flows between country $i$ and country $j$ in sector $s$ are such that $X_{i j, s}=\lambda_{i j, s} E_{j, s}$. Thus tariff revenues can be expressed as

$$
T_{j}=\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}}{1+t_{i j, s}} \lambda_{i j, s}\left(\beta_{j, s}\left(Y_{j}+D_{j}+T_{j}\right)+\sum_{k=1}^{S} \alpha_{j, s k} R_{j, k}\right)
$$

or, after rearrangements,

$$
T_{j}=\frac{\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}}{1+t_{i j, s}} \lambda_{i j, s}\left(\beta_{j, s}\left(Y_{j}+D_{j}\right)+\sum_{k=1}^{S} \alpha_{j, s k} R_{j, k}\right)}{1-\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}}{1+t_{i j, s}} \lambda_{i j, s} \beta_{j, s}}
$$

Substituting for $T_{j}$ in the expression for expenditure at the sector level, we get

$$
\begin{equation*}
E_{j, l}=\frac{\beta_{j, l}\left(Y_{j}+D_{j}+\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}}{1+t_{i j, s}} \lambda_{i j, s} \sum_{k=1}^{S} \alpha_{j, s k} R_{j, k}\right)}{1-\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}}{1+t_{i j, s}} \lambda_{i j, s} \beta_{j, s}}+\sum_{k=1}^{S} \alpha_{j, l k} R_{j, k} . \tag{10}
\end{equation*}
$$

In turn, sector-level revenues are given by

$$
\begin{equation*}
R_{i, s}=\sum_{j=1}^{n}\left(1+\frac{t_{i j, s}}{\sigma_{s}}\right)^{\delta_{s}} \frac{\lambda_{i j, s}}{1+t_{i j, s}} E_{j, s} \tag{11}
\end{equation*}
$$

whereas total income is

$$
\begin{equation*}
Y_{i}=\sum_{s=1}^{S}\left(1-\alpha_{i, s}\right) R_{i, s} \tag{12}
\end{equation*}
$$

For given trade imbalances $\left\{D_{i}\right\}$ and tariffs $\left\{t_{i j, s}\right\}$, a trade equilibrium can be described by bilateral expenditure shares at the sector-level, $\left\{\lambda_{i j, s}\right\}$, sector-level expenditures, $\left\{E_{i, s}\right\}$, sector-level revenues, $\left\{R_{i, s}\right\}$, and income levels, $\left\{Y_{i}\right\}$, such that Equations (7)-(12).

Welfare. First note that Equation (8), Equation (9), and $\varepsilon_{s}=\left(\sigma_{s}-1\right)\left(1+\eta_{s}\right)$ imply

$$
\lambda_{i j, s}=\frac{\left(\phi_{i j, s} c_{i, s}\right)^{-\varepsilon_{s}}\left(\frac{E_{j, s}}{c_{i, s}}\right)^{\delta_{s} \eta_{s}}\left(\frac{R_{i, s}}{c_{i, s}}\right)^{\delta_{s}} \chi_{i j, s}}{P_{j, s}^{-\varepsilon_{s}}}
$$

Setting $i=j$ and using the fact that $\phi_{j j, s}=1$, we get

$$
P_{j, s}=\lambda_{j j, s}^{1 / \varepsilon_{s}}\left(\frac{R_{j, s}}{c_{j, s}}\right)^{-\delta_{s} / \varepsilon_{s}}\left(\frac{E_{j, s}}{c_{j, s}}\right)^{-\delta_{s} \eta_{s} / \varepsilon_{s}} c_{j, s} \chi_{j j, s}^{-1 / \varepsilon_{s}}
$$

Using Equation (7), we therefore have

$$
P_{j, s}=B_{j, s}\left(\prod_{k=1}^{S} P_{j, k}^{\alpha_{j, k s}}\right)^{1+\delta_{s}\left(1+\eta_{s}\right) / \varepsilon_{s}}
$$

where

$$
B_{j, s} \equiv \lambda_{j j, s}^{1 / \varepsilon_{s}}\left(\frac{R_{j, s}}{Y_{j}}\right)^{-\delta_{s} / \varepsilon_{s}}\left(\frac{E_{j, s}}{Y_{j}}\right)^{-\delta_{s} \eta_{s} / \varepsilon_{s}} Y_{j}^{\left(1-\alpha_{j, s}\right)\left(1+\delta_{s}\left(1+\eta_{s}\right) / \varepsilon_{s}\right)-\delta_{s}\left(1+\eta_{s}\right) / \varepsilon_{s}} \chi_{j j, s}^{-1 / \varepsilon_{s}}
$$

