# Online Appendix for
# Managing Expectations: Instruments vs. Targets

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**Abstract**

This Appendix collects: the second, neoclassical application of our model; the details of the robustness exercises discussed in Section 5; and a variety of auxiliary results.

## A A Neoclassical Application

Our primary application, spelled out in Section 6, concerns monetary policy and output gaps in a Keynesian economy. The micro-foundation offered here differs in its approach (Neoclassical), policy instrument (taxation), and key decision (investment).

### A.1 Primitives

There are three periods, $t \in \{0,1,2\}$. A continuum of firms or entrepreneurs, $i \in [0,1]$, choose investment at $t = 1$. Investment is an intermediate input that enters the production of a final good at $t = 2$, along with the labor supply by a representative worker. The first period, $t = 0$, identifies only the time of policy announcement. We now review each of these ingredients in turn.

**Final good production at $t = 2$.** The final-good firm operates at $t = 2$. Define the following constant elasticity of substitution (CES) aggregator of the intermediate goods

$$X \equiv \left( \int x_i^{1-\frac{1}{\varepsilon}} \, \mathrm{d}i \right)^{\frac{\varepsilon}{\varepsilon-1}} \tag{71}$$

where $\varepsilon \in (1,\infty)$ is the inverse elasticity of substitution. The final goods firm operates with a Cobb-Douglas technology over this intermediate and labor with capital share $\alpha$:

$$Q = X^{\alpha} N^{1-\alpha} \tag{72}$$

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The firm operates competitively and has the following revenue

$$Q - wN - \int p_i x_i \,\mathrm{d}i \tag{73}$$

where $w$ is the wage and the $(p_i)_{i\in[0,1]}$ are the prices of the intermediates.

The final goods firm's demand for intermediates is

$$p_i = \alpha Q X^{\frac{1}{\varepsilon}-1} x_i^{-\frac{1}{\varepsilon}} \tag{74}$$

and their demand for labor is

$$w = (1-\alpha)\frac{Q}{N} \tag{75}$$

**Representative worker.** A representative worker lives only in period $t = 2$. They consume at level $C_w$ and supply $N$ hours of labor, to maximize their utility, given by

$$\mathcal{U}_w = \log C_w - \frac{1}{1+\phi} N^{1+\phi} \tag{76}$$

where $\phi > 0$ parameterizes the Frisch elasticity of labor supply. Their income equals the sum of labor earnings, taxed at rate $z$, and a transfer $T$:

$$C_w \le w(1-z)N + T$$

The transfer rebates the tax, or $T = zwN$. Labor supply has the following simple form:

$$w(1-z) = N^{\phi} C_w \tag{77}$$

which, combined with (75), fully characterizes the labor market.

**Entrepreneurs and investment at $t = 1$.** At $t = 1$, the entrepreneur is endowed with one unit of the final good, which they choose how to allocate between current consumption, $c_{i,1}$, and investment, $x_i$, into the next-period production of an intermediate good. The technology is linear, and the productivity is set to one for simplicity, so the quantity of the intermediate good produced and sold at $t = 2$ is also $x_i$. The entrepreneur's consumption at $t = 2$ is given by the proceeds of selling the intermediate good, or $c_{i,2} = p_i x_i$. His preferences are linear:

$$\mathcal{U}_e = c_{i,1} + c_{i,2}$$

It follows that

$$\mathcal{U}_e = 1 - x_i + p_i x_i.$$

where $p_i$ is given by (74). The entrepreneur therefore chooses investment, $x_i$, so as to maximize the net payoff $p_i x_i - x_i$, subject to (74). This yields the following optimality condition:

$$x_i^{\frac{1}{\varepsilon}} = \alpha(1-\varepsilon^{-1})\mathbb{E}_i\left[X^{\alpha+\frac{1}{\varepsilon}-1} N^{1-\alpha}\right] \tag{78}$$

**Policy preferences.** For simplicity, the policymaker cares only about the representative worker's welfare but cannot directly transfer consumption from the entrepreneurs to the worker. Instead, she must use the single available tax instrument, the proportional tax on the second-period final good, to balance two objectives: encourage investment by the entrepreneurs so as to increase aggregate the consumption of workers; and collect tax revenue so as to pay for a public good.

The policymaker wishes to maximize the utility of the worker, but also has an external value for tax revenues $rwN$. Assume that external benefit is linearly separable in the policymaker's preferences, has a log functional form, and has weight $\xi$ relative to the worker's welfare. The policymaker thus wishes to maximize

$$\mathscr{W} = \log C_w - \frac{N^{1+\phi}}{1+\phi} + \xi \log(rwN). \tag{79}$$

The primitive shock of interest in our model will be $\xi$, which is a pure shifter of the policymaker's preferences.

