(1) Dampening GE: from Micro to Macro

(2) Forward Guidance without Common Knowledge

George-Marios Angeletos* **Chen Lian****

Chicago Booth: April 10, 2017

*MIT and NBER, **MIT

Motivation

- GE effects key to macroeconomics (and elsewhere)
  - limit usefulness of PE intuitions
  - limit usefulness of micro-based estimates a la Mian-Sufi
  - drive interpretations of business cycles
  - drive policy predictions (and some puzzles)

Motivation

- GE effects key to macroeconomics (and elsewhere)
  - limit usefulness of PE intuitions
  - limit usefulness of micro-based estimates a la Mian-Sufi
  - drive interpretations of business cycles
  - drive policy predictions (and some puzzles)
- But: how potent/fast is GE adjustment?
  - standard practice assumes “perfect” and “instantaneous”
  - hardwired in solution concept + info assumptions

This talk, part 1: Dampening GE

- Goals/contribution:
  - formalize notion "GE Adjustment is Weak / Takes Times"
  - reduce gap between micro and macro elasticities
- Context: simple, abstract, Walrasian economy (Econ101)
- Two alternative methods:
  1. depart from REE concept
  2. relax Common Knowledge
- Clarify similarities/differences of two methods

This talk, part 2: Forward Guidance

- Topical Context: NK economy at ZLB
- Identify and attenuate GE effects
- Disentangle
  - PM's ability to manage expectations of policy
  - PM's ability to manage expectations of inflation, income, etc
- Lessen FG puzzle – and paradox of flexibility, too
- Justify "front-loading" in monetary and fiscal policy

Dampening GE: from Micro to Macro

Contribution

- Goals:
  - formalize notion "GE Adjustment is Weak / Takes Times"
  - reduce gap between micro and macro elasticities

- How?

- Define and characterize an appropriate "frictionless benchmark"
  - assumes REE+CK, replicates Arrow-Debreu

- Depart from this benchmark in two possible ways
  - replace REE with cognitive/behavioral concepts
  - maintain REE but remove CK

Framework

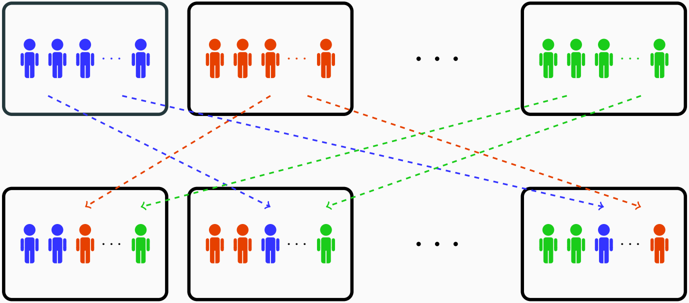
- Elementary Walrasian economy
  - with decentralized and sequential trading
- Two periods: "morning" and "afternoon"
  - proxy for "present" vs "future"
  - t and $t+1$ in multi-period extension
- Three goods = numeraire + morning good + afternoon good
  - numeraire can be land, capital, money,

    or leisure as in Lagos-Wright, Guerrieri-Lorenzoni, etc

Framework

- A continuum "marketplaces" $m \in [0, 1]$
  - can, but do not have to, be correlated with geographic regions
- Every agent trades in a single marketplace in each period
  - $t = 1$: morning good against numeraire
  - $t = 2$: afternoon good against numeraire
- But: agents may "relocate" as time passes
  - GE interdependence across markets

Framework



Demand, Supply, and Fundamentals

- in the paper: details of micro-foundations
  - preferences, endowments, technology
- here: work with (log-linearized) demand and supply functions
  - morning markets: $c_m = D(\cdot) \quad q_m = S(\cdot)$
  - afternoon markets: $c_m^* = D^*(\cdot) \quad q_m^* = D^*(\cdot)$
- "fundamentals" = exogenous shifter to demand and supply
  - θ_m = average fundamental of agents originally in marketplace m
  - $\bar{\theta}$ = economy-wide average fundamental

Standard Practice vs This Paper

- Elementary assumptions: for every m,

  (1.a) individual rationality and market clearing

  (1.b) knowledge of (θ_m, p_m) in *own* marketplace

  (1.c) same subjective beliefs *within* each marketplace

  (1.d) knowledge that (1.a)–(1.c) hold in *own* marketplace

Standard Practice vs This Paper

- Elementary assumptions: for every m,

  (1.a) individual rationality and market clearing

  (1.b) knowledge of (θ_m, p_m) in *own* marketplace

  (1.c) same subjective beliefs *within* each marketplace

  (1.d) knowledge that (1.a)–(1.c) hold in *own* marketplace

- Standard practice: the above plus

  (2) Common Knowledge (CK) of entire $\{\theta_m, p_m\}_{m \in [0,1]}$

  (3) Rational Expectations Equilibrium (REE)

  $\Rightarrow$ · CK of individual rationality and of market clearing
  · subjective beliefs = objective expectations
  · identical beliefs both within and across marketplaces