Taking logs yields

$$
\ln P_{j, s}=\ln B_{j, s}+\sum_{k} \tilde{\alpha}_{j, k s} \ln P_{j, k}
$$

where $\tilde{\alpha}_{j, k s} \equiv \alpha_{j, k s}\left(1+\delta_{s}\left(1+\eta_{s}\right) / \varepsilon_{s}\right)$. In matrix notation, this leads to

$$
\left(I d-\tilde{A}_{j}\right) \ln \mathbf{P}_{j}=\ln \mathbf{B}_{j}
$$

where $\tilde{A}_{j} \equiv\left\{\tilde{\alpha}_{j, s k}\right\}$, and where $\ln \mathbf{P}_{j}$ and $\ln \mathbf{B}_{j}$ are $S \times 1$ vectors with typical element $\ln P_{j, s}$ and $\ln B_{j, s}$, respectively. Inverting the previous system of equations, we obtain

$$
P_{j, s}=\prod_{k=1}^{S} B_{j, k}^{\tilde{a}_{j, s k}}
$$

where $\tilde{a}_{j, s k}$ is the $(s, k)$ entry of the matrix $\left(I-\tilde{A}_{j}\right)^{-1}$. One can check that

$$
\sum_{k=1}^{s}\left[\left(1-\alpha_{j, k}\right)\left(1+\delta_{k}\left(1+\eta_{k}\right) / \varepsilon_{k}\right)-\delta_{k}\left(1+\eta_{k}\right) / \varepsilon_{s}\right] \tilde{a}_{j, s k}=1
$$

which implies

$$
P_{j, s}=Y_{j} \prod_{k=1}^{S}\left(\lambda_{j j, k}^{-1}\left(\frac{R_{j, k}}{Y_{j}}\left(\frac{E_{j, k}}{Y_{j}}\right)^{\eta_{k}}\right)^{\delta_{k}} \chi_{j j, k}\right)^{-\frac{\tilde{j}_{j, k k}}{\varepsilon_{k}}}
$$

Under the assumption that upper-level utility functions are Cobb-Douglas, we get

$$
P_{j}=Y_{j} \prod_{s=1}^{S} \prod_{k=1}^{S}\left(\lambda_{j j, k}^{-1}\left(\frac{R_{j, k}}{Y_{j}}\left(\frac{E_{j, k}}{Y_{j}}\right)^{\eta_{k}}\right)^{\delta_{k}} \chi_{j j, k}\right)^{-\frac{\beta_{j, s} \tilde{\eta}_{j, s k}}{\varepsilon_{k}}}
$$

This implies that real consumption, $C_{j}=\left(Y_{j}+D_{j}+T_{j}\right) / P_{j}$, can be expressed as

$$
\begin{gathered}
C_{j}=\left(Y_{j}+D_{j}+\frac{\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}}{1+t_{i j, s}} \lambda_{i j, s}\left(\beta_{j, s}\left(Y_{j}+D_{j}\right)+\sum_{k=1}^{S} \alpha_{j, s k} R_{j, k}\right)}{1-\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}}{1+t_{i j, s}} \lambda_{i j, s} \beta_{j, s}}\right) \\
\times \frac{1}{Y_{j}} \prod_{s=1}^{S} \prod_{k=1}^{S}\left(\lambda_{j j, k}^{-1}\left(\frac{R_{j, k}}{Y_{j}}\left(\frac{E_{j, k}}{Y_{j}}\right)^{\eta_{k}}\right)^{\delta_{k}} \chi_{j j, k}\right)^{\frac{\beta_{j, s} \tilde{a}_{j, k}}{\varepsilon_{k}}}
\end{gathered} .
$$

Counterfactual Analysis. In Sections 4.1, 4.2, 4.3, and 5.1, we are interested in counterfactual changes in import tariffs from $\mathbf{t} \equiv\left\{t_{i j, s}\right\}$ to $\mathbf{t}^{\prime} \equiv\left\{t_{i j, s}^{\prime}\right\}$ as well as counterfactual changes in trade imbalances, which we model as changes from $\varkappa \equiv\left\{\varkappa_{j}\right\}$ to $\varkappa^{\prime} \equiv\left\{\varkappa_{j}^{\prime}\right\}$.