Objective (79) can be justified as follows. Let the underlying government budget constraint take the form $rwN \geq g$, where $g$ is the exogenous and random level of government spending. Provided that $g$ is bounded away from zero, we can re-write this constraint as $\log(rwN) \geq \log g$. This yields (79) as the Lagrangian of this problem, with $\xi$ being positive and increasing in $g$. And since there is a one-to-one mapping between $g$ and $\xi$, we can treat the latter as the "fundamental" for our purposes.

### A.2 Benchmark with REE and Optimal Policy

Assume rational expectations. Let us first characterize implementable equilibria indexed by the tax rate $z$. In such an equilibrium, the agent will conjecture that $x_{-i} = x_i \equiv X$. Since everything is now known, we can pull $X$ out of the expectation and solve to get

$$X_i = X = (\alpha(1-\varepsilon^{-1}))^{\frac{1}{1-\alpha}} N$$

It is immediate that output is linear in labor:

$$Q = X^{\eta} N^{1-\eta} = (\alpha(1-\varepsilon^{-1})))^{\frac{\alpha}{1-\alpha}} N$$

and finally note from the worker's budget constraint that consumer income equals consumer spending, or $C_w = wN = (1-\alpha)Q$.

Setting labor supply to labor demand gives

$$N = (1-z)^{\frac{1}{1+\phi}} \tag{80}$$

which corresponds to output level

$$Q = (\alpha(1-\varepsilon^{-1})))^{\frac{\alpha}{1-\alpha}} (1-z)^{\frac{1}{1+\phi}} \tag{81}$$

and investment level

$$X_i = X = (\alpha(1-\varepsilon^{-1}))^{\frac{1}{1-\alpha}} (1-z)^{\frac{1}{1+\phi}}$$

The policymaker chooses one of the implementable allocations, as described by (80) and (81), to maximize its objective function (79). We appeal to standard arguments to write the problem in the "dual" form as a function of the policy instrument. Substituting out the production function gives the following representation of the policy problem:

$$\max_{r}\left\{(1+\xi)\log\left((1-\alpha)X^{\alpha}N^{1-\alpha}\right)-\frac{N^{1+\phi}}{1+\phi}+\xi\log((1-\alpha)z)\right\} \tag{82}$$

and further substituting in the implementability constraints for $(X, N)$ gives the following up to constants:

$$\max_{z}\left\{(1+\xi)\log(1-z)+z+\xi(1+\phi)\log z\right\} \tag{83}$$

The first-order condition is

$$1+\frac{\xi(1+\phi)}{z}=\frac{(1+\xi)}{1-z} \tag{84}$$

There are two solutions to this, and the relevant one that corresponds to a minimum of the objective is

$$z^{*}(\xi)=\frac{1}{2}\left(\sqrt{\xi^{2}(2+\phi)^{2}+4(1+\phi\xi)}-\xi(2+\phi)\right) \tag{85}$$

which increases in $\xi$, the government's preference for raising revenue.

### A.3 Forward Guidance

At $t=0$, the policymaker learns its preference shifter $\xi$ and decides to levy a tax or subsidy. They have two options. The first is to announce and commit to a fixed level of the tax $z$ at $t=2$. The second is to commit to a given level of output, and adjust *ex post* the tax such that, for a pre-determined level of the capital stock, the output target is met. Observe that, under rational expectations, the two approaches are equivalent; but under non-rational expectations they may differ.

### A.4 Mapping to the Abstract Behavioral Equations

Now consider a more general model in which agents do not form rational expectations, because of either limited information or various behavioral biases. The fixed-point equation (78) can no longer be solved without expectations. To make progress, we will take log-linear approximations around a case in which the government preference shock is at a steady-state value, or $\xi=\bar{\xi}$, and the tax is set at the (rational-expectations-implementation) optimum that achieves the second-best, or $z=\bar{z}=z^{*}(\bar{\xi})$. Let $(\bar{Q},\bar{N},\bar{X})$ denote output, labor, and investment evaluated at this point. Let $Y=\log Q-\log\bar{Q}$, $k_i=(\log x_i-\log\bar{X})$, $K=\int_i k_i\,\mathrm{d}i$, and $n=\log N-\log\bar{N}$ be log deviations of the first two quantities. Further, define

$$\tau=\frac{1}{1+\phi}\log\left(\frac{1-z}{1-\bar{z}}\right)\approx-\frac{1}{1+\phi}(z-\bar{z})$$

as a convenient transformation of the tax, which is higher when the tax is relatively low and lower when the tax is relatively high.

