SMALL SAMPLE BIAS AND ADJUSTMENT COSTS

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Abstract—The response of most stock variables (e.g., capital, housing, consumer durables, and prices) to exogenous impulses involves a dynamic—or "short-run"—reaction, and a target—or "long-run"—reaction. The difference between these two is typically attributed to some form of adjustment cost. In this paper I argue that the small sample problems of cointegrating procedures used to estimate the "long"-run component are particularly severe when adjustment costs are important. More precisely, elasticity estimates will tend to be biased downward. I illustrate the empirical relevance of this by showing that the target elasticity of capital with respect to its cost is severely *downward* biased when estimated with conventional OLS cointegration procedures. Once this is corrected, the elasticity of the U.S. capital–output ratio to the cost of capital is found to be *large* and close to (minus) one.

THE response of most stock variables (e.g., capital, housing, consumer durables, and prices) to exogenous impulses involves a dynamic —or "short run"—reaction, and a target—or "long run"—reaction.[1] The difference between these two is typically attributed to some form of adjustment cost. In this paper I concentrate on the small sample problems of estimating the target relationship. In particular, I focus on the estimation of the relation between the stock of capital and its cost.

Most theories founded on microeconomic arguments suggest a strong relationship between the cost of capital and investment. With few exceptions (e.g., Feldstein, 1982, and Auerbach and Hassett, 1990), the empirical evidence has typically indicated the opposite. After an extensive review of academic and non-academic research and discussions, Shapiro (1986) calls this finding, or the lack of it, "an embarrassment of neoclassical theory." And Blanchard (1986) writes " . . . it is well known that to get the user cost to appear at all in the investment equation, one has to display more than the usual amount of economet-

ric ingenuity, resorting most of the time to choosing a specification that simply forces the effect to be there."

That discussion, however, refers mainly to the dynamic relationship between capital and its cost. Before disposing of neoclassical theory, one would like to know whether the target relationship is consistent with the basic implication of this theory or not. After all, the dynamic relationship is likely to be cluttered by adjustment costs of many sorts, and for many policy questions "long-run" responses are at least as important as short-run effects. The natural way to estimate these target relationships is with cointegration methods. In this paper I argue that the often disregarded small sample biases of OLS-cointegration methods are particularly harmful in any model in which adjustment costs are important. In particular, I show that the estimates of the long-run response of capital to changes in its cost are severely *downward* biased.

Econometricians have recently developed methods with better small sample properties.[2] In this paper I use one of these methods—due to Stock and Watson (1993)—and find that after the biases are reduced, the target elasticity of the stock of capital to its cost is approximately unity; large relative to most estimates.

This introduction is followed by three sections. Section I shows the downward small sample bias inherent in conventional cointegrating equations when adjustment costs are important, and then presents results for the U.S. long-run demand for capital. Section II supports the claims of section I with Monte Carlo evidence and explores the robustness of the results to variations in the properties of the regressors and to measurement error. Section III concludes.

I. Small Sample Bias

Let K_t^*, the "frictionless" or "desired" stock of capital at time t, be a fixed linear function of an observable random walk vector X_t (more gen-

[1] My frequent use of the concept of *target* relationship instead of the *long-run* relationship is aimed at including the case in which the cost of capital is stationary.

[2] See, e.g., Banerjee et al. (1986), Johansen (1988), Hansen and Philips (1988), Philips and Hansen (1989), Stock and Watson (1993), Saikkonen (1991), Phillips and Loretan (1991).

[52]

eral processes are discussed later):

$$K_t^* = \alpha X_t,$$

where α is a vector of constants. This is the *target* (and in this case, the *long-run*) relationship between the stock of capital and the variables in X.

Due to adjustment costs, the observed stock of capital differs from the desired stock of capital. Defining the discrepancy between the actual stock of capital, K_t, and its frictionless counterpart by Z_t, $(Z_t \equiv K_t - K_t^*)$, it is straightforward to see that

$$K_t \equiv K_t^* + Z_t \tag{1}$$

or

$$K_t = \alpha X_t + Z_t. \tag{2}$$

Adjustment costs models in general imply that Z_t is stationary; it follows from the theory of cointegrated processes that the vector α can be estimated consistently by ordinary least squares (OLS). However, one would like to estimate α not only consistently, but also with an "acceptable" degree of small sample bias. This, OLS will not do; the same adjustment cost theory that justifies the use of cointegrated analysis also implies that K_t^* and Z_t will be strongly negatively correlated. The small sample implication of this correlation is that OLS estimates of α are biased towards zero;[3] this problem becomes more severe as the importance of adjustment costs rises.