In order to analyze the consequences of such counterfactual changes, we can again use the exact hat algebra. The "hat" counterparts of Equations (7)-(12) are given by

$$
\begin{gather*}
\hat{c}_{i, s}=\hat{Y}_{i}^{1-\alpha_{s}} \prod_{k=1}^{S} \hat{P}_{i, k}^{\alpha_{i, k s}}  \tag{13}\\
\hat{P}_{i, k}=\left(\sum_{l=1}^{n} \lambda_{l i, k}\left(\frac{\hat{R}_{l, k}}{\hat{c}_{l, k}}\left(\frac{\hat{E}_{i, k}}{\hat{c}_{l, k}}\right)^{\eta_{k}}\right)^{\delta_{k}}\left(\hat{\phi}_{l i, k} \hat{c}_{l, k}\right)^{-\varepsilon_{k}}\right)^{-\frac{1}{\varepsilon_{k}}},  \tag{14}\\
\hat{\lambda}_{i j, s}=\frac{\left(\frac{\hat{R}_{i, s}}{\hat{c}_{i, s}}\right)^{\delta_{s}}\left(\hat{\phi}_{i j, s} \hat{c}_{i, s}\right)^{-\varepsilon_{s}} \hat{c}_{i, s}^{-\delta_{s} \eta_{s}}}{\sum_{l} \lambda_{l j, s}\left(\frac{\hat{R}_{l, s}}{\hat{c}_{l, s}}\right)^{\delta_{s}}\left(\hat{\phi}_{l j, s} \hat{c}_{l, s}\right)^{-\varepsilon_{s}} \hat{c}_{l, s}^{-\delta_{s} \eta_{s}}} \tag{15}
\end{gather*}
$$

$$
\begin{gather*}
\hat{E}_{j, s} E_{j, s}=\frac{\beta_{j, s}}{1-\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}^{\prime}}{1+t_{i j, s}^{\prime}} \hat{\lambda}_{i j, s} \lambda_{i j, s} \beta_{j, s}}  \tag{16}\\
\times\left(\hat{w}_{j} Y_{j}+\hat{\varkappa}_{j} \hat{Y}_{j}^{\mu} D_{j}+\sum_{i, l=1}^{n}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, l}^{\prime}}{1+t_{i j, l}^{\prime}} \hat{\lambda}_{i j, l} \lambda_{i j, l} \sum_{k=1}^{S} \alpha_{j, l k} \hat{R}_{j, k} R_{j, k}\right)+\sum_{k=1}^{S} \alpha_{j, s k} \hat{R}_{j, k} R_{j, k} \\
\hat{R}_{i, s} R_{i, s}=\sum_{j=1}^{n}\left(1+\frac{t_{i j, s}^{\prime}}{\sigma_{s}}\right)^{\delta_{s}} \frac{\hat{\lambda}_{i j, s} \lambda_{i j, s}}{1+t_{i j, s}^{\prime}} \hat{E}_{j, s} E_{j, s}  \tag{17}\\
\hat{Y}_{i} Y_{i}=\sum_{s=1}^{S}\left(1-\alpha_{i, s}\right) \hat{R}_{i, s} R_{i, s} \tag{18}
\end{gather*}
$$

In addition, our choice of numeraire implies

$$
\begin{equation*}
\sum_{i=1}^{n} \hat{Y}_{i} Y_{i}=1 \tag{19}
\end{equation*}
$$

Given data on the initial trade equilibrium, Equations (13)-(19) provide a system of $n \times$ $S+n \times S+n \times n \times S+n \times S+n \times S+n$ independent equations that can be solved for $n \times S+n \times S+n \times n \times S+n \times S+n \times S+n$ unknowns, $\left\{\hat{c}_{i, s}\right\},\left\{\hat{P}_{i, s}\right\},\left\{\hat{\lambda}_{i j, s}\right\},\left\{\hat{E}_{i, s}\right\},\left\{\hat{Y}_{i}\right\}$, and $\left\{\hat{R}_{i, s}\right\}$, as a function of the counterfactual changes in tariffs and trade imbalances.

Once changes in $\left\{\hat{c}_{i, s}\right\},\left\{\hat{P}_{i, s}\right\},\left\{\hat{\lambda}_{i j, s}\right\},\left\{\hat{E}_{i, s}\right\},\left\{\hat{Y}_{i}\right\}$, and $\left\{\hat{R}_{i, s}\right\}$ are known, changes in real consumption, $C_{j}=\left(Y_{j}+D_{j}+T_{j}\right) / P_{j}$, can be computed as

$$
\begin{equation*}
\hat{C}_{j}=\frac{\left(Y_{j}+\widehat{D_{j}}+T_{j}\right)}{\hat{Y}_{j}} \times \prod_{s=1}^{S} \prod_{k=1}^{S}\left(\hat{\lambda}_{j j, k}^{-1}\left(\frac{\hat{R}_{j, k}}{\hat{Y}_{j}}\left(\frac{\hat{E}_{j, k}}{\hat{Y}_{j}}\right)^{\eta_{k}}\right)^{\delta_{k}}\right)^{\frac{\beta_{j, s} \tilde{j}_{j, k k}}{\varepsilon_{k}}} \tag{20}
\end{equation*}
$$

