

# Moderation of an Ideological Party<sup>1</sup>

Vlasta Pokladniková

Muhamet Yildiz<sup>2</sup>

MIT

First Version: May, 1998

This Version: March, 2007

<sup>1</sup>We thank Daron Acemoglu, Estelle Cantillon, Stephen Coate, Avner Greif, Philip Keefer, Casey Rothschild, Michael Schwarz, Murat Sertel, Robert Wilson, two anonymous referees, and the seminar participants at Harvard, MIT, NYU, Rochester, and Stanford.

<sup>2</sup>Corresponding author: MIT Department of Economics, 50 Memorial Drive, Cambridge, MA 02142. E-mail: myildiz@mit.edu.

## Abstract

It is a common fear in many countries that ideological parties will come to power through elections but will implement extreme policies. Many countries cope with this problem by overriding the election results when such parties are elected. We demonstrate that the alternative approach of containing these parties within the democratic system is more effective. We show that, as the probability of state's intervention in the next elections increases, an ideological party implements a more extreme policy in equilibrium. This hurts the median voter. Our main result shows that from the median voter's perspective, the optimal intervention scheme can be implemented by committing not to intervene and adjusting election times appropriately. That is, elections are a better incentive mechanism than the threat of a coup.

*Journal of Economic Literature Classification Numbers:* C72, D72

*Keywords:* Moderation, Ideological Parties, Elections, Coup

# 1 Introduction

How should a democratic regime defend itself against those political parties that would come to power through democratic channels but would implement their extreme policies—policies inconsistent with the state’s fundamental principles—or even end the democratic regime in order to establish their own ideological system? One commonly employed defense is that of direct confrontation: the state bans such parties, prosecutes individuals who form such parties, or intervenes in election results whenever such parties come to power. This approach is apparently taken by many countries, such as Algeria, Turkey, and several Latin American countries. The organization of the state in these countries facilitates the state intervention. For example, in Turkey, the national intelligence agency is controlled by the army, which has carried out several coups; the army has its own courts and is allowed to defend the system against internal enemies according to its internal code. Moreover, there are state security courts that regularly outlaw such parties (mainly Kurdish or Islamic) and prosecute their leaders.<sup>1</sup> Similar roles and means are given to the army in many Latin American countries (Huntington, 1957 and Huntington, 1968; Chapter 4), where the coups have been frequently used against the leftist parties. An alternative approach to defending the state against such ideological parties aims to contain them within the system by allowing them to come to power, empowering the elected offices, and minimizing the nondemocratic interference in government (see Huntington (1968) and Rawls (1993)). Within a simple formal model, this paper shows that this latter approach may provide better incentives—as elections are better incentive mechanisms than threats of intervention—and that the incentives within the system are undermined by possibility of future intervention.

We consider a simple two-period model with a single policy issue: a real number is to be chosen in each period. There is an ideological party (IP), whose preferences (i.e., type) are its private information. The alternative to IP is a fixed policy. In each period, there is an election between IP and its alternative. If IP wins, it implements a policy for that period, which is observable. Otherwise, the alternative is implemented. After the second-period election, if IP wins, the state overrides the election results with

---

<sup>1</sup>Some of these laws have been changed recently as a part of “democratization reform,” a precondition for Turkey’s membership in the European Union.

some probability  $q$  and implements the alternative. Notice that, as in the citizen-candidate models (Osborne and Slivinski, 1996; Besley and Coate, 1997), IP has intrinsic policy preferences. When  $q$  is independent of earlier policy choices and we regard  $1 - q$  as a discount factor, our model becomes a stripped down version of the model by Alesina and Cukierman (1990), who show in their model that ideological parties choose to implement moderate policies in order to win the next election. The probability of a coup differs from a simple discount factor in that (i) it may vary as we change the past policies, (ii) it is usually endogenously determined in equilibrium, and, most importantly for this paper, (iii) it can be changed through the changes in the organization of the state, thus permitting a discussion of the optimal coup scheme.

When  $q$  is constant, each equilibrium can be summarized with two parameters  $a$  and  $b$ , where  $a < b$ . IP wins the second election if and only if its policy in the first period is not more extreme than  $a$ , and IP chooses to implement such a policy in the first period if IP is not more extreme than  $b$ . In the first period, if its type is in between  $a$  and  $b$ , IP chooses to moderate and implements  $a$ ;<sup>2</sup> otherwise, it implements the policy it finds best. Here,  $a$  is the least moderate policy that the median voter expects IP of a moderate type to implement in order to signal its type convincingly, and  $b$  is the most extreme type who is willing to moderate in order to win the next election.

These parameters change with respect to the probability  $q$  of a state intervention in the next election as follows. If  $q$  decreases, then winning the next elections becomes more important, and hence IP's gain from moderation increases regardless of its type. It turns out that this typically has two effects: first,  $a$  is lower—i.e., voters expect a moderate IP to implement more moderate policies. Second, despite this, some more extreme types of IP are now willing to moderate—i.e.,  $b$  is higher. A lower  $a$  and a higher  $b$  mean that now the set of types of IP who choose to moderate at the first period is larger, and they moderate more. Hence, every type of IP responds to a lower

---

<sup>2</sup>See the references in Alesina and Cukierman (1990) for empirical evidence for this form of moderation. This form of moderation is also very familiar in Turkish political life. Pro-Islam Welfare Party, which was outlawed after the army's intervention in February 28<sup>th</sup>, 1998, was accused of *Takiyye*, the practice of hiding one's beliefs in order to survive (and perhaps to change the conditions in the future). (See Yavuz (1997) for the history of Welfare Party.) On November 3<sup>rd</sup>, 2002, another pro-Islam party (AKP) has been elected and has been implementing moderate policies.

second period coup probability ( $q$ ) by implementing (if anything) a more moderate policy.<sup>3</sup> This remains true when  $q$  depends on the first-period policy: when  $q$  decreases uniformly across first-period policies, IP implements more moderate policies in the first period. These results suggest that the state's interventionist organization might even be causing the polarized political spectrum in the above countries. We further show that the median voter gains from such moderation, provided that it does not lead IP to implement policies on the other side of the median, an event that will be referred to as overmoderation.

Now suppose that, by choosing an appropriate organization of the state, the probability of intervention can be made to be any function  $q$  of the first period policy. Suppose also that we want to maximize the welfare of the median voter (for normative reasons that will be explained in Section 4). What, then, is the optimal  $q$ —i.e., the  $q$  that maximizes the median voter's payoff? We show that the optimal  $q$  is a constant function: there exists some  $q^*$  in  $(0, 1)$  associated with some equilibrium  $e^*$  such that for every function  $\tilde{q}$  and every associated equilibrium  $\tilde{e}$ , the median voter prefers the equilibrium  $e^*$  under constant probability  $q^*$  of intervention to the equilibrium  $\tilde{e}$  under  $\tilde{q}$ . The parameters for equilibrium  $e^*$  are  $a^*$  and  $b^*$ , where  $a^*$  is the median voter's ideal policy and  $b^*$  is such that the median voter would be indifferent between IP and the alternative if he just knew that IP is not more extreme than  $b^*$ . We show that there cannot be any equilibrium (under any function  $q$ ) in which IP moderates when it is more extreme than  $b^*$ . Therefore,  $e^*$  both leads to the ideal policy of the median voter in the first period *and* allows moderation for any type that could possibly moderate. (Of course, some of these extreme types that moderate are much worse than the alternative for the median voter. Since the alternative is implemented after a coup, increasing coup schemes have the advantage of making these policies less likely to be implemented in the second period. As explained below, however, these schemes cause larger inefficiencies and yield lower payoffs for the median voter.) To implement this  $e^*$ ,  $q^*$  must be very small: for any  $q < q^*$ , IP loses the next election in

---

<sup>3</sup>In contrast, the discount factor has an ambiguous impact on the level of moderation in Alesina and Cukierman (1990). This difference stems partly from their assumption that expectations are linear. The equilibrium expectations are necessarily nonlinear, as the party's strategies (in both papers) are nonlinear in its type. Partly, we avoid ambiguity by formulating our result in terms of sets of equilibria.

any equilibrium, even if it implements the median voter's ideal policy. In reality, such a probability of intervention could be implemented by committing not to intervene and adjusting the election times so that the discount rate of IP is  $1 - q^*$ . It is in this sense elections are optimal.

It is rather surprising that  $q^*$  is constant—not increasing. One may naively have thought that the optimal probability  $q^*$  would be increasing, as such a scheme would lead IP to moderate more. It turns out that this is not desirable in equilibrium. This is because for a given  $a$ , when IP's type is in between  $a$  and some  $\alpha > a$ , IP will implement policies that are more moderate than  $a$  in order to decrease the probability of intervention. Then, when the median voter observes that  $a$  is implemented, he knows that IP is more extreme than  $\alpha$ . In order for him to vote for IP,  $b$  must be smaller. Therefore, in such equilibria, we will either have over-moderation of relatively moderate types or non-moderation of relatively extreme types—and typically both. To put it differently, elections are imperfect mechanisms in the sense that the median voter cannot commit to action plans that may turn out to be interim suboptimal. Schemes that elicit more information undermine the electoral incentives by revealing information that renders certain favorable plans interim suboptimal.

We take individuals' political preferences as given. These preferences have deeper economic, ethnic and cultural roots, and are interrelated to the ideology of political parties (Duverger, 1954; Sartori, 1976; Powel, 1986; Remmer, 1991). The latter factors will clearly affect the parties behavior—at least through the voters' preferences. In the countries mentioned above, ideologies are delineated by clear socioeconomic lines; the coups have been consistently against the parties that advocate the interests of lower classes. Most notably, economic factors such as poverty and income inequality affect these preferences and can empirically explain the coups well (Acemoglu and Robinson, 2006; see also Jackman, 1978; Johnson et al, 1984; Remmer, 1991; O'Kane, 1993).

In addition to the large empirical literature on coups and extremism, there is a sizeable informal literature, most notably Huntington (1968), that address the main issue addressed in this paper. The formal game theoretical literature is relatively small. In addition to Alesina and Cukierman (1990), Acemoglu and Robinson (2000, 2001, 2006) analyze economic theories of coup and political transition, and Wantchekon (1999) and Ellman and Wantchekon (2000) analyze a game theoretical model of voting

under the threat of a coup. Also, Banks (1990), Harrington (1993), Coate (2004), and Callander (2004) explore the role of campaign promises in signaling the candidates' ideology.

In the next three sections, we formalize our ideas within a simple model in which probability of coup is exogenously given function of the first-period policy. In Section 6, we present two extensions; in particular we endogenize the probability of coup. Section 7 concludes. Proofs are in the appendix.

