Target Zones and Realignments

By Giuseppe Bertola and Ricardo J. Caballero*

Recent contributions emphasize that the presence of exchange-rate target zones has important effects on the within-band behavior of exchange rates when agents are forward-looking. We find that the implications of available models are inconsistent with European exchange-rate data, and we suggest that the frequent realignments occurring in the period we consider may be responsible for this. We construct a model in which the likelihood of a realignment in the near future increases as the exchange rate approaches the limits of its fluctuation band and show that its implications are broadly consistent with the evidence. (JEL F31, F33)

The literature on stochastic models of target zones initiated by Paul Krugman (1991) has introduced new techniques to the economic literature and has familiarized readers with interesting and quite general insights. Yet, the positive and normative implications of these models remain largely unexplored. On the positive side, the new techniques could help interpret the historical performance of exchange-rate regimes, from the gold standard, to Bretton Woods, to the Exchange Rate Mechanism (ERM) of the European Monetary System. This motivates our work in this paper. Following Lars E. O. Svensson (1991b), we explore the implications of these models for the conditional and unconditional behavior of observable quantities (exchange rates and interest rates) and argue that these implications are fairly robust to the details of the model's specification. Like Gordon Bodnar (1989), we find that data from the early period of ERM operations are strikingly inconsistent with models of fully credible exchange-rate

bands and that frequent realignments had a primary role in determining the behavior of exchange rates within the ERM bands in that period.

We proceed to develop a formal model of discrete, repeated realignments.[1] Though simple and based on ad hoc assumptions, the class of models we propose appears to be flexible enough to allow an interpretation of real-world data in terms of a few intuitive parameters. The characterization of ERM-like policy regimes that emerges from the data is quite different from that implied by stylized models à la Krugman, and we argue in the conclusions that our results should be taken into account in further theoretical work on nominal-exchange-rate regimes.

The empirical implications of target-zone models are summarized in Section I, and Section II finds them to be inconsistent with French and German exchange- and

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[1]As opposed to the one-time collapse of an exchange-rate regime studied in Willem H. Buiter and Vittorio U. Grilli (1989), Francisco Delgado and Bernard Dumas (1990), Kenneth A. Froot and Maurice Obstfeld (1991), and Krugman (1991), Marcus H. Miller and Paul A. Weller (1989) analyze realignments in the context of a stochastic Dornbusch model and independently derive some of the implications discussed in the present paper. The empirical evidence presented here can be interpreted in light of Miller and Weller's qualitative results as well as in terms of our formal analysis.

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interest-rate data from the 1979–1987 period. A simple model of stochastic realignments is introduced in Section III and found to be broadly consistent with the qualitative features of ERM data. Other elements of realism are introduced in Section IV, and Section V concludes by discussing the relevance of our results and speculating on future extensions.

I. Band-Policy Regimes and Observable Variables

We denote with $\{x_t\}$ the log exchange-rate process and follow the literature in assuming that it satisfies an asset-pricing relationship:

$$(1) \qquad x_t = f_t + \frac{\alpha}{dt} E_t \{ dx_t \}.$$

Here, $E_t\{\cdot\}$ denotes an expectation taken conditionally on the information available at time t, and $\{f_t\}$ is the *fundamental* determinant of exchange rates. The interpretation given to f_t will not be crucial for most of what follows, and it depends, of course, on the specific model of exchange-rate determination one has in mind. It has become customary to interpret it as the stochastically varying log-quantity of money in an otherwise static monetary model of exchange-rate determination; with this interpretation, the parameter α represents the semielasticity of money demand to interest rates. However, the only crucial assumption in (1) is that exchange rates depend both on current observable variables, summarized by the indicator f, and on their own expected rate of change, with α indexing the importance of the latter effect. This feature is largely uncontroversial: as long as money can be used as a store of value, its price should reflect expectations of future events if it is determined by market interaction between forward-looking agents.

Formal assumptions about the exogenous behavior of f are needed to obtain quantitative inferences from the model's solution. We shall assume that *in the absence of intervention* fundamentals follow a Brownian-motion process,

$$(2) \qquad df_t = \vartheta \, dt + \sigma \, dW_t.$$

Many results derived below would be qualitatively correct for much more general processes, and we shall note when this is the case.

Equation (1) simply requires that the $\{x_t\}$ path be self-consistent and consistent with the stochastic characteristics of the $\{f_t\}$ process. For the process in (2) and a large class of similar continuous-time processes, it can be shown that the convergent solution to the forward-looking equation (1) has the form of an expected discounted integral of future $\{f_t\}$ realizations:

$$(3) \qquad x_t = \frac{1}{\alpha} \int_t^\infty E_t \{ f_\tau \} e^{-(\tau - t)/\alpha} \, d\tau$$

where $E_t\{\cdot\}$ denotes an expectation conditional on the relevant information available at time t. If the dynamics of $\{f_t\}$ were always given by (2), then we would have $E\{f_\tau | f_t\} = f_t + \vartheta(\tau - t)$ for all t, all $\tau \geq t$, and integrating (3),

$$(4) \qquad x_t = f_t + \alpha \vartheta.$$

By (4), exchange rates follow dynamics similar to those in (2) if (2) applies at all times. In a way, then, calling f the *fundamental* and assuming the (sub-, super-) martingale dynamics in (2) is a matter of definition, not of substance. The assumptions in (1) and (2) provide an agnostic starting point for a fairly general analysis of exchange-rate regimes, if we can be convinced that nominal exchange rates would follow (log) random walks in the absence of any official concern about their levels and variability—when they would be determined by, for example, the authorities' desire to raise seigniorage revenue.

In general, exchange-rate targeting could be modeled in terms of the drift and standard deviation parameters of the $\{f_t\}$ process. Recent contributions, however, have

modeled exchange-rate intervention in non-linear fashion. The authorities' concern for nominal-exchange-rate variability is assumed to take the form of a *fluctuation band* for the fundamental process: $\{f_t\}$ is allowed to fluctuate according to (2), but it is infrequently regulated so as to prevent it from wandering too far from some central, desired value. Such infrequent control will be dubbed *intervention*. Once again, its nature will depend on the underlying economic assumptions and need not have the very specific meaning (e.g., sterilized, unsterilized) it has in policy-oriented international economics. The crucial assumption needed to proceed is that the probability distribution of intervention, whatever it is, is uniquely determined by the current $\{f_t\}$ level—to imply that fundamentals follow a strong Markov process in levels and that the exchange rate is, by (3), a function of the current fundamental level, $x_t = x(f_t)$. If intervention determines well-defined limits on the size of f excursions from some central value, the exchange-rate process $\{x_t\}$ will be similarly restrained in its deviations from some central level. The assumption of infrequent, nonlinear intervention is therefore appealing on grounds of realism: real-life exchange-rate arrangements typically specify sharp limits on the realized values of x, rather than on other measures of its variability.

Since intervention is infrequent, we can investigate exchange-rate behavior when no intervention is taking place and (2) describes the local dynamics of f_t. Using Itô's stochastic change-of-variable formula to obtain an expression for the expectation appearing in (1),

$$(5) \quad \frac{1}{dt} E_t\{dx_t\} = x'(f_t)\vartheta + x''(f_t)\tfrac{1}{2}\sigma^2$$

we find that $x(f)$ must solve the functional equation

$$x(f) = f + \alpha x'(f_t)\vartheta + \alpha x''(f_t)\tfrac{1}{2}\sigma^2.$$

All solutions to (5) can be written in the

form

$$(6) \quad x(f) = f + \alpha\vartheta + A_1 e^{\lambda_1 f} + A_2 e^{\lambda_2 f}$$

where

$$\lambda_1 = \frac{-\vartheta + \sqrt{\vartheta^2 + 2\sigma^2/\alpha}}{\sigma^2} > 0$$

$$\lambda_2 = \frac{-\vartheta - \sqrt{\vartheta^2 + 2\sigma^2/\alpha}}{\sigma^2} < 0$$

and where A_1 and A_2 are constants of integration to be determined by the boundary conditions satisfied by $x(f)$ at times of intervention.

