

NOTES, COMMENTS, AND LETTERS TO THE EDITOR

The Revelation of Information and Self-Fulfilling Beliefs*

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At a Rational Expectations Equilibrium (REE), individuals are assumed to know the map from states to prices. This hypothesis has two components, that agents agree (consensus) and that they have point expectations (degeneracy). We consider economies where agents' beliefs are described by a joint distribution on states and prices, and these beliefs are fulfilled at equilibrium. Beliefs are self-fulfilling if every price in the support of the distribution is an equilibrium price. The corresponding equilibria are Beliefs Equilibria (BE). The further restriction that agents have the same beliefs results in Common Beliefs Equilibria (CBE). We study the relationship between BE, CBE, and REE, thus isolating the role of consensus and of degeneracy in achieving rational expectations. *Journal of Economic Literature* Classification Numbers: D51, D82. © 1997 Academic Press

1. INTRODUCTION

One of the remarkable properties of competitive equilibrium is its economy in terms of informational requirements imposed on participants

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in the market. Agents are only expected to know, or observe, the prices of goods, in addition to their own characteristics. While the informational requirement is minimal, it is often thought to be too demanding relative to the operation of actual markets. The prices that agents need to observe include those which arise at all dates, and all contingencies. Nevertheless, the allocations achieved by overall competitive equilibria—or Arrow–Debreu allocations—can be implemented by a simpler, and more realistic structure of markets, if agents have “rational expectations”, as shown by Radner [17, 18]. This is at some cost; in making the transition, we must give up the informational economy of the competitive paradigm.

The rational expectations hypothesis requires that agents know the map from states to prices. In sequence economies, states correspond to date–event pairs. In economies with asymmetric information, states correspond to information signals received by agents. In either case, the requirement that all markets are open simultaneously has to be replaced by the assumption that agents form expectations about prices which would realize in such markets. Expectations are fulfilled at equilibrium, which, along with market clearing, defines a Rational Expectations Equilibrium (REE). The hypothesis itself makes strong assumptions about what agents know, and what they agree upon. The “map from states to prices” summarizes very large amounts of information, about preferences, endowments, and technological possibilities available to all participants in each state and date. While it is natural to assume that agents know prices in the realized state of the world, the assumption that they know the map is much stronger. Formally, the Rational Expectations Hypothesis (REH) amounts to the assumption that price expectations are single-valued, and coincident across agents. Agents agree on the realization of prices in each state (*consensus*) and simultaneously believe that there is a single possible price in each state (*degenerate beliefs*).

We are concerned with REE in economies with asymmetric information. Radner [18] showed that at a REE, prices generically reveal all information, in economies where information, or signals, take finitely many values. In addition, these fully revealing equilibria coincide with the competitive equilibria of the corresponding symmetric information economy, and inherit there usual properties, such as local uniqueness.¹

This note has three objectives. First, to formally disentangle the impact of consensus and degenerate beliefs from standard market clearing and rationality assumptions. Second, to demonstrate that in an asymmetric information economy, each restriction has bite. Economies with unique rational expectations equilibria may have a continuum of equilibria

¹ These results were extended to the infinite dimensional case in Allen [1].

without point expectations. Finally, we demonstrate that generic revelation results rely on the point expectations assumption: economies with a unique symmetric information equilibrium that is fully revealing can have a continuum of partially revealing equilibria.

To understand the role of consensus and point expectations, it is necessary to start with a set of restrictions on individual rationality which are weaker than the REH. How should one describe the behaviour of agents who do not necessarily know the map from states to prices, even though market clearing is common knowledge? We do this by describing agents as being uncertain about prices. Their beliefs are this represented by a joint probability distribution on states and prices. It is then possible (under the maintained assumption of expected utility maximization) to associate successively stronger hypotheses about the rationality of beliefs, or expectations, in terms of the restrictions imposed on this joint distribution. This construction allows us to ask what restrictions are imposed by individual rationality, and whether further restrictions are necessary to obtain the REH. At the same time, we need to pay attention to the characteristics of the resulting competitive equilibria. It is important to understand whether weaker hypotheses of rationality enlarge the set of equilibria, and alter their characteristics. In other words, can REE be the outcome of restrictions weaker than the REH?