## 2 Model

There are two dates,  $t \in \{0, 1\}$ , and for each date a policy  $x$ , which is a real number, is to be implemented as a policy. The main actors are an *ideological party* (denoted by IP) and the *median voter* (denoted by MV), representing a group of voters.<sup>4</sup> We write  $u_0(x)$  for the median voter's per-period payoff from policy  $x$  and assume that  $u_0$  is strictly concave and maximized at 0, the ideal policy for the median voter. The alternative to IP is a given policy  $s$ , which is the ideal policy of the state.<sup>5</sup> We assume that  $s < 0$ . The justification for this assumption is that the median voter's ideal policy is likely to differ from the policy that is best for the representative bureaucrat (representing the state officials who can carry out a coup), which can also be observed from the voting data in the above countries. (When  $s = 0$ , the problem becomes trivial; IP loses the elections.)

The order of the events is as follows.

1. There is an election at  $t = 0$ . If the median voter votes for IP, then IP wins.

---

<sup>4</sup>Under a mild single-crossing property, if the voters do not play weakly dominated actions, then the median voter's vote is indicative of whether IP wins an election. Our results remain intact in such a model (see our working paper).

<sup>5</sup>The fixed alternative  $s$  can be replaced with non-ideological parties, which do not have intrinsic policy preferences but choose a policy in order to win the elections, as in usual Downsian models, so that they can extract a known rent from governing. The median voter bears a cost  $\gamma$  for the rent or corruption. The analysis of this alternative specification is identical to that of our model, where the cost of corruption plays the same role as the difference between the ideal points of the state and the median voter;  $\gamma$  replaces  $u_0(0)$ .

2. If IP wins, then it chooses a policy  $x_0$ , which becomes public information; otherwise  $s$  is implemented.
3. At  $t = 1$ , there is another election as in period 0.
4. If IP wins the election at  $t = 1$ , then there will be a coup with probability  $q(x_0) \in (0, 1)$ , yielding  $s$ .
5. If IP wins and there is no coup, then IP chooses some  $x_1$ ; otherwise  $s$  is chosen.

The ideal policy for IP is denoted by  $z$ , which is referred to as IP's type. We assume that  $z$  is IP's private information. The cumulative distribution function (CDF) of  $z$  is denoted by  $F$ , and the probability density function (pdf) is denoted by  $f$ . For simplicity, we assume that  $z > s$ ,  $\Pr(z < 0) = 0$ , and  $f(z) > 0$  for each  $z > 0$ . Here,  $F$  represents the median voter's belief at the beginning; after observing IP's choice  $x_0$ , he updates his beliefs. We assume that, if IP loses the election at  $t = 0$ , MV adheres to his initial beliefs. We write  $E$  and  $E[\cdot|\cdot]$  for the unconditional and conditional expectations, respectively.

Assuming that the agents care only about the policy implemented, we write  $w(x, z)$  for the per-period benefit of any policy  $x$  for IP of any type  $z$ , where  $w$  is a twice continuously differentiable and strictly concave function, maximized at  $z$ . We normalize  $u_0$  and  $w$  so that  $u_0(s) = w(s, z) = 0$  for each  $z$ . Each agent maximizes the sum of his two per-period benefits. Everything described above is common knowledge. Throughout the paper, we will assume:

- A1.** Both  $w$  and the logarithm of  $w$  are supermodular:  $\partial^2 w(x, z) / \partial x \partial z > 0$  and  $\partial^2 \log(w(x, z)) / \partial x \partial z > 0$  whenever they are defined.
- A2.**  $E[u_0(z)] < 0$ .

A1 will play a crucial role in our monotone comparative statics and in separating IP's types (see Lemma 3). A2 states that the median voter would not vote for IP if he had no information about IP and believed that IP would implement the policy that it finds best.



**Example (Euclidean preferences)** Let the utility of an individual with ideal policy  $y$  from a policy  $x$  be  $-v(x - y)$  where  $v$  is an even and strictly convex function. After the normalization  $u_0(s) \equiv w(s, y) \equiv 0$  (by an additive constant), we have

$$u_0(x) = v(s) - v(x) \text{ and } w(x, y) = -v(x - y) + v(s - y) \quad (1)$$

at each  $x$  and  $y$ . One can easily check that these functions satisfy our assumptions whenever the mapping  $\zeta \mapsto 1/v(\zeta)$  is convex for  $\zeta > 0$  as in the canonical case.

Notice that  $x_0$  is a function of  $z$ . If IP comes to power at  $t = 0$ , then  $x_1$  is a function of  $x_0$  and  $z$ , and the median voter's second-period vote is conditioned on  $x_0$ ; otherwise,  $x_1$  is a function of  $z$ . A sequential equilibrium  $e^*$  is a pair of a *sequentially rational* strategy profile and posterior beliefs for MV (after observing  $x_0$ ) that are *consistent* with the strategy profile. That is, at each history each player maximizes his expected utility given his beliefs at that history and given that that history is reached, and the median voter's beliefs are derived through Bayes' rule at each  $x_0$  that is implemented by IP of some type  $z$ . We will write  $SE(q)$  for the set of sequential equilibria. For any  $e \in SE(q)$ , we write  $U_0(e, q)$  for the median voter's expected payoff at the node "IP comes to the power at  $t = 0$ " when equilibrium  $e$  is played and the probability of a coup is  $q$ . Notice that the median voter's ex ante payoff is  $\max\{U_0(e, q), 0\}$ .

**Remark** Our model has only two periods, but in our working paper we have shown that the analysis here extends to the stationary equilibria of multiple- or infinite-period models with slight modification—regardless of whether the state intervention is permanent, as in the case of the 1936 military intervention in Spain that led to decades of civil war and dictatorship, or transient, as in Turkey.

### 3 Moderation of Policy

In this section, under the restrictive assumption that  $q'$  is not too high, we characterize the sequential equilibria that satisfy the intuitive criterion described below. We show that as the level of  $q$  increases, the incentives of IP for winning the next election decreases, and the set of equilibria moves in a direction where the IP moderates less.

Observe that, at any sequential equilibrium  $e^*$ , the policy chosen by IP at  $t = 1$  is

$$x_1^*(z) = z \quad \text{for each } z. \quad (2)$$

That is, in the last period, IP implements the policy that it finds best. Therefore, after the history that IP comes to power at  $t = 0$ , there is a signaling game: IP with private information  $z$  chooses some  $x_0(z)$ ; observing  $x_0$ , MV votes—for or against IP. We require in this section that the substrategy profile in this signaling game passes the Intuitive Criterion of Cho and Kreps (1987), defined in the appendix. We write  $SE^*(q)$  for the set of sequential equilibria that satisfy this requirement.

The payoffs in the signaling game above are

	If IP wins	If IP loses
payoff of IP of type $z$	$w(x_0, z) + \delta(x_0) w(z, z)$	$w(x_0, z)$
payoff of MV	$u_0(x_0) + \delta(x_0) u_0(z)$	$u_0(x_0)$

where future benefits are discounted by the effective discount rate  $\delta(x_0) = 1 - q(x_0)$ , because with probability  $q(x_0)$ , there will be a coup and everyone will get 0.<sup>6</sup> Shifting  $q$  upwards is equivalent to shifting  $\delta$  downwards, which weakens the incentives of IP. Consequently, such a shift will result in more extreme policies by IP in the first period. For example, when  $q$  is identically 1, IP will always implement its ideal policy, leading to most extreme first-period policy. On the other hand, when  $q$  is identically 0, all types will have incentive to choose a more moderate policy in the first period if it results in IP winning the next elections. In that case, median voter will vote for IP only if  $x_0 = s$ , and all but very extreme types will choose  $s$  in the first period. That is, IP will "overmoderate" and pick policies that are closer to the status quo than the median voter's ideal policy. We will now establish this monotonicity result under the following assumption, which we will maintain only in this section.

**Assumption 1** *The probability  $q(x_0)$  of a coup is a twice differentiable, weakly increasing function of  $x_0$ . The function*

$$\hat{W}(x, z) = w(x, z) + (1 - q(x)) w(z, z) \tag{3}$$

*has a unique maximizer  $\hat{x}(z)$  and supermodular, and  $\hat{x}(z) \rightarrow \infty$  as  $z \rightarrow \infty$ .*

---

<sup>6</sup>This is because we have assumed that individuals are indifferent between IP's defeat and a coup. In reality, a coup has its social cost, and voters would prefer  $s$  being implemented without a coup to the one with a coup. In such cases, those who oppose IP may threaten the voters by a coup (see Wantchekon, 1999, and Ellman and Wantchekon, 2000).

Supermodularity ensures that  $\hat{x}$  is increasing. That is, if there were no election in the future, more extreme types would choose more extreme policies. This assumption is satisfied when the slope of  $q$  is not too high:  $q'(x_0) \partial w(z, z) / \partial z \leq \partial^2 w(x_0, z) / \partial x_0 \partial z$ . In general, this is a restrictive assumption, and we will *not* assume it in our main result, presented in the next section. That result will show that optimal  $q$  will not depend on  $x_0$  and hence will satisfy Assumption 1. Under this assumption, the next result characterizes the sequential equilibria that satisfy our requirement.

**Theorem 1** *Under Assumption 1, IP chooses policy  $x_0^*$  in an equilibrium at  $t = 0$  if and only if*

$$x_0^*(z) = \begin{cases} \hat{x}(z) & \text{if } z \leq \alpha, \\ a & \text{if } \alpha < z \leq b, \\ z & \text{otherwise} \end{cases} \quad \forall z \quad (4)$$

for parameters  $a$  and  $b$  that satisfy the conditions

$$w(a, b) = q(a) w(b, b), \quad (5)$$

$$u_0(b) \leq 0, \quad (6)$$

and

$$\int_{\alpha}^b u_0(z) f(z) dz \geq 0, \quad (7)$$

where  $\alpha$  is defined by  $\partial w(a, \alpha) / \partial a = q'(a) w(\alpha, \alpha)$ . In any such equilibrium, IP wins the election at  $t = 1$  if and only if  $x_0 \leq a$ .

The equilibrium behavior and conditions in this result are straightforward. The cost of choosing a moderate policy for extreme types of IP is higher. Under the intuitive criterion, there then exists a level  $a$  of first period policy such that the median voter votes for IP if and only if  $x_0 \leq a$ . The best response of IP in the first period is as in (4). If IP is very extreme, i.e.,  $z$  is larger than some  $b$  that is determined in equilibrium, then its best response is to choose  $x_0(z) = z$ . In that case, IP's extreme type is revealed, and IP loses the next election. All the other types choose a policy  $x_0(z) \leq a$  and win the next election. Some very moderate types, with  $z < \alpha$  for some  $\alpha < b$ , choose a policy level that is even more moderate than  $a$ . These types also reveal their types, but the median voter finds them better than the alternative  $s$ .

These types may choose  $x_0(z) < z$ , in order to decrease the probability of coup; they will choose  $x_0(z) = z$  when the coup probability does not depend on  $x_0$ .

