Abstract: I give conditions under which changes in private spending are accommodated in general equilibrium exactly like changes in aggregate fiscal expenditure. Under such demand equivalence, researchers can use time series evidence on fiscal multipliers to recover the general equilibrium “missing intercept” of shocks to private spending identified in the cross section. I apply this method to deficit-financed stimulus checks, and find a) a large direct consumption spending response, and b) a fiscal multiplier of one and so a missing intercept close to zero. I also discuss the robustness of this aggregation approach to empirically plausible violations of demand equivalence.
1 Introduction

A large literature in macroeconomics tries to estimate the aggregate effects of shocks to consumption and investment expenditure.\(^1\) For most of these demand shifters, the experimental ideal—exogeneity at the macro level—is not attainable. In response, researchers increasingly leverage the cross-sectional variation available in micro data. Appealingly, because micro estimates rely exclusively on cross-sectional information, they do not require macroeconomic identification restrictions. The well-known shortcoming is that micro estimates miss general equilibrium effects that affect all micro units (price changes, labor demand, ...), and thus do not give macro counterfactuals—what I call a “missing intercept” problem.

So how can researchers aggregate their micro estimates into macro counterfactuals? The familiar Keynesian cross suggests one line of attack: if changes in private demand propagate similarly to changes in public expenditure, then simply scaling cross-sectionally identified private spending impulses by estimates of aggregate fiscal multipliers may approximately give a true macro effect. Back-of-the-envelope aggregation along these lines is already popular in policy practice (see Reichling & Whalen, 2012) and some academic work (e.g. Hausman, 2016). A second, largely separate literature instead turns micro estimates into macro counterfactuals through rich structural models (e.g. Kaplan & Violante, 2018).

This paper offers a hybrid perspective: I use structural models to gauge the informativeness of fiscal spending shocks for the general equilibrium effects of private demand shifters, clarify the conditions under which those fiscal shocks solve the missing intercept problem, and finally assess the plausibility of these conditions. The analysis proceeds in three steps. First, in the context of a relatively general structural macro model, I give a set of restrictions on economic primitives ensuring “demand equivalence”—that is, identical changes in private and public demand eliciting identical general equilibrium feedback. Second, I leverage this equivalence result to propose a measurement strategy. Under demand equivalence, time-series estimates of the aggregate effects of a given change in fiscal expenditure for free allow us to aggregate particular shifters of private spending: those that (i) induce the same time path of aggregate net excess demand; (ii) are financed just like the fiscal spending change; and (iii) occur in the same macroeconomic environment. To illustrate my measurement strategy, I combine a) cross-sectional evidence on household consumption following stimulus check receipt and b) time series evidence on fiscal spending shocks to estimate the aggregate effects

\(^1\)Examples include stimulus checks (Parker et al., 2013), redistribution (Jappelli & Pistaferri, 2014), credit tightening (Mian et al., 2013; Guerrieri & Lorenzoni, 2017), and bonus depreciation (Zwick & Mahon, 2017).
of stimulus check policies. My estimates of a fiscal multiplier of one suggest a “missing intercept” close to zero, and so a macro counterfactual close to cross-sectional spending responses. Third, I discuss the robustness of this approach to violations of demand equivalence. I find that most plausible violations lead to an upward bias: aggregation via fiscal multipliers understates the degree of general equilibrium crowding-out, and so overstates the macro causal effect. Using structural model simulations, I present general conditions under which the bias is likely to be small, and find them to be satisfied in my stimulus check application.

In the first part of the paper I identify conditions under which shocks to private spending—e.g., stimulus checks or credit tightenings—propagate in general equilibrium just like changes in public expenditure. The key building block result is that, in standard business-cycle models, linearized impulse responses to aggregate shocks can be characterized implicitly as solutions to a linear infinite-horizon system of market-clearing conditions. Private and public spending shocks thus induce identical general equilibrium effects as long as they perturb the same market-clearing conditions by the same amount. In a rich heterogeneous-agent model, this abstract exclusion restriction maps into three substantive economic assumptions. First, households and government consume the same final good. If so, identical changes in private or public spending lead to identical excess demand for that common good. Second, households and government borrow and lend at the same interest rate. The identical expansions in private and public demand then induce the same fiscal deficit (in net present value terms), and so can in principle be financed using identical paths of future taxes. In particular, if the net excess demand path has zero net present value (as is the case for non-policy shifters like credit tightenings), then the identical change in public spending can be purely deficit-financed, with no direct tax response. Third, household labor supply does not respond differentially to the two shocks; sufficient conditions for this are either the absence of wealth effects in labor supply or fully demand-determined employment. Under these three restrictions, for any given shock to private spending, I prove that there exists a public spending shock that solves the missing intercept problem—that is, the response of aggregate consumption to the private demand shifter is equal to the sum of a) the shifter’s direct effect on consumer spending and b) the total response of consumption to the public spending shock. The constructive proof reveals the properties of this public spending shock: it must (i) induce the same path of excess demand and (ii) be associated with the same path of future taxes as the private demand shifter. My focus on linearized equilibria furthermore automatically imposes a third condition (iii): both shocks occur in the same macro environment, including in particular the same response of the monetary authority.
Next, I show how to combine this theoretical demand equivalence insight with empirical time series and cross-sectional evidence on public and private spending shock transmission. Cross-sectional regressions of private spending on a demand shifter (e.g., check receipt) recover that shifter’s direct effect on private demand. Semi-structural time series methods on the other hand have been applied widely to estimate the aggregate effects of fiscal purchases. Demand equivalence tells us how to put the two together: time series fiscal multiplier estimates for free give us the general equilibrium effects of any shifter of private demand that satisfies my conditions (i) - (iii)—same net excess demand, same financing, and same macro environment. Can we in practice find cross-sectionally and time series identified shocks that align in this very particular way? I first of all present two strategies to ensure (i), the alignment of net excess demand paths.

1. Given any cross-sectionally identified demand path, researchers can search the set of fiscal spending experiments studied in previous work to find the one—or a linear combination of several—that induces as similar an aggregate excess demand path as possible.

2. Given any public spending path estimate from time series experiments, researchers can try to figure out what kind of shock to private demand would induce that same spending path. I show that one can do so by either running a large number of cross-sectional regressions or (more plausibly) by specifying a partial equilibrium model of private spending behavior.

With demand paths matched using either of these strategies, the next step is to compute the sum of the a) cross-sectional and b) time series estimated consumption impulse responses. Conditions (ii) and (iii) then tell us how to interpret this sum: under demand equivalence, the sum is a valid general equilibrium counterfactual for a private demand shifter that (ii) was financed just like the identified fiscal shock and (iii) occurred in the same macro environment.

The chief appeal of this methodology is that it promises to allow researchers to estimate the aggregate effects of shocks and policies for which no credible macroeconomic experiment is available. Its limitations are, first, that it requires the combination of very particular pieces of cross-sectional and time series variation; and second, that demand equivalence itself rests on restrictive assumptions. The remainder of the paper discusses these challenges.

I first showcase the method’s applicability through a study of “stimulus check” policies. While increasingly popular as a policy stimulus tool, relatively little is known about their aggregate effects, simply because of a lack of plausibly exogenous time series variation. Cross-sectional micro evidence suggests that one-off checks lead to a large but short-lived expansion in private spending (Parker et al., 2013). I connect this finding to the time series evidence on a
similarly short-lived expansion in public spending, following Ramey (2011). The expansion is
deficit-financed, largely accommodated by the monetary authority, and approximately moves
output one-for-one, with only small effects on private spending. By demand equivalence, I
put these two pieces of evidence together and conclude that one-off, deficit-financed checks
briefly but significantly stimulate aggregate consumption, with the overall response close to
the direct effect estimated using micro data alone. To illustrate my second strategy for the
alignment of net excess demand paths, I then consider the fiscal proxy-SVAR of Caldara &
Kamps (2017). Their methodology identifies a persistent, deficit-financed increase in fiscal
purchases, again largely accommodated by the monetary authority. I combine cross-sectional
estimates of household spending behavior with some basic consumer theory (following Wolf,
2021) to conclude that the identified spending path can be equivalently induced via a string
of stimulus checks sent to households. I then aggregate as before, and again find a missing
intercept path close to zero throughout.

In the third part of the paper I discuss the extent to which this “demand equivalence”
solution to the missing intercept problem is robust to empirically plausible violations of its
strong assumptions. The nature of the exercise is as follows: I consider structural models
violating demand equivalence, simulate data, implement my method, and then report the er-
ror. My main laboratory is a rich estimated heterogeneous-agent New Keynesian (“HANK”) model. In this setting, demand equivalence fails only because of short-term wealth effects
in labor supply. However, consistent with both micro evidence (Cesarini et al., 2017) and
the results of much previous modeling (Christiano, 2011a), I find the inaccuracy resulting
from this channel to be negligible. Several further model extensions—including relative price
movements between public and private consumption bundles, productive benefits of public
spending, and openness of the economy—unsurprisingly all tend to increase the approxima-
tion error. Interestingly, I find that the sign of the error is common for almost all of those
extensions: fiscal multiplier estimates now miss some of the general equilibrium crowding-out
relevant for private demand shocks, and so my procedure tends to overstate the general equi-
librium response of private spending. I conclude that the output of the demand equivalence
approximation should generally be interpreted as giving an upper bound for actual general
equilibrium counterfactuals. This bound will be particularly tight if: the spending shock is
transitory, muting wealth and relative price effects; there is little leakage of spending abroad
(e.g., because economy is quite closed); and the researcher uses time series fiscal multiplier
estimates that capture changes in public consumption (and not productive investment). I
verify all of these conditions in my application to stimulus checks.
I conclude with a brief discussion of the scope of the demand equivalence approach. First, while my main application is to uniform stimulus checks, the method applies without change to many other shifters of consumer demand, including targeted transfers (Andreolli & Surico, 2021), household deleveraging (Guerrieri & Lorenzoni, 2017) or increases in inequality (Auclert & Rognlie, 2018). To illustrate, I show that a fiscal contraction today offset by an expansion in the future identifies the missing intercept of a temporary increase in earnings inequality. Second, I extend the equivalence theory to shifters of investment demand and provide an application to bonus depreciation stimulus.

LITERATURE. My analysis relates and contributes to several strands of literature.

First, the demand equivalence approach to the missing intercept problem connects two empirical literatures. A fast-growing line of work uses variation at the individual or regional level to estimate spending responses to policy changes and other shocks (e.g. Mian & Sufi, 2009; Parker et al., 2013; Zwick & Mahon, 2017). As all of these studies control for macro fluctuations through time fixed effects, they are silent on any possible general equilibrium feedback. I give formal conditions under which a second literature—that on the aggregate effects of changes in government spending—can be informative about this “missing intercept.” Comprehensive literature summaries are Hall (2009) and Ramey (2018).

Second, the consumption and investment demand equivalence results elaborate on the familiar Keynesian cross intuition of a common “demand multiplier” (Reichling & Whalen, 2012; Hausman, 2016). Building on Auclert & Rognlie (2018) and Auclert et al. (2018), I give sufficient conditions under which there exist aggregate public spending shocks that solve the missing intercept problem for private demand shifters, and in particular characterize the properties required of these public spending shocks. In contemporaneous and independent work, Guren et al. (2020) and Chodorow-Reich et al. (2019) use the reverse logic to strip out the local general equilibrium effects present in cross-regional regressions.

Third, the proposed methodology naturally complements existing strategies for the estimation of (policy) counterfactuals in macroeconomics. In its reliance on general exclusion restrictions rather than parametric structural models, it is semi-structural in the same way as conventional Structural Vector Autoregressive (SVAR) analysis (Sims, 1980). The general idea also connects with the “sufficient statistics” approach that is common in public finance (Chetty, 2009) and increasingly widespread in macroeconomics (Auclert et al., 2018; Nakamura & Steinsson, 2018). I show that, across a particular family of structural models, certain moments—the cross-sectional and time series estimates required by my methodology—are
fully informative about the desired macroeconomic counterfactual, thus obviating the need for model solution (Andrews et al., 2018).

OUTLINE. Section 2 establishes the consumption demand equivalence result. In Section 3, I show how to connect this theoretical result with cross-sectional and time series estimates of dynamic causal effects, and present an application to stimulus checks. Section 4 critically assesses the output of the proposed procedure. Applications to other shifters of consumer demand as well as the extension to investment are discussed in Section 5. Section 6 concludes, and supplementary details, proofs and further applications are all relegated to the appendix.

2 Consumption demand equivalence

This section builds on the familiar Keynesian cross logic to develop an equivalence result for the general equilibrium propagation of shocks to private consumption and to public spending. In Sections 2.1 and 2.2, I discuss the restrictions on economic primitives needed for demand equivalence in a standard structural business-cycle model. Section 2.3 complements this particular model-based analysis with a general abstract formulation of shock equivalence as a set of exclusion restrictions on (linearized) aggregate equilibrium conditions.

2.1 Model

I begin by presenting a particular model environment, general enough to nest many seminal contributions to quantitative business-cycle analysis. The purpose of the model is twofold: First, it allows me to present economically interpretable sufficient conditions for demand equivalence in a familiar, canonical environment. Second, the model will form the backbone of my critical assessment of the demand equivalence methodology in Section 4.

Time is discrete and runs forever, \( t = 0, 1, \ldots \). The economy is populated by households, firms, and a government. There is no aggregate uncertainty, but households and firms are allowed to face idiosyncratic risk. I study perfect foresight transition paths back to steady state after one-time unexpected aggregate innovations at time 0; for vanishingly small innovations, these transition paths are equivalent to standard impulse response functions.

\[ \text{2} \]

The environment nests conventional estimated New Keynesian models (e.g. Smets & Wouters, 2007), models with uninsurable household income risk (Aiyagari, 1994; McKay et al., 2016), and models with rich real and financial firm-level investment frictions (Khan & Thomas, 2013; Winberry, 2018). A thorough assessment of its generality and limitations is relegated to Section 4.
computed from the first-order perturbation solution of an otherwise identical model with aggregate risk.\footnote{This result is an implication of certainty equivalence coupled with Taylor’s theorem (Boppart et al., 2018). For ordinary business-cycle fluctuations, such first-order perturbations offer an accurate characterization of the model’s global dynamics (e.g. Fernández-Villaverde et al., 2016; Ahn et al., 2017; Auclert et al., 2019).} Anticipating my main application, I will focus on two such innovations: first, stimulus checks sent to households, and second, a transitory expansion in government spending. Section 2.3 shows how the equivalence result extends to generic policy and non-policy shifters of consumption demand (e.g., changes in borrowing constraints).

**Notation.** The realization of a variable $x$ at time $t$ along the equilibrium perfect foresight transition path will be denoted $x_t$, while the entire time path will be denoted $x = \{x_t\}_{t=0}^\infty$. Hats denote deviations from the deterministic steady state, bars denote steady-state values, and tildes denote logs. I study two structural shocks indexed by $s \in \{\tau, g\}$—stimulus checks and government spending. I write individual shock paths as $\varepsilon_s$, and use subscripts $\varepsilon$ for transitions after a generic path $\varepsilon \equiv (\varepsilon^*_\tau, \varepsilon^*_g)'$. I reserve $s$ subscripts for pure stimulus check or government spending shocks—that is, shock paths with $\varepsilon_u = 0$ for $u \neq s$.

**Households.** A unit continuum of households $i \in [0,1]$ has preferences over consumption $c_{it}$ and labor $\ell_{it}$. The real relative price of the consumption bundle in terms of the economy’s numeraire is denoted $p^c_t$. Households are subject to idiosyncratic productivity risk $e_{it}$, and can self-insure by investing in liquid nominal bonds $b^h_{it}$, with nominal returns $i^b_t$ and subject to a borrowing constraint $b$. Borrowing incurs an additional penalty $\kappa b \geq 0$. Income consists of labor earnings as well as (potentially type-specific) lump-sum transfers $\tau_{it}$ and dividend income $d_{it}$. Total hours worked $\ell_{it}$ are determined by demands of a unit continuum $k \in [0,1]$ of price-setting labor unions, as in Erceg et al. (2000); the problem of labor unions will be considered later. Given a path of prices, transfers, dividends, hours worked and inflation $(\pi_t)$, the consumption-savings problem of household $i$ is thus

$$\max_{\{c_{it}, b^h_{it}\}} \mathbb{E}_0 \left[ \sum_{t=0}^\infty \beta^t u(c_{it}, \ell_{it}) \right] \tag{1}$$

such that

$$p^c_t c_{it} + b^h_{it} = (1 - \tau_t) w_t e_{it} \ell_{it} + \frac{1 + i^b_{t-1} + \kappa_t \mathbb{1}_{b^h_{it-1} < 0} b^h_{it-1}}{1 + \pi_t} b^h_{it-1} + \tau_{it} + d_{it}, \quad b^h_{it} \geq b$$

Labor productivity $e_{it}$ follows a (stochastic) law of motion with $\int e_{it} di = 1$ at all times.
Because of frictions in the labor market, household hours worked are determined by labor unions, as in Erceg et al. (2000) and Auclert et al. (2018). Worker $i$ provides $\ell_{ikt}$ units of labor to union $k$, giving total hours worked for household $i$ of $\ell_i t = \int_k \ell_{ikt} dk$. The total effective amount of labor intermediated by union $k$ is $\ell_{kt} \equiv \int_i \ell_{ikt} di$; each union then sells its labor services to a competitive labor packer at price $w_{kt}$. The labor packer aggregates union-specific labor to aggregate labor services,

$$\ell^h_t \equiv \left( \int_k \ell_{kt}^{\epsilon_w^{-1} w} dk \right)^{\epsilon_w}$$

sold at the aggregate wage index $w_t$, and where $\epsilon_w$ denotes the elasticity of substitution between different types of labor. Union $k$ chooses its wage rate $w_{kt}$ subject to wage-setting adjustment costs, and satisfies the corresponding demand for its labor services. I assume that it does so by demanding a common amount of hours worked from its members. \(^4\) Since the wage-setting problem is standard, I relegate details to Appendix B.1. For the purposes of the analysis here, it suffices to note that union behavior can be summarized through a wage New Keynesian Phillips curve—effectively, an aggregate labor supply relation.

**Fiscal Policy.** The fiscal authority consumes a bundle with real relative price $p^g_t$. Fiscal consumption $g_t$ and total lump-sum transfers $\tau_t \equiv \int_0^1 \tau_{it} di$ are financed through debt issuance and taxes on labor income. The government flow budget constraint is

$$1 + i_t b_{t-1} + p^g_t g_t + \tau_t = \tau_{ew} \ell_t + b_t$$

I assume that total government spending $g = g(\epsilon_g)$ and the discretionary part of stimulus checks $\tau^x = \tau^x(\epsilon_r)$ follow exogenous processes. A government debt financing rule is mapping from spending targets $(g, \tau^x)$, initial nominal debt $b_{t-1}$ and a path of prices and quantities $(w, \ell, i^h, \pi, p^g)$ into the endogenous part of transfers $\tau^e$ such that $\tau = \tau^e + \tau^x$, the flow government budget constraint holds at all periods $t$, and $\lim_{t \to \infty} \tilde{b}_t = 0$. That is, lump-sum taxes adjust in response to fiscal outlays—both outright expenditure and stimulus checks—to ultimately return government debt to its steady-state level. I emphasize that all results below extend without change to the alternative assumption of outlays financed with time-varying

\(^4\)A uniform hiring rule is the natural assumption in sticky-wage heterogeneous-household models, but is of course awkward in the flexible-wage limit, as it then does not nest the alternative natural case of flexible labor supply for each individual household. I consider a model without unions in Appendix E.3.
distortionary taxes \( \tau_e \); the key restriction for demand equivalence will be that stimulus checks and spending increases are financed using identical paths of taxes, distortionary or not.

**Rest of the economy.** Since my focus is on the equivalence of private and public expansions in demand, I only sketch the rest of the model, with a detailed outline provided in Appendix B.1. The corporate sector is populated by three sets of firms: a unit continuum of heterogeneous, perfectly competitive intermediate goods producers; a unit continuum of monopolistically competitive retailers with nominal price rigidities; and aggregators for final (private and public) consumption and investment goods. Intermediate goods producers accumulate capital, hire labor, issue risk-free debt, and sell their composite intermediate good, possibly subject to capital adjustment costs as well as generic constraints on equity and debt issuance. Retailers purchase the intermediate good, costlessly differentiate, monopolistically set prices, and sell their differentiated good on to the competitive aggregators.

The last remaining entity in the model is the monetary authority. This monetary authority sets nominal rates on liquid bonds \( i^b_t \) following some (Taylor-type) rule.

**Equilibrium.** I assume that there exists a unique deterministic steady state.\(^5\) To allow interpretation of perfect foresight transition paths as conventional first-order perturbation solutions, I impose that the economy is indeed initially in steady state, and then study perfect foresight transition equilibria back to the initial deterministic steady state. The definition of equilibrium perfect foresight transition paths is then standard (see Appendix B.1); I discuss an extension to transition paths with other starting points in Appendix C.1.

### 2.2 The equivalence result

I now formalize the Keynesian cross intuition of a commonality in general equilibrium propagation. A precise statement of such equivalence first of all requires a definition of direct (or partial equilibrium) responses and indirect (or general equilibrium) effects.

I assume that the consumption-savings problem (1) has a unique solution for any path of prices, quantities and shocks faced by households. Aggregating the solutions across households, we obtain a consumption function \( c = c(s^h; \xi) \), where \( s^h = (i^h, \pi, w, \ell, \tau_e, d, p^c) \) collects household income, saving returns and prices—objects that adjust in general equilibrium. The

\(^5\)More precisely, I make implicit assumptions on functional forms and parameter values that guarantee that there is a unique deterministic steady state. In all numerical exercises, I have verified the uniqueness of the steady state and the (local) existence and uniqueness of transition paths.
total impulse response of consumption to the shock path $\varepsilon$ is simply

$$
\hat{c}_\varepsilon \equiv c(s^h; \varepsilon) - c(s^h; 0)
$$

I decompose this aggregate impulse response into two parts: a direct “partial equilibrium” impulse and an indirect “general equilibrium” feedback part.\(^6\)

**Definition 1.** Let the direct (partial equilibrium) response of consumption to a shock path $\varepsilon$ be defined as

$$
\hat{c}^{PE}_\varepsilon \equiv c(s^h; \varepsilon) - c(s^h; 0)
$$

Similarly, let the indirect (general equilibrium) feedback be

$$
\hat{c}^{GE}_\varepsilon \equiv c(s^h; 0) - c(s^h; 0)
$$

It is immediate that, to first order, the aggregate impulse response admits a simple additive decomposition into partial equilibrium response and general equilibrium feedback:

$$
\hat{c}_\varepsilon = \hat{c}^{PE}_\varepsilon + \hat{c}^{GE}_\varepsilon
$$

For example, for a stimulus check policy, the direct response captures the effect of the stimulus check $\tau(x; \varepsilon)$ on spending in isolation, while the indirect effect contains both the tax financing and all other general equilibrium effects (e.g., labor demand, prices, . . . ).

The remainder of this section establishes properties of the decomposition (5)—the desired formalization of the simple Keynesian cross intuition. Section 3 will then connect theory and measurement, linking the components of (5) to measurable objects and so in particular to the “missing intercept” (or aggregation) problem of cross-sectional causal effect estimates.

**Demand equivalence & its implications.** To state a demand equivalence result in the model of Section 2.1, I require three additional restrictions on model primitives.

The first assumption restricts goods bundles in the economy.

---

\(^6\)My definition of the partial equilibrium consumption response abstracts from endogenous partial equilibrium adjustments in earnings. I do so for three reasons. First, many empirical estimates of household spending responses to sudden income changes are actually interpretable as such netted spending elasticities (e.g. see Auclert, 2019, footnote 34). Second, in models with union-intermediated labor supply, replicating cross-sectional regressions differences out labor responses (see Proposition 2). Third, microeconomic evidence suggests that short-run wealth effects are weak (Cesarini et al., 2017). Nevertheless, in Appendix E.3, I repeat my analysis in an alternative model without unions, but with a non-standard preference parameterization allowing for weak short-run wealth effects (as in Jaimovich & Rebelo, 2009; Gál et al., 2012).
Assumption 1. Households and government consume a single, homogeneous final good. It follows that $p^c_t = p^g_t = 1$ for all $t$.

The second assumption relates to the interest rates faced by households and government: all agents must borrow and lend at a common interest rate.

Assumption 2. There is no borrowing wedge ($\kappa_b = 0$), so households and government borrow and lend at the same interest rate $i^b_t$.