Common knowledge of rationality and market clearing imposes the restriction that beliefs are concentrated on market clearing prices. We consider, first, such “Beliefs Equilibria (BE)” which result from profiles of beliefs which satisfy this property, and no further restrictions. As we have noted before, consensus needs to be imposed as a restriction; we call the outcomes “Common Beliefs Equilibria (CBE)” where agents agree on the joint distribution of prices and states, and these beliefs are self-fulfilling by definition. Rational Expectations Equilibria (REE) are CBE where the beliefs of every agent are degenerate, so that they assign probability one to a single price in each state.

We formally define these solution concepts in Section 2. In Section 3, we demonstrate a robust class of economies which possess a continuum of CBE, though each has a unique REE. This demonstrates that the restriction of degeneracy is necessary to obtain rational expectations equilibria. While the unique REE is fully revealing, the CBE are not. In the concluding Section 4, we discuss the relation to alternative approaches.

2. FRAMEWORK

The framework is essentially that of Radner [18]. There are H agents and L goods. Each agent h observes a signal $s^h \in \mathcal{S}^h$, so that uncertainty

is represented by a discrete state space $\mathcal{S} = \mathcal{S}^1 \times \dots \times \mathcal{S}^H$, with typical element s .² Preferences of agents can be state dependent: utility functions are $u^h: \mathfrak{R}_+^L \times \mathcal{S} \rightarrow \mathfrak{R}$, for $h=1, \dots, H$. For simplicity, we assume agents know their own endowments: endowments are $e^h: \mathcal{S}^h \rightarrow \mathfrak{R}_+^L$, for $h=1, \dots, H$. Let $\Delta(\mathcal{S})$ be the class of probability distributions on \mathcal{S} . For each $\beta \in \Delta(\mathcal{S})$, and $x: \mathcal{S} \rightarrow \mathfrak{R}_+^L$, define

$$V^h(\beta; x) = \sum_{s \in \mathcal{S}} \beta(s) U^h(x(s); s)$$

as the expected utility of agents h from the state-dependent consumption bundle x if she holds belief β about states.

We assume that agents share a common prior distribution on the fundamental state space; call this $\pi \in \Delta(\mathcal{S})$. In addition, agent h observes her private signal, s^h , and the vector of prices p . Agent h does not observe other agents' signals, $s^{-h} = (s^1, \dots, s^{h-1}, s^{h+1}, \dots, s^H)$. But since prices can vary across states, she can infer information about s^{-h} from p . Suppose that conditional beliefs for agent h were given by $\beta^h: \mathcal{S}^h \times \mathfrak{R}_{++}^{L-1} \rightarrow \Delta(\mathcal{S})$, so that $\beta^h(\cdot | s^h, p)$ is h 's probability distribution over states if he observed s^h and p . To start with, fix a vector of such conditional beliefs, $\beta = (\beta^1, \dots, \beta^h, \dots, \beta^H)$, to obtain equilibria. We can then impose restrictions for these beliefs to be consistent with equilibrium.

Agent h 's net trade $z^h(s^h, p | \beta^h)$ maximizes

$$V^h(\beta^h(\cdot | s^h, p); z^h + e^h),$$

subject to the budget constraints $p \cdot z^h \leq 0$. Prices, p , clear markets, so that

$$\sum_{h=1}^H z^h(s^h, p | \beta^h) = 0,$$

for each $s \in \mathcal{S}$. Note that the true state s is typically unobserved at the time of choosing z^h . Let $\mathcal{P}_C(\beta, s)$ be the set of prices which clear markets in state s given beliefs β , i.e.,

$$\mathcal{P}_C(\beta, s) = \left\{ p \in \mathfrak{R}_{++}^{L-1} \mid \sum_{h=1}^H z^h(s^h, p | \beta^h) = 0 \right\}.$$

The Rational Expectations Hypothesis assumes, at this point, that every agent knows a deterministic map from states to prices. This map can then

² In [9], we allowed for uncountable signals and states of the world which were not revealed by agents' private information. We focus on this simpler state space in order to clarify definitions.

be used to *derive* β . We start with a hypothesis weaker than the REH: each agent's belief is described by a conditional probability distribution on prices given states, written as $\delta^h(p|s)$, to allow for non-trivial uncertainty about prices.

