# Durability, Deadline, and Election Effects in Bargaining*

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**Abstract**

We propose a tractable model of bargaining with optimism. The distinguishing feature of our model is that the bargaining power is *durable* and changes only due to important events such as elections. Players know their current bargaining powers, but they can be optimistic that events will shift the bargaining power in their favor. We define *congruence* (in political negotiations, *political capital*) as the extent to which a party's current bargaining power translates into its expected payoff from bargaining. We show that durability increases congruence and plays a central role in understanding bargaining delays, as well as the finer bargaining details in political negotiations. Optimistic players delay the agreement if durability is expected to increase in the future. The applications of this durability effect include *deadline* and *election effects*, by which upcoming deadlines or elections lead to ex-ante gridlock. In political negotiations, political capital is highest in the immediate aftermath of the election, but it decreases as the next election approaches.

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**Keywords:** bargaining power, congruence, optimism, delay, deadline effect, election effect, durability.

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# 1 Introduction

One of the most robust empirical regularities in bargaining is the phenomenon called *the deadline effect*: the agreement is delayed until the very last minute before the deadline (see e.g., Roth, Murnighan, and Schoumaker, 1988)). The labor negotiations are settled only at the "eleventh hour" before a strike starts and the litigants pursue costly negotiations only to reach an "agreement on the steps of the courthouse." These are all well-known by the practitioners of negotiation. Recently, general public also witnessed dramatic examples of deadline effect in the political arena. The Democratic and Republican leaders reached an agreement to raise the debt ceiling on July 31 2011 and passed a law only on August 2, 2011, under the threat of a US Treasury default on August 3, 2011. In "fiscal cliff" negotiations of late 2012, they reached an agreement on a new tax law late on the new year eve, in order to avert an across-the-board tax increase starting in the new year (the president signed the bill on January 2, 2013).

Deadlines are not the only sources of political gridlock. Elections seem to be another factor. Mayhew (1991) shows that the US Congresses between 1947-1990 enacted 25% fewer important laws on average when they convened in the two years before a presidential election compared to the two years after. Binder (2000) notes as an example that House Republicans were reluctant to negotiate over tax cuts in late 1999, after President Bill Clinton vetoed their initial proposal, in the hopes of regaining the presidency. Their beliefs were in fact vindicated, and under the presidency of George W. Bush, Republicans passed a sweeping tax cut legislation in 2001 shortly after the election.

One rationale for gridlock, proposed by many authors, is *optimism*: players might be holding out since they both perceive there will be a better opportunity to strike deal (see Yildiz (2011) for a survey of the literature). But why does optimism lead to gridlock at certain times, such as before deadlines or elections, but not at other times, such as after elections? When gridlock is avoided, how does optimism affect the finer details of bargaining outcomes—such as the players' shares from an agreement? To address these questions, we develop a bargaining model with the key feature that the bargaining power is somewhat *durable.* We show that durability plays a central role in understanding bargaining delays driven by optimism, as well as the nature of agreement outcomes in political negotiations.

Our model features two risk-neutral players, say Ann and Bob, who are negotiating in order to divide a dollar. Ann's bargaining power at time $t$, $\pi_t^{Ann} \in [0, 1]$, determines Ann's share of the surplus from agreeing today rather than negotiating for one more period. Bob's bargaining power is the residual, $\pi_t^{Bob} = 1 - \pi_t^{Ann}$. In sequential bargaining, the bargaining power corresponds to the probability of making a take-it-or-leave-it offer at time $t$; there will be another offer in the next round if the offer is rejected. We view the bargaining power as capturing in reduced form the fundamental bargaining strength of the parties. For example, in legal negotiations, the bargaining power of a litigant might reflect the extent to which the available evidence supports her case. In congressional negotiations, in which players correspond to parties, the bargaining power might capture the extent to which the President and the Congressmen support the party on the negotiated issue. We specify the bargaining power as an exogenous stochastic process in continuous time, and characterize how it translates into endogenous bargaining outcomes.

We capture players' optimism by allowing them to have subjective beliefs about the bargaining power process. At time $t$, Ann may expect her bargaining power at a future time $t^* > t$ to be $3/4$ on average, while, symmetrically, Bob expects his own bargaining at time $t^*$ to be $3/4$ on average. Observe that Bob is optimistic about his own bargaining power relative to Ann, who expects Bob's bargaining power at time $t^*$ to be only $1/4$ on average. Likewise, Ann is optimistic about her bargaining power relative to Bob. We quantify players' optimism by the extent to which the sum of their expectations of their own bargaining powers exceeds 1. In this example, their optimism about time $t^*$ is measured by $3/4 + 3/4 - 1 = 1/2$.

Our key assumption is that the bargaining power process is somewhat durable, which also puts some discipline on optimism. Specifically, we assume the bargaining power changes only due to important events. In political negotiations, the bargaining power would naturally change due to elections, but it could also change due to major events that might influence the public opinion—such as political scandals or international conflict. The players observe—and therefore agree on—the current value of the bargaining power. They also agree on how frequently—and in the case of elections, when—important events happen. But they have subjective and optimistic beliefs that the events will shift the bargaining power in their favor. This setup ensures that the players cannot hold very optimistic beliefs about the short run, especially if the bargaining power is very durable. The players can hold optimistic beliefs as they consider the more distant future.

Our model lends itself to a tractable and intuitive solution. A player's expected payoff from bargaining can be written as a weighted average of the player's current bargaining power, and a second term that depends on the players' beliefs about the long-run levels of the bargaining power. We refer to the weight on the current bargaining power as *congruence*. When players reach agreement, congruence captures the extent to which the agreement shares reflect their current bargaining powers. In political negotiations, congruence can also be thought of as a form of *political capital*: the extent to which a player that has "won" the last election (and thus, has higher current power) receives a higher payoff from bargaining. If the political capital is high, and players reach agreement, then the player in power implements the outcomes in its favor.

Our model reveals a simple cost-benefit analysis of gridlock: the parties delay the agreement if their optimism about their shares in a future agreement is higher than the cost of waiting. Optimism about shares is in turn equal to the product of optimism about bargaining power and congruence—which translates bargaining power into shares. Therefore, to understand bargaining delays, it is necessary to understand what causes high optimism, and more subtly, what causes high congruence.

Durability turns out to be a key determinant of both optimism and congruence. It decreases optimism and increases congruence. Consequently, agreement is delayed in run up to times when durability increases. To see the rationale, suppose bargaining power becomes more durable at some $t^*$ and the durability remains constant thereafter. We show that the players reach an agreement at $t^*$, and the congruence at $t^*$ is an increasing function of the durability level at $t^*$. Durability not only reduces optimism, pushing the players towards agreement, but it also ensures that the current bargaining power has a high influence on the agreement outcomes. In contrast, prior to $t^*$, the players can hold highly optimistic beliefs about the bargaining power at $t^*$—because the bargaining power is less durable there. The combination of high congruence at $t^*$ and high level of optimism about $t^*$ leads to a high level of optimism about the shares at $t^*$, enticing players to wait until $t^*$. We refer to this phenomenon as the *durability effect.*

For a concrete example, imagine Ann and Bob's bargaining power becomes constant starting at time $t^*$; that is, $\pi_t^{Ann} = \pi_{t^*}^{Ann}$ and $\pi_t^{Bob} = \pi_{t^*}^{Bob}$ for each $t \geq t^*$. At time $t^*$ onwards, the future bargaining powers are known. Hence, at $t^*$, they reach an agreement, giving $\pi_{t^*}^{Ann}$ to Ann and $\pi_{t^*}^{Bob}$ to Bob. Congruence at time $t^*$ is one, the highest possible level. Now, imagine that the bargaining power is less durable before $t^*$, and players are highly optimistic about $t^*$ at some $t < t^*$: they both expect their bargaining powers at time $t^*$ to be 3/4, as described earlier. Suppose the time $t$ value of receiving one dollar at time $t^*$ is more than 2/3 dollars. Then, Ann believes she can obtain more than $2/3 \times 3/4 = 1/2$ dollars simply by waiting until time $t^*$. Similarly, Bob believes he can obtain more than 1/2 dollars by waiting until time $t^*$. Clearly, there is no division of the dollar at time $t$ that can satisfy both players' optimistic expectations, and they disagree at $t$.

Times such as $t^*$ at which durability increases and leads to high congruence are common in practice. Elections provide a natural application. In political negotiations, the bargaining power might change considerably depending on which party will win an upcoming the election. Moreover, the parties' post-election bargaining powers are unlikely to change significantly for a considerable while (e.g., until the next major election). In view of these observations, we establish an *election effect*: optimistic parties disagree before the election, and agree after the election with terms that are congruent with the interests of the winning party—consistent with Republicans passing a tax cut legislation in 2001.

Less obviously, we show that deadlines provide another application of the durability effect. Suppose $t^*$ corresponds to a time at which a possibly stochastic deadline becomes likely to arrive. If the players do not agree by the time the deadline arrives, then they receive zero. In this setting, we establish a deadline effect: optimistic players delay agreement before time $t^*$, and agree at time $t^*$ with terms that are congruent with their bargaining powers—as in the earlier example. Intuitively, even though the bargaining power after time $t^*$ is not durable in the strict sense of the word, it is highly durable in the sense that it is unlikely to change much before the stochastic deadline arrives.

Our unifying explanation for gridlock with optimism is, then, an increase in *the effective durability* of bargaining power. Deadlines and elections are two (seemingly distinct) phenomena both of which increase effective durability and lead to high congruence. Consistent with this intuition, we find that the severity of gridlock in these settings depends on—among other aspects—how much the effective durability increases. For instance, the deadline effect is more prominent when the deadline is less uncertain, and the election effect is more prominent when the bargaining power is more durable in non-election times. We also establish additional comparative statics for deadline and election effects, linking the severity of gridlock to the players' optimism, their cost of delay, and—for the case of elections—the parties' relative popularity with voters.

While deadlines and elections can both cause gridlock, elections are often associated with more structure. For instance, they are held periodically at fixed intervals. We use our model to analyze political negotiations with periodically repeated elections, and show that congruence plays a central role also in this context. Most importantly, we show that congruence (or political capital) is highest immediately after the election, but it gradually declines over the election cycle. As time passes, longer-run factors such as the upcoming election also start to affect the agreement shares. As the next election draws closer, the players reach what might be called "compromise outcomes." The party in power is forced to leave a sizeable share of the surplus to the other party. Intuitively, the party without the power has a credible threat to delay the agreement until the next election, which enables it to extract some surplus. As the election draws even closer, the threat becomes real due to optimism and the election effect, which induces disagreement until after the election.

Our final analysis concerns political negotiations in which a stronger election—that resets the bargaining power with a higher probability—periodically alternates with a weaker election. We view this setting as capturing (in reduced form) some important features of the political cycle in the US. In one application, the stronger election can be thought of as the four-year elections that feature presidential and congressional elections, whereas the weaker election captures the midterm elections that feature only congressional elections. In another application, the stronger election captures an election in which the incumbent president cannot run for the office due to a binding two-term limit, whereas the weaker election is one in which the incumbent can rerun. The latter election is arguably associated with less optimism because the incumbent is generally thought to have an advantage in elections, which reduces the chance of a close election (see Section 3).

With alternating elections, we find that there is a longer period of delay before the stronger election compared to the weaker election. Moreover, political capital (or congruence) is also higher in the aftermath of the stronger election. Intuitively, the stronger election constitutes a greater change in durability, which leads to more severe gridlock in its run-up and a greater political capital in its aftermath. In the context of presidential elections with term limits, this result suggest a lame duck effect: we predict that presidents that cannot be reelected are associated with lower political capital—or more compromise—as well as a longer period of delay before the next presidential election. We test the latter prediction by extending Mayhew's (1991) analysis of legislative gridlock. The US Congresses that convene before presidential elections with a binding term limit seem to enact fewer important laws—consistent with our lame duck effect—but the result is not statistically significant due to small sample size.

**Literature Review** Existing bargaining models that build upon Rubinstein (1982) specify an explicit bargaining protocol, e.g., who makes an offer and when. Several papers, such as Yildiz (2003, 2004a) and Ali (2006), assume optimistic beliefs about the protocol. In these models, the bargaining power is implicitly determined by the protocol. Moreover, the implied bargaining power is often highly nondurable. For example, in Rubinstein's (1982) alternating-offer model, the bargaining power shifts from one side to other every period. This is also the case in the setups for the main results of Yildiz (2003) and Ali (2006), where the bargaining power is assumed to be serially independent. Our methodological innovation is to start with a bargaining model with an explicit bargaining power process, defined in real time, and allow players to hold optimistic beliefs about it. This allows us to model durability of bargaining power transparently and study the impact of durability in equilibrium.

It turns out that durability is a necessary ingredient for obtaining high congruence, and ultimately, for *robust* bargaining delays with optimism. We show that assuming serially independent (and thus, nondurable) bargaining power as in Yildiz (2003) leads to congruence that is bounded from above by the cost of one-period delay. Intuitively, without any impact on the future bargaining power, the current bargaining power can only affect the allocation of the current gain from trade, which is the cost of delaying agreement until the next period. This one-period cost is typically low except for one extreme scenario: a deterministic deadline. In this case, the cost of delaying agreement beyond the deadline is equal to the whole pie, which yields a high congruence of one at the deadline. Under optimism, this may lead to long delays as in the example above, which is similar to a two-period example in Yildiz (2003). In particular, one can obtain a deadline effect for deterministic deadlines using models with nondurable bargaining power. Nonetheless, that is a singular case. In all other cases, including cases that involve stochastic deadlines, the one-period cost of delay is vanishingly small in the continuous-time limit in which we allow the players negotiate frequently. Therefore, there cannot be any delay in the continuous-time limit. In particular, the deadline effect obtained in models with nondurable bargaining power is highly fragile, and disappears if the deadline is stochastic. This is problematic because stochastic deadlines appear to be common in practice (see Footnote 4 in Section 3).

A strand of literature focuses on the role of learning in generating bargaining delays (see, for instance, Yildiz (2004a), Thanassoulis (2010), Galasso (2012)). For instance, Yildiz

(2004a) shows that, when players learn about their future bargaining power, optimism leads to delay because each optimistic player $i$ waits for information in the hopes that information vindicates $i$ and persuades the other party $j$ to agree to $i$'s terms. Such a persuasion motive plays an implicit yet important role in our election effect. Intuitively, the election reveals to the parties their future bargaining powers, which in turn ensures that they reach agreement with high congruence. On the other hand, persuasion does not play an apparent role in our deadline effect. There, the parties reach agreement with high congruence in view of the high time discounting due to the deadline, even if they continue to disagree about the evolution of their bargaining powers. Thus, our durability effect provides a distinct mechanism for delay, which in some special cases (such as the election effect) can be mapped into the persuasion effect from the previous literature.

Our election effect is related to a political science literature that analyzes the sources of gridlock in legislative politics (see Binder (2003) for a review). In recent and parallel work, Ortner (2013) formalizes an alternative mechanism for gridlock before elections based on the idea that the terms of an agreement might affect the parties' prospects in the upcoming election. This "electoral concerns hypothesis" is complementary to our election effect with some differences that we discuss in Section 4.3 (see Remark 4). A key distinction is that electoral concerns can play a role only when the negotiated issue is visible and salient for voters' decisions, while optimism can cause a gridlock regardless of such visibility. Going beyond gridlock, we also analyze the nature of agreement outcomes in political negotiations. Our notion of political capital, as well as our results about its evolution over typical election cycles, appear to be new.

**Outline** Section 2 introduces our bargaining model and characterizes the equilibrium. This section also provides a closed form solution for a baseline scenario, which illustrates that congruence is increasing in effective durability. Section 3 establishes the durability effect, and obtains the deadline effect as its corollary. This section also establishes the comparative statics of the deadline effect. Section 4 is devoted to elections, establishing election effect, the related comparative statics, and the results about the periodic elections. Section 5 analyzes the more general determinants of delay in our framework. Section 6 discusses the extensions of our results to general bargaining processes. Section 7 concludes. The appendix contains the omitted proofs and some extensions of our baseline analysis.

# 2 Model and Equilibrium

In this section, we introduce our model and characterize its unique equilibrium. To simplify the exposition, we focus on a tractable and parsimonious bargaining power process. In our working paper, we analyze more general bargaining power processes and obtain the analogues of many of our results. We discuss these generalizations in Section 6.

## 2.1 Model

Consider two risk-neutral players, $i \in \{1, 2\}$, who negotiate over a continuum of times, $t \in \mathbb{R}_+ = [0, \infty)$, in order to pick some $x \in [0, 1]$. The players can strike a deal only at times on a grid $T = \{0, 1/n, 2/n, .., \}$, where $n = 2^m$ is a large integer. If the players strike a deal at time $t$, then they respectively receive the payoffs, $u_1(x) = x$ and $u_2(x) = 1 - x$, at that time. We assume that the players discount the future payoffs at a common, time-varying discount rate $r(t)$, which is bounded away from zero and $\infty$. We use the time-varying discount rate to capture stochastic deadlines (see Section 3). We also define the discount factor between times $t$ and $s$ as $\delta_{t,s} = e^{-\int_t^s r(\tilde{s})d\tilde{s}}$. The expected payoff of player $i$ at time $t$ from reaching an agreement at time $s$ is simply $\delta_{t,s} u_i(x)$.

Our key object is a player's *bargaining power*, denoted by $\pi_t^i$. As in the standard bargaining literature, we define the bargaining power as the probability that player $i$ makes a take-it-or-leave-it offer in a sequential bargaining game. We take the bargaining power $(\pi_t^1, 1 - \pi_t^2)_{t \in \mathbb{R}_+}$ as an exogenously given continuous-time stochastic process, and explore how it translates into actual bargaining outcomes in equilibrium. Formally, at each time $t \in T$, player $i$ is recognized as the proposer with probability $\pi_t^i$. The recognized player offers some $x \in [0, 1]$. If the other player accepts the offer, then the game ends, picking $x$. Otherwise, the game continues to the next period. We investigate the subgame perfect equilibrium of this game.

As usual, the model with take-it-or-leave-it offers provides a convenient approach to capture bargaining strength. However, we interpret the bargaining power more broadly as capturing the fundamental factors that affect how players split a given amount of surplus. In fact, as we will see, the bargaining power in our model is exactly equal to the fraction of the surplus from agreement a player gets (in addition to her continuation value from delay).

We focus on bargaining power processes that satisfy certain reasonable properties. We assume that, at any time $t$, players know the current realization of the bargaining power, $(\pi_t^1, 1 - \pi_t^2)$. Importantly, we also assume that the bargaining power is somewhat durable. More specifically, the bargaining power remains constant at its current level until some important event that affects the bargaining power occurs. These two assumptions ensure that there is considerable discipline on players' *short-run* beliefs.

Formally, consider a Poisson process with time-varying arrival rate $\lambda(t)$, which is assumed to be piecewise continuous. At each arrival, a new pair $(\pi^1, 1 - \pi^1)$ of bargaining powers is drawn from a fixed distribution (independently from earlier values of the bargaining power and the deadline). The bargaining powers remain constant as $(\pi_t^1, \pi_t^2) = (\pi^1, 1 - \pi^1)$ until the next arrival. The players agree about the arrival process for simplicity (common $\lambda(t)$), but they might disagree about how an arrival will affect their bargaining powers. Let $H^i$ denote the distribution of $\pi^i$ according to player $i$. We write $\bar{\pi}^i = \int \pi dH^i(\pi)$ for the expected value of bargaining power $\pi_t^i$ upon arrival according to $i$.

To explore the effects of optimism, we assume

$$\overline{y} \equiv \bar{\pi}^1 + \bar{\pi}^2 - 1 > 0.$$

Note that the expected value of $\pi_t^1$ upon arrival is $\bar{\pi}^1$ according to player 1 while it is only $1 - \bar{\pi}^2$ according to player 2. Hence, $\bar{y} = \bar{\pi}^1 - (1 - \pi^2)$ is a measure of long-run optimism. It captures the amount by which a player over-estimates her own bargaining power upon arrival with respect to the other player. We define *optimism at time* $t$ about time $s \geq t$ analogously as

$$y_{t,s} = E_t^1 \left[\pi_s^1\right] + E_t^2 \left[\pi_s^2\right] - 1. \tag{1}$$

Here, $E_t^i \left[\pi_s^i\right]$ denotes a player's expectation about his bargaining power at a future date $s \geq t$. In this setup, this is a weighted average of her current bargaining power, $\pi_t^i$, and the long-run expectation, $\bar{\pi}^i$. In particular,

$$E_t^i \left[\pi_s^i\right] = \left(1 - \Lambda_{t,s}\right) \pi_t^i + \Lambda_{t,s} \bar{\pi}^i \tag{2}$$

where

$$\Lambda_{t,s} = 1 - e^{-\int_t^s \lambda(t')dt'} \tag{3}$$

denotes the probability of an arrival over the interval $[t, s]$. Combining (1) and (2), the players' optimism can also be written as

$$y_{t,s} = \Lambda_{t,s} \overline{y}. \tag{4}$$

Eqs. (2-4) illustrate the key features of our model. Players have a perpetual tendency to be optimistic ($\overline{y} > 0$). However, this tendency is countered by the current realities ($\pi_t^i$), and the rate at which these realities change ($\Lambda_{t,s}$). Observe that the players' beliefs about the short run largely reflect the current bargaining power—especially if the important events do not happen very frequently. As the players consider the more distant future, their beliefs are disconnected from the current bargaining power. Their optimism increases and eventually approaches $\overline{y}$. These features of the model are consistent with recent survey evidence from Case, Shiller, and Thompson (2012)—albeit from a very different context. They find that homebuyers in the US are typically informed and not very optimistic about their home price changes over the next year, but they are quite optimistic that the price will increase considerably over the next ten years.

Our model has some extreme features that do not play an important role beyond providing analytical tractability. For instance, the assumption that the bargaining power is completely reset upon arrival is unrealistic. In our working paper, we extend our main results to general bargaining processes (see Section 6). The key feature of our model is that the bargaining power is known to be somewhat durable. *The durability rate* in this model, which we define for more general processes in our working paper, turns out to be $1/\lambda(t)$—the inverse of the rate at which important events occur. Observe from (2) that the players' beliefs depend on the current and the future values of the durability rate, $[1/\lambda(t)]_{t'=t}^{\infty}$. These values are largely unrestricted in our model (except that $\lambda(t)$ is deterministic and piecewise continuous). Hence, although simplified in some dimensions, the model is sufficiently rich to capture various economically interesting scenarios, which we explore in subsequent sections.