where

$$
\begin{aligned}
& \left(Y_{j} \widehat{+D_{j}}+T_{j}\right)=\frac{1}{Y_{j}+D_{j}+T_{j}} \\
\times & \left(\hat{Y}_{j} Y_{j}+\hat{\varkappa}_{j} \hat{Y}_{j}^{\mu} D_{j}+\frac{\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{i j, s}^{\prime}}{1+t_{i j, s}^{\prime}} \hat{\lambda}_{i j, s} \lambda_{i j, s}\left(\beta_{j, s}\left(\hat{Y}_{j} Y_{j}+\hat{\varkappa}_{j} \hat{Y}_{j}^{\mu} D_{j}\right)+\sum_{k=1}^{S} \alpha_{j, s k} \hat{R}_{j, k} R_{j, k}\right)}{1-\sum_{i=1}^{n} \sum_{s=1}^{S}\left(\frac{\sigma_{s}-1}{\sigma_{s}}\right)^{\delta_{s}} \frac{t_{j i, s}^{\prime}}{1+t_{i j, s}^{\prime}} \hat{\lambda}_{i j, s} \lambda_{i j, s} \beta_{j, s}}\right)
\end{aligned}
$$

We are now ready to discuss how counterfactual results in Section 4.1, 4.2, 4.3, and 5.1 are constructed.

Throughout Section 4, we assume that there are no tariffs in the initial equilibrium, $t_{i j, s}=0$ for all $i, j$, and $s$. In Sections 4.1-4.3, we focus on counterfactual changes in import tariffs using alternative models that are all special cases of the model presented here. For each exercise, we first modify the WIOD data such that overall trade is balanced country-by-country. Formally, for each model, we first compute the counterfactual changes in $\left\{\hat{c}_{i, s}\right\},\left\{\hat{P}_{i, s}\right\},\left\{\hat{\lambda}_{i j, s}\right\},\left\{\hat{E}_{i, s}\right\},\left\{\hat{Y}_{i}\right\}$, and $\left\{\hat{R}_{i, s}\right\}$ associated with setting $\hat{\varkappa}_{j}=0$ for all $j$, i.e. removing all trade imbalances, by using Equations (13)-(19), and use the resulting dataset without trade imbalances to conduct the counterfactual tariff analysis.

Section 4.1 focuses on the one-sector Armington model: $\delta_{s}=0, \alpha_{j, s}=0$, and $s=$ 1. Section 4.2 extends the analysis to multi-sector models, $s>1$, intermediate goods, $\alpha_{j, s}>0$, and monopolistic competition, $\delta_{s}=1$. Section 4.3 considers the effects of import tariffs that are heterogeneous across sectors, i.e. $t_{i j, s}^{\prime}$ varies across sectors $s$. In all cases, starting from the equilibrium without trade imbalances, we first use Equations (13)-(19) to compute the changes in $\left\{\hat{c}_{i, s}\right\},\left\{\hat{P}_{i, s}\right\},\left\{\hat{\lambda}_{i j, s}\right\},\left\{\hat{E}_{i, s}\right\},\left\{\hat{Y}_{i}\right\}$, and $\left\{\hat{R}_{i, s}\right\}$ associated with a given counterfactual change in import tariffs from $\mathbf{t} \equiv\left\{t_{i j, s}\right\}$ to $\mathfrak{t}^{\prime} \equiv\left\{t_{i j, s}^{\prime}\right\}$. We then compute the associated change in real consumption using Equation (20). These are the welfare numbers reported in Figure 1, Figure 2, Table 2, and Table 3.

In Section 5.1, we again use the one-sector Armington model: $\delta_{s}=0, \alpha_{j, s}=0$, and $s=1$. Unlike in Section 4, we do not remove trade imbalances by setting $\hat{\varkappa}_{j}=0$ for all $j$. Rather we start from the trade equilibrium with trade imbalances, as observed in the WIOD, and directly compute the changes in $\left\{\hat{c}_{i, s}\right\},\left\{\hat{P}_{i, s}\right\},\left\{\hat{\lambda}_{i j, s}\right\},\left\{\hat{E}_{i, s}\right\},\left\{\hat{Y}_{i}\right\}$, and $\left\{\hat{R}_{i, s}\right\}$ associated with a given counterfactual change in import tariffs from $\mathbf{t} \equiv\left\{t_{i j, s}\right\}$ to $\mathbf{t}^{\prime} \equiv\left\{t_{i j, s}^{\prime}\right\}$ using Equations (13)-(19). We do so under the assumption that trade imbalances are proportional to domestic GDP, $\mu=1$, and under the assumption that trade imbalances are proportional to World GDP. The associated change in real consumption under each assumption is computed using Equation (20) and plotted in Figure 3.