A *belief* for agent h is a mapping $\delta^h: \mathcal{S} \rightarrow \Delta(\mathfrak{R}_{++}^{L-1})$. For each $s \in \mathcal{S}$, $\delta^h(\cdot|s)$ is a conditional probability distribution on the $L-1$ vector of relative prices p . Let $\delta \equiv (\delta^1, \dots, \delta^H)$ be a profile of such beliefs; we will assume that they have common support and write

$$\mathcal{P}_*(\delta, s) = \{p \in \mathfrak{R}_{++}^{L-1} \mid \delta^h(p|s) > 0\} \quad \text{for all } h = 1, \dots, H.$$

Given h 's beliefs δ^h , we can deduce her posterior beliefs β^h by Bayes rule. Define β_δ^h by

$$\beta_\delta^h(s|p, s^h) = \frac{\pi(s|s^h) \delta^h(p|s)}{\sum_{s' \in \mathcal{S}} \delta^h(p|s') \pi(s'|s^h)}, \quad (2.1)$$

whenever $\sum_{s' \in \mathcal{S}} \delta^h(p|s') \pi(s'|s^h) > 0$; because of the common support assumption above, we will not be evaluating β_δ^h when this condition is not satisfied. Write $\delta = (\delta^1, \dots, \delta^h, \dots, \delta^H)$ and $\beta_\delta = (\beta_{\delta^1}^1, \dots, \beta_{\delta^H}^H)$. We are now in a position to define our solution concepts.

DEFINITION 2.1. Beliefs profile δ is *self-fulfilling* if, for each $s \in \mathcal{S}$, markets clear at every p in the common support of $\delta^h(s)$, i.e.,

$$\mathcal{P}_*(\delta, s) \subseteq \mathcal{P}_C(\beta_\delta, s) \quad \text{for all } s \in \mathcal{S}.$$

Notice that, because of our common support assumption on δ , conditional beliefs β are always uniquely defined by (2.1) for all s in $\mathcal{P}_*(\delta, s)$. On the other hand, if we dropped the common support assumption, the definition would be sensitive to beliefs following the realizations of unexpected prices.

DEFINITION 2.2. Beliefs profile δ is *degenerate* if there exists $\phi: S \rightarrow \mathfrak{R}_{++}^{L-1}$ such that $\mathcal{P}_*(\delta, s) = \{\phi(s)\}$ and thus, for all h ,

$$\delta^h(p|s) = \begin{cases} 1, & \text{if } p = \phi(s) \\ 0, & \text{otherwise.} \end{cases}$$

DEFINITION 2.3. A *Beliefs Equilibrium* is a self-fulfilling beliefs profile. A *Common Beliefs Equilibrium* is a Beliefs Equilibrium with $\delta^h = \delta$ for $h = 1, 2, \dots, H$. A *Rational Expectations Equilibrium* is a common beliefs equilibrium with degenerate beliefs.

Remarks. • Our first definition explores the rationality of beliefs held by agents. These are theories about prices, which are an outcome of the economic system. If they live in a world where prices clear markets, this should be incorporated into their model of the world. Self-fulfilling beliefs support only equilibrium prices in each state.

- Self-fulfilling beliefs may not be strong enough to yield rational expectations equilibria. Definition 2.3 indicates the successive strengthening which is required to achieve that. At a rational expectations equilibrium, agents' beliefs agree, and, in addition, these beliefs are degenerate.