*Remark* 1 (Relationship with Existing Bargaining Models). If *the grid of negotiation times $T$ is fixed*, then our game is a random-proposer model (Binmore 1987; Merlo and Wilson 1995). The closest model is provided by Yildiz (2003), who also allows the players to have subjective beliefs about the recognition to make an offer. The first distinguishing property of our model is that the probability $\pi_t^i$ is publicly observable. In the existing models, the players only observe the proposer (or the state that deterministically determines the proposer). These models can be captured in our general framework (discussed in Section 6) by taking $\pi_t^i \in \{0, 1\}$.[1] More importantly, we take the bargaining power as a function of *the real time*, independently of how frequently players come together to negotiate. In fact we will often focus on the solution in the continuous time limit as $n \to \infty$. This approach is particularly useful to model the durability of the bargaining power. In Section 5, we show that durability plays an important role in generating bargaining delays. In contrast, in existing models, the bargaining power varies with $n$, often leading to a highly non-durable bargaining power as $n \to \infty$. For example, the main results in Yildiz (2003) assume that the recognition process is serially independent, so that players do not learn about future bargaining power from the current one. This can be thought of as a special (limit) case of our model in which $\pi_t^i \in \{0, 1\}$ and $\Lambda_{t,s} = 1$ for each $t, s \geq t$ (i.e. durability rate $1/\lambda(t)$ is zero everywhere in continuous time.[2]

## 2.2 Characterization of Equilibrium

Let the random variable $V_t^i$ denote the continuation value of player $i$ at time $t$ after $\pi_t^1$ is revealed but before the proposer at time $t$ is recognized. By individual rationality, $V_t^i$ is restricted to be in $[0, 1]$. Given a subsequent negotiation time $s \in T$, we define

$$W_{t,s} \equiv \delta_{t,s} \left( E_t^1 \left[ V_s^1 \right] + E_t^2 \left[ V_s^2 \right] \right) \tag{5}$$

[1] Since the bargaining power in these models is defined only implicitly, alternative interpretations might sometimes be more appropriate. For example, the typical random proposer model can be thought of as another special case of our model in which $\pi_t^i = \pi^i$, that is, player $i$ makes an offer with known probability $\pi^i$ in every period. Yildiz (2004a) assumes that $\pi^i$ in the previous example is unknown and players hold optimistic beliefs about it. This model can be interpreted as players having a durable and *unobservable* bargaining power, about which they obtain public signals as they observe who makes an offer.

[2] Yildiz (2003) allows the new bargaining powers to be drawn from a different distribution, while we fix that distribution. This difference is not relevant for our comparisons in Section 5.

as the sum of players' perceived payoffs from delaying agreement (or waiting) until time $s$. Note that $W_{t,t+1/n}$ captures players' total perceived payoff from waiting until the next negotiation time. Hence, $1 - W_{t,t+1/n}$ captures players' perceived surplus from agreeing at time $t$.

First suppose $W_{t,t+1/n} < 1$, so that the surplus at time $t$ is positive. Then, it is easy to check that the players reach an agreement with the proposer receiving the full surplus. Hence, player $i$'s expected payoff before the proposer is recognized is

$$V_t^i = \pi_t^i \left(1 - W_{t,t+1/n}\right) + \delta_{t,t+1/n} E_t^i \left[V_{t+1/n}^i\right].$$

In particular, players split the surplus according to their bargaining powers, $\pi_t^1$ and $\pi_t^2$.

Next suppose the surplus is negative, that is, $W_{t,t+1/n} > 1$. In this case, there cannot be an agreement that satisfies both players' expectations, as the sum of their continuation values from delay exceeds 1. Hence, there will be disagreement at such $t$ regardless of the proposer. Player $i$'s continuation value is

$$V_t^i = \delta_{t,t+1/n} E_t^i \left[V_{t+1/n}^i\right].$$

Finally, if $W_{t,t+1/n} = 1$, then the surplus is zero and the players are indifferent to agree.

In general, the equilibrium is obtained by combining the three cases, yielding a unique subgame-perfect Nash equilibrium up to the indifference in the last case. Our next result characterizes the equilibrium further using our specification of the bargaining power process. We write $\delta_t = \delta_{t,t+1/n}$ for the one period discount rate, $\Lambda_t = \Lambda_{t,t+1/n}$ for the one period arrival probability [cf. (3)], $W_t = W_{t,t+1/n}$ for the players' perceived total payoff from waiting for one period [cf. (5)].

**Proposition 1** (Characterization and Uniqueness of Equilibrium). *In any subgame-perfect Nash equilibrium, the continuation value of any player $i \in \{1,2\}$ at any time $t \in T$ is*

$$V_t^i = K_t \pi_t^i + (S_t - K_t) \frac{\overline{\pi}^i}{1+\overline{y}} \qquad \forall i, t \tag{6}$$

*where the deterministic weights $K_t$ and $S_t$ are the unique solutions to the difference equations*

$$\begin{aligned} K_t &= \max\{1 - W_t, 0\} + \delta_t (1 - \Lambda_t) K_{t+1/n}, & (7)\\ S_t &= \max\{1, W_t\}, & (8)\\ W_t &= \delta_t \left(S_{t+1/n} + \Lambda_t \overline{y} K_{t+1/n}\right). & (9)\end{aligned}$$

*At any time $t$, the players agree if $W_t < 1$ and disagree if $W_t > 1$.*

The proposition establishes several properties of equilibrium. First, there exists a unique equilibrium payoff vector. Second, the expected payoff of a player $i$ can be decomposed into a combination of her current bargaining power, $\pi_t^i$, and her expected long-run bargaining power, $\overline{\pi}^i$, as in (6). We refer to the weight on the current bargaining power, $K_t$, as the *congruence*, as this determines the extent to which a player's bargaining power at time $t$ translates into her expected payoff. In political negotiations, congruence can also be thought of as *political capital*: the extent to which a player that has high current bargaining power also receives a high payoff from bargaining (see Section 4). We have written the weight on the long-run bargaining power as $(S_t - K_t) / (1 + \bar{y})$, which ensures $V_t^1 + V_t^2 = S_t$. We refer to $S_t$ as *the size of the pie*, that is, the sum of players' (expected) payoffs.

Third, the functions $K_t$ and $S_t$ are the unique solutions to the difference equations (7-9). The size of the pie, $S_t$, is the maximum value of agreement and disagreement outcomes. To understand the determinants of $K_t$, note that solving the equations forward yields:

$$K_t = \sum_{\{s \in T | s \geq t\}} \max\{1 - W_s, 0\} \, \delta_{t,s} \left(1 - \Lambda_{t,s}\right). \tag{10}$$

That is, the congruence is a discounted sum of the gains from agreement, $\max\{1 - W_s, 0\}$, because the bargaining power translates those gains into actual payoffs. The contribution of the future gains depend not only on the discount factor, $\delta_{t,s}$, but also on the probability that the current bargaining power will remain unchanged, $1 - \Lambda_{t,s}$. The effects of the events after a change in bargaining power are captured by the other term, $S_t - K_t$.

Finally, equation (9) characterizes the value of waiting—and thus, the players' agreement decisions—in terms of the functions $K_t$ and $S_t$. The value of waiting depends on the size of the surplus in the next period, as well as the optimism, $\Lambda_t \overline{y}$, and the congruence, $K_{t+1/n}$. Intuitively, waiting is more valuable if optimism about the bargaining power is large and the bargaining power will translate into a greater payoff in the next period.

Note that (7-9) represents a *deterministic* system of equations. That is, although the continuation values and equilibrium shares are stochastic (as they depend on the realizations of $\pi_t^i$), the functions $K_t, S_t$, and $W_t$ are deterministic. In particular, whether there is agreement (i.e., $W_t < 1$) or disagreement (i.e., $W_t > 1$) at any instant is deterministic. Hence, the settlement date is known at the beginning of the game.

We next study the solution to the deterministic the difference equations (7-9). We will analyze cases in which the piecewise continuous functions $r$ and $\lambda$ might be stationary, or might change over time in view of the arrival of deadlines or various elections. We solve the equilibrium corresponding to these cases in discrete time, that is, for a fixed $n$ with the corresponding grid $T = \{0, 1/n, 2/n, .., \}$. However, to simplify the exposition in the main text, we often describe the equilibrium variables such as $K_t$ in the continuous time limit, that is, as $n \to \infty$ (when the limit exists).

## 2.3 Equilibrium in a Stationary Environment

We start by analyzing a baseline specification in which $r(t)$ and $\lambda(t)$ are constant.

**Stationary Model.** Suppose $r(t) = r^0$ and $\lambda(t) = \lambda^0$ for each $t$, where $r^0$ and $\lambda^0$ are positive constants that capture the baseline levels of the discount rate and the arrival rate.

This case is useful to illustrate how a combination of durability and time discounting leads to high congruence. To state the result, we define *the effective durability rate* as the product of the instantaneous durability rate with the discount rate:

$$\rho(t) = 1/\lambda(t) \times r(t). \tag{11}$$

We let $\rho^0 = r^0/\lambda^0$ denote the baseline level of the effective durability rate. We also define *the stationary congruence* as a function of the effective durability rate:

$$k(\rho) \equiv \frac{\rho}{\rho + 1 + \overline{y}}. \tag{12}$$

**Proposition 2** (Stationarity)**.** *Consider the Stationary Model. Players reach agreement at each time. In the continuous-time limit, the congruence is* $\lim_{n\to\infty} K_t = k(\rho^0)$.

*Proof.* We conjecture that there is agreement at all dates: $W_t < 1$ and $S_t = 1$ for each $t$. Using (9), the one period gain from agreement can be written as

$$1 - W_t = 1 - \delta_t \left(1 + \Lambda_t \overline{y} K_{t+1/n}\right).$$

Substituting this into (7), and rearranging terms, we obtain

$$K_t = 1 - \delta_t + \hat{\delta} K_{t+1/n}$$

where

$$\hat{\delta} = \delta_t \left(1 - \Lambda_t \left(1 + \overline{y}\right)\right). \tag{13}$$

Since the environment is stationary, $K_t = K_{t+1} \equiv K^{stat}$, yielding

$$K_t = K^{stat} = \frac{1 - \delta_t}{1 - \hat{\delta}} \to k\left(\rho^0\right). \tag{14}$$

It is easy to verify that $W_t = \delta_t \left(1 + \Lambda_t \overline{y} K^{stat}\right) < 1$. □

The result shows that the players in the stationary model reach agreement immediately. This is perhaps surprising since a naive view could posit that a sufficiently high level of optimism, $\overline{y}$, could lead to disagreement. Intuitively, if there were disagreement at some time, there would be disagreement at all times since the environment is stationary and the solution is deterministic. Since the players cannot disagree forever, there is agreement at all times. Optimism affects the agreement shares, as illustrated by (12), but it need not generate delays (see Section 5 for an extension in which there are delays with stationarity).

The result also implies that higher effective durability increases congruence. In particular, the function $k(\cdot)$ is increasing [cf. (12)]. Thus, with greater effective durability, the agreement shares reflect relatively more the current bargaining power. This leaves less room for long-run factors, including the players' optimism [cf. (6)]. In fact, as $\rho^0 \to \infty$, the congruence approaches one and optimism has no impact on agreement shares.

To obtain an intuition, note that Eq. (14) can be equivalently written as a sum:

$$K^{stat} = \sum_{s \geq t} (1 - \delta_s) \hat{\delta}^{n(s-t)} \tag{15}$$

(since $\delta_s = \delta_t$ for each $s$). That is, the congruence (over an agreement region) can be viewed as a sum of the "frictionless" gains from agreement that would obtain in the absence of disagreement, $1 - \delta_s$. The effect of optimism is now captured as a reduction of the discounting term, $\hat{\delta} = \delta_t (1 - \Lambda_t (1 + \bar{y})) < \delta_t (1 - \Lambda_t)$. Intuitively, the earlier representation in (10) accounted for optimism as a drag on the (endogenous) gains from agreement, whereas the new representation pushes the effect into discounting. This representation is useful and enables us to obtain a closed form solution.

Eq. (15) illustrates that a rapidly arriving deadline, captured by high $r^0$ and low $\delta_s$, leads to high congruence because it increases the gains from agreement in the short run. Optimism cannot overturn this outcome since the parties cannot be optimistic about the short run (by assumption). High durability, captured by high $\lambda^0$ and low $\Lambda_t$, leads to a very similar outcome for a slightly different reason. In this case, the gain from agreement in any time interval can be relatively small. However, since the bargaining power is durable, the discounting in the sum largely reflects time discounting or deadlines ($\hat{\delta} \simeq \delta_t$). The discounted sum of these small gains translates into high congruence. Optimism cannot overturn this outcome because it is disciplined by high durability.

There is, in fact, a deeper connection between the disciplining roles of deadlines and durability. To see this, consider the change of variable, $t = \Delta / r^0$, where $\Delta$ captures the payoff relevant distance of time $t$ from time 0 (since $e^{-r^0 t} = e^{-\Delta}$). The continuous time limit of Eq. (15) can then be written as:

$$\lim_{n \to \infty} K^{stat} = \int_0^\infty e^{-\Delta\left(1 + (1+\bar{y})/\rho^0\right)} d\Delta.$$

Thus, when written in terms of the payoff relevant distance, the congruence becomes a function of the single variable, $\rho^0 = r^0 / \lambda^0$, as opposed to two separate variables, $r^0$ and $\lambda^0$. Intuitively, in view of the relation, $t = \Delta / r^0$, a high discount rate, $r^0$, shortens the

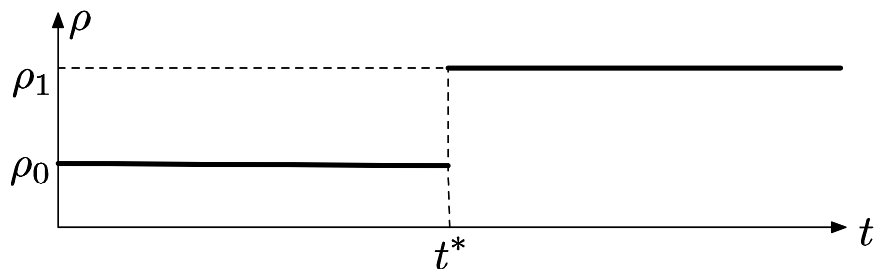


Figure 1: The effective durability rate in Section 3.

time intervals over which players face equivalent trade-offs. Over short time intervals, the bargaining power cannot change very much even if the bargaining power is not very durable. Hence, the case in which $r^0$ is high is mathematically equivalent to a hypothetical case in which $r^0$ is lower and the durability rate, $1/\lambda^0$, is proportionally higher. This point applies more generally beyond the stationary environment. The continuous time payoffs in our model often depend on the effective durability rate, $\rho(t) = r(t)/\lambda(t)$.

# 3 Durability and Deadline Effects

In this section, we present our main result on the durability effect, and obtain a robust deadline effect as its corollary. We establish these results using the following special case of our model.

**Increasing Durability Model:** Imagine that the arrival and the discount rates $\lambda(t)$ and $r(t)$ are stationary at their baseline levels up to some negotiation time $t^* \in T$. They change at $t^*$ and remain stationary at a new level after $t^*$. That is,

$$\lambda(t) = \begin{cases} \lambda^1 & \text{if } t \geq t^* \\ \lambda^0 & \text{otherwise} \end{cases} \qquad r(t) = \begin{cases} r^1 & \text{if } t \geq t^* \\ r^0 & \text{otherwise} \end{cases} \qquad \rho(t) = \begin{cases} \rho^1 & \text{if } t \geq t^* \\ \rho^0 & \text{otherwise} \end{cases}. \tag{16}$$

Suppose $\rho^1 > \rho^0$ so that the effective durability rate $\rho^1 = r^1/\lambda^1$ after $t^*$ is higher than the effective durability rate $\rho^0 = r^0/\lambda^0$ before $t^*$ as in Figure 1. We further assume

$$\overline{y}k\left(\rho^1\right) > \rho^0. \tag{17}$$

It can be checked from (12) that $\rho > k(\rho)\overline{y}$ for each $\rho$. Thus, condition (17) requires $\rho^1$ to be sufficiently larger than $\rho^0$. The increase in effective durability can be driven by an increase in the discount rate, $r(t)$, or the durability rate, $1/\lambda(t)$. As we formalize below, the increase in $r(t)$ captures the arrival of a stochastic deadline. The increase in $1/\lambda(t)$ can capture several other applications. For example, a pending reform, such as a labor law or tort reform, might increase the durability of the bargaining power of individual parties in related negotiations, such as wage negotiations or pre-trial negotiations. The players' bargaining power is less durable before the law is enacted—since there could be last minute changes in the law—and it arguably becomes more durable after the law is enacted—since it takes time to enact a new law.

Our next result shows that these types of increase in effective durability induces delays. To state the result, we define the following function:

$$w(\Delta, K, \eta) = e^{-\Delta}\left(1 + \left(1 - e^{-\Delta/\rho^0}\eta\right) K\bar{y}\right). \tag{18}$$

To understand this function, imagine that the congruence at some later negotiation time $t^*$ is given by $K$. Consider an earlier negotiation time, $t^* - \Delta/r^0$,which has payoff relevant distance $\Delta$ from $t^*$ (since $e^{-r^0(t^*-t)} = e^{-\Delta}$). Imagine that the probability that the bargaining power remains unchanged between these times is given by $e^{-\Delta/\rho^0}\eta$. Here, $e^{-\Delta/\rho^0}$ captures the effect of arrivals due to the baseline specification, and $\eta \leq 1$ captures the effect of additional arrivals (if any) beyond the baseline specification. Then, $w(\Delta, K, \eta)$ captures the value of waiting from time $t^* - \Delta/r^0$ until time $t^*$.[3] The players are willing to wait as long as $w(\Delta, K, \eta) > 1$. Our next result shows that players choose to wait in the run-up to time $t^*$.

**Proposition 3** (Durability Effect)**.** *Consider the Increasing Durability Model. There exists a negotiation time $\bar{t} \leq t^*$, such that players agree at each $t < \bar{t}$, disagree at each $t \in [\bar{t}, t^*)$, and agree at $t^*$ and thereafter. In the continuous time limit,* $\lim_{n\to\infty} \bar{t} = \max\left(0, t^* - \bar{\Delta}/r^0\right)$*, where $\bar{\Delta} > 0$ denotes the unique positive solution to $w\left(\bar{\Delta}, k\left(\rho^1\right), 1\right) = 1$.*

The result implies that the disagreement threshold, $\bar{t}$, is strictly below $t^*$ as long as $n$ is sufficiently large. Hence, if the effective durability increases to $\rho^1$ from $\rho^0$ at some $t^*$, then there is a strict period of inactivity prior to $t^*$ during which the players must disagree in equilibrium. In the continuous time limit, the payoff relevant length of the disagreement period is given by $\bar{\Delta}$ that equates the value of waiting to 1.

**Durability Effect** As a special case, suppose the discount rate is constant, $r^0 = r^1 = \hat{r}$, but the durability rate $1/\lambda$ increases sufficiently at time $t^*$, so that condition (17) holds. Then, Proposition 3 implies that there is a period of disagreement before $t^*$, establishing the durability effect.

**Deadline Effect** As another special case, suppose a deadline arrives starting at time $t^*$ with a constant hazard rate $\alpha > 0$, so that the probability of deadline arriving before time

[3] To derive this expression, note that the representation in (6) implies, $E_t^1\left[V_{t^*}^1\right] + E_t^1\left[V_{t^*}^1\right] = S_{t^*} + K_{t^*}\left(E_t^1\left[\pi_{t^*}^1\right] + E_t^1\left[\pi_{t^*}^1\right] - 1\right) = S_{t^*} + y_{t,t^*}K_s$. Hence, by (5), we have $W_{t,t^*} = \delta_{t,t^*}\left(S_{t^*} + y_{t,t^*}K_{t^*}\right)$. We then obtain $W_{t,t^*} = w(\Delta, K, \eta)$ by substituting $\delta_{t,t^*} = e^{-\Delta}$, $S_{t^*} = 1$, $K_{t^*} = K$, and $y_{t,t^*} = \Lambda_{t,t^*}\bar{y} = \left(1 - e^{-\Delta/\rho^0}\eta\right)\bar{y}$.

$t \geq t^*$ is $1 - e^{-\alpha(t-t^*)}$. If the deadline arrives at $t$, and players have not agreed before time $t$, then negotiations end at time $t$ with each player receiving 0. Then, the discount rate is

$$r(t) = \begin{cases} \hat{r} + \alpha & \text{if } t \geq t^* \\ \hat{r} & \text{otherwise} \end{cases} \quad (19)$$

where $\hat{r}$ is the baseline discount rate. The corresponding effective durability rates are given by $\rho^0 = \hat{r}/\lambda^0$ and $\rho^1 = (\hat{r} + \alpha)/\lambda^1$. Suppose the deadline arrival rate $\alpha$ is sufficiently large (and the baseline effective durability rate $\rho^0$ is not too large) so that condition (17) holds. Then, Proposition 3 implies the players wait for $t^*$ to reach an agreement, establishing the deadline effect.[4]

We describe a sketch of the proof for Proposition 3 (completed in the appendix). Note that the environment becomes stationary at $t^*$ with effective durability rate $\rho^1$. Hence, Proposition 2 applies starting at $t^*$ after replacing $\rho$ with $\rho^1$. In particular, players reach agreement at time $t^*$ with congruence in the continuous time limit given by $k(\rho^1)$. Next note that the value of waiting at time $t^* - \Delta/r^0$ until time $t^*$ is given by $w(\Delta, k(\rho^1), 1)$. By (18), this expression can be approximated around $\Delta = 0$ as

$$w\left(\Delta, k\left(\rho^1\right), 1\right) \simeq 1 + \left(\frac{k(\rho^1)\,\overline{y}}{\rho^0} - 1\right)\Delta.$$

Under condition (17), $w(\Delta) > 1$ for sufficiently small $\Delta$. This suggests that, in the continuous time limit, there is a period of delay before $t^*$. The length of delay is characterized by solving $w(\Delta, K(\rho^1), 1) = 1$. The appendix shows that a similar argument also applies for any finite $n$.

Intuitively, the low effective durability prior to $t^*$ implies there is little discipline on beliefs at time $t$, so that players can be optimistic about their bargaining powers at time $t^*$. In contrast, the high effective durability following $t^*$ implies there is high congruence at $t^*$. The combination of undisciplined optimism before time $t^*$ and the increase in congruence at time $t^*$ makes waiting valuable, and induces players to delay agreement. Note also that the increase in the effective durability rate $\rho = r/\lambda$ can come from either an increase in the durability rate $1/\lambda$ or from an increase in the deadline arrival rate $\alpha$. Hence, durability and deadline effects are two sides of the same coin.

How costly are the delays generated by deadline or durability effects? To get a sense of magnitudes, consider the payoff relevant length of the delay region, $\bar{\Delta}$. The total cost of delaying the agreement until time $t^*$, as opposed to agreeing at time $t = t^* - \bar{\Delta}/r$, is given

[4] Bargaining deadlines in practice are often uncertain, captured by a stochastic deadline. For example, in the recent US debt ceiling negotiations, the deadline can be thought of as the time at which the Treasury will reach the statutory debt limit. In practice, this time is quite uncertain since the Federal government expenses are not entirely predictable. For another example, in legal negotiations, such as plea bargaining, the deadline can be thought of as the time at which the court will reach a judgement, an uncertain deadline.

by $1 - e^{-\bar{\Delta}}$. Using (18) and Proposition 3, the total cost is bounded from above:

$$1 - e^{-\bar{\Delta}} \leq \frac{\bar{y}}{1 + \bar{y}}. \tag{20}$$

There are parameters under which the upper bound is attained (e.g., $\lambda^0 \cong 0$ and $k(\rho^1) \cong 1$). Hence, the cost of delay, measured as a fraction of the total pie, is in the same ballpark as players' optimism about their long-run bargaining power, $\bar{y}$. When parties are highly optimistic, so that $\bar{y} = 1$, the cost can be as large as half of the total pie.

