ELSEVIER

Journal of Development Economics
Vol. 46 (1995) 295–316

JOURNAL OF
Development
ECONOMICS

Inflation dynamics and the parallel market for foreign exchange

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Received February 1990; final version received November 1993

Abstract

Pinto (1990) in World Bank Economic Review 3, 321–338, showed that unification of official and 'parallel' market exchange rates may lead to an increase in steady-state inflation, because of the fiscal impact of real official exchange rate changes. This paper shows how this, and other comparative static and stability results in Pinto (1990), are reversed under the assumption that official exchange rate devaluation reduces money creation in the economy. It is argued that this was the case for Uganda in the 1980s, and we give a simple rule of thumb for estimating when unification will increase or decrease steady-state inflation.

Keywords: Inflation; Parallel market; Exchange rate

JEL classification: E31; F31; F32

* This paper is based on work carried out during 1987–1989 while the author was working as an Economist with the Macroeconomic Analysis section of the Ministry of Planning and Economic Development, Uganda, under the sponsorship of the Overseas Development Institute, London; some of that work was reported in Morris (1989a, 1989b). It incorporates many helpful discussions with colleagues at Ministries of Planning and Finance and the Bank of Uganda. The views expressed are those of the author alone.

SSDI 0304-3878(94)00065-4

1. Introduction

The existence of a 'parallel' or black market for foreign exchange suggests that the official exchange rate overvalues domestic currency, leading to allocative inefficiencies in the economy. Therefore a number of countries have attempted to eliminate the parallel market premium either by floating the exchange rate or by attempting to adjust the official exchange rate towards the parallel market rate.

Khoras and Pinto (1989) and Pinto (1990) suggest that this may be a dangerous policy. Suppose that Government is making large foreign debt payments (denominated in foreign currency). Consider first the impact of a one-off devaluation of the exchange rate. This will lead to an increase in the domestic currency cost of those debt payments, which must be funded by increased credit to the Government. If this is monetized, inflation will increase until the real devaluation is reversed, and the economy returns to the previous steady state.

Now consider what happens if Government unifies the exchange rates by floating the official exchange rate. The same inflationary effect will be at work. Now the economy will converge to a new steady state with a higher real official exchange rate, but with permanently higher inflation generating an inflation tax that funds the permanently higher cost of servicing foreign debt.

In Sections 4 and 5, the experience of Uganda under different exchange rate regimes is discussed. Uganda seems to have had a different recent experience because of a different structure of the Government budget. In Uganda, external inflows of foreign capital easily outweigh debt obligations. Government receives a large proportion of resources in 'program aid' – grant or concessionary loans which provide foreign exchange to the Government to sell to the private sector, often for productive inputs, sometimes even for consumer goods. The local currency counterpart of these program aid funds then goes to support the Government budget. The budget then stands to gain from devaluations of the budget. The budgetary situation of Uganda seems typical of many African countries.

I show that the different fiscal structure leads to radically different policy implications: in particular, unifying the exchange rate makes sense both for the traditional reason that it improves allocative efficiency *and* because it reduces inflation. The trade-off identified by Pinto is no longer in operation. Indeed, it turns out that a fixed crawl regime is inherently unstable, which also appears to be consistent with Uganda's experience.

I address these issues (in Section 3) by solving the model of Pinto (1990) not only for the case where devaluation leads to increased money creation, but also for the opposite case. Thus (like Pinto) the analytical model does not explicitly model the allocative gains from unification, but focusses on identifying the macroeconomic implications. Although the local dynamics are complex and sometimes hard to interpret, many key results of Pinto (1990) are reversed under the different fiscal assumptions. If a constant crawl exchange rate regime was stable before, now it is unstable. If unifying the exchange rates increased inflation before, now it reduces inflation.

Section 2 provides a stylized account of the budget, monetary accounts and balance of payments of a country like Uganda, with an official exchange rate which is not the marginal price of foreign exchange in the economy. We want to identify exactly when the key coefficient, representing the effect of devaluation on money creation, is positive or negative. I argue three fiscal and credit policy variables accompanying exchange rate policy can be used to identify the effect, with exogenous variables (such as foreign debt obligations and program aid) washed out in the accounting. For those agricultural exports for which Government sets the domestic purchase price for farmers, suppose that proportion α of the increased domestic value (in domestic currency terms) following devaluation is passed on to farmers. For those imports for which Government sets the domestic price for consumers, suppose that proportion β of the increased domestic value (in domestic currency terms) is passed on to consumers. Finally, suppose that proportion γ of the increased cost to the private sector of purchasing official foreign exchange at the official exchange rate is reflected in increased credit to the private sector. Now let k_E equal α times the dollar value of exports purchased domestically at Government prices minus β times the dollar value of imports sold at Government determined prices minus $(1-\gamma)$ times the dollar value of the private sector's allocation of official foreign exchange. Then devaluation has a negative affect on money creation if and only if $k_E < 0$.

The analysis of this paper is complementary to that of Kaufmann and O'Connell (1992), which also emphasized how, for certain fiscal environments, parallel market unification can improve macroeconomic performance. This paper provides a full dynamic analysis of that interaction, and illustrates its importance in Uganda. Kaufmann and O'Connell provide a more detailed but static analysis of the situation of Tanzania.

2. Monetary expansion and the official exchange rate

This section looks at the determinants of money growth as a function of the price level, the official exchange rate, and some key fiscal and credit policy variables. Because this paper is concerned with economies where the official exchange rate does not affect any transaction in the economy at the margin, its importance is solely as a determinant of implicit taxes on those forced to sell foreign exchange to the central bank at the official rate, and the implicit taxes on those allocated foreign exchange at the (cheap) official rate. This section thus consists solely of accounting identities which capture the net effect of changes in these rents on money creation.

I would like to identify the impact of changes in the official exchange rate on money creation. In identifying that impact, I will be making both explicit and implicit assumptions about how (as a matter of Government policy) different components of the budget change in response to the devaluation.

Consider first money creation to finance the Government deficit. This is equal to the increase in Government credit on the assumption that money creation is the sole source of financing. Suppose Government holds domestic items of local expenditure and taxes constant in real (domestic currency) terms, at the same time holding items of Government expenditure and receipts in foreign currency (dollar) constant in dollar terms (assuming negligible dollar inflation). Then increasing the official exchange rate will increase the budget deficit and thus the flow of credit to the Government if Government has a *deficit* on its foreign exchange account. The result is reversed if Government has a *surplus* on the foreign exchange account.

This result is obviously going to depend on exactly which items are denominated in local currency or dollars. Two critical items are those exports and imports whose domestic prices are determined by the Government. In Uganda, there were essentially one such export, coffee, and one such import, petroleum products ('oil'), and in the analysis that follows I refer only to these two. Food imports will be another extremely important example of Government price-controlled imports in other African countries. The price paid to farmers for coffee and the price consumers must pay for oil is determined by the Government. The Government budget will then show the difference between the world price at the official exchange rate and the domestic price as an explicit tax for the export, coffee; while the difference between the domestic price and the world price at the official exchange rate is an explicit tax for the import, oil.