In equilibrium, IP of a type  $z > \alpha$  faces two options: (i) it can either implement policy  $a$  at  $t = 0$ , win the elections at  $t = 1$ , and thereby (if there is no coup) implement  $z$  at  $t = 1$ , or (ii) it can implement  $z$  at  $t = 0$  and lose the next elections, yielding the policy  $s$  at  $t = 1$ . (No other strategy can be a best response.) Its payoffs for these two strategies are  $w(a, z) + (1 - q(a))w(z, z)$  and  $w(z, z)$ , respectively. Hence, its *net gain from moderation* is

$$R(a, z) = w(a, z) - q(a)w(z, z). \quad (8)$$

Now IP of a given type  $z > a$  can rationally choose to moderate if and only if  $R(a, z) \geq 0$ . Since IP of type  $b$  chooses to moderate (i.e.,  $x_0^*(b) = a$ ), we must have  $R(a, b) \geq 0$ . Likewise, for any  $z > b$ , since  $x_0^*(z) = z$ , we must have  $R(a, z) \leq 0$ . Since  $R$  is continuous, we therefore have  $R(a, b) = 0$ , which is equivalent to the equilibrium condition (5). When the median voter observes  $x_0 \geq b$ , he learns that  $z = x_0$ . Since he does not vote for IP when he observes  $x_0 = b$ , it must be that he finds  $s$  at least as good as  $b$ , yielding equilibrium condition (6). When he observes that  $x_0 = a$ , he only learns that  $\alpha < z \leq b$ . In that case, his expected payoff from voting for IP is

$$E[u_0(z)|x_0^*(z) = a] \equiv \frac{1}{F(b) - F(\alpha)} \int_{\alpha}^b u_0(z)f(z)dz. \quad (9)$$

Since he votes for IP, it must be that this payoff is at least as high as the payoff from the alternative,  $s$ . This yields the equilibrium condition (7).

When  $q$  does not depend on  $x_0$ , the equilibrium behavior is simpler:

$$x_0^*(z) = \begin{cases} a & \text{if } z \in [a, b], \\ z & \text{otherwise} \end{cases} \quad \text{and} \quad x_1^*(z) = z \quad \forall z;$$

(6) becomes  $\int_a^b u_0(z)f(z)dz \geq 0$ . In the remainder of this section we will maintain the following assumption, which holds when  $q'$  is not too high.

**Assumption 2**  $R(a, b)$  is increasing in  $a$  at each  $(a, b)$  with  $R(a, b) = 0$  and  $a < b$ .

By Theorem 1, we can summarize equilibria with two real numbers, namely,  $a$ —the most extreme policy level median voter tolerates at  $t = 0$ , and  $b$ —the most extreme

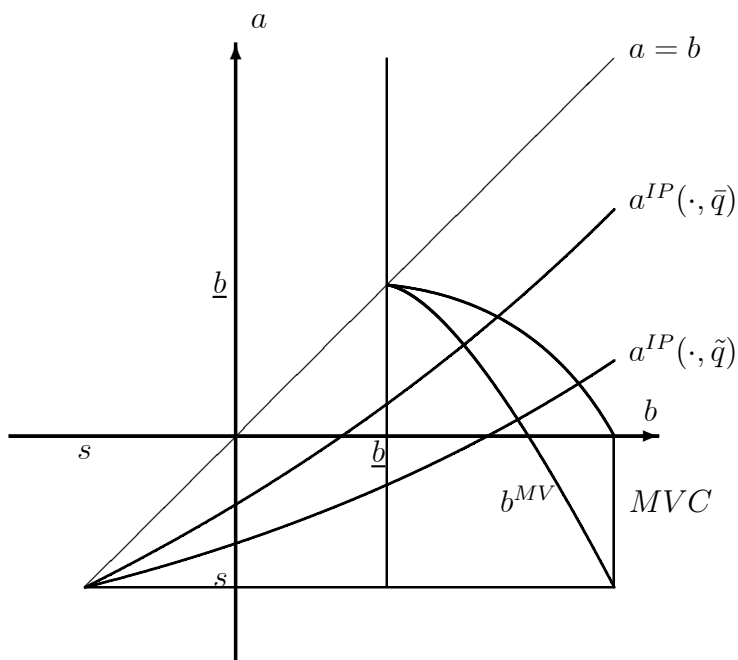


Figure 1: The set of equilibrium parameters.

type who can afford to implement  $a$  in order to win the next election, after which it chooses its best policy. (The parameter  $\alpha$  is a function of  $a$ .) The set of equilibrium parameters are characterized by conditions (5), (6), and (7). The next result presents simplified versions of these conditions.

**Lemma 1** *Under Assumptions 1 and 2, for each  $b > s$ , there is a unique solution  $a^{IP}(b, q)$  to (5), and  $a^{IP}(b, q)$  is increasing in  $b$ . The set of all equilibrium parameters is*

$$SEP(q) = \{(a, b) | a = a^{IP}(b, q), \underline{b} \leq b \leq \bar{b}(q)\} \quad (10)$$

where  $u_0(\underline{b}) = 0$  and  $\bar{b}(q)$  is the unique solution to  $E[u_0(z) | \alpha(a^{IP}(b, q)) \leq z \leq b] = 0$ .

The equilibrium parameters are plotted in Figure 1. The set of these parameters is simply the graph of  $a^{IP}$  within a region determined by the median voters equilibrium conditions. The lower bound is given by  $b \geq \underline{b}$ . The upper bound is obtained by setting  $\int_{\alpha(a)}^b u_0(z) f(z) dz = 0$ , which has a unique solution  $b^{MV}(a, q')$ ; the solution depends only on  $a$  and  $q'$ . The graph of  $b^{MV}$  bounds the region from above and is below the curve  $MVC$  defined by  $\int_a^b u_0(z) f(z) dz = 0$ . The latter corresponds to the case that  $q$  does not depend on  $x_0$ .

**Moderation of Policy** The equilibrium response to the changes in the level of the coup probability is determined by the incentives of IP. These incentives are summarized by  $a^{IP}$ , where  $a^{IP}(z, q)$  is the most moderate policy level IP of type  $z$  is willing to choose in order to win the next election. Now suppose that we increase the probability of coup by amount  $\Delta$  everywhere, so that the new coup probability is  $\tilde{q}(x_0) = q(x_0) + \Delta$ . Now at  $a = a^{IP}(b, q)$ , we have  $w(a, b) < \tilde{q}(a) w(b, b)$ . In order to have an equality, under Assumption 2, we must increase  $a$ , i.e.,

$$a^{IP}(b, \tilde{q}) > a^{IP}(b, q).$$

That is, the increase in the probability of coup weakens the incentives of IP. Since  $\underline{b}$  and  $b^{MV}$  are not affected by such a change, the equilibrium parameters shifts in the direction that  $a$  increases and  $b$  decreases. Since  $a$  is higher, IP needs to moderate less in order to convince the median voter that it is not too extreme. Despite this, fewer types of IP choose to moderate in equilibrium, as  $b$  is lower now. All in all, each type of IP chooses (if anything) less moderate policies. This is established in the next result.

**Theorem 2** *Under Assumptions 1 and 2, for  $\tilde{q} = q + \Delta$ , where  $\Delta \geq 0$  is a constant,*

1. *for each equilibrium  $e \in SE^*(q)$  with policy  $x_t^*$ , there exists an equilibrium  $\tilde{e} \in SE^*(\tilde{q})$  with policy  $\tilde{x}_t^*$ , such that*

$$\tilde{x}_t^*(z) \geq x_t^*(z) \text{ for all } z \text{ and } t;$$

2. *for each equilibrium  $\tilde{e} \in SE^*(\tilde{q})$  with policy  $\tilde{x}_t^*$ , there exists an equilibrium  $e \in SE^*(q)$  with policy  $x_t^*$ , such that*

$$\tilde{x}_t^*(z) \geq x_t^*(z) \text{ for all } z \text{ and } t.$$

Intuitively, as the probability  $q$  of a coup decreases, the discount rate  $\delta$  increases. For this reason, given any required level of moderation in order to win the next elections, the gain from moderation for IP of any given type increases, and hence IP of more extreme types can afford to chose moderate policies. In that case, the voters expect IP of a “moderate type” to implement more moderate policies in order to signal its type convincingly. This typically results in two changes:

1.  $a$  gets lower, and IP is required to implement more moderate policies in order to win the next election;
2.  $b$  gets higher, and IP of some more extreme types can now afford to implement even this more moderate required policy.

Consequently, at each date, if IP comes to power, IP of any given type implements (if anything) some more moderate policy.<sup>7</sup> As  $q$  approaches 1 everywhere, the set of equilibrium parameters  $SEP(q)$  approaches  $\{(\underline{b}, \underline{b})\}$ , where there is no moderation, i.e., IP always implements  $z$ . In that case, since  $E[u_0(z)] < 0$ , IP loses the election at  $t = 0$ . On the other hand, as  $q$  approaches 0 everywhere,  $SEP(q)$  approaches the set  $\{s\} \times [\underline{b}, \bar{b}(0)]$ , where IP is required to imitate its alternative  $s$  in order to win the next election.

**Median Voter’s Welfare** Now suppose that we decrease the probability of coup from  $\tilde{q}$  to  $q$  and this results in a change in equilibrium as in Theorem 2. The next result establishes that the change benefits the median voter, provided that this does not result in “overmoderation,” i.e.,  $x_0^*$  remains non-negative. That is, the median voter would prefer the state to commit to a low probability of a coup in order to entice IP to moderate, so long as it does not lead IP to overly moderate and implement policies that are close to the status quo—a contingency that is usually ignored in public discourse.

**Theorem 3** *Under Assumptions 1 and 2, let  $q$  and  $\tilde{q}$  be such that  $\tilde{q}(x_0) = q(x_0) + \Delta$  for some  $\Delta > 0$ . Let  $e \in SE^*(q)$  and  $\tilde{e} \in SE^*(\tilde{q})$  be such that  $\tilde{x}_0^*(z) \geq x_0^*(z) \geq 0$  for all  $z \geq 0$ , where  $x_0^*$  and  $\tilde{x}_0^*$  are first-period policies in  $e$  and  $\tilde{e}$ , respectively. The median voter prefers  $(e, q)$  to  $(\tilde{e}, \tilde{q})$ :  $U_0(e, q) \geq U_0(\tilde{e}, \tilde{q})$ .*

Since the change from  $(\tilde{e}, \tilde{q})$  to  $(e, q)$  decreases the first-period policy without making it negative, it brings the first period policy closer to the ideal policy of the median

---

<sup>7</sup>In reality, coup may be carried out as soon as IP comes to power, before it chooses a policy, as in Algeria. In that case, we may not observe a more extreme policy when  $q$  is high. This is equivalent to banning such parties. Such a ban may push the constituents of IP outside of the democratic system, and they may try to change the policy by force, as it happened in Algeria. Sometimes, as in Iran, those attempts do succeed, and we observe extreme policies implemented.

voter. It does not effect the second-period policy of IP. Such a change alone would benefit the median voter. The only difficulty is that some of those extreme types who choose to moderate will implement policies that are strictly worse than  $s$  in the second period. Higher probability of coup makes this less likely. The equilibrium conditions (6) and (7) ensure that the former effect dominates the latter.