Proposition 3 has several testable implications. For instance, we have noted that the enactment of a major law, such as a labor reform, might increase the durability of the bargaining power in negotiations among individual parties affected by the law, such as wage negotiations between firms and their workers. Proposition 3 would then predict that pending major laws induces gridlock in related negotiations among individual parties. Intuitively, before the law is enacted, the parties can be optimistic about the exact terms that will be implemented. After the law is enacted, the terms are set for the near future, which would lead to durable bargaining power and high congruence. The combination of optimism before the enactment and durability after the enactment induces delay.

Another testable implication of Proposition 3 is the deadline effect. We next analyze when the deadline effect more prominent and obtain additional predictions. Recall that $\bar{\Delta}$ is the solution to the equation $w(\Delta, k(\rho^1), 1) = 1$. Our analysis in the appendix shows that the solution, $\bar{\Delta}$, is increasing in any change that increases the value of waiting evaluated at the (pre-change) length of delay, $w(\bar{\Delta}, k(\rho^1), 1)$. Our next result combines this observation with Eq. (18) to establish the comparative statics.

**Proposition 4.** *Given the deadline described in* (19)*, the length of delay, $\bar{\Delta}$, is*

1. *decreasing in the durability rate before the deadline arrival $1/\lambda^0$, and increasing in the durability rate during deadline arrival $1/\lambda^1$,*
2. *increasing in the deadline arrival rate $\alpha$,*
3. *increasing in players' long-run optimism $\bar{y}$,*
4. *decreasing in the discount rate $\hat{r}$ provided that the deadline is sufficiently firm (i.e., $\alpha$ is sufficiently high).*

The first part shows that the durability rate before and during the deadline arrival period have different effects on delays. Intuitively, greater durability during the deadline arrival period increases congruence, which in turn exacerbates delays. In contrast, greater durability before the deadline arrival disciplines optimism and mitigates delays.

The second part suggests that there might be a silver lining to setting an uncertain or soft deadline in negotiations. An uncertain deadline, which we capture with low $\alpha$, leads to lower congruence at time $t^*$, $k(\rho^1)$ (recall that $\rho^1 = (\hat{r} + \alpha)/\lambda^1$). Lower congruence might be interpreted as more compromise by the player that has the higher bargaining power. Depending on the context, this effect might be desirable in itself. Moreover, low congruence at time $t^*$ also induces a shorter delay, because the players' optimism about bargaining powers at time $t^*$ translates relatively less into optimism about payoffs.

The third part links optimism to delays. Optimism has a direct effect that tends to increase the value of waiting. However, optimism after time $t^*$ also has an indirect effect that tends to reduce this value by reducing the congruence $k(\rho^1)$. Eq. (18) illustrates that the net effect is governed by the product $k(\rho^1)\,\overline{y}$. In our model, the net effect is positive (see (12)), which implies that optimism leads to longer and costlier delays.

The last part considers the effect of the discount rate $\hat{r}$, which captures players' cost of delay. Higher cost of delay generates a direct effect that tends to reduce the value of waiting. However, higher $\hat{r}$ also generates an indirect effect that tends to increase this value via greater congruence, $k(\rho^1)$ (recall that $\rho^1 = (\hat{r} + \alpha)/\lambda^1$). The net effect is in general ambiguous. If the deadline were deterministic, the indirect effect would be absent because $k(\rho^1)$ would be equal to 1 regardless of $\hat{r}$. Likewise, as long as the deadline is sufficiently firm, the direct effect dominates, and greater $\hat{r}$ leads to shorter and less costly delays.

*Remark* 2 (Other Models of the Deadline Effect). Several theoretical papers establish a deadline effect using ingredients and mechanisms that are quite different than in our paper. Spier (1992) shows that, in a pre-trial negotiation with incomplete information, the settlement probability will be a U-shaped function of time, consistent with the deadline effect. Recently, Fanning (2013) obtains deadline effect in the incomplete-information model of Abreu and Gul (2000). He shows that, when the deadlines are stochastic as in our paper, the hazard rate of settlement is an affine function of the hazard rate of the deadline—as a consequence of the indifference condition in the war of attrition. A similar U-shaped function arises in a recent paper by Wasserman and Yildiz (2016) due to learning motives under optimism. Ma and Manove (1993) develop a model in which delay is not costly and a player can wait as much as she wants before making an offer. They show that the player waits until the deadline and makes a last minute take-it-or-leave it offer. Roth, Murnighan, and Schoumaker (1988) informally discuss a possible explanation based on the idea that there is no cost of delay except for a cost at the end due to a slight uncertainty about the deadline.

# 4 Political Negotiations

In this section, we use variations in the durability rate $\lambda(t)$ to establish our results about political negotiations. We start with a baseline setting with a single election, which is useful to illustrate the basic election effect and its comparative statics. We then consider a richer setting in which elections are periodically held, and characterize the bargaining outcomes

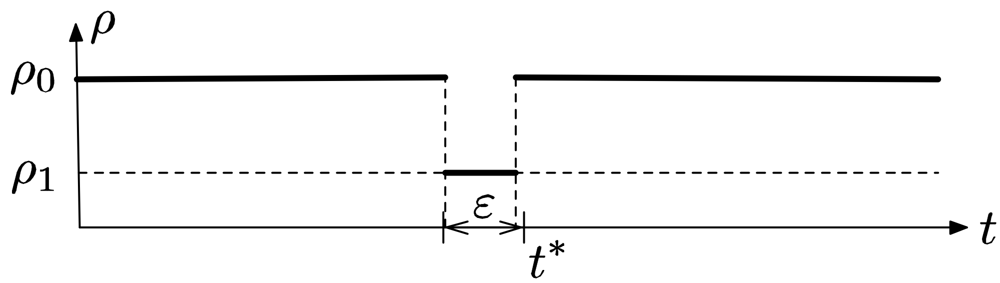


Figure 2: Effective durability rate for a single election.

over the election cycle. We also analyze the outcomes when a stronger election periodically alternates with a weaker election.

## 4.1 Election Effect

**Single Election Model:** Suppose the discount rate is constant at a baseline rate, $r(t) = r^0$, throughout. Imagine the arrival rate $\lambda(t)$ is constant everywhere except for a short period of "election date" at which it increases. That is,

$$\lambda(t) = \begin{cases} \lambda^0 + \frac{\lambda^E}{\varepsilon(n)} & \text{if } t^* - \varepsilon(n) < t < t^* \\ \lambda^0 & \text{otherwise} \end{cases}, \tag{21}$$

where $t^* \in T$ is a negotiation time, $\lambda^E > 0$ is a constant, and $\varepsilon(n)$ is a parameter that only depends on the negotiation frequency. Note that the effective durability rate starts at $\rho^0 = r^0/\lambda^0$, dips down to $\rho^1 = r^0/\left(\lambda^0 + \frac{\lambda^E}{\varepsilon}\right)$ over a period of $(t^* - \varepsilon(n), t^*)$ and switches back to the original level as in Figure 2. The parameter $\varepsilon(n) > 0$ captures the length of the election period. We assume that the election takes place over a short time. Formally, $\varepsilon(n) < 1/n$ for each $n$ so that the election starts after the negotiation time $t^* - 1/n$ and ends before the subsequent negotiation time $t^*$; this assumption is made for expositional simplicity.

The election model is meant to capture political negotiations for which the baseline durability rate is likely to be relatively high. In this context, the arrival rate $\lambda^0$ reflects major events that might influence the public opinion (and ultimately affect the parties' bargaining strength) such as political scandals, terrorist attacks, mass protests, or international conflict. In contrast, the higher arrival rate $\lambda^0 + \frac{\lambda^E}{\varepsilon}$ reflects the impact of (one or more) elections in which many political offices are contested and can change hands. Our normalization ensures that the probability that bargaining power changes due to the election is given by $1 - e^{-\lambda^E}$. Note that this expression is independent of the length of the election, $\varepsilon(n)$, and it is increasing in the parameter, $\lambda^E$. We refer to $\lambda^E$ as *the strength* of the election, and use it to capture various comparative statics (see Section 4.3).

How do upcoming elections affect the dynamics of negotiation? Observe that the main impact of elections comes from the fact that elections lower the overall durability and thereby increases the room for optimism. Indeed, at any negotiation time, $t < t^*$, the probability that the bargaining powers remain unchanged until $t^*$ is $e^{-\lambda^0(t^*-t)}e^{-\lambda^E}$. Hence, the overall optimism about the bargaining power at $t^*$ is $y_{t,t^*} = \left(1 - e^{-\lambda^0(t^*-t)}e^{-\lambda^E}\right)\bar{y}$. Thus, the election lowers the discipline on optimism. Indeed, note that $y_{t,t^*} \geq \left(1 - e^{-\lambda^E}\right)\overline{y} > 0$ for each negotiation time $t < t^*$. That is, players have significant optimism regardless of how soon the upcoming election will take place. Our next result establishes that this leads to delays in the run-up to the election.

**Proposition 5** (Election Effect)**.** *Consider the Single Election Model. There exists a negotiation time, $\bar{t} < t^*$, such that players agree at each $t < \bar{t}$, disagree at each $t \in [\bar{t}, t^*)$, and agree at $t^*$ and thereafter. In the continuous time limit, $\lim_{n\to\infty} \bar{t} = \max\left\{0, t^* - \bar{\Delta}/r^0\right\}$, where $\bar{\Delta} > 0$ denotes the unique positive solution to $w\left(\bar{\Delta}, k\left(\rho^0\right), e^{-\lambda^E}\right) = 1$.*

A dip in durability due to an election generates a delay prior to the election, illustrating the election effect. The sketch proof of Proposition 5 is similar to the proof of Proposition 3. The congruence at $t^*$ is $k\left(\rho^0\right)$ by Proposition 2. The survival probability of the bargaining power during the election is given by $e^{-\lambda^E}$. Thus, the value of waiting from a time prior to the election, $t = t^* - \overline{\Delta}/r^0$, until time $t^*$ is now given by, $w\left(\Delta, k\left(\rho^0\right), \eta\right)$, where $\eta = e^{-\lambda^E}$. Evaluating this expression at $\Delta = 0$, we obtain [cf. Eq. (18)]

$$w\left(0, k\left(\rho^0\right), e^{-\lambda^E}\right) = 1 + \left(1 - e^{-\lambda^E}\right)\overline{y}k\left(\rho^0\right) > 1.$$

Hence, there is a period of delay before the election. The length of delay is characterized by solving $w\left(\overline{\Delta}, k\left(\rho^0\right), e^{-\lambda^E}\right) = 1$.

Intuitively, the players are optimistic about their likelihood of "winning" the election. More specifically, they both believe the bargaining power will be reset during the election to a new value that is on average in their favor. Hence, there is little discipline on the players' optimism in the run-up to an election. In addition, there is some (typically, high) congruence after the election in view of durability, $k\left(\rho^0\right) > 0$. It follows that the players disagree before the election in the hope that they will get a better deal after the election.

Hence, similar to the durability and deadline effects, the election effect also stems from an increase of effective durability. The election effect further illustrates the discipline on optimism at a prior time is determined by the "weakest link" of effective durability following that time. In particular, note that there is little discipline at time $t < t^*$ for beliefs at time $t^*$ despite the fact that the bargaining power is quite durable over most of the interval $[t, t^*]$. Put differently, if there is a period of transience, such as an election, durability in the rest of that period does not create much discipline.

We next establish comparative statics for the payoff relevant length of the delay region,

$\bar{\Delta}$, which also captures the cost of delay, $1 - e^{-\bar{\Delta}}$. When the baseline bargaining power is highly durable, i.e., when $\rho^0 \to \infty$, the cost of delay has a closed form solution:

$$1 - e^{-\bar{\Delta}} = \frac{(1 - e^{-\lambda^E})\overline{y}}{1 + (1 - e^{-\lambda^E})\overline{y}}.$$

As in deadline and durability effects, the cost due to the election effect can be as large as half of the total pie. Moreover, the cost is increasing in the players' optimism due to the election, $(1 - e^{-\lambda^E})\overline{y}$. Our result establishes these and other comparative statics more generally. As before, the delay is increasing in any change that increases the value of waiting evaluated at the (pre-change) length of delay, $w\left(\bar{\Delta}, k\left(\rho^1\right), e^{-\lambda^E}\right)$.

**Proposition 6.** *The length of delay $\bar{\Delta}$ before an election is increasing in the strength of the election $\lambda^E$, players' long-run optimism, $\overline{y}$, and the baseline durability rate, $1/\lambda^0$.*

This proposition establishes three results. First, a stronger election, in which the bargaining power changes with greater probability, induces longer and costlier delays. Intuitively, a stronger election implies a greater drop in durability, which facilitates greater optimism about post-election bargaining powers. This result generates several testable implications that we discuss further in Section 4.3. Second, optimism increases the length of delay, as in the deadline effect. Third, a greater baseline durability rate also increases the length of delay. To understand this result, note that greater $1/\lambda^0$ affects the length of disagreement in two ways. First, it lowers optimism before the election (through the $e^{-\Delta/\rho^0}$ term in (18)), thereby shortening the delay. More importantly, it increases the rate $k\left(\rho^0\right)$ at which post-election bargaining powers translate into agreement shares, increasing the delay. The proof in the appendix shows that the latter effect dominates. High $1/\lambda^0$ could be thought of as capturing politically stable democracies in which most of the important changes to bargaining power happen during elections—as opposed to unstable political settings in which the bargaining power can also change considerably in non-election times. Under this interpretation, the third part suggests the election effect is more prominent in politically stable democracies.

*Remark* 3 (Popularity with Voters and Endogenous Optimism). Our baseline model assumes that the arrival during an election resets the bargaining power to a new level drawn from a *fixed* distribution. Consequently, the optimism about the bargaining power after the election depends only on the strength of the election, $\lambda^E$, and the optimism parameter, $\bar{y}$. This specification is tractable, but it does not speak to some important features of elections in practice. In Appendix A, we analyze a richer model that provides many intuitive comparative statics. Specifically, we assume that the election resets the bargaining power as

$$\pi^1_{t^*} = G\left(Z_{t^*}\right)$$

where $Z_t$ is a publicly observable Brownian motion, representing the relative popularity of player 1, and $G$ is a symmetric S-shaped function, capturing the idea that results are more sensitive to the vote shares in close elections. We assume that the players are optimistic about the drift, by taking $\mu^i$ as the drift of $Z_t$ according to $i$ and assuming that $\mu^1 > \mu^2$. We show that there is delay prior to election at some $t$ close to $t^*$ if and only if

$$\left(1 - e^{-\lambda_E}\right) G'\left(Z_t\right)\left(\mu^1 - \mu^2\right)\left(t^* - t\right) > r\left(t^* - t\right). \tag{22}$$

Here, the left side is the parties' optimism about post-election bargaining powers, $y_{t,t^*}$. Note that $y_{t,t^*}$ depends on the parties' optimism about the drifts in popularity and the time left until the election, as well as how popularity translates into bargaining power. The right side is the cost of delay, which depends on the discount rate and the time left until the election. Condition (22) leads to a key observation: the players delay agreement when the election is sufficiently close ($|Z_t| < \bar{Z}$ for some $\bar{Z}$), but they reach immediate agreement otherwise. Intuitively, since $G(\cdot)$ is an S-shaped function, optimism is decreasing in the absolute value of the relative popularity, $|Z_t|$. There is greater optimism about the bargaining power when the election results are expected to be close ($|Z_t|$ near zero) and no optimism when a landslide is expected (i.e., $|Z_t|$ is large), verifying common sense.

This analysis has an important implication for comparing elections with and without an incumbent candidate in the race. Empirically, incumbents tend to have an advantage in the election (see, for instance, Mayhew (2008)). We show that the incumbency advantage makes elections less likely to be close and reduces the likelihood of delay. In Section 4.3, we capture the incumbency advantage in our baseline model (in reduced form) by assuming that an election with an incumbent is relatively weak (low $\lambda_E$). Note that a weaker election in our baseline model, like greater incumbency advantage, leads to a weaker election effect.

## 4.2 Periodic Elections

In Section 4.1, we focused on a single election for simplicity. In practice, elections are often held periodically at fixed time intervals. For instance, the US presidential elections are held every four years. We next extend our baseline election model to analyze the bargaining outcomes over a typical election cycle.

**Periodic Election Model:** Fix $r(t) = r^0$ for all $t$ and consider the arrival rate:

$$\lambda(t) = \begin{cases} \lambda^0 + \frac{\lambda^E}{\varepsilon(n)} & \text{if } t \in \left(kt^* - \varepsilon(n), kt^*\right) \\ \lambda^0 & \text{if } t \in \left[(k-1)t^*, kt^* - \varepsilon(n)\right] \end{cases}, \text{ for } k \in \{1, 2, \ldots\}. \tag{23}$$

Here, $t^* \in T$ denotes a negotiation time, which implies that $kt^* \in T$ is also negotiation time for each $k \in \{1, 2, ...\}$. Elections are now held periodically slightly before these negotiation times. Thus, the effective durability periodically dips to a low level $\rho^1 = r^0 / \left(\lambda^0 + \frac{\lambda^E}{\varepsilon(n)}\right)$, as

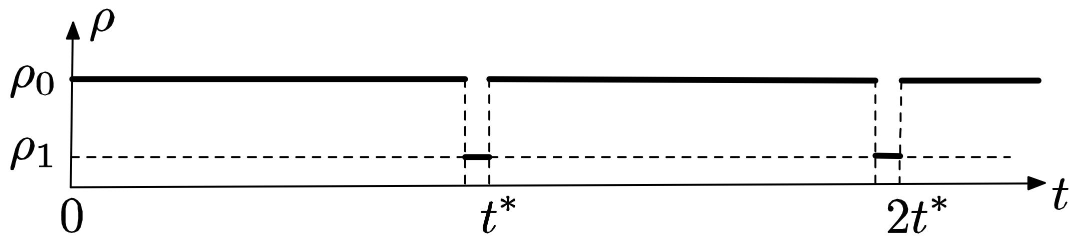


Figure 3: Effective durability rate for periodic elections.

illustrated in Figure 3. We continue to assume $\varepsilon(n) < 1/n$ for each $n$.

To characterize the equilibrium with periodic elections, we define

$$\mathbf{K}_0(\Delta) = \frac{1 - e^{-\left(r^0 t^* - \Delta\right)/k(\rho_0)}}{1 - e^{-(r^0 t^* - \Delta)/k(\rho_0)} e^{-(1+1/\rho^0)\Delta} e^{-\lambda^E}} k(\rho_0).$$

As we will see, $\mathbf{K}_0(\Delta)$ describes the congruence as a function of the length of delay before elections.

**Proposition 7** (Periodic Elections)**.** *Consider the Periodic Election Model. The equilibrium behavior is periodic with cycle $t^*$. There exists a negotiation time $\bar{t} \in (0, t^*]$, such that players agree at each $t \in [0, \bar{t})$ and disagree at each $t \in [\bar{t}, t^*)$. Over the agreement region, $t \in [0, \bar{t})$, $K_t$ is strictly decreasing in $t$. In the continuous time limit, $\lim_{n\to\infty} \bar{t} = t^* - \bar{\Delta} r^0$, and $\lim_{n\to\infty} K_0 = \mathbf{K}_0\left(\bar{\Delta}\right)$, where $\bar{\Delta}$ is the unique positive solution to $w\left(\bar{\Delta}, \mathbf{K}_0\left(\bar{\Delta}\right), e^{-\lambda^E}\right) = 1$.*

The result characterizes the equilibrium with periodic elections. The players disagree in the run-up to every election, and agree immediately after the election, consistent with the election effect. As before, the length of delay is characterized by setting the value of waiting equal to 1. The congruence after an election, $K_0$, is jointly determined with the length of delay. The new result is that the congruence, $K_t$, is strictly decreasing over the range of the election cycle on which there is agreement. Recall that we referred to $K_t$ in this context as a form of political capital: the extent to which the party that "won" the last election can implement outcomes that it prefers (controlling for its bargaining power). The result says that political capital is highest immediately after the election, and it gradually declines as the next election approaches.

We present a sketch proof for Proposition 7 (formalized in the appendix), which is useful to understand the intuition. Recall from (15) that, over an agreement region, the congruence can be described as a discounted sum in which the effect of optimism is captured by the discounting term $\hat{\delta} = \delta_t (1 - \Lambda_t (1 + \bar{y}))$. Using similar steps, we obtain for each $t \in [0, \bar{t})$:

$$\begin{aligned} K_t &= \sum_{s \in [t, \bar{t})} (1 - \delta_s) \hat{\delta}^{n(s-t)} + \hat{\delta}^{n(\bar{t}-t)} K_{\bar{t}} \\ &= K^{stat}\left(1 - \hat{\delta}^{n(\bar{t}-t)}\right) + \hat{\delta}^{n(\bar{t}-t)} K_{\bar{t}}. \end{aligned} \quad (24)$$

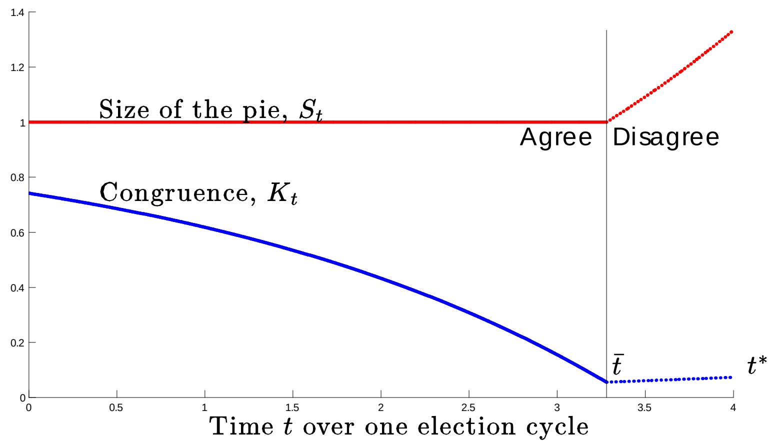


Figure 4: The equilibrium values of $K_t$ and $S_t$ with periodic elections for a particular parameterization.

Here, $K^{stat} = \frac{1-\delta_t}{1-\hat{\delta}}$ describes the congruence that would obtain in the stationary model (see (14)). In the present model with elections, the congruence is a weighted average of $K^{stat}$ and the congruence at the disagreement threshold, $K_{\bar{t}}$. The latter term incorporates the effect of the upcoming elections. In fact, applying (7) over the disagreement region, we obtain

$$K_{\bar{t}} = e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} e^{-\lambda^E} K_0. \tag{25}$$

That is, $K_{\bar{t}}$ is lowered by the arrivals due to the election (captured by $e^{-\lambda^E} < 1$) as well the delay caused by the election (captured by $e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} < 1$). Combining (24) and (25), $K_0$ can be solved in closed form. Doing so reveals that $K_0 < K_t^{stat}$, which in turn implies $K_{\bar{t}} < K_t^{stat}$. Hence, the sum in (24) is a weighted average of two terms, with the weight on the larger term declining as $t$ approaches $\bar{t}$. This establishes that $K_t$ is declining over the agreement range. The proof in the appendix completes the argument and derives the continuous time limit of the solution.