"Smooth pasting" provides such conditions for the case in which intervention occurs only at the boundaries of the exchange-rate band and is infinitesimal in size (see Krugman, 1991). As shown by Robert P. Flood and Peter M. Garber (1991), the obvious requirement that the exchange rate never jump in an expected way provides boundary conditions for more general intervention policies, which allow for intervention of discrete size at points strictly in the interior of the bands. Whenever $A_1 < 0$ and $A_2 > 0$, which is certainly the case if authorities are firmly committed to defending the band, the relationship between exchange rates and fundamentals has the S shape depicted in Figure 1: as the fundamental deviates from its central level in, say, the positive direction and the $e^{\lambda_1 f}$ term becomes larger, the exchange rate becomes a flatter function of fundamentals, and its level is reduced by expected intervention in the near future. Even though no intervention is actually taking place, existence of exchange-rate bands (however wide) stabilizes exchange-rate behavior within the bands, a phenomenon dubbed the "honeymoon effect" by Krugman (1991).

Since intervention is designed to impose sharp limits on exchange-rate excursions, the $x(f)$ function has a proper maximum and a proper minimum, where $x'(f)$ vanishes. Thus, the A_1 and A_2 constants in (6)
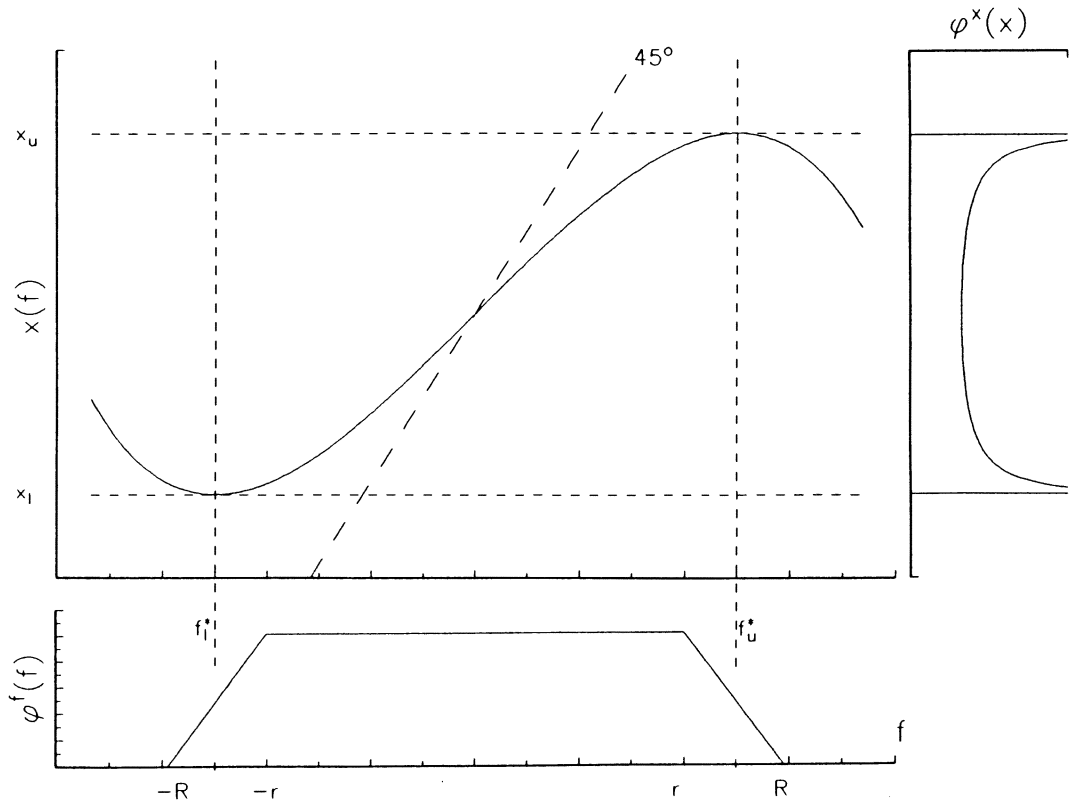
FIGURE 1. THE EXCHANGE-RATE/FUNDAMENTALS RELATIONSHIP AND ERGODIC DISTRIBUTIONS

are uniquely determined by the exchange-rate band: the nonlinear conditions

$$(7) \qquad x'(f_u^*) = 0 \qquad x(f_u^*) = x_u$$

$$x'(f_l^*) = 0 \qquad x(f_l^*) = x_l$$

are in general sufficient to determine A_1 and A_2 from knowledge of the exchange-rate limits (x_l, x_u).

By Itô's rule, $\{x_t\} = \{x(f_t)\}$ is a diffusion process, with stochastic differential

$$(8) \qquad dx_t = \left[x'(f_t)\vartheta + \tfrac{1}{2}\sigma^2 x''(f_t) \right] dt$$

$$+ \, x'(f_t)\sigma \, dW_t.$$

Equations (6) and (7) characterize many different intervention policies: all those that defend the declared band with probability

1. Since f_t appears in equation (8), the behavior of the exchange rate within its band depends on the specific form of intervention under consideration and the $\{f_t\}$ process it produces. Svensson (1991b) examines the case of infinitesimal intervention at the band limits (i.e., when $R = f^*$ and $r = f^*$ in Fig. 1) and exploits the relationship in (8) to derive implications for several observables.

By (8), the instantaneous variability of x_t is proportional to the absolute value of the $x(f)$ function's slope. It is clear from Figure 1 that $|x'(f)|$ is large in the middle of the band and declines to zero at its limits, where the exchange rate must be locally predictable.

Under uncovered interest parity, the interest-rate differential should equal the expected log exchange-rate change in equa-

tion (5). In the interior of a target zone, interest-rate differentials should quite intuitively predict appreciation when x nears the upper limit of the band and should predict depreciation when it nears the lower limit.[2]

The function relating exchange rates and fundamentals in the infinitesimal-intervention case is single-valued and admits an inverse $f(x)$ (though this is not expressible in closed form). Thus, the long-run density of exchange rates within the band, $\phi^x(x)$, can be derived by a change of variable,

$$(9) \qquad \phi^x(x) = \frac{\phi^f(f(x))}{x'(f(x))}$$

whenever the long-run distribution of fundamentals, $\phi^f(f)$, is known. In the case of infinitesimal control at the barriers, the latter is a truncated exponential leaning in the direction of drift or is uniform if the drift is zero. As shown in Svensson (1991b), the resulting distribution for exchange rates is U-shaped. This is again apparent from Figure 1 and equation (9): as $x'(f)$ vanishes at the margins of the band, where the fundamental's long-run density is positive, the exchange-rate distribution must tend to a spike at both of those points.