Standard Practice vs This Paper

- Elementary assumptions: for every m,

  (1.a) individual rationality and market clearing

  (1.b) knowledge of (θ_m, p_m) in *own* marketplace

  (1.c) same subjective beliefs *within* each marketplace

  (1.d) knowledge that (1.a)–(1.c) hold in *own* marketplace

- Standard practice: the above plus

  (2) Common Knowledge (CK) of entire $\{\theta_m, p_m\}_{m\in[0,1]}$

  (3) Rational Expectations Equilibrium (REE)

  $\Rightarrow$ · CK of individual rationality and of market clearing
  · subjective beliefs = objective expectations
  · identical beliefs both within and across marketplaces

- Our paper: maintain (1) but relax (2) and/or (3)

  $\Rightarrow$ preserve PE but modify GE

Shocks and Outcomes

- Aggregate shock

$$\bar{\theta}_{old} \longrightarrow \bar{\theta}_{new} = \bar{\theta}_{old} + \Delta\bar{\theta}$$

- Allow for differential exposure and local shocks

$$\Delta\theta_m = (1 + \delta_m)\Delta\bar{\theta} + z_m$$

- Question: how do morning quantities/prices respond to $\Delta\bar{\theta}$?

- Answer: depends on how shock affects, for every m,
  - demand and supply in *own* marketplace, for given beliefs
  - beliefs of outcomes in *other* marketplaces, for given fundamentals

- Preliminary step: market clearing with arbitrary subjective beliefs

Morning and Afternoon Markets

Lemma. For every m, the morning outcomes satisfy

$$\begin{pmatrix} q_m \\ p_m \end{pmatrix} = F\left(\hat{E}_m[\bar{p}^*] \ , \ \theta_m \right)$$

where $\hat{E}_m[\cdot]$ denotes the subjective belief in marketplace m and $\bar{p}^*$ denotes the average afternoon price.

Morning and Afternoon Markets

Lemma. For every m, the morning outcomes satisfy

$$\begin{pmatrix} q_m \\ p_m \end{pmatrix} = F\left(\hat{E}_m[\bar{p}^*] \ , \ \theta_m \right)$$

where $\hat{E}_m[\cdot]$ denotes the subjective belief in marketplace m and $\bar{p}^*$ denotes the average afternoon price.

Lemma. For every m, the afternoon outcomes satisfy

$$\begin{pmatrix} q_m^* \\ p_m^* \end{pmatrix} = F^* \left(\rho q_m + (1-\rho)\bar{q} \ , \ \rho\theta_m + (1-\rho)\bar{\theta} \right)$$

where ρ is mass of native agents, $1-\rho$ is mass of agents relocating from other marketplaces,, and $\bar{x}$ denotes economy-wide aggregates

Two-Way Feedback

- Two-way feedback:

$$\text{afternoon outcomes} = \text{function(realized morning outcomes)}$$
$$\text{morning outcomes} = \text{function(beliefs of afternoon prices)}$$

- What's next? How beliefs are formed and adjust to $\Delta\bar{\theta}$

Benchmark and Variants

- Frictionless Benchmark: REE + CK
  - replicates Arrow-Debreu despite sequential and decentralized trading
- Type-1 Variants: replace REE with solution concepts that capture
  - Tatonnement dynamics [main example]
  - Cobweb dynamics
  - Level-k Thinking
  - Reflective Equilibrium as in Garcia-Schmidt and Woodford (2015)
  - Discounting as in Gabaix (2016)
- Type-2 Variant: maintain REE but drop CK
  - in line with literature on HOB reviewed in our handbook chapter

Frictionless Benchmark

Assumption. Rational expectations plus commonly-shared information

Lemma. There exists a function $\mathcal{T}$ such that REE satisfies

$$\bar{p}^* \;=\; \mathcal{T}\left(\, \mathbb{E}[\bar{p}^*]\,,\; \bar{\theta}\,\right)$$

Also, $\alpha \equiv \frac{\partial \mathcal{T}}{\partial \hat{p}} \in (0,1)$, which means that $\mathcal{T}$ is a contraction mapping.

Corrolary. The equilibrium exists and is unique. Furthermore,

$$\mathbb{E}[\bar{p}^*] = \bar{p}^* = \mathcal{P}(\bar{\theta}).$$

Micro vs Macro, and PE vs GE

Proposition

There exists scalars ϵ^{micro} and ϵ^{Macro} such that

$$\Delta\bar{q} = \epsilon^{Macro}\Delta\bar{\theta}$$
$$\Delta q_m = \Delta\bar{q} + \epsilon^{micro}\left(\delta_m\Delta\bar{\theta} + z_m\right)$$

- PE vs GE

$$\epsilon^{micro} = \text{PE} \qquad \epsilon^{Macro} = \text{PE} + \text{GE}$$

  - PE effect = holding beliefs of $\bar{p}^*$ fixed = as if idiosyncratic shock
  - GE effect = adjustment in beliefs of $\bar{p}^*$

Micro vs Macro, and PE vs GE

Proposition

There exists scalars ϵ^{micro} and ϵ^{Macro} such that

$$\Delta\bar{q} = \epsilon^{Macro}\Delta\bar{\theta}$$
$$\Delta q_m = \Delta\bar{q} + \epsilon^{micro}\left(\delta_m \Delta\bar{\theta} + z_m\right)$$