Intuitively, relative to the stationary model, periodic elections reduce the congruence at agreement times through two effects. First, an upcoming election reduces durability, which lowers $K_t$. If the bargaining power will change soon, then its current level is less relevant for payoffs. Second, an upcoming election also induces disagreement, which further lowers $K_t$ at agreement times. Even though the players currently reach agreement, if they will soon start to disagree, then the current bargaining power matters less for payoffs. These effects imply that the congruence is highest immediately after an election, and it declines as the next election looms closer.

Figure 4 plots the equilibrium for a particular parameterization. At the beginning of the cycle, political capital (or congruence) is highest. The player that wins the election is able to implement the outcomes in its favor. As time passes, the political capital is depleted, and players start to reach agreement outcomes that could be thought of a "compromise." In a compromise outcome, the agreement shares reflect long-run factors [cf. (6)] relatively more than the identity of the player who won the last election. The winner is forced into the compromise because the other player has a credible threat to delay the agreement until after the next election. As the election draws even closer, the threat becomes real (due to optimism) and the agreement is actually delayed. Hence, Proposition 7 adds to our list of testable predictions by establishing that the nature of the bargaining outcomes systematically change over the election cycle, with the outcomes first steering towards agreement with compromise, before they eventually feature disagreement.

## 4.3 Alternating Elections

In the above model, we assumed that all elections have the same strength. In practice, there are also systematic variations in the strength of periodically repeated elections. In the US, the Congressional elections take place every two years, and the presidential elections take place every four years. Hence, the political cycle is one in which the four-year joint elections alternate with the midterm elections that feature only congressional elections. The latter elections are arguably weaker, and have a smaller impact on the parties' bargaining powers.

A similar variation in strength also applies to the alternating US presidential elections. By law, the president is not allowed to be reelected more than two times. This suggests a longer (eight year) political "supercycle" in which the presidential election in which the incumbent cannot run for the office, alternates alongside with a presidential election in which the incumbent can also run.[5] The latter elections arguably represent a smaller drop in durability because the bargaining power associated with the president's office will not change in case the incumbent wins the election. As we argued in Section 3, these elections are also associated with less optimism (and less gridlock) on average in view of the incumbent's popularity advantage in the election. We capture both features by assuming that the presidential election with an incumbent is weaker (smaller $\lambda^E$) than an election without one.

We next extend our model to analyze the bargaining outcomes over alternating elections with heterogeneous strength.

**Alternating Election Model:** Fix $r(t) = r^0$ for all $t$ and consider the arrival rate:

$$\lambda(t) = \begin{cases} \lambda^0 + \frac{\lambda^{E,odd}}{\varepsilon(n)} & \text{if } t \in (kt^* - \varepsilon(n), kt^*) \text{ and } k \text{ is odd} \\ \lambda^0 + \frac{\lambda^{E,even}}{\varepsilon(n)} & \text{if } t \in (kt^* - \varepsilon(n), kt^*) \text{ and } k \text{ is even} \\ \lambda^0 & \text{if } t \in [(k-1)t^*, kt^* - \varepsilon(n)] \end{cases}, \text{ for } k \in \{1, 2, \dots\}. \quad (26)$$

[5] This mapping is not perfect as it assumes that the incumbent presidents always stay for two full terms. We could make the mapping more realistic by introducing stochasticity into the alternation process. We work with the deterministic specification for tractability.

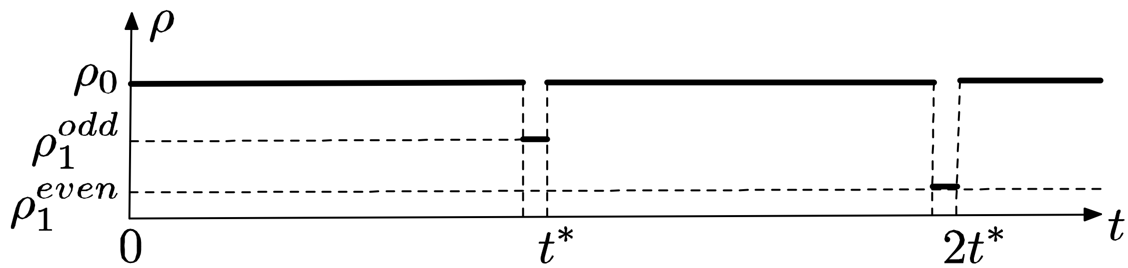


Figure 5: Effective durability rate for alternating elections.

In particular, there are now two types of elections, with strengths respectively given by $\lambda^{E,odd}$ and $\lambda^{E,even}$. The elections alternate over time with a fixed time distance. Without loss of generality, we assume that $\lambda^{E,odd} < \lambda^{E,even}$, so that the elections that take place in even periods are stronger. This implies that the effective durabilities during the elections also satisfy $\rho^{0,even} < \rho^{0,odd} < \rho^0$. Thus, the effective durability dips more in even periods relative to the level it falls in odd periods, as illustrated in Figure 5.

We assume the parameters satisfy two additional conditions:

$$k\left(\rho^0\right)\overline{y} \leq e^{r^0 t^*} - 1, \tag{27}$$

$$e^{-\lambda^{E,even}} \leq \overline{y}/\left(1+\overline{y}\right). \tag{28}$$

Condition (27) says that the distance between the two elections is sufficiently large. This ensures that delay before an election does not extend beyond the other election. Condition (28) says that the stronger of the two elections is sufficiently strong. This condition does not play an important role beyond facilitating analytical tractability.[6] For simplicity, we also focus on the cases with large $n$, that is, $n > \bar{n}$ where $\bar{n}$ is characterized in the appendix.

**Proposition 8** (Alternating Elections)**.** *In the Alternating Election Model, the equilibrium behavior is periodic with cycle $2t^*$. There exist $\bar{t}^{odd} \in (0, t^*]$ and $\bar{t}^{even} \in (t^*, 2t^*]$, such that players agree on the intervals, $[0, \bar{t}^{odd})$ and $[t^*, t^* + \bar{t}^{even})$, and disagree at remaining times. Moreover, the solution features $t^* - \bar{t}^{odd} < 2t^* - \bar{t}^{even}$ and $K_{t^*} < K_0$.*

This result characterizes the equilibrium with alternating elections with heterogeneous strength. The players disagree in the run-up to either election, and agree immediately after the election. Moreover, the stronger (even) elections feature a longer delay in their run-up and greater congruence in their aftermath. Figure 6 illustrates the equilibrium for a particular parameterization. Note that there is a longer period of disagreement before the stronger election compared to the weaker election. Moreover, the congruence (or the political capital) is also greater after the stronger election.

[6]In our numerical simulations, these comparative statics continue to hold even if we drop condition (28). See the proof of Lemma 4 in the online appendix for a detailed explanation of the role this condition plays in our our analysis.

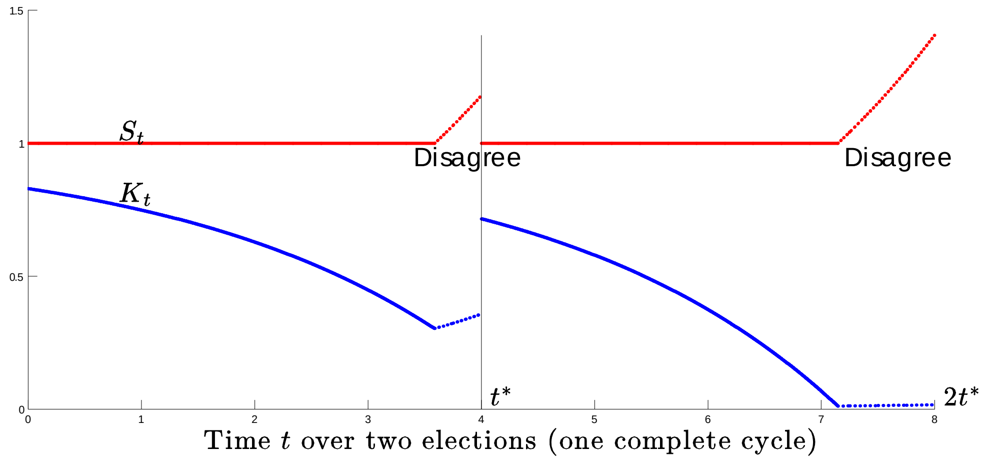


Figure 6: The equilibrium values of $K_t$ and $S_t$ with two alternating elections for a particular parameterization. The second election is stronger ($\lambda^{E,even} > \lambda^{E,odd}$).

The comparative statics for delay can be understood from Proposition 6 (first part), which illustrates in a single election context that a stronger election leads to greater optimism and longer delay. With alternating elections, there are additional subtleties regarding the endogenous determination of congruence, but these do not undo the basic effect identified in Proposition 6. To understand the comparative statics for congruence, consider the limit, $\lambda^{E,odd} \simeq 0$, so that there is effectively only the strong election held with a period of $2t^*$. In this hypothetical scenario, Proposition 7 would apply and imply that the congruence at the beginning of the election cycle is greater than the congruence in the middle, $K_0 > K_{t^*}$. Proposition 8 says that the presence of a weaker election in the middle does not undo this comparison. The congruence after the strong election (or at the beginning of the "strong election cycle") is greater than the congruence after the weaker election (or at the beginning of the "weak election cycle").

Proposition 8 generates various testable implications regarding the length of disagreement before various elections, as well as the terms of the agreement after elections. For instance, when applied to the political cycle with alternating midterm and four-year elections, the result predicts that there will be more gridlock in the run-up to the four-year elections compared to the midterm elections. The result also predicts that the winner of the four-year elections will have greater political capital than the winner of the midterm elections.

Likewise, when applied to the political supercycle with alternating presidential elections, the result predicts that there will be more gridlock in the run-up to the presidential elections in which the incumbent president cannot run due to the two-term limit. Moreover, the result also predicts that the party that has the presidency will have more political capital when the president is elected for the first time compared to when she is elected for a second time. An incumbent president approaching the end of his/her term limit is sometimes referred to

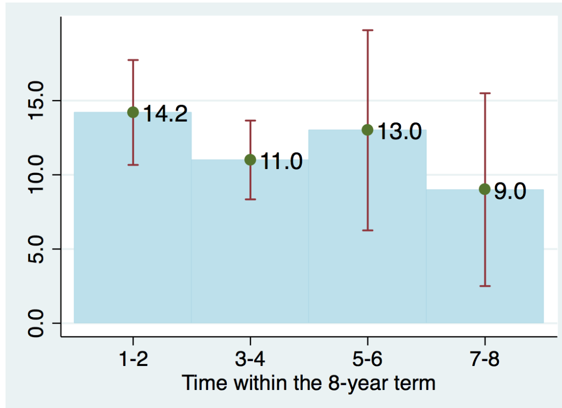


Figure 7: The average number of important laws enacted by the US Congresses between 1953-2012, as a function of the time within the 8-year presidential term. The capped lines illustrate the 5% confidence intervals around the mean.

as a *lame duck*. Using these terminology, Proposition 8 implies that lame duck presidents are associated with smaller political capital—or more compromise—in agreement outcomes, as well as a greater likelihood of observing disagreement outcomes.

We next present preliminary evidence from legislative politics in the US that is consistent with this lame duck effect. We categorize the 30 US Congresses between 1953-2012 based on the time frame they fall within the maximum 8-year presidential term. More specifically, we use "years 1-2" to label the US Congresses during the first two years of a first presidential term, "years 5-6" to label the US Congresses during the first two years of a second presidential term, and so on. We measure (the inverse of) legislative gridlock by the number of important laws enacted by the Congress, as categorized by Mayhew (1991) (see the online appendix for details of our data). Figure 7 illustrates that an average congress in years 3-4 of a presidency enacted about 14% fewer laws than a congress in years 5-6, which suggests a positive yet somewhat weak election effect during the first term. In contrast, an average congress in years 7-8 enacted 37% fewer important laws than a congress in years 1-2, which suggests a stronger election effect during the second term—consistent with the lame duck effect. However, these results are not statistically significant. This is expected, partly because we only have 30 data points in total divided across four categories, and partly because there are many more factors excluded from our analysis that might also affect legislative gridlock (see Binder (2003) for a review). We therefore view our findings as preliminary evidence, which should be subject to closer empirical scrutiny as the relevant data becomes available.

*Remark* 4 (Comparison with Electoral Concerns). The parties' concerns with their performance in the upcoming election provide an alternative hypothesis for bargaining delays before elections. In recent work, Ortner (2013) formalizes and theoretically analyzes this "electoral concerns hypothesis." In Ortner's model, the terms of agreement can affect the parties' relative popularity, and thereby, their probability of winning the next election. The winner of the election receives a nontransferable benefit, which introduces electoral concerns into negotiations. Ortner shows that electoral concerns can lead to a delay only if they conflict with the parties' desired policies. Specifically, if implementing each party's most favorable policy would also increase the popularity of that party (perhaps because the parties and their potential voters share similar preferences), then the parties find a way to compromise and reach immediate agreement. On the other hand, if one of the parties faces a trade-off, then electoral concerns can lead to delays. For example, if the majority party would have to sacrifice its popularity with voters by passing its favorite policy, then the party might prefer to postpone the bill until the post-election period. A second example arises when a particular party loses its popularity with voters because of bargaining delays, regardless of how the delay arises. In that case, the other party may obstruct agreement in order to improve its popularity.

We view the electoral concerns hypothesis as mainly complementary to our election effect, with two main differences. First, our mechanism for gridlock arguably applies more broadly because it requires fewer assumptions than the key assumption, optimism. In contrast, electoral concerns apply only if the negotiated issue is sufficiently visible and salient for voters. For instance, such concerns might be invoked to investigate delays in appointing a judge to the supreme court. However, they are unlikely to be relevant for less salient issues such as the appellate court appointments. Even for the salient issues, however, the electoral concerns do not immediately ensure delay. As Ortner's (2013) analysis illustrates, delay requires additional and somewhat subtle conditions.[7] Second, we make several testable predictions regarding the strength of the election effect, some of which do not necessarily follow from electoral concerns. For instance, our election effect causes more gridlock when there is a lame duck president—as weakly corroborated by our empirical analysis above. In contrast, one can plausibly envision that a lame duck president is concerned more with his own legacy than the results of the next election, which could create less gridlock through the electoral concerns channel. In addition to these differences regarding gridlock, our analysis also characterizes how the agreement outcomes evolve over typical political cycles.

# 5 What Causes Delay?

One of our main contributions is to provide a new mechanism for delay, the durability effect, which provides the deadline and election effects as its special cases. In this section, we inspect the more general determinants of delay in our framework. We show that some

[7] In the second example, gridlock is assumed to make one of the parties less popular *relative to the other party*. While it is intuitive that gridlock will reduce the popularity of the parties, it is not clear that this will benefit the other party—especially the one that obstructs the bills. It is more plausible that disillusioned voters go to third parties or outside/anti-establishment candidates (as int he case of 2016 US presidential elections). If this is the case, then one could expect electoral concerns to increase the parties' incentive to reach an agreement.

durability of bargaining power is necessary to obtain robust bargaining delays with optimism. We also illustrate that nonstationarity of the environment—while essential in our baseline framework—is not necessary once we consider a natural generalization of the model.

In our model, delay before some future time $t^*$ is driven by a combination of optimism about the time, $y_{t,t^*}$, and high congruence at the time, $K_{t^*}$. This induces the players to be optimistic about their bargaining shares in the future, $y_{t,t^*}K_{t^*}$, and induces them to wait. An increase in durability at time $t^*$ ensures that both conditions are satisfied. Our next result shows that some form of durability is also necessary to obtain delays. We capture nondurability by assuming that the one-period arrival probability is equal to one, $\Lambda_t = 1$ for each $t$, that is, the bargaining power is reset in every period [cf. (3)].

**Proposition 9** (Immediate Agreement without Durability)**.** *Assume $\Lambda_t = 1$ for all $t$. Then,*

$$K_t = \max\{1 - W_t, 0\} \leq 1 - \delta_t \qquad (\forall t),$$

*and there is immediate agreement in the continuous-time limit (i.e. $\lim_{n\to\infty} t_n^A = 0$ where $t_n^A$ is the first time with agreement).*

The first part of the result immediately follows from Proposition 1, which also applies in the limit case with non-durable bargaining power. The displayed equality is obtained by substituting $\Lambda_t = 1$ in (7). The inequality is obtained by noting that $W_t = \delta_t \left(S_{t+1/n} + \Lambda_t \bar{y} K_{t+1/n}\right) \geq \delta_t$. Intuitively, when the bargaining power is highly non-durable and changes in every period, the current bargaining power can affect only the gain from agreement in the current period. This gain is equal to the cost of waiting until the next period, which is bounded from above by the loss due to one-period time discounting, $1 - \delta_t$. The one-period loss is vanishingly small in the continuous-time limit in which the parties can frequently negotiate. This implies that the continuous-time limit features low congruence, $\lim_{n\to\infty} K_t = 0$ for each $t$, which in turn leads to the second part. Absent congruence, the parties cannot be optimistic about their shares from bargaining at the agreement time, that is, $\lim_{n\to\infty} y_{0,t_n^A} K_{t_n^A} = 0$, which leads to a vanishingly short period of delay, $t_n^A$. (Results with vanishing delays has also been established by Ali (2006) and Ortner (2013) in other setups, which implicitly assume non-durable bargaining power.)

Proposition 9 reinforces the key role that durability plays for our results on gridlock. It also helps to differentiate our paper from the related literature on bargaining with optimism. As discussed before, Yildiz (2003) analyzes delays with optimism under a special structure in which the bargaining power is serially independent over time—and thus, highly non-durable. This structure ensures that the players' optimism satisfies $y_{t,s} = y_{0,s}$ for each $t < s$, that is, the players' optimism about a future time $s$ is the same at all earlier times. Yildiz (2003) relates delays to a sudden drop in optimism about the future times. Specifically, if the optimism changes smoothly throughout ($y_{0,s} - y_{0,s+1/n} \leq (1 - \delta_t)/\delta_t$ for each $s$ and $t < s$), then he shows that there is immediate agreement. Conversely, if there is a future time $s = t^*$ at which the optimism drops sufficiently fast ($y_{0,t^*} - y_{0,t^*+1/n}$ is sufficiently large), then there is a period of delay before $t^*$. Proposition 9 reveals that the delay obtained in Yildiz (2003) is vanishingly short in the continuous-time limit. A drop in optimism at time $t^*$ is necessary for delays, as emphasized in Yildiz (2003), but it is by itself not sufficient. One also needs high congruence at time $t^*$, which requires some form of durability. We further show that high effective durability, combined with the observability of the bargaining power, provides an intuitive structure—with various applications—that delivers high congruence.

One feature of our model is that there is immediate agreement under stationarity (see Proposition 2). From here, it might be tempting to conclude that a nonstationary environment is necessary for delays. This conclusion would be incorrect. In Appendix A, we generalize the model to the case in which the durability rate can also be stochastic, $\lambda_s(t)$, where the state $s \in S$ follows a Markov chain. We then present an example in which the environment is stationary (all variables are time-independent), and yet there are delays in some of the states. Specifically, we take $S = \{H, L\}$ where $H$ denotes a state with high effective durability and $L$ denotes a state with low effective durability, and establish a condition under which the optimistic players disagree at state $s = L$ and agree at state $s = H$. The condition for the delay is more likely to hold when the effective durability in state $H$ is greater, and when the transition from state $L$ to state $H$ is more rapid. Intuitively, at the low-durability state, the players wait for the bargaining power to become more durable in order to reach an agreement. Thus, the example reinforces our earlier explanation that the delay in our model—the durability effect—is driven by an increase in effective durability. Nonstationarity does not play an important role beyond enabling us to capture the increase in effective durability using a tractable model. The example also illustrates that the increase in effective durability does not have to be deterministic but it should be expected by both parties with sufficiently high probability.

# 6 General Model

In our working paper, we allow bargaining power $\pi_t$ to be any stochastic process with piecewise continuous paths almost surely. We show that there (generally) exists a unique subgame-perfect Nash equilibrium up to the indifference discussed in Section 2. Moreover, the continuation value process $V_t^i$ is the unique solution to a system of stochastic difference equations. Under the common-prior assumption, the solution is straightforward: $V_t^i$ is the discounted average value of the future bargaining powers. Absent the common-prior assumption, the equation system is in general not tractable. The special structure we consider in this paper enables us to replace the stochastic difference equations with a more tractable and intuitive system of deterministic difference equations (see Proposition 1).

While we cannot solve the general model in closed form, we are able to generalize the main results and insights. The key step is to introduce a notion of the durability of bargaining power as a condition on the stochastic process $\pi_t$. Our notion of durability requires each player's expectation of the future bargaining power to be Lipschitz-continuous in time. This means that: (i) the expectation for nearby future times is almost the same as the current bargaining power, (ii) and as one considers more distant times, the rate at which the expectation changes is bounded from above. We define the inverse of this allowed rate of change (or the Lipschitz constant) as *the durability rate.* The special structure we analyze in this paper satisfies this general durability condition with the durability rate given by $1/\lambda(t)$.

With this definition, durability induces high congruence more generally. Specifically, we show that, when the bargaining power is highly durable over a sufficiently long time interval (from today onwards), the players' expected payoffs from bargaining reflect their current bargaining powers—resulting in high congruence. Moreover, by slightly strengthening the durability condition, we also show that the players reach agreement with high probability. This generalizes the main insights from the stationary model (Proposition 2).

A more subtle insight from our analysis also applies more generally: What matters for congruence and agreement is *effective durability*, which reflects a combination of durability and time discounting (cf. (11)). Specifically, if the time discounting is sufficiently large, e.g., in view of a deadline, then the above results hold even if the bargaining power is durable over a short time interval (from today onwards). Even though the bargaining power might not be durable in the strict sense of the word, it is sufficiently durable in payoff-relevant terms, which leads to similar bargaining outcomes. Thus, as in the stationary model, deadlines and durability provide similar disciplining roles on beliefs and bargaining outcomes.

Armed with these results, our working paper also establishes the analogues of the durability, deadline, and election effects in the more general setup. If the optimism about time $t^*$ at a prior time $t$ is sufficiently high (compared to an inverse measure of the durability after time $t^*$), then there is disagreement at $t$, generalizing the durability effect. We obtain the deadline effect as a special case of this result after observing that the arrival of a deadline starting at time $t^*$ increases the effective durability after time $t^*$ without affecting optimism before time $t^*$. We also obtain the election effect as a special case after observing that elections schedules just before time $t^*$ allow for optimism about time $t^*$ at prior times without affecting the effective durability after time $t^*$.

# 7 Conclusion

In this paper, we develop a tractable model of bargaining with optimism. The distinguishing feature of our model is that the bargaining power is somewhat *durable* and changes only due to important events (such as elections). Players know their current bargaining powers, but they can be optimistic that events will shift the bargaining power in their favor. The durability of bargaining power—the rate at which important events happen—provides a natural discipline on the player's beliefs. We define *congruence* (in political negotiations, *political capital*) as the extent to which a player's current bargaining power translates into expected payoffs from bargaining. We show that durability influences congruence and plays an important role in understanding bargaining gridlock driven by optimism, as well as the finer details of bargaining outcomes in political negotiations.