We show next that the Flood and Garber (1991) model of intramarginal, discrete intervention policies has similar implications for observables, considering a fully symmetric case for simplicity and comparability to Svensson's results. We assume $\vartheta = 0$ and let the intervention policy regulate the $\{f_t\}$ process at four fixed points, symmetric around zero, so as to maintain the exchange rate within predetermined bands. These symmetry assumptions reduce (6) to a simpler expression:

$$(10) \qquad x(f) = f - Ae^{\lambda f} + Ae^{-\lambda f}$$

$$\lambda = \sqrt{2/(\alpha\sigma^2)}\ .$$

We denote the distance of the intervention points from zero with R, and we denote the size of the intervention with $R - r > 0$. When $\{f_t\}$ reaches $-R$, it instantaneously moves to $-r$; similarly, when $\{f_t\}$ reaches R, it moves to r.

We can then compute the A constant that is consistent with the regulation policy under consideration. In the case of discrete interventions, the proper boundary conditions are (as in Flood and Garber [1991])

$$(11) \quad x(r) = x(R) \qquad x(-r) = x(-R).$$

That is, the exchange rate should not jump when the fundamental does, given that the intervention points are common knowledge and that the fundamental sample path is continuous both before and after a discrete intervention. Using (10) in (11) yields the solution:[3]

$$(12) \quad A = \frac{R - r}{\left(e^{-\lambda r} - e^{-\lambda R}\right)^2 - \left(e^{\lambda r} - e^{\lambda R}\right)^2}\ .$$

Equation (8) allows a characterization of expected and unexpected x_t movements when f_t is subject to discrete interventions. The drift and standard deviation of the exchange-rate process no longer depend on x_t alone, however, because the inverse of the $x(f)$ function is not single-valued. Denoting with $\bar{x}$ the upper limit of the exchange-rate fluctuation band, it is apparent from Figure 1 that two different fundamental levels correspond to each exchange rate in the intervals $(-\bar{x}, x(-r))$ and $(x(r), \bar{x})$.[4] Note,

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[2] In Figure 1, $x'(f_t)$ approaches zero at both limits; thus, it is the sign of $x''(f_t)$ that dominates as the exchange rate approaches the limits of the bands. The S-shaped function $x(f)$ is concave near the upper bound and convex near the lower bound.

[3] Not surprisingly, this expression converges to the one in Svensson (1991b) as $r \to f^*$, $R \to f^*$, and only marginal intervention is allowed.

[4] It may be interesting to note that the effects of "fundamental" movements on the exchange rate are perverse at the extremes of the fundamental fluctuation band: whereas an increase in f would cause depreciation in a pure float, it may be accompanied by appreciation if it makes large corrective actions more likely in the immediate future. The result does depend on the somewhat artificial assumption of deterministic intervention points and is quite likely to be eliminated when the possibility of realignments is taken into account (see Section III).

however, that the position and shape of the $x(f)$ function is uniquely determined by the exchange-rate fluctuation band: whatever the size of interventions, the $x(f)$ function must satisfy smooth pasting at the limits of the exchange-rate band [equation (7)]. Let f^* denote the fundamental level that corresponds to $\bar{x}$. Using (10) and smooth-pasting at $-f^*$ and f^*, it is easy to check that the pairs of f points (f_1 and f_2) in (r, R) which yield the same exchange rate must solve

$$(13) \quad e^{-\lambda f^*} + e^{\lambda f^*}$$

$$= \frac{1}{(f_1 - f_2)\lambda}$$

$$\times \left[(e^{\lambda f_1} - e^{\lambda f_2}) - (e^{-\lambda f_1} - e^{-\lambda f_2}) \right].$$

This relationship has a symmetric counterpart for points in $(-R, -r)$, and both are easily solved numerically. In particular, (13) defines the locus of (R, r) pairs that result in the same exchange-rate limits $(-\bar{x}, \bar{x})$ as a policy of infinitesimal intervention at f^*.

Instantaneous exchange-rate variability is once again proportional to the steepness of the $x(f)$ function, by (8), and interest-rate differentials should still be given by (5) if uncovered interest parity holds. The expressions in (5) and (8) now depend on the position of the fundamentals within their band as well as on that of the exchange rate, but the qualitative implications are by and large unchanged. Since $x'(f)$ is still flat at the margins of the exchange-rate band, all variability must vanish there, and x_t must be expected to appreciate when it is close to its upper limit.

Consider next the characteristics of $\{x_t\}$ in the long run. The asymptotic distribution of the driftless, jump-regulated Brownian-motion process followed by $\{f_t\}$ is easily derived using the results in the Appendix:

$$\phi^f(f) = \begin{cases} 0 & \text{for } f \leq -R \\ (R+f)/(R^2 - r^2) & \text{for } -R \leq f \leq -r \\ 1/(R+r) & \text{for } -r \leq f \leq r \\ (R-f)/(R^2 - r^2) & \text{for } r \leq f \leq R \\ 0 & \text{for } R \leq f. \end{cases}$$

The density is flat on the $(-r, r)$ range, where it resembles the uniform distribution of fundamentals in the absence of drift derived by Svensson (1991b) for the case of marginal intervention. Over the $(-R, -r)$ and (r, R) intervals, the density has constant slope and reaches zero at $-R$ and R. Intuitively, the likelihood of observing $f_t = R$ must be zero, as any attempt to reach that point triggers a move back to r; and over the (r, R) range the probability density must decrease, since points farther from r are less likely to be reached from the right than from the left. Similar considerations apply to the $(-R, -r)$ range.

Though fundamentals tend to concentrate in the middle of their fluctuation band, the exchange rate's long-run distribution is quite different. To see this, consider that the change-of-variable formula (9) needs to be amended to account for both the fundamental values mapping into the same exchange rates in the outer ranges [denoted $f_1(x)$ and $f_2(x)$ and computed by (13)]:

$$\phi^x(x) = \begin{cases} 0 & \text{for } x < -\bar{x} \\ \dfrac{\phi^f(f_1(x))}{|x'(f_1)|} + \dfrac{\phi^f(f_2(x))}{|x'(f_2)|} & \text{for } -\bar{x} \leq x \leq x(-r) \\ \dfrac{\phi^f(f(x))}{x'(f)} & \text{for } x(-r) \leq x \leq x(r) \\ \dfrac{\phi^f(f_1(x))}{|x'(f_1)|} + \dfrac{\phi^f(f_2(x))}{|x'(f_2)|} & \text{for } x(r) \leq x \leq \bar{x} \\ 0 & \text{for } \bar{x} \leq x. \end{cases}$$

The effects of discrete intervention on the exchange rate's long-run occupation density are quite minor, and not surprisingly so: since $x(f)$ is flat at $-\bar{x}$ and $\bar{x}$, the exchange-rate density still approaches infinity there (see the right-hand panel in Fig. 1). The fundamentals are more likely to be observed in the central portion of their band than near its boundaries, but this does not imply that the same should be true for the exchange rate: in fact, the exchange rate's distribution would be unaffected by the size of interventions if $x(f)$ were symmetric in the regions surrounding $-f^*$ and f^*. This is not the case, because a hyperbolic sine function is not symmetric around its peak and trough, but for reasonable values of λ the asymmetry is inconsequential.

We conclude that available models of fully credible target-zones have similar implications for observable processes. Further, it is unlikely that even more general assumptions about the behavior of $\{f_t\}$ would have a significant impact on our findings. The implication would be qualitatively correct for processes more general than (2) and, in particular, for mean-reverting fundamental processes. More general fully credible intervention policies could also be allowed for, perhaps specifying size distributions for intervention at every point. Still, (1) must always hold true: thus, if the exchange rate is below the level given by (6), it must be possible that in the next instant it will be above that level. At the limits of a credible fluctuation band, the latter possibility is ruled out, and the implications follow.