- Beliefs are theories about prices in each information state, s . Agreement on beliefs δ is not sufficient for agreement on posterior probabilities, β . This last is typically true only at fully revealing equilibria (formally defined below).

- The property that beliefs are self-fulfilling is true of the entire profile; one person's theory is fulfilled by the simultaneous actions of all agents, which depend on their beliefs.

- We have specified that self-fulfilling beliefs have support entirely on market-clearing prices. This definition could be weakened. Anderson and Sonnenschein [3] require beliefs to contain equilibrium prices, i.e., $\mathcal{P}_* \cap \mathcal{P}_C \neq \emptyset$, so that rationality imposes the relatively weak restriction that agents never observe prices which they had thought impossible. They may nevertheless continue to assign positive probability to impossible events. Clearly, our equilibria continue to be admissible in this definition. We explore the implications of the tighter notion.

- We do not impose the stronger restriction that $\mathcal{P}_* = \mathcal{P}_C$, i.e., that every market clearing price is expected to occur with positive probability. Recall that the REH requires that agents coordinate their belief on a single equilibrium, even though the economy can possess multiple equilibria in some states.

- If beliefs are self-fulfilling, every price in the support is an equilibrium price. Given this, the auctioneer may choose to randomize in any way over the equilibria, because market clearing must be achieved state by state, and not in expected or in probabilistic terms. In the case of CBE, this is really a demonstration that common beliefs can be fulfilled, and not necessarily that they will be.

- Our definition assumed that agents' joint information revealed the true state. If we did not make this simplifying assumption, we would want to impose the requirement that the price randomization was measurable with respect to the state space. By contrast, McAllister [13] studies equilibria where prices are correlated with information not known to any agent.

DEFINITION 2.4. Belief profile δ is fully revealing if an outside observer could deduce all signals from prices, i.e., $\mathcal{P}_*(\delta, s) \cap \mathcal{P}_*(\delta, s') = \emptyset$ if $s \neq s'$.

The question of whether equilibria are fully revealing is important in evaluating the solution concepts. The claim that REE are fully revealing can be understood as the property of a particular beliefs profile. It is entirely possible that a different, and non-degenerate, beliefs profile is not fully revealing at equilibrium, in the same economy. In the next section, we show that this is true for a class of economies with asymmetric information. As a consequence, there is a continuum of CBE which are distinct from the unique REE.

It is important to know whether non-trivial CBE, or BE, exist in a given economy. We do not have characterizing conditions: but some properties are worth remarking upon.

First, imagine an economy where every beliefs profile is fully revealing. Clearly, market participants can recover the information s from prices, so that posterior probabilities are degenerate. It follows that BE, and CBE, can only contain randomizations over REE. If, in addition, the economy has a unique REE for each $s \in \mathcal{S}$, the REE is the unique BE. This property can only hold with strong restrictions on preferences and information structure. A weaker requirement is that every common belief is fully revealing. In an earlier version of this paper [9], we exhibited a class of economies with this property: CBE and REE coincide, though the set of BE is larger. A similar phenomenon occurs in the economy studied by Maskin and Tirole [12]. Finally, the set of CBE is larger than randomizations over REE if some common beliefs are not fully revealing. We will study this phenomenon in the next section.

3. COMMON BELIEF EQUILIBRIA

We will construct a class of economies where there is a unique rational expectations equilibrium, but there is a continuum of common belief equilibria. This example has two sided uncertainty—i.e., there are two agents, each of whom knows something which the other does not.

There are two agents, 1 and 2, and two goods, x and y . Individual and aggregate endowments are state-independent. Endowments are $e_1 = e_2 = (1, 1)$. Preferences are state-dependent:

$$u^h(x, y, s) = a^h(s) \ln(x) + (1 - a^h(s)) \ln(y).$$

Agent 1 observes $s^1 \in \mathcal{S}^1 \in \{H, L\}$. Agent 2 observes $s^2 \in \mathcal{S}^2 \in \{H, L\}$. Thus, $\mathcal{S} = \mathcal{S}^1 \times \mathcal{S}^2$ contains four elements. We parameterize this economy

by the pair $(\pi, \mathbf{a}) \in \Delta(S) \times (0, 1)^8$ where $\mathbf{a} = \{a^h(s) : h \in \{1, 2\} \text{ and } s \in \mathcal{S}\}$. Normalize prices to lie in the unit simplex, so that the price of good x is p while the price of good y is $1 - p$.