The model leads to a simple cost-benefit analysis of gridlock: the parties delay the agreement if their optimism about the bargaining shares in the future is higher than cost of waiting. The latter optimism is measured by the multiplication of optimism about bargaining power and the congruence at the future date. The question of gridlock then turns into the question of what causes high congruence. We show that some degree of durability is essential to obtain high congruence and robust bargaining delays. We also establish a general *durability effect* by which an increase in effective durability—a combination of durability and time discounting—leads to high congruence and ex-ante delays. As applications of the durability effect, we establish deadline and election effects by which an upcoming deadline or election leads to ex-ante delays in bargaining. The deadline effect is more prominent when players are more optimistic and when the deadline is less uncertain. For firm deadlines, the effect is less prominent when players perceive a greater cost of delay. The election effect is more prominent when the players are more optimistic, when the election is stronger (more likely to shift the bargaining power), when the bargaining power is more durable during non-election times (i.e., in more stable democracies), when the election is closer (in terms of the parties' popularities), and when there is not an incumbent in the race.

Our analysis of political negotiations reveals that political capital (or congruence) changes systematically over typical election cycles. With periodic elections, political capital is highest in the immediate aftermath of the election. The parties agree and give a large share to the party that just won the election. As the next election approaches, political capital declines. The parties reach "compromise solutions," giving some surplus to the party with low current bargaining power, before they eventually start to disagree in view of the upcoming election. We also analyze political cycles in which a stronger election that changes the bargaining power with greater probability alternates with a weaker election. The stronger election leads to a greater political capital in its aftermath, as well as a more severe gridlock in its run-up. This result suggests that the four-year elections in the US are associated with more gridlock compared to the midterm elections. The result also suggests a lame-duck effect: presidents that are not eligible to be reelected are associated with less political capital and more gridlock, relative to presidents that can be reelected. Our empirical analysis of legislative gridlock in the US lends some preliminary support to the election and lame duck effects. We leave a more complete empirical test of the predictions of our theory for future work.

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# A Appendix: Extensions

This appendix contains the omitted extensions of our baseline model. Appendix B conatains the omitted proofs. The online appendix contains the description of the data used in Section 4.3 and the proofs of the lemmas used in Appendix B.

## A.1 Election Model with Popularity and Endogenous Optimism

In our election model, the arrival during an election resets the bargaining power to a new level drawn from a fixed distribution. Consequently, the optimism about the bargaining power after the election depends on the probability of reset, $1 - e^{-\lambda^E}$, which we interpreted as the strength of the election, and optimism conditional on reset, $\bar{y}$. This specification is tractable and enables us to obtain several results. However, it does not speak to some important features of elections in practice such as the parties' relative popularities with voters, and how the elections map those popularities into bargaining power. In this section, we sketch a richer model in which we explicitly account for these features. The model illustrates that closer elections—in which the parties' popularity is close to one another—are associated with greater optimism and a more prominent election effect. The model also illustrates that the incumbency advantage reduces the severity of the election effect, which motivates our reduced form modeling of the incumbency effect in the main text.

Consider the single-election model and set baseline arrival probability to zero, so that $\lambda(t) = \lambda_E/\varepsilon$ for $t \in [t^* - \varepsilon, t^*]$ and $\lambda(t) = 0$ otherwise, where $\varepsilon$ is vanishingly small. Hence, the bargaining power can change only at the election date. As before, the probability of reset during the election is $1 - e^{-\lambda^E}$. The difference is that, in case of a reset, the bargaining power is set according to

$$\pi_t^1 = G(Z_t).$$

Here, $Z_t$ is an adapted (observable) stochastic process, and $G : \mathbb{R} \to [0, 1]$ is an increasing function.

The process, $Z_t$, captures the relative popularity of party 1 (compared to party 2). The baseline model can be thought of as the extreme case in which $Z_t$ is iid. In this section, we suppose each player $i$ believes $Z_t$ follows a Brownian motion with variance $\sigma^2$ and drift $\mu^i$, that is,

$$dZ_t = \mu^i dt + \sigma dB_t.$$

We capture optimism by assuming $\mu^1 > \mu^2$: that is, player 1 believes its relative popularity will increase faster than (or decrease slower than) what player 2 believes.

The function $G(\cdot)$ maps popularity into relative bargaining power. The mapping depends in part on how the election translates relative popularity into political offices. For instance, in the US congressional elections, the seats (in the aggregate) are allocated according to a mapping that is (roughly) linear in popularity. In contrast, in the US presidential elections, the presidency is allocated according to a mapping that is (roughly) binary in popularity. In addition, the mapping $G(\cdot)$ also depends on how the political system translates the political offices into bargaining strength. For instance, obtaining a majority of seats in the US House or the US Senate provides a large advantage to the majority party. However, obtaining at least 40% of the seats in the Senate is also important, since it allows the senators from the minority party to filibuster legislative action.

These considerations suggest that, for a variety of reasons, the function $G(\cdot)$ is steeper when the parties are close in popularity ($|Z_t| \simeq 0$) as opposed to when one party is more popular than another ($|Z_t|$ high). Therefore, we take $G(\cdot)$ to be an S-shaped function: that is, $G'(Z_t)$ is single peaked with maximum obtained at $Z_t = 0$, and it also satisfies $\lim_{Z_t \to -\infty} G'(Z_t) = \lim_{Z_t \to \infty} G'(Z_t) = 0$. We also assume the mapping is symmetric for the two parties, $G(Z_t) = -G(-Z_t)$. Since $\pi_t^1 = G(Z_t)$

represents the bargaining power, this also implies $G(0) = \frac{1}{2}$. An example function that satisfies all of these assumptions is $G(Z_t) = e^Z / \left(e^Z + 1\right)$.

The rest of the model is unchanged. We next characterize the solution. After the election, the parties reach agreement with $K_{t^*} = 1$ (since the baseline durability rate is set to one). Thus, there is disagreement at some time $t < t^*$ as long as

$$e^{-r(t^*-t)} \left(1 + y_{t,t^*}(Z_t)\right) > 1 \tag{29}$$

where $y_{t,t^*}(Z_t)$ is the level of optimism about the election date $t^*$ at time $t$, which depends now on $Z_t$ as well as $t$ and $t^*$. In particular,

$$y_{t,t^*}(Z_t) = \left(1 - e^{-\lambda_E}\right) \bar{y}_{t,t^*}(Z_t)$$

where

$$\bar{y}_{t,t^*}(Z_t) = E^1 \left[G(Z_{t^*}) | Z_t\right] - E^2 \left[G(Z_{t^*}) | Z_t\right].$$

In our baseline example, the analogous expression was $y_{t,t^*} = \left(1 - e^{-\lambda_E}\right) \bar{y}$. In this extension, optimism depends on the strength of the election effect, as before, but it also depends on the current level of popularity, $Z_t$, as well as the current time, $t$.

We use Ito's lemma to characterize the solution further. Note that party $i$ believes the bargaining power evolves according to the process

$$dG(Z_t) = \left(\mu^i G'(Z_t) + \frac{1}{2}\sigma^2 G''(Z_t)\right) dt + \sigma G'(Z_t)\, dB_t.$$

Since this is a nonlinear diffusion process, the party's expectations for a future time $t^*$ does not have a simple closed form solution. However, for times close to the election time, $t \simeq t^*$, the expectation is approximately equal to $G(Z_t) + \left(\mu^i G'(Z_t) + \frac{1}{2}\sigma^2 G''(Z_t)\right)(t^* - t)$. Thus, optimism shortly before the election is approximately

$$y_{t,t^*} \simeq \left(1 - e^{-\lambda_E}\right) G'(Z_t) \left(\mu^1 - \mu^2\right)(t^* - t). \tag{30}$$

Note that, since $G(\cdot)$ is an S-shaped function, optimism is decreasing in the absolute value of the relative popularity, $|Z_t|$, and approaches 0 as $|Z_t| \to \infty$. Intuitively, there is greater optimism about the bargaining power when the election results are expected to be close ($|Z_t|$ near zero) and no optimism when a landslide is expected (i.e., $|Z_t|$ is large).

Combining Eqs. (29) and (30) (and using $t^* - t \simeq 0$) provides condition (22) in the main text that characterizes delay. If optimism is sufficiently large, so that, $\left(1 - e^{-\lambda_E}\right)\left(\mu^1 - \mu^2\right) > r / G'(0)$, then there is delay as long as

$$|Z_t| < \bar{Z} \equiv \left(G'\right)^{-1} \left(\frac{r}{\left(1 - e^{-\lambda_E}\right)\left(\mu^1 - \mu^2\right)}\right). \tag{31}$$

That is, the players delay agreement when the election is sufficiently close ($|Z_t| < \bar{Z}$) but they reach immediate agreement otherwise. Hence, closer elections are associated with greater optimism and a more prominent election effect. Note also that the cutoff level of popularity that induces disagreement, $\bar{Z}$, is increasing in the discount rate, $r$, and decreasing in the strength of the election, $\lambda_E$, as well as the degree of optimism about the underlying process, $\mu^1 - \mu^2$. Hence, the analogues of the comparative statics in Proposition 3 also apply in this case. Unlike in the main text, however, the disagreement period prior to the election is no longer deterministic. Whether there is an agreement depends on the evolution of relative popularity, $Z_t$. As the election approaches, the parties receive information about how close the election will be. Shortly before the election, they might reach agreement or disagreement depending on the resolution of uncertainty.

Condition (31) can also be used to analyze the effect of an incumbent candidate on the incidence of gridlock. To this end, suppose the unconditional (objective) distribution of the popularity on the election eve is given by a Normal distribution, $Z_t \sim N\left(\iota, \bar{\sigma}^2\right)$, with mean $\iota$ and variance $\bar{\sigma}^2 > 0$. If player 1 (resp. player 2) is an incumbent, then we set $\iota > 0$ (resp. $\iota < 0$). This captures the idea that the incumbent candidates often have an advantage in popularity. The case without an incumbent is captured by $\iota = 0$. Suppose also that the parties' optimism, $\mu^1 - \mu^2$, is the same regardless of whether there is an incumbent in the race. Then, using (31), the likelihood of delay with an incumbent can be written as

$$\Pr\left(|Z_t| < \bar{Z}; \iota\right) = \Phi\left(\frac{\bar{Z} - \iota}{\bar{\sigma}^2}\right) - \Phi\left(\frac{-\bar{Z} - \iota}{\bar{\sigma}^2}\right),$$

where $\Phi(\cdot)$ denotes the cdf for the standard normal distribution. The effect of the incumbency advantage, $\iota$, on delay is then given by

$$\frac{d}{d\iota}\left(\Pr\left(|Z_t| < \bar{Z}; \iota\right)\right) = \frac{1}{\bar{\sigma}^2}\left(\phi\left(\frac{-\bar{Z} - \iota}{\bar{\sigma}^2}\right) - \phi\left(\frac{\bar{Z} - \iota}{\bar{\sigma}^2}\right)\right), \tag{32}$$

where $\phi(\cdot)$ denotes the pdf for the standard normal distribution. This expression is negative when $\iota > 0$ and positive when $\iota < 0$. Thus, the likelihood of delay is maximized when there is no incumbent, $\iota = 0$.

To understand this result, imagine raising the incumbency effect further for a party that already has some incumbency advantage ($\iota > 0$). This has two counteracting effects on the likelihood of delay. On the one hand, it tends to mitigate delay by increasing the probability that the incumbent will win by a sufficiently large margin ($Z_t > \bar{Z}$). On the other hand, it tends to exacerbate delay by reducing the probability that the incumbent will lose by a large margin ($Z_t < -\bar{Z}$). Eq. (32) illustrates that the first effect dominates. This is intuitively because the incumbent is more likely to win by a large margin than to lose by the same margin. Hence, the incumbency effect reduces the likelihood of delay by increasing the chance of a landslide election on average (equivalently, by reducing the chance of a close election). In the main text, we have captured this effect in reduced form via a smaller probability of reset, $1 - e^{-\lambda^E}$, which reduces the length of the delay by reducing the average optimism about the election.

## A.2 Stochastic Durability

In our model, the arrival rate, $\lambda(t)$, is deterministic although it can vary with time. We next describe an extension of the model in which the arrival rate can also be stochastic. We use the extended model to show that the our main result, the durability effect, is not driven by the non-stationarity of the environment per se, but instead by an increase in effective durability.

Suppose the arrival rate, $\lambda_s(t)$, can depend on a state $s \in S$ in addition to time. To be more specific, suppose the arrival rate between the negotiation times $t$ and $t + 1/n$ is still fixed but it is given by the level, $\lambda_s(t)$, that might depend on the state realization $s \in S$ at time $t$. Thus, the one period arrival rate is now given by,

$$\Lambda_{s,t} = 1 - e^{-\int_t^{t+1/n} \lambda_s(t')dt'}.$$

The state space, $S$, has finite elements and follows a Markov chain. We let $\zeta_{s'|s}$ denote the probability that the state at the next negotiation time, $t + 1/n$, is equal to $s'$ given state $s$ at time $t$. Our next result generalizes Proposition 1 to this case (the proof is omitted).

**Proposition 10.** *For each time $t \in T$, state $s \in S$, and player $i \in \{1,2\}$, the continuation value of a player is given by,*

$$V^i_{s,t} = K_{s,t}\pi^i_t + (S_{s,t} - K_{s,t})\frac{\bar{\pi}^i}{1+\overline{y}} \qquad \forall i,t,s,$$

*where the weights $K_t(s)$ and $S_t(s)$ are the unique solutions to the difference equations*

$$\begin{aligned} K_{s,t} &= \max\{1 - W_{s,t}, 0\} + \delta_t (1 - \Lambda_{s,t}) \sum_{s' \in S} \zeta_{s'|s} K_{s',t+1/n}, \\ S_{s,t} &= \max\{1, W_{s,t}\}, \\ W_{s,t} &= \delta_t \sum_{s' \in S} \zeta_{s'|s} \left(S_{s',t+1/n} + \Lambda_{s,t}\overline{y}K_{s',t+1/n}\right). \end{aligned}$$

*At any time $t$ and state $s$, the players agree if $W_{s,t} < 1$ and disagree if $W_{s,t} > 1$.*

The main difference in this case is that the difference equations that determine $K_{s,t}$ and $W_{s,t}$ are stochastic and reflect the potential future changes in durability. The analysis becomes particularly tractable when the discount rate $r$ is constant, and the durability rate can be written as only a function of the state, $\lambda_s$ (i.e., does not depend on time). In this case, the difference equations do not depend on time, and thus, the solution can also be written as a function the state, $K_s$ and $W_s$. The following example characterizes the solution further for a special case.

*Example* 1. Suppose $r$ is constant, $\lambda_s$ depends only on the state $s$, and there are two possible states, $s \in \{H, L\}$, with $\lambda_H > \lambda_L$. Thus, the effective durability $\rho_H = r/\lambda_H$ at state $H$ is higher than the effective durability $\rho_L = r/\lambda_L$ at state $L$. There is a positive probability of switching from state $L$ to state $H$, that is, $\zeta(H|L) = 1 - e^{-\sigma/n}$. Note that we have specified the probability in terms of the switch rate, $\sigma$, which will enable us to take the continuous time limit. For simplicity, there is zero probability of switching from state $H$ to state $L$, that is, $\zeta(L|H) = 0$.

Since state $H$ is an absorbing state, Proposition 2 applies at this state. The parties reach agreement with congruence $K_H = k(\rho_H)$. The sum of the expected payoffs is $S_H = 1$. At state $L$, the equilibrium is characterized by the difference equations in Proposition 10 after dropping the time subscripts. We conjecture an equilibrium in which there is disagreement at this state (i.e., $W_L > 1$). Then, the difference equation for congruence implies,

$$K_L = \delta(1 - \Lambda_L)\left(e^{-\sigma/n}K_L + \left(1 - e^{-\sigma/n}\right)K_H\right),$$

where $\delta = e^{-r/n}$ and $\Lambda_L = 1 - e^{-\lambda_L/n}$. Solving for $K_L$ and taking the limit, we obtain,

$$\lim_{n\to\infty} K_L = \frac{\sigma}{r + \sigma + \lambda_L} k(\rho_H).$$

The difference of the value of waiting implies,

$$W_L = \delta e^{-\sigma/n}(W_L + \Lambda_L\overline{y}K_L) + \delta\left(1 - e^{-\sigma/n}\right)(1 + \Lambda_L\overline{y}K_H).$$

Here, we have used $S_L = W_L$ and $S_H = 1$. Substituting the expressions for $K_H$ and $K_L$ and taking the limit, we obtain,

$$\lim_{n\to\infty} W_L = \frac{\sigma}{r+\sigma}\left(1+\frac{\lambda_L}{r+\sigma+\lambda_L}\bar{y}k\left(\rho_H\right)\right).$$

In the continuous-time limit, there is disagreement at state $L$ (i.e., $\lim_{n\to\infty} W_L > 1$) if and only if,

$$\bar{y}k\left(\rho_H\right) > \rho_L + \left(1+\rho_L\right)\frac{r}{\sigma}. \tag{33}$$

This verifies the conjecture and completes the characterization (for sufficiently large $n$).

Note that the left side of Eq. (33) is increasing in the level of optimism, $\bar{y}$, and in the effective durability at high-durability state, $\rho_H$. The right-hand side is increasing in the effective durability in low-durability state, $\rho_L$, and in the ratio, $r/\sigma$. Hence, there is disagreement at state $L$ as long as $\bar{y}$ and $\rho_H$ are sufficiently large and $\rho_L$ and $r/\sigma$ are sufficiently low. Intuitively, at the low-durability state, the players wait for the bargaining power to become more durable in order to reach an agreement. In contrast, if the bargaining power were more durable at state $L$ (i.e. If $\rho_L \geq \rho_H$), then players would also agree at state $L$.[8] Thus, the delay in the example is caused by an expected increase in durability.

The example provides an analogue of the durability effect established in Proposition 3 for the case in which durability rate is stationary (or time-independent) but stochastic. Condition (33) is the analogue of the condition (17), $\bar{y}k\left(\rho_H\right) > \rho_L$, that was necessary and sufficient in the deterministic case. In this case, we have an additional term, $\left(\rho_L+1\right)r/\sigma$, in the lower bound. Intuitively, not only the durability at state $H$ should be sufficiently higher than at state $L$, but there should also be sufficiently fast transition to state $H$ (high $\sigma$).

Importantly, the example illustrates that non-stationarity of the environment per se is not essential to obtain bargaining delays. The delay in Proposition 3 is caused by an expected increase in the effective durability at time $t^*$. Note also that, unlike in Proposition 3, the delay in this example can be arbitrarily long depending on the realization of uncertainty.

# B Appendix: Proofs

**Proof of Proposition 1.** Combining the agreement and the disagreement cases described in Section 2.2, the players' payoffs are characterized as the solution to the difference equation

$$V_t^i = \pi_t^i \max\left\{0, 1-W_{t,t+1/n}\right\} + \delta_{t,t+1/n} E_t^i\left[V_{t+1/n}^i\right]. \tag{34}$$

We claim that the payoffs described by Eqs. (6-8) solve this difference equation at all times. Since $V$ is the limit of the solutions to the models truncated at finite dates $\bar{t}$, it suffices to prove the claim for the truncated problem. Thus, consider the truncated model with a firm deadline at $\bar{t}$. We prove the claim by induction. Since $V_{\bar{t}}^i = \pi_{\bar{t}}^i$ and $\delta_{\bar{t}} = 0$, Eq. (6-8) holds at $t = \bar{t}$ with $K_{\bar{t}} = 1$, $S_{\bar{t}} = 1$, and $W_{\bar{t}} = 0$. Towards an induction, assume the equations hold for time $t+1/n$ onwards,

[8] This follows by observing the inequalities,

$$\rho_L + \left(1+\rho_L\right)r/\sigma > \rho_L > \bar{y}k\left(\rho_L\right) \geq \bar{y}k\left(\rho_H\right).$$

and consider time $t$. Then, the expected payoff of $i$ from waiting is

$$\begin{aligned} E_t^i\left[V_{t+1/n}^i\right] &= K_{t+1/n} E_t^i\left[\pi_{t+1/n}^i\right] + \frac{S_{t+1/n} - K_{t+1/n}}{1+\overline{y}} \overline{\pi}^i \\ &= K_{t+1/n}\left(1-\Lambda_t\right)\pi_t^i + \left(K_{t+1/n}\Lambda_t + \frac{S_{t+1/n} - K_{t+1/n}}{1+\overline{y}}\right)\overline{\pi}^i. \end{aligned}$$

Plugging this expression into (5), we obtain (9) for time $t$. Plugging the same expression into (34), we also obtain

$$\begin{aligned} V_t^i &= \max\{1-W_t, 0\}\pi_t^i + \delta_t E_t^i\left[V_{t+1/n}^i\right] \\ &= \left(\max\{1-W_t,0\} + \delta_t\left(1-\Lambda_t\right)K_{t+1/n}\right)\pi_t^i + \delta_t\left(K_{t+1/n}\Lambda_t\left(1+\overline{y}\right) + S_{t+1/n} - K_{t+1/n}\right)\frac{\overline{\pi}^i}{1+\overline{y}}. \end{aligned}$$

Hence, $V_t^i$ is also a weighted average of $\pi_t^i$ and $\frac{\overline{\pi}^i}{1+\overline{y}}$. We obtain (7) and (8) by setting the weights equal to $K_t$ and $S_t - K_t$, respectively. □

We define the variables

$$\delta^0 = e^{-r^0/n} \text{ and } \Lambda^0 = 1 - e^{-\lambda^0/n}, \tag{35}$$

to denote the one period discount rate and the arrival rate when $r(t)$ and $\lambda(t)$ are constant at their baseline levels $r^0$ and $\lambda^0$. Given some $\delta_t$ and $\Lambda_t$, we also define *the effective durability in discrete time* as,

$$\bar{\rho}\left(\delta_t, \Lambda_t\right) = \frac{1-\delta_t}{\delta_t \Lambda_t}. \tag{36}$$

Here,$\bar{\rho}\left(\delta_t, \Lambda_t\right)$ is the discrete time analogue of the effective durability, $\rho(t)$, which plays a central role in the analysis. It can be checked that

$$\lim_{n\to\infty} \bar{\rho}\left(\delta^0, \Lambda^0\right) = \rho^0. \tag{37}$$

It can also be checked that

$$\bar{\rho}\left(\delta^0, \Lambda^0\right) > \rho^0. \tag{38}$$

Hence, the effective durability rate is always higher in discrete time, and approaches its continuous time counterpart from above. Recall also from (18) that $w(\Delta, K, \eta) = e^{-\Delta}\left(1 + \left(1 - e^{-\Delta/\rho^0}\eta\right)K\overline{y}\right)$. In the online appendix, we prove the following lemma, establishing useful properties of the function $w(\cdot, K, \eta)$ for our subsequent proofs.

**Lemma 1.** *Suppose $\eta = 1$ and $K\overline{y} > \rho^0$, or $\eta < 1$ and $K > 0$.*

*(i) There exists a unique solution to the equation $w\left(\bar{\Delta}, K, \eta\right) = 1$.*

*(ii) At the solution $\Delta = \bar{\Delta}$, $w$ is strictly decreasing in $\Delta$; $w(\Delta, K, \eta) > 1$ is for each $\Delta \in \left(0, \bar{\Delta}\right)$, and $w(\Delta, K, \eta) < 1$ for each $\Delta > \bar{\Delta}$.*

**Proof of Proposition 3.** Note that the environment becomes stationary at $t^*$ with effective durability rate $\rho^1$. Hence, Proposition 2 applies starting at $t^*$ after replacing $\rho^0$ with $\rho^1$. As we show in the proof of Proposition 2 (for fixed $n$), players reach agreement at time $t^*$ (and thereafter) with congruence $K_{t^*} = k(\bar{\rho}(\delta^1, \Lambda^1))$, where $\delta^1 = e^{-r^1/n}$ and $\Lambda^1 = 1 - e^{-\lambda^1/n}$.