Heuristically, the problem can be described as follows: Economic theory implies that in general, in any given sample the frictionless stock of capital fluctuates more than the actual stock of capital; thus, equation (1) can only be satisfied if K^* and Z are negatively correlated. However, the normal equations of OLS determine that in the same sample, the estimates of both quantities, $\hat{K}^*$ and $\hat{Z}$, are orthogonal, which implies that the variance of $\hat{K}^*$ is less than the variance of K. OLS accomplishes this by biasing the estimate of α towards zero.

To see this more rigorously, note that most slow adjustment models—whether they emphasize convex adjustment costs, aggregation of non-homogeneous units facing irreversibility constraints, or lumpiness—imply a relationship between the actual and frictionless stock of capi-

tal of the form:

$$K_t = \frac{1}{\theta_t(1)} \theta_t(L) K_t^*.$$

To simplify the exposition, I suppress the coefficients' time dependence; however, no important argument hinges on this simplification. It is also the case that except for some infrequent "echo effects," it is not too restrictive to assume that $\theta_i \geq 0$ for all i.

Simple algebraic steps (see the appendix) yield:

$$Z_t = -\phi(L)\Delta K_t^*, \tag{3}$$

with $\phi_j \equiv (\sum_{i=j+1}^{\infty} \theta_i)/\theta(1)$, $\Delta K_t^* = K_t^* - K_{t-1}^*$, and $\sum_j \phi_j^2 < \infty$.

To avoid a burdensome notation, I illustrate the main issues of this subsection through the simplest possible partial adjustment model:

$$I_t = \lambda(K_t^* - K_{t-1}),$$

where $I_t \equiv K_t - K_{t-1}$.[4] In this framework $\phi_j = (1 - \lambda)^{1-j}$ for $j = 0, 1, \ldots, \infty$.

Suppose for a moment that the sequence $\{K_t^*\}_{t=0}^T$ is actually observed; the question is then how far from one is the estimated value of γ in the simple linear regression:

$$K_t = \gamma K_t^* + Z_t. \tag{4}$$

Letting $\hat{\gamma}$ denote the OLS estimator, yields:

$$\hat{\gamma} - 1 = \left(\frac{K^{*\prime}K^*}{T}\right)^{-1} \frac{K^{*\prime}Z}{T},$$

and consistency follows immediately, because $(K^{*\prime}K^*/T)$ goes to infinity as the sample size approaches infinity, while $(K^{*\prime}Z/T)$ remains bounded. More importantly, it is clear that the OLS estimator is biased since K^* and Z are correlated, both contemporaneously and at lags. In the next section I compute the small sample bias through Monte Carlo experiments; for expository purposes, I now report a simpler index of the bias, which approximates the expectation of a ratio by the ratio of the expectations:

$$B(T, \lambda) \equiv \frac{E_0[K^{*\prime}Z]}{E_0[K^{*\prime}K^*]},$$

where T is the number of observations and $E_0[\cdot]$

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[3] Obviously, the bias of individual coefficients in α need not be downward, if α contains more than one element.

[4] Note that if the model is expressed in logs, I_t does not represent net investment but the log ratio of current to lagged capital.