Thus Government decides whether what proportion of the payments to coffee farmers for their coffee and payments by consumers for their oil are denominated in local or foreign currency. Suppose the Government passes on to farmers proportion α of the increased value of coffee exports in domestic currency terms; and suppose the Government passes on proportion β of the increased cost of oil and sold at Government determined prices to consumers; α and β are thus policy variables of the Government.

Focussing exclusively on the Government budget, however, misses a critical element of the story. Private sector credit may also vary in response to changes in the official exchange rate. Indeed, an argument is often made that exchange rate auctions in Uganda in 1982/84 and Zambia failed at least partly because private sector credit increased to finance the increased domestic price of purchasing foreign exchange. Whether or not there is an economic rationale for such increases (and there may be), allocatees of foreign exchange lose rents (implicit subsidies) if the official exchange rate increases, and they seek compensation in the form of an increase in (rationed) credit. In the analysis that follows, it is assumed that the Central Bank increases credit to allocatees of foreign exchange by some proportion γ of the increased cost of buying dollars as the official exchange rate changes.

Now we will put all this together to get a single number, a function of the three policy variables α, β, and γ, which determines the impact of exchange rate changes on money creation. A third component of money creation is accumulation of foreign reserves which must be financed with domestic currency, but we see in what follows that this washes out in the accounting, since the allocation of foreign exchange to the private sector is (by an identity) the difference between the Government's budget surplus in foreign exchange and accumulation of foreign reserves.

The following are stylized versions of the monetary accounts, balance of payments, and Government budget of Uganda and countries in a similar situation. 'Shillings' are the domestic currency, 'dollars' are the foreign currency, P is the price level in shillings, and E is the nominal official exchange rate in shillings per dollar.

[1] *Monetary Accounts* (in shillings)

$$\mathrm{d}M/dt = \mathrm{d}M_G/\mathrm{d}t + \mathrm{d}M_P/\mathrm{d}t + \mathrm{d}r^*/\mathrm{d}tE$$

$\mathrm{d}M/dt$	$\mathrm{d}M_G/\mathrm{d}t$	$\mathrm{d}M_P/\mathrm{d}t$	$\mathrm{d}r^*/\mathrm{d}tE$
Increase in total money stock	Increase in government credit	Increase in private sector credit	Increase in foreign currency reserves

[2] *Balance of Payments* (in dollars)

$$\mathrm{d}r^*/\mathrm{d}t = c^* + g^* - d^* - o^* - a^*$$

$\mathrm{d}r^*/\mathrm{d}t$	c^*	g^*	d^*	o^*	a^*
Increase in foreign currency reserves	Coffee exports	External assistance	Debt and other government foreign currency private	Oil imports	Allocation of foreign exchange to the sector

Sources of foreign currency are coffee exports (of total dollar value c^*) and external assistance (g^*). Imports are divided into those imports which are sold through the Government (o^*) and those through the private sector (a^*).

[3] *Government Budget* (in shillings)

$$\mathrm{d}M_G/\mathrm{d}t = (s-t)P + (d^* - g^*)E + \left[(c_P P + \alpha c^* E) - c^* E\right]$$

$\mathrm{d}M_G/\mathrm{d}t$	$(s-t)P$	$(d^* - g^*)E$	$(c_P P + \alpha c^* E)$	$c^* E$
Increase in government credit	Local currency deficit	Foreign currency deficit	Coffee payments	Coffee receipts

$$+ \left[o^* E - (o_P P + \beta o^* E)\right]$$

$o^* E$	$(o_P P + \beta o^* E)$
Oil payments	Oil receipts

In the Government budget, local taxes (t) and expenditure (s) are assumed to be held constant in real terms. Similarly, foreign currency receipts (g^*) and expenditure (d^*) are held constant in dollar terms. Government receives (as taxes) the difference between revenue from sale of exports (c^* in dollar terms) and payments to farmers. The payment to farmers comes in two parts: a fixed real component c_P and the proportion α of the domestic currency value of the coffee. Similarly, Government receives (as taxes) the difference between the amount paid by consumers for oil products and the cost of imports. The price consumers pay is divided into a real component, o_P, and a proportion β of the domestic currency value of the imports.

[4] *Private Sector Credit*

$$\underset{\substack{\text{Increase in}\\\text{credit to}\\\text{private}\\\text{sector}}}{\mathrm{d}M_P/\mathrm{d}t} = a_P P + \gamma a^* E$$

The real flow of private sector lending is equal to a fixed real component, a_P, and some proportion γ of the domestic value of the private sector's allocation of foreign exchange.

Now substituting [2], [3] and [4] in [1] gives:

$$\mathrm{d}M/\mathrm{d}t = k_E E + k_P P,$$

where $k_E = \alpha c^* - \beta o^* + (1 - \gamma)a^*$ and $k_P = c_P - o_P + s - t + a_P$

Thus to determine, for given fiscal and credit policy variables α, β, and γ, whether an increase in the nominal exchange rate will increase or decrease nominal money creation, it is necessary to identify (in dollar terms) the value of exports and imports with Government controlled prices, and private sector allocations of foreign exchange.

This is a simplistic analysis of the Government budget. I excluded microeconomic impacts of the official exchange rate devaluation, on the grounds that the parallel market represented the relevant marginal price for most transactions in the economy. But, for example, the decision whether to smuggle exports or deliver them to official channels will be influenced by the official exchange rate, so the official exchange rate may have an indirect impact on trade flows. Items such as customs revenue may be collected in local currency but may be ad valorem taxes on the value (at the official exchange rate) of imports. Such taxes might be expect to rise in line with an official devaluation. By excluding these effects, I am implicitly assuming that Government chooses to hold the domestic value of domestic denominated taxation constant.

In Section 4, I will make this analysis more concrete by looking at budget numbers for Uganda in 1989. In Section 3, this money equation is one of three in a simple dynamic model based on Pinto (1990).

3. A model of inflation and the parallel market exchange rate

The economy is described by three equations: (1) nominal money creation depends on the price level and the official exchange rate; (2) a portfolio choice equation gives demand for local currency ('shillings') and parallel market foreign exchange ('dollars'), as a function of inflation; and (3) a 'balance of payments' equation clears the parallel market for foreign exchange. It is assumed that the domestic price level (P) is always equal to the parallel market exchange rate (in shillings/dollar). This framework seems to be the simplest possible to capture the interactions between the structure of the budget, the parallel market for foreign exchange, and inflation. Elements of other models describing production and smuggling decisions, which do not effect the dynamics, are omitted. [1] The model in this section closely follows Pinto (1990), where a more detailed discussion can be found. The key difference is the money equation, [2] where Pinto assumed $k_P = 0$, and $k_E > 0$. All the results in this section for that case were shown in Pinto (1986, 1990, 1991).