## 4 Trade Imbalances, Tariff Revenues, and Gains from Trade

In Sections 2 and 3 in the text we have ignored trade imbalances and tariff revenues when computing the gains from trade. Using the exact hat algebra of the previous section, one can investigate how a move to autarky that also involves the elimination of trade imbalances and tariff revenues-i.e., $D_{j}^{\prime}=T_{j}^{\prime}=0$-would affect the magnitude of the gains from trade.

According to Equation (20), the change in real consumption associated with such a
counterfactual scenario would be given by

$$
\frac{C_{j}^{A}}{C_{j}}=\frac{Y_{j}}{Y_{j}+D_{j}+T_{j}} \prod_{s=1}^{S} \prod_{k=1}^{S}\left(\lambda_{j j, k}\left(\frac{R_{j, k}^{A} / R_{j, k}}{Y_{j}^{A} / Y_{j}}\left(\frac{E_{j, k}^{A} / E_{j, k}}{Y_{j}^{A} / Y_{j}}\right)^{\eta_{k}}\right)^{\delta_{k}}\right)^{\frac{\beta_{j, s} \tilde{\eta}_{j, s k}}{\varepsilon_{k}}}
$$

Letting $e_{j, k} \equiv E_{j, k} / E_{j}, r_{j, k} \equiv R_{j, k} / R_{j}, v_{j} \equiv Y_{j} / R_{j}$ and $\rho_{j} \equiv R_{j} / E_{j}$, and noting that $r_{j, k}^{A}=e_{j, k^{\prime}}^{A}$ we then have

$$
\frac{C_{j}^{A}}{C_{j}}=\frac{Y_{j}^{A}}{Y_{j}+D_{j}+T_{j}} \prod_{s, k}\left(\lambda_{j j, k}\left(\frac{r_{j, k}}{b_{j, k}}\left(\frac{e_{j, k}}{b_{j, k}} \rho_{j}\right)^{\eta_{k}}\right)^{-\delta_{k}}\right)^{\frac{\beta_{j, s} \tilde{a}_{j, k k}}{\varepsilon_{k}}}
$$

with

$$
b_{j, k} \equiv v_{j} e_{j, k}^{A} / v_{j}^{A}=v_{j} \sum_{l=1}^{S} \beta_{j, l} a_{j, k l}
$$

where the equality follows from Equation (6). In turn, we get

$$
\begin{equation*}
G_{j}=1-\frac{Y_{j}^{A}}{Y_{j}+D_{j}+T_{j}} \prod_{s, k}\left(\lambda_{j j, k}\left(\frac{r_{j, k}}{b_{j, k}}\left(\frac{e_{j, k}}{b_{j, k}} \rho_{j}\right)^{\eta_{k}}\right)^{-\delta_{k}}\right)^{\frac{\beta_{j, s} \tilde{a}_{j, s k}}{\varepsilon_{k}}} \tag{21}
\end{equation*}
$$

When trade imbalances and tariff revenues are ignored, i.e. $D_{j}=T_{j}=0$, and trade in goods is balanced, i.e., $R_{j}=E_{j}$, the previous expression simplifies into Equation (29) in the text.

Quantitatively, tariff revenues are a very small share of income in most countries (see footnote 25), so ignoring $T_{j}$ does not affect the magnitude of the gains from trade significantly. Ignoring trade imbalances, in contrast has sizable effects on the magnitude of the gains from trade through its direct effect on expenditure. However, it is not clear whether one should focus on real consumption or real income in the presence of trade imbalances. In a fully specified intertemporal model, trade deficits are not lump-sum transfers: they are paid for by future trade surpluses. If we focused on real income, $Y_{j} / P_{j}$, rather than real consumption, $C_{j}$, then the only effect of trade imbalances is through $\rho_{j}$. Quantitatively, the implications of this adjustment are minimal.

## 5 Modeling Import Tariffs

In footnote 29 we have discussed the difference between modeling import tariffs as "costshifters" and as "demand-shifters". The goal of this section is to discuss the sensitivity of our results to these considerations.