Regardless of the fundamental's drift and ergodic distribution, the smooth-pasting property of a fully credible intervention policy would still imply low exchange-rate variability in the neighborhood of the target zone's edges; low volatility, in turn, should keep the exchange rate in that neighborhood for a relatively long time once it gets there. Thus, the exchange rate's long-run density should still be relatively large at the margins of the fluctuation band.

II. Some Facts

The results summarized above suggest that exchange rates should be more variable when they are far from the band's boundaries, that an interest rate differential should decrease as an exchange rate approaches its upper boundary, and that in the long run exchange rates should be more frequently near the limits of the band than in the middle of it. Although "fundamentals" are unobservable unless one is willing to rely on a specific model of floating exchange-rate determination, the structure outlined above is common to many such models and its implications are, in principle, quite easy to verify on exchange- and interest-rate data from the many instances of regimes which purport to maintain exchange-rate fluctuations within sharp limits.

Figures 2–6 consider evidence from the first eight years of the European Exchange
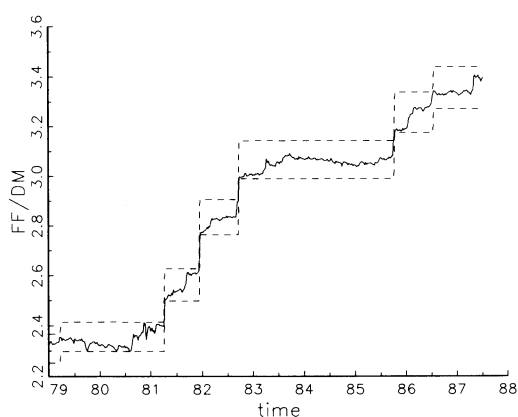

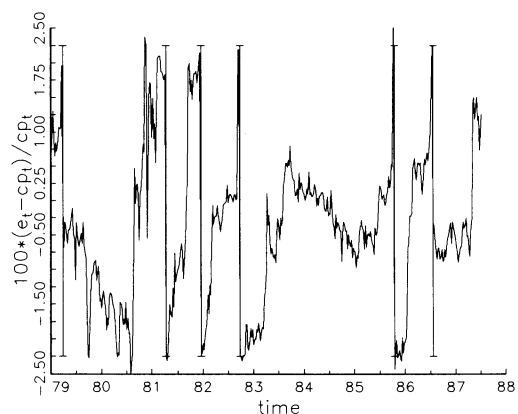FIGURE 2. FRENCH-FRANC/DEUTSCHE-MARK EXCHANGE RATE: ACTUAL PATH

FIGURE 3. FRENCH-FRANC/DEUTSCHE-MARK EXCHANGE RATE: DEVIATIONS FROM CENTRAL PARITY

Rate Mechanism. The data are the weekly closing spot French-franc/deutsche-mark (FF/DM, henceforth) exchange rate and the one-month Euromarket interest-rates differential, between April 6, 1979, and December 31, 1987. Throughout the period under consideration, ERM fluctuation bands were ±2.25 percent for the FF/DM exchange rate.

Figure 2 shows the path of the FF/DM exchange rate, and Figure 3 reports the history of deviations of this rate from the current central parity. Figure 4 reports a histogram for the relative frequency of such deviations, dividing the fluctuation band in eight equal parts, and Figure 5 plots the
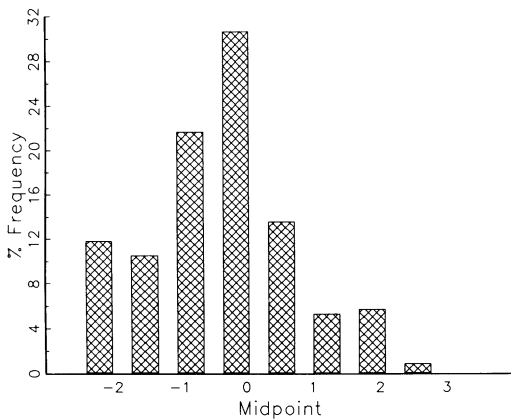
FIGURE 4. FREQUENCY OF EXCHANGE-RATE
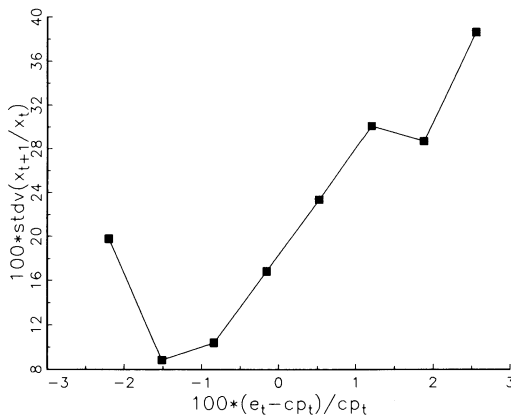OBSERVATIONS IN EIGHT SUBINTERVALS OF THE
BAND

FIGURE 5. STANDARD DEVIATION OF
EXCHANGE-RATE CHANGES IN EIGHT
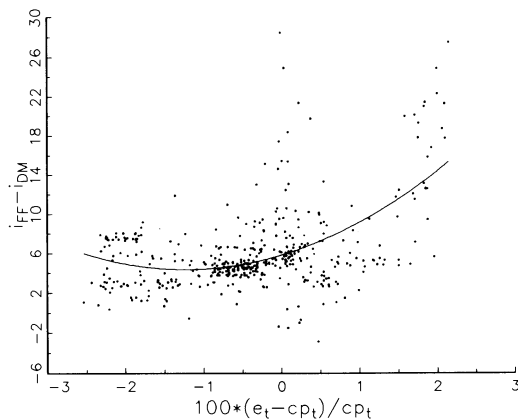SUBINTERVALS OF THE BAND

FIGURE 6. INTEREST-RATE DIFFERENTIALS
VERSUS EXCHANGE-RATE DEVIATION FROM
CENTRAL PARITY

annualized weekly standard deviation of the percentage changes in the exchange rate within each of these intervals (excluding the change when a realignment takes place between observations). Figure 6 plots interest-rate differentials against the deviation of the relevant exchange rate from the current central parity, with a quadratic interpolating line.

The rough evidence in the figures is inconsistent with the predictions of the simple model above. The exchange rate is most frequently observed in the interior of its fluctuation band and is more variable when approaching the upper limit of the band; interest-rate differentials tend to predict further depreciation of the French franc when the exchange rate approaches the upper limit. The data also indicate that the bands do not represent a firm commitment to intervention. In fact, during the $8\frac{1}{2}$-year period the FF/DM bilateral exchange rate was realigned six times. In the following, we discuss how the basic models of Section I can be amended to make them consistent with the evidence.

III. A Simple Model of Realignments

The implications of an S-shaped relationship between (not better specified) fundamentals and exchange rates appear to be quite robust to details of model specification and quite easy to falsify by inspection of the exchange-rate data considered above. Observation of the data also suggests an important source of inconsistency between theory and reality: in the early period of the European Exchange Rate Mechanism, within-bands exchange-rate developments were dwarfed by relatively frequent and large realignments. In what follows, we propose a simple model of discrete exchange-rate intervention that allows for stochastic realignments. It turns out that these extensions can actually reverse the model's implications for observables, pointing to a possible reconciliation of theory and certain features of the data considered above.[5]

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[5]Krugman (1991) shows that imperfect credibility of exchange-rate bands may reduce, but not eliminate,

To clarify the mechanisms through which realignments affect the workings of the previous sections' models, we first propose and solve a stylized, symmetric example. Assume again that $\vartheta = 0$ and that authorities only intervene at prespecified, common-knowledge points $c_t - \bar{f}$, $c_t + \bar{f}$, where c_t stands for the (no longer immutable) central parity applying up to time t. Intervention may now take one of two forms: when f reaches either of the boundaries, the authorities may either bring it back to the center of the current (c_t) or do the opposite (double its distance from c_t) and declare a new fluctuation band for the fundamental process, adjoining the current one, with center $c_{t(+)} \equiv c_t \pm 2\bar{f}$ and unchanged width.