3.1. Money creation

Nominal money creation ($\mathrm{d}M/\mathrm{d}t$) is assumed to depend on the price level (P) and the official exchange rate (E). The origins of this relation were discussed in detail in the previous section. Recall that prices are determined by the parallel market exchange rate, so there is no direct cost push effect of E on P.

$$\mathrm{d}M/\mathrm{d}t = k_P P + k_E E.$$

If we write m for the real money stock (M/P), e for the real official exchange rate (E/P) and $\hat{P}$ for the rate of inflation [$(\mathrm{d}P/\mathrm{d}t)/P$], this equation can be given in real terms as

$$\mathrm{d}m/\mathrm{d}t = k_P + ek_E - m\hat{P}. \tag{1}$$

3.2. Parallel market foreign asset accumulation

The stock of parallel market dollars is f. The parallel market exchange rate is equal to the price level (P). Private wealth (w), in dollars at the parallel market exchange rate, or in real terms (deflated by the price level) is thus $m + f$. Now suppose that some function $\phi(w)$ of the stock of private wealth is spent on imported parallel market goods at any one time. If the supply of foreign exchange on the parallel market (e.g. from smuggled exports) is x^*, then the equation for parallel market foreign asset accumulation is:

$$\mathrm{d}f/\mathrm{d}t = x^* - \phi(m + f), \quad \phi'(\cdot) > 0. \tag{2}$$

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[1] Pinto (1986) incorporates the smuggling decision in the parallel market balance of payments. The results that follow would remain essentially the same if this was incorporated in this paper.

[2] There are also expositional differences: 'real' variables are deflated by the price level (not the nominal exchange rate), and the real exchange rate ($e = E/P$) is an analyzed instead of parallel market premium (P/E). Symbols used throughout the paper are collected in Appendix A.

3.3. Portfolio balance

Domestic money and parallel market dollars are the only assets in the economy. The proportion of total wealth held in domestic money is a decreasing function of the rate of inflation (which is the rate of nominal depreciation of domestic currency on the parallel market). Let

$$m = \mu(\hat{P})w, \quad \text{with } \mu'(\hat{P}) < 0 \quad \text{and} \quad 0 < \mu(\hat{P}) < 1.$$

Thus the proportion of wealth held as in domestic currency is a decreasing function of the rate of inflation. Now

$$m/f = \mu(\hat{P})/\left[1 - \mu(\hat{P})\right].$$

Since $\mu(\cdot)/[1 - \mu(\cdot)]$ is strictly decreasing, we can define function $\lambda(\cdot)$ such that

$$\hat{P} = \lambda(m/f), \quad \lambda'(\cdot) < 0.$$

Now since $e = E/P$, if the rate of change of the official exchange rate E is $\hat{E}$,

$$\mathrm{d}e/\mathrm{d}t = e\left[\hat{E} - \lambda(m/f)\right]. \tag{3}$$

3.4. Structure of money demand

Define the elasticity of money demand $\eta(\cdot)$ by:

$$\eta(\hat{P}) = -\hat{P}\mu'(\hat{P})/\mu(\hat{P}) > 0.$$

Some results in what follow will depend on the shape of money demand and its implications for inflation tax. Inflation tax will be proportional to $I(\hat{P}) = \hat{P}\mu(\hat{P})$. Say inflation tax is *hump-shaped* if it is increasing from $\hat{P} = 0$ to some critical level $\hat{P}^*$ beyond which it decreases, i.e. if (i) $I'(0) > 0$, (ii) $I''(\hat{P}) < 0$ for all $\hat{P} \geqslant 0$, and (iii) $I'(\hat{P}^*) = 0$. These conditions on I translate into conditions on the elasticity as follows:

$$\text{(i)}\ \eta(0) < 1$$

$$\text{(ii)}\ \eta(\hat{P}) + \hat{P}\eta'(\hat{P}) \geqslant 0 \quad \text{for all} \quad \hat{P} \geqslant 0$$

$$\text{(iii)}\ \eta(\hat{P}^*) = 1$$

3.5. The fixed crawl regime

One exchange rate policy is to fix the rate of nominal exchange rate crawl at some $\hat{E}$. Eqs. (1), (2) and (3) then define the dynamics of the three variables f, m, and e.

Theorem (Fixed Crawl Regime). Suppose $k_E \neq 0$ and there exists $e \epsilon (0,1)$ such that $k_P + ek_E = \hat{E}\mu(\hat{E})\phi^{-1}(x^*)$. Then:

1. Steady State: There exists a unique steady state of the system with $df/dt = dm/dt = de/dt = 0$, and non-zero e: [3]

$$m^* = \mu(\hat{E})\phi^{-1}(x^*)$$
$$f^* = \left[1 - \mu(\hat{E})\right]\phi^{-1}(x^*)$$
$$e^* = \left\{\left[\hat{E}\mu(\hat{E})\phi^{-1}(x^*)\right] - k_P\right\}/k_E$$

and equilibrium inflation $\hat{P} = \hat{E}$.

2. Comparative Statics. $de^*/d\hat{E} = \mu(\hat{E})(1 - \eta(\hat{E}))\phi^{-1}(x^*)/k_E$.

3. Stability of Steady State. There exists some η_0, such that:
If $k_E < 0$, and $\eta(\hat{E}) > \eta_0$, then the steady state is locally totally stable. [4]
If $k_E < 0$ and $\eta(\hat{E}) < \eta_0$, then the steady state is unstable. [5]
If $k_E > 0$, the steady state is locally saddlepoint stable. [6]

The steady-state conditions (1) tell us that the real exchange rate adjusts in order to ensure that real money creation equals the real inflation tax,

$$k_P + k_E e^* = \hat{E}\mu(\hat{E})\phi^{-1}(x^*).$$

Notice that by assumption, there exists $0 < e^* < 1$ solving this equation.

The comparative statics condition (2) tells us that if $\eta(\hat{E}) < 1$, increasing the rate of exchange rate crawl, $\hat{E}$, will increase (decrease) the real exchange rate if $k_E < 0$ ($k_E > 0$).

The stability results (3) are proved in Appendix B. These results tell us that depending on the parameter values, the dynamics around the steady state are likely to be quite different.

Consider first the case where $k_E > 0$. This is the case studied by Pinto (1990), since he assumed a government deficit which was fixed in dollar terms. In this case, the dynamics are saddlepoint stable, so that if one variable is allowed to 'jump' to an equilibrium path, the economy would respond to a deviation from the steady state by jumping back onto a unique path back to the steady state. Since prices are assumed to be able to jump in response to a shock, this saddlepoint stability property suggests that the steady-state equilibrium will be robust to shocks.

[3] There is also a steady state with $dM/dt = de/dt = dF/dt = 0$ and $e^* = 0$. This is discussed below and in Appendix E.

[4] All roots of the 3×3 dynamic matrix have negative real parts.

[5] There are two roots with positive real parts and one root with a negative real part.

[6] There are two roots with negative real parts and one root with real part positive.