Aggregate production is log-linear, or $Y=(1-\alpha)n+\alpha K$. And, up to log deviations, labor is the same as the rescaled tax: $n=\tau$. Hence we recover the abstract model's equation

$$Y = (1-\alpha)\tau + \alpha K \tag{86}$$

in which $\alpha$ has a structural interpretation as the capital share of income. The direct effect of policy, with weight $1-\alpha$, comes entirely through the expansion of labor demand.

Let us now turn to the investment decision (78). To a log-linear approximation, it is

$$k_i = \varepsilon \mathbb{E}_i[Y] + (1-\varepsilon)\mathbb{E}_i[K]$$

After substituting in equilibrium $K$ from the production function, this simplifies to

$$k_i = \left(1-\gamma\right)\mathbb{E}_i[\tau] + \gamma \mathbb{E}[Y] \tag{87}$$

for feedback parameter

$$\gamma \equiv \frac{1}{\alpha} - \varepsilon\left(\frac{1}{\alpha} - 1\right) \tag{88}$$

This parameter is in the domain $(-\infty, 1]$, reaching the latter for $\varepsilon = 1$. It is positive if and only if $\varepsilon < 1 + \alpha$, or the aggregate demand externality is sufficiently strong relative a threshold that decreases in the capital share of income. Economically this means that the force of the aggregate demand externality, which works only through the accumulation of capital in the model, offsets the GE force of resource scarcity in the labor market.

## A.5 Policy Objective

We approximate objective (79) around the aforemenioned steady-state with the second-best policy. This results in the following loss function:

$$(1+\xi)(Y-Y^*)^2 + (1+\phi)^2\left(\phi(1-\bar{r}) + \xi(1-\bar{r})^2)\right)(\tau - \tau^*)^2 \tag{89}$$

which maps to our abstract problem for target weight

$$\chi \equiv \frac{1+\xi}{(1+\phi)^2(\phi(1-\bar{r}) + \xi(1-\bar{r})^2) + 1 + \xi} \tag{90}$$

and ideal points

$$Y^* = \tau^* = \theta \equiv (1 - z^*(\xi))^{\frac{1}{1+\phi}} \tag{91}$$

The policymaker cares both about hitting the second-best level of output and the second-best level of the policy instrument. The former measures the payoffs to the policymaker via both the household's consumption and the additional amount of tax revenue for a fixed tax rate. The latter measures the benefit of setting the right tax and not additionally distorting labor supply relative to the second-best benchmark.

## B Level-k Thinking

The key mechanism in the main analysis is agents' under-forecasting of others' responses to the policy message: as demonstrated in Lemma 2, $\bar{\mathbb{E}}[K]$ moves less than $K$ in response to variation in $\hat{X}$. One could recast this as the consequence of agents' bounded ability to calculate others' responses or to comprehend the GE effects of the policy.

A simple formalization of such cognitive or computational bounds is Level-k Thinking. This concept represents a relaxation of the part of Assumption 3 that imposes common knowledge of rationality: agents play rationally themselves, but question the rationality of others. In particular, this concept is defined recursively by letting the level-0 agent make an exogenously specified choice (this is the completely irrational agent), the level-1 agent play optimally given the belief that others are level-0 (this agent is rational but believes that others are irrational), the level-2 agent play optimally given the belief that others are level-1, and so on, up to some finite order $k$. Level-k Thinking therefore imposes a pecking order, with every agent believing that others are less sophisticated than herself in the sense that they base their beliefs on fewer iterations of the best responses than she does.

To see the implications of this concept in our context, assume all agents think to the same order $k \geq 1$ and let the "base case" (level-0 behavior) correspond to $K = 0$. Because level-$k$ agents believe that all other agents are of cognitive order $k-1$, the expectation of $K$ is now given by

$$\bar{\mathbb{E}}[K] = (1-\delta_X)\sum_{h=0}^{k-1}(\delta_X)^h\hat{X} = (1-(\delta_X)^k)\hat{X}. \tag{92}$$

For *even* $k$ and $\delta_X \in (-1,1)$, this always implies a dampened response of beliefs to the fundamental. Outcomes $K = ((1-\delta_X)+\delta_X(1-(\delta_X)^k))\hat{X}$ have dampened response to $\hat{X}$ for $\delta_X > 0$ and amplified response for $\delta_X < 0$. These distortions remain monotone in the extent of strategic interaction in either direction, $|\delta_X|$. Intuitively, higher $|\delta_X|$ puts higher weight on agents' faulty reasoning. As such our core results readily extend to this case.