The third assumption restricts the economy’s labor market. In response to the partial equilibrium increase in consumption demand $\hat{c}^{PE}_t$ induced by a given stimulus check policy, the average marginal utility of consumption declines, and so sticky-wage unions may try to bargain for higher wages. I denote the desired adjustment in aggregate hours worked at unchanged wages by $\hat{\ell}^{PE}_t$, defined formally in Appendix B.1. My third assumption provides two possible sufficient conditions to guarantee that $\hat{\ell}^{PE}_t = 0$.

Assumption 3. Either household preferences are such that there are no wealth effects in labor supply, or wages are perfectly sticky (i.e., wage adjustment costs are infinitely large).

These three assumptions are sufficient for the following demand equivalence result.

Proposition 1. Consider a stimulus check policy $\epsilon^\tau$, and suppose that Assumptions 1 to 3 hold. Then, for a fiscal spending policy $\epsilon^g$ such that (i) $\hat{c}^g = \hat{c}^{PE}_t$ (identical net excess demand) and (ii) $\hat{\tau}^e_g = \hat{\tau}^e_\tau$ (identical tax response), we have that, to first order,

$$\hat{c}_\tau = \hat{c}^{PE}_\tau + \hat{c}^g = GE \text{ feedback}$$

(6)

Under Assumptions 1 to 3, shocks to private and public net excess demand induce the exact same general equilibrium feedback effects. Proposition 1 presents the key implication of such demand equivalence that is relevant for the analysis in this paper: the response of aggregate consumption to a fiscal spending shock with the properties (i) and (ii) at the same time gives the general equilibrium feedback effects associated with the stimulus check policy $\epsilon^\tau$. Figure 1 provides a graphical illustration: in a quantitative HANK model satisfying Assumptions 1 to 3, the general equilibrium feedback effects on consumption after a stimulus check policy (orange line, left panel) and a fiscal spending expansion (orange line, right panel) are exactly the same. In the chosen model parameterization, interest rates and tax financing induce some general equilibrium crowding-out, while Keynesian employment effects lead to crowding-in, with the latter effect dominating slightly.
Demand Equivalence Illustration, Sticky-Wage HANK Model

Figure 1: Consumption impulse response decompositions after stimulus check and government spending shocks in the estimated HANK model of Section 4.1, but with fully rigid wages. The direct response and the indirect general equilibrium feedback are computed following Definition 1.

Finally, I emphasize that my focus on linearized equilibria also implicitly imposes a further condition (iii) on the two shocks: they occur in the same macroeconomic environment, including for example the same monetary policy response. This assumption will be important in communicating the results of my measurement strategy in Section 3.

Proof Sketch. My proof of the demand equivalence decomposition in (6) leverages the “sequence-space” approach to equilibrium characterization developed in Auclert & Rognlie (2018) and Auclert et al. (2018).

The basic idea of the argument is the following. Equilibria even in the rich model of Section 2.1 can be characterized as a system of several aggregate prices and quantities adjusting to clear several markets. Assumptions 1 to 3 simply turn out to be sufficient to ensure that, for any given private spending shock, a public expenditure shock with properties (i) and (ii) will perturb the same market-clearing conditions by the same amount, thus eliciting the same general equilibrium adjustment and so implying (6). First, Assumption 1—in conjunction with requirement (i) on the fiscal shock, \( \tilde{g} = \tilde{c}^{PE} \)—ensures that the private and public demand shocks lead to the same excess demand pressure for the common final good. Second, since by Assumption 2 households and governments borrow and lend at identical rates, this common excess demand path can in principle be financed using identical paths of future
taxes. Property (ii) of the fiscal spending shock ensures that this is indeed the case. Third, Assumption 3—a restriction on household behavior—ensures that the consumption increase induced by stimulus checks does not lead to any direct adjustment in hours worked.\footnote{In general equilibrium, however, hours worked can and generally will respond to both shocks.}

Overall, my statement of demand equivalence in Proposition 1 offers two key insights relative to previous work. First, it explicitly characterizes the properties of the fiscal shock required for the decomposition in (6) to hold. These properties will take center stage in the connection of theory to measurement in Section 3. Second, it reveals that not all assumptions necessary to arrive at a Keynesian cross-type equilibrium characterization (as e.g. in Auclert et al., 2018) are also necessary for a demand equivalence result. Notably, neither the absence of investment nor the assumption of a fixed real rate of interest are required.

### 2.3 Extension to general exclusion restrictions

In the analysis so far I have used a particular shifter of consumer spending—stimulus checks—and a particular structural model—the general framework of Section 2.1—to present demand equivalence as a set of restrictions on economic primitives. As it turns out, however, many of the restrictions implicit in this framework are in fact unnecessary. To make this point, this section complements the previous discussion with an abstract statement of shock equivalence in terms of exclusion restrictions in a linearized equilibrium representation. Throughout, I continue to use the same notational conventions as in my baseline model.

A general statement of consumption demand equivalence requires only two ingredients: an aggregate consumption function \( c = c(s^h; \varepsilon_d) \) and a (differentiable) system of equations characterizing equilibrium aggregates \( H(x; \varepsilon_d, \varepsilon_g) = 0 \), where \( \varepsilon_d \) and \( \varepsilon_g \) are generic shocks to private and public spending, respectively, and the inputs to household consumption \( s^h \) are determined as part of the set of aggregates \( x \). Demand equivalence is then simply a set of exclusion restrictions on derivatives of the equilibrium mapping \( H(\bullet) \): as long as

\[
\frac{\partial H}{\partial \varepsilon_d} \times \varepsilon_d = \frac{\partial H}{\partial \varepsilon_g} \times \varepsilon_g
\]

(7)

it follows immediately that, to first order,

\[
\hat{c}_d = \hat{c}^{PE} + \hat{c}_g = \begin{cases} \hat{c}^{PE} \text{ PE response} \\ \hat{c}_g \text{ GE feedback} \end{cases}
\]

(8)
exactly as in Proposition 1.\footnote{An equilibrium is a solution of $\frac{\partial H}{\partial \mathbf{x}} \times \mathbf{x} + \frac{\partial H}{\partial \mathbf{e}_d} \times \mathbf{e}_d + \frac{\partial H}{\partial \mathbf{e}_g} \times \mathbf{e}_g = \mathbf{0}$. In stating (8) I am assuming that this system has a solution for $\mathbf{e}_d$; it then follows from my assumptions that the same path of $\mathbf{x}$ also solves the system for $\mathbf{e}_g$. Equilibrium uniqueness would require further assumptions on $\frac{\partial H}{\partial \mathbf{x}}$ (e.g., invertibility).} Condition (7) is a general exclusion restriction on the equilibrium system: both shocks must enter all equilibrium equations symmetrically. The proof of Proposition 1 works because, under my imposed restrictions, the equilibrium can be cast in a form consistent with (7).\footnote{Casting my results as exclusion restrictions on equilibrium representations suggests a connection to the identification of systems of simultaneous equations. This connection is explored in Guren et al. (2020).} In Appendices C.2 and C.3 I give examples of other shocks and models that can be written in this form. First, I extend the model of Section 2.1 to allow for time preference shocks as a generic non-policy consumption shifter. Equivalence obtains under the same restrictions as those discussed in Section 2.2; in particular, since non-policy private demand shifters $\mathbf{e}_d$ necessarily induce spending paths $\hat{c}^{PE}_d$ with zero net present value, requirement (ii) in the equivalence proof now dictates that the equivalent public spending shock $\mathbf{e}_g$ is purely deficit-financed, with $\mathbf{T}^e$ moving only because of general equilibrium effects, and not directly because of the spending shock $\mathbf{e}_g$. Second, I consider several examples of popular models beyond the familiar New Keynesian tradition—including for example models with non-rational expectation formation of firms and households—, and show that they still all fit into the general semi-structure of (7).

SUMMARY & OUTLOOK. This section has presented conditions ensuring the commonality of general equilibrium effects of private and public spending shocks. I emphasize, however, that this equivalence result by itself says nothing about the strength of that common general equilibrium feedback: in the illustrative HANK model example of Figure 1, general equilibrium effects are relatively weak; in Appendix C.4, I instead show two other examples, one with full crowding-out, the other with very strong amplification.

The appeal of the demand equivalence result is instead that it tells us how to measure those common general equilibrium effects, however strong (or weak) they may turn out to be. I now turn to this empirical measurement strategy.

3 Solving the missing intercept problem

This section shows how the demand equivalence result together with time series evidence on the dynamic causal effects of fiscal purchases can be used to solve the “missing intercept” aggregation problem for cross-sectionally identified consumption demand shifters. Section 3.1...
begins by tying the theoretical decomposition in (8) to empirically measurable objects. Section 3.2 uses this mapping between theory and empirics to propose a measurement strategy, and Section 3.3 applies the method to estimate the aggregate effects of stimulus checks.

### 3.1 From theory to measurement

The demand equivalence decomposition (8) has two parts: a) the direct (or partial equilibrium) response of household consumption to some demand shifter $\hat{c}^P$ and b) the dynamic causal effect of a particular change in fiscal purchases on household consumption $\hat{c}_g$ (which under demand equivalence equals the general equilibrium term $\hat{c}^{GE}_g$). Each of those two components can be tied to objects estimated in previous empirical work.

**Micro regressions.** Cross-sectional regressions of household-level consumption on idiosyncratic shock exposure promise to identify part a): the direct consumption response.

To make this claim precise, I return to the model of Section 2.1, but now assume that the transfer stimulus received by household $i$ is $\varepsilon_{\tau it} = \xi_{\tau it} \times \varepsilon_{\tau t}$, where $\xi_{\tau it}$ is i.i.d. across households and time (and uncorrelated with any household characteristics), with $\mathbb{E}(\xi_{\tau it}) = 1$ and $\text{Var}(\xi_{\tau it}) > 0$. Given this heterogeneity in shock exposure, I can study regressions run on the cross-section of households. A typical cross-sectional regression takes the form

$$
\begin{align*}
c_{it+h} &= \alpha_i + \delta_t + \beta_{\tau h} \times \varepsilon_{\tau it} + u_{it+h}, \quad h = 0, 1, 2, \ldots 
\end{align*}
$$

where $\alpha_i$ and $\delta_t$ are individual and time fixed effects. It is straightforward to show that, under my assumptions, regressions such as (9) estimate average household-level causal effects that are interpretable as direct partial equilibrium shock responses, as claimed.\footnote{Formally, for Proposition 2, I consider the first-order perturbation solution of the model in Section 2.1 with aggregate shocks $\varepsilon_{st}$, $s \in \{\tau, g\}$. This ensures that all regression estimands are well-defined.}\footnote{Note that the regression (9) is run at the household level. This is important: cross-regional regressions (e.g. as in Mian et al., 2013) contain local general equilibrium effects and so do not identify my notion of direct effects. I extend my approach to such cross-regional regressions in the companion note Wolf (2019).}

**Proposition 2.** Suppose an econometrician observes a panel of consumption $\{c_{it}\}$ and shock exposure $\{\varepsilon_{\tau it}\}$. Then the ordinary least-squares estimand of $\beta_\tau \equiv (\beta_{\tau 0}, \beta_{\tau 1}, \ldots)'$ satisfies

$$
\beta_\tau = \hat{c}^{PE}_\tau
$$

Note that common general equilibrium effects are absorbed by the time fixed effect. It is in light of this result that I refer to $\hat{c}^{GE}_\tau$ as the “missing intercept.”
FISCAL MULTIPLIERS. The dynamic causal effects of changes in fiscal purchases on macroeconomic outcomes can be estimated using the conventional semi-structural macroeconometric toolkit (see Ramey, 2016, for a review). Under demand equivalence, such time series analysis promises to identify part b): the “missing intercept” of general equilibrium effects.

Most previous work has relied on one of three possible sets of identifying assumptions. First, researchers may have access to direct, “narrative” measures of aggregate fiscal policy shocks (Ramey, 2011). Second, outside information—either in the form of direct zero or sign restrictions (Blanchard & Perotti, 2002; Mountford & Uhlig, 2009) or via measures of other shocks (Caldara & Kamps, 2017)—can help researchers pin down government spending rules and so identify spending shocks. Third, professional forecast errors of government spending may be interpreted as orthogonal to any (known) rules-based spending components, and so again serve as an instrumental variable for fiscal shocks (Ramey, 2011; Drautzburg, 2020). In all of these cases, the desired counterfactuals can then be estimated using Vector Autoregressions (VARs) or Local Projections (LPs). Importantly, across this list of identifying assumptions, estimated fiscal multipliers tend to lie at around 1 (Ramey, 2018).

The central implication of demand equivalence is that these fiscal multiplier estimates are actually informative about the propagation of a larger menu of structural shocks. Any identified aggregate fiscal shock is associated with an ( estimable) implied path of fiscal purchases $\hat{g}_g$ and underlying financing (taxes and/or deficits). Under demand equivalence, this shock in general equilibrium propagates exactly like any shock to private spending—say, stimulus checks—with (i) the same excess demand path $\hat{g}_g = \hat{c}_d^{PE}$ and (ii) the same tax response, and furthermore (iii) occurring in the same macro environment. It follows that conventional time series estimates of fiscal multipliers actually contain much more information than commonly believed: for a suitable private demand shifter $\varepsilon_d$, they for free give the general equilibrium component of the consumption response $\hat{c}_d^{GE}$ as well as full counterfactuals for all other macroeconomic aggregates (employment, investment, inflation, . . . ).

THE MATCHING PROBLEM. The previous discussion has revealed that, through the lens of the theory of demand equivalence, cross-sectional evidence on private demand shifters and time series estimates of public spending shock propagation are useful complements: when put together in accordance with Proposition 1, they fully characterize aggregate general equilibrium counterfactuals for the private demand shifters, allowing researchers to estimate the causal effects of these shifters even in the absence of exogenous macro variation.

To make this potentially powerful insight operational, however, we must confront an
important challenge: the demand equivalence result only allows us to combine particular cross-sectional and time series identified shocks—those that satisfy the conditions (i) - (iii). This is challenging; for example, nothing guarantees that any given time series and cross-sectional causal effect estimates give demand paths that are aligned as required by (i). The next subsection presents two solutions to this problem of aligning the shocks.

3.2 Matching time series & cross-sectional evidence

I begin with the first requirement: how can we find net excess demand paths \( \hat{g}_g \) and \( \hat{c}_{PE} \) that are at least approximately aligned? Linearity gives one degree of freedom, so we can slightly relax the matching requirement to the proportionality condition \( \hat{g}_g \propto \hat{c}_{PE} \). I consider two possible approaches to ensuring this alignment of spending paths.

1. From cross-section to time series. The first approach begins with some given cross-sectional spending estimate \( \hat{c}_{PE} \), and then tries to find the fiscal spending experiment that solves the “missing intercept” problem for that particular shock. To do so, suppose that a researcher has used one or several of the semi-structural time series identification approaches discussed in Section 3.1 to estimate the aggregate effects of a menu of \( n_k \) different kinds of government spending shocks \( \{ \varepsilon_{gk} \}_{k=1}^{n_k} \) with implied spending paths \( \hat{g}_{gk} \). A linear projection of \( \hat{c}_{PE} \) on the space spanned by those shock paths gives

\[
\hat{c}_{PE} = \sum_{k=1}^{n_k} \gamma_k \times \hat{g}_{gk} + \text{error} \tag{11}
\]

If the error is sufficiently small, we may consider the weighted average

\[
\sum_{k=1}^{n_k} \gamma_k \times \hat{c}_{gk} \tag{12}
\]

as a promising candidate to learn about the “missing intercept” \( \hat{c}_{d}^{GE} \). Of course, by construction, using this weighted average of fiscal shocks will only solve the missing intercept problem up to the aggregate effects of a shock that equals the error term.\(^\text{12}\) Section 3.3 will provide a concrete example of a case in which this matching error is indeed very small, implying that the proposed approach to aggregation is promising.

\(^\text{12}\)See Appendix E.8 for a further discussion of this point. I there also use a structural model to study the inaccuracy associated with demand matching errors of similar magnitude to those that I observe in my empirical stimulus check application in Section 3.3.
2. *From time series to cross-section.* The second approach proceeds the other way around, beginning with a path \( \hat{\mathbf{g}}_g \) identified in the time series, and then asking what kind of private demand shock—e.g., what sequence of stimulus checks—would induce the same spending response. Heuristically, given a private demand shock \( \varepsilon_d \) and the consumption function \( c(\bullet) \) defined in Section 2.2, we can in principle recover the required shock path as

\[
\varepsilon_d \equiv C_d^{-1} \times \hat{\mathbf{g}}_g
\]  

where \( C_d \equiv \frac{\partial c(s^h, \varepsilon)}{\partial \varepsilon_d} \), assumed to be invertible. It follows from Proposition 2 that cross-sectional analysis is informative about (weighted averages of) columns of \( C_d \). Running a large number of cross-sectional regressions for different paths of private demand shocks \( \varepsilon_d \) would thus in principle allow researchers to construct this inverse mapping. More realistically, researchers may combine empirical regression estimates with theoretical restrictions on the shape of \( C_d \). Importantly, such restrictions would only require the researcher to take a stance on a *partial equilibrium* model of household consumption decisions—all general equilibrium effects would still be captured semi-structurally by the time series estimates of \( \hat{\mathbf{c}}_g \). My second application in Section 3.3 will provide an example of this approach.

Having addressed the demand path matching problem through either of these two strategies, I propose to proceed and construct the demand equivalence approximation as

\[
\tilde{c}_d = \tilde{c}_{dPE} + \sum_{k=1}^{n_k} \gamma_k \times \tilde{c}_{gk}
\]

with the weights \( \gamma_k \) as in (11) under the first approach, or we set \( \tilde{c}_d = \tilde{c}_{dPE} + \tilde{c}_g \) with \( \tilde{c}_{dPE} = \hat{\mathbf{g}}_g \) for the second approach, giving a counterfactual for the shock \( \varepsilon_d \) derived in (13).

Conditions (ii) and (iii) then affect the *interpretation* of this construction. Under demand equivalence, my sum is interpretable as a valid general equilibrium counterfactual for a shock \( \varepsilon_d \) that is associated with the same aggregate movements in taxes as the identified public demand shock(s) *and* occurred in the same macroeconomic environment. For example, if the private demand shifter is stimulus checks, and the matched time series \( \varepsilon_g \) is persistently deficit-financed and largely accommodated by the monetary authority, then the estimated counterfactual will apply to similarly deficit-financed, accommodated stimulus checks.\(^{13}\)

\(^{13}\) Note that, for a non-policy demand shifter, \( \tilde{c}_{dPE} \) necessarily has zero net present value, so the relevant comparison is a change in fiscal purchases financed by a reversal tomorrow, with taxes changing *only* because of general equilibrium effects. I consider an example of such a shock in Appendix F.2.
3.3 Two applications to stimulus checks

I now apply the demand equivalence approach to estimate general equilibrium counterfactuals for a popular fiscal policy tool: stimulus checks. The study of such stimulus checks is well-suited to illustrate the proposed approach, for two reasons. First, even though stimulus checks as a policy tool are increasingly popular, there are few estimates of their aggregate effects. Intuitively, the core estimation challenge is that there is little-to-no exogenous time series variation in those payments. Second, a wealth of micro data has allowed researchers to estimate the direct spending response of households to the receipt of (small) lump-sum gains, giving the required micro identification.

In the remainder of this section I construct macro counterfactuals for two kinds of stimulus check experiments: one-off checks and a more persistent sequence of checks. The micro and macro parts of my analysis leverage canonical contributions to the relevant cross-sectional and time series literatures: Parker et al. (2013) and Broda & Parker (2014) for the response of consumption to lump-sum income receipts, and Ramey (2011) and Caldara & Kamps (2017) for the aggregate general equilibrium effects of fiscal purchases.

**One-off stimulus checks.** For a one-off, one-quarter stimulus check sent to everyone, the policy’s direct effect on partial equilibrium net excess spending is given as

\[ \hat{c}_{t,0}^{PE} = MPC_{t,0} \times \hat{\tau}_0 \]

where

\[ MPC_{t,0} \equiv \int_0^1 \frac{\partial c_{it}}{\partial \tau_0} \, di \]

is the average marginal propensity to consume at time \( t \) out of an income gain at time 0.

Several recent studies have used rich household spending data to estimate objects that are either exactly or approximately interpretable as the desired average MPC (e.g. Johnson et al., 2006; Parker et al., 2013; Jappelli & Pistaferri, 2014; Fagereng et al., 2018). A common finding in this literature is that households spend most of a (small) one-time income receipt on impact, and that the spending response then decays back to zero quickly. In particular, the point estimates of Parker et al. (2013) and Broda & Parker (2014) suggest that, following a one-off stimulus check, total consumption expenditures increase by around 50 cents on the dollar on impact and 20 cents in the subsequent quarter. Translated to the size of the 2008 stimulus check policy, this corresponds to around 1.5 per cent of total personal consumption expenditure on impact, and 0.6 per cent in the following quarter. In the left panel of Figure 2,
Figure 2: The left panel shows direct consumption responses to the stimulus check (green) vs. direct government spending response to identified spending shock (black), with 16th and 84th percentile confidence bands (grey), quarterly frequency. Estimated consumption responses from Parker et al. (2013) and Broda & Parker (2014), extrapolated for horizons beyond $t = 1$. The right panel shows the response of consumption to the fiscal spending shock.

the two green x’s show those two direct consumption responses $\hat{c}_{t,0}^{FE}$ and $\hat{c}_{t,1}^{FE}$; the solid green line extrapolates those MPCs simply by imposing a constant geometric rate of decay for MPCs from $t = 1$ onwards, together with the restriction that (discounted) MPCs sum to 1, consistent with the lifetime household budget constraint.\(^{14}\)

Taking as given that net excess demand path, we now need to find a similarly transitory fiscal spending expansion. Most time series experiments feature rather persistent increases in fiscal purchases, with the notable exception of the professional forecast errors studied in Ramey (2011). Those forecast errors are appealing as a measure of fiscal expenditure shocks because (1) professional forecasts are implicitly controlling for the rules-based component of fiscal expenditure and (2) the large information set of forecasters limits concerns of shock non-invertibility (Leeper et al., 2013). Furthermore, and most importantly for the purposes of the analysis here, they are known to induce a very transitory uptick in total fiscal purchases.

\(^{14}\)I discuss the mapping between empirical regressions of transfer stimulus receipt and my notion of a direct spending response in Appendix D.1. That appendix also elaborates on my extrapolation. As I emphasize there, a constant rate of decay of MPCs is (roughly) consistent with standard incomplete-market models (Auclert et al., 2018; Wolf, 2021) as well as other empirical evidence (e.g. Fagereng et al., 2018).
I study their propagation to the rest of the economy using a specification very similar to that in Ramey (2011), on a sample from 1981:Q3 (when forecast errors are first available) to 2008:Q4 (to ensure a stable macroeconomic regime). I then embed the shock in a recursive VAR, in line with the population results of Plagborg-Møller & Wolf (2021) and the finite-sample recommendations of Li et al. (2021). Since my choice of further macro controls, variable definitions, data construction and estimation procedure are all standard and in line with previous work, I relegate further details to Appendix D.3.

The left panel of Figure 2 reveals that, as required by the demand matching condition, the estimated increase in fiscal purchases closely mirrors the spending expansion implied by the stimulus check policy, with the targeted $\hat{c}_P^{FE}$ always remaining within the confidence bands for the estimated $\hat{g}_P$. Furthermore, the corresponding estimates for government debt and taxes reported in Appendix D.3 reveal the increase in fiscal purchases to be persistently deficit-financed. Finally, I there also show that the spending expansion was largely accommodated by the monetary authority, with nominal interest rates responding very little. It follows that the consumption response to the fiscal experiment $\hat{c}_P$, displayed in the right panel of Figure 2, promises to at the same time tell us about the missing general equilibrium effects of a deficit-financed, one-off stimulus check policy with little monetary offset.

Figure 3: Consumption and output responses to a stimulus check shock, quarterly frequency. The full consumption response is computed following the exact additive decomposition of Proposition 1, while the output response is simply equal to the response after a government spending shock. The grey areas again correspond to 16th and 84th percentile confidence bands.
Figure 3 puts all the pieces together to present full general equilibrium counterfactuals for stimulus checks. The left panel begins by implementing the demand equivalence decomposition in (6), simply summing a) the micro-estimated direct spending response $\hat{c}^PE_\tau$ and b) the response of consumption to the fiscal shock $\hat{c}_g$. Since the direct spending effect is large, while the response of private consumption to the fiscal spending expansion is muted (with some crowding-out over time), the estimated aggregate effect of the policy turns out to be close to the micro-estimated direct effect. Thus, perhaps surprisingly, the various price and multiplier effects cited in previous empirical and theoretical work seem to roughly cancel. The right panel shows the corresponding response of output, which by demand equivalence is the same for the fiscal spending expansion and the stimulus checks. Here I find a significant (if short-lived) response, with output on impact rising by somewhat less than 1 per cent, and then returning to baseline. Overall, my estimates suggest that deficit-financed stimulus checks provide meaningful stimulus to aggregate consumption and output.