We shall say that in the former case authorities "defend" the current parity, and we assume that this occurs with probability $1 - p$ whenever $f_t = c_t \pm \bar{f}$. In the latter case, authorities "realign" the central parity; thus, p is the probability of a realignment when the exchange rate is at the margin of the fluctuation band. The bivariate process $\{f_t, c_t\}$ is then jointly Markov, and the bubbleless exchange rate must be a function of the current level of fundamentals and the current central parity, $x_t = x(f_t; c_i)$. In the interior of every band, the function $x(f; c)$ must have the symmetric form

$$(14) \quad x(f; c) = f + Ae^{-\lambda(f-c)} - Ae^{\lambda(f-c)}$$

for λ as defined in (10). To determine A, we impose that the exchange rate not be

expected to change at times when intervention is known to be imminent. Thus, when $f = c_t + \bar{f}$, it must be true that

$$(15) \quad px(c_t + 2\bar{f}; c_t + 2\bar{f}) + (1 - p)x(c_t; c_t)$$
$$= x(c_t + \bar{f}; c_t).$$

The left-hand side of (15) weighs with the respective probabilities the two possible exchange rates just after the decision to defend or realign the band is revealed. To prevent risk-neutral agents from earning unbounded returns per unit time, this expected exchange rate must equal that prevailing at the instant the fundamentals reach the barrier and the authorities' decision is called for.

Using (14) in (15), we obtain

$$A = \frac{(1 - 2p)\bar{f}}{e^{\lambda \bar{f}} - e^{-\lambda \bar{f}}}.$$

In the models of the previous sections we had $A > 0$, producing the familiar S-shaped relationship. Here, $A \geq 0$ only if $0 \leq p \leq \frac{1}{2}$: when $p = 0$, the band is always defended, and the model is a special case of that in Flood and Garber (1991); as p rises, the S shape becomes steeper and coincides with the 45° line (free float) when $p = \frac{1}{2}$.

If $p > \frac{1}{2}$, the $x(f)$ function is everywhere steeper than it would be under free floating: the exchange rate diverges from the 45° line as it nears the boundaries, because the expectation of the least discounted f realizations in (4) increases as the positive expected jump in fundamentals that takes place there $[(2p - 1)\bar{f}]$ becomes more likely in the near future.

Figure 7 shows the $x(f; c)$ function over contiguous zones when $p = 0.6$ and $p = 0.9$. As p rises the pre- and postrealignment exchange-rate bands begin to overlap (for $p \geq \frac{1}{2}$, the extent of overlap is $p - \frac{1}{2}$). More generally, as p rises, a wider exchange-rate band becomes consistent with the same fundamental band (note that when $p \geq \frac{1}{2}$, $\bar{x} = 2p\bar{f}$); or, if p is large, only a narrow fundamental band is consistent with a given exchange-rate band. Higher likelihood of re-

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their qualitative stabilizing effects. However, the case he considers is that of a potential collapse of the exchange regime, a once-and-for-all abandonment of all intervention. In the model below, as in ERM reality, exchange rates do not revert to a free float when the band is not defended: rather, a new set of bands is defined. Realignments are relatively infrequent, recurring stochastic regime shifts. Miller and Weller (1989) model realignments in a framework that is qualitatively similar to the one we adopt, and Svensson (1991a, b) allows for realignments that occur with equal probability regardless of where the exchange rate is in the band; this turns out not to affect the observable implications of fully credible intervention policies.
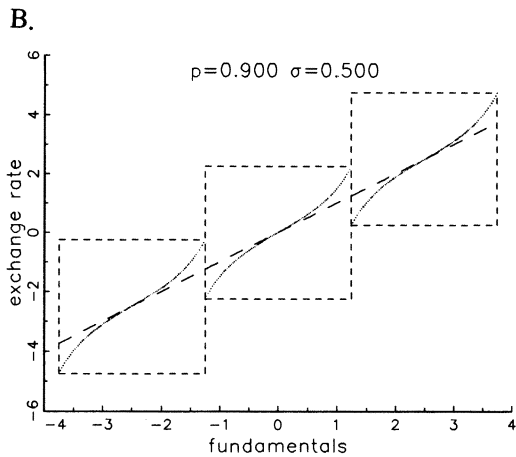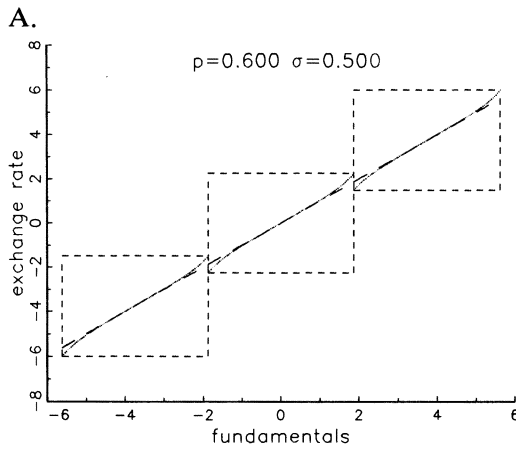
A.

B.

FIGURE 7. EXCHANGE-RATE/FUNDAMENTALS
RELATIONSHIP WHEN REALIGNMENTS ARE
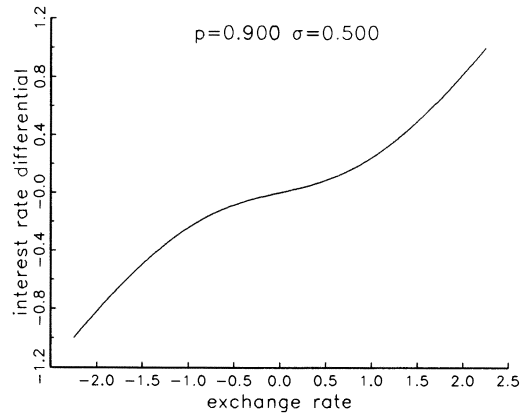POSSIBLE: A) $p = 0.6$, $\sigma = 0.5$; B) $p = 0.9$, $\sigma = 0.5$

FIGURE 8. EXPECTED DEPRECIATION WHEN
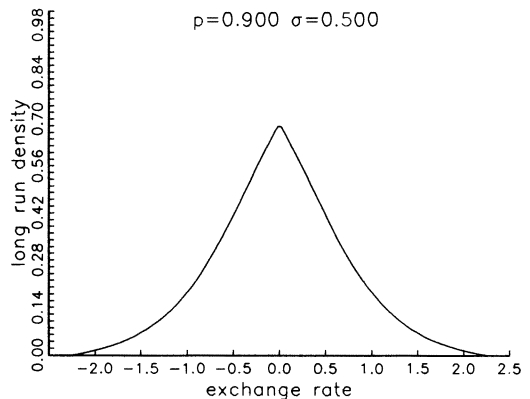REALIGNMENTS ARE POSSIBLE: $p = 0.9$, $\sigma = 0.5$

FIGURE 9. LONG-RUN DISTRIBUTION WHEN
REALIGNMENTS ARE POSSIBLE: $p = 0.9$, $\sigma = 0.5$

alignments produces shorter, more nervous "honeymoons."