The equivalence, however, breaks down for any odd number $k$ because Level-k Thinking displays a peculiar, "oscillatory" behavior in games of strategic substitutability. In our context, this problem emerges with target communication, precisely because this induces a game of strategic substitutability.

Let us explain. For any given announcement, an agent wants to invest more when he expects others to investment less. Because the level-0 agent is assumed to be completely unresponsive, a level-1 agent expects $K$ to move *less* than in the frictionless benchmark and thus moves *more* himself. A level-2 agent then expects $K$ to move *more* than in the frictionless benchmark and therefore chooses to move *less* himself. That is, whereas $k = 0$ amplifies the actual response of investment relative to rational expectations, $k = 1$ attenuates it. The left panel of Figure 1 shows that this oscillatory pattern continues for higher $k$, and that this oscillation with target communication is the only qualitative difference between the present specification and that studied as our baseline.

We are not aware of any experimental evidence of this oscillatory pattern. We suspect that it is an unintended "bug" of a solution concept that was originally developed and tested in the experimental literature

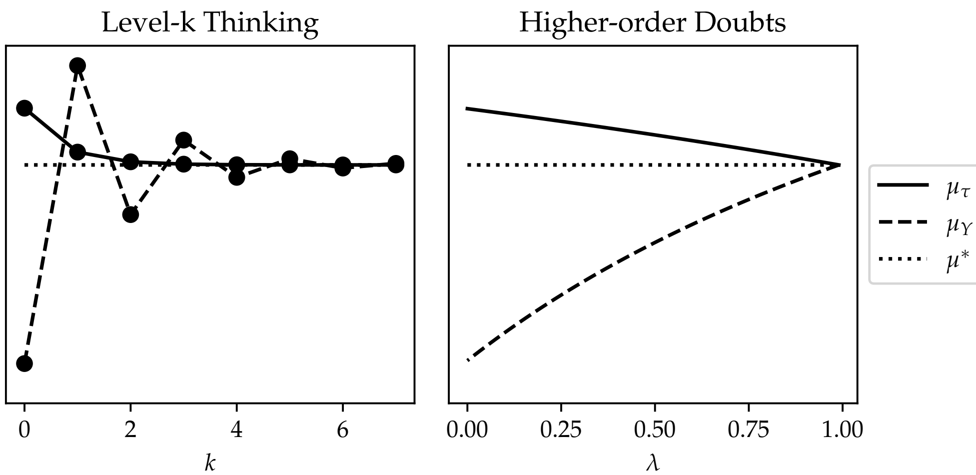


Figure 1: The implementability coefficients $\mu_\tau$ and $\mu_Y$ under Level-$k$ Thinking (left) and our main specification (right).

primarily for games of strategic complementarity and may not be applicable to games of strategic substitutability without appropriate modification. Seen from this perspective, the formalization adopted in the main text captures the essence of Level-k Thinking while bypassing its "pathological" feature.

The same goal can be achieved with a "smooth" version of Level-k Thinking along the lines of Garcıa-Schmidt and Woodford (2019). The concept of "cognitive discounting" introduced in Gabaix (2020) works in a similar manner, too, because it directly postulates that the subjective expectations of endogenous variables such as $K$ move less than the rational expectations of it.

# C Communicating Other Objects

Our focus on communicating $\tau$ or $Y$ seemed natural for applications. But, for completeness, we discuss here the possibilities of committing to a target for the aggregate action $K$ or communicating the realized value of $\theta$ along with (or perhaps instead of) a policy plan.

## C.1 Communicating a target for $K$

Consider the scenario in which the policymaker commits to a target for $K$, instead of a value for $\tau$ or $Y$. This option may be impractical if $K$ stands for a complex set of decisions that is hard to measure. But even abstracting from such measurement issues, this option is not well-posed in our model.

Consider in particular the specification studied in Section 3.3 and let the policymaker announce and commit to a value $\hat{K}$ for aggregate investment. Assume that first-order beliefs about investment are correct ($\bar{\mathbb{E}}[K] = \hat{K}$) and higher-order beliefs are sticky around zero ($\bar{\mathbb{E}}^h[K] = \lambda^{h-1}\hat{K}$). For the announcement to be fulfilled in equilibrium, it must be the case that

$$\hat{K} = (1-\delta_X)\bar{\mathbb{E}}[X] + \delta_X\bar{\mathbb{E}}[K] = (1-\delta_X)\bar{\mathbb{E}}[X] + \delta_X\hat{K}$$

for either fundamental $X \in \{\tau, Y\}$. The only first-order beliefs compatible with this announcement, then, are $\bar{\mathbb{E}}[\tau] = \bar{\mathbb{E}}[Y] = \bar{\mathbb{E}}[K] = \hat{K}$: on average (and, in fact, uniformly), agents believe that equilibrium will be $\tau = Y = K$. This is an ideal scenario for the policymaker.