A STRING OF STIMULUS CHECKS. For my second illustration I proceed the other way around, beginning with the time-series fiscal multiplier estimates of Caldara & Kamps (2017). Those authors estimate a conventional fiscal policy VAR and show that conclusions on fiscal multipliers hinge critically on the systematic feedback from economic activity to fiscal purchases. The key contribution is to estimate this feedback not through outright restrictions on the fiscal rule (as in Blanchard & Perotti, 2002), but through proxy variables for other aggregate shocks as valid instrumental variables. I replicate their baseline specification, and show the response of fiscal purchases to the identified fiscal spending shock $\hat{g}_\tau$ in the left panel of Figure 4.\textsuperscript{15} Fiscal purchases increase persistently for several quarters, and then gradually return to baseline. I normalize the impact spending response to equal 1 per cent of steady-state (quarterly) consumption, or around $150 per household.

It remains to do the inversion step: what sequence of stimulus check payments to households would induce a net excess spending path $\hat{c}^{PE}_\tau$ akin to that in the left panel of Figure 4? By (13), we need to learn about $C_{-1}:$ the inverse of the mapping from sequences of transfers into sequences of consumer spending. I construct an estimate of this mapping by combining the empirical evidence in Parker et al. (2013) and Broda & Parker (2014) on spending responses to checks with consumer theory reviewed in Auclert et al. (2018) and Wolf (2021).

Standard models of the household consumption-savings decision with binding liquidity constraints imply that lump-sum stimulus checks today are spent gradually. I show in Wolf

\textsuperscript{15}Details of the replicated VAR specification are provided in Appendix D.3.
Figure 4: Direct spending response and general equilibrium consumption counterfactuals for a sequence of stimulus checks. The transfer shock path (purple line) is recovered as $C_{t}^{-1} \times \hat{g}$, with $C_{t}$ as in (15), while the full consumption response is computed following the exact additive decomposition of Proposition 1. The grey areas again correspond to 16th and 84th percentile confidence bands.

(2021) that this gradual spending response is very well-described by a simple three-parameter profile: $(\omega, \omega\xi, \omega\xi^2, \omega\xi^3, \ldots)$. Matching this spending profile to the evidence reported by Broda & Parker gives $\{\omega = 0.5, \xi = 0.7, \theta = 0.59\}$—the first column of $C_{t}$.$^{16}$ Furthermore, since Broda & Parker fail to find significant pre-treatment effects, and also consistent with results in other settings (e.g., Kueng, 2018; Ganong & Noel, 2019; Baugh et al., 2021), I for my baseline analysis assume the absence of any anticipation effects related to future transfer receipt. The direct household spending response map $C_{t}$ is then simply given as

\[
C_{t} = \omega \times \begin{pmatrix}
1 & 0 & 0 & \ldots \\
\xi \theta & 1 & 0 & \ldots \\
\xi \theta^2 & \xi \theta & 1 & \ldots \\
\vdots & \vdots & \vdots & \ddots
\end{pmatrix}
\]  

(15)

$^{16}$As discussed above, Broda & Parker estimate the first two entries of the first column to be around 0.5 and 0.2. I then discipline the rate of decay by requiring a lifetime MPC of 1 (i.e., the column sums to 1, assuming zero rate of interest), exactly as I did for my MPC extrapolation in the first application.
While my discussion in the remainder of this section will rely on (15), I in Appendix D.1 present results for different degrees of household foresight, following Gabaix (2020), Auclert et al. (2019) and Wolf (2021).\(^{17}\)

The purple line in Figure 4 shows the implied equivalent transfer path. Inverting \(C_t\), we find it as

\[
\hat{\tau}_t = \frac{1}{\omega} \times \left[ \bar{c}^{PE}_{\tau,t} - \xi (1 - \xi) \sum_{\ell=1}^{t} \theta^{\ell-1} \bar{c}^{PE}_{\tau,t-\ell} \right]
\]

The displayed path has the expected shape: it scales in \(1/MPC = 1/\omega\), features a significant news component, but is less persistent than the spending path \(\hat{c}^{PE}_t\) that it engineers. In particular, given the assumed absence of anticipatory spending effects, the time-0 check is equal to \(1/MPC \times \$150 = \$300\), i.e., exactly twice as large as the desired impact spending response. For stronger anticipation effects the impact stimulus check can be somewhat smaller, as I show in the supplementary discussion of Appendix D.1.

Finally, it remains to construct the full general equilibrium counterfactual for aggregate consumption, reported in the right panel of Figure 4. Caldara & Kamps (2017) estimate fiscal multipliers slightly in excess of one; as I show in Appendix D.3, their estimates correspond to persistently deficit-financed spending expansions with limited monetary offset. Summing a) the direct spending impact and b) the response of consumption to the fiscal spending shock, we see that the transfer shock depicted in the left panel induces a persistent increase in household consumption, with the full response slightly larger than the direct effect.

**Summary.** This section has illustrated the practical feasibility of the demand equivalence approach with applications to an increasingly popular policy tool: stimulus checks. By putting together standard pieces of cross-sectional and time series evidence, I conclude that such policies significantly stimulate aggregate consumption, with the estimated causal effect quite close to the direct micro estimates—a “missing intercept” close to zero.

How should we interpret this finding? By the theoretical analysis in Section 2, we know immediately that any structural model satisfying demand equivalence and estimated to match the cross-sectional and time series empirical evidence reviewed here will invariably arrive at that same conclusion. It thus remains to discuss the plausibility of the demand equivalence assumption itself. I do so in the next section.

\(^{17}\)An alternative interpretation of (15)—consistent with forward-looking households—would be the following. On the one hand, unconstrained households behave in line with the permanent income hypothesis, so their consumption responds by very little. On the other hand, constrained households are in fact fully constrained, so they cannot respond to stimulus check news, and spend realized income gradually.
4 How plausible is demand equivalence?

The methodology and results presented in Section 3 are exactly valid only under the strong conditions required for demand equivalence. These conditions, however, are unlikely to hold in practice. I thus in this section apply the proposed methodology to models violating demand equivalence, and ask whether the estimated counterfactuals at least approximately equal the true model-implied causal effects.

I proceed in three steps. First, in Section 4.1, I consider an estimated HANK model, enriched to feature many of the bells and whistles of the quantitative business-cycle literature. Second, in Section 4.2, I further extend this baseline environment with various additional frictions specifically designed to break demand equivalence, and discuss the likely sign and magnitude of the error. Finally in Section 4.3 I summarize the results from my model laboratories in the form of recommendations for applied practice.

4.1 Estimated business-cycle models

My first model laboratory is an estimated HANK model, rich enough to feature many of the frictions popular in the quantitative business-cycle literature (e.g. Smets & Wouters, 2007; Justiniano et al., 2010). Models of this sort are routinely used for quantitative policy evaluation, and so in particular are a natural candidate for a fully structural solution to the aggregation (missing intercept) problem.

I build on the general framework of Section 2.1, and continue to impose Assumptions 1 and 2, but now relax Assumption 3. Demand equivalence in this generalized setting thus fails only because of the labor supply channel.

Proposition 3. Consider a stimulus check policy $\varepsilon_{\tau}$, and suppose that Assumptions 1 and 2 hold. Then, for a fiscal spending policy $\varepsilon_{g}$ such that (i) $\hat{g}_{g} = \hat{c}_{\tau}^{PE}$ (identical net excess demand) and (ii) $\hat{r}_{g}^{e,PE} = \hat{r}_{\tau}^{e,PE}$ (identical direct tax response), we have that, to first order,

$$\hat{c}_{\tau} = \hat{c}_{\tau}^{PE} + \hat{c}_{g} + \text{error}(\ell_{\tau}^{PE})$$

18Formally, I consider an econometrician with access to infinitely large samples of cross-sectional and time-series model-generated data. Using the data, the econometrician implements the method of Section 3.

19In stating Proposition 3, I have relaxed the equal financing assumption to one of equal direct financing, where the direct tax response is defined analogously to Definition 1. With Assumption 2 and $\hat{g}_{g} = \hat{c}_{\tau}^{PE}$, such equal direct financing is still feasible. Identical overall financing—i.e., $\hat{r}_{g} = \hat{r}_{\tau}$—however is generally not feasible without Assumption 3. This is because differences in general equilibrium feedback imply that the other inputs to the fiscal budget may not respond identically to the two shocks.
where the error function is characterized in Appendix G.3 and is equal to 0 if $\ell^{\text{PE}} = 0$.

My choice to only relax Assumption 3 is motivated by previous work: contributions to the quantitative business-cycle literature rarely depart from the common-goods assumption and feature households borrowing and lending in government bonds, but usually do not impose Assumption 3 (for canonical examples see Christiano et al., 2005; Smets & Wouters, 2007).

**Estimation.** I provide a brief outline of the model and my estimation strategy here, and relegate further details to Appendix B.2.

The household block is as described in Section 2.1, while the rest of the economy is designed to be as close as possible to the canonical model of Justiniano et al. (2010). First, I allow for investment adjustment costs, variable capacity utilization, and a rich monetary policy rule. Second, I extend the economy to be subject to a standard menu of aggregate shocks: to total factor productivity and the marginal efficiency of investment, to household patience, to wage mark-ups, to government spending, and to monetary policy. The only purpose of these additional shocks is to allow the model to fit aggregate U.S. business-cycle dynamics reasonably well, opening the door for a conventional likelihood-based estimation approach (An & Schorfheide, 2007). I calibrate the model’s steady state using targets familiar from the HANK literature (e.g. Kaplan et al., 2018). Importantly, because household self-insurance is severely limited, the average MPC is high, at around 30% quarterly out of a lump-sum 500$ income gain. Model parameters governing dynamics are then estimated using likelihood methods on a standard set of macroeconomic aggregates.20 The key exception is the degree of wage stickiness which—in light of its centrality to my results—is directly calibrated to be consistent with recent micro evidence (Grigsby et al., 2019; Beraja et al., 2019), with wage re-sets every 2.5 quarters on average. Most of the results in the remainder of this section refer to the estimated model’s posterior mode.

**Results.** I subject the economy to a one-off stimulus check policy, and consider a researcher that uses the methodology of Section 3 to estimate that policy’s aggregate effects.21

---

20Specifically, I include measures of output, inflation, a short-term interest rate, consumption, investment, and hours worked—six observables for six shocks.

21I compute the response of the endogenous component of taxes to the stimulus check policy, $\hat{\tau}_t^c$, using the particular rule (B.7). I then set $\hat{\tau}_t^c \propto \hat{\tau}_t^e$, with the factor of proportionality chosen so that $\lim_{t \to \infty} \hat{\theta}_t \to 0$ after the fiscal spending shock $\varepsilon_g$. With Assumption 3 this specification would ensure identical overall tax financing, exactly as in Proposition 1. I maintain this specification of fiscal rules for all of Section 4.
Approximate Demand Equivalence, Estimated HANK Model

Figure 5: Consumption impulse response decompositions and demand equivalence approximation in the estimated HANK model, with details on the parameterization in Appendix B.2. The direct response and the indirect general equilibrium feedback are computed following Definition 1.

Results are displayed in Figure 5: the left panel decomposes the response of aggregate consumption to the stimulus check into direct partial equilibrium (green) and indirect general equilibrium (orange) effects, while the right panel compares the actual model-implied causal effect (grey) with the output of the my procedure (dashed black).

The main take-away from Figure 5 is that the demand equivalence approximation remains excellent, with the grey and black dashed lines in the right panel close to each other throughout.22 The left panel first of all reveals that general equilibrium effects in the estimated model are small throughout, reflecting largely offsetting interest rate, tax financing, and Keynesian amplification effects. Following a similarly short-lived and deficit-financed fiscal spending expansion, the same forces imply that aggregate consumption barely moves, giving the small approximation error displayed in the right panel. The intuition for the sign and magnitude of that approximation error is simple. Following receipt of the stimulus check, households consume more. Given their lower marginal utility of consumption, they would optimally like to work less, thus in general equilibrium depressing aggregate output and consumption. This labor supply channel is absent after an increase in (unvalued) fiscal purchases, so the demand equivalence approximation overstates the response of consumption.

22At its largest, the associated error equals just below three per cent of the true peak consumption response.
to the transfer stimulus. However, even with strong wealth effects, this channel is largely irrelevant quantitatively: as long as wages are at least moderately sticky, labor is mostly demand-determined in the short run, so transitory shifts in labor supply do not matter much. This finding is consistent with conventional wisdom in the business-cycle literature (e.g. Christiano, 2011a,b): at least for relatively transitory fluctuations, hours worked in conventional (New Keynesian) business-cycle models are largely demand-determined.

EXTENSIONS & OTHER MODELS. The results in Figure 5 are neither special to the posterior mode of my estimated model, nor to the particular HANK setting considered here.

First, in Appendix E.1, I randomly draw model parameters from large supports, solve the implied model, and compute the approximation accuracy. The analysis reveals that, of all estimated parameters, only the degrees of price and wage rigidity have a material impact on the accuracy of the approximation, as expected. Second, in Appendix E.2, I use the demand equivalence approximation to construct counterfactuals for private demand shocks in the popular model of Justiniano et al. (2010), solved at the posterior mode. Since wages are even stickier there, the approximation is in fact better than in my estimated HANK model, with the approximation error now barely visible.

DISCUSSION. The analysis in this section has demonstrated that demand equivalence is, at least approximately, a feature of standard quantitative models of business-cycle fluctuations. Intuitively, the features added to such models to ensure agreement with time series aggregates—for example investment adjustment costs, variable capacity utilization or price- and wage-indexing—are entirely consistent with demand equivalence. In particular, the common goods and financing assumptions are imposed regularly, and the labor supply channel is generally found to be quantitatively insignificant. We can thus strengthen the conclusions of Section 3: conventional business-cycle models, when estimated to be consistent with a) cross-sectional evidence on consumer spending behavior, b) time-series evidence on aggregate fiscal multipliers, and c) the behavior of standard U.S. time series aggregates, are likely to be in close agreement with my semi-structural stimulus check causal effect estimates presented in the two applications in Section 3.3.

While promising, this result is however only a first step to gauging the empirical relevance of the demand equivalence approximation. All three assumptions required for Proposition 1—and not just the labor supply restriction—are likely to be violated in practice, so I now extend the estimated baseline model in several directions to understand better why and how the approximation can fail.
4.2 Breaking equivalence

In this section I consider a large number of model extensions, each designed to challenge the quality of the demand equivalence approximation by breaking one or several of Assumptions 1 to 3. For each model variant, I begin with the baseline estimated HANK model of Section 4.1, and then add another friction to further break demand equivalence. My discussion here will be largely focussed on the sign and size of the bias induced by each of those model extensions, with model and calibration details relegated to Appendix B. Results are reported in Figure 6, which plots the demand equivalence error for each of my experiments, defined as

$$\text{error} = \frac{\hat{c}_{PE} + \hat{c}_g - \hat{c}_r}{\hat{c}_{r0}}$$  \hspace{1cm} (17)

Note that (17) does not normalize the approximation errors to be positive. The fact that the errors displayed in Figure 6 all turn out to be positive is thus not an artifact of normalization, but in fact a key result.

Labor supply & wealth effects. Four experiments—the baseline HANK model, fixed wages, a model with flexible prices and wages, and a model with household preferences that imply weak wealth effects—illustrate the role of Assumption 3 on labor supply in breaking demand equivalence. As shown previously in Figures 1 and 5, in the estimated HANK model, the demand equivalence approximation is very accurate even with moderately sticky wages, and exact in the case of fully rigid wages. The purple line in Figure 6 shows that, with (nearly) flexible prices and wages, the quality of the approximation deteriorates sharply: because of quite strong wealth effects in labor supply, households cut hours worked following transfer receipt, and so the demand equivalence approximation—which misses these wealth effects—significantly overstates the response of aggregate consumption.

How material is this particular threat to the demand equivalence approach? I have already emphasized that, for relatively transitory shocks (such as one-off stimulus checks), even moderately sticky wages are enough to mute the labor supply channel. Other pieces of macro and micro evidence suggest the same conclusion. First, on the macro side, standard time series estimation exercises usually call for near-zero wealth effects in labor supply (Schmitt-Grohé & Uribe, 2012; Born & Pfeifer, 2014). Second, on the micro side, quasi-experimental evidence at the household level suggests that, at least in response to moderately sized lump-sum transfer receipts, hours worked and earnings drop by an order of magnitude less than
spending increases (e.g., Cesarini et al., 2017; Fagereng et al., 2018). The yellow line in Figure 6 shows that, with fully flexible wages but household preferences adjusted to feature such data-consistent weak wealth effects, the approximation error is again small.

**Figure 6**: Errors (relative to the true impact consumption response) of the demand equivalence approximation in several model extensions. Details for all extensions and their parameterizations are relegated to Appendix B.

**Many Goods**. Heterogeneity in public and private consumption baskets is an obvious threat to demand equivalence: without the restriction of a common final good (Assumption 1), changes in public and private purchases may set in motion very different general equilibrium effects. First, relative prices will move in response to sectoral spending shocks (Ramey & Shapiro, 1998). Second, if goods differ in their factor incidence (e.g., capital vs. labor income), and if factor income covaries with household characteristics (e.g., households with little non-labor income have high MPCs), then general equilibrium effects will

---

23 Coibion et al. (2020) document similarly small earnings responses after stimulus payments in the COVID-19 recession. Mogstad et al. (2021) in contrast estimate significantly larger marginal propensities to earn, but their estimates come from much larger lottery wins (> $30,000). Consistent with this finding, I in Section 4.3 recommend that researchers should only apply the demand equivalence approximation to moderately sized spending shocks.
necessarily be shock-specific (Alonso, 2017; Baqaee, 2015).

To gauge the importance of these channels, I construct the demand equivalence approximation in a multi-good model in which: goods differ in their labor intensity; real relative prices fluctuate in response to (sectoral) shocks; and government expenditure is concentrated on the relatively more labor-intensive good. The cyan line in Figure 6 shows that these model extensions further reinforce the (still-present) wealth effect baseline error, with the positive bias now even more pronounced. The logic is as follows. First, the real relative price of the private consumption bundle naturally increases by more after stimulus checks than after an increase in fiscal purchases. Demand equivalence thus misses one channel of general equilibrium crowding-out. Second, since in my model MPCs out of labor income exceed those out of capital income, fiscal purchases have larger general equilibrium multipliers.

While positive throughout, the error is again moderate, at around double of the baseline model. First, even with prices adjusting every 2.5 quarters on average, transitory spending shocks induce only small relative price fluctuations, so the price channel is almost completely irrelevant. Second, and consistent with both Alonso (2017) and Baqaee (2015), I find that plausible differences in MPCs and factor incidence are not enough to yield sizable differences in multipliers. In the data, the average consumption good has a labor share of around of 0.4, while the network-adjusted labor share of government consumption is around 0.65. Even assuming an average quarterly MPC out of labor income of around 0.4, and an MPC out of any residual income of 0.1, the resulting second-round demand difference from spending on the two goods would only be around 7.5 cents for every dollar of spending.\footnote{Arguably, this is an upper bound for the likely size of the effect, since heterogeneity in MPCs by skill implies the opposite conclusion: Government expenditure is concentrated on relatively high-skilled labor (Baqaee, 2015); if MPCs out of skilled labor are smaller, then the gap displayed in Figure 6 shrinks.}

**PRODUCTIVE GOVERNMENT PURCHASES.** As a second violation of the common-good assumption, I extend my baseline HANK model to allow for productive benefits of government spending, with the stock of government “capital” $k_t^g \equiv (1 - \delta)k_{t-1}^g + g_t$ directly entering the production function of firms. I calibrate the model to match empirical evidence on public investment multipliers (Leduc & Wilson, 2013; Gechert, 2015).

The orange line in Figure 6 reveals that productive benefits of government purchases can quite materially undermine the quality of the demand equivalence approximation. The approximation error is positive throughout, reflecting the fact that government purchases increase the economy’s productive capacity and so crowd-in consumption—an amplification...
channel missing for transfer stimulus. In fact, since the productive benefits are long-lived, the error remains quite persistently elevated even for transitory shocks.

**Open Economy.** As a third violation of the common-good assumption, I consider an open-economy version of my HANK model. In this environment, private consumption purchases in response to stimulus checks partially leak abroad, while government purchases are assumed to fall exclusively on domestic goods. By equating government purchases $\hat{g}$ and private expenditure on the domestic good $\hat{c}^{H,PE}$, the demand equivalence approach can still ensure identical net excess demand for the domestic good; however, because of demand leakage, the government purchases $\hat{g}$ induce a strictly smaller deficit (in net present value terms) than the stimulus check policy $\varepsilon$, thus breaking demand equivalence.

The pink line in Figure 6 shows the approximation error for an open economy with a home bias of 0.89 (matching the U.S.). As expected, openness increases the approximation error relative to the baseline economy: because of the lack of demand leakage, government spending is cheaper than the equivalent stimulus check, so taxes rise by less, leading to less general equilibrium crowding-out. However, given the substantial degree of home bias, it is not surprising that the error remains close to the baseline model throughout.

**Interest Rates.** The third key assumption required for exact demand equivalence is that of identical borrowing and lending rates for household and government (Assumption 2). This assumption is necessarily violated in models with multiple savings vehicles, such as the two-asset model of Kaplan & Violante (2014) and Kaplan et al. (2018). Here, and in contrast to the other sources of bias, it is not clear ex-ante in which direction a violation of this assumption will bias the approximation: if household returns are high relative to government returns (e.g., due to credit card debt or savings in equity), then taxes need to increase by less to finance private relative to public spending, and so the demand equivalence approximation is biased downward. Conversely, if returns are low (e.g., due to bank intermediation), then the bias is positive.

To get a sense of the likely magnitude of the implied approximation error, the green line in Figure 6 shows results for a two-asset model in which households pay an intermediation fee on liquid deposits, giving a positive bias and thus reinforcing the always-present labor supply error. I find that, even for an (implausible) quarterly return gap of 1.25 per cent, the error remains moderate, peaking at around 7 per cent of the true impact consumption response. To see why, suppose that, in response to a transfer stimulus, direct (partial equilibrium) household spending increases by $1 for one year. My approximation compares the aggregate
effects of this shock to those of an identical expansion in public spending. Even if annual household and government discount rates differ by $4 \times 1.25 = 5$ per cent, the difference in present discounted values of the two spending expansions is just five cents—small compared to the initial size of the stimulus. The implied difference in tax financing is thus also small, and so the approximation remains quite accurate. Thus, even for large return gaps (in either direction), the bias is comparable in magnitude to the (small) labor supply-related bias.

### 4.3 Summary and recommendations for practice

The results in Sections 4.1 and 4.2 shed light on the appeals and limitations of the proposed demand equivalence approach. Key necessary conditions for its accuracy include: transitory and relatively small shocks, ensuring that wealth effects in labor supply as well as real relative price movements are indeed negligible; a fiscal time series experiment that does not pick up productive government investment; and a relatively closed economy, or more generally a private spending shock that verifiably fell largely onto domestically produced goods. Interest rate effects or sectoral heterogeneity in spending multipliers on the other hand appear somewhat less likely to materially threaten the accuracy of the approximation. Thus, if these necessary conditions are satisfied, then the output of the demand equivalence approximation can be interpreted as a quite tight upper bound to the actual general equilibrium response of private spending to the private demand shifter.

To summarize, while substantial care is necessary in applying the demand equivalence approach, I have also argued that it can be highly informative under the right circumstances. In particular, stimulus checks—the main application of Section 3, and a topic of much policy interest—appear well-suited: the stimulus is relatively short-lived, and wealth effects are known to be quite weak; the U.S. economy is relatively closed; and I made sure to use fiscal spending experiments that do not capture (productive) government investment.