FIGURE 1.—SMALL SAMPLE BIAS MEASURE
STATIC OLS
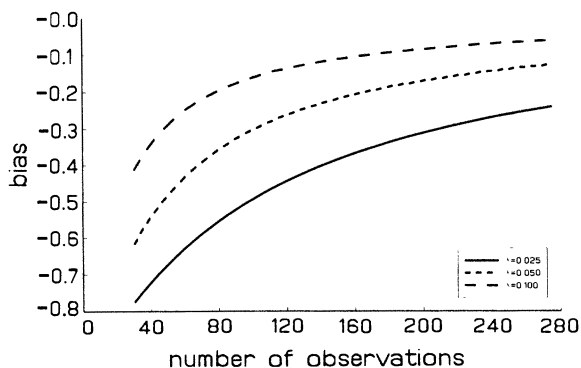
FIGURE 2.—SMALL SAMPLE BIAS MEASURE
DYNAMIC OLS
$T = 120$
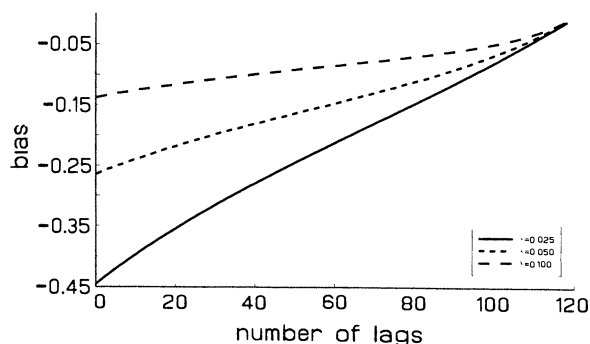
denotes the expectation conditional on the K^*'s and Z's observed up to time 0. Simple algebra shows that, for the example presented, this expression reduces to:

$$B(T, \lambda) = -\left(\frac{2}{T+1}\right)\left(\frac{1-\lambda}{\lambda}\right)$$
$$\times \left\{1 - \left(\frac{1-\lambda}{\lambda}\right)\left(\frac{1-(1-\lambda)^T}{T}\right)\right\}.$$

Figure 1 plots this function against the number of observations for three different values of the adjustment costs parameter, λ. It is apparent from this figure that the downward small sample bias can be substantial, especially as adjustment costs become more important (λ falls).

Stock and Watson (1993) provides a simple procedure to reduce small sample biases in cointegrated systems; they notice that by adding leads and lags of the first difference of the integrated regressor (here K^*) on the right hand side of equation (4), the small sample bias is reduced as these new regressors capture part of the correlation between K^* and Z. An operational problem of this procedure, however, is that for variables with strong dynamic behavior, as is the case of the stock of capital, running out of degrees of freedom is a serious consideration. I partially circumvent this problem by noticing that if the source of the dynamic relationship is adjustment costs, then lags are likely to contribute substantially more than leads; i.e., adjustment costs are likely to dominate feedbacks from investment to the cost of capital. Later I confirm this conjecture in the context of my investment example.

The question now becomes: how far from one is the estimate of γ in equation (6) below?

$$K_t = \gamma K_t^* + \sum_{i=0}^{N-1} \beta_i \Delta K_{t-i}^* + \tilde{Z}_t, \qquad (6)$$

where N is the number of lags included (counting the contemporaneous first difference), the β_i are coefficients to be estimated, and $\tilde{Z}_t = Z_t - \sum_{i=0}^{N-1} \beta_i \Delta K_{t-i}^*$. Defining the bias function as before, it is possible to show that

$$B(T, N, \lambda) = -\left(\frac{2}{T+N+1}\right)\left(\frac{1-\lambda}{\lambda}\right)$$
$$\times \left\{1 - \left(\frac{1-\lambda}{\lambda}\right)\right.$$
$$\left. \times \left(\frac{1-(1-\lambda)^{T-N}}{T-N}\right)\right\}.$$

Figure 2 plots this function against the number of lags, for $T = 120$ and different values of λ. Clearly the small sample bias is reduced as more lags are added.

A. Results

Since the frictionless stock of capital is not directly observable, some theoretical restrictions must be imposed in order to make this concept operational. Many alternative models (e.g., the simplest neoclassical framework) yield an equation for the capital/output ratio of the form:

$$K_t^* - Y_t = \alpha_0 + \alpha_r R_t, \qquad (7)$$

where all the variables are in logs, Y_t is some measure of aggregate value added, and R_t is an

TABLE 1.—TARGET COST OF CAPITAL ELASTICITY

N (LAGS)	0	5	9	13	17	21	25	29
α_r	-0.404	-0.547	-0.626	-0.686	-0.751	-0.814	-0.870	-0.933
	(0.099)	(0.098)	(0.091)	(0.085)	(0.080)	(0.080)	(0.075)	(0.068)
AIC	0.814	0.769	0.765	0.750	0.673	0.460	0.306	0.063
SIC	0.863	0.940	1.034	1.116	1.137	1.022	0.965	0.819

Note: All equations include a constant. Autocorrelation-consistent standard errors in parentheses. Quarterly data 1957:1–87:4. AIC: Akaike information criterion. SIC: Schwarz information criterion.