It turns out, however, that a rational agent who doubts the attentiveness of others will doubt that other agents play the announcement, or that $K = \hat{K}$. If a given agent $i$ thinks that agent $j$ plays $k_j = \hat{K}$, she is implicitly taking a stand on agent $j$'s beliefs about $\tau$ and $Y$. Specifically, agent $i$ believes that agent $j$ is following her best response (here, written with $X = \tau$ ), namely

$$\mathbb{E}_i[k_j] = (1-\delta_\tau)\mathbb{E}_i\mathbb{E}_j[\tau] + \delta_\tau \mathbb{E}_i\mathbb{E}_j[K]$$

We have assumed that $\mathbb{E}_i[k_j] = \hat{K}$ and $\mathbb{E}_i\mathbb{E}_j[K] = \lambda\hat{K}$. This produces the following restriction on second-order beliefs about $\tau$:

$$\mathbb{E}_i\mathbb{E}_j[\tau] = \frac{1-\lambda\delta_\tau}{1-\delta_\tau}\hat{K}.$$

This has a simple interpretation: to rationalize aggregate investment being $\hat{K}$ despite the fact that fraction $(1-\lambda)$ of agents were inattentive to the announcement, agent $i$ thinks that a typical other agent has *over*-forecasted the policy instrument $\tau$.

At the same time, agent $i$ knows that, like himself, all attentive agents expect $\tau$ to coincide with $\hat{K}$. And since agent $i$ believes that the fraction of attentive agents is $\lambda$, the following restriction of second-order beliefs also has to hold:

$$\mathbb{E}_i\mathbb{E}_j[\tau] = \lambda\hat{K}.$$

When $\lambda = 1$ (rational expectations), the above two restrictions are jointly satisfied for any $\hat{K}$. When instead $\lambda < 1$, this is true only for $\hat{K} = 0$. This proves the claim made in the text that, as long as $\lambda < 1$, there is no equilibrium in which is infeasible to announce and commit to any $\hat{K}$ other than 0 (the default point).

In a nutshell, the problem with communicating $K$ is that the policymaker has no direct control over it. From this perspective, output communication worked precisely because the policymaker had some plausible commitment. Agents could rationalize $Y = \hat{Y}$ regardless of their beliefs about $K$ because there always existed some level of $\tau$ that implemented $\hat{Y}$.

### C.2 Expanding the message space

Return to the case in which the policymaker commits to a value for $\tau$ or $Y$ (conditional on $\theta$), but allow her to provide an additional message of the form

$$m = a\theta + b\varepsilon$$

where $a, b \in \mathbb{R}$ and $\varepsilon$ is an arbitrary random variable. This could capture a perfect or imperfect signal of the fundamental, a "justification for the policy choice," or some other arbitrary message. Let Assumption 4 apply to the vector $(\theta, X)$.

It is obvious that the additional message plays no role in the best response (7) or the expansion (23); does not enter the expression for $K$; and thus does not affect the implementability constraint. Hence the implementable sets are the same as the ones given for instrument and target communication in Proposition 1. The messages provide no extra flexibility.

### C.2.1 Communicating only $\theta$

What about communicating a message *without* a policy plan? In particular, what if the policymaker communicates *only* the value of $\theta$? In general, agents may have no idea what $\theta$ means, or how to map its announced value to an expectation for $\tau$ and $Y$. So, unless additional assumptions are made, this scenario is ill-posed.

One way to close this scenario is to assume that the agents have knowledge of the policymaker's *entire* problem, namely her objective as given in (3), her set of options (pick a value for $\tau$ or one for $Y$), and her beliefs about the structure of the economy. The agents could then use this knowledge along with the announcement of $\theta$ to figure out the policymaker's choices. This would only replicate the outcomes of our baseline analysis, in a indirect and uninteresting way.

This is is true, of course, insofar as the policymaker's problem remains the same as in our baseline analysis: the policymaker is still committing to a value for $\tau$ or $Y$, although "secretly" so. If, instead, the policymaker lacks commitment, they will expect her to play a different strategy. This takes us to the territory of Section G, where we explain why commitment is essential for regulating the bite of bounded rationality.