Our interest in high realignment probabilities is motivated by the qualitative consistency of an inverted-S shape with ERM evidence on exchange-rate behavior within the bands. In the neighborhood of the band's margins, the $x(f)$ function is steepest, to imply large exchange-rate volatility. Rather than reverting to central parities, expected depreciation diverges (Fig. 8), implying that the interest-rate spread should rise rather than decline as the exchange rate approaches the upper limit of the current band. It is easy to show, using the results in the Appendix, that the long-run distribution of $\{f_t - c_t\}$ is symmetric and triangular in the case under consideration; and the long-run distribution of the exchange rate has more weight in the center than near the boundaries of the band when $p > \frac{1}{2}$ (Fig. 9).

IV. Other Elements of Realism

While the symmetric case studied in the previous section is not meant to be realistic, its implications are qualitatively consistent with the evidence in Section II. Additional

realistic features are easily introduced in our approach to modeling realignments. We propose some simple and quite obvious extensions in this section: still assuming fundamental bands to be $2\bar{f}$ wide around every central parity, we allow for $\vartheta \neq 0$, for arbitrary (though still deterministic) intervention and realignment sizes, and for asymmetric probabilities of realignment.

Specifically, when $f = c_{t(-)} + \bar{f}$, the authorities may declare a new fundamental center $c_{t(+)} = c_{t(-)} + D_d$ and at the same time increase fundamentals by $D_d - 2\bar{f} + K_d$. Note that $D_d - 2\bar{f}$ is the least possible change in fundamentals consistent with the new parity and that both D_d and K_d must be positive. This is known to occur with probability p_d (d for devaluation); alternatively, with probability $1 - p_d$, the authorities might defend the current central parity, through a fundamental jump of size $-R_d$.

Conversely, when $f = c_{t(-)} - \bar{f}$, the probability is p_r (r for revaluation) that a realignment to the new central parity $c_{t(+)} = c_{t(-)} - D_r$ will occur, accompanied by a negative jump of size $-D_r + 2\bar{f} - K_r$ in the fundamental process ($K_r \geq 0$, $D_r \geq 0$). With probability $1 - p_r$, the band will be defended, and a fundamental jump of size $R_r > 0$ will take place.

The functional relationship among x, f, and c can then be written

$$(16) \quad x(f;c) = f + \alpha\vartheta + A_1 e^{\lambda_1(f-c)}$$
$$+ A_2 e^{\lambda_2(f-c)}$$

with boundary conditions:

$$(17) \quad p_d x\left(c_t + \bar{f} + D_d - 2\bar{f} + K_d; c_t + D_d\right)$$
$$+ (1 - p_d) x\left(c_t + \bar{f} - R_d; c_t\right)$$
$$= x\left(c_t + \bar{f}; c_t\right)$$
$$p_r x\left(c_t - \bar{f} - D_r + 2\bar{f} - K_r; c_t - D_r\right)$$
$$+ (1 - p_r) x\left(c_t - \bar{f} + R_r; c_t\right)$$
$$= x\left(c_t - \bar{f}; c_t\right).$$

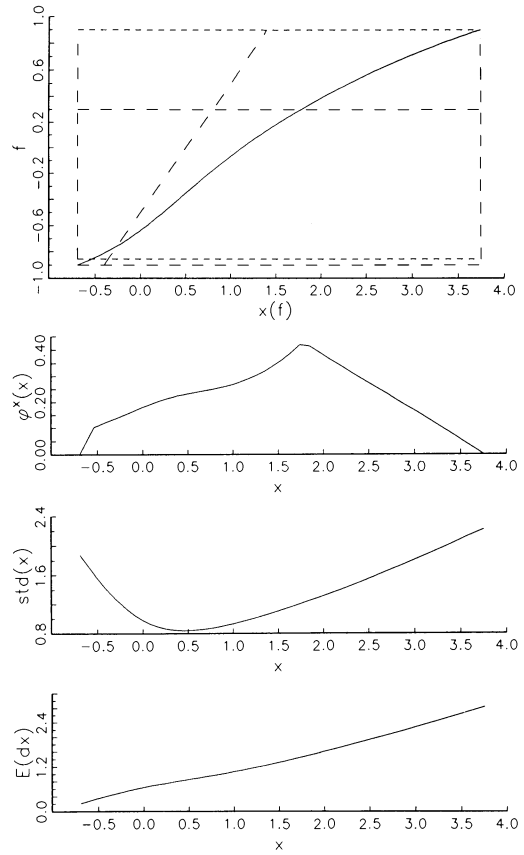Using (16) in (17), we obtain a linear system of equations in A_1 and A_2. The long-run

FIGURE 10. MODEL IMPLICATIONS WITH REALISTIC PARAMETERS: $\sigma = 0.5$, $\alpha = 1.0$, $\vartheta = 0.5$, $\bar{f} = 0.9$, $P_d = 0.9$, $D_d = 4.5$, $R_d = 0.045$, $K_d = 0.6$, $P_r = 0.4$, $D_r = 2.7$, $R_r = 0.6$, $K_r = 0.045$

distribution of the deviations of fundamentals from their current central point is not difficult to obtain in closed form as well (see the Appendix).

Given a set of parameter values, we can derive the model's implications for observables. More interestingly, real-world data can be used for inference about the behavior of unobserved "fundamentals" in ERM countries. At a qualitative level, in fact, the simple model outlined here appears to be capable of replicating most of the features emerging from the evidence on FF/DM exchange and interest rates discussed in Section II. Figure 10 reports the parameter values we have chosen and the resulting

functional relationships; the remainder of this section discusses the role of various parameters in producing the satisfactory fit emerging from a comparison of Figures 2–6 and Figure 10.

The orders of magnitude of several parameters can be gauged from relatively informal considerations. In the ERM, the French franc has never been revalued against the deutsche mark; thus p_r is not likely to be as large as p_d. The within-band behavior of the exchange rate suggests the presence of a positive fundamental drift, further reducing the likelihood of a revaluation realignment. Typical realignments do not yield overlapping exchange-rate bands; thus, D_d should be larger than $\bar{f}$. The exchange rate is near the bottom of its new fluctuation band just after a realignment, implying a small K_d in our model.

The sample contains little information about the behavior of the FF/DM exchange near the bottom of its fluctuation band, except for the fact that the histograms show relatively large accumulation there. At the top of the band $x(f;c)$ should be quite steep for the model to be consistent with the evidence on long-run occupancy, exchange-rate variability, and interest-rate differentials. We briefly discuss the role of several parameters in determining the shape of $x(f)$ at the top of the band.