PROPOSITION 3.1. (1) *For generic choice of (π, \mathbf{a}) there is a unique REE, which is fully revealing.* (2) *There is an open set of (π, \mathbf{a}) for which there exist a continuum of partially revealing CBE. In particular, there is a continuum of partially revealing CBE if $\pi(s) > 0$ for each $s \in \mathcal{S}$ and*

$$\begin{aligned} a^1(HH) &> a^1(LH) > a^1(HL) > a^1(LL) \\ a^2(HH) &> a^2(HL) > a^2(LH) > a^2(LL). \end{aligned} \quad (3.1)$$

(1) is well-known from [18]. We include the proof since we use it in proving (2). Note that prices can always be inferred from beliefs as follows. Suppose that in state s , agent h 's expected value of a^h is $\alpha^h = \mathbf{E}^h a^h$; then his demand for the first good is α^h/p . Markets clear if and only if $p = \frac{1}{2}(\alpha^1 + \alpha^2)$.

LEMMA 3.2. *A fully revealing REE exists if and only if*

$$a^1(s) + a^2(s) \neq a^1(s') + a^2(s') \quad \text{for all } s \neq s'. \quad (3.2)$$

Proof. At any fully revealing equilibrium, $p(s) = \frac{1}{2}(a^1(s) + a^2(s))$. Now restriction (3.2) ensures that $p(s) \neq p(s')$ for all $s \neq s'$, and the result follows. ■

Now for each $\pi \in \Delta(\mathcal{S})$, let $\alpha_s^h(\pi)$ be the expected value of a^h if π is the prior probability distribution on states and agent h observes signal s , i.e.,

$$\alpha_H^1(\pi) = \frac{\pi(HH) a^1(HH) + \pi(HL) a^1(HL)}{\pi(HH) + \pi(HL)}; \quad (3.3)$$

$$\alpha_L^1(\pi) = \frac{\pi(LH) a^1(LH) + \pi(LL) a^1(LL)}{\pi(LH) + \pi(LL)}; \quad (3.4)$$

$$\alpha_H^2(\pi) = \frac{\pi(HH) a^2(HH) + \pi(LH) a^2(LH)}{\pi(HH) + \pi(LH)}; \quad (3.5)$$

$$\alpha_L^2(\pi) = \frac{\pi(HL) a^2(HL) + \pi(LL) a^2(LL)}{\pi(HL) + \pi(LL)}; \quad (3.6)$$

LEMMA 3.3. *A non-revealing REE does not exist if*

$$\alpha_H^1(\pi) \neq \alpha_L^1(\pi) \quad \text{and} \quad \alpha_H^2(\pi) \neq \alpha_L^2(\pi). \quad (3.7)$$

Proof. $\alpha_H^1(\pi) = \alpha_L^1(\pi)$ is necessary for 1's demand not be revealed (at any equilibrium price); $\alpha_H^2(\pi) = \alpha_L^2(\pi)$ is necessary for 2's demand not to be revealed. ■

Since restrictions (3.2) and (3.7) hold for generic choice of (π, \mathbf{a}) , Lemmas 3.2 and 3.3 together show part 1 of Proposition 3.1. In proving the existence of a continuum of CBE, it is useful to first fully characterize *completely non-revealing equilibria*: say that an REE is completely non-revealing if $p(s) = p^*$, for all $s \in \mathcal{S}$.