## D Convergent Higher-Order Beliefs

Most of our analysis restricts $\alpha < \frac{1}{2-\gamma}$ so as to guarantee that $-1 < \delta_X < 1$ for both form of forward guidance. The following technical lemma states and verifies this claim:

**Lemma D.1.** *Let $\gamma \in (-\infty, 1]$ and $\alpha \in (0,1)$. The following statements are equivalent: $|\delta_X| < 1$ for $X \in \{\tau, Y\}$; $\gamma > 2 - 1/\alpha$; and $\alpha < 1/(2-\gamma)$.*

*Proof.* This is a very simple calculation. Note that $\delta_Y < 0$ for any $\gamma \leq 1$, so $\delta_Y < 1$ is guaranteed. The condition $\delta_Y > -1$ re-arranges to

$$\delta_Y = -(1-\gamma)\frac{\alpha}{1-\alpha} > -1$$

This re-arranges to $1-\gamma < 1 - \frac{1}{\alpha}$ given $\alpha \in (0,1)$ and the previous re-arranges to $\gamma > 2 - 1/\alpha$. Finally, solving for $\alpha$ gives $\alpha < 1/(2-\gamma)$. Thus we have shown that $|\delta_Y| < 1$, $\gamma > 2-1/\alpha$; and $\alpha < 1/(2-\gamma)$ are interchangeable statements.

Note next that $\delta_\tau < 1$ is guaranteed by $\alpha \in (0,1)$. $\delta_\tau > -1$ requires $\gamma > -1/\alpha$. But this is implied by $\gamma > 2 - 1/\alpha$ and hence by $\delta_Y > -1$. □

This allows the characterization of beliefs and behavior by repeated iteration of the best responses. In particular, in Section 3 it guarantees that the joint of Assumptions 2 and 3 replicates the REE benchmark; in Online Appendix B, it guarantees that the Level-k outcome converges to the REE outcome as agents become "infinitely rational" ($k \to \infty$); and in Sections 3.3 and 5.3, it guarantees that Assumptions 4 and 5 yield the corresponding PBE outcomes. On a more technical level, restricting $\delta_Y > -1$ allows us to maintain $\mu_Y > 0$, which is important to the proofs of Theorem 2 and Proposition 3. This is established below:

**Lemma D.2** (Sign of $\mu_Y$)**.** *Fix a value for $\alpha$ and a domain $\gamma \in G$, with $0 \in G$, such that $\delta_Y > -1$. Then $\mu_Y > 0$ on the same domain for all values of $\lambda$.*

*Proof.* Note that $\mu_Y > 0$ when $\kappa_Y < 1/\alpha$. This reduces the following condition: to

$$\gamma\alpha(\lambda - \alpha) < 1 - \alpha(2 - \lambda).$$

Let's consider three cases of this.

First, assume that $\lambda > \alpha$. The above condition can then be rewritten as

$$\gamma < 1 + \frac{(1-\alpha)^2}{\alpha(\lambda - \alpha)},$$

which is obviously true for any $\gamma < 1$.

Next, consider $\lambda = \alpha$. The condition becomes

$$\alpha(2 - \alpha) < 1$$

which is always true for $\alpha = \lambda \in (0, 1)$.

Finally, consider $\lambda < \alpha$. In this case, the condition is

$$\gamma > \frac{1 + \alpha(\lambda - 2)}{\alpha(\lambda - \alpha)}$$

A strictly tighter condition is the same evaluated at $\lambda = 0$, which re-arranges to

$$\gamma > \frac{1}{\alpha}\left(2 - \frac{1}{\alpha}\right) \tag{93}$$

Note then that the restriction $\delta_Y > -1$ encodes the following restriction for fixed $\alpha$ and all $\gamma \in G$:

$$\gamma > 2 - \frac{1}{\alpha}, \tag{94}$$

as calculated in Lemma D.1. Evaluating this at $\gamma = 0$, which is within the domain $G$ over which the condition must apply, gives $\alpha < 1/2$. When $\alpha < 1/2$, the right-hand-side of conditions (93) and (94) are both negative, and (94) implies (93). Clearly, this must apply for all $\gamma \in G$ and $\alpha$ such that (94) holds. This completes the proof. □

We think, for all the reasons above, it is most reasonable to restrict to $|\delta_X| < 1$. But for completeness we discuss here what happens otherwise. Consider first an "adversarial" selection of outcome in this case. This will only strengthen the case for the main results. For $-1/\alpha < \gamma < 2 - 1/\alpha$, we have $\delta_Y < -1$ and $\delta_\tau \in (-1, 1)$. Instrument communication would clearly be preferred to prevent arbitrarily poor outcomes under target communication. To use an analogy which applies directly in our extension that considers policy rules (Section G), this is like picking a policy that obeys the Taylor principle over one that does not. For $\gamma < -1/\alpha$, we have both $\delta_Y < -1$ and $\delta_\tau < -1$, so theory lacks a clear prediction under either communication strategy.