## 4 Optimal Coup Scheme

Imagine the founders of the state, designing its organization. If these designers want to make sure that the ideal state policy, namely  $s$ , is implemented, they either choose a very high probability  $q \cong 1$  of coup, i.e., essentially a non-democratic state, or choose a very low probability  $q \cong 0$  of coup, a state that is committed to democracy. In either case, IP will lose the elections at  $t = 0$ , and  $s$  will be implemented throughout, because IP is assumed to be worse than the status quo for the median voter in expectation.

Alternatively, they may be motivated by some normative considerations that respect the future generations' preferences. These preferences may be very different from the founders' preferences, and the median voter's ideal policy in the future may differ from  $s$ , the policy that is best for the representative state official who can carry out a coup. If the future generations had the opportunity to determine the state's organization for their time through ideal democratic means, they would clearly choose an organization that would maximize the median voter's payoff given the constraints imposed by the environment, as many median-voter theorems suggest. Nevertheless, the citizens in the future may not have enough power to choose the organization of the state. Foreseeing this, the founders may therefore want to institute an organization to maximize the median voter's payoff in a way that requires little input from future voters. We will now show that the optimal coup scheme for the median voter is a constant  $q^*$ , which can be implemented by making sure that there will be no coups and setting election frequencies appropriately. (The optimal frequencies depend on the median voter's preferences.) We must emphasize that this result does not assume any condition on coup functions, including Assumptions 1 and 2.

**Theorem 4** *The optimal coup scheme for the median voter is a constant. That is, there exists  $q^* \in (0, 1)$  with equilibrium  $e^* \in SE^*(q^*)$  such that for every integrable*



function  $\tilde{q} : (-\infty, \infty) \rightarrow [0, 1]$  and every  $\tilde{e} \in SE(\tilde{q})$ ,

$$U_0(e^*, q^*) \geq U_0(\tilde{e}, \tilde{q}).$$

The inequality states that the median voter prefers equilibrium  $e^*$  and constant probability  $q^*$  of a coup to every coup scheme  $\tilde{q}$  and every associated equilibrium  $\tilde{e}$ . Recall that when the agents discount the future, we can implement such constant probability by making sure that coup does not occur and adjusting election times appropriately. Moreover, as will be clear in a moment, increasing coup schemes are typically inefficient. Therefore, elections are better incentive schemes than the threat of a coup.

The equilibrium  $e^*$  is defined by the parameters  $(a^*, b^*) \in SEP(q^*)$  where  $a^* = 0$  and  $b^*$  is the unique intersection of the curve  $MVC$  with the horizontal axis (see Figure 1);  $b^*$  is defined by  $E[u_0(z) | 0 \leq z \leq b^*] = 0$ . The optimal probability of a coup is given by IP's optimization condition (5):  $q^* = w(a^*, b^*) / w(b^*, b^*)$ . Notice that, among all equilibria in  $SE^*(q^*)$ ,  $e^*$  is the equilibrium in which the median voter is most lenient towards IP, i.e.,  $a \leq a^*$  for each  $(a, b) \in SEP(q^*)$ . But  $q^*$  is so small that even in this lenient equilibrium the median voter has very high expectations and will not vote for IP in the next elections if IP implements  $x_0 > 0$ . For any  $q < q^*$ , IP loses the next elections even when it implements the ideal policy  $x_0 = 0$  of the median voter.

In our proof, which is in the appendix, we show that for any  $(\tilde{e}, \tilde{q})$  as in the theorem, there exists  $\tilde{b} \leq b^*$  such that IP wins the elections at  $t = 1$  if  $z < \tilde{b}$  and loses the elections at  $t = 1$  if  $z > \tilde{b}$ .<sup>8</sup> This already shows that  $e^*$  both induces the best possible first period policy *and* allows moderation for all types that can moderate in any equilibrium. To elaborate further, let us compare equilibrium outcomes for each  $z$ . When  $z \geq b^*$ , the equilibrium outcome is the same in both equilibria. If  $\tilde{b} < z < b^*$ , in  $(e^*, q^*)$ , IP chooses  $x_0^*(z) = 0$  at  $t = 0$ , yielding the highest possible payoff  $u_0(0) > 0$  for the median voter, and  $x_1^*(z) = z$  at  $t = 1$ , yielding a payoff of  $(1 - q^*)u_0(z) < 0$ , as there will be a coup with probability  $q^*$ . On the other hand, in  $(\tilde{e}, \tilde{q})$ , IP chooses  $\tilde{x}_0(z) = z$  and loses the next elections yielding the very low payoff of  $u_0(z) < 0$  for the

---

<sup>8</sup>In Figure 1,  $\tilde{b}$  is given by the intersection of the graphs of  $a^{IP}(\cdot, \tilde{q})$  and  $b^{MV}$ . As shown in the figure, when  $\tilde{q}$  is increasing, typically  $\tilde{b} < b^*$ .

median voter, and leaving him clearly worse off. When  $z \leq \tilde{b}$ , in the first period IP implements again the best policy for the median voter in  $e^*$ , and in both equilibria  $e^*$  and  $\tilde{e}$ , IP wins the next elections and implements  $x_1^*(z) = \tilde{x}_1(z) = z$ . Now, although the median voter gets the best possible payoff at  $t = 0$  in  $(e^*, q^*)$ , there is a potential advantage of  $\tilde{q}$ , as  $\tilde{q}(\tilde{x}_0(z))$  may be increasing in  $z$ , making more extreme policies less likely to be implemented in the second period. It turns out that this advantage is small compared to all of these inefficiencies introduced—as our result establishes.

**Remark:** Optimality of constant coup probability relies on the modeling assumption that coup occurs at the time of the next election. In general, coup can remove IP from power earlier. Since there is no social cost of coup in our model, in that case the optimal coup scheme would be a step function:  $q^*$  if  $x_0 \leq 0$ , and 1 otherwise. That is, the types who would lose the next election are removed from power through a coup without waiting for the next election. Coups come with high social cost in reality, however. If this cost is higher than the benefit of removing IP couple years earlier, then the optimal coup scheme will be as in our model: state commits not to intervene and the elections are held in appropriate frequency.

**Remark:** In this section we assumed that by varying the organization of the state, the designers can pick different coup functions. This is a reasonable assumption because whether a coup will be successful depends on how strong different agencies, such as intelligence, police, and military, are and how aligned their incentives are. For example, in Turkey, where the military leaders played a central role in its organization since the beginning, the police force outside of the military was weak, and the intelligence agency was controlled by the military. Under such an organization, the military coups are likely to succeed, and  $q$  is high. On the other hand, in Soviet Union, under the fear of counterrevolution, these agencies were all designed to be strong and independent. In that case, the probability of a successful coup is lower. While  $q$  can be affected by organizational design, it is not clear that all functions are implementable. This does not affect our result, however. As we discussed above, a constant coup function can be implemented by committing to not having a coup and holding the elections in appropriate frequency. In that case, the optimal coup scheme will remain to be as in this paper even some coup functions are not implementable.

## 5 Robustness to Alternative Models

### 5.1 Soldier as the Guardian of the State

In previous sections, we have taken the probability of a coup to be an *exogenous* function of earlier policy choices, so that we could show the direction of the causality—that higher probability of a coup in the future makes today’s policies more extreme. We now consider a natural model of a coup where the probability of a coup is endogenous.

We consider a model in which there is a coup leader; after the elections at  $t = 1$ , he decides whether to intervene and override the election results. We assume that the coup leader has strictly concave, twice continuously differentiable utility function  $u_s$  that is maximized at  $s$ . This is consistent with our earlier interpretation that the state’s ideal policy,  $s$ , is the ideal policy of the representative official who can carry out a coup. We again normalize  $u_s$  by setting  $u_s(s) = 0$ . Intervention costs  $c$  to the coup leader, and  $c$  will be known by the coup leader at  $t = 1$  and is not known before. The CDF and pdf of  $c$  are denoted by  $G$  and  $g$ , respectively.

In equilibrium, at  $t = 1$ , IP implements  $x_1^*(z) = z$ , and the coup leader intervenes if and only if the expected cost from this policy, which is  $-E[u_s(z) | x_0^*(z) = x_0]$ , exceeds the cost of intervention,  $c$ . Hence, the probability of a coup from IP’s point of view is

$$q^*(x_0) = G(-E[u_s(z) | x_0^*(z) = x_0]), \quad (11)$$

which is a non-decreasing function of  $x_0$  when  $x_0^*$  is non-decreasing. Notice that the functions  $x_0^*$  and  $q^*$  are simultaneously and endogenously determined in equilibrium. Our objective is to understand how the function  $x_0^*$  varies as we vary  $G$ , the distribution of the cost. We will show that if the coup becomes more costly to the coup leader—in the statistical sense below—then IP will implement a more moderate policy at  $t = 0$  in equilibrium.

For simplicity, we assume that  $G(\underline{c}) = 0$ ,  $G(\bar{c}) = 1$ , and  $g(c) > 0$  for each  $c \in [\underline{c}, \bar{c}]$ , for some minimum and maximum costs  $\underline{c}$  and  $\bar{c}$  where  $\underline{c} > 0$ . We also consider another CDF  $\hat{G}$  for cost  $c$  with associated pdf  $\hat{g}$ , minimum cost  $\hat{\underline{c}} \in (\underline{c}, \bar{c})$ , and maximum cost  $\hat{\bar{c}} > \bar{c}$  where

$$\hat{G}(c) = G(c) - G(\hat{\underline{c}}) \quad (\forall c \in [\hat{\underline{c}}, \hat{\bar{c}}]),$$

$\hat{G}(\underline{c}) = 0$ ,  $\hat{G}(\bar{c}) = 1$ , and  $\hat{g}(c) > 0$  for each  $c \in [\underline{c}, \bar{c}]$ . Notice that

$$\hat{g}(c) = g(c) \quad (\forall c \in [\underline{c}, \bar{c}]), \quad (12)$$

so that by changing  $G$  to  $\hat{G}$ , we simply move all the probability mass at low costs  $[\underline{c}, \hat{c})$  to high costs  $(\bar{c}, \bar{c}]$ , making the coup costlier in this statistical sense. We define  $\underline{z}$  and  $\bar{z}$  by  $G(-u_s(\underline{z})) = \underline{c}$  and  $G(-u_s(\bar{z})) = \bar{c}$ , so that there will be no coup if it is known that  $z < \underline{z}$ , and there will be a coup with probability 1 if IP's type is known to be greater than  $\bar{z}$ . We define  $\hat{z}$  and  $\bar{\hat{z}}$  similarly for  $\hat{G}$ . Recall that  $\underline{b}$  is defined by  $u_0(\underline{b}) = 0$ , so that the median voter votes for (resp., against) IP if he knows that  $z < \underline{b}$  (resp.,  $z > \underline{b}$ ).