Most of our analysis in the text is conducted under the assumption that tariffs act as cost shifters, i.e., are imposed before markups. Under perfect competition, there are no markups. So this assumption is, of course, innocuous. Under monopolistic competition, the prices charged and quantities sold by a given firm in a given market are not affected by the way tariffs are modeled, but profits do depend on this modeling choice. To see this in a simple way, let $q_{j}(p)$ denote demand in market $j$ and consider a monopolistically competitive firm from $i$ exporting to $j$ with productivity $z$ and unit cost $c_{i} / z$, trade cost $\tau_{i j}$, and tariff $t_{i j}$. If tariffs are modeled as cost shifters then profits of such a firm are the same as profits with trade cost $\phi_{i j}$. In contrast, if tariffs are modeled as demand shifters, then profits are given by

$$
\pi(p)=\frac{p q_{j}(p)}{1+t_{i j}}-\frac{c_{i} \tau_{i j} q_{j}(p)}{z}
$$

The fact that $\left(1+t_{i j}\right)$ and $\tau_{i j}$ enter separately in this expression implies that entry and marketing decisions will be differently affected by tariffs and trade costs in this case. As a consequence, the expression for $P_{i j}$ in (14) now becomes

$$
P_{i j}=\phi_{i j} c_{i}^{p}\left(\left(\frac{E_{j}}{\left(1+t_{i j}\right) c_{i j}^{x}}\right)^{\frac{\delta}{1-\sigma}} \frac{\phi_{i j} c_{i}^{p}}{P_{j}}\right)^{\eta}\left(\frac{R_{i}}{c_{i}^{e}}\right)^{\frac{\delta}{1-\sigma}} \xi_{i j} .
$$

Under the assumption that $c_{i}^{p}=c_{i i}^{x}=c_{i}^{e}=Y_{i}$ and $R_{i}=Y_{i}$, the gravity equation in turn becomes

$$
\begin{equation*}
X_{i j}=\frac{\left(Y_{i} \phi_{i j}\right)^{-\varepsilon}\left(\left(1+t_{i j}\right) c_{i j}^{x}\right)^{-\delta \eta} \chi_{i j}}{\sum_{l=1}^{n}\left(Y_{l} \phi_{l j}\right)^{-\varepsilon}\left(\left(1+t_{l j}\right) c_{l j}^{x}\right)^{-\delta \eta} \chi_{l j}} E_{j} . \tag{22}
\end{equation*}
$$

Expressions for $\hat{C}_{j}$ and $G_{j}$ are not affected, so the modeling of tariffs does not affect welfare conditional on the trade elasticity $\varepsilon$ and $\eta$ as well as $\hat{\lambda}_{j j}$ and $T_{j}$. However, Equation (22) shows that the trade elasticity is now different depending on the nature of trade costs. The trade elasticity with respect to physical trade costs continues to be $\varepsilon$, but the trade elasticity with respect to tariffs is $\varepsilon+\delta \eta$. This, in principle, matters for how one calibrates a gravity model. For example, since Caliendo and Parro (2010) use variation in tariffs to estimate the trade elasticity, this would pin down different parameters depending on
whether $\delta \eta=0$ or $\delta \eta>0$. To be concrete, a trade elasticity of 5 obtained by Caliendo and Parro (2010) implies that $\varepsilon=5$ under perfect competition, but $\varepsilon+\eta=5$ under monopolistic competition if import tariffs act as demand-shifters rather than cost-shifters.

## 6 Data

All trade and input-output data used in our quantitative analysis are from the World Input-Output Database (WIOD) for the year 2008; see Timmer (2012).

The World Input-Output Database covers 41 regions: 40 countries and the Rest of the World. In our chapter we use two aggregation schemes. The first one, which we refer to as our basic aggregation, is used in all sections except Section 4.2. The basic aggregation is characterized by 34 regions of which 33 are large countries and the 34th region is an aggregate of the Rest of the World and the smallest countries in the original dataset. The basic aggregation is described in Table 1. The second aggregation scheme is a 10-region aggregation, which we use in Section 4.2. The 10-region aggregation represents an aggregation of neighboring countries into larger regional clusters, so that the ratio of output of the largest cluster ("North America") to the smallest ("Indian Ocean") is below 10. A detailed description of this aggregation scheme is provided in Table 2.

The World Input-Output Database covers 35 sectors. Due to differences in sector classifications across countries, some sectors in some countries are associated with both zero output and consumption. For example, the sector "Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel" for China and the sector "Leather, Leather and Footwear" for Sweden have zero output and zero consumption. To avoid the problems associated with zero output or zero consumption, while keeping the data as disaggregated as possible, we aggregated the data to 31 sectors, all with non-zero outputs and consumption in all countries. This is our basic sectoral aggregation, which we use in all sections except Section 4.2 . In this section, we instead aggregate to 16 sectors; see Table 3 for details. Table 3 also includes the trade elasticities used for each sector in the two aggregation schemes.