Consider now what happens if $k_E < 0$ and $\eta(\hat{E}) < \eta_0$. Then we have the instability discussed in the introduction. Even allowing for the ability for prices to jump to bring the economy back into equilibrium, the dynamics will be unstable. If the economy is ever perturbed away from the steady state, there will be a tendency for the deviation to become ever larger.

The easiest way to explain this instability is to consider an extreme case where it is possible to solve explicitly for the dynamics. I consider the extreme case where the elasticity of money demand is zero, so that prices adjust instantly to monetary growth and the quantity theory holds continuously. This case is solved in Appendix C. The real exchange rate evolves as follows:

$$e(t) = \frac{1}{\dfrac{1}{e^*} + \left\{\dfrac{1}{e_0} - \dfrac{1}{e^*}\right\} \exp^{-e^* k_E t}}$$

where e_0 is the initial value of the real exchange rate e, and e^* is the steady-state real exchange rate $[m^* \hat{E} - k_P]/k_E$. We can see the dynamics clearly in this case. As long as $k_E > 0$, any deviation of e_0 from e^* decreases through time; but if $k_E < 0$, then there is an explosive path away from the steady state. In Section 5, it is argued that exchange rate policy certainly seemed to have this characteristic during one period in Uganda.

Finally, consider what happens when $k_E < 0$ and $\eta(\hat{E}) > \eta_0$, so that the equilibrium is totally locally stable. In this case, all dynamic paths in the neighborhood of the steady state are directed back to the steady state, and there is no need for prices to 'jump' in order to get onto that path.

3.6. *The unified exchange rate regime*

As in Pinto (1990), we can compare the constant exchange rate crawl with a unified exchange rate regime. Suppose now that E is set continuously equal to P, so that $e = 1$ always. Our dynamic system reduces to two equations:

$$\mathrm{d}m/\mathrm{d}t = k_P + k_E - m\lambda(m/f), \tag{4}$$

$$\mathrm{d}f/\mathrm{d}t = x^* - \phi(m+f). \tag{5}$$

Theorem (Unified Regime). Suppose $k_E \neq 0$ *and* $0 < k_P + k_E \leqslant \hat{P}^*\mu(\hat{P}^*)\phi^{-1}(x^*)$. *Then:*

1. Steady State. A steady state of the system (4), (5) with $\mathrm{d}f/\mathrm{d}t = \mathrm{d}m/\mathrm{d}t = 0$ solves:

$$m^* = \mu(\hat{P})\phi^{-1}(x^*)$$

$$f^* = \left[1 - \mu(\hat{P})\right]\phi^{-1}(x^*)$$

with $m^* = k_E + k_P$.

2. Stability of Steady State. A steady state is locally saddlepoint stable if $1 < \eta(\hat{P})$, and locally completely stable otherwise.

In the constant crawl regime, the real exchange rate adjusted to equate the real increase in credit with real inflation tax. In the unified regime, inflation adjusts to bring about the same equilibrium.

The stability result is proved in Appendix C. It is noteworthy that in the case where the elasticity of money demand is low, the steady state of the fixed crawl regime was unstable, whereas under the unified regime, it has become stable. The intuition for this result is the following. The instability arose under the fixed exchange regime because a foreign exchange surplus in the Government deficit was funding a local currency deficit. Real devaluation of the official exchange rate led to a deteriorating budget leading to inflation and further real devaluation of the official exchange rate. By preventing the divergence of the official exchange rate from the parallel rate and domestic prices, this vicious unstable cycle is broken. Allowing the exchange rate to float leads to no other such instability.

If inflation tax is hump-shaped (as discussed above) there will be two steady states of the above system: one with inflation $\hat{P}$ less than $\hat{P}^*$ (the inflation rate which maximizes $\hat{P}\mu(\hat{P})$) and saddlepoint stable, and one greater than $\hat{P}^*$, which is completely stable.

Theorem (Comparison of Regimes). If money demand is hump-shaped, $\hat{E} < \hat{P}^*$ and the economy converges to the low equilibrium in the unified regime, then inflation is higher (lower) in the unified regime than in the fixed crawl regime if $k_E > 0$ ($k_E < 0$).

To see why, note that:

$$\hat{P}\mu(\hat{P})\phi^{-1}(x^*) = k_P + k_E,$$

$$\hat{E}\mu(\hat{E})\phi^{-1}(x^*) = k_P + e^* k_E < k_P + k_E.$$

Under the assumption of the theorem both $\hat{E}$ and inflation in the unified regime $\hat{P}$ are less than $\hat{P}^*$, so $\hat{P}\mu(\hat{P})$ is increasing in $\hat{P}$.

3.7. Discussion

We have analyzed the comparative statics and local stability of steady states under two regimes: a fixed nominal exchange rate crawl and a floating exchange rate. The dynamics of the model are complex, and we must take care not to over-interpret the local stability conditions. [7] Our purpose has been to highlight

[7] In Appendix E, a steady state in the fixed exchange rate crawl regime with $e = 0$ is studied; since this steady state is locally stable for $\hat{E}$ such that $\hat{E}\mu(\hat{E}) < k_P$, it follows that the $0 < e^* < 1$ steady state is not globally stable.

the key role of assumptions about money creation. Comparative static results in Pinto (1990) are in many cases directly reversed as a result of different budget assumptions. If we wish to control inflation, and if the elasticity of money demand is quite low – the Quantity Equation is found to hold more or less continuously – then the best exchange rate policy will depend on the structure of the money creation equation discussed in Section 2. If the exchange rate policy is to be accompanied by a set of fiscal and credit policies which imply money creation *increases* with nominal devaluation ($k_E > 0$), then inflation will be minimized by an exchange rate with a fixed rate of crawl, with a parallel market premium in the steady state ($e^* < 1$). But if the exchange rate policy is to be accompanied by fiscal and credit policies which imply money creation *decreases* with nominal inflation ($k_E < 0$), then inflation will be minimized by a unified exchange rate regime (with $e = 1$).

The next section examines, for the case of Uganda, the determinants of the critical parameter k_E, while Section 5 discusses how the Uganda experience conforms to the dynamic model of this section.

4. Fiscal and exchange rate policy, Uganda 1989

This section reports calculations from Morris (1989b) on the impact of changes in the official exchange rate on money creation in Uganda, based on 1989/90 budget estimates. [8]

[1] *Balance of Payments* (in million US dollars)

$\mathrm{d}r^*/\mathrm{d}t$	=	c^*	+	g^*	−	d^*	−	o^*	−	a^*
0	=	285	+	440	−	370	−	80	−	275
Increase in foreign currency reserves		Coffee exports		External assistance		Debt and other government foreign currency payments		Oil imports		Allocation of foreign exchange to the private sector

The US$440 million of external assistance includes US$240 million of project aid (with expenditure corresponding items in d^*). The foreign exchange allocated to the private sector includes US$200 million of program assistance.

[8] Uganda (1989b). Some figures were not in the form required for this analysis and were derived from other Government sources. These are planned, not actual, figures, but they thus represent the perceived policy choice facing Government.