We first assume that  $\bar{\hat{z}} < \underline{b}$  and analyze the case that the equilibrium policy  $x_0^*$  is strictly increasing on  $(\underline{z}, \bar{z})$ . In that case, at each  $x_0$  after which the median voter votes against IP, there would have been a coup with probability 1 anyway, and hence the elections are irrelevant to our problem. The main issue is the signalling between IP and the coup leader. Even in this case, making the coup costlier leads to moderation. Towards establishing this, we first derive a differential equation that governs the equilibrium behavior.

**Lemma 2** *Assume  $\bar{z} < \underline{b}$  and  $x_0^*$  is strictly increasing on  $(\underline{z}, \bar{z})$ . Then, on  $(\underline{z}, \bar{z})$ ,  $x_0^*$  is a solution to the first-order ordinary differential equation*

$$\frac{dx_0}{dz} = -\frac{w(z, z)g(-u_s(z))u'_s(z)}{\partial w(x_0, z)/\partial x_0} \equiv \Phi(x_0, z) \quad (13)$$

with the boundary condition

$$\lim_{z \uparrow \bar{z}} x_0(z) = z. \quad (14)$$

**Proof.** Since  $x_0^*$  is strictly increasing, by (11),  $q^*$  is differentiable on  $(\underline{z}, \bar{z})$ , and

$$\frac{\partial q^*}{\partial x_0} = -\frac{g(-u(z, s))u'_s(z)}{dx_0^*/dz}.$$

By substituting this into the first-order condition,

$$\frac{\partial}{\partial x_0} [w(x_0, z) + (1 - q^*(x_0))w(z, z)] = \frac{\partial w}{\partial x_0} - w(z, z)\frac{\partial q^*}{\partial x_0} = 0,$$

we obtain (13). Moreover, when  $z > \bar{z}$ ,  $q^*(x_0^*(z)) = 1$ , and hence  $x_0^*(z) = z$ . Since IP's payoff must be continuous in his type, this yields (14). ■

Lemma 2 has two important implications for us. Firstly, by (13) and (14), any equilibrium policy as in the lemma must be continuous on  $(\underline{z}, \infty)$ . Secondly, since  $\Phi$  is continuously differentiable when  $x_0 < z$ , the graphs of distinct solutions to (13) for possibly distinct boundary conditions never intersect each other at any  $(z, x_0)$  with  $x_0 < z$ . These lead to our next result.

**Theorem 5** *Assume  $\bar{z} < \underline{b}$ , and let  $x_0^*$  and  $\hat{x}_0^*$  be equilibrium policies under  $G$  and  $\hat{G}$ , respectively, that are strictly increasing on  $(\underline{z}, \bar{z})$  and  $(\hat{z}, \bar{z})$ , respectively. Then,*

$$\hat{x}_0^*(z) < x_0^*(z) \quad (\forall z \in (\hat{z}, \bar{z})). \quad (15)$$

**Proof.** Notice that  $x_0^*(z) = z$  for any  $z \geq \bar{z}$ . For any  $z \in (\underline{z}, \bar{z})$ , since  $\lim_{x_0 \rightarrow z} \Phi(x_0, z) = \infty$ , we must have  $x_0^*(z) < z$ . Similarly,  $\hat{x}_0^*(z) < z$  for any  $z \in (\hat{z}, \bar{z})$ . Hence,  $\hat{x}_0^*(\bar{z}) < \bar{z} = x_0^*(\bar{z})$ . This implies (15). For, otherwise, since  $\hat{x}_0^*$  and  $x_0^*$  are continuous on  $(\hat{z}, \bar{z}]$ , by the Mean Value Theorem, we must have  $\hat{x}_0^*(z) = x_0^*(z) < z$  for some  $z \in (\hat{z}, \bar{z})$ . But by (12) and Lemma 2, both  $\hat{x}_0^*$  and  $x_0^*$  are solutions to (13)—for some distinct boundary conditions, leading to a contradiction. ■

When coups become statistically costlier to the state, certain extreme types (in  $(\bar{z}, \bar{z})$ ) moderate due to the new incentive to decrease the probability of a coup, while they they did not have any incentive to moderate before. The incentive compatibility condition (13) then dictates that less extreme types also implement more moderate policies. Consequently, IP implements a more moderate policy, except for some types whose ideal policies are so close to that of the state that there is no coup. This remains true when  $\bar{z} > \underline{b}$ .

**Theorem 6** *Assume that  $w(x, z)/w(z, z)$  is non-decreasing in  $z$  and  $\bar{z} > \underline{b}$ . Then, for any non-decreasing, piece-wise continuous equilibrium policy  $x_0^*$  under  $G$ , there is an equilibrium policy  $\hat{x}_0^*$  under  $\hat{G}$  such that*

$$\hat{x}_0^*(z) \leq x_0^*(z) \quad (16)$$

*at each  $z$  with  $\hat{G}(E[-u_s(z') | x_0^*(z') = x_0^*(z)]) > 0$ . The above inequality is strict whenever  $\hat{G}(E[-u_s(z') | x_0^*(z') = x_0^*(z)]) \in (0, 1)$  and IP of type  $z$  wins the elections at  $t = 1$  under the original equilibrium.*

## 5.2 Hitler Syndrome

It is a common fear that an ideological party may come to power and end the democratic regime in order to establish its own ideological system (see Dyzenhaus (1997) and the references therein.<sup>9</sup>) Now, in the case that IP comes to power at  $t = 0$ , before the election at  $t = 1$ , we allow IP to try a coup that will succeed with some small probability  $p$  and will cost  $C$  to IP. If IP's coup is successful, IP cancels the election at  $t = 1$  and implements a policy  $x_1$ , which will be  $z$  in equilibrium. If its coup is unsuccessful, it loses the election. We will assume that the probability of coup is a constant  $q \in (0, 1)$ . The following result summarizes the equilibrium behavior and shows that an increase in  $q$  leads to more extreme policies.

**Theorem 7** *IP implements  $x_0^*$  in equilibrium if and only if*

$$x_0^*(z) = \begin{cases} a & \text{if } a \leq z \leq \min\{b, c\}, \\ z & \text{otherwise} \end{cases} \quad (\forall z),$$

where  $w(a, b) = qw(b, b)$ ,  $w(a, c) = (p + q)w(c, c) - C$ ,  $\int_a^b u_0(z)f(z)dz \geq 0$ , and  $u_0(b) \leq 0$ . IP wins the election at  $t = 1$  if and only if  $x_0 \leq a$  and it has not attempted a coup. IP attempts a coup if and only if  $z \geq \max\{c, d\}$  where  $w(d, d) = C/p$ . When  $q$  decreases, IP implements more moderate policies in equilibrium. In that case, the probability that IP tries a coup, namely  $\Pr(z > \max\{c, d\})$ , also weakly decreases.

**Sketch of Proof.** IP of a type  $z > a$  has the following options: it can either (i) choose  $x_0 = a$  and not try a coup, which yields  $w(a, z) + (1 - q)w(z, z)$ , (ii) choose  $x_0 = z$  without attempting a coup, which yields  $w(z, z)$ , or (iii) choose  $x_0 = z$  and try a coup, which yields  $(1 + p)w(z, z) - C$ . We already know that the option (i) is at least as good as (ii) if and only if  $z \leq b$ . The option (ii) is at least as good as the option (iii) if and only if  $z \leq d$ . Finally, (i) is at least as good as (iii) if and only if  $w(a, c) - (p + q)w(c, c) \geq -C$ . By Lemma 3 in the appendix, this inequality holds if and only if  $z \leq c$ . In summary, (i) is a best response when  $z \leq \min\{b, c\}$ ; (ii) is a best response when  $b \leq z \leq d$ , and (iii) is a best response when  $z \geq \max\{c, d\}$ . This is the strategy of IP that is described in the Theorem. In equilibrium, IP's policy remains essentially unchanged — except for a new cutoff value. The new cutoff value,

---

<sup>9</sup>Kalaycioglu and Sertel (1995) call this Hitler syndrome.

$c$ , is defined by the same equation, except that the the total probability of a coup is  $p + q$  instead of  $q$  and the constant on the right-hand side is  $-C$  instead of 0. These differences are irrelevant for our comparative statics. ■

Define  $a^*$  by  $w(a^*, d) = qw(d, d)$ . When  $a < a^*$ , we have  $b < c < d$ . In that case, if IP is of some type  $z \in (b, d)$ , it does not try a coup even though it reveals its extreme type by implementing  $z$ . Hence, the incentive for moderation for the relevant types (with  $z \leq b$ ) is as before, and the median voter draws the same inferences when he observes that  $x_0 \leq b$ . Therefore the equilibrium policy is the same as in the basic model. This is the case when the cost of a coup relative to the probability of success,  $C/p$ , is high. When  $C/p$  is low, we have  $a > a^*$ , and hence  $b > c > d$ . In that case, IP attempts a coup whenever it does not moderate. Any decrease in  $q$  entices IP to moderate more, decreasing the probability of a coup by IP.

## 6 Conclusion

How should a democratic regime defend itself against extreme ideological parties? Should it institute an organization that can easily intervene in political process when such parties come to power—as in a Praetorian society, or should it give incentive to ideological parties to stay in the system by empowering the elected officials—as in a consolidated democracy? Within a simple model, we show that elections are a better incentive scheme than the threat of a coup, as the possibility of an intervention in the future undermines the electoral incentives, leading to a more polarized political spectrum. This remains true even when we consider a Praetorian system; an ideological party implements a more moderate policy when we make it more difficult for the state to intervene in the future (in a case when the ideological party wins automatically when there is no coup).

## A Appendix—Proofs

In this appendix,  $\mathbb{R}$  and  $\mathbb{R}_+$  denote the sets of all real numbers and all non-negative real numbers, respectively. Given any  $f : \mathbb{R}^n \rightarrow \mathbb{R}$ ,  $f_i$  denotes the partial derivative with respect to  $i$ th coordinate, and  $f_{ij}$  denotes  $(f_i)_j$ . We also write  $Z = (s, \infty)$  for the set of IP's

types. The following lemma will play a central role in our proofs; recall that  $R(a, z) = w(a, z) - q(a)w(z, z)$ .