Finally, I emphasize that my conclusions here are also informative for researchers who wish to use structural models to solve the “missing intercept” aggregation problem. To the extent that a structural modeler finds a missing intercept path far from zero, we know from the results in this paper that this finding must be tied either to fiscal multipliers far from one—if the model is close to standard business-cycle modeling practice—or to departures from demand equivalence, with Section 4.2 providing a list of the most important ones. I hope that these insights will prove useful in relating and interpreting the results from aggregation exercises in various existing studies (e.g. Kaplan & Violante, 2018; Auclert & Rognlie, 2018; Auclert et al., 2019).
5 Extensions

My theoretical and empirical analysis so far has been largely restricted to the stimulus check application. This section discusses the wider scope of the demand equivalence approach. First, in Section 5.1, I give examples of other shocks and policies covered by the consumption equivalence result. Second, in Section 5.2, I discuss the conditions required for an analogous investment demand equivalence result, and apply it to study another well-known fiscal stimulus policy: investment bonus depreciation.

5.1 Other consumer spending shocks

As argued in Section 2.3, the consumption demand equivalence result—and so the measurement strategy of Section 3—applies to any shifter of private household spending, not just uniform stimulus checks. In this section I discuss two examples. First, as another instance of a policy-induced shifter, I consider stimulus checks targeted at certain sub-populations, consistent with recent U.S. policy design. Second, I study the effects of a short-lived increase in income inequality as an example of a non-policy shifter of household spending.

**Targeted Transfers.** Consider a one-off stimulus check policy \( \varepsilon_t \) targeted at some sub-population \( T \subseteq [0, 1] \) of households. Proceeding analogously to the discussion in Section 3.3, we get the direct consumption response as

\[
\hat{c}^{PE}_{tT} \equiv \left[ \dfrac{\# \text{ of recipients}}{|T|} \right] \times \dfrac{MPC_{t0}}{\text{MPC of recipients}} \times \dfrac{\varepsilon_t}{\text{check size}}
\]

where \( |T| \equiv \int_{i \in T} di \) and

\[
MPC_{t0}^T = \dfrac{1}{|T|} \int_{i \in T} \dfrac{\partial c_{it}}{\partial \tau_{0}} di
\]

The direct response is thus estimable using information on household MPCs in the targeted sub-group, and so general equilibrium counterfactuals can be estimated as in Section 3.3. The demand equivalence approach can thus be used to gauge the effects of stimulus check policies even if the desired targeting has no historical precedent—we only need the corresponding net excess demand path to have been studied before in a time series experiment.\(^{25}\)

\(^{25}\)For the inverse step from fiscal multiplier estimates to equivalent targeted stimulus, the researcher would need to use the response matrix \( C^T_t \) for the targeted sub-population.
Income inequality. Examples of much-studied non-policy shifters of private consumer demand include a tightening of financial constraints (e.g. Guerrieri & Lorenzoni, 2017), changes in income inequality (e.g. Auclert & Rognlie, 2018) and changing tastes, e.g. due to infection risk associated with the consumption of certain goods (Beraja & Wolf, 2020). Such shocks are covered by the more general decomposition in (8): they induce some zero net present value perturbation \( \hat{c}_{d}^{PE} \) of net excess consumption demand, and so full general equilibrium counterfactuals can be constructed with knowledge of \( \hat{c}_{g} \) for a purely deficit-financed fiscal spending experiment with \( \hat{g}_{g} = \hat{c}_{d}^{PE} \)—i.e., a change in spending today that is financed with a spending reversal in the future.

I apply the demand equivalence approach to temporary increase in (labor) income inequality in Appendix F.2. The analysis proceeds in two steps, leveraging my second approach to demand matching discussed in Section 3.2. First, I construct a linear combination of fiscal spending experiments that has zero net present value and so can be entirely deficit-financed; that is, I consider a contraction of government spending today offset through an equivalent increase in the future. Second, I use a standard partial equilibrium consumption-savings problem to show that a short-lived increase in labor income inequality would engineer a very similar time path of aggregate net excess demand. My analysis thus naturally complements that in Auclert & Rognlie (2018): I use a partial equilibrium model to predict the direct effect on demand (like they do), but then I leverage demand equivalence together with time series fiscal policy shock evidence to solve the missing intercept problem, rather than relying on a full structural model. Overall my findings echo those of the stimulus check application: general equilibrium effects are relatively weak, so the total response of consumption is close to the shock's direct effect.

5.2 Investment

The demand equivalence logic can also be leveraged to estimate general equilibrium counterfactuals for shifters of investment demand. In this section I first sketch the conditions required for investment demand equivalence and then discuss an application to bonus depreciation stimulus. Details for theory and application are provided in Appendices A.2 and F.1.

Theory: investment demand equivalence. I again use the model of Section 2.1. Anticipating the empirical application, I augment the model to feature investment tax credit shocks \( \varepsilon_{q} \)—shocks that reduce the cost of capital purchases by intermediate goods producers.
at time $t$ by an amount $\tau qt = \tau q(\varepsilon q)$.\footnote{More generally, my results can be interpreted as applying to any kind of shock that appears as a reduced-form wedge in firm investment optimality conditions (Chari et al., 2007).} I define direct (partial equilibrium) responses and indirect (general equilibrium) feedback for firm investment exactly analogously to Definition 1, using the implied aggregate investment function $i(\bullet)$. As before, the question is: under what restrictions on primitives does the response of investment to a suitably chosen fiscal spending experiment give the “missing intercept” $\tilde{i}^{GE}q$?

The proof strategy is identical to that in Section 2: I characterize equilibrium response paths as a system of market-clearing conditions, and then impose enough restrictions on this system to ensure that the investment tax credit as well as a suitable fiscal experiment perturb the same equations by the same amount. In my model, the investment tax credit policy has three direct effects. First, investment responds; since investment invariably boosts the future productive capacity of the economy, production also increases, so the induced partial equilibrium net excess demand path for the final output and investment good is $\tilde{i}^{PE}q - \tilde{y}^{PE}q$. Second, the policy may be redistributive: the cost of financing is borne by taxed households, but the benefits accrue to households receiving dividend payments. The two groups need not be the same. Third, more investment and so more capital will increase the marginal product of labor, pushing up firm labor demand.\footnote{Strictly speaking, the output response $\tilde{y}^{PE}q$ may also appear as a wedge in the monetary policy rule. The restrictions required to rule out this policy effect are relatively standard and so not further discussed here; details are provided in Appendix A.2.}

Matching the first effect is straightforward: we simply need to consider a fiscal spending expansion with

$$\tilde{g}q = \tilde{i}^{PE}q - \tilde{y}^{PE}q$$

For the other two effects I require additional exclusion restrictions. To rule out heterogeneous distributional implications of tax financing and dividend payments, I assume that household income risk is perfectly insurable, thus effectively imposing a representative-household structure. This restriction also implies that Ricardian equivalence holds, so the precise timing of the policy financing is irrelevant. Next, to ignore the labor demand response, I assume that labor supply is perfectly flexible, either because of a large Frisch elasticity of labor supply, or again because labor is fully demand-determined. Under those two additional restrictions on primitives, a fiscal experiment satisfying (18) indeed gives the “missing intercept” of the investment response, in the sense that

$$\tilde{i}q = \tilde{i}^{PE}q + \tilde{i}y$$

(19)
Analogously we can also recover the output response as

\[ \hat{y}_q = \hat{y}^{PE}_q + \hat{y}_g \]  

(20)

I formally state the equivalence result and its assumptions in Appendix A.2. Crucially, the proposition requires no additional restrictions on the production side of the economy: firms can face a rich set of real and financial frictions, including (convex and non-convex) capital adjustment costs as well as a generic set of constraints on equity issuance and borrowing. A detailed discussion of nested (heterogeneous-firm) models is provided in Appendix C.5.

Application: bonus depreciation. I apply the investment demand equivalence result to estimate general equilibrium counterfactuals for bonus depreciation stimulus—that is, the ability to tax-deduct investment expenditure at a faster rate.\(^{28}\) I focus on bonus depreciation for three reasons. First, it is popular: with conventional monetary policy constrained by an effective lower bound, bonus depreciation has arguably become the go-to tool for investment stimulus. Second, previous empirical work has leveraged heterogeneity in firm exposure to the stimulus to estimate its direct effect on investment, \(\hat{i}^{PE}_q\) (Zwick & Mahon, 2017; Koby & Wolf, 2020)—the key empirical input needed for my approach. Third, given the endogeneity of bonus depreciation policies to wider macroeconomic conditions, I am not aware of any studies that credibly identify the aggregate causal effects of such policies.

I only provide a brief summary of the results here, with details provided in Appendix F.1. Overall, my findings closely echo those of Section 3.3. Micro data indicate a large response of investment: following a one-quarter bonus depreciation shock worth around 8 cents (a shock similar in magnitude to the stimulus of 2008-2010), investment demand increases by around 16 per cent on impact, and then remains elevated. Combining the investment demand equivalence result in (19) with evidence on fiscal multipliers, I conclude that the increase in investment demand is accommodated through a sharp immediate increase in output as well as a smaller and somewhat delayed drop in consumption.

6 Conclusion

How can researchers learn about the “missing intercept” of cross-sectionally identified shifters of private spending? In this paper I ask whether—consistent with a simple Keynesian cross

\(^{28}\)In the absence of firm-level financial frictions, such accelerated bonus depreciation schedules are isomorphic to the investment tax credits covered by the investment equivalence result (see Winberry, 2018).
intuition—time series estimates of aggregate fiscal expenditure shocks can solve this aggregation problem. I give a set of restrictions on economic primitives under which aggregation via such “demand equivalence” is indeed possible, show how to operationalize this result, and discuss its empirical plausibility. In an application to deficit-financed stimulus checks, I find that cross-sectionally identified spending estimates are likely to be close to full general equilibrium counterfactuals, corresponding to a missing intercept close to zero.

I leave several avenues for future research. First, to be widely applicable, the demand equivalence approach requires time series estimates for a wide menu of fiscal spending experiments with different persistence and financing. More research on fiscal multipliers is thus welcome: it promises to not only tell us narrowly about those fiscal multipliers, but also about the propagation of many other shocks and policies. By the same token, running more cross-sectional regressions will help researchers construct the inverse mapping from demand path to primitive private spending shock. Second, other interesting macro shocks covered by the demand equivalence approach include firm uncertainty (Bloom, 2009; Bloom et al., 2018), shocks to firm credit conditions (Khan & Thomas, 2013) and household debt relief (Auclert et al., 2019). The methodology developed here could be applied to all of them. Third, I emphasize that the general conceptual idea of this paper—to leverage equivalence in the general equilibrium propagation of different shocks and policies—is not necessarily limited to fiscal spending and demand amplification, and so may help to solve the missing intercept problem in other contexts as well.
A Appendix

A.1 Proof of consumption demand equivalence

I begin by writing the equilibrium of the full baseline model as a dynamic system of market-clearing conditions.

Lemma A.1. Consider the structural model of Section 2.1. Under Assumption 1, a perfect foresight equilibrium is a sequence of nominal interest rates $i_b$, aggregate output $y$, wages $w$ and the endogenous part of tax rebates $\tau^e$ such that

$$
\begin{align*}
    c(s^h(x); \varepsilon) + i(s^f(x); \varepsilon) + g(\varepsilon) &= y(s^f(x); \varepsilon) \\
    \ell^h(s^u(x; \varepsilon)) &= \ell^f(s^f(x; \varepsilon)) \\
    y(s^f(x; \varepsilon)) &= y \\
    \tau^e(s^f(x; \varepsilon)) &= \tau^e \\
\end{align*}
$$

where $x = (i^b, y, w, \tau^e)$, $s^h = (i^b, \pi, w, \ell^h, d)$, $s^u = (\pi, w, c)$, $s^f = (i^b, w, \pi)$, and the consumption, production, investment, labor demand and labor supply functions $c(\bullet)$, $y(\bullet)$, $i(\bullet)$, $\ell^h(\bullet)$ and $\ell^f(\bullet)$ are derived from optimal firm, household and union behavior, and $\tau^e(\bullet)$ is the fiscal rule.

Proof. See Appendix G of the Online Appendix. \hfill \Box

A perfect foresight equilibrium is thus, to first order, a solution to the system of linear equations

$$
\begin{align*}
    \left( \frac{\partial c}{\partial x} \times \hat{x} + \frac{\partial c}{\partial \varepsilon} \times \varepsilon \right) + \left( \frac{\partial i}{\partial x} \times \hat{x} + \frac{\partial i}{\partial \varepsilon} \times \varepsilon \right) + \frac{\partial g}{\partial \varepsilon} \times \varepsilon &= \left( \frac{\partial y}{\partial x} \times \hat{x} + \frac{\partial y}{\partial \varepsilon} \times \varepsilon \right) \\
    \left( \frac{\partial \ell^h}{\partial x} \times \hat{x} + \frac{\partial \ell^h}{\partial \varepsilon} \times \varepsilon \right) &= \left( \frac{\partial \ell^f}{\partial x} \times \hat{x} + \frac{\partial \ell^f}{\partial \varepsilon} \times \varepsilon \right) \\
    \left( \frac{\partial y}{\partial x} \times \hat{x} + \frac{\partial y}{\partial \varepsilon} \times \varepsilon \right) &= J_2 \times \hat{x} \\
    \left( \frac{\partial \tau^e}{\partial x} \times \hat{x} + \frac{\partial \tau^e}{\partial \varepsilon} \times \varepsilon \right) &= J_4 \times \hat{x}
\end{align*}
$$

where $J_i$ denotes the infinite-dimensional generalization of the selection matrix selecting the $i$th entry of a vector $x_t$. Assuming equilibrium existence and uniqueness, there exists a unique linear map $H$ such that

---

29Existence and uniqueness of a bounded transition path for representative-agent models can be shown as usual. For the heterogeneous-agent models, I have verified existence and uniqueness for particular numerical examples, using the conditions of Blanchard & Kahn (1980).
\[ \hat{x} = H(\text{GE adjustment}) \times \left( \begin{array}{c}
\frac{\partial c}{\partial \varepsilon} + \frac{\partial h}{\partial \varepsilon} + \frac{\partial g}{\partial \varepsilon} - \frac{\partial y}{\partial \varepsilon} \\
\frac{\partial c}{\partial \varepsilon} + \frac{\partial h}{\partial \varepsilon} - \frac{\partial \nu}{\partial \varepsilon} \\
\frac{\partial y}{\partial \varepsilon} - \frac{\partial \nu}{\partial \varepsilon} \\
\frac{\partial \nu}{\partial \varepsilon} - \frac{\partial \nu}{\partial \varepsilon} \\
J_2 - \frac{\partial y}{\partial \varepsilon} \\
J_4 - \frac{\partial e}{\partial \varepsilon}
\end{array} \right) \times \varepsilon \]

where \( H \) is a left inverse of

\[ \begin{pmatrix}
\frac{\partial y}{\partial \varepsilon} - \frac{\partial c}{\partial \varepsilon} - \frac{\partial h}{\partial \varepsilon} \\
\frac{\partial c}{\partial \varepsilon} + \frac{\partial h}{\partial \varepsilon} - \frac{\partial \nu}{\partial \varepsilon} \\
\frac{\partial y}{\partial \varepsilon} - \frac{\partial \nu}{\partial \varepsilon} \\
\frac{\partial \nu}{\partial \varepsilon} - \frac{\partial \nu}{\partial \varepsilon} \\
J_2 - \frac{\partial y}{\partial \varepsilon} \\
J_4 - \frac{\partial e}{\partial \varepsilon}
\end{pmatrix} \]

The assumed existence and uniqueness of the equilibrium ensures that this left inverse is in fact unique. Now consider stimulus check and government spending shocks. To reduce unnecessary clutter, I use the notation \( \frac{\partial}{\partial \varepsilon} \) (rather than the generic \( \frac{\partial}{\partial \varepsilon} \)) to denote derivatives for a shock path where only entries of shock \( s \) are non-zero. By definition of the firm policy functions (see Appendix B.1), we know that \( \frac{\partial h}{\partial \varepsilon} = \frac{\partial y}{\partial \varepsilon} = \frac{\partial \nu}{\partial \varepsilon} = 0 \), and similarly that \( \frac{\partial h}{\partial e_g} = \frac{\partial y}{\partial e_g} = \frac{\partial \nu}{\partial e_g} = 0 \). We also know that \( \frac{\partial h}{\partial e_g} = 0 \), and by Assumption 3 \( \frac{\partial h}{\partial e_g} = 0 \). The two direct shock responses are then

\[ \begin{pmatrix}
\frac{\partial c}{\partial \varepsilon} \\
0 \\
0 \\
0 \\
0 \\
0 \\
0
\end{pmatrix} \times \varepsilon_r = \begin{pmatrix}
0 \\
0 \\
\tilde{c}_{e,PE} \\
0 \\
0 \\
0 \\
0
\end{pmatrix}, \quad \text{and} \quad \frac{\partial c}{\partial \varepsilon} \times \varepsilon_g = \begin{pmatrix}
0 \\
0 \\
\tilde{c}_{g,PE} \\
0 \\
0 \\
0 \\
0
\end{pmatrix} \times \varepsilon_g
\]

The impulse response paths of consumption thus satisfy

\[ \hat{c}_r = \frac{\partial c}{\partial \varepsilon} \times \varepsilon_r + \frac{\partial c}{\partial \varepsilon} \times H \times \begin{pmatrix}
\hat{c}_{e,PE} \\
0 \\
0 \\
\tilde{c}_{e,PE}
\end{pmatrix}, \quad \text{and} \quad \hat{c}_g = 0 + \frac{\partial c}{\partial \varepsilon} \times H \times \begin{pmatrix}
\hat{g}_g \\
0 \\
0 \\
\tilde{g}_{g,PE}
\end{pmatrix} \]

respectively. By Assumption 2 we know that, if \( \hat{c}_{e,PE} = \hat{g}_g \), then setting \( \tilde{c}_{g,PE} = \tilde{g}_{g,PE} \) is consistent with \( \lim_{t \to \infty} \hat{b}_t = 0 \), since \( \hat{e}_r \) and \( \hat{g}_g \) by construction have the same net present value. This establishes that, if \( \hat{c}_{e,PE} = \hat{g}_g \) and \( \tilde{c}_{g,PE} = \tilde{g}_{g,PE} \), then \( \hat{c}_{GE} = \hat{c}_g \). (6) then follows. Finally note that, given the assumed fiscal financing rule \( \tau^f(\bullet) \), \( \hat{c}_{e,PE} = \hat{c}_{e,PE} \) also implies \( \tilde{c}_g = \hat{c}_g \)—the stated property (ii) of the fiscal spending shock.

\[ \sum_{t=0}^{\infty} \left( \frac{1}{1 + \tau^f} \right)^t \hat{v}_{r,t} = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \tau^f} \right)^t \tilde{v}_{r,t} \]

Combining the two: \[ \sum_{t=0}^{\infty} \left( \frac{1}{1 + \tau^f} \right)^t \hat{g}_{g,t} = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \tau^f} \right)^t \tilde{g}_{g,t} \]
A.2 Details on investment demand equivalence

I begin with the restrictions needed for an exact investment demand equivalence result. The first assumption is again that of a single common final good.

**Assumption A.1.** *A single final good is used for investment and (government) consumption.*

In imposing this first restriction, I implicitly assume that all meaningful capital adjustment costs are internal to the firm, and that the aggregate supply of capital (out of the common final good) is perfectly elastic. This assumption is consistent with the empirical findings in House & Shapiro (2008), Edgerton (2010) and House et al. (2017).

The second assumption rules out any heterogeneous distributional implications associated with dividend and tax payments following the firm subsidy and the equivalent fiscal spending change.

**Assumption A.2.** *All households* $i \in [0,1]$ *have identical preferences, receive equal lump-sum government rebates* $\tau_t$ *and firm dividend income* $d_t$, *and face no idiosyncratic earnings risk.*

The third assumption allows me to ignore the labor demand response.

**Assumption A.3.** *Labor supply is perfectly elastic, either because the Frisch elasticity of labor supply is infinite (linear labor disutility), or because wages are perfectly sticky. Furthermore, the period household felicity function is separable in consumption and hours worked.*

Finally, I require an additional restriction on monetary policy feedback. If the monetary authority directly responds to the level of aggregate output, then the increase in production associated with the investment subsidy will induce a contractionary monetary response. I rule this out by assuming that the monetary authority targets the output gap (as for example in Justiniano et al., 2010), or does not respond at all to fluctuations in output.

**Assumption A.4.** *The monetary authority’s interest rate rule does not include an endogenous response to fluctuations in the level of aggregate output.*

Under Assumptions A.1 to A.4 I can prove the following demand equivalence result.

**Proposition A.1.** *Consider an investment stimulus policy* $\varepsilon_q$, *and suppose that Assumptions A.1 to A.4 hold. Then, for a fiscal spending policy* $\varepsilon_g$ *such that* $\hat{g}_g = \hat{\gamma}_q^{PE} - \hat{\gamma}_q^{PE}$, *we have that, to first order,*

$$\hat{i}_q = \hat{\gamma}_q^{PE} + \hat{i}_g$$

**Proof.** See Appendix G of the Online Appendix.

It is immediate from the proof of Proposition A.1 that all results extend to generic investment “wedges” (Chari et al., 2007).
References


Online Appendix for:
The Missing Intercept:
A Demand Equivalence Approach

This online appendix contains supplemental material for the article “The Missing Intercept: A Demand Equivalence Approach”. I provide (i) further details for the various structural models used in the paper, (ii) several additional results on exact demand equivalence, (iii) details on the cross-sectional and time series experiments used to construct my semi-structural counterfactuals, (iv) results on approximation accuracy in models where demand equivalence fails, and (v) two further applications of my empirical methodology. The end of this appendix contains further proofs and auxiliary lemmas.

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded “B.”—“G.” refer to the main article.
## Contents

**B** Model details
B.1 Rest of the baseline economy and equilibrium definition .................................. 54  
B.2 Estimated HANK model ..................................................................................... 59  
B.3 Model extensions of Section 4.2 .......................................................................... 64  

**C** Further results on demand equivalence
C.1 General transition paths ....................................................................................... 71  
C.2 Generic consumption demand shifters .................................................................. 71  
C.3 Exact equivalence beyond the baseline model ...................................................... 72  
C.4 Range of outcomes for the “missing intercept” ...................................................... 75  
C.5 Nested models for investment demand equivalence .............................................. 76  

**D** Empirical appendix
D.1 Cross-sectional consumption elasticities ............................................................... 77  
D.2 Cross-sectional investment elasticities .................................................................... 81  
D.3 Time series fiscal policy estimates ....................................................................... 81  

**E** Approximation accuracy
E.1 Random parameter draws .................................................................................... 87  
E.2 Other estimated business-cycle models ................................................................. 88  
E.3 Labor supply ......................................................................................................... 90  
E.4 Multi-sector economy ........................................................................................... 91  
E.5 Useful government spending .................................................................................. 91  
E.6 Open Economy ...................................................................................................... 92  
E.7 Interest rates ......................................................................................................... 93  
E.8 Imperfect demand matching .................................................................................. 93  

**F** Further applications
F.1 Bonus depreciation ............................................................................................... 95  
F.2 Income redistribution ........................................................................................... 98  

**G** Further proofs and auxiliary lemmas
G.1 Proof of Lemma A.1 ............................................................................................. 101
B Model details

This appendix provides additional details on the structural models considered in the main body of the paper. In Appendix B.1 I begin by outlining the full baseline model and offering a formal definition of equilibrium transition paths. Appendix B.2 discusses estimation of the baseline HANK model for Section 4.1. Finally, in Appendix B.3, I give modeling details for the various extensions considered in Section 4.2.

B.1 Rest of the baseline economy and equilibrium definition

Recall that the model is populated by households, firms, and the government. Whenever there is no risk of confusion, I replace the full decision problems of agents by simple conditions characterizing their optimal behavior.\(^{31}\) Since for much of the paper I impose the one-good restriction of Assumption 1, I here present the equilibrium for this baseline case, and relegate a discussion of the notationally involved multi-good extension to Appendix B.3.1.