TABLE 2.—TARGET COST OF CAPITAL ELASTICITY

N (LEADS)	0	5	9	13	17	21	25	29
α_r	-0.404	-0.470	-0.487	-0.488	-0.494	-0.487	-0.470	-0.460
	(0.099)	(0.096)	(0.086)	(0.072)	(0.062)	(0.074)	(0.074)	(0.071)
AIC	0.814	0.758	0.795	0.834	0.900	0.940	0.991	1.057
SIC	0.863	0.929	1.064	1.200	1.367	1.487	1.650	1.814

Note: All equations include a constant. Autocorrelation-consistent standard errors in parentheses. Quarterly data 1957:1–87:4. AIC: Akaike information criterion. SIC: Schwarz information criterion.

index of the cost of capital (the data are fully described in the appendix). The conventional cointegrating equation (static OLS) is

$$K_t - Y_t = \alpha_0 + \alpha_r R_t + Z_t, \qquad (8)$$

where K_t is the log of the stock of capital (equipment). The results for the United States during the period 1957:1–1987:4 are presented in table 1. The pattern is clearly consistent with the discussion above (see figure 2), suggesting that the small bias of the conventional cointegration equation is large and that the correction procedure is quite effective; bringing about a large elasticity of the target stock of capital to changes in its cost. In selecting the number of lags, the Akaike criterion (AIC) seems to perform better than the more conservative Schwarz criterion (SIC).[5]

Table 2 is the analogue of table 1 but leads replace lags. It is clear from comparing these tables that, for investment, leads are not as efficient as lags in correcting the small sample bias problem. My conjecture is that this is likely to happen whenever adjustment costs are large.

II. Monte Carlo Evidence and Robustness

Here I report Monte Carlo evidence in support of the claims in the previous section, and discuss the consequences of stationary regressors and measurement error.

[5] Except at very long lags, where both provide the right answer.

A. Monte Carlo Evidence

In a finite sample, whether the regressors are stationary or not is not the appropriate question to determine the expected discrepancy between estimates and truth; what matters is the size of the regressors' variance relative to their covariance with the residual. Furthermore, if adjustment costs are important, Z_t is likely to have strong serial correlation; thus cointegration tests between the capital/output ratio and the cost of capital are unlikely to be very informative either. It is not surprising, then, that the main points and results of this paper do not depend on the integrated nature of the regressors. To illustrate this, I will use two representations (estimated) of the joint U.S. output–cost of capital process.

I will start by assuming that the cost of capital is non-stationary, and estimate the joint system:[6]

$$\Delta Y_t = 0.33\Delta Y_{t-1} + e_{y,t}$$

$$\Delta R_t = 1.65\Delta Y_{t-1} - 0.21\Delta R_{t-2} + e_{r,t}$$

with variance covariance of the innovations, Σ:

$$\Sigma = \begin{bmatrix} 0.0001 & 0.0002 \\ 0.0002 & 0.0078 \end{bmatrix}.$$

Using this representation I generate 1,000 random samples. On each sample I estimate the static (conventional cointegration) and dynamic

[6] Under the maintained assumption that Y and R are non-stationary, the no-cointegration null between them cannot be rejected. Initially, I included two lags of ΔY and ΔR on each equation; the equations reported and finally used contain only those coefficients that are significant.

TABLE 3.—MONTE CARLO RESULTS ($\alpha_r = -1$)

	$\lambda = 0.025$				$\lambda = 0.200$			
	$N = 0$	$N = 5$	$N = 15$	$N = 25$	$N = 0$	$N = 5$	$N = 15$	$N = 25$
$\hat{\alpha}_r$	-0.56	-0.64	-0.76	-0.83	-0.82	-0.93	-0.99	-1.00
$\sigma_{\hat{\alpha}_r}$	0.23	0.21	0.18	0.17	0.12	0.06	0.03	0.04
Rejections	0.76	0.70	0.65	0.58	0.55	0.39	0.16	0.09

Note: $T = 120$, Replications = 1,000.
All equations include a constant. Rejections: Fraction of rejections of the true parameters (one-sided tests with nominal size equal to 5%). The covariance matrices are autocorrelation consistent, estimated using an AR(1) approximation to the errors.