**Lemma 3** *Take any  $\bar{z}$  with  $R(a, \bar{z}) \leq 0$ . Then,  $R(a, \cdot)$  is decreasing on  $[\bar{z}, \infty)$ , and therefore  $R(a, z) < 0$  at each  $z > \bar{z}$ .*

**Proof.** Take any  $z \in (a, \infty)$  with  $R(a, z) \leq 0$ . Notice that  $R$  is differentiable with respect to  $z$ . We **claim** that  $R_2(a, z) < 0$ . Since this will be true for arbitrary  $z' \in (a, \infty)$  with  $R(a, z') \leq 0$ , this will imply that  $R(a, z') < 0$  (and hence  $R_2(a, z') < 0$ ) thereafter, which will prove the lemma. In order to prove our **claim**, we note that, since  $R(a, z) = w(a, z) - q(a)w(z, z) \leq 0$ , we have  $q(a)w(z, z)/w(a, z) \geq 1$ . Since  $w$  is log-super-modular (i.e.,  $\partial^2 \log(w(x, z))/\partial x \partial z > 0$ ),  $\frac{\partial \log(w(x, z))}{\partial z} = \frac{w_2(x, z)}{w(x, z)}$  is increasing in  $x$ , and hence we have

$$\frac{w_2(a, z)}{w(a, z)} < \frac{w_2(z, z)}{w(z, z)} \leq \frac{w_2(z, z)}{w(z, z)} \frac{q(a)w(z, z)}{w(a, z)} = \frac{q(a)w_2(z, z)}{w(a, z)}.$$

Thus  $w_2(a, z) < q(a)w_2(z, z)$ , and therefore  $R_2(a, z) = w_2(a, z) - q(a)w_2(z, z) < 0$ . ■

## A.1 Proof of Theorem 1

**Sufficiency of conditions (5), (6), and (7).** The characterizing conditions for equilibrium as in our theorem are (i) IP wins the elections at  $t = 1$  iff  $x_0 \leq a$ , and (ii)  $x_0^*$  is a best response to (i).

Condition (i) is equivalent to (6) and (7). Proof: The condition is equivalent to  $E[u_0(z) | x_0] \geq 0 \iff x_0 \leq a$ , which implies (6) and (7) as special cases. But (6) and (7) are also sufficient: We have  $u_0(z) > u_0(b)$  at each  $z \in [0, b)$ , because  $u_0$  is single-peaked with a maximum at 0. Thus, if both (6) and (7) hold, then  $u_0(\alpha) \geq 0$ , hence we have  $u_0(z) \geq \min\{u_0(\alpha), u_0(s)\} \geq 0$  at each  $z \leq \alpha$ , showing that  $x_0 \leq a \Rightarrow E[u_0(z) | x_0] \geq 0$ . Since  $b \geq 0$  and  $u_0$  is decreasing on  $\mathbb{R}_+$ , (7) also implies that  $u_0(z) < 0$  at each  $z > b$ , showing that  $x_0 > a \Rightarrow E[u_0(z) | x_0] < 0$ .

Lemma 3 implies that (5) is sufficient for  $x_0^*$  to be a best response. Proof: if we had  $R(a, z) \leq 0$  at any  $z \in [a, b)$ , then by Lemma 3 we would also have  $R(a, b) < 0$ , which is false by definition. Hence, we need to have  $R(a, z) > 0$  at each  $z \in [a, b)$ , which in turn implies that  $x_0^*(z) = \min\{a, \hat{x}(z)\}$  is a best response at each  $z \in [a, b)$ . For any  $z < a$ ,  $\hat{x}(z) < a$  is a best response. On the other hand, by the second part of Lemma 3, since  $R(a, b) = 0$ , we have  $R(a, z) < 0$  at each  $z > b$ , which implies that now  $x_0^*(z) = z$  is a best response.



**Converse** We now define intuitive criterion and show that all sequential equilibria that pass this test are as described above. Fix any equilibrium  $e \in SE^*(q)$  with first-period policy  $\tilde{x}_0$ . We will show that there exists  $a$  such that IP wins the next election if  $\tilde{x}_0(z) \leq a$ , and loses if  $\tilde{x}_0(z) > a$ ; therefore  $\tilde{x}_0 = x_0^*$  for some  $(a, b) \in SEP(q)$ . Write  $A$  for the set of policy levels  $x$  such that IP wins the next election if it implements  $x$  at time 0. The expected benefit to IP is

$$W(x, z) = \begin{cases} w(x, z) + (1 - q(x))w(z, z) & \text{if } x \in A, \\ w(x, z) & \text{otherwise.} \end{cases} \quad (17)$$

Observe that under Assumption 1,  $\hat{x}(Z) = Z$ . We also observe some basic properties of  $e$ :

**Lemma 4** (1)  $(A \cap Z) \cup \{s\}$  is closed. (2)  $A \cap Z \neq \emptyset$ . (3)  $\tilde{x}_0(z) = \hat{x}(z)$  whenever  $\hat{x}(z) \in A \cap Z$ . (4)  $Z \setminus A \neq \emptyset$ .

**Proof.** (1) Otherwise, we would have  $x_n \rightarrow x$  for some sequence with  $x_n \in A \cap Z$  and  $s < x \notin A$ . But then the best-response correspondence would be empty at  $z$  with  $\hat{x}(z) = x$ . (2) Otherwise we would have  $\tilde{x}_0(z) \equiv z$ , which would imply that  $0 \in A$ . (3) See (17). (4) Otherwise, we would have  $\tilde{x}_0(z) \equiv \hat{x}(z)$  by part 3, and thus  $z \notin A$  for each  $\hat{x}(z) > \underline{b}$ . ■

**The Intuitive Criterion** for our game is defined as follows. For every  $x$  and  $z$ , define

$$I(x, z) = \hat{W}(x, z) - W(\tilde{x}_0(z), z).$$

Note that  $I(x, z)$  is the best increment IP of type  $z$  can get by implementing  $x$  at  $t = 0$ . Take any  $x \in Z \setminus \tilde{x}_0(Z)$ , which is not implemented in equilibrium, and therefore by Lemma 4.3,  $x \notin A$ . We write  $\tilde{Z}(x) = \{z \in Z \mid I(x, z) < 0\}$  for the set of types who would never want to deviate to  $x$ . Equilibrium  $e$  fails the Intuitive Criterion if  $u_0(z) > 0$  at each  $z \in Z \setminus \tilde{Z}(x)$ , i.e., IP wins the election when it implements  $x$  no matter how voters interpret this as long as they are convinced that IP is not of some type  $z$  in  $Z \setminus \tilde{Z}(x)$ . Under the new voter response, type  $z$  with  $\hat{x}(z) = x$  would like to deviate to  $x$ .

**Lemma 5** If  $x^* \notin A$  for some  $x^*$ , then  $x \notin A$  for every  $x \geq x^*$ .

**Proof.** Take any  $x^* \in Z \setminus A$ , and write  $\bar{x} = \min\{x \in A \mid x \geq x^*\}$  and  $\underline{x} = \max\{x \in A \mid x \leq x^*\}$ , where we use the convention that  $\min \emptyset = \infty$  and  $\max \emptyset = -\infty$ . By Lemma 4.1,  $\bar{x}$  exists and is greater than  $x^*$ . We will show that  $\bar{x} = \infty$ . Suppose that  $\bar{x} < \infty$ , i.e., our lemma is false. Then,  $\bar{x} \in A \cap Z$ . By supermodularity of  $\hat{W}$ , there exists  $z_0 \in (\hat{x}^{-1}(\underline{x}), \hat{x}^{-1}(\bar{x})) \equiv (\underline{z}, \bar{z})$  such that

$$W(\underline{x}, z) \geq W(\bar{x}, z) \iff z \leq z_0. \quad (18)$$

Hence,  $\tilde{x}_0^{-1}(\bar{x}) \subseteq [z_0, \infty)$ . Likewise, there exists  $z_1 \in (z_0, \bar{z})$ , such that  $\tilde{x}_0^{-1}(\bar{x}) \supseteq (z_1, \bar{z})$ . But, since  $\bar{x} \in A$ ,  $E[u_0(z) | x_0(z) = \bar{x}] \geq 0$ , and thus  $u_0(z_0) > 0$ . Therefore, by continuity, there exists  $z_2 > z_0$  such that

$$u_0(z) > 0 \quad \forall z < z_2. \quad (19)$$

Moreover, by (18) and continuity, there exists  $x \in (\underline{x}, \hat{x}(z_0))$  such that  $0 > \hat{W}(x, z) - W(\bar{x}, z) \geq I(x, z)$  for each  $z \geq z_2$ , i.e.,  $\tilde{Z}(x) \supseteq [z_2, \infty)$ . In summary, we have  $x \in Z \setminus \tilde{x}_0(Z)$  (by definition) with  $u_0(z) > 0$  for each  $z \in Z \setminus \tilde{Z}(x)$  (by (19)), showing that  $e$  fails the Intuitive Criterion, a contradiction. ■

**To complete the proof of the theorem**, define  $a \equiv \sup A$ . By Lemmas 4.4 and 5,  $a \in Z$ , and in fact,  $A \cap Z = (s, a]$ . Then, by Lemma 4.3,  $x_0(z) = \hat{x}(z)$  whenever  $z \leq \hat{x}^{-1}(a) \equiv \alpha$ , where  $\alpha \geq a$ . Since  $a > s$ , there exists (a unique)  $b > a$  such that  $R(a, b) = 0$ . [If there were no such  $b$ , since  $R(a, a) > 0$ , by Lemma 3 we would have  $R(a, z) > 0$  at each  $z > a$ , and hence we would have  $x_0(z) = a$  at each  $z > \alpha$ , therefore  $E[u_0(z) | x_0(z) = a] = E[u_0(z) | z \geq \alpha] \leq E[u_0(z)] < 0$ , which contradicts that  $a \in A$ . By Lemma 3,  $b$  must be unique.] Now by Lemma 3, we have  $R(a, z) > 0$  at each  $z \in [\alpha, b)$  and  $R(a, z) < 0$  at each  $z > b$ . Hence  $x_0^*$  (defined in the statement of the theorem) is the only best response, and therefore  $\tilde{x}_0 = x_0^*$ . ■

## A.2 Proof of Lemma 1

There exists  $a^{IP}(b, q)$  because  $R$  is continuous,  $R(s, b) \leq 0$  and  $R(b, b) \geq 0$ . It is unique because existence of a second solution would imply a third solution in between where  $R$  is decreasing in  $a$ . By implicit function theorem,  $\partial a^{IP}(b, q) / \partial b = -R_2(a^{IP}, b) / R_1(a^{IP}, b) > 0$ ; the inequality is by Lemma 3 and by the assumption that  $R$  is increasing in  $a$ . Thus,  $a^{IP}$  is increasing in  $b$ . Consider (6):  $u_0(b) \leq 0$ . We must have  $b > 0$ , and  $u_0$  is decreasing on this region. Hence we have (6) if and only if  $b \geq \underline{b}$  where  $\underline{b}$  is defined by  $u_0(\underline{b}) = 0$ . Now, consider (7):  $E[u_0(z) | \alpha \leq z \leq b] \geq 0$ . Given any  $(a, b)$  and  $(a', b')$  with  $(a, b) \geq (a', b') \geq (s, \underline{b})$ , if  $(a, b)$  satisfies (7), so does  $(a', b')$ . Therefore, given the fact that  $(a, b) \geq (s, \underline{b})$ , the set of parameters that satisfy (7) is the region under the graph of  $b^{MV}$ . Since  $a^{IP}$  is strictly increasing in  $b$ , the graphs of  $a^{IP}$  and  $b^{MV}$  have unique intersection (with  $b = \bar{b}(q)$ ), which is the maximum of equilibrium parameters. ■