LEMMA 3.4. *A completely non-revealing REE exists if and only if $\alpha_H^1(\pi) = \alpha_L^1(\pi)$ and $\alpha_H^2(\pi) = \alpha_L^2(\pi)$.*

Proof. $\alpha_H^1(\pi) = \alpha_L^1(\pi)$ is necessary for 1's demand to be non-revealing; $\alpha_H^2(\pi) = \alpha_L^2(\pi)$ is necessary for 2's demand to be non-revealing. But if both hold, there is a non-revealing equilibrium with $p(s) = (\alpha^1 + \alpha^2)/2$, where $\alpha^1 = \alpha_H^1(\pi) = \alpha_L^1(\pi)$ and $\alpha^2 = \alpha_H^2(\pi) = \alpha_L^2(\pi)$. ■

The restrictions of Lemma 3.4 are clearly non-generic, but we need to identify strictly positive distributions with these restrictions, as part of our construction: write Δ_C for such *confounding distributions*, i.e.,

$$\Delta_C = \{ \psi \in \Delta_{++}(\mathcal{S}) : \alpha_H^1(\psi) = \alpha_L^1(\psi) \text{ and } \alpha_H^2(\psi) = \alpha_L^2(\psi) \}.$$

LEMMA 3.5. *Suppose condition (3.1) holds. Then Δ_C contains a continuum of probability distributions.*

Proof. We first show that $\psi \in \Delta_C$ if and only if $\psi \in \Delta_{++}(\mathcal{S})$ and there exists $\alpha_*^1, \alpha_*^2 \in (0, 1)$ such that

$$\frac{\psi(HH)}{\psi(HL)} = \frac{\alpha_*^1 - a^1(HL)}{a^1(HH) - \alpha_*^1}, \quad \frac{\psi(LL)}{\psi(LH)} = \frac{a^1(LH) - \alpha_*^1}{\alpha_*^1 - a^1(LL)}, \quad (3.8)$$

$$\frac{\psi(HH)}{\psi(LH)} = \frac{\alpha_*^2 - a^2(LH)}{a^1(HH) - \alpha_*^2}, \quad \frac{\psi(LL)}{\phi(HL)} = \frac{a^2(HL) - \alpha_*^2}{\alpha_*^2 - a^2(LL)}, \quad (3.9)$$

and

$$f^1(\alpha_*^1) = f^2(\alpha_*^2), \quad (3.10)$$

where

$$f^1(\alpha_*^1) = \left(\frac{\alpha_*^1 - a^1(HL)}{a^1(HH) - \alpha_*^1} \right) \left(\frac{a^1(LH) - \alpha_*^1}{\alpha_*^1 - a^1(LL)} \right),$$

$$f^2(\alpha_*^2) = \left(\frac{\alpha_*^2 - a^2(LH)}{a^1(HH) - \alpha_*^2} \right) \left(\frac{a^2(HL) - \alpha_*^2}{\alpha_*^2 - a^2(LL)} \right).$$

To show “if” observe that (3.8) implies $\alpha_H^1(\psi) = \alpha_L^1(\psi) = \alpha_*^1$ while (3.9) implies $\alpha_H^2(\psi) = \alpha_L^2(\psi) = \alpha_*^2$. To show “only if” observe that $\alpha_H^1(\psi) = \alpha_L^1(\psi)$ implies there exists α_*^1 satisfying (3.8); $\alpha_H^2(\psi) = \alpha_L^2(\psi)$ implies there exists α_*^2 satisfying (3.9); and (3.8) and (3.9) together imply (3.10).

Now observe that f^1 is continuous with respect to α_*^1 , and strictly positive only on the interval $(a^1(HL), a^1(LH))$, and $f^1(t) \rightarrow 0$ as $t \rightarrow a^1(HL)$ and as $t \rightarrow a^1(LH)$. Similarly, f^2 is continuous with respect to α_*^2 , and strictly positive only on the interval $(a^2(LH), a^2(HL))$, and $f^2(t) \rightarrow 0$ as $t \rightarrow a^2(LH)$ and as $t \rightarrow a^2(HL)$. Thus for each $0 < t < \min\{\max_{t' \in \mathfrak{R}_{++}} f^1(t'), \max_{t' \in \mathfrak{R}_{++}} f^2(t')\}$, we can find $\alpha_*^1, \alpha_*^2 \in (0, 1)$, with $f^1(\alpha_*^1) = f^2(\alpha_*^2) = t$, and construct ψ from (3.8) and (3.9). Thus there exists a continuum of probability distributions in Δ_C . ■