[2] *Government Budget* (in billion shillings, at 200 shillings/$)

$$\underset{\substack{\text{Increase in}\\\text{government}\\\text{credit}}}{\underset{60}{\mathrm{d}M_G/\mathrm{d}t}} = \underset{\substack{\text{Local}\\\text{currency}\\\text{deficit}}}{\underset{155-64}{(s-t)P}} + \underset{\substack{\text{Foreign}\\\text{currency}\\\text{deficit}}}{\underset{74-88}{(d^*-g^*)E}} + \Big[\underset{\substack{\text{Coffee}\\\text{payments}}}{\underset{50}{(c_P P+\alpha c^* E)}} - \underset{\substack{\text{Coffee}\\\text{receipts}}}{\underset{57}{c^* E}}\Big]$$

$$+ \Big[\underset{\substack{\text{Oil}\\\text{payments}}}{\underset{16}{o^* E}} - \underset{\substack{\text{Oil}\\\text{receipts}}}{\underset{26}{(o_P P+\beta o^* E)}}\Big]$$

Thus Uganda has a highly skewed budget. Large amounts of program aid are funding a deficit on local expenditure. We can use the formula of Section 2 to identify the impact of devaluation on money creation:

$$k_E = 285\alpha - 80\beta + 275(1-\gamma).$$

Thus if Government devalued the exchange rate to 400 shillings per dollar, but kept payments to farmers constant in real terms ($\alpha = 0$), oil prices constant in real terms ($\beta = 0$), and credit to the private sector constant in real terms ($\gamma = 0$), the (immediate) nominal budget deficit would improve by 275 million $\times$ (400 − 200) = 55 billion shillings, i.e. the deficit would be reduced from 60 billion to 5 billion. The effect on the steady state and dynamics can be analyzed by looking at the results of Section 3 for the case $k_E > 0$.

The purpose of this section was to show how it is possible in practice to identify the fiscal impact of exchange rate policy. Kaufmann and O'Connell (1992) carried out a very similar analysis of what they called the 'monetary effect' of exchange rate policy for Tanzania. They found that, for Tanzania, the sign of the effect changed as a result of increased inflows of foreign exchange.

5. Inflation and the parallel market, Uganda, 1982–1989

In this section, the experience of Uganda is briefly reviewed in the light of the macroeconomic model presented in this paper. For a more detailed discussion, see Morris (1989a). Fig. 1 shows the behavior of prices and the parallel market and official exchange rates. [9]

Three historical stages need to be distinguished. From 1982 through 1984, there was a period of relatively stable Government (the 'Obote II' era), with an IMF sponsored foreign exchange rate auction setting the official exchange rate. [10] Foreign exchange continued to trade at a premium on a parallel market, reflecting

[9] All series were compiled by the author from assorted government sources, e.g. Uganda (1989a).

[10] There was also a second (lower) official exchange rate being used for some transactions during this period.

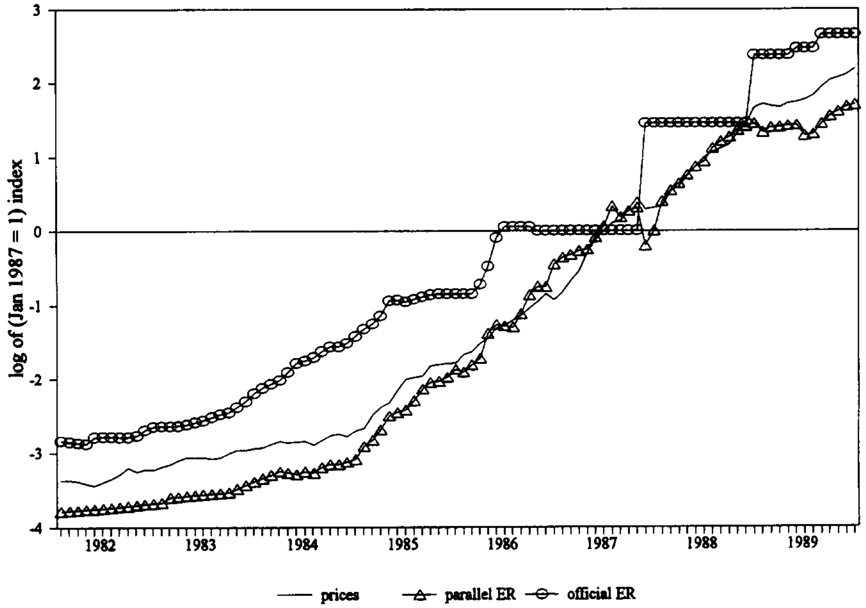

Fig. 1. Prices and exchange rates 1982–1989.

tax avoidance of the large 'informal' sector and also the avoidance of the large local currency deposit requirements to participate in the official auction. From 1984, increasing political instability led to increased military spending, rising real deficits and the eventual abandonment of the foreign exchange auction in May 1985. The Obote Government was replaced in January 1986 by that of the National Resistance Movement (NRM). In May 1987, a currency reform was implemented. During the period 1986–1989, a fixed exchange rate policy was maintained, although large, irregular devaluations resulted from a failure to control inflation.

Let us first look at the key assumption of the model that the price level moves with the parallel market exchange rate, with the official exchange rate relevant only for shifting rents. This pattern is shown in the medium term. Thus, Fig. 2 shows during the fixed exchange rate regime of 1986–1989, the real exchange rate fluctuates much more than the real parallel market exchange rate.

This is consistent with the evidence that (1) the parallel market rate represented the marginal cost of traded goods (other than petroleum products) in the economy; [11] and (2) it is remarkably difficult to isolate a distinct pattern for traded

[11] The rule was well illustrated by occasional exceptions. In 1987/88, the Government arranged the sale of surplus cooking oil donated by the United States Agency for International Development at a price determined by the official exchange rate. For many months, cooking oil sold well below its usual real price (determined by the price of Kenyan cooking oil and the black market exchange rate). A similar phenomenon was observed with Cuban car tires obtained in barter deals. In each case, prices rose back to normal levels when it became clear that supplies would not persist at the official prices.