### A.3 Proof of Theorem 2

(See Figure 1 for the illustration.) We will work with equilibrium parameters where  $(a, b) \in SEP(q)$  and  $(\tilde{a}, \tilde{b}) \in SEP(\tilde{q})$  correspond to  $e$  and  $\tilde{e}$ , respectively. Since  $\tilde{x}_1^*(z) = x_1^*(z) = z$ , we only need to establish that  $\tilde{x}_0^*(z) \geq x_0^*(z)$ . In Theorem 1,  $\alpha$  and  $\hat{x}_0(z)$  only depend on  $a$  and the derivative of the coup probability, which is same under  $q$  and  $\tilde{q}$ . Then, by (4),  $\tilde{x}_0^*(z) \geq x_0^*(z)$  for each  $z$  if and only if  $\tilde{a} \geq a$  and  $\tilde{b} \leq b$ . Note that  $SEP(q)$  is an ordered set with maximal member  $(\bar{a}(q), \bar{b}(q))$  where  $\bar{a}(q) = a^{IP}(\bar{b}(q), q)$ . To show part 1, take any  $(a, b) \in SEP(q)$ . If  $b \leq \bar{b}(\tilde{q})$ , then  $\tilde{a} = a^{IP}(b, \tilde{q})$  and  $\tilde{b} = b$  satisfy our requirements because  $a^{IP}(b, \tilde{q}) \geq a^{IP}(b, q) = a$ . So, assume that  $b > \bar{b}(\tilde{q})$ . Now, if  $\bar{a}(\tilde{q}) \equiv a^{IP}(\bar{b}(\tilde{q}), \tilde{q}) < a^{IP}(b, q) \equiv a$ , then  $(\bar{a}(\tilde{q}), \bar{b}(\tilde{q})) < (a, b)$  contradicting the maximality of  $(\bar{a}(q), \bar{b}(q))$  and the fact that  $b^{MV}$  is the same under  $q$  and  $\tilde{q}$ . Hence,  $\bar{a}(q) \geq a$ . Thus,  $(\tilde{a}, \tilde{b}) = (\bar{a}(q), \bar{b}(q))$  satisfies our requirement. To show part 2, take any  $(\tilde{a}, \tilde{b}) \in SEP(\tilde{q})$ . Since  $b^{MV}$  is decreasing (and same function under both  $q$  and  $\tilde{q}$ ) and  $a^{IP}(\cdot, \tilde{q}) \geq a^{IP}(\cdot, q)$ , we have  $\bar{b}(\tilde{q}) \leq \bar{b}(q)$ . Then,  $a = a^{IP}(\tilde{b}, q)$  and  $b = \tilde{b}$  satisfy our requirements. ■

### A.4 Proof of Theorem 3

Consider a change from  $(\tilde{a}, \tilde{b}, \tilde{q})$  to  $(a, b, q)$  as in the theorem and compare the equilibrium outcomes from the median voter's point of view for each possible type  $z$  of IP. Write  $\alpha$  and  $\tilde{\alpha}$  for the cutoff values in Theorem 1. If  $z \leq \alpha$ , then IP chooses  $\hat{x}(z)$  in both periods in both equilibria, which only depends on the slope. Hence, the only change is now we have a lower probability of a coup, in which case the inferior policy  $s$  is implemented in the second period rather than  $z \leq \alpha$ . (Recall that  $u_0(z) > u_0(\alpha) \geq 0 = u_0(s)$  for  $z < \alpha$ .) We refer to the welfare effect of decreasing the probability of implementing  $s$  after a coup as the direct effect. Consider the case  $\alpha < z < \tilde{\alpha}$ . Before the change, the policies chosen in the first and the second periods were both  $z$ , but now IP chooses  $a < z$  in the first period, benefitting the median voter. The lower probability of coup also benefits the median voter. Now consider the case that  $\tilde{\alpha} \leq z \leq \tilde{b}$ . Before the change, IP was choosing the policies  $\tilde{a}$  and  $z$  in the first and the second periods, respectively. Under  $q$ , it chooses  $a < \tilde{a}$  in the first period and  $z$  in the second period, benefitting the median voter once again. Now the direct effect of lowering the probability of a coup depends on  $z$ . This effect is positive for lower values of  $z$ , and negative for the higher values of  $z$ . But by the equilibrium condition (7), the expected value  $E[u_0(z) | \tilde{\alpha} \leq z \leq \tilde{b}]$  of this direct effect is non-negative. Next consider the case  $\tilde{b} < z \leq b$ . Before the change, IP would have chosen  $z$  in the first period and lost the elections, after which  $s$  would have been implemented, yielding the negative payoff of  $u_0(z)$  for the median

voter. After the change, IP chooses  $a$  in the first period and  $z$  in the second period. The payoff of the median voter is  $u_0(a) + (1 - q(a))u_0(z)$ . Now the median voter gets the negative payoff  $u_0(z)$  only with probability  $1 - q(a)$ , and also gets the positive payoff  $u_0(a)$ ; his overall payoff is therefore higher. Finally, if  $z > b$ , nothing has changed—IP chooses  $z$  in the first period and  $s$  is implemented in the second period. Moderation benefits the median voter at each region; hence it benefits him in expectation. ■

## A.5 Proof of Theorem 4

The proof consists of three steps.

*Step 1* (Construction of  $(e^*, q^*)$ ): For each  $q \in (0, 1)$ , we have maximal equilibrium  $(\bar{a}(q), \bar{b}(q))$  — at the intersection of  $MVC$  and the graph of  $a^{IP}(\cdot, q)$ . Since  $MVC$  is connected,  $\lim_{q \rightarrow 0} \bar{a}(q) = \underline{b} > 0$ , and  $\lim_{q \rightarrow 1} \bar{a}(q) = s < 0$ , there exists  $q^* \in (0, 1)$  such that  $\bar{a}(q^*) = a^* \equiv 0$ . We have  $b^* = \bar{b}(q^*)$ , where

$$E[u_0(z) | 0 \leq z \leq b^*] = 0. \quad (20)$$

*Step 2* (Existence of  $\tilde{b} \leq b^*$ ): For any  $z$  and  $z' < z$ , in equilibrium  $\tilde{e}$ , if IP wins the next elections when its type is  $z$ , it wins the next elections if its type is  $z'$ , too. This is because if IP wins at  $z$ , we must have  $w(\tilde{x}_0(z), z) - \tilde{q}(\tilde{x}_0(z))w(z, z) \geq 0$ , and by Lemma 3, we have  $w(\tilde{x}_0(z), z') - \tilde{q}(\tilde{x}_0(z))w(z', z') \geq 0$ , showing that IP prefers implementing  $\tilde{x}_0(z)$  to losing the next election at  $z'$ . Define  $\tilde{b}$  as the supremum of  $z$ 's at which IP wins the next elections in equilibrium  $\tilde{e}$ . We have just established that IP wins the next election if  $z < \tilde{b}$ , and it loses when  $z > \tilde{b}$ . Now for any  $z < \tilde{b}$ , the median voter votes for IP when he observes  $\tilde{x}_0(z)$ ; hence  $E[u_0(z) : x_0 = \tilde{x}_0(z)] \geq 0$ . Integrating both sides over  $x_0 < \tilde{x}_0(\tilde{b})$ , and observing that  $\Pr(z < 0) = 0$ , we obtain that  $E[u_0(z) : 0 \leq z < \tilde{b}] \geq 0$ . By (20), this yields  $\tilde{b} \leq b^*$ .

*Step 3* (Main): Compute that

$$\begin{aligned} U_0(e^*, q^*) &= \int_0^{b^*} u_0(0) dF(z) + (1 - q^*) \int_0^{b^*} u_0(z) dF(z) + \int_{b^*}^{\infty} u_0(z) dF(z) \\ &= \int_0^{b^*} u_0(0) dF(z) + \int_{b^*}^{\infty} u_0(z) dF(z), \end{aligned}$$

where the last equality is by (20). Using (20) one more time, we compute that

$$\begin{aligned} U_0(\tilde{e}, \tilde{q}) &= \int_0^{\tilde{b}} u_0(\tilde{x}_0(z)) dF(z) + \int_0^{\tilde{b}} (1 - \tilde{q}(\tilde{x}_0(z))) u_0(z) dF(z) + \int_{\tilde{b}}^{\infty} u_0(z) dF(z) \\ &= \int_0^{\tilde{b}} u_0(\tilde{x}_0(z)) dF(z) - \int_0^{\tilde{b}} \tilde{q}(\tilde{x}_0(z)) u_0(z) dF(z) + \int_{b^*}^{\infty} u_0(z) dF(z). \end{aligned}$$

Hence,

$$\begin{aligned} U_0(e^*, q^*) - U_0(\tilde{e}, \tilde{q}) &= \int_0^{\tilde{b}} [u_0(0) - u_0(\tilde{x}_0(z))] dF(z) + \int_{\tilde{b}}^{b^*} u_0(0) dF(z) \\ &\quad + \int_0^{\tilde{b}} \tilde{q}(\tilde{x}_0(z)) u_0(z) dF(z). \end{aligned}$$

Since  $\tilde{b} \leq b^*$  and  $u_0(0) \geq u_0(\cdot)$ , the first two terms are clearly non-negative. It thus suffices to show that  $I \equiv \int_0^{\tilde{b}} \tilde{q}(\tilde{x}_0(z)) u_0(z) dF(z) \geq 0$ . But at each  $x_0 \in \tilde{x}_0([0, \tilde{b}])$ , the median voter votes for IP when he observes that  $\tilde{x}_0(z) = x_0$ , and hence

$$\int_{\tilde{x}_0(z)=x_0} \tilde{q}(\tilde{x}_0(z)) u_0(z) dF(z) \geq 0.$$

By integrating both sides over  $x_0$ , we obtain  $I \geq 0$ ,<sup>10</sup> completing the proof. ■

## A.6 Proof of Theorem 6

Take any equilibrium with  $(x_0^*, q^*)$  under  $G$  as in the hypothesis. We will construct an equilibrium with  $(\hat{x}_0^*, \hat{q}^*)$  under  $\hat{G}$  that satisfies the desired inequality. Since  $x_0^*$  is non-decreasing, there exists an  $a$  such that the median voter votes for IP iff  $x_0 \leq a$ . If  $a = s$ , then there exists  $b$  such that  $x_0^*(z) = s$  if  $z \leq b$  and  $x_0^*(z) = z$  otherwise. Moreover,  $G(E[-u_s(z)|z \leq b]) = 0$ . Then,  $\hat{G}(E[-u_s(z)|z \leq b]) = G(E[-u_s(z)|z \leq b]) = 0$ , and hence the equilibrium above remains an equilibrium under  $\hat{G}$ . Now assume that  $a > s$ . Without loss of generality, assume also that there exists  $b$  such that  $x_0^*(b) = a$  and  $x_0^*(z) > a$  at each  $z > b$ . For each  $z > b$ ,  $x_0^*(z) = z$ , and we also set  $\hat{x}_0^*(z) = z$ . By the hypothesis, there exist  $z_0, z_1, \dots, z_N$  with  $z_0 \leq \hat{z} < z_1 < \dots < z_N = b$  such that  $x_0^*$  is continuous on each interval  $I_n \equiv (z_n, z_{n+1})$ . For each  $n$ ,  $x_0^*$  is either constant on  $I_n$  or is strictly increasing on  $I_n$ ,<sup>11</sup> with possible exception that  $x_0^*$  is constant on  $(z_0, \hat{z})$  and is strictly increasing on  $(\hat{z}, z_1)$ , a possibility that can be avoided. If  $x_0^*$  is constant on  $I_0$ , we assume without loss of generality that  $x_0^*$  is not constant on an open interval that strictly contains  $I_0$ .