## E Connection to Poole (1970)

Our baseline model included exogenous shocks to the preferences of the policymaker but excluded such shocks from conditions (1) and (2). This is without loss of generality if the other shocks are common knowledge and observed by the policymaker. These assumptions are extreme, but common in the Ramsey policy paradigm. In our context, they guarantee that implementability results remain true provided that the quantities $(\tau, Y)$ are re-defined to be "partialed out" from the extra shocks.

A more plausible scenario, perhaps, is that other shocks are unobserved and the policymaker cannot condition on them. This introduces into our analysis similar considerations as those in Poole (1970). The latter focused on how two different policies—fixing the interest rate or fixing the money supply—differed in their robustness to external shocks. Primitive shocks (to supply and demand) had different effects on the policy objective (output gap) depending on the slope of the model equations and the policy choice. Poole could do comparative statics of optimal policy in these slopes as well as the relative variance of the shocks.

Such "Poole considerations" can be inserted into our framework and will naturally affect the choice between fixing $\tau$ and fixing $Y$. However, such consideration matter even in the REE benchmark and, roughly speaking, are separable from the mechanism we have identified in our paper.

**Shocks to output.** Consider now a model in which output contains a random component:

$$Y = (1-\alpha)\tau + \alpha K + u,$$

where $u$ is drawn from a Normal distribution with mean 0 and variance $\sigma_u^2$, is orthogonal to $\theta$, and is unobserved by both the policymaker and the private agents. In this case, announcing and committing to a value for $Y$ stabilizes output at the expense of letting the tax distortion fluctuate with $u$. Conversely, announcing and committing to a value for $\tau$ stabilizes the tax distortion at the expense of letting output fluctuate with $u$. It follows that, even in the frictionless benchmark ($\lambda = 1$), the policymaker is no more indifferent between the two. In particular, target communication is preferable if and only if the welfare cost of the fluctuations in $Y$ exceeds that of the fluctuations in $\tau$, which is in turn is the case whenever $\chi$ is high enough.[1]

While these possibilities are interesting on their own right, they are orthogonal to the message of our paper. Indeed, the shock considered above does not affect the strategic interaction of the private agents under either the form of forward guidance: Lemma 1 remains the same. By the same token, when $\lambda = 1$, the sets of the implementable $(\tau, Y)$ pairs remain invariant to $\gamma$, even though they now depend on the realization of $u$. It then also follows that, as long as $\lambda = 1$, the optimal communication strategy does not depend on $\gamma$.

[1] The above scenario has maintained that the ideal level of output is $Y^{\text{fb}} = \theta$. What if instead $Y^{\text{fb}} = \theta + u$? This could correspond to a micro-founded business-cycle model in which technology shocks that have symmetric effects on equilibrium and first-best allocations. Under this scenario, it becomes desirable to let output fluctuate with $u$, which in turn implies that instrument communication always dominates target communication with rational expectations. A non-trivial trade off between the two could then be recovered by adding unobserved shocks to the tax distortion. The optimal strategy is then determined by the relative variance of the two unobserved shocks and the relative importance of the resulting fluctuations, along the lines of Poole (1970).

As soon as $\lambda < 1$, the implementability sets and the optimal communication strategy start depending on $\gamma$, for exactly the same reasons as those explained before. To make this more clear, note that the implementable set for instrument communication is

$$\left\{(\tau, Y) : Y = \mu_\tau^{-1}\tau + u\right\}$$

which means the policymaker, free to choose announcement $\tau = r_\tau\theta$, can implement $(\tau, Y)$ pairs of the form $(r_\tau\mu_\tau^{-1}\theta, r_\tau\theta + u)$.

Consider a policymaker who must commit ex ante, before the realization of $\theta$, to either instrument or target communication and a mapping from $\theta$ to their announcement $\hat{X}$. This is a slightly different assumption than our main analysis, but an appropriate translation of the classic Poole problem. We could just as easily have assumed contingency on $\theta$ but *not* $u$, with the minor change that optimal policy now depends on the realization of $\theta$ in place of its ex ante variance.

The appropriate translation of the loss function is

$$\mathscr{L}_\tau \equiv \min_{r_\tau\in\mathbb{R}} \left[\sigma_\theta^2\left[(1-\chi)(r_\tau - 1)^2 + \chi(r_\tau/\mu_\tau - 1)^2\right] + \chi\sigma_u^2\right]$$

where $(\sigma_u^2, \sigma_\theta^2)$ are the respective variances of $u$ and $\theta$. To re-iterate, were policy contingent on realized $\theta$, the same would apply with $\theta^2$ in place of $\sigma_\theta^2$.