LEMMA 3.6. *Suppose that $\pi(s) > 0$ for all $s \in \mathcal{S}$, and \mathbf{a} satisfies (3.1) and (3.2). There exist a continuum of partially revealing CBE.*

Proof. The REE is $p(s) = (a^1(s) + a^2(s))/2$. Let $\psi \in \Delta_C$ be any confounding distribution. Write $q(\psi)$ for the state-independent price which results if ψ is the prior distribution. For each s , let $\varepsilon_k(s) = k(\psi(s)/\pi(s))$, where $k < \max_{s \in \mathcal{S}} (\pi(s)/\psi(s))$. It follows that $(\pi(s) \varepsilon_k(s))/(\sum_{s \in \mathcal{S}} \pi(s) \varepsilon_k(s)) = \psi(s)$ by construction. Now consider the common beliefs equilibrium

$$\delta(p|s) = \begin{cases} p(s), & \text{with probability } 1 - \varepsilon_k(s) \\ q(\psi), & \text{with probability } \varepsilon_k(s). \end{cases}$$

This is a CBE for each $k \in (0, \max_{s \in \mathcal{S}} (\pi(s)/\psi(s)))$, as long as $q(\psi) \neq p(s)$ for each s . Individuals observe either $p(s)$ or $q(\psi)$ as the price of good 1. In the first case, the price reveals the true state, and $p(s)$ clears markets. If they observe $q(\psi)$, their posterior probabilities result in non-revealing price $q(\psi)$, whatever the true state. By construction, each $\psi \in \Delta_C$ generates a class of CBE indexed by k . By Lemma 3.5, there are a continuum of such ψ . Further, it is possible to choose a continuum of such ψ with $q(\psi) \neq p(s)$ for each s and distinct $q(\psi)$,³ so that the equilibria have different price supports. ■

In constructing a CBE, we use the fact that some beliefs profiles are not fully revealing. Specifically, there are beliefs such that $q \in \mathcal{P}_*(\delta, s)$ for all $s \in \mathcal{S}$. The resulting CBE are randomizations over REE and such non-revealing prices. This randomization needs to be correlated with s , so that

³ To see the latter, recall that for any ψ satisfying (3.8)–(3.10), we have $q(\psi) = (\alpha_*^1 + \alpha_*^2)/2$. This equation is not co-linear with (3.8)–(3.10). We can thus find a continuum of solutions (α_*^1, α_*^2) giving distinct values of ψ and $q(\psi)$.

the confounding distribution is the equilibrium posterior distribution. Each such distribution has the property that net trades, and thus prices, are insensitive to private signals: it is individually rational to ignore private information, which is valuable only if combined with the information held by others. The crucial property of these economies is condition (3.1), which guarantees that confounding distributions exist. A different restriction, such as $(a^1(s) - a^1(s'))(a^2(s) - a^2(s')) > 0$ for all $s, s' \in \mathcal{S}$, would ensure that confounding is impossible.

This example is robust to perturbations of the utility functions and endowments. Thus we have generated a robust class of economies where there is a unique rational expectations equilibrium which is fully revealing, but there are a continuum of partially revealing common belief equilibria.

4. REMARKS AND FURTHER ISSUES

4.1. *Sunspots*

By allowing agents to be uncertain about the mapping from payoff relevant states to prices, we allow for *extrinsic* uncertainty, i.e., sunspots in the sense introduced by Cass and Shell [7]. It is well-known that equilibria with price uncertainty, such as CBE, are formally equivalent to some equilibrium of a model with sunspots. It suffices to index the variation in prices within each state by a random variable, and call this variable the realization of a sunspot. Thus one contribution of this note is to exhibit yet another setting in which sunspots matter. Our result fits well with the intuition that “market imperfections” are required for sunspots. Disagreement on probabilities is one such imperfection; at CBE, this disagreement is caused by differences of information and not by differences in priors. Non-revelation of information by prices allows disagreement, while disagreement allows non-revealing distributions to prevail at equilibria.