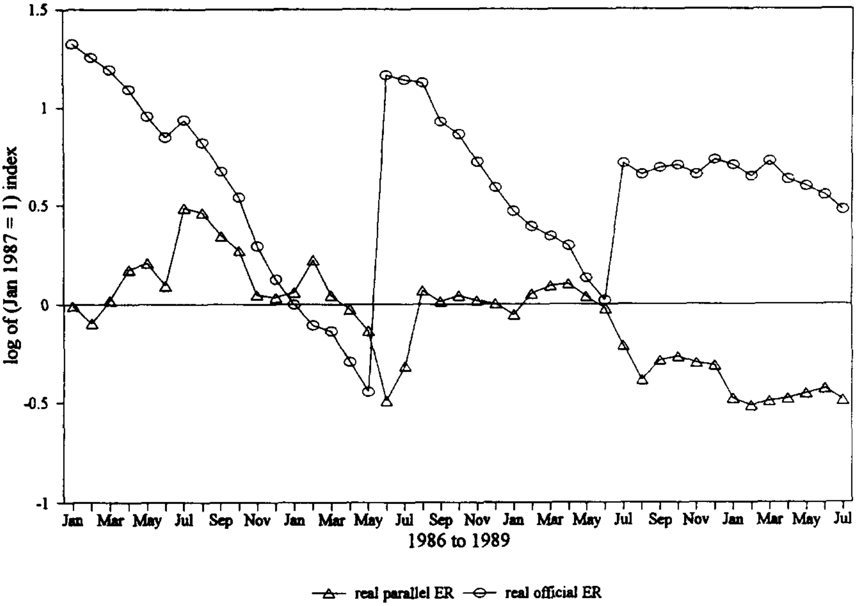

Fig. 2. Real exchanges rates 1986–1989.

and non-traded goods in the price data. [12] There were large short-run fluctuations in the real parallel market exchange rate: Morris (1989a) suggests that 'overshooting' occurred as the parallel market adjusted faster to shocks than the goods market.

The structure of the budget, outlined for fiscal year 1989/90 in Section 4, was essentially the same during the 1982–89 period, except that foreign funding dried up during the height of the civil war in between the Obote and NRM Governments (1985–1986). In both the preceding and following years, budgetary conditions were favorable for an attempt to unify the official exchange rate with the black market rate.

The model of Section 3 suggested that, in the presence of budgetary conditions like Uganda's, money creation would be *decreasing* in the official exchange rate. This meant, under assumptions about money demand plausible for Uganda, the steady-state rate of inflation would be lower under a unified regime than under an attempt at a fixed crawl. Thus, Fig. 3 shows that inflation was relatively low and stable during the 1982–84 flexible exchange rate regime, and high and variable from 1986–1989.

[12] All the price data are for the capital Kampala. It may make more sense to think of the model in this paper as a model of a small monetized sector in an economy that had reverted to a remarkable degree of subsistence in response to a decade and a half of civil war.

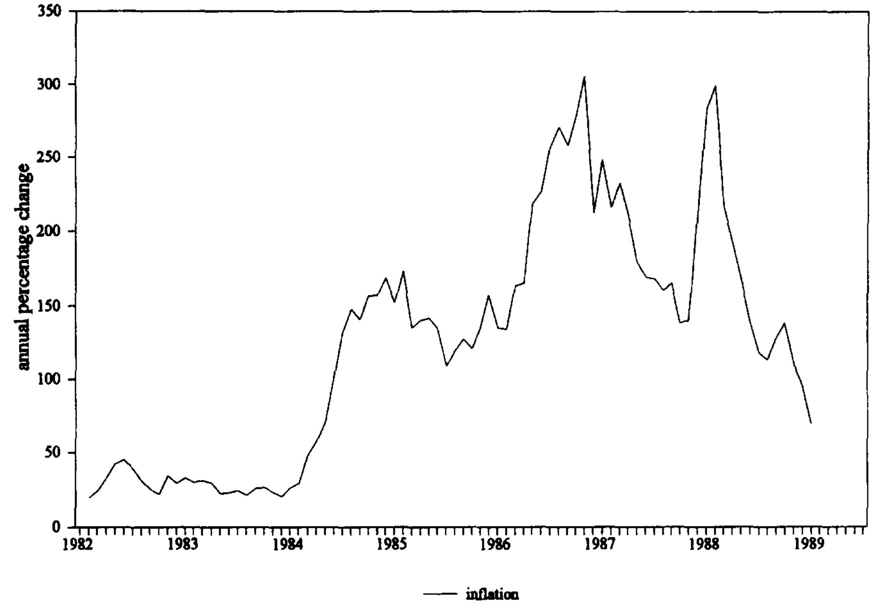

Fig. 3. Inflation 1982–1989.

In addition, the model suggested that the steady state with a fixed crawl regime would be inherently unstable. The possibility was also noted that the economy would converge to a zero real exchange rate steady state: increasing inflation would lower the real exchange rate, increasing real money creation, leading to a downward spiral to a zero real exchange rate. This is certainly consistent with the perception of policy makers during the period that if an initially balanced budget was allowed to get temporarily out of control, it would get more and more difficult to get it back on track. Attempts at balanced budgets and balanced credit policy apparently failed at least in part because an attempt was being made to stay in an unstable equilibrium.

Since 1989, exchange rate policy has moved to a managed float as the parallel market for foreign exchange was legalized and the official exchange rate set to track the now legal parallel rate. Inflation has dropped to the comparatively low rates (20% to 30% annual rates) experienced in 1982/84, from the high 120% to 300% rates experienced in the 1986–1989 period.

While Uganda's experience is suggestive, it should be made clear that it is hard to directly test econometrically the model presented here. The model relates financial variables, not all of them observable (e.g. the stock of parallel market dollars held by the private sector) and none of them conceivably exogenous. Data on real variables that might be expected to enter the model and identify the system (GDP) are essentially worthless for this period in Uganda. Regime shifts are many and dramatic in Uganda. Kuleesa (1990) attempts an econometric analysis of some of the macroeconomic relationships discussed in this paper. The results are consistent with the model here, but not conclusive. Chhibber and Shafik (1990) estimate a model, using annual data, for Ghana, a country with a very similar set of macroeconomic conditions to Uganda. They find that devaluation tends to reduce inflation. Aron and Elbadawi (1992) estimate the determinants of the parallel market premium in Zambia.

6. Conclusion

The key idea of the dynamic model of Section 3 is more general than the parallel market model presented here. [13] Consider any economy with a fixed nominal exchange rate crawl, which is initially in a steady state, with a constant real exchange rate and inflation equal to the fixed crawl. Monetary policy is held fixed, with real money creation some function of the real exchange rate. Suppose a monetary shock temporarily increases inflation. If real money creation is a decreasing function of the real exchange rate, the monetary shock will lower the real exchange rate, increasing inflation, leading to a (locally) explosive path away from the steady state. Conversely, if real money creation is an increasing function of the real exchange rate, budget shocks will tend to be damped.

This paper examined this effect in a class of economies where we believe it will be especially pronounced, and important in policy terms. The paper builds on work of Pinto (1990), by calculating comparative static and dynamic results in his models under different assumptions about money creation. Pinto emphasized the importance of the fiscal policy accompanying exchange rate policy changes. This paper reiterates that importance and gives a simple rule of thumb (in Section 2) for when unification will reduce steady-state inflation as a function of both fiscal and private sector credit policies.