Next consider a negotiation time $t = t^* - \Delta/r^0 < t^*$. Using (4) and (6), the value of waiting until time $t^*$ can be written as

$$W_{t,t^*} = e^{-\Delta}\left(1 + y_{t,t^*} K_{t^*}\right) = w(\Delta, K_{t^*}, 1).$$

Recall also that $K_{t^*} = k(\bar{\rho}(\delta^1, \Lambda^1)) > k\left(\rho^1\right)$ since $\bar{\rho}\left(\delta^1, \Lambda^1\right) > \rho^1$ [cf. (38)]. Combining this observation with condition (17), we also have $K_{t^*}\bar{y} > \rho^0$. Then, Lemma 1 implies that there is a unique positive solution, $\tilde{\Delta}_n > 0$, to the equation, $w(\Delta, K_{t^*}, 1) = 1$.

Let $\tilde{t} = t^* - \tilde{\Delta}_n / r^0$ denote the time corresponding to delay, $\tilde{\Delta}_n$. We define $\bar{t} = t^* - \bar{\Delta}_n / r^0 \in T$ as the first negotiation time that is weakly greater than $\tilde{t}$. Lemma 1 (part (ii)) also implies that $W_{t,t^*} = w(\Delta, K_{t^*}, 1) \geq 1$ for each $t \in [\bar{t}, t^*)$. Hence, there is disagreement at times $t \in [\bar{t}, t^*) \cap T$. Note also that $\bar{t} - 1/n < \tilde{t}$. Then, Lemma 1 (part (ii)) also implies that $W_{\bar{t}-1/n} = w(\bar{\Delta} + r^0/n, K_{t^*}, 1) < 1$. Hence, there is agreement at time $\bar{t} - 1/n$.

It remains to characterize the continuous time limit. First recall that $\lim_{n\to\infty} K_{t^*} = k(\bar{\rho}(\delta^1, \Lambda^1)) = k\left(\rho^1\right)$. Next note that the sequence $\left\{\tilde{\Delta}_n\right\}_n$ has a convergent subsequence (since it lies in a compact set). Let $\bar{\Delta}$ denote the limit of an arbitrary convergent subsequence. Then, taking the limit of the equation $w(\tilde{\Delta}_n, k(\bar{\rho}(\delta^1, \Lambda^1)), 1) = 1$ over this subsequence, we obtain the equation $w(\bar{\Delta}, k\left(\rho^1\right), 1) = 1$. Lemma 1 implies that there is a unique positive solution, $\bar{\Delta} > 0$, to this equation. Since all convergent subsequences have the same limit, we obtain $\lim_{n\to\infty} \tilde{\Delta}_n = \bar{\Delta}$. Recall that we defined $\bar{t}$ as the first negotiation time that weakly exceeds $\tilde{t}$. This implies $\bar{t} = \max\left(0, t^* - \tilde{\Delta}_n r^0 + f_n\left(t^* - \tilde{\Delta}_n r^0\right)\right)$, where $f_n\left(\tilde{t}\right)$ is an "error function" which satisfies $f_n\left(\tilde{t}\right) < 1/n$ for each $\tilde{t} \in \mathbb{R}$. Taking the limit, we obtain $\lim_{n\to\infty} \bar{t} = \max\left(0, t^* - \bar{\Delta}/r^0\right)$, completing the proof. □

**Proof of Proposition 4.** By Lemma 1, $\bar{\Delta}$ is increasing in any change that increases the value of waiting evaluated at the solution, $w\left(\bar{\Delta}, k\left(\rho^1\right), 1\right)$. Substituting $\rho^0 = \hat{r}/\lambda^0$ and $\rho^1 = \left(\hat{r} + \alpha\right)/\lambda^1$ into (18), we obtain

$$w\left(\Delta, k\left(\rho^1\right), 1\right) = e^{-\Delta}\left(1 + \left(1 - e^{-\Delta\lambda^0/\hat{r}}\right) k\left(\left(\hat{r} + \alpha\right)/\lambda^1\right)\bar{y}\right) \equiv f\left(\Delta, \hat{r}\right).$$

Here, the second equality defines the function $f\left(\Delta, \hat{r}\right)$. Then, the first two parts follow because $f\left(\bar{\Delta}, \hat{r}\right)$ is decreasing in $1/\lambda^0$, and increasing in $1/\lambda^1$ and $\alpha$ (since $k\left(\rho^1\right)$ is increasing in $\rho^1$). The third part also follows after observing that $k\left(\rho^1\right)\bar{y}$ is increasing in $\bar{y}$. It remains to prove the last part. By the Implicit Function Theorem, we have, $\frac{d\bar{\Delta}}{d\hat{r}} = -\left(\frac{\partial w}{\partial \Delta}|_{\Delta=\bar{\Delta}}\right)^{-1} \frac{\partial f}{\partial \hat{r}}|_{\Delta=\bar{\Delta}}$. Since $\frac{\partial w}{\partial \Delta}|_{\Delta=\bar{\Delta}} < 0$, the derivative has the same sign as

$$\frac{\partial f}{\partial \hat{r}}|_{\Delta=\bar{\Delta}} = -\frac{1}{\hat{r}^2}\lambda^0 e^{-\bar{\Delta}\lambda^0/\hat{r}} e^{-\bar{\Delta}} k\left(\rho^1\right)\bar{y} + \left(1 - e^{-\bar{\Delta}\lambda^0/\hat{r}}\right)\bar{y}\frac{dk\left(\rho^1\right)}{d\rho^1}\frac{d\rho^1}{dr}.$$

As $\alpha \to \infty$, we have $k\left(\rho^1\right) \to 1$ and $\frac{dk\left(\rho^1\right)}{d\rho^1} \to 0$. Hence, the first term on the right hand side remains strictly negative whereas the second term approaches zero. Let $\bar{\alpha} < \infty$ sufficiently large so that $\frac{\partial f}{d\hat{r}}|_{\Delta=\bar{\Delta}} < 0$ for each $\alpha > \bar{\alpha}$. Then, we also have $\frac{d\bar{\Delta}}{d\hat{r}} < 0$ for each $\alpha > \bar{\alpha}$. □

**Proof of Proposition 5.** The proof closely parallels the proof of Proposition 3 with minor differences. In this case, the environment becomes stationary at $t^*$ with effective durability rate $\rho^0$. Hence, the players agree at each time $t^*$ (and thereafter) with $K_{t^*} = k(\bar{\rho}(\delta^0, \Lambda^0)) > k\left(\rho^0\right)$.

Consider a negotiation time $t = t^* - \Delta/r^0 < t^*$. The value of waiting is now given by

$$W_{t,t^*} = e^{-\Delta}\left(1 + \left(1 - e^{-\Delta/\rho^0}\eta\right)K_{t^*}\right) = w(\Delta, K_{t^*}, \eta),$$

where we use the notation $\eta = e^{-\lambda^E}$ to describe the survival probability corresponding to the election. Since $\eta < 1$ and $K_{t^*} > 0$, Lemma 1 implies there is a unique positive solution, $\bar{\Delta}_n > 0$, to the equation $w(\Delta, K_{t^*}, \eta) = 1$. Let $\bar{t} = t^* - \bar{\Delta}_n/r^0$. Then, the same steps as in the proof of Proposition 3 yield disagreement at each $t \in [\bar{t}, t^*) \cap T$ and agreement at $\bar{t} - 1/n$. We also obtain $\lim_{n\to\infty} \bar{t} = \max\left(0, t^* - \bar{\Delta}/r^0\right)$, where $\bar{\Delta}$ denotes the unique solution to the equation $w(\Delta, k\left(\rho^0\right), \eta) = 1$. □

**Proof of Proposition 6.** Now, $\bar{\Delta}$ is increasing in any change that increases

$$w\left(\Delta, k\left(\rho^0\right), e^{-\lambda^E}\right) = e^{-\Delta}\left(1 + \left(1 - e^{-\Delta\rho^0}e^{-\lambda^E}\right)k\left(\rho^0\right)\bar{y}\right).$$

Then, $\bar{\Delta}$ is increasing in $\lambda^E$ and $\bar{y}$ because $w\left(\Delta, k\left(\rho^0\right), e^{-\lambda^E}\right)$ is increasing in $\lambda^E$ and $\bar{y}$; recall that $k\left(\rho^0\right)\bar{y}$ is increasing in $\bar{y}$. To show that $\bar{\Delta}$ is increasing in $1/\lambda^0$, we will show that $\frac{d\bar{\Delta}}{d\rho^0} > 0$; recall that $\rho^0 = r^0/\lambda^0$. By the Implicit Function Theorem, $\frac{d\bar{\Delta}}{d\rho^0} = -\left(\frac{\partial w}{\partial \Delta}|_{\Delta=\bar{\Delta}}\right)^{-1}\frac{\partial w\left(\bar{\Delta}, k\left(\rho^0\right), e^{-\lambda^E}\right)}{\partial \rho^0}$. By Lemma 1, we have $\frac{\partial w}{\partial \Delta}|_{\Delta=\bar{\Delta}} < 0$, and we show in the online appendix that

$$\frac{\partial w\left(\bar{\Delta}, k\left(\rho^0\right), e^{-\lambda^E}\right)}{\partial \rho^0} = e^{-\bar{\Delta}}\bar{y}\bar{\Delta}\frac{k\left(\rho^0\right)}{\left(\rho^0\right)^2}\left(\frac{e^{\bar{\Delta}} - 1}{\bar{\Delta}}\frac{1 + \bar{y}}{\bar{y}} - e^{-\bar{\Delta}/\rho^0}\eta\right) > 0. \tag{39}$$

Therefore, $\frac{d\bar{\Delta}}{d\rho^0} > 0$, completing the proof. □

The following lemmas facilitate subsequent proofs. The first lemma provides an upper bound for congruence in the election models analyzed in Sections 4.2 and 4.3. The second lemma is an analogue of Lemma 1 in this context. The proofs of the lemmas are relegated to the online appendix.

**Lemma 2.** *Suppose $\delta_t = \delta^0$ and $\Lambda_t \geq \Lambda^0$ for each $t \in T$. Then, $K_t \in \left[0, k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)\right]$ for each $t \in T$.*

**Lemma 3.** *Consider the function $f(\Delta) \equiv w(\Delta, \mathbf{K}(\Delta), \eta)$ where $\eta < 1$ and $\mathbf{K}(\cdot)$ is a function that satisfies $K'(\Delta) < 0$ for each $\Delta \in [0, \Delta^{\max}]$ and $\mathbf{K}(\Delta^{\mu a x}) = 0$. Then, the equation $f(\Delta) = 1$ has a unique solution over the range $[0, \Delta^{\max}]$. Moreover, $\bar{\Delta} \in (0, \Delta^{\max})$.*

**Proof of Proposition 7.** The equilibrium is periodic since otherwise we could construct multiple equilibria, violating the uniqueness result. We next define the threshold time, $\bar{t}$. If there is no agreement date (with $W_t < 1$), we let $\bar{t} = 0$. Otherwise, we let $\bar{t} - 1/n \in T$ be the last agreement time ($W_{\bar{t}-1/n} < 1$) within $[0, t^*)$. By definition, there is disagreement ($W_t \geq 1$) over the range $t \in [\bar{t}, t^*)$ (which could be empty).

We next claim that there is agreement ($W_t < 1$) over the interval $[0, \bar{t})$. If $\bar{t} = 0$, then the interval is empty and the statement is true. Suppose $\bar{t} > 0$ and consider $t \in [0, \bar{t}) \cap T$. We prove the claim by backward induction. Note that there is agreement at time $\bar{t} - 1/n$. Suppose there is agreement at some $t + 1/n \in [0, \bar{t}) \cap T$ and consider the prior negotiation time. Using Eq. (9) along with $S_{t+1/n} = 1$ (since there is agreement), we obtain

$$W_t = \delta^0 \left(1 + \Lambda^0 \bar{y} K_{t+1/n}\right) \leq \delta^0 \left(1 + \Lambda^0 \bar{y} k \left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)\right) < 1.$$

Here, the first inequality follows from Lemma 2, which applies to this model and implies $K_{t+1/n} \leq k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)$. The second inequality follows after substituting $k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right) = \frac{1-\delta^0}{1-\delta^0+\delta^0\Lambda^0(1+\bar{y})}$ [cf. (14)]. Hence, there is also agreement at time $t$, completing the proof of the claim.

We next characterize the congruence levels. We use the notation $\eta = e^{-\lambda^E} \in (0,1)$. Since there is disagreement over the range $[\bar{t}, t^*)$, Eq. (7) implies

$$K_{\bar{t}} = e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} \eta K_{t^*} = e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} \eta K_0.$$

This expression implies in particular that $\bar{t} > 0$. Next consider an agreement time $t \in [0, \bar{t})$. Substituting $W_t = \delta^0 \left(1 + \Lambda^0 \bar{y} K_{t+1/n}\right)$ into (7), we obtain

$$K_t = 1 - \delta^0 + \delta^0 \left(1 - \Lambda^0 \left(1 + \bar{y}\right)\right) K_{t+1/n} = 1 - \delta^0 + \hat{\delta} K_{t+1/n},$$

where $\hat{\delta} = \delta^0 \left(1 - \Lambda^0 \left(1 + \bar{y}\right)\right)$ (see (13)). Solving the difference equation forward, for each $t \in [0, \bar{t})$, we obtain

$$K_t = \frac{1-\delta^0}{1-\hat{\delta}} \left(1 - \hat{\delta}^{n(\bar{t}-t)}\right) + \hat{\delta}^{n(\bar{t}-t)} K_{\bar{t}} = k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right) \left(1 - \hat{\delta}^{n(\bar{t}-t)}\right) + \hat{\delta}^{n(\bar{t}-t)} K_{\bar{t}}. \tag{40}$$

Here, the second equality substitutes $k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right) = \frac{1-\delta^0}{1-\hat{\delta}}$. This establishes Eq. (24) in the main text. Substituting for $K_{\bar{t}}$, we obtain

$$K_t = \left(1 - \hat{\delta}^{n(\bar{t}-t)}\right) k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right) + \hat{\delta}^{n(\bar{t}-t)} e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} \eta K_0$$

for each $t \in [0, \bar{t})$. Applying this expression for $t = 0$, we obtain

$$K_0 = \frac{1 - \hat{\delta}^{n\bar{t}}}{1 - \hat{\delta}^{n\bar{t}} e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} \eta} k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right).$$

Note that $K_0 < k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)$ since $\eta < 1$. This in turn implies $K_{\bar{t}} = e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} \eta K_0 < k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)$. Hence, by (40), $K_t$ is strictly decreasing in $t$ over the agreement range $[0, \bar{t})$.

Let $\bar{\Delta}_n = r^0 \left(t^* - \bar{t}\right)$ denote the payoff relevant distance of the threshold time, $\bar{t}$. Since there is disagreement over the range $[\bar{t}, t^*)$ and agreement at $\bar{t} - 1/n$, we have $W_{\bar{t}, t^*} \geq 1 > W_{\bar{t}-1/n, t^*}$. Using $W_{t,t^*} = w\left(r^0 \left(t^* - t\right), K_{0,n}, \eta\right)$, the threshold, $\bar{\Delta}_n$, satisfies

$$w\left(\bar{\Delta}_n, K_{0,n}, \eta\right) \geq 1 > w\left(\bar{\Delta}_n + r^0/n, K_{0,n}, \eta\right). \tag{41}$$

From the above analysis, $K_{0,n}$ can also be written as a function of $\bar{\Delta}_n$:

$$K_{0,n} = \frac{1 - \hat{\delta}^{n\left(t^* - \bar{\Delta}_n/r^0\right)}}{1 - \hat{\delta}^{n\left(t^* - \bar{\Delta}_n/r^0\right)} e^{-(1+1/\rho^0)\bar{\Delta}_n} \eta} k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right). \tag{42}$$

The equilibrium for any finite $n$ is characterized as the solution to Eqs. $(41 - 42)$.

It remains to characterize the continuous time limit. Note that the sequence $\left\{K_{0,n}, \bar{\Delta}_n\right\}_n$ is bounded (since $K_{0,n} \in [0,1]$ and $\bar{\Delta}_n \in \left[0, t^* r^0\right]$). Thus, it has at least one limit point denoted by $\left(K(0), \bar{\Delta}\right)$. Taking the limit of Eqs. (41) and (42), and using $\lim_{n\to\infty} \hat{\delta}^n = e^{-\left(r^0+\lambda^0(1+\bar{y})\right)} = e^{-r^0/k\left(\rho^0\right)}$, we obtain the system

$$
\begin{aligned}
w\left(\bar{\Delta}, K(0), \eta\right) &= 1, \\
K(0) &= \frac{1 - e^{-\left(r^0 t^* - \bar{\Delta}\right)/k(\rho_0)}}{1 - e^{-\left(r^0 t^* - \bar{\Delta}\right)/k(\rho_0)} e^{-(1+1/\rho^0)\bar{\Delta}} e^{-\lambda^E}} k(\rho_0).
\end{aligned} \tag{43}
$$

By Lemma 3, this system has a unique and interior solution, $\bar{\Delta} \in (0, \Delta^{\max})$. Since there is a unique limit point, we conclude that $\lim_{n\to\infty} \left\{\bar{\Delta}_n\right\}_n = \bar{\Delta}$ and $\lim_{n\to\infty} K_{0,n} = \mathbf{K}_0\left(\bar{\Delta}\right)$. This also implies $\lim_{n\to\infty} \bar{t} = t^* - \bar{\Delta}/r^0$, completing the proof. □

The following lemma facilitates the subsequent proof. The lemma establishes the uniqueness and the comparative statics of the solution to an equation system, which is the analogue of the system in (43) for the case of alternating elections. The equation system of interest can be written as $F\left(\bar{\mathbf{x}}|\eta^{odd}, \eta^{even}\right) = 0$ where $F\left(\cdot|\eta^{odd}, \eta^{even}\right) : X \to \mathbb{R}^4$ is a vector valued function over the domain

$$
X = \left\{\mathbf{x} = \left(K_0, \Delta^{odd}, K_{t^*}, \Delta^{even}\right) \in \mathbb{R}^4 \quad | \quad K_0, K_{t^*} \in \left[0, k\left(\rho^0\right)\right], \Delta^{odd}, \Delta^{even} \in \left[0, t^* r^0\right]\right\}.
$$

The components of the function $F\left(\cdot|\eta^{odd}, \eta^{even}\right)$ are defined by

$$
\begin{aligned}
F_1(\mathbf{x}) &= K_0 - k^{sum}\left(\Delta^{odd}, K_{t^*}, \eta^{odd}\right) \\
F_2(\mathbf{x}) &= 1 - w\left(\Delta^{odd}, K_{t^*}, \eta^{odd}\right) \\
F_3(\mathbf{x}) &= K(t^*) - k^{sum}\left(\Delta^{even}, K_0, \eta^{even}\right) \\
F_4(\mathbf{x}) &= 1 - w\left(\Delta^{even}, K_0, \eta^{even}\right).
\end{aligned} \tag{44}
$$

where

$$
k^{sum}\left(\Delta, K^{next}, \eta\right) = \left(1 - e^{-\left(r^0 t^* - \Delta\right)/k\left(\rho^0\right)}\right) k\left(\rho^0\right) + e^{-\left(r^0 t^* - \Delta\right)/k\left(\rho^0\right)} e^{-\left(1+1/\rho^0\right)\bar{\Delta}} \eta K^{next}. \tag{45}
$$

**Lemma 4.** *Suppose condition* (27) *holds. Then:*

*(i) For each $\eta^{odd}, \eta^{even} \in (0,1)$, the system $F\left(\bar{\mathbf{x}}|\eta^{odd}, \eta^{even}\right) = 0$ has a unique solution over $X$, which is also interior, that is, $K(0), K(t^*) \in (0,1)$ and $\bar{\Delta}^{odd}, \bar{\Delta}^{even} \in \left(0, t^* r^0\right)$.*

*(ii) Suppose, in addition, that condition* (28) *holds and $\eta^{odd} > \eta^{even}$. Then, the solution satisfies $\bar{\Delta}^{even} > \bar{\Delta}^{odd}$ and $K(0) > K(t^*)$.*

The proof is relegated to the online appendix. We briefly explain the proof since the analysis is not trivial. The first part relies on the Poincare-Hopf Index Theorem. We show that the Jacobean matrix evaluated at any interior solution, $\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}$, has a positive determinant. Moreover, the vector valued function $F$ points "outwards" on the boundary of its domain, $X$. In view of these observations, the Poincare-Hopf Theorem implies that there is a unique solution.

The second part of the lemma uses the Implicit Function Theorem (together with tedious algebra) to characterize how the solution changes in the inverse strength of the odd election $\frac{d\bar{\mathbf{x}}}{d\eta^{odd}}$.

In particular, we show

$$\frac{dK(0)}{d\eta^{odd}} > 0, \frac{d\overline{\Delta}^{odd}}{d\eta^{odd}} < 0, \frac{dK(t^*)}{d\eta^{odd}} < 0, \text{ and } \frac{d\overline{\Delta}^{even}}{d\eta^{odd}} > 0.$$

That is, reducing the strength of the odd election (increasing $\eta^{odd} = e^{-\lambda^{odd}}$) decreases $\overline{\Delta}^{odd}$ and $K(t^*)$, and increases $K(0)$ and $\overline{\Delta}^{even}$. We also observe that if the two elections were equally strong, then the model would be the same as the periodic election model and the solution would be symmetric, that is, $K(0)|_{\tilde{\eta}^{odd}=\eta^{even}} = K(t^*)|_{\tilde{\eta}^{odd}=\eta^{even}}$ and $\overline{\Delta}^{odd}|_{\tilde{\eta}^{odd}=\eta^{even}} = \overline{\Delta}^{even}|_{\tilde{\eta}^{odd}=\eta^{even}}$. Combining these observations implies that, when $\eta^{odd} > \eta^{even}$, the odd election (which is the weaker election) is associated with a smaller delay in its run-up, $\overline{\Delta}^{odd} < \bar{\Delta}^{even}$, and a smaller congruence in its aftermath, $K(t^*) < K(0)$.

**Proof of Proposition 8.** We let $\eta^{odd} = e^{-\lambda^{E,odd}}, \eta^{even} = e^{-\lambda^{E,even}} \in (0,1)$ denote the survival probabilities with respect to odd and even elections. In this case, the uniqueness of the solution implies that the outcomes are periodic with cycle $2t^*$. We define the thresholds $\bar{t}^{odd}$ and $\bar{t}^{even}$ similar to the proof of Proposition 7. If there is no agreement date (with $W_t < 1$) over $[0, t^*)$, then we let $\bar{t}^{odd} = 0$. Otherwise, we let $\bar{t}^{odd} - 1/n$ be the last agreement date over $[0, t^*)$. Note that in both cases, there is disagreement over the range, $[\bar{t}^{odd}, t^*)$ (which could be empty). We define $\bar{t}^{even}$ symmetrically over the range, $[t^*, 2t^*)$.