Households & unions. The household consumption-savings problem was described in Section 2.1. Since I for now consider a simpler one-good economy, we have that \(p^c_t = 1\) \(\forall t\). For all simulations I specialize household preferences to be of a standard separable form:

\[
u(c, \ell) = c^{1-\gamma - 1} - \gamma \ell^{\frac{1+\phi}{1+\psi}} \tag{B.1}\]

It remains to specify the problem of a wage-setting union \(k\). A union sets wages and labor to maximize weighted average utility of its members, taking as given optimal consumption-savings behavior of each individual member household, exactly as in Auclert et al. (2018). Following the same steps as those authors, it can be shown that optimal union behavior is summarized by a standard non-linear wage-NKPC:

\[
\pi^w_t (1 + \pi^w_t) = \frac{\epsilon_w}{\theta_w} \ell^h_t \left[ \int_0^1 \left\{ -u_t(c_{it}, \ell^h_t) - \frac{\epsilon_w - 1}{\epsilon_w} (1 - \tau_t) w_t e_{it} u_t(c_{it}, \ell^h_t) \right\} dt \right] \\
+ \beta \pi^w_{t+1} (1 + \pi^w_{t+1}) \tag{B.2}
\]

where \(1 + \pi^w_t = \frac{w_t}{w_{t-1}} \times \frac{1}{1+\pi_t}\), \(\epsilon_w\) is the elasticity of substitution between different kinds of labor,

\(^{31}\)I do so because many of the problems considered here (in particular those of price-setting entities) are notationally involved, but at the same time extremely well-known and so require no repetition.
and \( \theta_w \) denotes the Rotemberg adjustment cost. Given prices \((\pi, w)\) as well as a consumption path \(c\), (B.2) provides a simple restriction on total labor supply \(\ell^h\).\(^{32}\) Note that, without idiosyncratic labor productivity risk and so common consumption \(c_{it} = c_t\), the derived wage-NKPC (B.2) is to first order identical to the standard specification in Erceg et al. (2000). An extension to partially indexed wages, as in Smets & Wouters (2007) or Justiniano et al. (2010), is straightforward and omitted in the interest of notational simplicity. Note that, in the special case of preferences as in (B.1), (B.2) simplifies to

\[
\pi_t(1 + \pi_t^w) = \frac{\epsilon_w}{\theta_w} \ell^h \left[ \chi(\ell^h) \frac{1}{\epsilon} - \frac{\epsilon_w - 1}{\epsilon_w} (1 - \tau_t) w_t \int_0^1 e_{it} c_{it}^{-\gamma} di \right] + \beta \pi_{t+1}^w (1 + \pi_{t+1}^w)
\]

and so, in log deviations,

\[
\hat{\pi}_t^w = \kappa_w \times \left[ \frac{\zeta_h}{\varphi} \ell_t - (\bar{w}_t - \gamma \hat{c}_t^*) \right] + \beta \hat{\pi}_{t+1}^w
\]

where \(\kappa_w\) is a function of model parameters and \(c^*_t\) satisfies

\[
c^*_t \equiv \left[ \int_0^1 e_{it} c_{it}^{-\gamma} di \right]^{-\frac{1}{\gamma}}
\]

Together, the consumption-savings problem and the general wage-NKPC (B.2) characterize optimal household and union behavior. I assume that the solutions to each problem exist and are unique, and summarize the solution in terms of aggregate consumption, saving and union labor supply functions \(c^h(s^h; \xi), b^h(s^h; \xi), \text{ and } \ell^h(s^u)\), where \(s^h = (i^h, \pi, w, \ell, \tau^e, d)\) and \(s^u = (\pi, w, c)\).\(^ {33}\) In particular, the union problem gives

\[
\ell^{PE} \equiv \ell^h(\bar{\pi}, \bar{w}, c(s^h; \xi)) - \bar{\ell}^h
\]

To state and prove the equivalence results, I will impose the high-level assumption that all of those infinite-dimensional functions are at least once differentiable in their arguments.

\(^{32}\)In the special case \(\theta_w \rightarrow \infty\), equation (B.2) is vacuous, so then I instead simply assume that \(\ell^h = \ell^l\).

\(^{33}\)Formally, the input to the union problem is the “virtual” consumption aggregate in (B.5) (see Auclert et al. (2018) for a further discussion). In a slight abuse of notation, the dependence on \(c\) in the equations here is a shorthand for dependence on overall household consumption decisions given \((s^h; \xi)\).
Firms. I first study the problem of each of the three types of firms in isolation. I assume that all firms discount at the common rate $1 + r_t^b \equiv \frac{1 + \kappa_{t-1}^b}{1 + \pi_t}$.

1. Intermediate goods producers. The problem of intermediate goods producer $j$ is to

$$\max_{\{d_{jt}, y_{jt}, \ell_{jt}, k_{jt}, i_{jt}, u_{jt}, b_{jt}\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \left( \prod_{q=0}^{t-1} \frac{1}{1 + i_q^b} \right) d_{jt}^I \right]$$

such that

$$d_{jt}^I = p^I_t y_{jt} - w_t \ell_{jt} - \phi(k_{jt}, k_{jt-1}, i_{jt}, i_{jt-1}) - b_{jt}^f + \frac{1 + i_{t-1}^b b_{jt-1}^f}{1 + \pi_t}$$

$$y_{jt} = y(e_{jt}, u_{jt} k_{jt-1}, \ell_{jt})$$

$$i_{jt} = k_{jt} - [1 - \delta(u_{jt})] k_{jt-1}$$

$$-b_{jt}^f \leq \Gamma(k_{jt-1}, k_{jt}, \pi_{jt})$$

$$d_{jt}^I \geq d$$

The physical adjustment cost function $\phi(\bullet)$ is general: it may be convex and continuously differentiable, but it may also feature a fixed-cost component or partial irreversibility. Firms can vary capital utilization, with higher utilization leading to faster depreciation, i.e. $\delta'(\bullet) > 0$. The solution to the firm problem gives optimal production $y(\bullet)$, labor demand $\ell^f(\bullet)$, investment $i(\bullet)$, intermediate goods producer dividends $d^I(\bullet)$, capital utilization rates $u(\bullet)$ and liquid corporate bond savings $b^f(\bullet)$ as a function of nominal returns $i^b$, inflation $\pi$, wages $w$, and the intermediate goods price $p^I$.

2. Retailers. A unit continuum of retailers purchases the intermediate good at price $p^I_t$, costlessly differentiates it, and sells it on to a final goods aggregator. Price setting is subject to a Rotemberg adjustment cost. As usual, optimal retailer behavior gives rise to a standard NKPC as a joint restriction on the paths of inflation and the intermediate goods price. In log-linearized form:

$$\tilde{\pi}_t = \frac{\epsilon_p \epsilon_{p_t} - 1}{\theta_p \epsilon_p} \times \tilde{p}_t + \beta \tilde{\pi}_{t+1}$$

34Along a perfect foresight transition path, discounting at $1 + r_t^b$ is equivalent to discounting at the (common) stochastic discount factor of all households with strictly positive asset holdings.
where $\epsilon_p$ denotes the substitutability between different kinds of retail goods, and $\theta_p$ denotes the Rotemberg adjustment cost. In an equivalent (to first-order) Calvo formulation, the slope of the NKPC instead is given as

$$\kappa_p = \frac{(1 - \frac{1}{1+\bar{r}}\phi_p)(1 - \phi_p)}{\phi_p}$$

where $1 - \phi_p$ is the probability of a price re-set. A further extension to partially indexed prices, as in Smets & Wouters (2007) or Justiniano et al. (2010), is straightforward and omitted in the interest of notational simplicity. Total dividend payments of retailers are

$$d_t^R = (1 - p_t^I)y_t$$

3. **Aggregators.** Aggregators purchase retail goods and aggregate them to the composite final good. They make zero profits.

Total dividend payments by the corporate sector are given as

$$d_t = d_t^f + d_t^R$$

Using the restriction on the intermediate goods price implied by optimal retailer behavior, aggregate dividends can thus be obtained solely as a function of $s^f = (i^b, w, \pi)$. As before, I will assume that these aggregate firm block-level functions are at least once differentiable in their arguments.

**Government.** I denote the fiscal financing rule by $\tau^e = \tau^e(w, \ell, i^b, \pi, p^g, \tau^x, g)$. This rule must imply that, with debt evolving in accordance with the government budget constraint (2), we have $\lim_{t \to \infty} \hat{b}_t = 0$. This in particular implies that the path $\tau^e$ is such that the following log-linearized lifetime government budget constraint holds:

$$\sum_{t=0}^{\infty} \left( \frac{1}{1 + \bar{r}^b} \right)^t \bar{b} \left( i_{t-1}^b - \hat{\tau}_t \right) + \sum_{t=0}^{\infty} \left( \frac{1}{1 + \bar{r}^b} \right)^t \bar{p}^g \bar{g} \left( \hat{p}_t^g + \hat{\bar{g}}_t \right) + \sum_{t=0}^{\infty} \left( \frac{1}{1 + \bar{r}^b} \right)^t \bar{\tau} \left( \frac{\bar{\tau}^e}{\bar{\tau}^e} + \frac{\bar{\tau}^x}{\bar{\tau}^x} \right) = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \bar{r}^b} \right)^t \tau_t \bar{w} \bar{\ell} \left( \hat{w}_t + \hat{\ell}_t \right)$$

(B.6)
It remains to describe central bank behavior. In line with standard empirical practice I assume that the nominal rate on bonds $\tilde{i}^b$ is set according to the conventional Taylor rule

$$\tilde{i}^b_t = \rho_m \tilde{i}^b_{t-1} + (1 - \rho_m) \left( \phi \tilde{π}_t + \phi_y \tilde{y}_t + \phi_d \tilde{y}_{t-1} \right)$$

**Market-Clearing.** Equating liquid asset demand from households and intermediate goods producers, as well as liquid asset supply from the government, we get

$$b^h_t + b^f_t = b_t$$

Equating labor demand and supply:

$$\ell^f_t = \ell^h_t$$

Finally, aggregating all household, firm and government budget constraints, we obtain the aggregate output market-clearing condition for the single final good:\(^{35}\)

$$c_t + i_t + g_t = y_t$$

**Equilibrium.** All results in this paper rely on the following equilibrium definition.

**Definition 2.** Given initial distributions $\mu_0^h = \bar{\mu}^h$ and $\mu_0^f = \bar{\mu}^f$ of households and intermediate goods producers over their idiosyncratic state spaces, an initial real wage $w_{-1} = \bar{w}$, price level $p_{-1}$, and real government debt $b_{-1} = \bar{b}$, as well as exogenous shock paths $\{ \varepsilon_i \}_{i=0}^{\infty}$, a recursive competitive equilibrium is a sequence of aggregate quantities $\{ c_t, \ell^h_t, \ell^f_t, b^h_t, b^f_t, b_t, y_t, i_t, d_t, k_t, g_t, \tau_t \}_{t=0}^{\infty}$ and prices $\{ \pi_t, i^b_t, w_t \}_{t=0}^{\infty}$ such that:

1. **Household Optimization.** Given prices and government rebates, the paths of aggregate consumption $c = c(s^h ; \varepsilon)$, labor supply $\ell^h = \ell^h(s^u)$, and asset holdings $b^h = b^h(s^h ; \varepsilon)$ are consistent with optimal household and wage union behavior.

2. **Firm Optimization.** Given prices, the paths of aggregate production $y = y(s^f ; \varepsilon)$, investment $i = i(s^f ; \varepsilon)$, capital $k$, labor demand $\ell^f = \ell^f(s^f ; \varepsilon)$, dividends $d = d(s^f ; \varepsilon)$ and asset holdings $b^f = b^f(s^f ; \varepsilon)$ are consistent with optimal firm behavior. Furthermore, the path of inflation is consistent with optimal retailer behavior.

---

\(^{35}\)So as to not excessively clutter market-clearing conditions with various adjustment cost terms, I assume that adjustment costs are ex-post rebated lump-sum back to the agents facing the adjustment costs. Of course, all subsequent equivalence results are unaffected by this rebating. An alternative interpretation is that adjustment costs are instead just perceived utility costs, as in Auclert et al. (2018).
3. Government. The liquid nominal rate is set in accordance with the monetary authority’s Taylor rule. The government spending, lump-sum tax, and debt issuance paths are jointly consistent with the government’s budget constraint, its exogenous laws of motion for spending and discretionary transfers, and its financing rule $\tau^e(\bullet)$.

4. Market Clearing. The goods market clears,

$$c_t + i_t + g_t = y_t$$

the bond market clears,

$$b^h_t + b^f_t = b_t$$

and the labor market clears,

$$\ell^h_t = \ell^f_t$$

for all $t = 0, 1, 2, \ldots$.

B.2 Estimated HANK model

Much of my analysis builds on a particular estimated one-asset HANK model. This section provides details on the model, the solution algorithm, my approach to likelihood-based estimation, and the final parameterization used to generate the results in Section 4.

**Model outline.** The model is a particular variant of the rich baseline environment outlined in Section 2.1, consistent with Assumptions 1 and 2 but violating Assumption 3.

Households have preferences as in (B.1). To facilitate comparison with the standard New Keynesian business-cycle literature, I will throughout replace the virtual consumption aggregate (B.5) in the wage-NKPC with aggregate consumption $c_t$, thus giving me an entirely standard wage-NKPC (as in Hagedorn et al., 2019); results are, however, almost unchanged if I use $c^*_t$ instead.\(^{36}\) I furthermore slightly generalize the model of Section 2.1 to allow for stochastic death with probability $\xi$. All households receive identical lump-sum transfers (so $\tau_{it} = \tau_t \forall i$) but are heterogeneous in dividend payment receipts. In particular, I assume that the most productive households receive larger dividend payments, so that stock wealth is effectively concentrated among a small share of households.

\(^{36}\)Using $c_t$ has the advantage that union wage-setting is not affected by the distributional implications of the shock. However, since labor is largely demand-determined in the short run, even those distributional considerations have little effect on equilibrium hours worked.
The intermediate goods production block—in particular the production function $y(\cdot)$, the investment adjustment cost function $\phi(\cdot)$, and the capacity utilization depreciation rate $\delta(\cdot)$—is set up exactly as in Justiniano et al. (2010). For model estimation, I allow for structural shocks to output and investment productivity, monetary policy, government spending, household impatience, and wage mark-ups. All shocks are assumed to follow simple AR(1) processes. Finally I assume a fiscal financing rule of the form

$$\hat{\tau}_t^e = -(1 - \rho_r) \times \hat{b}_{t-1}$$

(B.7)

The endogenous part of transfers is cut in response to increases in $\hat{b}_t$. For plots of approximate equivalence results, I let transfer shocks be financed using this rule, and then assume that government spending shocks are financed using the same (potentially scaled) intertemporal tax profile, consistent with Assumption 2 (i.e., $\hat{\tau}_g^e \propto \hat{\tau}_t^e$). In particular, for all models in which Assumption 2 is imposed, the partial equilibrium financing paths of the two shocks will thus always be the same (since $\hat{g}_g = \hat{c}_r^{PE}$).

**Steady-state calibration.** Solving for the deterministic steady-state of the model requires specification of several parameters. On the household side, I need to set income risk and share endowment processes, specify preferences, and choose liquid borrowing limits as well as the substitutability between different kinds of labor. On the firm side, I need to specify production and investment technologies, as well as the substitutability between different kinds of goods. Finally, on the government side, I need to set taxes, transfers, and total bond supply. Government spending is then backed out residually. My preferred parameter values and associated calibration targets are displayed in Table B.1.

The first block shows parameter choices on the household side. For income risk, I adopt the 33-state specification of Kaplan et al. (2018), ported to discrete time. For share endowments, I assume that

$$d_{it} = \begin{cases} 0 & \text{if } e_{it}^p \leq \xi^p \\ \chi_0(e_{it}^p - \xi^p)^{\chi_1} \times d_t & \text{otherwise} \end{cases}$$

where $e_{it}^p$ is the permanent component of household $i$’s labor productivity. I set the cutoff $\xi^p$ so that the bottom half of households receive no dividends (consistent with the illiquid wealth distribution in the 2016 SCF), $\chi_1$ so that the top 10 per cent of households receive the same share of total dividends (and so total illiquid wealth) as in Kaplan et al. (2018), and then back
Table B.1: HANK model, steady-state calibration.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Households</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\rho_e, \sigma_e$</td>
<td>Income Risk</td>
<td>-</td>
<td>Kaplan et al. (2018)</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\xi^p, \chi_0, \chi_1$</td>
<td>Dividend Endowment</td>
<td>-</td>
<td>Illiquid Wealth Shares</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount Rate</td>
<td>0.97</td>
<td>B/Y</td>
<td>1.04</td>
<td>1.04</td>
</tr>
<tr>
<td>$r^b$</td>
<td>Average Return</td>
<td>0.01</td>
<td>Annual Rate</td>
<td>0.04</td>
<td>0.04</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Death Rate</td>
<td>1/180</td>
<td>Average Age</td>
<td>45</td>
<td>45</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Preference Curvature</td>
<td>1</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Labor Supply Elasticity</td>
<td>1</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\epsilon_w$</td>
<td>Labor Substitutability</td>
<td>10</td>
<td>Standard</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$b$</td>
<td>Borrowing Limit</td>
<td>0</td>
<td>McKay et al. (2016)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Firms</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Capital Share</td>
<td>0.2</td>
<td>Justiniano et al. (2010)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation</td>
<td>0.016</td>
<td>Total Wealth/Y</td>
<td>10.64</td>
<td>10.64</td>
</tr>
<tr>
<td>$\epsilon_p$</td>
<td>Goods Substitutability</td>
<td>16.67</td>
<td>Profit Share</td>
<td>0.06</td>
<td>0.06</td>
</tr>
<tr>
<td><strong>Government</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\tau_\ell$</td>
<td>Labor Tax</td>
<td>0.3</td>
<td>Average Labor Tax</td>
<td>0.30</td>
<td>0.30</td>
</tr>
<tr>
<td>$\tau/Y$</td>
<td>Transfer Share</td>
<td>0.05</td>
<td>Transfer Share</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$B/Y$</td>
<td>Liquid Wealth Supply</td>
<td>1.04</td>
<td>Government Debt/Y</td>
<td>1.04</td>
<td>1.04</td>
</tr>
</tbody>
</table>

Next, I set the average return on (liquid) assets in line with standard calibrations of business-cycle models. The discount and death rates are then disciplined through targets on the total amount of liquid wealth as well as average household age. For my baseline model, I further assume that households cannot borrow. All remaining parameters are set in line with conventional practice. The second block shows parameter choices on the firm side. I discipline the Cobb-Douglas production function $y = k^\alpha \ell^{1-\alpha}$ by setting $\alpha$ in line with Justiniano et al. (2010), identify goods substitutability by targeting the profit share, and finally back out the depreciation rate from my target of total wealth.

---

37 A natural alternative assumption would be to set $d_{it} = d_t$, as in McKay et al. (2016) or Auclert et al. (2018). This alternative choice of course changes impulse responses, but has little effect on the accuracy of the demand equivalence approximation.
(and so corporate sector valuation) in the economy as a whole. The third block informs the fiscal side of the model. The average government tax take, transfers, and debt issuance are all set in line with direct empirical evidence.

Importantly, because household self-insurance is severely limited, the average MPC in the economy is high, around 28% out of an unexpected 500$ income gain. As a result, the model can replicate the large (yet gradual) empirically observed consumption response to stimulus checks, as argued previously in Auclert et al. (2018).

**Dynamics: computational details.** I solve the model using a variant of the popular Reiter method (Reiter, 2009). In particular, I use a mixture of the methods developed in Ahn et al. (2017) and Bayer & Luetticke (2020) to reduce the dimensionality of the state space. Without dimensionality reduction, the number of idiosyncratic household-level states is too large to allow likelihood-based estimation. With dimensionality reduction, the number of states is reduced to around 300, making estimation feasible.

While the estimation relies on standard state-space methods, my displays of exact and approximate demand equivalence are instead computed in sequence-space. For the perfect foresight sequence-space solution I proceed similarly to Boppart et al. (2018) and Auclert et al. (2019); throughout, I verify that the results are not sensitive to the choice of transition endpoint $T$ or the step size used to approximate the Jacobians of the linearized system.

**Dynamics: estimation.** With two exceptions, I estimate the remaining model parameters (which exclusively govern dynamics around the deterministic steady state) using standard likelihood methods, as in An & Schorfheide (2007). The set of observables is: aggregate output ($y$), consumption ($c$), investment ($i$), inflation ($\pi$), the short-term nominal interest rate ($r^n_t$), and total hours worked ($\ell$). The construction of all series follows Justiniano et al. (2010), and my sample period is 1960:Q1—2006:Q4. Priors are reported in Table B.2.

The first exception is the transfer adjustment parameter $\rho_\tau$; since I do not include data on government debt, this parameter would likely be poorly identified. I thus simply set $\rho_\tau = 0.9$, in line with the VAR evidence documented in Galí et al. (2007) and Appendix D.3. Second, as it is central to my approximate equivalence results, I directly discipline the degree of wage stickiness from micro data. Exploiting the standard first-order equivalence of Calvo

---

More conventional higher values of $\alpha$ change impulse responses, but do not break demand equivalence. Similarly, the results also remain accurate with the low value of $\alpha$ entertained in Auclert & Rogalie (2018).

I thank Brian Livingston for help in assembling the data.
price re-sets and Rotemberg adjustment costs, it is easy to show that the slope parameter of the wage-NKPC (B.4) can be equivalently written as

\[ \kappa_w = \frac{(1 - \frac{1}{1 + \phi_w})(1 - \phi_w)}{\phi_w (e_{\frac{1}{\varphi}} + 1)} \]

where \(1 - \phi_w\) is the probability of wage adjustment in the quarter. I set the wage stickiness parameter consistent with the micro evidence in Grigsby et al. (2019) and Beraja et al. (2019), giving \(\phi_w = 0.6\)—price re-sets every 2.5 quarters.\(^{40}\)

The results of the estimation are displayed in Table B.2. Since they are not relevant for my purposes here, I omit estimates of shock persistence and volatility; some brief remarks on those follow at the end. I find the posterior mode using the csminwel routine provided by Chris Sims; for accuracy of the demand equivalence approximation beyond the mode parameterization of the model, see the discussion in Appendix E.1.\(^{41}\)

On the whole, the results are quite consistent with the parameter estimates in Justiniano et al. (2010). Relative to their rich framework, the two central changes in my model are, first, the introduction of uninsurable income risk, and second, the absence of habit formation. The first change ties consumption and income more closely together, while the second leads

\(^{40}\)As it turns out, direct estimation of this parameter also yields \(\phi_w\) close to 0.6. Most previous work that has used time series data to estimate the degree of wage stickiness instead finds even larger numbers (e.g. Justiniano et al., 2010).

\(^{41}\)The optimization routine is available at http://sims.princeton.edu/yftp/optimise/.
to less endogenous persistence and worsens the Barro-King puzzle (Barro & King, 1984).
Jointly, these changes dampen the importance of impatience shocks as a driving force of consumption fluctuations, but also give a somewhat smaller role for investment efficiency shocks as a source of cyclical fluctuations. These findings are consistent with the intuition in Werning (2016) and the estimation results on the no-habit model in Justiniano et al. (2010). Ultimately, given the similarity in model environment and data sources, the similarity of the resulting parameter estimates should not come as a surprise. A more serious estimation exercise on the effects of micro heterogeneity on macro fluctuations would also leverage the advantages afforded by time series of richer micro data, and is left for future work.

Simplified model. The simplified HANK model considered for the illustration in Figure 1 is identical to the estimated model except for one change: I set $\phi_w = 1$. As a result, demand equivalence holds exactly.

B.3 Model extensions of Section 4.2

B.3.1 Multiple goods

The full model with multiple goods departs from the one-sector baseline in three ways. First, it features three goods—two consumption goods and an investment good. The household consumption basket $c_{it}$ now satisfies

$$c_{it} = c_{ii}^\nu c_{i2t}^{1-\nu}$$

I let the ideal price index of the consumption bundle be the numeraire of my economy (so that we again have $p_i^c = 1 \forall t$), and I denote the real relative prices of the two consumption goods by $x^1_t$ and $x^2_t$. Investment is only possible using the economy’s investment good, whose real relative price is denoted $x^I_t$. The government purchases each of the three goods, with potentially different spending multipliers for each, and the monetary authority responds to changes in consumer price inflation.

Second, household disutility over labor supply takes the same functional form as before, with $\ell^h_t$ now given as an aggregate of labor supply for each of the three goods:

$$\ell^h_t \equiv \left[ \chi_1 (\ell^h_{1t})^{\frac{\mu}{\varphi}} + \chi_2 (\ell^h_{2t})^{\frac{\mu}{\varphi}} + \chi_I (\ell^h_{It})^{\frac{\mu}{\varphi}} \right]^{\frac{\varphi}{\varphi + \mu}}$$

where $\{\chi_1, \chi_2, \chi_I\}$ govern disutility from work in each of the sectors. $\mu = 0$ corresponds
to perfect labor mobility across the sectors, while $\mu = 1$ corresponds to perfect immobility, with all labor types entering separately into household utility. For each type of labor, labor supply is intermediated by a unit continuum of sticky-wage unions. Optimal union behavior then gives the three log-linearized wage-NKPCs; proceeding as in Appendix B.1, we find

$$\hat{w}_t^m = \frac{\beta}{1 + \beta} \hat{w}_{t+1}^m - \kappa_w \left[ \hat{w}_t^m - \left(1 - \frac{\mu \gamma h}{\varphi \ell_t} + \frac{\mu \gamma m}{\varphi \ell_t} \right) - \gamma c_t \right]$$

$$- \frac{1}{1 + \beta} \hat{r}_t + \frac{\beta}{1 + \beta} \hat{r}_{t+1} + \frac{1}{1 + \beta} \hat{w}_{t-1}^m$$

for $m = 1, 2, I$. Note that, with $\mu = 0$ (i.e., perfect labor mobility), wages in all sectors are at all times equalized. Overall, household $i$ then receives $e_{it} w_t \ell_t$ worth of labor earnings, where $w_t$ is the aggregated wage index.