Now, since we consider the case  $\bar{z} > \underline{b}$ , in order for MV's strategy to be a best response,  $x_0^*$  must be constant on  $I_{N-1}$ . We have  $w(a, b) = q^*(a)w(b, b)$  where  $q^*(a) = G(E[-u_s(z)|z_{N-1} \leq z \leq b])$ . Set  $\hat{q}^*(a) = \hat{G}(E[-u_s(z)|z_{N-1} \leq z \leq b]) < q^*(a)$ . There

<sup>10</sup>More formally, we first assume that  $\tilde{x}_0$  is simple in this region, i.e.,  $\tilde{x}_0([0, \tilde{b}]) = \{x^1, \dots, x^n\}$ . Writing  $Z_k = \tilde{x}_0^{-1}(\{x^k\})$ , we obtain  $I = \sum_{k=1}^n \int_{Z_k} \tilde{q}(\tilde{x}_0(z)) u_0(z) dF(z) = \sum_{k=1}^n \tilde{q}(x^k) \int_{Z_k} u_0(z) dF(z) \geq 0$ . We then apply the usual machinery.

<sup>11</sup>If  $x_0^*$  is constant on  $(z', z'') \subset (z, \bar{z})$  and is not constant on any larger open interval, then  $q^*$  must jump at the end  $[z', z'']$ , requiring a jump for  $x_0^*$  as well.

exists  $\hat{a} < a$  such that  $w(\hat{a}, b) = \hat{q}^*(a)w(b, b)$ . Set  $\hat{x}_0^*(z) = \hat{a}$  on  $(z_{N-1}, z_N]$ . In the new equilibrium, MV votes for IP iff  $x_0 \leq \hat{a}$ .

We define  $\hat{x}_0^*$  on the other intervals inductively, so that  $\hat{x}_0^*$  is constant on an interval iff  $x_0^*$  is constant on that interval, and  $\hat{x}_0^* < x_0^*$ . For any  $n$  with  $0 < n < N$ , assume that  $\hat{x}_0^*$  is defined on  $I_n$  as desired. Assume that  $x_0^*$  is strictly increasing on  $I_{n-1}$  and is constant on  $I_n$ ; the other cases can be handled similarly. Notice that  $\lim_{z \uparrow z^n} x_0^*(z) = x_0^*(z_n)$ . Define policies  $x^n$  and  $\hat{x}^n$  by  $\{x^n\} = x_0^*(I_n)$  and  $\{\hat{x}^n\} = \hat{x}_0^*(I_n)$ , respectively. The indifference condition for  $z_n$  implies that

$$\begin{aligned} w(x_0^*(z_n), z_n) &= w(x^n, z_n) - [q^*(x^n) - q^*(x_0^*(z_n))]w(z_n, z_n) \\ &= w(x^n, z_n) - [G(E[-u_s(z_n) | z \in I_n]) - G(-u_s(z_n))]w(z_n, z_n). \end{aligned}$$

On  $(z_{n-1}, z_n]$ , define  $\hat{x}_0^*$  as the unique solution to the differential equation (13) in Lemma 2 with the boundary condition

$$w(\hat{x}_0^*(z_n), z_n) = w(x_0^*(z_n), z_n) + [w(\hat{x}^n, z_n) - w(x^n, z_n)] < w(x_0^*(z_n), z_n). \quad (21)$$

Since both  $x_0^*$  and  $\hat{x}_0^*$  are solution the the same differential equation (13) as in Lemma 2, (21) implies that  $\hat{x}_0^* < x_0^*$  on  $(z_{n-1}, z_n]$ . Notice that

$$\begin{aligned} \hat{q}^*(\hat{x}^n) - q^*(\hat{x}_0^*(z_n)) &= \hat{G}(E[-u_s(z_n) | z \in I_n]) - \hat{G}(-u_s(z_n)) \\ &= G(E[-u_s(z_n) | z \in I_n]) - G(-u_s(z_n)) \\ &= q^*(x^n) - q^*(x_0^*(z_n)). \end{aligned}$$

Hence, type  $z_n$  is indifferent between playing  $\hat{x}_0^*(z_n)$  and  $\hat{x}^n$  in the new purported equilibrium. The assumption that  $w(x, z)/w(z, z)$  is non-decreasing in  $z$  then implies that  $\hat{x}_0^*$  is a best response, yielding an equilibrium.

To complete the definition of  $\hat{x}_0^*$  and the proof, consider  $I_0$ . If  $x_0^*$  is strictly increasing on  $I_0$ , we define  $\hat{x}_0^*$  on  $I_0$  as above, and define  $\hat{x}_0^*$  on  $[s, \hat{z}]$ , by setting  $\hat{x}_0^*(z) = \min\{z, \lim_{z \downarrow \hat{z}} \hat{x}_0^*(z)\}$ . If  $x_0^*$  is constant on  $I_0$  and  $\hat{G}(E[-u_s(z_n) | z \in \bar{I}_0]) > 0$ , we define  $\hat{x}_0^*$  on  $I_0$  as in the previous paragraph; we set  $\hat{x}_0^*(z) = \min\{z, y\}$  on  $[s, z_0]$  where  $y$  is defined by  $w(y, z_0) = w(\hat{x}^0, z_0) - \hat{G}(E[-u_s(z) | z \in I_0])w(z_0, z_0)$ . If  $\hat{G}(E[-u_s(z_n) | z \in I_0]) = 0$ , then we set  $\hat{x}_0^*(z) = \min\{z, y\}$  on  $[s, z_1]$  where  $w(y, z_1) = w(\hat{x}^1, z_1) - \hat{G}(E[-u_s(z) | z \in I_1])w(z_1, z_1)$ .

## References

- [1] Acemoglu, Daron and James A. Robinson (2000) “Why Did the West Extend the Franchise? Growth, Inequality and Democracy in Historical Perspective”, *Quarterly Journal of Economics*, CXV, 1167-1199.
- [2] Acemoglu, Daron and James Robinson (2001): “A theory of political transitions,” *American Economic Review*, 91-4, 938-963.
- [3] Acemoglu, Daron and James Robinson (2006): *Economic Origins of Dictatorship and Democracy*, Cambridge University Press.
- [4] Alesina, Alberto (1988): “Credibility and Policy Convergence in a Two-party System with Rational Voters,” *American Economic Review*, 78, 796-806.
- [5] Alesina, Alberto and Alex Cukierman (1990): “The Politics of Ambiguity,” *The Quarterly Journal of Economics*, Vol. CV, Issue 4, pp. 829-850.
- [6] Banks, Jeffrey (1990): "A model of electoral competition with incomplete information," *Journal of Economic Theory* 50, 309-325.
- [7] Besley, Timothy and Stephen Coate (1997): “An Economic Model of Representative Democracy,” *Quarterly Journal of Economics*, 112-1, 85-114.
- [8] Callander (2004): “Political Motivations,” Northwestern University, mimeo.
- [9] Cho, In-Koo and David Kreps (1987) “Signaling games and stable equilibria,” *The Quarterly Journal of Economics*, Vol. CII, Issue 2, pp. 179-221.
- [10] Coate, Stephen (2004): “Political Competition with Campaign Contributions and Informative Advertising,” *Journal of the European Economic Association*, forthcoming.
- [11] Duverger, Maurice (1954): *Political Parties* (New York:John Wiley.)
- [12] Dyzenhaus, David (1997): “Legal Theory in the Collapse of Weimar: Contemporary Lessons?” *The American Political Science Review*, 91-1, 121-134.
- [13] Ellman, Matthew and Leonard Wantchekon (2000): “Electoral competition under the threat of political unrest,” *The Quarterly Journal of Economics*, 115-2, 499 – 531.
- [14] Harrington, Joseph E. (1993): “The impact of reelection pressures on the fulfillment of campaign promises,” *Games and Economic Behavior*, 5, 71-97.

- [15] Huntington, Samuel P. (1957): *The Soldier and the State: The Theory and Politics of Civil-Military Relations*, (New York: Vintage Books.)
- [16] Huntington, Samuel P. (1968): *Political Order in Changing Societies* (New Haven: Yale University Press.)
- [17] Jackman, Robert W. “The Predictability of Coups d’Etat: A Model with African Data,” *The American Political Science Review*, 72, 1262-1275.
- [18] Johnson, Thomas H., Robert O. Slater, and Pat Mc.Gowan (1984): “Explaining African Military Coups d’Etat, 1960-1982,” *The American Political Science Review*, 78-3, 622-640.
- [19] Kalaycıoğlu, Ersin and Murat Sertel (1995) *Türkiye için yeni bir seçim yöntemine doğru*, TUSIAD, Istanbul.
- [20] O’Kane, Rosemary H.T. (1993): “Coups d’Etat in Africa: A Political Economy Approach,” *Journal of Peace Research*, 30-3, 251-270.
- [21] Osborne, Martin J. and Al Slivinski (1996): “A model of Political Competition with Citizen Candidates,” *The Quarterly Journal of Economics*, 111-1, 65-96.
- [22] Powel, G. Bingham (1986): “Extremist Parties and Political Turmoil: Two Puzzles,” *American Journal of Political Science*, 30-2, 357-378.
- [23] Rawls, John (1993): *Political Liberalism* (New York: Columbia University Press.)
- [24] Remmer, Karen L. (1991): “The Political Impact of Economic Crisis in Latin America in the 1980s,” *The American Political Science Review*, 85-3, 777-800.
- [25] Sartori, Giovanni (1976): *Parties and Party Systems*, (New York: Cambridge University Press.)
- [26] Wantchekon, Leonard (1999) “Strategic Voting in Conditions of Political Instability: The 1994 Elections in El Salvador,” *Comparative Political Studies*, 32, 810-834.
- [27] Yavuz, Hakan (1997): “Political Islam and the Welfare Party,” *Comparative politics*, 30-1, 63-82.