—with $N = 5$, $N = 15$ and $N = 25$—OLS regressions, under the maintained assumption that the capital stock is described by the simple partial adjustment model (with different values of λ), and $\alpha_r = -1.0$. Each sample contains 200 observation but only the last 120 are used in estimation (thus 95 usable observations are left when $N = 25$; about the number of observations used in constructing the U.S. estimates reported in the previous tables. The results are presented in table 3. In descending order, the rows show the average α_r estimated, the standard deviation of the series of estimated coefficients, and the fraction of times in which the one sided test against downward biased (in absolute value) values is rejected with a nominal size of 5%.[7]

In spite of the restrictive nature of the adjustment cost model used to generate the data, the pattern of the estimates has the same shape as the actual estimates obtained in the previous section. Table 3 also shows that if adjustment costs are large (i.e., changes in $Z(t)$ are very persistent) there is still an important amount of downward bias left even after 25 lags are used.

B. Stationarity

In the limit, if some components of the vector X_t in equation (2) are stationary, their coefficients cannot be identified. In practice, however, samples are unfortunately finite and therefore, as said before, stationarity is not the relevant concept; instead, what matters is the variance of the regressors relative to their covariance with the residual in the equation. Thus, if the unit root assumption on the cost of capital variable is relaxed and its stationary representation is used instead, the small sample problem is increased

(for both the static and dynamic OLS representations) but not changed in any fundamental way.[8]

In this case, the estimated joint process is

$$\Delta Y_t = 0.33 \Delta Y_{t-1} + e_{y,t}$$
$$R_t = 2.10 \Delta Y_{t-1} + 0.85 R_{t-1} + e_{r,t}$$

with variance-covariance:

$$\Sigma = \begin{bmatrix} 0.0001 & 0.0002 \\ 0.0002 & 0.0073 \end{bmatrix}.$$

Generating data as above (1,000 replications) I obtain the results reported in table 4. Not surprisingly, the downward bias of the static OLS procedure is increased in this case. Interestingly, however, the lag-correction is also more effective in removing the bias than when the cost of capital is taken as non-stationary. The explanation for this lies in the fact that the correlation between R_t and Z_t is also reduced, since lagged changes in R have less impact on today's R.

C. Is the Increasing Coefficients Pattern an Artifact of the Procedure?

One may ask whether the increasing (in absolute value) feature of the coefficients as the number of (difference) lags rises is an artifact of the procedure. For this, instead of adding lagged values of ΔK^*, I successively add stationary components uncorrelated with ΔK^* (with variance equal to that of the ΔK^*). Table 5 shows that in this case the estimates of the coefficient α_r show no particular pattern, suggesting that the results presented above are not just the consequence of a bias built into the procedure.

D. Measurement Error

Here I add noise to the right hand side of the partial adjustment equation (i.e., it has the same

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[7] The standard errors are autocorrelation-consistent, with an AR(1) approximation.

[8] Of course, identification is not literally achieved unless more structure is given to the disturbance.

TABLE 4.—STATIONARY COST OF CAPITAL

	$\lambda = 0.025$				$\lambda = 0.200$			
	$N = 0$	$N = 5$	$N = 15$	$N = 25$	$N = 0$	$N = 5$	$N = 15$	$N = 25$
$\hat{\alpha}_r$	-0.40	-0.57	-0.77	-0.90	-0.65	-0.88	-1.00	-1.02
$\sigma_{\hat{\alpha}_r}$	0.16	0.17	0.18	0.21	0.11	0.07	0.05	0.07
Rejections	0.96	0.76	0.47	0.27	0.94	0.52	0.04	0.034

$T = 120$, Replications = 1,000.
All equations include a constant. Rejections: Fraction of rejections of the true parameters (one sided tests with nominal size equal to 5%). The covariance matrices are autocorrelation consistent, estimated using an AR(1) approximation to the errors.

TABLE 5.—NOISY CORRECTION

N	0	5	9	13	17	21	25	29
α_r	-0.404	-0.401	-0.405	-0.398	-0.397	-0.394	-0.394	-0.387
	(0.100)	(0.100)	(0.102)	(0.108)	(0.123)	(0.129)	(0.120)	(0.117)
AIC	0.814	0.859	0.879	0.899	0.930	0.981	0.994	1.009
SIC	0.863	1.030	1.146	1.263	1.391	1.539	1.650	1.761

Note: All equations include a constant. The covariance matrices are autocorrelation consistent, estimated using an AR(1) approximation to the errors. Quarterly data (57:1–87:4).