Third, there are separate production sectors for each of the three goods. Briefly, I simply repeat the production sector of the baseline model described in Appendix B.1 three times, but with good-specific final prices $x_t^m$ and potentially heterogeneous capital shares $\alpha_m$. All three sectors then purchase capital goods at price $x_I^t$, hire labor at cost $w_t^m$, and sell their own good at real price $x_t^m$.

**Parameterization.** I build on the parameterization of the estimated HANK model of Section 4.1, with one notable difference: a smaller degree of nominal price rigidities. In the model, the probability of price re-sets governs relative price movements after a demand shock for a specific good. I have included measures of relative prices in my VARs and find little response (see Figure D.3), similar to Nakamura & Steinsson (2014); however, Ramey & Shapiro (1998) show that, after large and persistent government spending shocks that move output by almost 4 per cent, relative prices move by 2.5 per cent. To be conservative, I thus choose a model calibration with $\phi_p = 0.6$, giving relative price responses consistent with the evidence of Ramey & Shapiro.

Next, I set $\mu = 1$ (i.e., fully sector-specific labor). I set the average capital share $\bar{\alpha} \equiv \alpha_1 \bar{m}/\bar{y} + \alpha_2 \bar{q}_2/\bar{y} + \alpha_I \bar{q}_I/\bar{y} = 0.2$ (as in my baseline model), and then set relative labor shares as in Alonso (2017, Table 3.3), giving $\alpha_1 = 0.48$, $\alpha_2 = 0.04$, $\alpha_I = 0.17$. The fraction of labor in each of the three sectors is set so that their relative sizes are also data-consistent; again following Alonso (2017), this gives $\bar{y}_1/\bar{y} = 0.29$, $\bar{y}_2/\bar{y} = 0.48$ $\bar{y}_3/\bar{y} = 0.23$. I recover $\bar{g}_I$ residually from the market-clearing condition for good $I$, and then set $\bar{g}_1 = \bar{g}_2 = \frac{1}{2}(\bar{g} - \bar{g}_I)$, with $\bar{g}$ set as before. Finally I then recover the weight $\nu$ in household preferences as $\nu = \frac{\bar{c}}{\bar{c}}$, and set the labor preference weights $\{\chi_1, \chi_2, \chi_I\}$ to clear the labor market given $\{\bar{w}, \bar{w}_2, \bar{w}_I\}$. 

65
B.3.2 Open economy

To study the role of demand leakage abroad I consider a small open economy version of my baseline HANK model, following Auclert et al. (2021). Consumers and firms in the home economy $H$ consume and invest using a final good bundle that consists of both domestic and foreign goods (indexed by $F$), while the government consumes only the domestic good. The domestic economy is small, so domestic policies do not affect the rest of the world.

The domestic consumption basket aggregates the home and foreign final good:

$$c_{it} = \left[ \phi \frac{1}{m} (c_{Hit})^{\eta_1 - 1} + (1 - \phi) \frac{1}{m} (c_{F_{it}})^{\eta_1 - 1} \right]^{\eta_1 - 1}$$

Here $\phi$ is the degree of home bias and $\eta_1$ is the elasticity of substitution between home and foreign goods. I let the price of the total consumption bundle be the numeraire, and denote the real relative prices of the domestic and foreign final good by $x_{Ht}$ and $x_{Ft}$, respectively. Log-linearized real relative prices thus satisfy

$$\phi \hat{x}_t^H + (1 - \phi) \hat{x}_t^F = 0$$

For simplicity I assume that the investment bundle purchased by intermediate goods producers also consists of the domestic and foreign final goods, with the same steady-state home share $\phi$ and elasticity of substitution $\eta_1$. The problem of domestic intermediate goods producers is then unchanged relative to the baseline economy. For retailers we now get a price-NKPC in inflation of the domestic good:

$$\hat{\pi}_t^H = \kappa_p (\hat{p}_t - \hat{x}_t) + \beta \hat{\pi}_{t+1}$$

where inflation and real relative prices are linked as

$$\hat{x}_t^H = (\hat{x}_t^H - \hat{x}_{t-1}^H) + \hat{\pi}_t$$

Let $e_t$ denote the nominal exchange rate. Since foreign prices are fixed we have that

$$\hat{e}_t - \hat{e}_{t-1} = (\hat{x}_t^F - \hat{x}_{t-1}^F) + \hat{\pi}_t$$

With nominal interest rates on foreign bonds also fixed, arbitrage dictates that

$$\hat{i}_t = \hat{e}_{t+1} - \hat{e}_t$$

(B.8)
Finally, foreign consumer and firm demand for the domestic final good satisfies

$$\tilde{c}^H_t = -\eta_2 (\tilde{x}^H_t - \tilde{x}^F_t)$$

$$\tilde{i}^H_t = -\eta_2 (\tilde{x}^H_t - \tilde{x}^F_t)$$

where $\eta_2$ elasticity of substitution between home and foreign goods in the foreign bundle.

I consider the same monetary policy rule as before, so the central bank stabilizes inflation in the domestic consumption bundle. The fiscal authority consumes only the domestic good, so in the government budget constraint we have $p^g_t = x^H_t$. Finally, the domestic bond market-clearing condition is dropped for the arbitrage relation (B.8). The model is closed by requiring domestic output market-clearing, which dictates that

$$c^H_t + c^H_{*t} + i^H_t + i^H_{*t} + g_t = y_t$$

**Parameterization.** I set $\phi = 0.89$, matching the domestic consumption share of the U.S. economy. For the elasticities of substitution I set $\eta_1 = \eta_2 = 2$, in line with previous work. All other model parameters are kept exactly as in the baseline HANK model of Section 4.1.

**B.3.3 Two-asset model**

To illustrate the role of interest rate effects I consider an extended model with two assets: a liquid asset and an illiquid asset, as in Kaplan et al. (2018).

Households invest in an illiquid asset with real return $r^h$ and a liquid asset with real return $r^h - \kappa_b$, where $1 + r^h = \frac{1 + \pi^h_t}{1 + \pi^h_t - 1}$. The household consumption-savings problem then is

$$\max_{\{c_t, b^h_{it}, a^h_{it}\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_t, \ell_t) \right]$$

such that

$$c_t + b^h_{it} + a^h_{it} = (1 - \tau_t) w_t e_t \ell_t + \left[ \frac{1 + i^h_t}{1 + \pi^h_t} - \kappa_b \right] b^h_{it-1} + \frac{1 + i^h_t}{1 + \pi^h_t} a^h_{it-1} + \phi_a(a^h_{it}, a^h_{it-1}; \zeta_t) + \tau_{it}$$

and

$$b^h_{it} \geq b, \quad a^h_{it} \geq a$$

where $\phi_a(\cdot, \cdot; \zeta)$ is the adjustment cost function for illiquid asset holdings. Similar to Bayer
et al. (2019), I assume that a randomly chosen fraction $\eta$ of households can freely adjust their illiquid wealth holdings ($\zeta = 1$), while the remaining households cannot adjust ($\zeta = 0$). The adjustment cost function can then be written as

$$
\phi_a(a', a) = \begin{cases} 
0 & \text{if } \zeta = 1 \\
\infty & \text{if } \zeta = 0
\end{cases}
$$

Returns in the economy are determined as follows. Both liquid and illiquid assets are issued by a mutual fund, which in turn owns all government debt and all claims to corporate profits in the economy. Let $\omega_t \equiv b_t^h + b_t^f + \alpha_t^h$ denote total funds managed by the mutual fund. Returns earned by the mutual fund $i_t^m$ then satisfy

$$
\omega_{t-1} \times \frac{1 + \frac{i_{t-1}^m}{1 + \pi_t}}{1 + \pi_t} = b_{t-1} \frac{1 + \frac{i_{t-1}^b}{1 + \pi_t}}{1 + \pi_t} + (d_t + v_t)
$$

where $v_t$ denotes the value of the corporate sector, which by arbitrage satisfies

$$
\frac{1 + \frac{i_{t-1}^b}{1 + \pi_t}}{1 + \pi_t} = \frac{v_t + d_t}{v_{t-1}}
$$

except possibly at $t = 0$. I assume that the mutual fund is competitive, and faces intermediation costs $\kappa_b$ to make assets liquid. It follows immediately that we must have $i_t^h = i_t^m$.

The rest of the economy is unchanged; in particular, firms still discount at $\frac{1 + \frac{i_{t-1}^m}{1 + \pi_t}}{1 + \pi_t}$, which in the absence of aggregate risk is equivalent to discounting at $\frac{1 + \frac{i_{t-1}^b}{1 + \pi_t}}{1 + \pi_t}$. The only change to Definition 2 is the new asset market-clearing condition:

$$
b_t^h + b_t^f + \alpha_t^h = b_t + v_t
$$

**Parameterization.** For simplicity, I keep all parameters governing dynamics identical to the estimated one-asset HANK model, and only re-calibrate the steady state. Table B.3 displays all parameters from the re-calibrated two-asset model that are different from those displayed in Table B.1 for the benchmark one-asset model.

To provide a stringent test of the demand equivalence approximation, I set the wedge between returns on household deposits and government debt to be an (arguably implausible) 1.25 per cent per quarter. Given this large difference, I then choose the adjustment probability $\eta$ to ensure a reasonable fit to total liquid and illiquid wealth in the U.S. economy.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
<th>Target</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\eta$</td>
<td>Probability of Adjustment</td>
<td>0.15</td>
<td>A/Y</td>
<td>9.70</td>
<td>10.64</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Discount Rate</td>
<td>0.98</td>
<td>B/Y</td>
<td>1.28</td>
<td>1.04</td>
</tr>
<tr>
<td>$r^h$</td>
<td>Return</td>
<td>0.015</td>
<td>Upper Bound</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\kappa_b$</td>
<td>Liquid Wedge</td>
<td>0.0125</td>
<td>Upper Bound</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation</td>
<td>0.025</td>
<td>Firm Valuation</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Table B.3:** 2-asset HANK model, steady-state calibration.

### B.3.4 Weak wealth effects

To further illustrate the role of household labor supply decisions in breaking demand equivalence, I consider a variant of the estimated HANK model without unions, but with weak wealth effects in labor supply.

Relative to the baseline model, this variant differs in three respects. First, the economy is now populated by a double unit continuum of households—a unit continuum of families $f \in [0, 1]$, and a unit continuum of households $i \in [0, 1]$ for each $f$. Each family is a replica of the unit continuum of households in the benchmark model, but shock exposures may be heterogeneous across families. I will explain the purpose of this artificial construction momentarily. Second, there are no unions—each household decides on its own labor supply. Third, I change household preferences. Similar to Jaimovich & Rebelo (2009) and Galí et al. (2012), I assume that

$$u_{f,t}(c_{i,f,t}, \ell_{i,f,t}) = \frac{c_{i,f,t}^{1-\gamma} - 1}{1-\gamma} - \frac{\ell_{i,f,t}^{1+\frac{1}{\varphi}}}{1 + \frac{1}{\varphi}}$$

where the preference shifter $\theta_{i,f,t}$ satisfies

$$\theta_{i,f,t} = x_{f,t}^{\gamma} \times c_{i,f,t}^{-\gamma}$$

The variable $x_{f,t}$ is central. To jointly ensure (i) arbitrarily weak short-run wealth effects in labor supply, (ii) homogeneous wealth effects in the cross section of households (both

---

42Households do not internalize the effect of their consumption on the shifter.
consistent with the estimates in Cesarini et al. (2017)), and (iii) direct earnings responses showing up in cross-sectional regressions, I assume that

\[ x_{ft} = x_{ft-1}^{1-\omega} \times c_{ft}^{\omega} \]

This preference specification is the simplest design with all three desired properties. First, by varying the parameter \( \omega \), I can control the strength of short-term wealth effects, exactly as in Galí et al. (2012). With \( \omega = 0 \) wealth effects are 0, and so Assumption 3 is satisfied. Second, solving for optimal household labor supply decisions, we get

\[ \chi_{\ell_{ft}} = w_{t}x_{ft}^{-\gamma} \quad \text{(B.9)} \]

If all “families” are equally affected by the shock, then everyone’s labor supply is identical, giving the desired homogeneity. Thus, for the first two requirements, the family construction is not necessary—we could simply replace \( c_{ft} \) by \( c_{t} \), giving the natural heterogeneous-agent analogue of the preferences in Galí et al. (2012). But third, with heterogeneous family-level shock exposures, cross-sectional regressions as in Proposition 2 will pick up direct earnings responses. In particular, let \( \ell_{h} = \ell_{h}(w, c) \) denote the mapping from wages and family consumption into family labor supply induced by (B.9). The micro regression estimand in (9) then satisfies

\[ \hat{c}_{PE}^{\tau} = \left( I - \frac{\partial c}{\partial \ell} \times \frac{\partial \ell_{h}}{\partial c} \right)^{-1} \times \left( \frac{\partial c}{\partial \tau} \cdot d\tau \right) \quad \text{(B.10)} \]

For my accuracy checks, I simply match this regression estimand with an identical expansion in aggregate government spending.

**Parameterization.** The parameters related to the sticky-wage block of the baseline model are irrelevant for this model variant; all other parameters are set exactly as before. Finally, the sole new model parameter is \( \omega \). To ensure consistency with empirical evidence, I set \( \omega = 0.043 \). As in Cesarini et al. (2017), this specification results in a peak measured cross-sectional (partial equilibrium) labor supply response of around 4\$ for every 100\$ response in consumption.
C Further results on demand equivalence

This appendix collects several supplementary theoretical results. In Appendix C.1 I show that my arguments apply without change to perturbations around arbitrary transition paths. Appendices C.2 and C.3 emphasize the generality of consumption demand equivalence by considering a larger family of shocks and models, and in Appendix C.4 I illustrate the range of general equilibrium outcomes consistent with exact equivalence. Finally, in Appendix C.5, I show that many popular heterogeneous-firm models of investment are nested by the investment demand equivalence result.

C.1 General transition paths

All equivalence results in this paper are stated for transition paths starting at the deterministic steady state. However, it is immediate from the proof of Proposition 1 (and similarly that of Proposition A.1) that nothing hinges on the starting point. Intuitively, the crucial restriction in my arguments is that they are valid to first order, but not that they only apply to particular expansion points. All results can thus equivalently be interpreted as applying to first-order perturbation solutions around a given (deterministic) transition path.

For example, initial states $\mu_0^b, \mu_0^f, w_{-1}$ and $p_{-1}$ could be such that the economy is in a deep recession or brisk expansion. The equivalence results would then apply to deviations from the unshocked transition path of the economy back to steady state. These deviations need not agree with impulse responses at steady state, but they remain tied together across different kinds of demand shocks.

C.2 Generic consumption demand shifters

Consumption demand equivalence extends without change to generic shifters of consumption demand. To establish this claim, I augment the baseline model to feature fluctuations in household patience as a simple reduced-form stand-in for various more plausibly structural shocks to spending (e.g. changes in borrowing constraints, redistribution, ...).

The discount factor of every household is now subject to an additional common shifter $\zeta_t$, with $\zeta = \zeta(\nu)$, giving preferences as

$$E_0 \left[ \sum_{t=0}^{\infty} \beta^t \zeta_t(u(c_{it}, \ell_{it}) \right]$$

(C.1)
Note that impatience shocks—shocks that just shift the intertemporal profile of private consumption spending—necessarily induce consumption response paths $\widehat{c}_v^{PE}$ with zero net present value. As a result, a government spending shock satisfying requirement (i)—i.e., $\widehat{g}_g = \overline{c}_v^{PE}$—also has zero net present value, and so need not be financed through any change in taxes or transfers. We can thus again have $\tau^e_g = \tau^e_v$, and the proof of Proposition 1 then applies without change to give

$$\widehat{c}_v = \overline{c}_v^{PE} + \widehat{c}_g = \text{GE feedback}$$

(C.2)

Importantly, the observed tax responses to the two shocks now reflect only general equilibrium feedback to taxes (e.g., through changes in inflation or labor tax revenue).

**C.3 Exact equivalence beyond the baseline model**

The proof of consumption demand equivalence applies to any model that satisfies the set of semi-structural exclusion restrictions in Section 2.3. This section briefly discusses several prominent examples of such models.

**DURABLES.** I extend the household consumption-savings problem to feature durable and non-durable consumption:

$$\max_{\{c_{it}, d_{it}^h, b_{it}^h\}} \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t u(c_{it}, d_{it}^h, \ell_{it}) \right]$$

(C.3)

such that

$$c_{it} + d_{it}^h + b_{it}^h = (1 - \tau_t)w_t c_{it} \ell_{it} + \frac{1 + i_{t-1}^b + \kappa_b^1 b_{t-1}^h < 0}{1 + \pi_t} b_{t-1}^h + (1 - \delta) d_{it-1}^h + \tau_{it} + d_{it} + \phi_d(d_{it-1}^h, d_{it}^h)$$

and

$$b_{it} \geq b - (1 - \theta)d_{it}$$

where $\phi_d(\bullet)$ is the durables adjustment cost function, $1 - \theta$ is the share of durable holdings that can be collateralized, and—in a slight abuse of notation—I only use the superscript $h$ to distinguish between household durables consumption $d_{it}^h$ and dividend receipts $d_{it}$. Note that this specification allows for all of the bells and whistles considered in quantitative studies of durable and non-durable consumption (e.g. as in Berger & Vavra, 2015): households
have potentially non-separable preferences over $c$ and $d^h$, adjustments in durables may incur additional costs, and households can borrow against their durable goods holdings.\footnote{In particular, this implies that the model can in principle be consistent with empirical evidence suggesting MPCs close to zero for some households (non-adjusters) and in excess of one for others (those pushed towards durables adjustment).}

Crucially, I assume that the common final good $y_t$ can be costlessly turned into the durable good $d^h_t$, so that the aggregate resource constraint becomes

$$y_t = c_t + d^h_t - (1 - \delta)d^h_t + i_t + g_t$$

where $e_t$ is household expenditure. The equilibrium definition in Appendix B.1 thus generalizes straightforwardly, with aggregate household expenditure now replacing pure (non-durable) consumption expenditure. Defining a PE-GE decomposition for total household expenditure as in Definition 1, we can easily show that the demand equivalence result still applies, now for the aggregated household expenditure path $e$:

**Corollary C.1.** Extend the structural model of Section 2.1 to feature durable goods, as in Problem (C.3). Consider a stimulus check policy $\varepsilon$, and suppose that Assumptions 1 to 3 hold. Then, for a fiscal spending policy $\varepsilon$ such that (i) $\hat{g}_\varepsilon = \hat{e}^{PE}_\varepsilon$ (identical net excess demand) and (ii) $\hat{\tau}_g = \hat{\tau}_e$ (identical tax response), we have that, to first order,

$$\hat{e}_\varepsilon = \hat{e}^{PE}_\varepsilon + \hat{e}_g = GE \text{ feedback}$$

As argued in Beraja & Wolf (2020), consumption dynamics in models with durables generally look very different from those in models with only non-durable consumption. Corollary C.1 reveals, however, that this change in aggregate outcomes is in fact completely orthogonal to demand equivalence.

**Preferences.** My baseline structural model assumes time separability in household preferences. It is, however, immediate that general forms of time non-separability are similarly nested: as long as the consumption block of the model admits aggregation to some aggregate consumption function $c(\bullet)$, the equivalence proof goes through unchanged. My approximate equivalence results for the model of Justiniano et al. (2010)—with habit formation as a very simple form of non-separability—illustrate this claim.
Valued government spending. In my baseline model, households do not value government expenditure. However, it is immediate from the proof strategy for consumption demand equivalence that this assumption is stronger than necessary—the key restriction is that the aggregate consumption function $c(\bullet)$ does not directly depend on government consumption. A possible sufficient condition is that government spending enters the per-period felicity function in an additively separable fashion,

$$\bar{u}(c, \ell, g) = u(c, \ell) + v(g)$$

This is for example the case under a CES preference specification

$$u(c, \ell, g) = \left[\phi^\rho c^{1-\rho} + (1 - \phi)^\rho g^{1-\rho}\right]^\frac{1}{1-\rho} - 1 - \chi \frac{\ell^{1+\frac{1}{\rho}}}{1 + \frac{1}{\rho}},$$

with $\rho = \gamma$.

Expectation formation. All of the models considered in this paper impose rational expectation formation for households and firms. An attractive alternative is the sticky information structure in Auclert et al. (2019). For a simple example, suppose that only the consumption-savings problem of households is subject to a sticky information friction, with a fraction $1 - \theta$ of households updating their information at each point in time $t$. Then, for every input $p$ to the consumption-savings problem, the sticky information consumption derivative map $C_p \equiv \frac{\partial \xi}{\partial p}$ is related to the original derivative map $C_p^*$ via

$$C_p = \begin{pmatrix} C_p^*(1, 1) & (1 - \theta)C_p^*(1, 2) & (1 - \theta)C_p^*(1, 3) & \cdots \\ C_p^*(2, 1) & (1 - \theta)C_p^*(2, 2) + \theta C_p^*(1, 1) & (1 - \theta)C_p^*(2, 3) + \theta (1 - \theta)C_p^*(1, 2) & \cdots \\ C_p^*(3, 1) & (1 - \theta)C_p^*(3, 2) + \theta C_p^*(2, 1) & \cdots & \cdots \\ \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

Since the proof of Proposition 1 relies only on the existence of these linear maps (and not their shape), it follows immediately that all results extend without change to such behavioral model economies.
C.4 Range of outcomes for the “missing intercept”

Proposition 1 asserts that private and public spending shocks induce the same general equilibrium effects, but is silent on the strength of this common general equilibrium feedback. In this section I give two extreme examples, one with full general equilibrium crowding-out, and one with strong general equilibrium amplification.

The first example is a variant of the baseline model of Section 2.1, restricted to feature flexible prices and wages, labor-only production, and household preferences as in Greenwood et al. (1988). In this model, an income tax rebate does not move aggregate output, consumption, or labor. The argument is well-known and straightforward: Given a rebate path \( \hat{\tau} \), consider an interest rate path \( \hat{r} \) such that, at \((\hat{\tau}, \hat{r})\) and facing steady-state wages forever, households are willing to consume steady-state consumption \( \bar{c} \) forever. But then the output and labor markets clear by construction, and so we have indeed found an equilibrium. Thus, in this model, interest rate feedback fully crowds out any partial equilibrium perturbations to consumption demand.

The second example is quantitative. I consider the estimated New Keynesian business-cycle model of Justiniano et al. (2010), but now assume that preferences are as in Greenwood et al. (1988). Results are reported in Figure C.1.

Demand Equivalence, GHH in Justiniano et al. (2010)

![Figure C.1: Consumption impulse response decompositions after equally large, one-off impatience and government spending shocks in the model of Justiniano et al. (2010) with GHH preferences. The direct response and the indirect general equilibrium feedback are computed following Definition 1.](image-url)
It is immediate that this model satisfies all assumptions in Proposition 1, and so exact demand equivalence holds. Given strong complementarities in consumption and labor supply, the extra production induced by the demand shock will lead to yet more consumption demand, setting in motion a strong general equilibrium feedback cycle (see Auclert et al., 2020, for an analytical characterization).

C.5 Nested models for investment demand equivalence

Exact investment demand equivalence holds in the popular structural models of Khan & Thomas (2008), Khan & Thomas (2013), Winberry (2018), and Bloom et al. (2018). I verify this claim by checking that each of the assumptions necessary for the result is in fact satisfied.

First, in all of those models, capital adjustment costs are internal to the firm, so Assumption A.1 holds. Second, each model is closed with a simple representative household with linear labor disutility, so Assumptions A.2 and A.3 hold. Finally, since none of these models feature nominal rigidities, Assumption A.4 is irrelevant.\(^{44}\)

\(^{44}\)Well-known heterogeneous-firm models with nominal rigidities include Ottonello & Winberry (2018) and Koby & Wolf (2020). In both cases Assumption A.4 is satisfied.
D Empirical appendix

This appendix provides additional details on the empirical results that I use as an input to my measurement strategy. Appendix D.1 discusses estimates of the direct (partial equilibrium) consumption response to stimulus checks, Appendix D.2 does the same for the investment response to tax credits, and Appendix D.3 offers supplemental information on the VAR-based identification of government spending shock transmission.