Appendix A: Symbols used

In dollars

a^* private sector allocation of foreign exchange
c^* coffee exports
d^* debt and other Government payments
f stock of dollars on the parallel market
g^* external assistance
o^* oil imports

[13] For example, the comparative static and dynamic results of Khan and Lizondo (1987) will be similarly reversed under alternative assumptions about the budget.

r^* foreign currency reserves
x^* parallel market exports

Nominal

P price level
E official exchange rate
M money stock
M_G credit to Government
M_P credit to the private sector

In real terms (domestic)

e real official exchange rate (E/P)
m real money stock (M/P)
s domestic expenditure
t domestic taxation
c_P real component of payment to coffee farmers
o_P real component of domestic price of oil imports
a_P real component of private sector credit

Policy variables

α proportion of domestic currency value of coffee passed on to farmers
β proportion of domestic currency value of oil passed on to consumers
γ proportion of domestic currency value of private sector's allocation of foreign exchange reflected in increased private sector credit

Others

$\phi(w)$ expenditure function
$\hat{E}$ fixed exchange rate crawl $([\mathrm{d}E/\mathrm{d}t]/E)$
$\hat{P}$ rate of inflation $([\mathrm{d}p/\mathrm{d}t]/p)$
$\hat{P}^*$ rate of inflation maximizing $\mu(\hat{P})$
$\zeta(m/f)$ elasticity of inflation w.r.t. portfolio ratio (m/f)
$\eta(\hat{P})$ elasticity of money w.r.t. inflation
$\lambda(m/f)$ inflation as a function of portfolio ratio (m/f)
$\mu(\hat{P})$ money demand function

Appendix B: Stability of the fixed crawl regime

Dynamic equations:

$$\mathrm{d}m/\mathrm{d}t = k_P + ek_E - m\lambda(m/f) \quad \text{(B.1)}$$

$$\mathrm{d}e/\mathrm{d}t = e\left[\hat{E} - \lambda(m/f)\right] \quad \text{(B.2)}$$

$$\mathrm{d}f/\mathrm{d}t = x^* - \phi(m+f) \quad \text{(B.3)}$$

Matrix of local dynamics:

$$\begin{pmatrix} \mathrm{d}m/\mathrm{d}t \\ \mathrm{d}e/\mathrm{d}t \\ \mathrm{d}f/\mathrm{d}t \end{pmatrix} = \begin{pmatrix} -\lambda-(m/f)\lambda' & k_E & (m^2/f^2)\lambda' \\ -(e/f)\lambda' & \hat{E}-\lambda & (em/f^2)\lambda' \\ -\phi' & 0 & -\phi' \end{pmatrix} \begin{pmatrix} m-m^* \\ e-e^* \\ f-f^* \end{pmatrix}$$

Using steady-state condition $\hat{E} = \lambda(m/f)$, we can see that

$$\text{DETERMINANT } (D) = -k_E\lambda'\phi'(e/f)[1+(m/f)]$$

$$\text{TRACE } (T) = -\left[\hat{E}+(m/f)\lambda'+\phi'\right]$$

$$\text{SUM OF PRINCIPAL MINORS } (S)$$

$$= \phi'\{\hat{E}-\eta[1+(m/f)+k_E(e/m\phi'))]$$

let $\zeta(m/f) = -(m/f)\lambda'/\hat{E} = [1-\mu(\hat{E})]/\eta(\hat{E})$.
Observe that

$$\text{(i)}\, D>0 \Leftrightarrow k_E>0$$

$$\text{(ii)}\, T>0 \Leftrightarrow \zeta>1+\phi'/\hat{E}$$

$$\text{(iii)}\, \zeta>1+\phi'/\hat{E} \Rightarrow S<0$$

Recall that $D>0$ implies that either one or three real parts of roots of the characteristic equation are positive. But either $T<0$ (implying at least one negative real part) or $T>0$ (implying $S<0$ and thus at least one negative real part). Therefore $D>0$ implies exactly one root with positive real part and that the steady-state solution is saddlepoint stable.

Now suppose $D<0$. Either one or three real parts of roots are negative. We can use the modified Routh–Hurwitz conditions to test for complete stability (three negative real parts). [14] Complete stability of matrix **A** requires:

$$\text{(i)Determinant } (\mathbf{A})<0$$

$$\text{(ii)Trace } (\mathbf{A})<0$$

$$\text{(iii)the determinant of the bialternate matrix } (\mathbf{A}\cdot\mathbf{I})<0$$

This gives conditions:

$$\text{(i)} \Leftrightarrow k_E<0$$

$$\text{(ii)} \Leftrightarrow \zeta<1+\phi'/\hat{E}$$

$$\text{(iii)} \begin{vmatrix} -\hat{E}-(m/f)\lambda' & em\lambda'/f2 & -(m^2/f^2)\lambda' \\ 0 & -\left[\phi'-\hat{E}-(m/f)\lambda'\right] & k_E \\ \phi' & -(e/f)\lambda' & -\phi' \end{vmatrix} <0$$

$$\Leftrightarrow f(\zeta)>0$$

[14] Murata (1977), p. 92, theorem 11.

where

$$f(\zeta)=\hat{E}[\phi'+k_E e/m+(m/f)\phi']\zeta^2 - \left[\phi'(\phi'+\hat{E})(1+(m/f))+\hat{E}\phi'+k_E e\left((\hat{E}/m)-(\phi'/f)\right)\right] \times\zeta+\left[\phi'+\hat{E}\right].$$

Note that $f(0)=[\phi'+\hat{E}]>0$, $f(1+\phi'/\hat{E})<0$. Therefore there exists $\zeta_0 \epsilon (0,1+\phi'/\hat{E})$ such that

$$f(\zeta)>0 \quad \text{for all } \zeta<\zeta_0$$
$$f(\zeta)<0 \quad \text{for all } \zeta>\zeta_0$$

Now $k_E<0$ and $\zeta<\zeta_0$ implies complete local stability (all real parts of roots negative); (b) $k_E<0$ and $\zeta>\zeta_0$ implies instability (real parts of two roots positive).

Note that $\zeta=(1-\mu(\hat{E}))/\eta(\hat{E})$. Therefore there exists $\eta_0>\hat{E}(1-\mu(\cdot))/[\hat{E}+\phi'(\cdot)]$ such that if $k_E<0$ and $\eta>\eta_0$ implies complete local stability (all real parts of roots negative); (b) $k_E<0$ and $\eta<\eta_0$ implies instability (real parts of two roots positive).

Appendix C: Quantity theory case

Suppose $\mu(\hat{P})=\mu$, for all $\hat{P}$. Then f is constant, m must jump to equilibrium value $m^*=\mu\phi^{-1}(X)$. $\mathrm{d}m/\mathrm{d}t=0$, so $e(k_P+ek_E)=m(e\hat{E}-\mathrm{d}e/\mathrm{d}t)$.

We can solve this equation explicitly to give the dynamic path for the real exchange rate over time $(e(t))$. The initial price level is determined by the nominal money stock $(p_0=M_0/m^*)$, so the initial real exchange rate $e_0=em^*/M_0$:

$$e(t)=\frac{1}{\dfrac{k_E}{m^*\hat{E}-k_P}+\left\{\dfrac{1}{e_0}-\dfrac{k_E}{m^*\hat{E}-k_P}\right\}\exp^{-(m^*\hat{E}-k_P)t}}.$$

Substituting $e^*k_E=m^*\hat{E}-k_P$ gives:

$$e(t)=\frac{1}{\dfrac{1}{e^*}+\left\{\dfrac{1}{e_0}-\dfrac{1}{e^*}\right\}\exp^{-e^*k_E t}}.$$

(1) now if $k_E>0$, then the real exchange rate converges to the steady-state solution, $e^*=(m^*\hat{E}-k_P)/k_E$.