A more positive drift ϑ naturally yields higher expected depreciation for all levels of fundamentals and exchange rates. In addition, it reduces the convexity of an inverted S near the top: a shorter distance from the critical level has less impact on the probability of facing a realignment in the future when the drift is positive.[6] For the model to be consistent with evidence of increasing interest-rate differentials, high variability, and low long-run occupancy in the neighborhood of the upper bound, we then need to specify a small (positive) drift

of fundamentals within the bands. The variability of fundamentals, σ, should be small as well, not only in light of the evidence reviewed above, but also for consistency with the well-documented fact that short-run exchange-rate variability has been lower after the inception of the ERM. In the framework of the present model, lower variability cannot be due to "honeymoon" effects: by default, "fundamentals" (whatever they are) must be relatively stable.[7]

The probability of a realignment when the exchange rate hits the upper boundary, p_d, and the size of realignments (D_d) and intervention (R_d) have crucial roles in the model we propose. A larger p_d makes for a more convex $x(f)$ function near the upper limit of the band (as in Fig. 7), in accord with the empirical evidence. However, if realignments are not to occur with probability 1 when the upper boundary is reached —as is implied by the fact that the exchange rate does jump upward when realignments materialize—the authorities must take drastic action when they decide to defend the current parity: in the limit, if an attempt were made to implement a policy of infinitesimal interventions as in Krugman (1991), even a very small (but strictly positive) $p_d > 0$ would result in immediate realignment.[8] We set $p_d = 0.9$ in computing the functions plotted in Figure 10 to imply that the authorities would defend the French franc against a devaluation only one time out of ten (and we note from Fig. 2 that the downward jump observed in the spring of 1981 might be thought to correspond to one such instance). Not having a clear idea of

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[6] The leading term of $\partial x''(f)/\partial\vartheta$ when $f - c > 0$ is $(\partial\lambda_1/\partial\vartheta)\lambda_1 A_1 e^{\lambda_1(f-c)}[2 + \lambda_1(f - c)]$. When realignments are large or very likely, then $A_1 < 0$, and higher drift reduces the convexity of $x(f)$ since $\partial\lambda_1/\partial\vartheta$ and λ_1 are both positive.

[7] These results suggest that the precommitment motive emphasized by Francesco Giavazzi and Marco Pagano (1988) is more relevant to the ERM experience than the Krugman (1991) honeymoon effect.

[8] The Wiener process in (2) has infinite variation and hits the boundary infinitely frequently with probability 1 in arbitrarily short time if fundamentals are simply maintained at the boundary when they reach it. The probability of no realignment in arbitrarily short time is then $\lim_{n \to \infty}(1 - p_d)^n = 0$ at the boundary. Miller and Weller (1990) model realignments as Poisson events and express the realignment probability in terms of time units.

what the "fundamental" process should be taken to represent in the ERM reality, these large "interventions" are somewhat difficult to interpret. Perhaps, the very fact that authorities declare their intention to maintain the current parity when random events have brought the exchange rate to its upper boundary is enough to alter drastically the public's probability assessments about the future evolution of the relevant set of fundamental policy variables.

The assumptions of our stylized model are not meant to be a precise description of reality, of course. Rather, they try to capture the essence of the mechanisms underlying exchange-rate behavior in the ERM. In fact, the French franc was sometimes devalued when its exchange rate with the deutsche mark was not at the upper boundary of the fluctuation zone.[9] We believe, however, that our framework of analysis is still appropriate in this case. While realignments need not occur exactly at the margin of the band, qualitatively similar results would be obtained if, more generally, realignment were assumed to become more likely as fundamentals approach the limits of their fluctuation band. If our model were extended along theses lines, in fact, it might well be the case that most realignments would be observed when the exchange rate is relatively far from the edges of its fluctuation band, since very little time is spent in the neighborhood of the exchange-rate target zone's margin when the likelihood of realignments is very high there.

V. Concluding Comments

On the basis of simple empirical evidence, fully credible target-zone models are not appropriate for studying the early ERM period. We have shown that the theory can be reconciled with several aspects of reality by extending the technical apparatus to allow for recurring realignments.

[9]Intramarginal realignments were the rule, rather than the exception, in the Italian lira/DM case, which we discussed in earlier versions of this paper (Columbia University Working Paper No. 460, Princeton University Working Paper No. G-90-1, and CEPR Discussion Paper No. 398).

Different models may be appropriate for empirical work on different target zones, of course, and the thorough review of the evidence by Flood et al. (1990) suggests that target-zone models will need to be extended in many more directions than the one we consider.[10] Svensson (1991a) finds that Swedish interest-rate differentials tend to predict reversion toward exchange-rate central parities, indicating that devaluation risk may in fact be independent of the exchange rate's deviation from central parity in the Swedish target zone; and we limited our own evidence-gathering to the pre-1987 period because the nature of the European Exchange Rate Mechanism is widely believed to have changed in the more recent period of smaller and less frequent realignments. The post-1987 sample of exchange rates is too small to permit a separate analysis of the recent ERM experience using our approach, which is based on long-run qualitative properties. However, formal statistical estimation of our model is in principle possible: using numerical techniques to fit our model to French and German exchange-rate and interest-rate differentials, Leonardo Bartolini and Bodnar (1991) split the sample in mid-1986 and find evidence of greater credibility in the more recent period. To explain the large noise component in the relationship between interest-rate differentials and exchange rates (see Fig. 6), some allowance should be made for time-varying parameters in our model, possibly linking the likelihood and size of realignments to some measure of the reserve stocks available for intervention, along the lines of

[10]At a theoretical rather than empirical level, models of credible target zones are useful for the purpose of evaluating proposals for large-scale reform of the international monetary system (such as that in John Williamson and Miller [1987]). Normative analysis, however, would require firmer foundations than the ad hoc ones of available models. It is not clear why nominal-exchange-rate stability should be desirable, and it is even less clear that nonlinear policy rules should be the preferred way to bring it about. We know that costs with a kink at the no-adjustment point make inaction optimal in a range, but a satisfactory model of policy-making with this feature has not yet been developed.

Bertola and Caballero (1991).[11] Many more extensions are possible and should improve even further the realism of the model. Intramarginal interventions and realignments of random size can be introduced with relatively little effort, and adjustment efforts might be modeled in terms of different drifts and standard deviations for different regions in the exchange-rate band.

Much work remains to be done toward developing a satisfactory positive and normative theory of nominal-exchange-rate regimes. The message of data from the early ERM period, and of this paper, is simple: in terms of the simplified model of Section III, the S is inverted; the exchange rate spends most of the time near the center of the zone and is more volatile near the boundaries of the band; the interest-rate differential is increasing in the proximity of the exchange rate to the upper bound; and the fundamental has lower within-band flexibility the larger is the realignment probability. Inversion of the S reverses the "honeymoon" result, and realistically specified nonlinear policy interventions are found to produce larger within-band exchange-rate variability than a free float would for the same fundamental variability. On the other side of the coin, fundamentals need to be tightly controlled for an imperfectly credible target zone to be at all sustainable.

APPENDIX

Ergodic Distribution of Regulated Fundamentals

J. Michael Harrison (1985 pp. 89–93) and others provide a probabilistic treatment of the limiting behavior of reflected Brownian motion. In this appendix, we provide a simple analytical derivation of general regulated Brownian-motion limiting distributions, approximating the Wiener process by a binomial discrete random walk.

Define a process $\{z_t\} \equiv \{f_t - c_t\}$, the deviation of the fundamental process from its current central point. In the absence of intervention, $\{f_t\}$ follows equation (2) in the text, and c_t is constant; thus,

$$(A1) \qquad dz_t = \vartheta\, dt + \sigma\, dW_t.$$

A discrete-time analogue of (A1) is

$$(A2) \quad z_{t+dt}$$
$$= \begin{cases} z_t + dz & \text{with probability } \tfrac{1}{2}(1 + \vartheta\, dt / dz) \\ z_t - dz & \text{with probability } \tfrac{1}{2}(1 - \vartheta\, dt / dz). \end{cases}$$

As the (time, state)-space steps approach zero, (A2) approaches (A1), provided that the rates of convergence are such that $(dz)^2 = \sigma^2\, dt$.