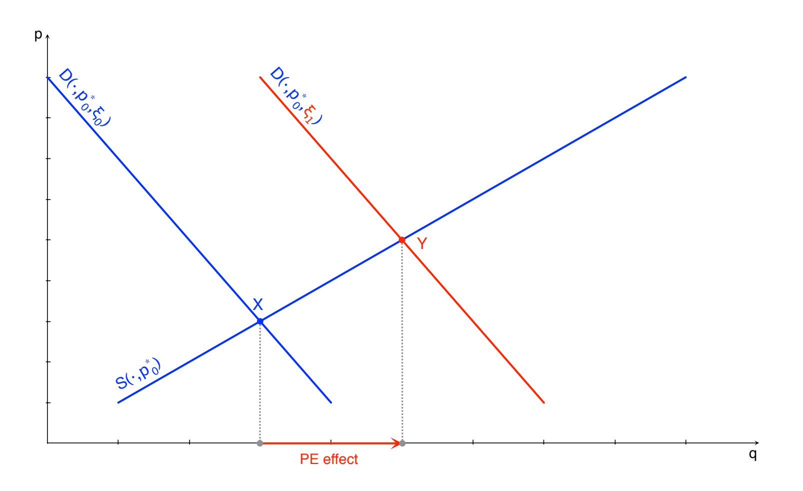
- PE vs GE

$$\epsilon^{micro} = \text{PE} \qquad \epsilon^{Macro} = \text{PE} + \text{GE}$$

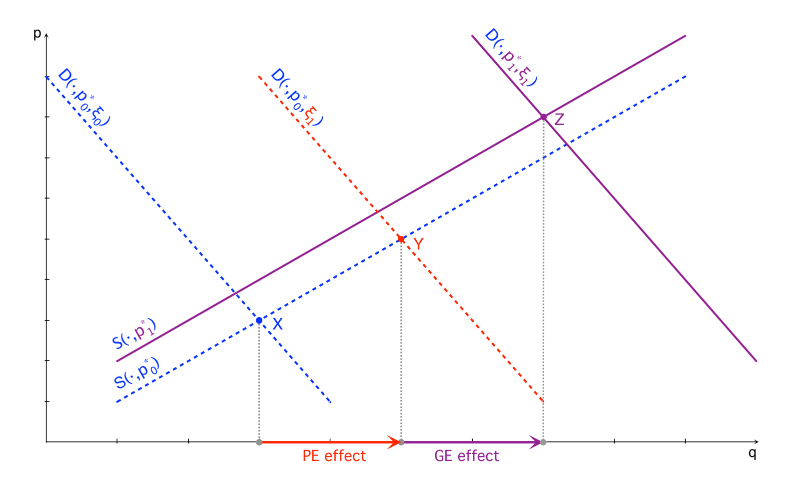
  - PE effect = holding beliefs of $\bar{p}^*$ fixed = as if idiosyncratic shock
  - GE effect = adjustment in beliefs of $\bar{p}^*$

- Two scenarios:
  - GE amplifies PE $\Leftrightarrow$ $\epsilon^{Macro} > \epsilon^{micro}$ (e.g., AD multipliers)
  - GE offsets PE $\Leftrightarrow$ $\epsilon^{Macro} < \epsilon^{micro}$ (e.g., competing for resources)

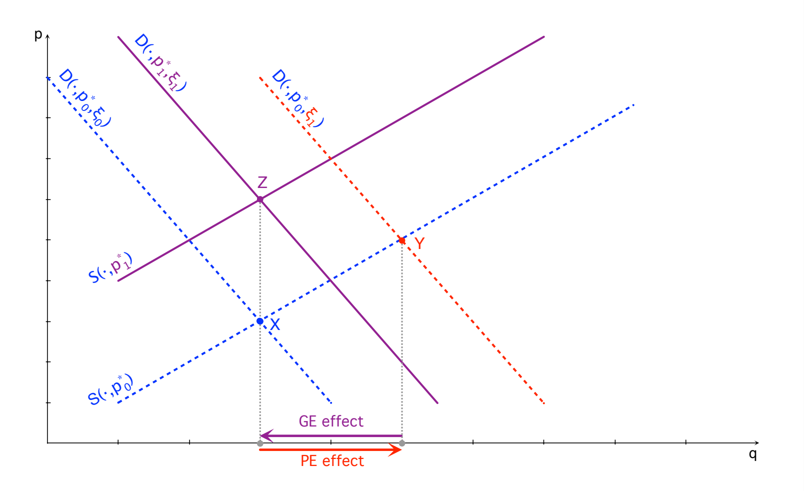
PE effect



GE offsets PE



GE attenuates PE



Connection to Empirical Work

- Recent empirical macro:
  - exploit cross-sectional variation in δ_m, exposure to aggregate shocks
  - avoid contamination of aggregate time series by other shocks
  - e.g., Mian-Sufi, Nakamura-Steinsson, Beraja-Hurst-Ospina, etc
- Mind the gap!
  - what is of interest: ϵ^{Macro}
  - what is estimated: ϵ^{micro}
- Problem: time fixed effect removes, not only the contaminating shocks, but also the GE effect of the shock of interest
- Our contribution (in the sequel): lessen the gap by dampening GE