Interpreting the price uncertainty in common belief equilibria as sunspots raises some questions. First, note that sunspots with generic and exogenous probabilities would not have expanded the set of equilibria in the example of Section 3. It is important that agents’ uncertainty about the price process (the “probability distribution over sunspots”) is part of the description of equilibrium. Second, the usual definition of rational expectations equilibrium requires that prices cannot reveal any information not known to any agent. In re-interpreting CBE as sunspot equilibria, we require that they do. Participants do not observe sunspot activity, but prices depend on them. Finally, the re-interpretation also implies that sunspots are correlated with intrinsic, or payoff-relevant, uncertainty.

4.2. *The Rational Expectations Hypothesis*

One contribution of this note was to break down the rational expectations hypothesis into its component parts: (i) common knowledge of market clearing and rationality; (ii) consensus and (iii) degenerate beliefs. Our approach follows MacAllister [13]. We focussed on situations when assuming (i) and (ii), (i.e., common belief equilibria) fails to imply (iii) (i.e., rational expectations equilibria). In a different setting, with symmetric information but dynamic trading, Guesnerie [10] has identified conditions where (i) implies (ii) and (iii) (i.e., common knowledge of market clearing and rationality alone imply rational expectations).

The attempt to break the definition of rational expectations equilibrium into those three component parts mirrors game theory results which show that Nash equilibrium can be justified by (i) common knowledge of rationality; (ii) consensus and (iii) independent beliefs [6]. Assuming (i) and (ii) alone implies only correlated equilibrium [4]. A number of authors have examined the relationship between correlated equilibria and sunspots ([12] and [14]).

4.3. *Partial Revelation and Noisy Rational Expectations*

Adding exogenous “noise” to the economy allows the existence of “noisy rational expectations equilibria” with partial revelation (e.g., [2, 3]). Such models have been criticized for introducing noise in arbitrary ways, often at odds with full rationality. Common belief equilibria can be interpreted as noisy rational expectations equilibria, where the noise is endogenous and consistent with full rationality.

It is hard to construct robust examples of rational expectations equilibria which convey some, but not all information [5]. Partial revelation arises naturally in common belief equilibria.⁴

4.4. *Determinacy and Existence*

Our results suggest that in economies where non-degenerate CBE exist, they are likely to be indeterminate, because market-clearing conditions

⁴ Two recent papers also use non-degeneracy of equilibrium beliefs to allow non-revelation. DeMarzo and Skiadas [8] derive necessary and sufficient conditions on the asset structure and preferences for prices to be fully revealing, when neither consensus nor degenerate beliefs are assumed. Pietra and Siconolfi [15] use non-degenerate beliefs (i.e., sunspots) to construct robust finite state economies with partial revelation. Their result exploits the existence of sunspots equilibria in the full revelation economy, and they obtain non-revelation with one-sided asymmetric information. By contrast, the economy we discuss in Section 3 has not sunspot equilibria if all information is revealed, but we *require* two-sided asymmetry of information to obtain non-revelation.

impose a relatively small number of restrictions on the equilibrium probability distributions.⁵

At the same time, there are economies for which CBE, and even BE, fail to exist. In an earlier version of this paper [9], we showed that the example due to Kreps [11] does not possess any beliefs equilibrium. The problem arises precisely because equilibrium beliefs must be supported entirely on market clearing prices. Intriguingly, that is a world where market clearing cannot be common knowledge at equilibrium, even *without* requiring consensus and degenerate beliefs.

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⁵ In an economy with nominal assets, Polemarchakis and Siconolfi [16] exploited indeterminacy in the symmetric information economy to construct robust non-revealing equilibria. By contrast, indeterminacy arises in CBE only because of non-revelation.

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