D.1 Cross-sectional consumption elasticities

I here provide further details for my estimates of direct consumption responses to stimulus check receipt. First, I begin by giving conditions under which the regressions of previous work can indeed be interpreted as giving such direct responses. Second, I provide robustness checks confirming that my first application in Section 3.3 is likely to capture the effects of a one-off stimulus check policy. And third, I construct the inverse mapping for the stimulus news shock application in Section 3.3 under alternative assumptions about household foresight.

Baseline estimates. Proposition 2 shows that, with truly exogenous cross-sectional heterogeneity in shock exposure, micro difference-in-differences regressions estimate direct partial equilibrium responses. In the empirical analysis of Johnson et al. (2006) and Parker et al. (2013), matters are slightly more subtle—all households are exposed to the shock, but exposure differs over time for exogenous reasons. Building heavily on Kaplan & Violante (2014), I here discuss how to interpret their regression estimands. Parker et al. estimate a differenced version of (9):

\[ \Delta c_{it} = \text{time fixed effects} + \text{controls} + \beta_0 ESP_{it} + \beta_1 ESP_{it-1} + u_{it} \]  

where $ESP_{it}$ is the dollar amount of the rebate receipt at time $t$. To establish that the regression estimands are interpretable as $MPC_{0,0}$ and $MPC_{1,0} - MPC_{0,0}$, respectively, consider again the structural model of Section 2.1, and suppose—roughly in line with the actual policy experiment (see Kaplan & Violante, 2014)—that a randomly selected fraction $\omega$ of households receive a lump-sum rebate at $t = 0$ ($\epsilon_{\tau i0} = 1$), and that the remaining households receive the same rebate at $t = 1$ ($\epsilon_{\tau i1} = 1$). The model analogue of regression (D.1) is then

\[ \Delta c_{it} = \delta_{\Delta t} + \beta_0 \epsilon_{\tau i t} + \beta_1 \epsilon_{\tau i t-1} + u_{it}, \quad t = 0, 1 \]  

77
Now suppose additionally that receipt of the rebate is a surprise for all households; in particular, it is a surprise at $t = 1$ for households who receive the delayed check. We can then follow exactly the same steps as in the proof of Proposition 2 to show that, to first order,

$$
\beta_0 = MPC_{0,0}, \quad \beta_1 = MPC_{1,0} - MPC_{0,0}
$$

If instead the delayed check was perfectly anticipated, then the regression estimands are $\beta_0 = MPC_{0,0} - MPC_{0,1}$ and $\beta_1 = MPC_{1,0} - MPC_{1,1}$, where $MPC_{t,1} \equiv \int_0^1 \frac{\partial c_i}{\partial \tau_1} d\tau$ is the response of consumption at $t$ to a rebate received at $t = 1$, but anticipated at $t = 0$.

Consistent with Broda & Parker (2014), Kueng (2018), Ganong & Noel (2019) and Baugh et al. (2021) (and also my analysis for the second stimulus check application in Section 3.3), I for my baseline analysis assume the complete absence of anticipation effects. I thus interpret the estimates of Parker et al. (2013) and Broda & Parker (2014) as giving me $MPC_{0,0} = 0.5$ and $MPC_{1,0} = 0.2$. I then extrapolate assuming a geometric rate of decay from $t = 1$ onwards; together with the requirement that

$$
\sum_{t=0}^{\infty} \left( \frac{1}{1 + \bar{r}} \right)^{t-1} MPC_{t,0} = 1
$$

This gives the green line in Figure 2, with $\bar{r} = 0$.

**No extrapolation and exact inversion.** The left panel of Figure 2 suggests that the identified aggregate fiscal expenditure shock of Ramey (2011) is well-suited to solve the missing intercept problem for short-lived stimulus checks. Figure D.1 illustrates two further arguments that re-affirm this conclusion.

First, the left panel uses the matrix $C_\tau$ from (15) and identified via the point estimates of Broda & Parker (2014) to construct the inverse mapping, from the spending path of Ramey (2011) to the exact equivalent transfer. As expected, the exact equivalent policy shows large payouts on impact, before then jumping to a (moderately noisy) zero. Second, the right panel compares the spending response of Ramey (2011) with the two point estimates of Parker et al. (2013) and Broda & Parker (2014), without any extrapolation. To ensure a lifetime MPC of 1 I scale the two MPCs so that they sum to 1; all future MPCs are then set to 0. Again as expected the two lines are very close, just now scaled up. To summarize, the results from my experiment in Section 3.3 can be robustly interpreted as giving us general equilibrium counterfactuals pertaining to a transitory stimulus check policy.
**Ramey (2011) vs. One-Off Stimulus Checks**

**Figure D.1:** The left panel shows the path $c_{\tau}^{-1} \times \hat{g}_g$ for the OLS point estimate of my Ramey (2011) identification. For the right panel I compare $\hat{g}_g$ with $c_{\tau}(1, 1)$ and $c_{\tau}(2, 1)$.

**Foresight and News Shock Inversion.** My inversion of the identified fiscal shock of Caldara & Kamps (2017) into transfer stimulus space in Section 3.3 relied on the myopic $c_{\tau}$ matrix in (15). I here present results for alternative degrees of household foresight.

To construct $c_{\tau}$ under different degrees of myopia, I follow Auclert et al. (2018) and Wolf (2021) and consider a simple partial-equilibrium consumption-savings problem with spenders, savers, and bonds in the utility function.\textsuperscript{45} I then calibrate this model to match my point estimates of $\{\omega, \theta, \xi\}$ discussed in Section 3.3. In the absence of myopia, this problem induces a consumption derivative map $\tilde{c}_{\tau}$. I then construct $c_{\tau}$ as

$$c_{\tau}(t, s) = \sum_{q=1}^{\min(t, s)} (\mathcal{E}_{q,s} - \mathcal{E}_{q-1,s}) \tilde{c}_{\tau}(t - q + 1, s - q + 1)$$

where

$$\mathcal{E} = \begin{pmatrix} 1 & \psi & \psi^2 & \ldots \\ 1 & 1 & \psi & \ldots \\ 1 & 1 & 1 & \ldots \\
\vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

\textsuperscript{45}To be precise, I consider the model in Appendices C.3 - C.4 of Wolf (2021).
Figure D.2: The green line shows the demand stimulus estimated by Caldara & Kamps (2017). The purple lines show the exact equivalent transfer sequence under alternative assumptions on $C_\tau$ and in particular the degree of household myopia $\psi$. And $\psi \in [0, 1]$ denotes the strength of household anticipation. For $\psi = 0$ this gives (15). For $\psi > 0$ we approximately—in a sense made precise in Wolf (2021)—find

$$C_\tau \approx \omega \times \begin{pmatrix}
1 & \xi\theta\psi & \xi(\theta\psi)^2 & \xi(\theta\psi)^3 & \ldots \\
\xi\theta & 1 & \xi\theta\psi & \xi(\theta\psi)^2 & \ldots \\
\xi\theta^2 & \xi\theta & 1 & \xi\theta\psi & \ldots \\
\xi\theta^3 & \xi\theta^2 & \xi\theta & 1 & \ldots \\
\vdots & \vdots & \vdots & \vdots & \ddots
\end{pmatrix}$$

Figure D.2 shows the exact equivalent transfer paths $\hat{\tau}$ for three different assumptions on the strength of anticipation effects $\psi$: full myopia ($\psi = 0$), fully forward-looking households ($\psi = 1$), and an intermediate case ($\psi = 0.5$). The results are intuitive: if households are forward-looking, then the impact transfer required to engineer the estimated demand stimulus can be smaller.\footnote{Note that all three lines necessarily have the same net present value, equal to the net present value of the induced demand path. Anticipation just somewhat affects the timing of the checks.}
D.2 Cross-sectional investment elasticities

Koby & Wolf (2020) generalize the static analysis of Zwick & Mahon (2017) and estimate dynamic projection regressions of the form

$$\hat{i}_{jt+h} = \alpha_j + \delta_t + \beta_{qh} \times z_{n(j),t} + u_{jt}$$ (D.3)

where $z_{n(j),t}$ is the size of the bonus depreciation investment stimulus for industry $n(j)$ of firm $j$. We estimate this regression on a quarterly Compustat sample spanning the years 1993–2017; this sample period features the two bonus depreciation episodes of 2001-2004 and 2008-2010, exactly as in Zwick & Mahon (2017). We then give sufficient conditions under which the estimands $\{\beta_{qs}\}$ are interpretable as the direct partial equilibrium response of investment to a one-time bonus depreciation stimulus. Given the estimated partial equilibrium path $\{\hat{i}_{PEqt}\}_{t=0}^3$, I recover the full partial equilibrium investment response by fitting a single Gaussian basis function, exactly as in Barnichon & Matthes (2018).

D.3 Time series fiscal policy estimates


RAMEY (2011). The first approach to the identification of aggregate government spending shocks relies on professional forecast errors. These forecast errors are treated as a valid IV for structural government spending shocks, and I study their propagation by ordering them first in a recursive VAR (Plagborg-Møller & Wolf, 2021).

My benchmark VAR consists of the log real per capita quantities of total government spending, total output (GDP), total (non-durable, durable and services) consumption, private fixed investment, total hours worked, and a measure of the federal average marginal tax rate (Alexander & Seater, 2009).\textsuperscript{47} All variables are defined and measured as in Ramey (2011). As further robustness checks, I also consider VARs with (i) total tax revenue in lieu of the marginal tax rate (following Caldara & Kamps, 2017), (ii) Greenbook defense spending forecast errors in lieu of professional forecaster errors (following Drautzburg, 2020), (iii)

\textsuperscript{47}The tax measure of Barro & Redlick (2011) includes state income taxes; given my focus on federal expenditure, I regard the Alexander & Seater series as more suitable for my purposes.
a log per capita measure of total federal debt, (iv) the federal funds rate as a measure of the monetary policy stance, and (v) a measure of the real relative price of the government consumption bundle.\footnote{I obtain the debt series from the tax shock replication data for Ramey (2016), deflating \textit{pubdebt} by \textit{pgdp}. For the real relative price series, I divide the implicit price deflator for federal government consumption expenditures and gross investment by the GDP deflator.}

I estimate all VARs in levels, with a quadratic time trend and four lags. The lag length selection is informed by standard information criteria, and is also consistent with the recommendation of Ramey (2016) in the postscript to her handbook chapter. For estimation of the model, I use a uniform-normal-inverse-Wishart distribution over the orthogonal reduced-form parameterization (Arias et al., 2018). Throughout, I display confidence bands constructed through 10,000 draws from the model’s posterior.

Figure D.3 shows the impulse responses of government spending, output, consumption, investment, the marginal tax rate, total federal debt, the real relative price of the government bundle, and the federal funds rate. As in most existing structural VAR work, I construct 16th and 84th percentile confidence bands; the output and tax responses, however, remain significant at the more conventional 95 per cent level. In line with most of the previous literature I find a significant positive output response (corresponding to around a unit multiplier), and a largely flat reaction of consumption. Total debt rises immediately and significantly, suggesting that the government spending expansion is debt-financed. In fact, I also find a delayed and persistent increase in labor income taxes, as well as a similarly timed increase in total tax revenues (not shown). Finally, I find that neither the real relative price nor the nominal interest respond much.

My central results—the 1-1 increase in output and the limited crowding-out of private expenditure—are robust to various changes in model specification. First, I have experimented with different sub-samples. Starting earlier (1971Q1) means that I need to link forecasts on real federal spending (available after 1981) to earlier forecasts of military spending. Depending on the set of included controls, the undershooting of consumption and investment is, in this earlier sample, usually more pronounced (similar to Ramey, 2011). However, the undershooting then goes hand-in-hand with a similar undershooting of spending itself, invalidating the required demand matching.\footnote{For demand matching I need to re-scale public and private demand shocks to be in dollar terms. This can be done using information on the GDP shares of consumption, investment, and government expenditure. I take those data from FRED, and then compute averages for the different shares across the sample period.} Continuing the sample to 2016Q4 means that

\footnote{Note, however, that—unlike the impact co-movement of fiscal spending and output—the dynamic undershooting of consumption and output is not statistically significant at the 95 per cent level. It is also somewhat}
I need to stop controlling for taxes, as my available measures only continue until 2009. Results in this expanded sample suggest that crowding-in is slightly stronger, consistent with standard intuition on zero lower bound effects. The results are, however, not particularly robust, similar to Ramey & Zubairy (2018) and Debortoli et al. (2019).\footnote{Note that Ramey & Zubairy (2018) also find little evidence on variation in fiscal multipliers by the state of the business cycle.} Second, replacing my benchmark measure of government spending forecast errors with Greenbook defense spending forecast errors leaves my results largely unchanged (see Figure D.4), with the main difference being that now there is some evidence of impact consumption crowding-in, similar to the aggregation exercise based on Caldara & Kamps (2017). Overall the similarity in results suggests that either (i) the benchmark VAR exercise itself is largely picking up the response to military spending forecast errors or (ii) multipliers are invariant to the spending type (similar to the conclusion in the meta study of Gechert (2015)). The high correlation between my baseline forecast error series and the defense forecast error series (0.74) provides dependent on the set of controls; for example, with most controls dropped, I instead find (again largely insignificant) over-shooting.
Figure D.4: See the captions for Figure 2 and Figure 3. I now use the defense forecast spending error series from Drautzburg (2020).

some evidence in favor of the former interpretation. Third, dropping the quadratic time trend has some effects on far-out impulse responses, but not on the short-run responses that I emphasize.

Caldara & Kamps (2017). My second approach leverages the analysis of Caldara & Kamps (2017). In a five-variable VAR, those authors identify a structural government expenditure equation by requiring its orthogonality to structural TFP, oil, and monetary policy shocks, and then furthermore imposing a simple temporal ordering on the fiscal expenditure and tax rules. Replicating their baseline specification gives me a series of structural government spending shocks $\varepsilon_g$. To study the propagation of those shocks to household consumption and government debt—two variables not included in the original VAR of Caldara & Kamps, but required for my analysis—I order the shock series first in a recursive VAR.

The first step of my analysis is the replication of Caldara & Kamps (2017). I consider the exact same five-variable system as they do (including output, the federal funds rate, CPI, government spending, and tax revenue), and point-identify it using the exact same external instruments. The results for the five variables in the system are displayed in the first five panels of Figure D.5, and they mirror the results in Caldara & Kamps (2017). Government spending increases persistently, output increases similarly persistently, prices and interest rates show little response, and tax revenue increases immediately.

I then estimate the impulse responses of consumption and debt by ordering the identified shock first in a recursive VAR with those variables, with all variable definitions exactly as in my Ramey (2011) specification. The last panel of Figure D.5 shows that debt increases
Figure D.5: Impulse responses after a one standard deviation innovation to the shock of Caldara & Kamps (2017), quarterly frequency. The grey areas correspond to 16th and 84th percentile confidence bands, constructed using 10,000 draws from the posterior distribution of the reduced-form VAR parameters.

Persistently, indicating that the fiscal expansion is persistently deficit-financed. In the interest of space I omit the consumption impulse response, as it is anyway implicit in the results displayed in Figure 4.\footnote{For completeness I have repeated the same exercise for investment. I find that investment responds little, consistent with my findings based on the alternative identification schemes of Ramey (2011) and Blanchard & Perotti (2002). These findings are available upon request.}

Blanchard & Perotti (2002). My third approach to the analysis of government spending shock propagation follows Blanchard & Perotti (2002). I estimate the exact same benchmark VAR as in the Ramey (2011) specification, but now consider the dynamic propagation of an innovation to the equation for government spending $g_t$ itself, rather than for its forecast error. This identification scheme is identical to the original approach of Blanchard & Perotti (2002), except for the fact that I now control implicitly for past professional forecast errors of government spending.

Similar to Caldara & Kamps (2017), I find that this alternative identification scheme iden-
tifies a government spending shock with a more persistent response of government spending itself. Qualitatively, the responses of other macroeconomic aggregates—in particular output, consumption and investment—look similar to those for my benchmark Ramey (2011) identification. Importantly, because both sets of impulse responses are identified in the same reduced-form VAR, I can easily account for joint uncertainty by drawing from the posterior of that reduced-form VAR, rotating forecast residuals in line with either my benchmark or the Blanchard-Perotti identification scheme, and then finding the best fit to net demand paths, following (11).
E Approximation accuracy

In this appendix I provide supplementary details to my assessment of the demand equivalence approximation in Section 4.

First, complementing the discussion in Section 4.1, Appendices E.1 and E.2 consider alternative parameterizations of the baseline estimated HANK model as well as other canonical estimated business-cycle models. Second, in Appendices E.3 to E.7, I present detailed results for the accuracy checks of Section 4.2. Finally, in Appendix E.8 I add one further experiment: I construct the demand equivalence approximation under the assumption that the demand matching in (11) is not exact. In all experiments I report the population estimands of the demand equivalence methodology, effectively assuming that the econometrician has access to infinitely large cross-sectional and time series samples.

E.1 Random parameter draws

The accuracy displayed in Figure 5 is not special to the particular (mode) parameterization of my estimated model, but a generic feature of standard business-cycle models with at least moderate wage and price stickiness. To illustrate this point, I proceed as follows: rather than setting the parameter values governing dynamics as in Table B.2, I randomly draw their values from uninformative uniform distributions over wide supports, as displayed in Table E.1. For each parameter draw, I compute the maximal demand equivalence error (in absolute value) relative to the true model-implied peak consumption response. This procedure is repeated for 10,000 draws from the joint uniform distributions in Table E.1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Lower Bound</th>
<th>Upper Bound</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_p$</td>
<td>Price Calvo Parameter</td>
<td>0.15</td>
<td>0.95</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>Capacity Utilization</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>Investment Adjustment Cost</td>
<td>0.5</td>
<td>5</td>
</tr>
<tr>
<td>$\rho_m$</td>
<td>Taylor Rule Persistence</td>
<td>0.15</td>
<td>0.95</td>
</tr>
<tr>
<td>$\phi_\pi$</td>
<td>Taylor Rule Inflation</td>
<td>1.1</td>
<td>2.5</td>
</tr>
<tr>
<td>$\phi_y$</td>
<td>Taylor Rule Output</td>
<td>0</td>
<td>0.5</td>
</tr>
<tr>
<td>$\phi_{dy}$</td>
<td>Taylor Rule Output Growth</td>
<td>0</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table E.1: Supports for uniform parameter draws in the HANK model.
Figure E.1: Kernel estimate of maximal error distribution, with parameters drawn randomly according to Table E.1 (orange). The grey lines show the same kernel density estimate when $\phi_p$ is fixed at its estimated posterior mode.

I find that the approximation accuracy is largely orthogonal to all parameters except for the price stickiness $\phi_p$. Figure E.1 provides a graphical illustration. The grey line shows a kernel density estimate of the error distribution when all parameters except for $\phi_p$ are drawn randomly. It is clear that the estimated parameters have little effect on approximation accuracy—most mass of the error distribution is concentrated around the error estimate at the posterior mode. If $\phi_p$ is also drawn randomly, then larger errors are more likely; however, given my calibrated moderate degree of wage rigidity, shifts in household labor supply still have limited aggregate effects, and so the maximal error remains relatively small.

E.2 Other estimated business-cycle models

Approximate consumption demand equivalence is not just a feature of my particular HANK model, but similarly holds in many canonical models of the previous business-cycle literature. In this section I illustrate this claim with two examples: (i) Justiniano et al. (2010) as an
Approximate Demand Equivalence, Justiniano et al. (2010)

In the estimated model of Justiniano et al. (2010), consumption demand equivalence fails only because Assumption 3 is not satisfied: wealth effects in labor supply are not zero, and hours worked are not fully demand-determined. However, prices and wages are estimated to be very sticky, and so—consistent with Christiano (2011a)—hours worked are still largely demand-determined, at least in the short run. This discussion suggests that demand equivalence should hold nearly exactly. Figure E.2 shows that this is indeed the case: for a consumption demand (impatience) shock with persistence $\rho_b = 0.1$ (and thus short-lived, similar to the stimulus check in my baseline HANK model), the error associated with the demand equivalence approximation is barely visible.

Schmitt-Grohé & Uribe (2012). The model of Schmitt-Grohé & Uribe (2012) similarly breaks consumption demand equivalence only through violation of Assumption 3. Wages and prices are now flexible, so labor is never demand-determined; however, near-exact demand equivalence still obtains because wealth effects in labor supply are essentially absent.

Figure E.2: Consumption impulse response decompositions and demand equivalence approximation in the model of Justiniano et al. (2010), solved at the posterior mode and for an impatience shock with persistence $\rho_b = 0.1$. The direct response and the indirect general equilibrium feedback are computed following Definition 1.
Figure E.3: Consumption impulse response decompositions and demand equivalence approximation for the HANK model with flexible prices & wages. The direct response and the indirect general equilibrium feedback are computed following Definition 1.

Adapted to the notation of this paper, household preferences are given as

\[ u(v) = \frac{v^{1-\sigma} - 1}{1 - \sigma} \]

where

\[ v_t = c_t - b c_{t-1} - \psi \ell_t^\theta s_t \]

and

\[ s_t = (c_t - b c_{t-1})^\gamma s_{t-1}^{1-\gamma} \]

As \( \gamma \to 0 \), there are no wealth effects in labor supply. Since both the Bayesian and frequentist estimation exercises in the paper give a very precise point estimate of \( \gamma = 0 \) (see their Table II), I conclude that Assumption 3 holds (essentially) exactly.

E.3 Labor supply

The “flexible price” error line in Figure 6 corresponds to an economy in which prices and wages re-set every 1.25 quarters on average (so \( \phi_p = \phi_w = 0.2 \)). Figure E.3 shows the corresponding full decomposition of aggregate impulse responses: there is significant general
Approximate Demand Equivalence, Multiple Goods

Figure E.4: Consumption impulse response decompositions and demand equivalence approximation for the HANK model with multiple goods. The direct response and the indirect general equilibrium feedback are computed following Definition 1.

equilibrium crowding out, and the simple demand equivalence approximation misses a large fraction of that crowding-out.

E.4 Multi-sector economy

Figure E.4 shows impulse response decompositions and the demand equivalence approximation for government purchases of (i) the labor-intensive consumption good (black) and (ii) the capital-intensive consumption good (purple). In both, relative price effects increase the approximation error relative to the baseline economy. General equilibrium MPC multiplier effects, however, increase the error for purchases of the labor-intensive good, and decrease it for purchases of the capital-intensive good. Figure 6 considers government purchases of the labor-intensive good (which give a larger bias), and is thus conservative.

E.5 Useful government spending

The demand equivalence approximation generally fails if government expenditure is useful, in the sense that it is either (non-separably) valued by households and/or productive.
PRODUCTIVE SPENDING. I provide the missing details for the discussion in Section 4.2. The intermediate goods production function is generalized to take the form

\[ y_{jt} = (k^g_{jt})^{\alpha_g} (u_{jt}k_{jt-1})^{\alpha} \ell_{jt}^{1-\alpha} \]

I set \( \alpha_g = 1 \), giving a two-year cumulative government spending multiplier that is around 30 per cent larger than the unit multiplier in the baseline model, roughly consistent with the empirical evidence reviewed in Leduc & Wilson (2013) and Gechert (2015). Results are reported in Figure 6.

VALUED SPENDING. In Appendix C.3 I discuss examples of household preferences over government expenditure that are consistent with exact demand equivalence. For a generic preference specification \( u(c, \ell; g) \), any change in government spending \( \hat{g} \) will induce a (zero net present value) consumption response path \( \hat{c}^{PE}_g \), defined exactly as in Definition 1. Following the same logic as in the proof of Proposition 1, it follows immediately that, if

\[ \hat{c}^{PE}_g + \hat{g} = \hat{c}^{PE}_\tau \]

then we have

\[ \hat{c}_\tau = \hat{c}^{PE}_\tau - \hat{c}^{PE}_g + \hat{c}_g \] (E.1)

Leeper et al. (2017) estimate that private and public consumption are (weak) complements. Given their preference specification, researchers could in principle back out \( \hat{c}^{PE}_g \) and thus implement the extended demand equivalence approximation of (E.1). However, since the estimated complementarity is small, I have—in unreported simulations—found a naive approximation (i.e., ignoring \( \hat{c}^{PE}_g \)) to actually not do much worse. These results are available upon request.