Method 1: Beyond REE

- Allow for *irrational* conjectures of how $\Delta\bar{\theta}$ affects
  - relevant future prices and/or
  - current activity in other marketplaces

- Conjectures, however, are not entirely arbitrary

- They are generated by cognitive processes that
  - capture Tatonnement, Level-k, or other plausible algorithms
  - can be indexed by "depth of reasoning"
  - converge to REE as depth of reasoning becomes infinite

Tatonnement

Assumption. Fix a $T \in (0, \infty)$. For all m,

$$\hat{E}_m[\bar{p}^*] = \hat{p}^* = \hat{P}^*(T)$$

where $\hat{P}^*$ is obtained by solving the following ODE:

$$\frac{d\hat{P}^*(t)}{dt} = N\left(\hat{P}^*(t), \bar{\theta}_{new}\right) \quad \forall t \geq 0$$

where N measures excess aggregate demand for afternoon goods

- Description of an instantaneous mental process
- t indexes iterations, not calendar time
- T defines depth of reasoning

Tatonnement: GE adjustment

Lemma. Price conjectures satisfy

$$\hat{p}^* \;=\; \bar{p}^*_{old} \;+\; w(T) \cdot (\bar{p}^*_{new} - \bar{p}^*_{old})$$

where w continuous and increasing, with $w(0) = 0$ and $w(\infty) = 1$, and $\bar{p}^*_{old}$ and $\bar{p}_{new}$ are the pre- and post-shock REE prices

- By varying cognitive depth T, vary extent of GE adjustment

Tatonnement: Spanning the Micro-Macro Gap

Proposition

Macro response given by

$$\Delta\bar{q} \;=\; \epsilon^{Tat} \cdot \Delta\bar{\theta}$$

where

$$\epsilon^{Tat} \;=\; \epsilon^{micro} \;+\; w(T) \cdot \left(\epsilon^{Macro} - \epsilon^{micro}\right)$$

- By varying T, span gap between ϵ^{micro} and ϵ^{Macro}
  - lower depth of reasoning $\Rightarrow$ ϵ^{Tat} closer to ϵ^{micro}

Method 2: Lack of Common Knowledge

- Key insight: lack of CK arrests adjustment in beliefs
  - recasts previous result from "off equilibrium" to "on equilibrium"
  - accommodates GE attenuation along REE
- Key applied lesson: Once again, span gap between ϵ^{micro} and ϵ^{Macro}

Method 2: Lack of CK

Assumption. Agents form Rational Expectations, but do not have CK of either the aggregate shock or the resulting outcomes in other marketplaces. Instead, info in market m modelled as a local signal

$$s_m = \Delta\bar{\theta} + v_m$$

where v_m is i.i.d., Normal, mean 0, variance $\sigma_v^2 > 0$

- HOB satisfy $\bar{\mathbb{E}}^h[\bar{\theta}] = \bar{\theta}_{old} + \lambda^h \Delta\bar{\theta}$ where $\lambda = \frac{1}{1+(\sigma_v^2/\sigma_\theta^2)} \in (0,1)$
- λ parametrizes degree of CK
- Note: HOB are anchored!

  $\bar{\mathbb{E}}^h[\bar{\theta}] \to \bar{\theta}_{old}$ as $h \to \infty$ regardless of how close λ is to 1

Equilibrium Characterization

Lemma. REE now satisfies

$$\bar{p}^* \;=\; \mathcal{T}\left(\, \bar{\mathbb{E}}[\bar{p}^*] \,,\; \bar{\theta} \,\right)$$

where $\mathcal{T}$ same contraction mapping as before

- Iterating $\Rightarrow$ equilibrium beliefs of prices $=$ HOB of fundamentals

$$\bar{\mathbb{E}}\left[\bar{p}^*\right] = \gamma(1-\alpha)\sum_{h=1}^{\infty}\alpha^{h-1}\bar{\mathbb{E}}^h\left[\bar{\theta}\right]$$

- Key implication (1): anchored HOB $\Rightarrow$ anchored price expectations
- Key implication (2): higher $|\alpha|$ implies
  - stronger GE under CK
  - but also more weight on anchored HOB without CK

Lack of CK as GE attenuation

Lemma. REE satisfies

$$\bar{\mathbb{E}}\left[\bar{p}^*\right] = \bar{p}^*_{old} + \pi(\lambda)\left(\bar{p}^*_{new} - \bar{p}^*_{old}\right)$$

where π is continuous and strictly increasing, $\pi(0) = 0$, and $\pi(1) = 1$.

- By varying degree of CK λ, vary extent of GE adjustment

Lack of CK: Spanning the Micro-Macro Gap

Proposition

There exists a scalar ϵ^{inco} such that

$$\Delta\bar{q} \;=\; \epsilon^{inco} \cdot \Delta\bar{\theta}$$

Furthermore,

$$\epsilon^{inco} \;=\; \epsilon^{micro} \;+\; \pi(\lambda) \cdot \left(\epsilon^{Macro} - \epsilon^{micro}\right)$$

where π is continuous and strictly increasing, $\pi(0) = 0$, and $\pi(1) = 1$.