For each country and sector, we construct trade flows, final demand, and intermediate purchases as follows. The World Input-Output Database contains information about purchases, $X_{i j, k s}$, of intermediate goods from sector $k$ and country $i$ in sector $s$ and country $j$, as well as the corresponding purchases for private consumption, government spending, and investment, which we denote by $X_{i j, k C,} X_{i j, k G}$, and $X_{i j, k I}$, respectively. In addition, WIOD data contain information for changes in inventories, which we denote by $X_{i j, k Q}$. In some cases, this is negative, reflecting a decline in inventories. If we treated $X_{i j, k Q}$ as
a part of the final demand vector, this results in some entries in the final demand vector being negative. To avoid this situation we separate changes in inventories into two vectors: one with positive entries, $X_{i j, k Q^{\prime}}^{+}$and one with negative entries, $X_{i j, k Q^{\prime}}^{-}$with $X_{i j, k Q}=$ $X_{i j, k Q}^{+}+X_{i j, k Q}^{-}$. We deal with these two terms differently. We treat $X_{i j, k Q}^{+}$as part of final demand, which is denoted by $X_{i j, k F}$ and defined as $X_{i j, k F} \equiv X_{i j, k C}+X_{i j, k G}+X_{i j, k I}+X_{i j, k Q}^{+}$. The term $X_{i j, k F}$ is the empirical counterpart of consumption, $C_{i j, k}$, in our model. In contrast, we interpret $\left|X_{i j, k Q}^{-}\right|$in the data as output that was produced in the previous period, stored and consumed in the current period. To incorporate $X_{i j, k Q}^{-}$in our static model, we therefore adjust the total output vector and matrix of inputs flows as if $\left|X_{i j, k Q}^{-}\right|$were produced (and consumed) in the current period. ${ }^{1}$ Given the previous adjustments, we finally compute sector by sector trade flows as $X_{i j, s}=\sum_{k} X_{i j, k s}+X_{i j, k F}$, final demand by country and sector as $F_{j, k}=\sum_{i} X_{i j, k F}$, and intermediate purchases as $X_{j, k s}=\sum_{i} X_{i j, k s}$.

Once sector-level trade flows $\left\{X_{i j, k}\right\}$ have been constructed, we can compute expenditure and revenue across countries and sectors as follows. Aggregate expenditure is $E_{j}=\sum_{i, s} X_{i j, s}$, whereas the overall share of expenditure on domestic goods is $\lambda_{j j}=$ $\sum_{s} X_{j j, s} / \sum_{i, s} X_{i j, s}$. At the sector level, expenditure is $E_{j, s}=\sum_{i} X_{i j, s}$, whereas the share of expenditure on domestic goods is $\lambda_{j j, s}=X_{j j, s} / \sum_{i} X_{i j, s}$. Similarly, aggregate revenue, i.e., gross output, is $R_{j}=\sum_{s} \sum_{l} X_{j l, s}$ and sector-level revenue is $R_{j, s}=\sum_{l} X_{j l, s}$. The associated expenditure and revenue shares are then given by $e_{j, s}=E_{j, s} / E_{j}$ and $r_{j, s}=R_{j, s} / R_{j}$.

Using final demand $\left\{F_{j, k}\right\}$, we can also compute final demand shares $\beta_{j, k}=F_{j, k} / \sum_{s} F_{j, s}$. Finally, using intermediate purchases $\left\{X_{j, k s}\right\}$ and revenues $\left\{R_{j, s}\right\}$, we can compute the share of intermediate goods from sector $k$ used in sector $s$ and country $j, \alpha_{j, k s}=X_{j, k s} / R_{j, s}$, as well as value added by sector $Y_{j, s}=R_{j, s}-\sum_{k} X_{j, k s}$. In a number of our simulations in Sections 3.4 and 4.2, we use an alternative measure of shares of intermediate goods, $\alpha_{j, k s}^{*}=\left(\sum_{k} X_{j, k s} / R_{j, s}\right) \times\left(E_{j, k} / E_{j}\right)$. Whenever we do this, we use $E_{j, s} / E_{j}$ rather than $F_{j, s} / \sum_{k} F_{j, k}$ as the empirical counterpart of $\beta_{j, s}$ in the model. This is necessary for the model with shares $\alpha_{j, k s}^{*}$ to match the aggregate data on $X_{j, k s}$ and $Y_{j, s}$.