E.6 Open Economy

I emphasize that the open economy error displayed in Figure 6 is small only because my model economy is fairly closed, with \( \phi = 0.89 \). While fitting for the U.S., such a calibration is clearly not appropriate for all economies; for example, setting \( \phi = 0.5 \) (which corresponds to typical calibrations for small, very open economies), the peak demand equivalence error is much larger, at around 15 per cent.
Approximate Demand Equivalence, Imperfect Matching

Figure E.5: Consumption impulse response decompositions and demand equivalence approximation in the estimated HANK model, with imperfect demand matching, following (E.2). The direct response and the indirect general equilibrium feedback are computed following Definition 1.

E.7 Interest rates

In Figure 6 I consider a two-asset model with a penalty on household liquid savings. That model is a particularly stringent test of the demand equivalence approximation: it pushes the approximation error upwards, reinforcing the labor supply bias and thus giving me the largest possible error.

I have instead also considered model variants with large borrowing wedges (e.g., reflecting credit card debt). With indebted households facing large effective rates of return, the interest rate channel now imparts a (small) negative bias, largely offsetting the small positive bias of the labor supply channel. Results for these model variants are available upon request.

E.8 Imperfect demand matching

The excess demand path in Figure 2 is matched well, but of course not perfectly. As discussed in Section 3.2, with imperfect demand matching, my aggregation procedure will be correct up to the general equilibrium effects of a shock that induces an aggregate net excess demand path equal to the matching error path. To gauge the distortions associated with moderate mis-matching of the kind observed in my empirical applications, I again consider the estimated HANK model of Section 4.1, but now do not assume perfectly matched excess demand paths;
instead, I construct the demand equivalence approximation for an inaccurately matched government spending path \( \hat{g}_g \) with

\[
\hat{g}_{gt} = (1 + \nu_t) \times \hat{c}_{rE}^{PE}
\]  

(E.2)

where \( \nu_t \sim N(0, \sigma^2_{\nu}) \). I set \( \sigma^2_{\nu} \) to get average errors identical in size to those displayed in Figure 2; this gives \( \sigma^2_{\nu} = 0.123 \).

I then construct the demand equivalence approximation for 10,000 draws of the error sequence \( \nu \), and for each compute the maximal prediction error relative to the peak true consumption response. I find that 95 per cent of prediction errors lie below 9.3 per cent, so the approximation remains quite accurate.\(^{53}\) The intuition is simple: Since the model only features relatively moderate general equilibrium amplification, prediction errors for consumption can only be large if the error in demand path matching itself is substantial. This error, however, is by construction small, and thus so are the overall approximation errors. To illustrate, Figure E.5 shows the quality of the demand equivalence approximation for one particular draw of the error sequence \( \nu \).

\(^{53}\)Most of the large approximation errors come from draws in which the \( \nu \)'s are so far from 0 that demand matching is clearly violated, so the results displayed here are actually an upper bound on likely inaccuracies.
F Further applications

This appendix presents two further applications of the demand equivalence methodology: bonus depreciation in Appendix F.1 and income redistribution in Appendix F.2.

F.1 Bonus depreciation

I use the demand equivalence approach to estimate aggregate general equilibrium counterfactuals for aggregate investment, output and consumption following an expansionary bonus depreciation stimulus policy.

DIRECT RESPONSE. My estimates of the direct response of investment to the shock rely heavily on Zwick & Mahon (2017) and Koby & Wolf (2020), who exploit cross-sectional firm-level heterogeneity in the exposure to bonus depreciation investment stimulus. In Koby & Wolf (2020), we estimate dynamic regressions akin to (9) and give sufficient conditions under which the regression estimands are identical to or at least informative about the desired partial equilibrium investment spending responses $\hat{i}_{q}^{PE}$. The discussion is largely analogous to that in Proposition 2 (see Appendix D.2 for details).

With the direct investment spending response $\hat{i}_{q}^{PE}$ thus measured, it remains to recover the corresponding output path $\hat{y}_{q}^{PE}$. In the absence of direct measurement of this path, I propose to construct it by imposing the same production function—a simple Cobb-Douglas production function in capital and labor, with decreasing returns to scale—and the same competition structure—the separation of intermediate goods producers, retailers and aggregators—as in my baseline structural model. Under those assumptions we get

$$\hat{y}_{q}^{PE} = \frac{\alpha \nu}{1 - (1 - \alpha) \nu} \times \hat{k}_{q,t-1}$$

(F.1)

I set $\alpha = 0.2$, $\nu = 1$ and $\delta = 0.016$, in line with standard modeling practice in general and my estimated HANK model in particular. A more robust approach, of course, would be to directly measure $\hat{y}_{q}^{PE}$. I leave such an extension to future work.

Note that (F.1) heavily leverages the fact that competition among intermediate goods producers is perfect, so nominal rigidities only matter in general equilibrium, via feedback through intermediate goods prices $p_{I,t}$. This assumption is popular in structural modeling (e.g. Ottonello & Winberry, 2018).
Investment Tax Credit, Impulse Responses

Figure F.1: Investment, output and consumption responses to an investment tax incentive shock, quarterly frequency, with the partial equilibrium net excess demand path matched to a linear combination of government spending shocks. “KW” refers to Koby & Wolf (2020); details are given in Appendix D.2. The investment and output responses are computed in line with (19) - (20), while the consumption response is just the response after the identified combination of government spending shocks. The grey areas again correspond to 16th and 84th percentile confidence bands.

of direct investment spending responses to a one-quarter bonus depreciation shock worth around 8 cents, a shock similar in magnitude to the stimulus of 2008-2010. The solid green line extrapolates the empirical estimates to a full response path using a Gaussian basis function, similar to Barnichon & Matthes (2018). I take this extrapolated path to be the empirical estimate of the full spending response path $\hat{r}^{PE}$.

Investment demand increases substantially and persistently in response to the stimulus. Since capital is pre-determined, and since all prices faced by firms (except for taxes and so effective capital goods prices) are fixed by the nature of the partial equilibrium exercise, output does not increase on impact, but instead only gradually increases over time. Together,
the investment and output responses translate into a more complicated intertemporal net excess demand profile, displayed in the top left panel: Net excess demand is large and positive on impact (due to higher investment demand), but turns negative over time, as additional capital becomes productive and so expands the productive capacity of the economy.

**Aggregation: the missing intercept.** Following (19), it remains to replicate the estimated net excess demand path through a suitable list of government spending shocks:

\[
\hat{I}_q^{PE} - \hat{Y}_q^{PE} = \sum_{k=1}^{n_k} \gamma_k \times \hat{g}_{g_k}
\]

It is unlikely that any single estimated spending shock can replicate the reversal documented in Figure F.1. Encouragingly, much previous work on fiscal multipliers actually estimates the effects of *delayed* increases in government spending (Ramey, 2011; Caldara & Kamps, 2017)—that is, government spending news shocks. In principle, combining these delayed spending responses with the immediate spending effect estimated in the first application in Section 3.3 should allow me to replicate the net demand effects of the investment tax credit.

To operationalize this insight, I consider the same VAR as before, but now study the responses to residualized innovations in both the instrument equation as well as the equation for government expenditure itself. The first innovation is simply the shock studied in Section 3.3, while the second innovation is similar to the popular recursive identification scheme of Blanchard & Perotti (2002), augmented to include forecast errors as a control for anticipation effects. Consistent with previous work, I find the effects of the Blanchard-Perotti shock to be delayed, and so a linear combination of the two shocks allows me to match the implied net excess demand path of the investment demand shock, as shown in the top left panel of Figure F.1. Further details on the empirical implementation—in particular the construction of standard errors for general equilibrium feedback—are provided in Appendix D.3.

**Macro counterfactuals.** All results for general equilibrium counterfactuals are displayed in Figure F.1. With the requirement that \( \hat{g}_g = \hat{I}_q^{PE} - \hat{Y}_q^{PE} \) satisfied, the investment and output panels implement the additive decompositions in (19) and (20), respectively. My main finding is that the substantial partial equilibrium investment demand responses estimated in Zwick & Mahon (2017) and Koby & Wolf (2020) also survive in general equilibrium. The increase in investment demand is accommodated through a sharp immediate
increase in output as well as a smaller and somewhat delayed drop in consumption.\textsuperscript{55} Taken together, the large direct investment spending responses estimated in micro data as well as extant evidence on the transmission of aggregate government spending shocks suggest that bonus depreciation investment incentives provide a sizable macroeconomic stimulus.

**F.2 Income redistribution**

I use the demand equivalence approach to estimate the response of aggregate consumption to a short-lived increase in (labor) income inequality.

**Direct response.** Similar to Auclert & Rognlie (2018), I recover the direct response of aggregate consumer net excess demand through a structural, partial equilibrium consumption-savings problem—the consumption-savings problem of my estimated HANK model of Section 4.1. Specifically, I change the household budget constraint to

\[
c_{it} + b_{it}^h = \left(1 - \tau_t\right)w_t e_{it} \ell_{it} (1 + \varepsilon_{z,t} z_{it}) + \frac{1 + \delta_{it-1}}{1 + \pi_t} b_{it-1}^h + \tau_{it} + d_{it}, \quad b_{it}^h \geq b
\]

where \(\varepsilon_{z,t}\) is an aggregate inequality shock, \(\int_0^1 e_{it} z_{it} di = 0\), and \(z_{it} \propto e_{it}\). A one-off shock \(\varepsilon_{z,0} > 0\) thus leads to a one-period increase in labor income inequality, with more productive households receiving a larger share of total aggregate income. Solving the partial-equilibrium consumption-savings problem for all households \(i\) given \(\varepsilon_{z,0}\) and then aggregating, we recover the direct response path \(\hat{c}_{PE}^z\) for the inequality shock \(\varepsilon_z\). I scale the shock to lead to a decline in consumption demand of 1 per cent on impact; the green lines in the top panels of Figure F.2 show the implied full consumption response path.

**Aggregation: the missing intercept.** Following the discussion in Section 5.1 it remains to replicate the net excess demand path \(\hat{c}_{PE}^z\) through a suitable list of government spending shocks:

\[
\hat{c}_{PE}^z = \sum_{k=1}^{n_k} \gamma_k \times \hat{g}_{k}
\] (F.3)

\textsuperscript{55}Note that, with technology fixed and capital pre-determined, the impact increase in output must reflect an increased use of the other factor of production: labor. This is entirely consistent with my assumption of weak wealth effects in the first part of the paper: for example, with Assumption A.3 holding because of sticky wages, hours worked will increase in \textit{general equilibrium} because the intermediate goods price has increased, pushing up firm labor demand at \(t = 0\).
Redistribution Shock, Impulse Responses

**Figure F.2:** Consumption, debt and tax responses to a redistribution shock, with the partial equilibrium net excess demand path matched to a linear combination of government spending shocks. The consumption response is computed in line with Proposition 1. The plot also shows the required demand matching and financing alignment. The dashed lines again correspond to 16th and 84th percentile confidence bands.

Note that, since the increase in inequality is a non-policy shock, the path $\hat{c}^{PE}$ has zero net present value, so any list of government spending shocks satisfying (F.3) also necessarily has zero net present value. The spending change itself thus in principle can be—and for our purposes in fact needs to be, by condition (ii)—purely deficit-financed. I proceed as in Appendix F.1 and use a combination of transitory and persistent fiscal spending changes to align the net excess demand paths (top left panel). The two bottom panels of Figure F.2 show that, as required, the fiscal expansion is deficit-financed, with little response of taxes.56

---

56Of course taxes can and generally will respond to both shocks *in general equilibrium*. With condition (i) of aligned demand paths satisfied, we know that the path $g_g$ requires no direct financing. Encouragingly,
MACRO COUNTERFACTUALS. The top right panel of Figure F.2 shows the desired aggregate general equilibrium consumption counterfactual. The macro-equivalent fiscal contraction leaves consumption largely unchanged on impact, before then leading to an increase in spending. We thus conclude that the temporary increase in inequality leads to a significant contraction of consumption on impact (mirroring the direct spending effect), before then leading to a delayed boom.

the bottom right panel of Figure F.2 gives no reason to believe that there was any kind of important direct tax financing.
G Further proofs and auxiliary lemmas

G.1 Proof of Lemma A.1

To prove Lemma A.1 I proceed in two steps. First, I show that all relevant inputs to the household and firm problems can be obtained as functions only of $x$ and $\varepsilon$. Second, I show sufficiency of the four equations in the statement of the result.

1. Given $(i^h, y)$, the Taylor rule of the monetary authority allows us to back out the path of inflation $\pi$. Thus all inputs to the firm problem are known, so indeed $s^f = s^f(x)$. We thus obtain $y$, $i$, and $\ell^f$. Setting $\ell = \ell^f$ and since $\tau^e \in x$, all inputs to the household problem are known, so indeed $s^h = s^h(x)$. We can thus also solve for the path of consumption, so that indeed $s^u = s^u(x; \varepsilon)$, and we finally recover union labor supply.

2. Optimal household, firm and government behavior is assured by assumption. It thus remains to check that (i) all markets clear; (ii) the input path of output is consistent with firm production; and (iii) the lump-sum tax path is consistent with the government budget constraint. Output and labor market-clearing are ensured by the first two equations in the statement of the lemma, and asset market-clearing then follows from Walras’ law. The third set of equations in the lemma statement then ensures consistency in aggregate production, while the fourth set—which uses that the only relevant endogenous quantities for the government budget constraint are $(i^b, \pi, w, \ell)$—ensures that the government budget constraint holds period-by-period and that $\lim_{t \to \infty} \hat{b}_t = 0$, by definition of $\tau^e(\bullet)$.

Together, 1. - 2. establish sufficiency of the conditions in the statement of Lemma A.1. Necessity is immediate, completing the argument. $\square$

G.2 Proof of Proposition 2

The proof proceeds in three steps. First, I show that aggregate impulse responses to the heterogeneous shocks $\{\varepsilon_{\tau i0}\}$ are identical to impulse responses to the common aggregate shock $\varepsilon_{\tau 0} \equiv \int_0^1 \varepsilon_{\tau i0}$. Second, I prove that $\hat{c}_{\tau i} - \hat{c}_\tau = (\xi_{\tau i0} - 1) \times \hat{c}_s^{PE} + \zeta_i$, where $\int_0^1 (\xi_{\tau i0} - 1) \zeta_i di = 0$. And third, I exploit standard properties of fixed-effects regressions to complete the argument. As in the proof of Proposition 1, I use the notation $\frac{\partial}{\partial \varepsilon_s}$ to denote derivatives for a shock path where only entries of shock $s$ are non-zero.

---

57 Note that the path of the intermediate goods price $p^I$ is obtained from the problem of retailers.
1. We study impulse responses to the shock path $\varepsilon_\tau \equiv e_1$, where $e_1 = (1, 0, 0, \ldots)'$. The direct partial equilibrium response of consumption to the shock is

$$\tilde{c}_r^{PE} \equiv \int_0^1 \frac{\partial c_i}{\partial \varepsilon_\tau} \times \xi_{\tau i0} \times \varepsilon_\tau di$$

where $c_i(\bullet)$ is the consumption function of individual $i$, defined analogously to the aggregate consumption function $c(\bullet)$. Since $\int_0^1 \xi_{\tau i0} di = 1$ and since $\xi_{\tau i0}$ is assigned randomly across households (and so does not correlate with $\frac{\partial c_i}{\partial \varepsilon_\tau} \times \varepsilon_\tau$ at any $t$), we have that

$$\hat{c}_r^{PE} = \int_0^1 \frac{\partial c_i}{\partial \varepsilon_\tau} \times \varepsilon_\tau di \times \left[ 1 + \int_0^1 (\xi_{\tau i0} - 1) di \right] = \int_0^1 \frac{\partial c_i}{\partial \varepsilon_\tau} \times \varepsilon_\tau di$$

The direct partial equilibrium response of aggregate consumption is thus identical to the response in an economy where all individuals $i$ face the common shock $\varepsilon_\tau$. The same argument applies to the desired partial equilibrium contraction in labor supply, $\hat{\ell}_r^{PE}$. But if direct partial equilibrium responses are the same, then general equilibrium adjustment is the same, and so all aggregates are the same.

2. Consumption of household $i$ along the transition path satisfies

$$\tilde{c}_{it} = \frac{\partial c_i}{\partial x} \times \tilde{x} + \frac{\partial c_i}{\partial \varepsilon_\tau} \times \xi_{\tau i0} \times \varepsilon_\tau$$

where $x$ was defined in Lemma A.1. We thus get

$$\tilde{c}_{it} - \tilde{c}_r = (\xi_{\tau i0} - 1) \times \frac{\partial c}{\partial \varepsilon_\tau} \times \varepsilon_\tau + \left( \frac{\partial c_i}{\partial x} - \frac{\partial c}{\partial x} \right) \times \tilde{x} + \xi_{\tau i0} \left( \frac{\partial c_i}{\partial \varepsilon_\tau} - \frac{\partial c}{\partial \varepsilon_\tau} \right) \times \varepsilon_\tau$$

Note that, since by definition we have $\int_0^1 \frac{\partial c}{\partial x} di = \frac{\partial c}{\partial x}$ and $\int_0^1 \frac{\partial c_i}{\partial \varepsilon_\tau} di = \frac{\partial c_i}{\partial \varepsilon_\tau}$, the residual term $\zeta_i$ must satisfy $\int_0^1 (\xi_{\tau i0} - 1) \zeta_i di = 0$.

3. By the standard properties of fixed-effects regression, we can re-write regression (9) as

$$\tilde{c}_{it+h} - \tilde{c}_{t+h} = \beta_{rh} \times (\xi_{it} - 1)\varepsilon_{rt} + u_{it+h} - u_{t+h}$$

(G.1)
By standard projection results, the estimand $\beta_\tau$ satisfies

$$\beta_\tau = \frac{\int_0^1 [(\xi_{\tau i0} - 1)\hat{c}_{\tau i} + \xi_i] (\xi_{\tau i0} - 1)di}{\int_0^1 (\xi_{\tau i0} - 1)^2di} = \hat{c}_{\tau}^{PE}$$

where I have used the fact that $\text{Var}(\xi_{\tau it}) > 0$.

\[\square\]

\section*{G.3 Proof of Proposition 3}

Following the same steps as in the proof of Proposition 1, but without imposing Assumption 3, we get the two direct shock responses as

$$\begin{pmatrix}
\frac{\partial c}{\partial \xi_{\tau}} \\
\frac{\partial \ell^h}{\partial \xi_{\tau}} \\
\frac{\partial \tau^e}{\partial \xi_{\tau}}
\end{pmatrix} \times \varepsilon_{\tau} = \begin{pmatrix} \hat{c}_{\tau}^{PE} \\ \hat{\ell}_{\tau}^{PE} \\ \hat{\tau}_{\tau}^{e,PE} \end{pmatrix}, \quad \text{and} \quad \begin{pmatrix}
\frac{\partial c}{\partial \xi_{g}} \\
\frac{\partial \ell^h}{\partial \xi_{g}} \\
\frac{\partial \tau^e}{\partial \xi_{g}}
\end{pmatrix} \times \varepsilon_{g} = \begin{pmatrix} \hat{g}_{g} \\ 0 \\ 0 \end{pmatrix}.$$

The general equilibrium response paths of consumption thus now satisfy

$$\hat{c}_{\tau} = \left(\frac{\partial c}{\partial \xi_{\tau}}\right)_{\hat{c}_{\tau}^{PE}} \times \varepsilon_{\tau} + \frac{\partial c}{\partial x} \times \mathcal{H} \times \begin{pmatrix} \hat{c}_{\tau}^{PE} \\ \hat{\ell}_{\tau}^{PE} \\ \hat{\tau}_{\tau}^{e,PE} \end{pmatrix}, \quad \text{and} \quad \hat{c}_{g} = 0 + \frac{\partial c}{\partial x} \times \mathcal{H} \times \begin{pmatrix} \hat{g}_{g} \\ 0 \\ 0 \end{pmatrix}$$

By properties (i) and (ii) of the fiscal spending shock, we can combine the two expressions above to get

$$\hat{c}_{\tau} = \hat{c}_{\tau}^{PE} + \hat{c}_{g} + \frac{\partial c}{\partial x} \times \mathcal{H} \times \begin{pmatrix} 0 \\ \hat{\ell}_{\tau}^{PE} \\ 0 \end{pmatrix} + \text{error(\hat{c}_{\tau}^{PE})}$$

In particular, the third term is immediately seen to be the general equilibrium response of consumption to a leisure shock leading to a desired union labor supply adjustment of $\hat{\ell}_{\tau}^{PE}$,
G.4 Auxiliary lemma for Proposition A.1

Lemma G.1. Consider the structural model of Section 2.1. Under Assumptions A.1 to A.4, all firm sector price inputs $s^f$ can be derived as functions only of the path of aggregate consumption $c$. Sequences of consumption $c$ and shocks $\varepsilon$ are part of a perfect foresight equilibrium if and only if

$$c + i(s^f(c); \varepsilon) + g(\varepsilon) = y(s^f(c); \varepsilon)$$

where the production and investment functions $y(\cdot)$, $i(\cdot)$ are derived from optimal firm behavior.

To prove Lemma G.1 I as before proceed in two steps. First, I show that all relevant inputs to the firm problem can be obtained as functions only of $c$ and $\varepsilon$. Second, I show sufficiency of the aggregate market-clearing equation.

1. By Assumptions A.2 and A.3, the household block admits aggregation to a single representative household with period felicity function $u(c) - v(\ell)$. Given $c$, the Euler equation of the representative household allows us to back out the path of real interest rates $r$. Given $r$, the Fisher equation and the Taylor rule of the monetary authority (by Assumption A.4) allow us to recover the paths of nominal interest rates $i^b$ and aggregate inflation $\pi$, and so by the NKPC of retailers we recover $p^i$. Next, given Assumption A.3, the wage-NKPC allows us to recover the path of real wages $w$. Together with $\varepsilon$ we thus have all inputs to the firm problem, and in particular indeed $s^f = s^f(c)$, as claimed.

2. Optimal firm and government behavior is assured by construction. Next, since the Euler equation and wage-NKPC hold, the only missing condition for household optimality is the lifetime budget constraint. But by assumption the aggregate market-clearing condition (G.2) holds at all times, so the household lifetime budget constraint must hold. Finally, the labor market clears by Assumption A.3.

Together, 1. - 2. establish sufficiency of the conditions in the statement of Lemma G.1. Necessity is immediate, completing the argument.
G.5 Proof of Proposition A.1

By Lemma G.1, a perfect foresight equilibrium is, to first order, a solution to

\[
\hat{c} + \frac{\partial i}{\partial c} \times \hat{c} + \frac{\partial i}{\partial \varepsilon} \times \varepsilon + \frac{\partial g}{\partial \varepsilon} \times \varepsilon = \frac{\partial y}{\partial c} \times \hat{c} + \frac{\partial y}{\partial \varepsilon} \times \varepsilon
\]

As before, we thus in general have

\[
\hat{c} = \mathcal{H} \times \left( \frac{\partial i}{\partial \varepsilon} \times \varepsilon - \frac{\partial y}{\partial \varepsilon} \times \varepsilon + \frac{\partial g}{\partial \varepsilon} \times \varepsilon \right)
\]

for a unique linear map \( \mathcal{H} \). Now again use the notation \( \frac{\partial}{\partial \varepsilon} \) to denote derivatives for a shock path where only entries of shock \( s \) are non-zero. In response to investment tax and government spending shocks, the response path of investment satisfies

\[
\hat{i}_q = \frac{\partial i}{\partial q} \times \varepsilon_q + \frac{\partial i}{\partial c} \times \mathcal{H} \times \left( \hat{i}^{PE}_q - \hat{y}^{PE}_q \right)
\]

and

\[
\hat{i}_g = 0 + \frac{\partial i}{\partial c} \times \mathcal{H} \times \hat{g}_q
\]

respectively. This establishes (19). The equations for output are exactly analogous. \( \square \)

G.6 Proof of Corollary C.1

It is straightforward to show that a generalization of Lemma A.1 holds for the system

\[
e(s^h(x); \varepsilon) + i(s^f(x); \varepsilon) + g(\varepsilon) = y(s^f(x); \varepsilon)
\]

\[
\ell^h(s^h(x); \varepsilon) = \ell^f(s^f(x); \varepsilon)
\]

\[
y(s^f(x); \varepsilon) = y
\]

\[
\tau^e(s^f(x); \varepsilon) = \tau^e
\]

where \( e \) is now the aggregated optimal household expenditure function for durable and non-durable consumption. Applying the same steps as in the proof of Proposition 1 to this new system, the result follows. \( \square \)