If (i) the regulation policy never allows it to leave some bounded state space, (ii) no state is absorbing, and (iii) the Markov chain is aperiodic, then the $\{z_t\}$ process has a unique, invariant, and ergodic probability distribution, since all its states are positive recurrent and aperiodic (see e.g., Sheldon M. Ross, 1983 p. 109). It is easy to verify that these conditions are satisfied, in the continuous time limit, by all the models discussed in this paper. To derive the ergodic distribution, we make use of its invariance property: at every point that is unaffected by intervention, the discrete steady-state probability density function should satisfy the balance equation

$$\phi(z) = \phi(z - dz)\tfrac{1}{2}(1 + \vartheta\, dt / dz)$$
$$+ \phi(z + dz)\tfrac{1}{2}(1 - \vartheta\, dt / dz).$$

Rearranging,

$$0 = [\phi(z + dz) - \phi(z)]$$
$$- [\phi(z) - \phi(z - dz)]$$
$$- \vartheta\frac{dt}{dz}\{[\phi(z + dz) - \phi(z)]$$
$$+ [\phi(z) - \phi(z - dz)]\}.$$

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[11] Bertola and Svensson (1991) propose an alternative model in which the likelihood of devaluations in the near future varies over time but does not depend on the fundamentals' position in their fluctuation band. Andrew K. Rose and Svensson (1991) find that this alternative model's implications appear to be consistent with some of the characteristics of French and German ERM data.

Dividing by dz and taking the limit, we find that $\phi(z)$ is continuously differentiable. Dividing by $(dz)^2$, and using $dt/(dz)^2 = \sigma^{-1}$, we have in the limit

$$(A3) \qquad \frac{\sigma^2}{2}\phi''(z) = \vartheta\phi'(z).$$

This is, in fact, the Kolmogorov transition equation satisfied by the time-invariant density function of Brownian motion with drift (see e.g., Harrison, 1985 p. 37; note that $\partial\phi/\partial t = 0$ in steady state).

The general solution of the functional equation (A3) has the form

$$(A4) \quad \phi(z) = \begin{cases} Az + B & \text{if } \vartheta = 0 \\ Ae^{\zeta z} + B & \text{if } \vartheta \neq 0 \end{cases}$$

where we define $\zeta \equiv 2\vartheta/\sigma^2$.

To determine which values of A and B are appropriate at every point, we study the boundary behavior of the stable density at the intervention points. Of course, the density must also satisfy the summing-up condition

$$(A5) \qquad \int_{-\bar{f}}^{\bar{f}} \phi(z)\,dz = 1.$$

Let the left- and right-hand-side derivatives of $\phi(\cdot)$ be denoted $\phi'_{(-)}(\cdot)$ and $\phi'_{(+)}(\cdot)$. Consider first the case of a reflecting barrier at, for example, $-\bar{f}$. When $z_t = -\bar{f}$, its discrete-time transitions can be written

$$z_{t+dt} = \begin{cases} -\bar{f}+dz & \text{with probability } \frac{1}{2}(1+\vartheta\,dt/dz) \\ -\bar{f} & \text{with probability } \frac{1}{2}(1-\vartheta\,dt/dz). \end{cases}$$

As it is possible to reach point f both from $f + dz$ and from f itself, the invariant distribution must satisfy the balance equation

$$\phi(-\bar{f}) = \phi(-\bar{f}+dz)\tfrac{1}{2}(1-\vartheta\,dt/dz)$$
$$+ \phi(-\bar{f})\tfrac{1}{2}(1-\vartheta\,dt/dz).$$

Right-continuity of $\phi(\cdot)$ at $-\bar{f}$ follows in the limit. Dividing by dz and taking the

limit, we obtain

$$(A6) \qquad \phi'_{(+)}(-\bar{f}) = \frac{2\vartheta}{\sigma^2}\phi(-\bar{f}).$$

If both boundaries are reflecting barriers, (A5) and (A6) determine a pair of (A, B) coefficients and yield the truncated exponential or uniform distribution derived in Harrison (1985) by probabilistic arguments.

Consider next the possibility of jumps: suppose that, from $-\bar{f}$, z jumps to $-\bar{f}+R_r$ with probability $1-p_r$ and that it jumps to $\bar{f}-K_r$ with probability p_r. In the discrete-time approximation, if $z_t = -\bar{f}+dz$, then

$$z_{t+dt}$$
$$= \begin{cases} -\bar{f}+2\,dz & \text{with probability } \frac{1}{2}(1+\vartheta\,dt/dz) \\ -\bar{f}+R_r & \text{with probability } (1-p_r)\frac{1}{2}(1-\vartheta\,dt/dt) \\ \bar{f}-K_r & \text{with probability } p_r\frac{1}{2}(1-\vartheta\,dt/dz). \end{cases}$$

Clearly, $\phi(-\bar{f}) = 0$. Now, z can reach $-\bar{f}+R_r$ not only from $-\bar{f}+R_r-dz$ and $-\bar{f}+R_r+dz$, but from $-\bar{f}+dz$ as well. The balance equation that yields invariance around $-\bar{f}+R_r$ is

$$\phi(-\bar{f}+R_r)$$
$$= \phi(-\bar{f}+R_r-dz)\tfrac{1}{2}(1+\vartheta\,dt/dz)$$
$$+ \phi(-\bar{f}+R_r+dz)\tfrac{1}{2}(1-\vartheta\,dt/dz)$$
$$+ (1-p_r)\phi(-\bar{f}+dz)\tfrac{1}{2}(1-\vartheta\,dt/dz).$$

In the limit, $\phi(z)$ is continuous at $-\bar{f}+R_r$ and $-\bar{f}$ but need not be continuously differentiable. Making use of $\phi(-\bar{f}) = 0$, the balance equation above can be rearranged to read

$$\phi(-\bar{f}+R_r) - \phi(-\bar{f}+R_r-dz)$$
$$= (1-p_r)[\phi(-\bar{f}+dz) - \phi(-\bar{f})]$$
$$+ [\phi(-\bar{f}+dz) - \phi(-\bar{f})]$$
$$+ \vartheta\frac{dt}{dz}\{[\phi(-\bar{f}+R_r-dz)$$
$$- \phi(-\bar{f}+R_r+dz)]$$
$$- [\phi(-\bar{f}+dz) - \phi(-\bar{f})]\}.$$

Dividing by dz, imposing $(dz)^2 = \sigma^2 dt$, and noting that the term in braces vanishes in the limit by continuity, we obtain

$$\phi'_{(-)}\left(-\bar{f} + R_r\right) = \phi'_{(+)}\left(-\bar{f} + R_r\right)$$
$$+ (1 - p_r)\phi'_{(+)}\left(-\bar{f}\right).$$

Similar steps lead to

$$\phi'_{(-)}\left(\bar{f} - K_r\right) = \phi'_{(+)}\left(\bar{f} - K_r\right)$$
$$+ p_r\phi'_{(+)}\left(-\bar{f}\right).$$

Analogous computations yield boundary conditions appropriate for all forms of intervention and realignment policies considered in the text (some algebraic complications arise when the return points reached after intervention coincide with each other or with the boundaries). In all cases, the stable density is piecewise exponential (if $\vartheta \neq 0$) or piecewise linear, and the boundary conditions and summing-up equation form a (rank-deficient) system of linear equations in a set of (A_i, B_i) coefficients applying to the different segments.

The approach outlined above can be straightforwardly adapted to allow for the possibility of intervention in the interior of $(-\bar{f}, \bar{f})$. Moreover, it is possible to allow for more complex dynamics for the fundamental process, as long as an analogue of (A4) can be found. For example, mean-reverting dynamics would yield a piecewise hypergeometric ergodic density.

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