- By varying λ, span gap between ϵ^{micro} and ϵ^{Macro}
  - less CK $\Rightarrow$ ϵ^{inco} closer to ϵ^{micro}

Equivalence: Lack of CK = Tatonnement

Corollary. $\forall T$ in Tatonnement, $\exists \lambda$ in incomplete-info variant s.t.

- rational expectations in latter = ad hoc conjectures in former
- same observables
- equal GE attenuation

The converse is also true.

- In this respect, removing CK is a substitute to relaxing REE
- Does this lesson extend to other relaxations of REE?

Cobweb and Level-k Thinking

- Recast Walrasian economy as a Beauty-Contest Game

$$\bar{p}^* = \mathcal{T}\left(\hat{E}[\bar{p}^*], \bar{\theta}\right) \Longleftrightarrow \bar{q} = \mathcal{BR}\left(\hat{E}[\bar{q}], \bar{\theta}\right)$$

- Cobweb dynamics $=$ iterating on $\mathcal{T}$
- Level-k Thinking $=$ iterating on $\mathcal{BR}$

Cobweb and Level-k Thinking

- Recast Walrasian economy as a Beauty-Contest Game

$$\bar{p}^* = \mathcal{T}\left(\hat{E}[\bar{p}^*], \bar{\theta}\right) \Longleftrightarrow \bar{q} = \mathcal{BR}\left(\hat{E}[\bar{q}], \bar{\theta}\right)$$

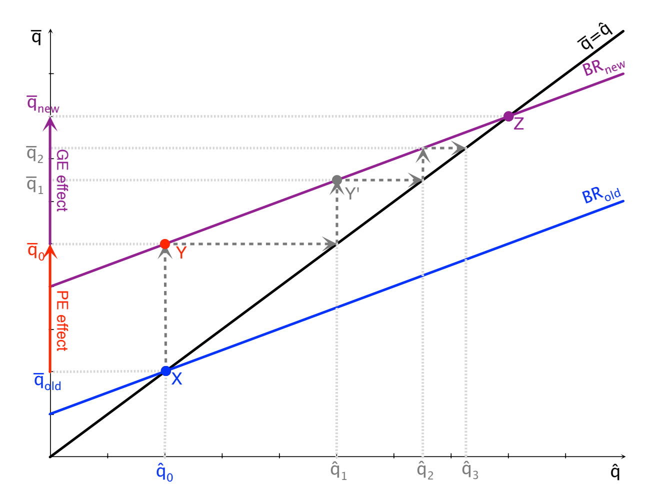
- Cobweb dynamics = iterating on $\mathcal{T}$
- Level-k Thinking = iterating on $\mathcal{BR}$

Corollary

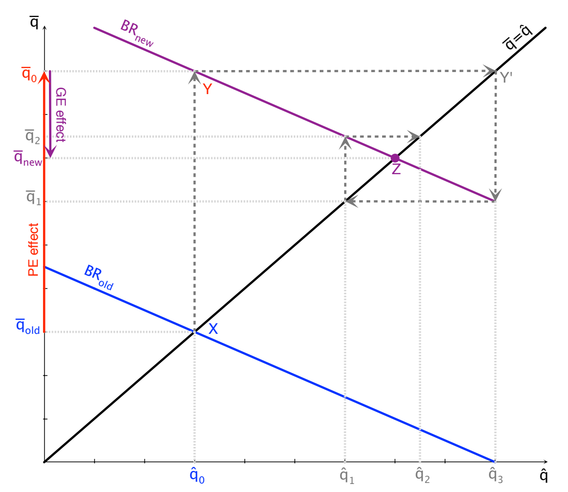
Cobweb dynamics = Level-k Thinking
but, in general, $\neq$ GE attenuation

- Reason: conjectures can "overshoot" when $\epsilon^{Macro} < \epsilon^{micro}$
  - when GE offsets PE, GE effect can be amplified

Scenario 1: GE amplifies PE, or Strategic Complementarity



Scenario 2: GE offsets PE, or Strategic Substitutability



Reflective Equilibrium as in Garcia-Schmidt and Woodford (2015)

- Similar to Level-k Thinking
- But, instead of iterating on BR, let conjecture solve ODE

$$\text{d[conjecture]} = \text{f(gap between conjecture and BR to it)}$$

- Perhaps unintentional: it cures "overshooting pathology"!