TABLE 6.—MEASUREMENT ERROR

	$\lambda = 0.025$				$\lambda = 0.200$			
	$N = 0$	$N = 5$	$N = 15$	$N = 25$	$N = 0$	$N = 5$	$N = 15$	$N = 25$
$\hat{\alpha}_r$	-0.57	-0.65	-0.76	-0.83	-0.82	-0.93	-0.98	-0.99
$\sigma_{\hat{\alpha}_r}$	0.23	0.21	0.18	0.17	0.12	0.06	0.03	0.04
Rejections	0.73	0.67	0.62	0.589	0.53	0.40	0.14	0.09

Note: Noise added: $I_t = \lambda(K_t^* - K_{t-1}) + 0.2\sigma_{\Delta Y} e_n$ with $e_n \sim N(0,1)$. $T = 120$, Replications = 1,000.
All equations include a constant. Rejections: Fraction of rejections of the true parameters (one-sided tests with nominal size equal to 5%). The covariance matrices are autocorrelation consistent, estimated using an AR(1) approximation to the errors.

serial correlation implications for the stock of capital than innovations in K^*) and recompute the results. The standard deviation of the noise is equal to 20% of that in ΔK^*. Comparing tables 3 and 6 shows that reasonable amounts of measurement error leave the main results virtually unchanged.

III. Conclusion

Econometricians have noticed for some time that cointegration relationships estimated with OLS are subject to small sample biases. In this short paper I have pointed out that this problem is particularly severe when adjustment costs are important. Since this is the case for most stock variables macroeconomists care about, I argue that these biases should be taken more seriously.

As an example, I have shown that correcting these biases increases the estimate of the elastic-

ity of capital with respect to its cost by a substantial amount.

APPENDIX

A. Derivation of Z

Using the formulae in the paper it is possible to write:[9]

$$Z_t = \frac{1}{\theta(1)}\theta(L)K_t^* - K_t^*$$

$$= \frac{1}{\theta(1)}[\theta(L)K_t^* - \theta(1)K_t^*]$$

$$= -\frac{1}{\theta(1)}\sum_{t=0}^{\infty}\theta_t(K_t^* - K_{t-t}^*).$$

But

$$K_t^* = \sum_{j=0}^{i-1}\Delta K_{t-j}^* + K_{t-i}^*$$

[9] See Beveridge and Nelson (1981).

hence

$$Z_t = -\frac{1}{\theta(1)} \sum_{i=1}^{\infty} \theta_i \sum_{j=0}^{i-1} \Delta K_{t-j}^* = -\frac{1}{\theta(1)} \sum_{j=0}^{\infty} \Delta K_{t-j}^* \sum_{i=j+1}^{\infty} \theta_i.$$

Defining

$$\phi_j = \frac{1}{\theta(1)} \sum_{i=j+1}^{\infty} \theta_i,$$

yields equation (3).

B. Data

The data are quarterly observations from 1957:1 to 1987:4. Capital corresponds to the fixed private equipment stock, and is constructed from the investment series of NIPA taking the starting value from the Department of Commerce of the Bureau of Economic Analysis capital stock series. Depreciation is assumed to be exponential and equal to 0.13 per year.

Output is real GNP from NIPA and the cost of capital is constructed along the lines of Auerbach and Hassett (1990). Two differences from theirs are that (i) I use the 3-month Treasury bill rate (using the dividend yield gives almost identical results), and (ii) I project the real perfect foresight (see explanation below) return on three lags of inflation (measured as the rate of change in the GNP deflator), the corporate tax rate and nominal T-bill rate.

The basic cost of capital series (before projection) is then:

$$R_t' = (r_t + \delta + 0.02)\frac{(1 - \Gamma_t)}{(1 - \tau_t)}\frac{P_{kt}}{P_t}$$

where $R_t' = e^{R_t}$, r_t is the T-bill rate minus the rate of change of the price of new capital, δ is the depreciation rate, Γ_t is a measure of the prefect foresight present value of tax credits (see Auerbach and Hassett, 1990), τ_t is the corporate tax rate, P_{kt} is the price of new capital (equipment investment deflator) and P_t is the GNP deflator. The 0.02 corresponds to an arbitrary risk premium.

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