With these definitions in place, the same steps as in the proof of Proposition 7 imply that there is agreement over the intervals $[0, \bar{t})$ and $[t^*, 2t^*)$ (which could be empty). Moreover, the congruences at election times satisfy

$$\begin{aligned} K_0 &= \left(1 - \hat{\delta}^{n\bar{t}^{odd}}\right) k\left(\bar{\rho}(\delta, \Lambda)\right) + \hat{\delta}^{n\bar{t}^{odd}} e^{-\left(r^0+\lambda^0\right)(t^*-\bar{t})} \eta^{odd} K_{t^*}, \\ K_{t^*} &= \left(1 - \hat{\delta}^{n(\bar{t}^{even}-t^*)}\right) k\left(\bar{\rho}(\delta, \Lambda)\right) + \hat{\delta}^{n(\bar{t}^{even}-t^*)} e^{-\left(r^0+\lambda^0\right)(2t^*-\bar{t}^{even})} \eta^{even} K_0. \end{aligned} \tag{46}$$

These equations also imply that $\bar{t}^{odd} > 0$ or $\bar{t}^{even} > t^*$ (since otherwise there would be a contradiction). We next establish more strongly that, if $n$ is sufficiently large, then $\bar{t}^{odd} > 0$ and $\bar{t}^{even} > t^*$. That is, there is an agreement date on each election cycle. To establish this, first recall that $\lim_{n\to\infty} k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right) = k\left(\rho^0\right)$ (see (37)). Recall also that condition (27) implies $1 + k\left(\rho^0\right)\overline{y} < e^{r^0 t^*}$. Then, there exists a sufficiently large $\bar{n}^{int}$ such that, for each $n$, the discrete time analogue of the same condition also holds, $1 + k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)\overline{y} < e^{r^0 t^*}$. Next let $n > \bar{n}^{int}$ and suppose, to reach a contradiction, that $\bar{t}^{odd} = 0$. Then, there is disagreement on the interval, $[0, t^*)$, and we have $W_{0,t^*} = w\left(r^0 t^*, K_{t^*}, \eta^{odd}\right) \geq 1$. Using (18) and $\left(1 - e^{-\Delta/\rho^0}\eta^{odd}\right) < 1$, this implies $e^{r^0 t^*} < 1 + K_{t^*}\overline{y}$. By Lemma 2, we also have $K_{t^*} \leq k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)$. This yields a contradiction to the condition, $1 + k\left(\bar{\rho}\left(\delta^0, \Lambda^0\right)\right)\overline{y} < e^{r^0 t^*}$, proving that $\bar{t}^{odd} > 0$. The same steps also imply that $\bar{t}^{even} > t^*$.

Since $\bar{t}^{odd} > 0$, there is agreement at time $\bar{t}^{odd} - 1/n$ and disagreement over the range, $[\bar{t}, t^*)$. Likewise, there is agreement at time $\bar{t}^{even} - 1/n$ and disagreement over the range, $[\bar{t}^{even}, 2t^*)$. In view these observations, the thresholds are characterized by the system

$$\begin{aligned} w\left(\bar{\Delta}_n^{odd}, K_0, \eta\right) &\geq 1 > w\left(\bar{\Delta}_n^{odd} + r^0/n, K_0, \eta\right) \\ w\left(\bar{\Delta}_n^{even}, K_{t^*}, \eta\right) &\geq 1 > w\left(\bar{\Delta}_n^{even} + r^0/n, K_{t^*}, \eta\right). \end{aligned} \tag{47}$$

Here, $\bar{\Delta}_n^{odd} = r^0\left(t^* - \bar{t}^{odd}\right)$ and $\bar{\Delta}_n^{even} = r^0\left(2t^* - \bar{t}^{even}\right)$ denote the payoff relevant distances for delay as before. The equilibrium for any finite $n$ is characterized by Eqs. $(46 - 47)$.

As before, we also characterize the continuous time limit. Taking the limit as $n \to \infty$, any limit point of the sequence $\left\{K_{0,n}^{odd}, \bar{\Delta}_n^{odd}, K_{t^*,n}^{even}, \bar{\Delta}_n^{even}\right\}_n$ solves the following system:

$$\begin{aligned} K(0) &= k^{sum}\left(\bar{\Delta}^{odd}, K(t^*), \eta^{odd}\right) \\ K(t^*) &= k^{sum}\left(\bar{\Delta}^{even}, K(0), \eta^{even}\right) \\ w\left(\bar{\Delta}^{odd}, K(t^*), \eta^{odd}\right) &= 1 \\ w\left(\bar{\Delta}^{even}, K(0), \eta^{even}\right) &= 1. \end{aligned}$$

Here, $k^{sum}(\cdot)$ denotes the function defined in (45). Thus, the equation system in the continuous time limit is given by $F\left(\bar{\mathbf{x}}|\eta^{odd}, \eta^{even}\right) = 0$ where $F\left(\cdot|\eta^{odd}, \eta^{even}\right)$ is the vector valued function defined in (44). By the first part of Lemma 4, there is a unique and interior solution, $K(0), K(t^*) \in (0,1)$ and $\bar{\Delta}^{odd}, \bar{\Delta}^{even} \in \left(0, t^* r^0\right)$. Since there is a unique limit point, we conclude that

$$\lim_{n\to\infty} K_{0,n} = K(0), \lim_{n\to\infty}\left\{\bar{\Delta}_n^{odd}\right\}_n = \bar{\Delta}^{odd}, \lim_{n\to\infty} K_{t^*,n} = K(t^*), \lim_{n\to\infty}\left\{\bar{\Delta}_n^{even}\right\}_n = \bar{\Delta}^{even}. \tag{48}$$

Next consider the comparative statics. By the second part of Lemma 4, the solution to the continuous time limit also satisfies $\bar{\Delta}^{even} > \bar{\Delta}^{odd}$ and $K(0) > K(t^*)$. In view of (48), there exists $\bar{n} > \bar{n}^{int}$ such that the discrete time solution also satisfies the same inequalities, $\bar{\Delta}_n^{even} > \bar{\Delta}_n^{odd}$ and $K_{0,n} > K_{t^*,n}$, for each $n > \bar{n}$. These inequalities also imply $\bar{t}^{even} - t^* < \bar{t}^{odd}$, which completes the proof of the proposition for each $n > \bar{n}$. □

**Proof of Proposition 9.** The first part is proven in the main text. To prove the second part, observe that the value of waiting for agreement time $t_n^A$ at time 0 is

$$W_{0,t_n^A} = \delta_{0,t_n^A}\left(S_{t_n^A} + \Lambda_{0,t_n^A}\bar{y}K_{t_n^A}\right) = \delta_{0,t_n^A}\left(1 + \bar{y}K_{t_n^A}\right) \leq \delta_{0,t_n^A}\left(1 + \bar{y}\left(1 - \delta_{t_n^A}\right)\right).$$

Here, the first equality is by substituting (6) to definition (5). The next equality is by $S_{t_n^A} = \Lambda_{0,t_n^A} = 1$, and the inequality is by the first part. Since $W_{0,t_n^A} \geq 1$ in equilibrium, we have $\delta_{0,t_n^A} \geq 1/\left(1 + \bar{y}\left(1 - \delta_{t_n^A}\right)\right)$. When $r$ is bounded, the right-hand limits to 1. Hence, $\delta_{0,t_n^A}$ limits to 1, and $t_n^A$ limits to 0. □

# Online Appendix for "Durability, Deadline, and Election Effects"

Alp Simsek and Muhamet Yildiz

## A Description of the Data Used in Section 4.3

Our data for gridlock comes from Mayhew (1991), who constructed a list of enactments of important laws in the US since 1947. The major measurement challenge here is to identify the important laws among the numerous laws that the congresses regularly pass. Mayhew constructs two separate lists—which he refers to as "sweeps"—that uses two related but distinct principles to establish importance. *Sweep One* is based on the contemporary judgement of journalists, who appraised the laws as they were passed. *Sweep Two* is based on the retrospective judgements by policy specialists—who assigned importance to the laws in the more recent writings. The second list has the advantage that importance is arguably best judged with hindsight, but the first list has the advantage of being available for a longer period of time. Mayhew also provides a master list that combines these two lists by taking a union (as opposed to the intersection). In his own empirical analysis, Mayhew uses this master list. Mayhew also argues that the master list provides a cleaner measurement than alternative ways of combining the two sweeps.[9]

Following Mayhew, we have also used the master list in our analysis. We have obtained the extended data set that covers the years between 1947 and 2012 from Mayhew's website.[10] We have linked this data with information on the US presidential elections and term limits. Specifically, we have coded each US Congress according to whether it corresponds to "years 1-2", "years 3-4", "years 5-6", or "years 7-8" of a presidency. There are some exceptions in the data, which we have addressed as follows:

- The two-term limit for the US presidents was established in the post-war years by a constitutional amendment. The amendment passed the Congress in 1947, and it was ratified in 1951. During this period, Harry Truman was the president. As the sitting president, he was exempted from the limit. Thus, there was some uncertainty about whether he would rerun in 1952. Truman decided not to run in 1952, and a new president (Eisenhower) took office in 1953. In view of these observations, we have started our analysis in 1953 (that is, we have excluded the congresses between 1947-1952).

- John F. Kennedy was assassinated in 1963. The vice president, Lyndon Johnson, took the office. We have coded the Kennedy-Johnson presidency in years 1963-1964 as "years 3-4" to ensure consistency with the electoral cycle.

- Lyndon Johnson won the election in 1964 and extended his presidency. We have coded Lyndon Johnson's presidency of 1965-1968 as "years 1-2" and "years 3-4," since Johnson was eligible to be reelected in 1968.

- During his second term as president, Richard Nixon resigned from the office on August 9, 1974. The vice president, Gerald Ford, took the office. We have coded the Nixon–Ford presidency in years 1973-1974 as "years 5-6". That is, we treated these years as if Nixon was the president throughout, because Ford took office towards the end of this period.

[9] See Mayhew, D. R. (1993), "Reply: Let's Stick with the Longer List," *Polity*, p.485-488.

[10] http://davidmayhew.commons.yale.edu/datasets-divided-we-govern/

- We have coded Ford's presidency of 1975-1976 as "years 3-4" since Ford was eligible to be reelected in 1976 (but not in 1980 had he won the election in 1976).

# B Omitted Proofs

**Proof of Lemma 1.** We first claim that $\frac{\partial w}{\partial \Delta}|_{\Delta=\bar{\Delta}} < 0$ for each $\bar{\Delta} > 0$ that solves $w\left(\bar{\Delta}, K, \eta\right) = 1$. Note that the sign of $\frac{\partial w}{\partial \Delta}$ is the same as the sign of $\frac{\partial \log w}{\partial \Delta}$. The latter derivative can be calculated as

$$\frac{\partial \log w}{\partial \Delta} = -1 + \frac{\left(e^{-\Delta/\rho^0}/\rho^0\right)\eta K\bar{y}}{1+\left(1-e^{-\Delta/\rho^0}\eta\right)K\bar{y}}.$$

When evaluated at a solution, the derivative becomes

$$\begin{aligned} \frac{\partial \log w}{\partial \Delta}|_{\Delta=\bar{\Delta}} &= -1 + \frac{e^{-\bar{\Delta}/\rho^0}/\rho^0}{1-e^{-\bar{\Delta}/\rho^0}\eta}\eta\frac{e^{\bar{\Delta}}-1}{e^{\bar{\Delta}}} \\ &< -1 + \frac{1}{e^{\bar{\Delta}/\rho^0}-1}\frac{1}{\rho^0}\left(1-e^{-\bar{\Delta}}\right) \\ &< -1 + \rho^0 \times \frac{1}{\rho^0} \times 1 = 0, \end{aligned}$$

proving the claim. Here, the first line uses $w\left(\bar{\Delta}, K, \eta\right) = 1$ to substitute for $K\bar{y}$; the second line uses $\eta \leq 1$ and rearranges terms; and the third line uses the inequalities $\frac{1}{e^{\Delta/\rho^0}-1} < \rho_0$ and $1 - e^{-\Delta} < 1$.

Since $\frac{\partial w}{\partial \Delta}|_{\Delta=\bar{\Delta}} < 0$ at each solution, the function, $w(\cdot, K, \eta)$, crosses the horizontal line $w = 1$ always from above. This implies that the equation, $w\left(\bar{\Delta}, K, \eta\right) = 1$, has at most one positive solution. To show that a solution exists, note that $\lim_{\Delta\to\infty} w(\Delta, K, \eta) = 0$. Hence, the solution exists (and is unique) as long as $w(\Delta, K, \eta) > 1$ for sufficiently small $\Delta > 0$. We next show that this is the case as long as the parametric conditions in the statement of the lemma hold. First consider the case, $\eta = 1$ and $K\bar{y} > \rho^0$. A Taylor approximation of the function $w(\Delta, K, \eta)$ around $\Delta = 0$ gives, $w(K, \Delta, 1) \simeq 1 + \frac{K\bar{y}}{\rho^0}\Delta$. Since $K\bar{y} > \rho^0$, this implies $w(\Delta, K, \eta) > 1$ for sufficiently small $\Delta > 0$. Next consider the case, $\eta < 1$ and $K > 0$. In this case, we have, $w(K, 0, \eta) = 1 + (1-\eta)K\bar{y} > 1$. This also implies $w(\Delta, K, \eta) > 1$ for sufficiently small $\Delta > 0$, and completes the proof of the first part.

Since $\frac{\partial w}{\partial \Delta}|_{\Delta=\bar{\Delta}} < 0$ and the solution is unique, we also have that $w(\Delta, K, \eta) > 1$ for each $\Delta < \bar{\Delta}$, and $w(\Delta, K, \eta) < 1$ for each $\Delta > \bar{\Delta}$, proving the second part. □

**Proof of Lemma 2.** The nonnegativity of $K_t$ follows from (10). To establish the upper bound, first consider some time $t$ with disagreement ($W_t \geq 1$). In this case, (7) implies

$$K_t \leq \delta_t (1-\Lambda_t) K_{t+1/n}.$$

Next consider some time $t$ with agreement. In this case, we have $W_t = \delta_t\left(S_{t+1} + \Lambda_t \bar{y} K_{t+1/n}\right) < 1$. Plugging this into Eq. (7), we obtain.

$$\begin{aligned} K_t &= 1 - \delta_t\left(S_{t+1} + \Lambda_t \bar{y} K_{t+1/n}\right) + \delta_t (1-\Lambda_t) K_{t+1/n} \\ &\leq 1 - \delta_t + \delta_t (1 - \Lambda_t (1+\bar{y})) K_{t+1/n}. \end{aligned}$$

Combining the two cases, we obtain the inequality,

$$\begin{aligned} K_t &\leq \max\left(\delta_t\left(1-\Lambda_t\right)K_{t+1/n}, 1-\delta_t+\delta_t\left(1-\Lambda_t\left(1+\bar{y}\right)\right)K_{t+1/n}\right), \\ &\leq \max\left(\delta^0\left(1-\Lambda^0\right)K_{t+1/n}, 1-\delta^0+\delta^0\left(1-\Lambda^0\left(1+\bar{y}\right)\right)K_{t+1/n}\right). \end{aligned}$$

Here, the second line uses the assumptions, $\delta_t = \delta^0$ and $\Lambda_t \geq \Lambda^0$. This inequality can be equivalently rewritten as,

$$K_{t+1/n} \geq \min\left(\frac{K_t}{\delta^0\left(1-\Lambda^0\right)}, f\left(K_t\right)\right) \text{ where } f\left(K_t\right) = \frac{K_t-\left(1-\delta^0\right)}{\delta^0\left(1-\Lambda^0\left(1+\bar{y}\right)\right)} \tag{49}$$

Note also that, using (14), we have,

$$f\left(K_t\right)\begin{cases} < K_t, & \text{if } K_t < k\left(\bar{\rho}\left(\delta^0,\Lambda^0\right)\right) \\ = K_t, & \text{if } K_t = k\left(\bar{\rho}\left(\delta^0,\Lambda^0\right)\right) \\ > K_t, & \text{if } K_t > k\left(\bar{\rho}\left(\delta^0,\Lambda^0\right)\right) \end{cases}.$$

Next, to reach a contradiction, suppose $K_t > k\left(\bar{\rho}\left(\delta^0,\Lambda^0\right)\right)$ for some $t$. Since $f\left(K_t\right) > K_t$ and $\frac{K_t}{\delta^0(1-\Lambda^0)} > K_t$, the inequality (49) implies $K_{t+1/n} > K_t$. Hence, $\{K_{\tilde{t}}\}_{\tilde{t}=t}^{\infty}$ is an increasing sequence. Since it is also bounded from above (by one), it has a limit point denoted by $K^*$. Applying the inequality (49) in the limit, we obtain $K^* \geq f\left(K^*\right)$. This in turn implies $K^* \leq k\left(\bar{\rho}\left(\delta^0,\Lambda^0\right)\right)$, and yields a contradiction to the assumption that $K_t > k\left(\bar{\rho}\left(\delta^0,\Lambda^0\right)\right)$. □

**Proof of Lemma 3.** Note that $\frac{\partial w(\Delta,\mathbf{K}(\Delta),\eta)}{\partial K} = e^{-\Delta}\left(1-e^{-\Delta/\rho^0}\eta\right)\bar{y} > 0$ for each $\Delta \geq 0$. Next note that,

$$f'\left(\Delta\right) = \frac{\partial w\left(\Delta,\mathbf{K}\left(\Delta\right),\eta\right)}{\partial \Delta} + \frac{\partial w\left(\Delta,\mathbf{K}\left(\Delta\right),\eta\right)}{\partial K}K'\left(\Delta\right).$$

Hence, the second term in the derivative is always negative. In view of the proof of Lemma 3, we also have that $\frac{\partial w(\Delta,\mathbf{K}(\Delta),\eta)}{\partial \Delta}|_{\Delta=\bar{\Delta}} < 0$ for each $\bar{\Delta} > 0$ that satisfies $w\left(\bar{\Delta},\mathbf{K}\left(\bar{\Delta}\right),\eta\right) = 1$. It follows that $f'\left(\bar{\Delta}\right) < 1$ for each $\bar{\Delta} > 0$ such that $f\left(\bar{\Delta}\right) = 1$. This in turn implies that there can be at most one positive solution to the equation, $f\left(\bar{\Delta}\right) = 1$. The result that there is a unique and interior solution, $\bar{\Delta} \in (0,\Delta^{\max})$, follows after observing that $f\left(0\right) = 1+\left(1-\eta\right)\mathbf{K}\left(0\right)\bar{y} > 1$ (since $\mathbf{K}\left(0\right) > 0$) and that $f\left(\Delta^{\max}\right) = e^{-\Delta^{\max}} < 1$ (since $\mathbf{K}\left(\Delta^{\max}\right) = 0$). □

We next prove Lemma 4. Recall that this lemma concerns the solution to the system, $F\left(\bar{\mathbf{x}}|\eta^{odd},\eta^{even}\right) = 0$, where $F\left(\cdot|\eta^{odd},\eta^{even}\right): X \to \mathbb{R}^4$ is a vector valued function over the domain,

$$X = \left\{\mathbf{x} = \left(K_0,\Delta^{odd},K_{t^*},\Delta^{even}\right) \in \mathbb{R}^4 \quad | \quad K_0, K_{t^*} \in \left[0, k\left(\rho^0\right)\right], \Delta^{odd},\Delta^{even} \in \left[0, t^* r^0\right]\right\},$$

The components of the function $F\left(\cdot|\eta^{odd},\eta^{even}\right)$ are defined by,

$$\begin{aligned} F_1(\mathbf{x}) &= K_0 - k^{sum}\left(\Delta^{odd}, K_{t^*}, \eta^{odd}\right) \\ F_2(\mathbf{x}) &= 1 - w\left(\Delta^{odd}, K_{t^*}, \eta^{odd}\right) \\ F_3(\mathbf{x}) &= K(t^*) - k^{sum}\left(\Delta^{even}, K_0, \eta^{even}\right) \\ F_4(\mathbf{x}) &= 1 - w\left(\Delta^{even}, K_0, \eta^{even}\right). \end{aligned}$$

where

$$k^{sum}\left(\Delta, K^{next}, \eta\right) = \left(1 - e^{-\left(r^0 t^* - \Delta\right)/k\left(\rho^0\right)}\right) k\left(\rho^0\right) + e^{-\left(r^0 t^* - \Delta\right)/k\left(\rho^0\right)} e^{-\left(1+1/\rho^0\right)\bar{\Delta}} \eta K^{next}.$$

The proof of Lemma 4 relies on the properties of the Jacobean matrix of $F$ evaluated at a solution $\bar{\mathbf{x}} = \left(K_0, \Delta^{odd}, K_{t^*}, \Delta^{even}\right)$. This can be written as,

$$\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}} = \begin{bmatrix} 1 & -\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd} & -\left(\frac{\partial k^{sum}}{\partial K}\right)^{odd} & 0 \\ 0 & -\left(\frac{\partial w}{\partial \Delta}\right)^{odd} & -\left(\frac{\partial w}{\partial K}\right)^{odd} & 0 \\ -\left(\frac{\partial k^{sum}}{\partial K}\right)^{even} & 0 & 1 & -\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{even} \\ -\left(\frac{\partial w}{\partial K}\right)^{even} & 0 & 0 & -\left(\frac{\partial w}{\partial \Delta}\right)^{even} \end{bmatrix}. \tag{50}$$

Here, $\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd}, \left(\frac{\partial w}{\partial \Delta}\right)^{odd}$ and $\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{even}, \left(\frac{\partial w}{\partial \Delta}\right)^{even}$ denote the derivatives of the functions $k^{sum}(\cdot), w(\cdot)$ evaluated respectively at the vectors $\left(\bar{\Delta}^{odd}, K(t^*), \eta^{odd}\right)$ and $\left(\bar{\Delta}^{even}, K(0), \eta^{even}\right)$. Before we prove Lemma 4, we state and prove another lemma that establishes various properties of these partial derivatives as well as the Jacobean matrix $\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}$. To state the result, we define the following auxiliary variables,

$$\zeta^j \equiv \left(\frac{\partial w}{\partial \Delta}\right)^j \left(\frac{\partial k^{sum}}{\partial K}\right)^j - \left(\frac{\partial w}{\partial K}\right)^j \left(\frac{\partial k^{sum}}{\partial \Delta}\right)^j \text{ for } j \in \{odd, even\}.$$

**Lemma 5.** *Suppose the derivatives are evaluated at some* $\bar{\mathbf{x}} \in X$ *that satisfies* $F\left(\bar{\mathbf{x}}|\eta^{odd},\eta^{even}\right) = 0$.
*(i) For each,* $j \in \{odd, even\}$*, we have,*

$$\begin{aligned} &\left(\frac{\partial k^{sum}}{\partial K}\right)^j \in (0,1), \left(\frac{\partial k^{sum}}{\partial \Delta}\right)^j < 0, \left(\frac{\partial w}{\partial \Delta}\right)^j < 0, \left(\frac{\partial w}{\partial K}\right)^j > 0 \\ \textit{and} &\left(\frac{\partial k^{sum}}{\partial \eta^{odd}}\right)^{odd} > 0, \left(\frac{\partial w}{\partial \eta^{odd}}\right)^{odd} < 0. \end{aligned}$$

*(ii)* $\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}$ *is nonsingular with* $\det\left(\frac{\partial F}{\partial \mathbf{x}}\right) > 0$.
*(iii) If parameters satisfy* $\eta^{even} \leq \frac{\bar{y}}{1+\bar{y}}$*, then* $\zeta^{even} > 0$.