Corollary

Reflective Equilibrium = Lack of CK = GE attenuation

Discounting as in Gabaix (2016)

Assumption.

$$\text{(perceived change in } \bar{p}^* \text{ or } \bar{q}) \; = \; \chi \cdot \text{(true change)}$$

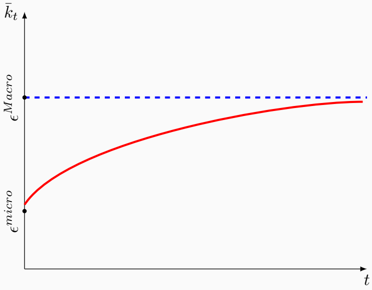
for some $\chi \in (0, 1)$

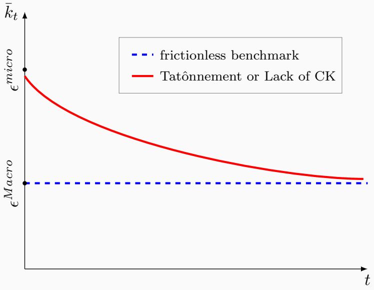
- Implication: GE is attenuated
- But note: one is free to assume $\chi > 1$, in which case GE is amplified
  - a priori, why is it more plausible/desirable to assume that behavioral agents expect the others to under-react than to over-react?
- Same caveat with $\epsilon-$equilibrium or Akerlof-Yellen
  - "near rationality" can either reduce or increase the micro-to-macro gap
- By contrast: with lack of CK, GE is *necessarily* attenuated

GE Takes Time

- Sought-after notion: GE takes time
  - GE effect is weak on impact, but gets stronger with time
  - gap between micro and macro elasticity is small only in SR
- Framework: example with trading over large number of periods
- Method 1: replace REE concept
  - requires additional assumption
  - that agents become "deeper thinkers" as time passes
- Method 2: relax CK assumption
  - follows naturally from endogenous learning
  - as agents observe past outcomes

GE Takes Time





Lessons and Applications

Take-Home Lessons

- GE attenuation = robust prediction of relaxing either REE or CK
- In this regard, lack of CK = substitute of relaxing REE
- But, some differences:
  1. immune to Lucas critique
  2. ties degree of attenuation to informational proximity
  3. no ad hoc "default" point
  4. naturally implies that attenuation decays with time
  5. easier to adapt to stationary settings with recurring shocks
- In our view, this tilts the balance in favor of lack of CK

Applications (in companion papers)

- Forward guidance [next]
- Ricardian Equivalence and Fiscal Policy
- AD and Keynesian Multipliers without Nominal Rigidity
- ...

Forward Guidance without Common Knowledge

Context

- A NK Economy at the ZLB
- Forward Guidance
  - promise to keep low rates in the future (after exiting ZLB)
- The Puzzle
  - quantitative large effect
  - effect increases with horizon
- It's all about GE
  - PE effect decreases with horizon

Main Findings

- Removing CK $\Rightarrow$
  - attenuates GE effects
  - anchors $E[y]$ and $E[\pi]$ for given $E[R]$

- In simple words:
  - *"Hard for PM to shift expectations of inflation and income even if she can manage expectations of policy"*

- Also: attenuation increases with horizon
  - lessens forward-guidance puzzle
  - justifies front loading' of either MP or fiscal stimuli

- Additional results
  - paradox of flexibility, discounted Euler/NKPC...

Roadmap

1. Framework
2. Review the Puzzle
3. Remove CK
    - recast IS and NKPC as Beauty Contests
    - unearth HOB
4. Revisit Forward Guidance
5. Robustness and Additional Results

Households

- Continuum of households/consumers
- Preferences

$$\mathcal{U}_i = \sum_{t=0}^{+\infty} \beta^t \left(\log c_{it} - \tfrac{1}{1+\epsilon} n_{it}^{1+\epsilon} \right),$$

- Budget constraint

$$c_{it} + s_{i,t} = a_{it} + w_{it} n_{it} + e_{it}$$

$$a_{i,t} = (1 + r_t)\, s_{i,t-1} = \frac{1 + R_{t-1}}{1 + \pi_t} s_{i,t-1}$$

  - incomplete markets in the sense of no risk-sharing
  - but no liquidity constraints (only for simplicity)

Firms

- Final goods produced by a competitive sector

$$y_t = \left(\int_0^1 \left(y_t^j \right)^{\frac{\varsigma - 1}{\varsigma}} dj \right)^{\frac{\varsigma}{\varsigma - 1}}$$

- Each variety j produced by a monopolist

$$y_t^j = l_t^j$$

- Nominal rigidity a la Calvo
  - fraction $1 - \theta$ changes price each period

Shocks

- Shocks to markups
  - μ_t^j at the firm level
  - μ_t at the aggregate level
- Shocks to wages
  - $w_t^j = w_t u_t^j$ at the firm level
  - $w_{it} = w_t \xi_{it}$ at the household level
- Modeling role of shocks: limit aggregation of information
- Note: henceforth work with log-linearized model

ZLB and Forward Guidance

- Let T index length of liquidity trap and horizon of FG
  - $t < T - 1$: ZLB binds and $R_t = 0$ for all
  - $t \geq T + \Delta$: "natural level" and $y_t = \pi_t = 0$
  - let $\Delta = 1$ for simplicity
- Forward guidance
  - policy announcement at $t = 0$ of likely R_T
  - modeled as $z = R_T + noise$
- Standard model: common knowledge of z

The Standard NK Model

- Nest standard NK model by imposing CK

$$y_t = -\left\{R_t + E_t\left[\pi_{t+1}\right]\right\} + E_t\left[y_{t+1}\right]$$
$$\pi_t = \kappa y_t + \beta E_t\left[\pi_{t+1}\right] + \mu_t$$

- *Question:* How does y_0 vary with $E_0[R_T]$?