(2) if $k_E<0$, then the real exchange rate diverges from e. If $e_0>e^*$, then the real exchange rate tends to 1 (i.e. a zero parallel market premium) in finite time. If $e_0<e^*$, then the exchange rate tends to zero, and inflation tends to k_P/m^*. [15]

[15] This corresponds to the general $e=0$ steady state discussed in Appendix E.

Appendix D: Unified regime dynamics

Matrix of local dynamics is:

$$\begin{pmatrix} \mathrm{d}m/\mathrm{d}t \\ \mathrm{d}f/\mathrm{d}t \end{pmatrix} = \begin{pmatrix} -\lambda-(m/f)\lambda' & (m^2/f^2)\lambda' \\ -\phi' & -\phi' \end{pmatrix} \begin{pmatrix} m-m^* \\ f-f^* \end{pmatrix}$$

$$\begin{aligned} \text{DETERMINANT} &= \phi'\left[\lambda+(m/f)\lambda'+(m^2/f^2)\lambda'\right] \\ &= \phi'\left[1-\eta(\hat{P})\right] \end{aligned}$$

i.e. DET $< 0 \Leftrightarrow \eta(\hat{P}) < 1$.

$$\begin{aligned} \text{TRACE} &= -\{\phi'+\lambda+(m/f)\lambda'\} \\ &= -\{\phi'+\lambda[1-(1-\mu(\cdot))/\eta]\} \\ &< 0 \quad \text{if } \eta > 1. \end{aligned}$$

Therefore the two equation $(\mathrm{d}m/\mathrm{d}t,\ \mathrm{d}f/\mathrm{d}t)$ system has one (two) negative roots if η is less than (more than) one.

Appendix E

If there exists $\hat{P}$ such that $\hat{P}\mu(\hat{P}) = k_P$, then there exists at least one steady state of the fixed exchange rate crawl model, with the real exchange rate equal to zero, [16] with the equilibrium inflation rate $\hat{P}$ solving

$$\begin{aligned} \hat{P}\mu(\hat{P}) &= k_P, \quad \text{and} \\ m^* &= \mu(\hat{P})\phi^{-1}(X) \\ f^* &= \left[1-\mu(\hat{P})\right]\phi^{-1}(X) \\ e^* &= 0. \end{aligned}$$

This is the outcome of an increasing spiral of inflation driving the value of the real exchange rate to zero, so that the deficit in local expenditure must be financed entirely by inflation tax. We can solve for local stability:
Matrix of local dynamics, using $e = 0$:

$$\begin{pmatrix} \mathrm{d}m/\mathrm{d}t \\ \mathrm{d}e/\mathrm{d}t \\ \mathrm{d}f/\mathrm{d}t \end{pmatrix} = \begin{pmatrix} -\lambda-(m/f)\lambda' & k_E & (m^2/f^2)\lambda' \\ 0 & \hat{E}-\lambda & 0 \\ -\phi' & \hat{0} & -\phi' \end{pmatrix} \begin{pmatrix} m-m^* \\ e-e^* \\ f-f^* \end{pmatrix}.$$

[16] If inflation tax is hump-shaped, there will be exactly two, one with $\eta^* < 1$ and one with $\eta^* > 1$.

We can separately solve the independent system in m and f. This is identical to the system in Appendix D. Thus there are 1 (2) negative roots if η is less than (more than) one. The third root is negative if $\hat{E} - \lambda < 0$, i.e. if $\hat{E} < \hat{P}$.
Thus:
(i) if $\eta(\hat{P}) > 1$ and $\hat{E} < \hat{P}$, then completely stable
(ii) if $\eta(\hat{P}) > 1$ *or* $\hat{E} < \hat{P}$, then saddlepoint stable
(iii) if $\eta(\hat{P}) < 1$ and $\hat{E} > \hat{P}$, then unstable.

If money demand is well-behaved, there are exactly two zero e steady states, one with $\eta < 1$ and one with $\eta > 1$. Thus the low-inflation equilibrium is saddlepoint stable if and only if $\hat{E} < \hat{P}$. In other words, unless the exchange rate crawl is higher that the rate of inflation necessary to finance the local deficit from inflation tax, the zero real exchange rate steady state will be locally stable. This means that even though the positive e steady state with $k_E > 0$ is locally stable, we know it is not globally stable.

References

Aron, J. and I. Elbadawi, 1992, Parallel markets, the foreign exchange auction, and exchange rate unification in Zambia, World Bank Policy Research, Working Paper no. 909.

Chhibber, A. and N. Shafik, 1990, Exchange reform, parallel markets, and inflation in Africa, World Bank PPR Working Paper Series no. 427.

Kaufmann, D. and S. O'Connell, 1992, Fiscal and monetary effect of the parallel premium: Theory and the Tanzanian case, Working Paper.

Khan, M. and J. Lizondo, 1987, Devaluation, fiscal deficits and the real exchange rate, The World Bank Economic Review 1, 357–374.

Khoras, H. and B. Pinto, 1989, Exchange rate rules, parallel market premia and fiscal deficits: The Bolivian hyperinflation, Review of Economic Studies 56, 435–448.

Kuleesa, M., 1990, The black market exchange rate and exchange rate policy: The Ugandan experience (1984–1990), I.M.F.

Lizondo, J.S., 1987, Exchange rate differential and balance of payments under dual exchange markets, Journal of Development Economics 26, 37–53.

Morris, S., 1989a, Macroeconomic features of the Uganda economy and some policy implications, part one: The relationship between money, prices and the parallel market exchange rate, Ministry of Planning and Economic Development, Discussion Paper, August.

Morris, S., 1989b, Macroeconomic features of the Uganda economy and some policy implications, part two: The impact of official exchange rate devaluation on Uganda, Ministry of Planning and Economic Development, Discussion Paper, September.

Murata, Y., 1977, Mathematics for stability and optimization of economic systems (Academic Press, New York).

Pinto, B., 1986, Fiscal deficits, inflation and parallel markets in Ghana: Monetarism in the tropics?, World Bank Country Policy Department, Discussion Article 1985-43, Washington D.C.

Pinto, B., 1990, Black market premia, exchange rate unification, and inflation in Sub-Saharan Africa, World Bank Economic Review 3, 321–338.

Pinto, B., 1991, Black markets for foreign exchange, real exchange rates and inflation, Journal of International Economics 30, 121–135.

Uganda, Government of, 1989a, Background to the Budget, 1989/1990.

Uganda, Government of, 1989b, Budget Estimates 1989/90.