The first part establishes the signs of the partial derivatives of $k^{sum}(\cdot)$ and $w(\cdot)$. The second part establishes the nonsingularity of the Jacobean matrix and characterizes the sign of its determinant. The last part characterizes the sign of the auxiliary variable, $\zeta^{even}$, under condition (28). As we will see, the second part is useful to establish the uniqueness of the solution (to prove the first part of Lemma 4), whereas the first and the third parts are useful to establish comparative statics (to prove the second part of Lemma 4).

**Proof of Lemma 5.** Without loss of generality, consider the case $j = odd$. The results for $j = even$ follow from a symmetric analysis.
**Part (i).** First consider the partial derivatives of the function $k^{sum}(\cdot)$. Note that,

$$\left(\frac{\partial k^{sum}}{\partial K}\right)^{odd} = e^{-\left(r^0 t^* - \bar{\Delta}^{odd}\right)/k\left(\rho^0\right)} e^{-\left(1+1/\rho^0\right)\overline{\Delta}^{odd}} \eta^{odd},$$

which implies $\left(\frac{\partial k^{sum}}{\partial K}\right)^{odd} \in (0,1)$. Note also that

$$\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd} = -e^{-\left(r^0 t^* - \overline{\Delta}^{odd}\right)/k\left(\rho^0\right)} \left[1 - e^{-\left(1+1/\rho^0\right)\overline{\Delta}^{odd}} \frac{\overline{y}\eta^{odd} K\left(t^*\right)}{\rho^0}\right].$$

Hence, $\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd} < 0$ if and only if the term in the brackets is positive. This term can be rewritten as,

$$\begin{aligned} 1 - \frac{\eta^{odd}}{\rho^0} e^{-\overline{\Delta}^{odd}/\rho^0} \left(e^{-\overline{\Delta}^{odd}} \overline{y} K\left(t^*\right)\right) &= 1 - \frac{1}{\rho^0} \frac{1 - e^{-\overline{\Delta}^{odd}}}{e^{\overline{\Delta}^{odd}/\rho^0}/\eta^{odd} - 1} \\ &> 1 - \frac{1}{\rho^0} \frac{1 - e^{-\overline{\Delta}^{odd}}}{e^{\frac{\overline{\Delta}^{odd}}{\rho^0}} - 1} > 1 - \frac{1}{\rho^0}\rho^0 = 0. \end{aligned}$$

Here, the first line substitutes for $e^{-\bar{\Delta}^{odd}} K\left(t^*\right) \overline{y}$ from $w\left(\bar{\Delta}^{odd}, K\left(t^*\right), \eta^{odd}\right) = 1$ [cf. (18)]. The second line uses the inequality $\eta^{odd} < 1$ as well as the inequality $\frac{1-e^{-\overline{\Delta}^{odd}}}{e^{\overline{\Delta}^{odd}/\rho^0}-1} < \rho^0$. This proves that $\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd} < 0$.

Next consider the partial derivatives of the function $w(\cdot)$. Note that,

$$\left(\frac{\partial w}{\partial K}\right)^{odd} = \left(1 - \eta^{odd} e^{-\bar{\Delta}^{odd}/\rho^0}\right) e^{-\bar{\Delta}^{odd}} \overline{y},$$

which implies $\left(\frac{\partial w}{\partial K}\right)^{odd} > 0$. Next note that

$$\begin{aligned} \left(\frac{\partial w}{\partial \Delta}\right)^{odd} &= -e^{-\bar{\Delta}^{odd}} - \left(1 - \eta^{odd} e^{-\bar{\Delta}^{odd}/\rho^0}\right) e^{-\bar{\Delta}^{odd}} K\left(t^*\right) \overline{y} + \frac{\eta^{odd}}{\rho^0} e^{-\bar{\Delta}^{odd}/\rho^0} e^{-\bar{\Delta}^{odd}} K\left(t^*\right) \overline{y} \\ &= -\left[1 - e^{-\left(1+1/\rho^0\right)\bar{\Delta}^{odd}} \frac{\eta^{odd}\overline{y} K\left(t^*\right)}{\rho^0}\right], \end{aligned}$$

where the second line uses $w\left(\bar{\Delta}^{odd}, K\left(t^*\right), \eta^{odd}\right) = 1$. We have established earlier that the bracketed term is positive, which implies $\left(\frac{\partial w}{\partial \Delta}\right)^{odd} < 0$.

Finally, note that the partial derivatives with respect to $\eta^{odd}$ satisfy,

$$\begin{aligned} \left(\frac{\partial k^{sum}}{\partial \eta^{odd}}\right)^{odd} &= e^{-\left(r^0 t^* - \bar{\Delta}^{odd}\right)/k\left(\rho^0\right)} e^{-\left(1+1/\rho^0\right)\overline{\Delta}^{odd}} K\left(t^*\right) > 0 \\ \text{and} \left(\frac{\partial w}{\partial \eta^{odd}}\right)^{odd} &= -e^{-\left(1+1/\rho^0\right)\bar{\Delta}^{odd}} K\left(t^*\right) \overline{y} < 0. \end{aligned}$$

**Part (ii).** The determinant of the Jacobean matrix in (50) can be calculated as,

$$\det\left(\frac{\partial F}{\partial \mathbf{x}}\right) = \left(\frac{\partial w}{\partial \Delta}\right)^{even}\left(\frac{\partial w}{\partial \Delta}\right)^{odd} - \zeta^{even}\zeta^{odd}.$$

We claim that

$$\left|\zeta^{j}\right| < \left|\left(\frac{\partial w}{\partial \Delta}\right)^{j}\right| \text{ for each } j \in \{odd, even\},$$

which in turn implies $\det\left(\frac{\partial F}{\partial \mathbf{x}}\right) > 0$.

To prove the claim, suppose $j = odd$ (the other case is symmetric). After substituting the expressions from part (i) and rearranging terms, we obtain

$$\begin{aligned}
\zeta^{odd} &= \left(\frac{\partial w}{\partial \Delta}\right)^{odd}\left(\frac{\partial k^{sum}}{\partial K}\right)^{odd} - \left(\frac{\partial w}{\partial K}\right)^{odd}\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd} \qquad (51)\\
&= e^{-\left(r^0 t^* - \bar{\Delta}^{odd}\right)/k\left(\rho^0\right)}\left[1 - e^{-\left(1+1/\rho^0\right)\bar{\Delta}^{odd}}\frac{\eta^{odd}\overline{y}K\left(t^*\right)}{\rho^0}\right]e^{-\bar{\Delta}^{odd}}\left(\overline{y} - \left(1+\overline{y}\right)\eta^{odd}e^{-\bar{\Delta}^{odd}/\rho^0}\right)\\
&= \left(-\left(\frac{\partial w}{\partial \Delta}\right)^{odd}\right)e^{-\left(r^0 t^* - \bar{\Delta}^{odd}\right)/k\left(\rho^0\right)}e^{-\bar{\Delta}^{odd}}\left(\overline{y} - \left(1+\overline{y}\right)\eta^{odd}e^{-\bar{\Delta}^{odd}/\rho^0}\right).
\end{aligned}$$

Here, the last line substitutes back the definition of $\left(\frac{\partial w}{\partial \Delta}\right)^{odd}$. Note that $\left(\overline{y} - \left(1+\overline{y}\right)\eta^{odd}e^{-\bar{\Delta}^{odd}/\rho^0}\right) \leq \overline{y} < 1$. Since $\eta^{odd}e^{-\bar{\Delta}^{odd}/\rho^0} < 1$, we also have $\left(\overline{y} - \left(1+\overline{y}\right)\eta^{odd}e^{-\bar{\Delta}^{odd}/\rho^0}\right) > \overline{y} - \left(1+\overline{y}\right) = -1$. Combining these observations implies $\left|\overline{y} - \left(1+\overline{y}\right)\eta^{odd}e^{-\bar{\Delta}^{odd}/\rho^0}\right| < 1$. Combining this inequality with the expression for $\zeta^{odd}$ shows that $\left|\zeta^{odd}\right| < \left|\left(\frac{\partial w}{\partial \Delta}\right)^{odd}\right|$.

**Part (iii).** Redoing the calculation in (51) for $j = even$, we obtain,

$$\zeta^{even} = \left(-\left(\frac{\partial w}{\partial \Delta}\right)^{even}\right)e^{-\left(r^0 t^* - \bar{\Delta}^{even}\right)/k\left(\rho^0\right)}e^{-\bar{\Delta}^{even}}\left(\overline{y} - \left(1+\overline{y}\right)\eta^{even}e^{-\bar{\Delta}^{even}/\rho^0}\right). \qquad (52)$$

Suppose the additional regularity condition, $\eta^{even} \leq \frac{\overline{y}}{1+\overline{y}}$, holds. Using $e^{-\bar{\Delta}^{even}/\rho^0} < 1$, this also implies $\overline{y} - \left(1+\overline{y}\right)\eta^{even}e^{-\bar{\Delta}^{even}/\rho^0} > 0$. This in turn implies $\zeta^{even} > 0$ and completes the proof of the lemma. □

**Proof of Lemma 4. Part (i).** We establish the uniqueness of the solution by applying the Poincare-Hopf Index Theorem. To this end, we first show that the function $F$ (when viewed as a vector field) points "outwards" on the boundaries of the box-constrained region, $X$. More specifically, we conjecture (and verify below) that the function satisfies,

$$\begin{aligned}
F_1 &> 0 \text{ when } K_0 = k\left(\rho^0\right) \qquad (53)\\
F_1 &\leq 0 \text{ when } K_0 = 0, \text{ with strict inequality if } \Delta^{odd} < t^* r^0, \qquad (54)\\
F_2 &> 0 \text{ when } \Delta^{odd} = t^* r^0, \qquad (55)\\
\text{and } F_2 &< 0 \text{ when } \Delta^{odd} = 0, \text{ with strict inequality if } K_{t^*} > 0. \qquad (56)
\end{aligned}$$

We also claim that symmetric conditions hold for $F_3$ and $F_4$ (by appropriately modifying the corresponding variables). Note that these conditions also rule out the boundary solutions to the system, $F = 0$. The first two conditions rule out the boundary solutions for $K_0$ except possibly when $\Delta^{odd} = t^* r^0$. The third condition rules out the boundary solution, $\Delta^{odd} = t^* r^0$, eliminating the remaining conditionality with respect to $K_0$. The fourth condition rules out the boundary condition, $\Delta^{odd} = 0$, except possibly when $K_{t^*} = 0$. The analogous conditions for $F_3$ and $F_4$ rule out the boundary solutions for $K_{t^*}$ and $\Delta^{even}$, including $K_{t^*} = 0$, eliminating the remaining conditionality.

In view of the conditions (53) – (56), a standard Poincare-Hopf Index Theorem applies to the vector valued function, $F$ (see, for instance, Simsek, Ozdaglar, Acemoglu (2008)). In particular, if $\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}$ is nonsingular for each solution, then we have,

$$1 = \sum_{\left\{\bar{\mathbf{x}} \in X \quad | \quad F\left(\bar{\mathbf{x}} | \eta^{odd}, \eta^{even}\right) = 0\right\}} sign\left(\det\left(\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}\right)\right). \tag{57}$$

Here, the left hand side is the Euler characteristic of the region $X$, which is equal to 1. The right hand side is the sum of the indices of $F$ at the solutions (which are all interior in view of the boundary conditions). The index of a solution is equal to the sign of the Jacobean matrix evaluated at the solution. By Lemma 5, the determinant is nonsingular with a strictly positive determinant, $\det\left(\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}\right) > 0$, at any solution $\bar{\mathbf{x}}$. Combining this with Eq. (57) implies that there is a unique interior solution.

It remains to establish the boundary conditions (53) – (56). Using the definition of $k^{sum}(\cdot)$ in (10), together with the inequality, $e^{-\left(1+1/\rho^0\right)\Delta}\eta K < k\left(\rho^0\right)$ (since $\eta < 1$ and $K \leq k\left(\rho^0\right)$), we have,

$$k^{sum}(\Delta, K, \eta) < \left(1 - e^{-\left(r^0 t^* - \Delta\right)/k\left(\rho^0\right)}\right) k\left(\rho^0\right) + e^{-\left(r^0 t^* - \Delta\right)/k\left(\rho^0\right)} k\left(\rho^0\right) = k\left(\rho^0\right).$$

In particular, $F_1|_{K_0 = k(\rho^0)} = k\left(\rho^0\right) - k^{sum} < 0$, proving (53). Note also that $k^{sum}(\Delta, K, \eta) \geq 0$, where the equality holds only if $\Delta = r^0 t^*$ and $K = 0$. In particular, $F_1|_{K_0=0} = -k^{sum} \leq 0$ with strict inequality if $\Delta^{odd} < t^* r^0$, proving (54). Next using the definition of $w(\cdot)$ in (18), we have,

$$w\left(t^* r^0, K, \eta\right) \leq w\left(t^* r^0, k\left(\rho^0\right), 0\right) < 1.$$

Here, the first inequality follows since $w(\cdot)$ is increasing in $K$ and decreasing in $\eta$, and the second inequality follows from condition (27). This implies $F_2|_{\Delta^{odd} = t^* r^0} = 1 - w > 0$, proving (55). Finally, note also that $w(0, K, \eta) = 1 + (1 - \eta) K \overline{y} > 1$, with strict inequality if $K > 0$. This implies, $F_2|_{\Delta^{odd} = t^* r^0} = 1 - w < 0$ with strict inequality if $K_{t^*} > 0$, proving (56). After relabeling, symmetric conditions also hold for $F_3$ and $F_4$. This completes the proof of the uniqueness of the solution.

**Part (ii).** To prove this part, we first establish the comparative statics of the solution with respect to the parameter, $\eta^{odd} = e^{-\lambda^{E,odd}}$, which provides an inverse measure of the strength of the odd election. Recall that $\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}$ is nonsingular with a positive determinant (cf. Lemma 5). Then, implicitly differentiating the equation, $F = 0$, we obtain,

$$\frac{d\bar{\mathbf{x}}}{d\eta^{odd}} = \left(\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}\right)^{-1} \left(-\frac{\partial F}{\partial \eta^{odd}}|_{\mathbf{x}=\bar{\mathbf{x}}}\right). \tag{58}$$

Recall also that the Jacobean matrix, $\frac{\partial F}{\partial \mathbf{x}}|_{\mathbf{x}=\bar{\mathbf{x}}}$, is given by (50). Taking the inverse of this matrix, the first two columns of the Jacobean matrix multiplied by the determinant, $\det\left(\frac{\partial F}{\partial \mathbf{x}}\right)\left(\frac{\partial F}{\partial \mathbf{x}}\right)^{-1}$, is given by,

$$\begin{bmatrix} \left(\frac{\partial w}{\partial \Delta}\right)^{odd}\left(\frac{\partial w}{\partial \Delta}\right)^{even} & (-1)\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd}\left(\frac{\partial w}{\partial \Delta}\right)^{even} \\ -\left(\frac{\partial w}{\partial K}\right)^{odd}\zeta^{even} & -\left(\frac{\partial w}{\partial \Delta}\right)^{even}+\left(\frac{\partial k^{sum}}{\partial K}\right)^{odd}\zeta^{even} \\ \left(\frac{\partial w}{\partial \Delta}\right)^{odd}\zeta^{even} & (-1)\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd}\zeta^{even} \\ (-1)\left(\frac{\partial w}{\partial K}\right)^{even}\left(\frac{\partial w}{\partial \Delta}\right)^{odd} & \left(\frac{\partial w}{\partial K}\right)^{even}\left(\frac{\partial k^{sum}}{\partial \Delta}\right)^{odd} \end{bmatrix}. \tag{59}$$

From the third part of Lemma 5, $\zeta^{even}$ is strictly positive. From the first part of the lemma, all of the partial derivatives with respect to $\Delta$ are strictly negative, and all of the partial derivatives with respect to $K$ are strictly positive. Thus, the signs of the entries of the columns in (59) can be calculated as,

$$\begin{bmatrix} + & - \\ - & + \\ - & + \\ + & - \end{bmatrix}.$$

Note also that the partial derivative vector, $-\frac{\partial F}{\partial \eta^{odd}}|_{\mathbf{x}=\bar{\mathbf{x}}}$, can be calculated as,

$$\left[\left(\frac{\partial k^{sum}}{\partial \eta^{odd}}\right)^{odd}, \left(\frac{\partial w}{\partial \eta^{odd}}\right)^{odd}, 0, 0\right]'. \tag{60}$$

Using the first part of Lemma 5, the corresponding signs are given by,

$$\begin{bmatrix} + & - & 0 & 0 \end{bmatrix}'.$$

Combining Eqs. (58) – (59), together with the calculated signs, we obtain the comparative statics,

$$\frac{dK(0)}{d\eta^{odd}} > 0, \frac{d\overline{\Delta}^{odd}}{d\eta^{odd}} < 0, \frac{dK(t^*)}{d\eta^{odd}} < 0, \text{ and } \frac{d\overline{\Delta}^{even}}{d\eta^{odd}} > 0. \tag{61}$$

Hence, reducing the strength of the odd election (increasing $\eta^{odd} = e^{-\lambda^{odd}}$) decreases $\overline{\Delta}^{odd}$ and $K(t^*)$, and increases $K(0)$ and $\overline{\Delta}^{even}$.

We next use these comparative statics to prove the result. Note that $\eta^{odd} = e^{-\lambda^{odd}} > \eta^{even} = e^{-\lambda^{even}}$ since $\lambda^{even} > \lambda^{odd}$. For each $\tilde{\eta}^{odd} \in \left[\eta^{even}, \eta^{odd}\right]$, let $\mathbf{X}\left(\tilde{\eta}^{odd}\right)$ denote the equilibrium corresponding to survival probabilities $\tilde{\eta}^{odd}$ and $\eta^{even}$. Using the Fundamental Theorem of Calculus, we have,

$$\mathbf{X}\left(\eta^{odd}\right) = \mathbf{X}\left(\eta^{even}\right) + \int_{\eta^{even}}^{\eta^{odd}} \frac{d\mathbf{x}}{d\eta^{odd}}\Big|_{\tilde{\eta}^{odd}=\eta^{even}} d\tilde{\eta}^{odd}.$$

Note that the comparative statics in Eq. (61) apply for each derivative term inside the integral. Consequently, we have,

$$\begin{aligned} K(0) &> K(0)|_{\tilde{\eta}^{odd}=\eta^{even}} \\ \overline{\Delta}^{odd} &< \overline{\Delta}^{odd}|_{\tilde{\eta}^{odd}=\eta^{even}} \\ K(t^*) &< K(t^*)|_{\tilde{\eta}^{odd}=\eta^{even}} \\ \overline{\Delta}^{even} &> \overline{\Delta}^{even}|_{\tilde{\eta}^{odd}=\eta^{even}} \end{aligned}$$

When $\tilde{\eta}^{odd} = \eta^{even}$, the elections are equally strong and the equilibrium is the same as in Proposition 7. In particular, we have $K(0)|_{\tilde{\eta}^{odd}=\eta^{even}} = K(t^*)|_{\tilde{\eta}^{odd}=\eta^{even}}$ and $\overline{\Delta}^{odd}|_{\tilde{\eta}^{odd}=\eta^{even}} = \overline{\Delta}^{even}|_{\tilde{\eta}^{odd}=\eta^{even}}$. Combining these observations, we obtain $\overline{\Delta}^{odd} < \overline{\Delta}^{even}$ and $K(0) > K(t^*)$, completing the proof of Lemma 4.

We conclude by providing an intuition for the comparative statics in (61), which is also useful to illustrate the role of the additional parametric condition (28). Intuitively, increasing the survival probability during the odd election (or reducing its strength) decreases the length of delay before this election, $\overline{\Delta}^{odd}$. The shortening of the delay in the first election cycle also increases the initial congruence, $K(0)$. The increase in $K(0)$ in turn increases the delay before the other election, $\overline{\Delta}^{even}$. The increase in delay in the other election cycle tends to reduce the initial congruence, $K(t^*)$. On the other hand, the increase in $K(0)$ tends to increase $K(t^*)$. Hence, there are counteracting forces on $K(t^*)$. The required parametric condition, $\eta^{even} \leq \frac{\bar{y}}{1+\bar{y}}$, ensures that the effect of $K(0)$ is relatively weak so that $K(t^*)$ declines. This is captured by Eqs. (58) – (59), which illustrate that $sgn\left(\frac{dK(t^*)}{d\eta^{odd}}\right) = -\zeta^{even}$. The parametric condition ensures that $\zeta^{even}$ is positive and thus $\frac{dK(t^*)}{d\eta^{odd}}$ is negative (see Lemma 5). The decline in $K(t^*)$ further contributes to the decline in $\overline{\Delta}^{odd}$ and reinforces the initial effect on $\overline{\Delta}^{odd}$.

Hence, the main role of the condition, $\eta^{even} \leq \frac{\bar{y}}{1+\bar{y}}$, concerns the knock-on effects of $\eta^{odd}$ on the $K(t^*)$ of *the previous election cycle* (which is at time distance $2t^*$). The condition ensures that these knock-on effects influence $\overline{\Delta}^{odd}$ in the same way as the direct effect. The knock-on effects are typically small, especially if the distance between the elections are large (Eq. (52) below illustrates that $\zeta^{even}$ is decreasing in $t^* r^0$). Consequently, our comparative statics results typically hold in numerical simulations even when the additional condition fails. We assume the condition only because it provides analytical tractability and enables us to formally establish the comparative statics. □

**Proof of (39).** We compute

$$\begin{aligned}\frac{\partial w\left(\bar{\Delta}, k\left(\rho^0\right), e^{-\lambda^E}\right)}{\partial \rho^0} &= e^{-\bar{\Delta}}\bar{y}\left(-\frac{\bar{\Delta}}{\left(\rho^0\right)^2}e^{-\bar{\Delta}/\rho^0}\eta k\left(\rho^0\right) + \left(1 - e^{-\bar{\Delta}/\rho^0}\eta\right)k'\left(\rho^0\right)\right) \\ &= e^{-\bar{\Delta}}\bar{y}\left(-\bar{\Delta}\frac{k\left(\rho^0\right)}{\left(\rho^0\right)^2}e^{-\bar{\Delta}/\rho^0}\eta + \frac{e^{\bar{\Delta}}-1}{k\left(\rho^0\right)\bar{y}}k'\left(\rho^0\right)\right).\end{aligned}$$

Here, the second line uses $w\left(\bar{\Delta}, k\left(\rho^0\right), \eta\right) = 1$ to substitute for the term, $\left(1 - e^{-\bar{\Delta}/\rho^0}\eta\right)$. Next note that $k\left(\rho^0\right) = \rho^0/\left(1+\rho^0+\bar{y}\right)$ implies $k'\left(\rho^0\right) = (1+\bar{y})\left(\frac{k\left(\rho^0\right)}{\rho^0}\right)^2$. After substituting this expression and rearranging terms, we obtain (39). In (39), the inequality follows since $e^{-\bar{\Delta}/\rho^0}\eta < 1$ and $\frac{e^{\bar{\Delta}}-1}{\bar{\Delta}} \geq 1$. □