The Standard NK Model

- Nest standard NK model by imposing CK

$$y_t = -\{R_t + E_t[\pi_{t+1}]\} + E_t[y_{t+1}]$$
$$\pi_t = \kappa y_t + \beta E_t[\pi_{t+1}] + \mu_t$$

- *Question:* How does y_0 vary with $E_0[R_T]$?
- *Answer:* There exists a function ϕ^* such that

$$y_0 = -\phi^*(T) \cdot E_0[R_T]$$

  - ϕ^* measures power of FG under CK
  - note: ϕ^* invariant to precision of z (credibility etc)

The Puzzle

- ϕ^* increases without bound as we increase T
  - the further into the future FG operates, the stronger its effect!
- ϕ^* quantitatively large even for T=few years
- Why? GE feedback between spending and inflation
  - as we iterate IS and NKPC backwards, effects keep pilling up!

What's Next

- Recast NK model as a multi-layer Beauty Contest
    - clarify GE mechanisms
    - recast them as dynamic strategic complementaries
    - unearth role of HOB
- Relax CK $=$ anchor expectations $=$ attenuate GE effects

Removing Common Knowledge

The IS Curve *with* Common Knowledge

$$y_t = -E_t\left[r_{t+1}\right] + E_t\left[y_{t+1}\right]$$

The IS Curve *WITHOUT* Common Knowledge

$$y_t = -\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[r_{t+k-1}]\right\} + (1-\beta)\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t\left[y_{t+k}\right]\right\}$$

The IS Curve *WITHOUT* Common Knowledge

$$y_t = -\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[r_{t+k-1}]\right\} + (1-\beta)\left\{\sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t\left[y_{t+k}\right]\right\}$$

- dynamic beauty contest among consumers
  - follows from PIH and $y = c$
  - modern version of Keynesian income multiplier
  - optimal c depends on E of future real rates and income
    $\Rightarrow$ equilibrium y depends on HOB of future R and π

The NK Philips Curve *with* Common Knowledge

$$\pi_t = \kappa y_t + \beta E_t \left[\pi_{t+1}\right] + \mu_t$$

The NK Philips Curve *WITHOUT* Common Knowledge

$$\pi_t = \kappa y_t + \kappa \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \, \bar{E}_t^f [y_{t+k}] \right\} + \frac{1-\theta}{\theta} \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \, \bar{E}_t^f [\pi_{t+k}] \right\} + \mu_t$$

The NK Philips Curve *WITHOUT* Common Knowledge

$$\pi_t = \kappa y_t + \kappa \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \, \bar{E}_t^f [y_{t+k}] \right\} + \frac{1-\theta}{\theta} \left\{ \sum_{k=1}^{+\infty} (\beta\theta)^k \, \bar{E}_t^f [\pi_{t+k}] \right\} + \mu_t$$

- dynamic beauty contest among firms
  - follows from optimal pricing with Calvo
  - optimal reset prices depend on expectations of future MC
    $\Rightarrow$ equilibrium π depends on HOB of future y and π

The NK Model without Common Knowledge

- IS = beauty contest *within* group of consumers
  - y as a function of HOB of R and π
- NKPC = beauty contest *within* group of firms
  - π as a function of HOB of y
- Overall equil = upper-layer beauty contest *between* two groups
  - feedback between y and π

Summary

- three GE effects
  - one "hidden" behind Euler/IS
  - another "hidden" behind NKPC
  - and third across the two
- standard practice: impose CK = maximize all GE effects
- our paper: relax CK = attenuate all GE effects

Revisiting Forward Guidance

Baseline Scenario

- Information structure
  - initial private signal

$$x_i = z + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0, \sigma_\epsilon^2)$$

  - ϵ_i can be interpreted as the product of rational inattention
  - limit with no learning (large shocks)

- Degree of CK indexed by $\lambda \in (0, 1]$
  - HOB satisfy $\bar{\mathbb{E}}^h[R_T] = \lambda \bar{\mathbb{E}}^{h-1}[R_T] \quad \forall h \geq 2$
  - consumers vs firms: λ_c vs λ_f
  - benchmark nested with $\lambda_c = \lambda_f = 1$

The Power of Forward Guidance

- *Question:* How does y_0 vary with $\bar{E}_0[R_T]$
- *Answer:* There exists a function ϕ such that

$$y_0 = -\phi\left(\lambda_c, \lambda_f, T\right) \cdot \bar{E}_0[R_T]$$

  - note: answer orthogonal to how much $\bar{E}_0[R_T]$ itself varies
  - comparison: ϕ vs ϕ^*