[^0]Table 1: Basic aggregation of regions

| Country's name | WIOD code | Basic aggregation |
| :---: | :---: | :---: |
| Australia | AUS | Australia |
| Austria | AUT | Austria |
| Belgium | BEL | Belgium |
| Brazil | BRA | Brazil |
| Canada | CAN | Canada |
| China | CHN | China |
| Czech Republic | CZE | Czech Republic |
| Germany | DEU | Germany |
| Denmark | DNK | Denmark |
| Spain | ESP | Spain |
| Finland | FIN | Finland |
| France | FRA | France |
| United Kingdom | GBR | United Kingdom |
| Greece | GRC | Greece |
| Hungary | HUN | Hungary |
| India | IDN | India |
| Indonesia | IND | Indonesia |
| Ireland | IRL | Ireland |
| Italy | ITA | Italy |
| Japan | JPN | Japan |
| Korea | KOR | Korea |
| Mexico | MEX | Mexico |
| Netherlands | NLD | Netherlands |
| Poland | POL | Poland |
| Portugal | PRT | Portugal |
| Romania | ROM | Romania |
| Russia | RUS | Russia |
| Slovakia | SVK | Slovakia |
| Slovenia | SVN | Slovenia |
| Sweden | SWE | Sweden |
| Turkey | TUR | Turkey |
| Taiwan | TWN | Taiwan |
| United States | USA | United States |
| Bulgaria | BGR |  |
| Cyprus | CYP |  |
| Estonia | EST |  |
| Latvia | LVA | Rest of the World |
| Lithuania | LTU | Rest of the World |
| Luximburg | LUX |  |
| Malta | MLT |  |
| Rest of the World | ROW |  |

Table 2: 10-region aggregation

| Country's name | WIOD code | 10-region aggregation |
| :--- | :--- | :--- |
| Australia | AUS |  |
| Japan | JPN | Pacific Ocean |
| Korea | KOR |  |
| Taiwan | TWN |  |
| Austria | AUT |  |
| Belgium | BEL |  |
| Germany | DEU | Western Europe |
| France | FRA |  |
| Luximburg | LUX |  |
| Netherlands | NLD |  |
| Bulgaria | BGR |  |
| Czech Republic | CZE |  |
| Estonia | EST |  |
| Hungary | HUN |  |
| Lithuania | LTU | Eastern Europe |
| Latvia | LVA |  |
| Poland | POL |  |
| Romania | ROM |  |
| Russia | RUS |  |
| Slovakia | SVK |  |
| Slovenia | SVN |  |
| Brazil | BRA | Latin America |
| Mexico | MEX |  |
| Canada | CAN | North America |
| United States | USA | China |
| China | CHN |  |
| Cyprus | CYP |  |
| Spain | ESP |  |
| Greece | GRC | Southern Europe |
| Italy | ITA |  |
| Malta | MLT |  |
| Portugal | PRT |  |
| Turkey | TUR |  |
| Denmark | DNK |  |
| Finland | FIN | Northern Europe |
| United Kingdom | GBR |  |
| Ireland | IRL |  |
| Sweden | SWE |  |
| India |  |  |
| Indonesia | RND |  |
| Rest of the World | ROW |  |

Table 3: Sectoral aggregations
WIOD Sector's description sector

| 1 |
| :--- |
| 2 |
| 3 |
| 4 |
| 5 |
| 6 |
| 7 |
| 8 |
| 9 |
| 10 |
| 11 |
| 12 |
| 13 |
| 14 |
| 15 |


| 14 | Electrical and Optical Equipment |
| :--- | :--- |
| 15 | Transport Equipment |

Manufacturing, Nec; Recycling
Electricity, Gas and Water Supply
Construction
Sale, Maintenance and Repair of Motor Vehicles and Motorcycles; Retail Sale of Fuel
Roolesale Trade and Commission Trade, Except of Motor Vehicles and Motorcycles
Hotels and Restaurants
Inland Transport
Water Transport
Other Supporting and Auxiliary Transport Activities; Activities of Travel Agencies
Post and Telecommunications
Financial Intermediation
Renting of M\&Eq and Other Business Activities
Education
Health and Social Work
Public Admin and Defence; Compulsory Social Security
Other Community, Social and Personal Services
Private Households with Employed Persons
16
8



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[^0]:    ${ }^{1}$ Formally, let's consider the matrix equation for the output and consumption balance:

    $$
    X=A X+F+I n v,
    $$

    where $X$ is the $(N \cdot S) \times 1$ vector of total output, $A$ is the $(N \cdot S) \times(N \cdot S)$ matrix of direct input coefficients, $F$ is $(N \cdot S) \times 1$ vector of final demand (which includes increases in inventories) and $I n v$ is the $(N \cdot S) \times 1$ vector of negative changes in inventories. One can express total output in the current period as $X=$ $(I-A)^{-1}(F+I n v)$. Now, setting all entries of the vector $I n v$ equal to zero we can compute the modified vector of total output as $\tilde{X}=(I-A)^{-1} F$, while the modified matrix of flows of intermediate goods between sectors and regions becomes $A \tilde{X}$. The vector of final demand $F$ remains unmodified, and we have $\tilde{X}=$ $A \tilde{X}+F$.