Main Results

1. Attenuation for any horizon
   - $\phi < \phi^*$
   - lower λ_c or $\lambda_f \Rightarrow$ lower ϕ

Main Results

1. Attenuation for any horizon
   - $\phi < \phi^*$
   - lower λ_c or $\lambda_f \Rightarrow$ lower ϕ

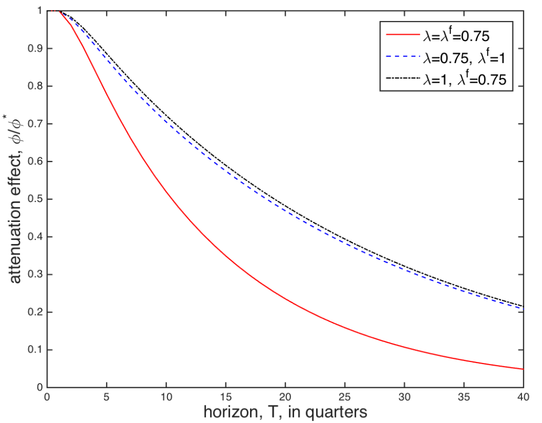
2. Attenuation effect increases with horizon
   - ratio ϕ/ϕ^* decreases in T
   - why? longer horizons = beliefs of higher order
   - iterating on the equations of the model = iterating on HOB
   - underscores sensitivity of related predictions

Main Results

1. Attenuation for any horizon
    - $\phi < \phi^*$
    - lower λ_c or $\lambda_f \Rightarrow$ lower ϕ
2. Attenuation effect increases with horizon
    - ratio ϕ/ϕ^* decreases in T
    - why? longer horizons = beliefs of higher order
    - iterating on the equations of the model = iterating on HOB
    - underscores sensitivity of related predictions
3. Attenuation effect grows without limit as $T \to \infty$
    - $\phi/\phi^* \to 0$ as $T \to \infty$, even if $\lambda \approx 1$
    - small friction can have large consequences
    - for λ_c small enough, $\phi \to 0$ in absolute, not only relative to ϕ^*

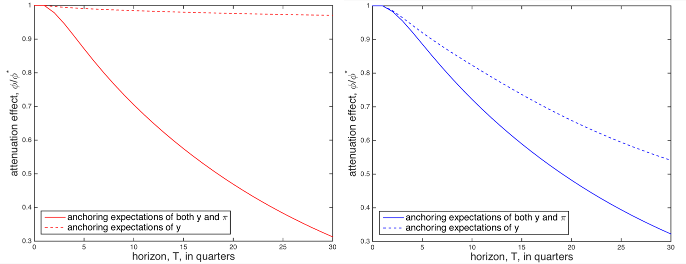
A Numerical Illustration

- Modest friction: $\lambda_c = \lambda_f = 0.75$
  - 25% prob that *others* have failed to hear announcement
  - all other parameters standard, as in McKay et al (2015)



A Decomposition

- Anchoring $E[y]$ vs $E[\pi]$: which one matters most?
  - consumers: most action comes from anchoring $E[\pi]$ (i.e., $E[r]$)
  - firms: anchoring $E[y]$ (i.e, $E[\text{realMC}]$) relatively more important



  - dashed line: $E[\pi]$ forced to be as in frictionless benchmark

Robustness

- Generalization to almost arbitrary information structures
  - learning through current and past outcomes
  - anticipation of future policy announcements
- Formalization of notion "longer horizons = beliefs of higher order"
  - corollary: argument for front-loading policy

Fiscal Stimuli: Back- vs Front-Loading

- Standard NK prediction:
  - fiscal stimuli work because they trigger inflation
  - better to back-load so as to “pile up” inflation effects
- Our twist:
  - such piling up = iterating HOB
  - not as potent when CK assumption is dropped
  - better to front-load so as to minimize coordination friction

Paradox of Flexibility

- In standard NK model, ϕ^* increases with $1 - \theta$
  - paradox: price flexibility makes MP more powerful
  - related: price flex exacerbates ZLB constraint/deflationary spiral
- Our twist: ϕ/ϕ^* decreases with $1 - \theta$
  - price flexibility increases attenuation effect
  - lack of CK lessens paradox of flexibility
- Intuition: same causes (GE effects), same resolution

Discounted Euler and Discounted NKPC

- *As if* representative agent discounts more heavily the future

$$
\begin{aligned}
y_t &= \Lambda_t E_t\left[y_{t+1}\right] - \left\{R_t - \lambda_t E_t\left[\pi_{t+1}\right]\right\} \\
\pi_t &= M_t \beta E_t\left[\pi_{t+1}\right] + m_t \kappa y_t
\end{aligned}
$$

  where $\Lambda_t, M_t, m_t \in (0,1)$.

- Reminiscent of McKay et al. (2015) and Gabaix (2016), but:
  - preserve micro-foundations and Euler at individual level
  - preserve Rational Expectations
  - learning $\Rightarrow$ discounting is "transitory"

- Discounting = manifestation of anchored but rational beliefs

Conclusion

Conclusion

- Relaxing Common Knowledge
  - structured (superior?) substitute to certain non-REE approaches
  - attenuates or slows down GE mechanisms
  - increases relevance of empirical work a la Mian-Sufi
  - lessens NK puzzles
  - favors front-loading
- Related ongoing work
  - revisit Ricardian equivalence and effects of G, deficits
  - a theory of Keynesian multipliers without nominal rigidity
  - ...