For target communication, the implementable set is

$$\left\{(\tau, Y) : \tau = \mu_Y Y - \frac{u}{1-\alpha}\right\}$$

which means that, for announcement $Y = r_Y\theta$, the policymaker can implement $(\tau, Y)$ pairs of the form $(r_Y\mu_Y\theta - u/(1-\alpha), r_Y\theta)$. The appropriate translation of the loss function is

$$\mathscr{L}_Y \equiv \min_{r_Y\in\mathbb{R}} \left[\sigma_\theta^2\left[(1-\chi)(r_Y - 1)^2 + \chi(r_Y/\mu_Y - 1)^2\right] + \frac{1-\chi}{(1-\alpha)^2}\sigma_u^2\right]\right]$$

Note that the extra terms that appear in the loss functions for $\sigma_u^2 > 0$ have no dependence on $\gamma$. To map to the loss functions plotted in Figure 1 as a function of $\gamma$, each loss function is shifted above, but neither "twists" or loses its monotonicity in $\gamma$.

Finally, note that in expectation both implementable sets are the same as the ones that are presented in Theorem 1. This demonstrates Proposition 8. In particular, when $\lambda = 1$ and $\mu_\tau = \mu_Y = 1$, the implementable sets are the same as the rational-expectations ones in Proposition 1. This demonstrates Proposition 7.

**Measurement errors and trembles.** The same logic as above applies if we introduce measurement errors in the policymaker's observation of $\tau$ and $Y$, or equivalently trembles in her control of these objects. To see this, consider a variant of our framework that lets the policymaker control either $\tilde{\tau}$ or $\tilde{Y}$, where

$$\tilde{\tau} = \tau + u_\tau, \qquad \tilde{Y} = Y + u_Y,$$

and the $u$'s are independent Gaussian shocks, orthogonal to $\theta$, and unpredictable by both the policymaker and the private agents. Instrument communication now amounts to announcing and committing to a value for $\tilde{\tau}$, whereas target communication amounts to announcing and committing to a value for $\tilde{Y}$.

By combining the above with condition (1), we infer that, under both form of forward guidance, the following restriction has to hold:

$$\tilde{Y} = (1-\alpha)\tilde{\tau} + \alpha K + \tilde{u},$$

where

$$\tilde{u} \equiv -(1-\alpha)u_{\tau} + u_Y.$$

At the same time, because the $u$'s are unpredictable, the best response of the agents can be restated as

$$k_i = (1-\gamma)\mathbb{E}_i[\tilde{\tau}] + \gamma \mathbb{E}_i[\tilde{Y}].$$

This maps directly to the version with unobserved shocks just discussed above if we simply reinterpret $\tilde{\tau}$, $\tilde{Y}$, and $\tilde{u}$ as, respectively, the actual tax rate, the actual level of output, and the unobserved output shock.

To sum up, the presence of unobserved shocks and measurement error can tilt the optimal strategy of the policymaker one way or another in manners already studied in the literature that has followed the lead of Poole (1970). This, however, does not interfere with the essence of our paper's main message regarding the choice of a communication strategy as a means for regulating the impact of strategic uncertainty and the bite of the considered forms of bounded rationality.

# F Inattention vs. Distorted Reasoning

Our main analysis allows people to imperfectly reason about equilibrium, which is the friction of interest, but abstracts from the possibility that people are inattentive to forward guidance. In this Online Appendix, we accommodate this possibility and study how it matters, or does not matter, for our paper's lessons. In particular, we show that our main result (Theorem 2) remains intact if we let people be *rationally* inattentive and maintain our working hypothesis that the policymaker aims at getting the economy as close as possible to the rational-expectations outcome. But we also explore what happens away from this case.

## F.1 Implementability and distortions

We start with a reduced-form specification that let us flexibly incorporate both inattention and imperfect equilibrium reasoning. A specific micro-foundation in terms of information and priors will be provided in the next subsection.

Maintain that higher-order beliefs have the structure from Section 3.3, or

$$\bar{\mathbb{E}}^h[X] = \lambda^{h-1}\bar{\mathbb{E}}[X],$$

for some $\lambda \in (0,1]$ and all $h \geq 2$. But now let first-order beliefs satisfy

$$\bar{\mathbb{E}}[X] = qX,$$

for some parameter $q \in (0,1]$. Our analysis so far is nested by $q = 1$. Inattention, rational or not, is introduced by letting $q < 1$. Alternatively, $q < 1$ can be interpreted as the main specification of "sparsity" employed in Gabaix (2014) and Gabaix (2020).

Behavior is still determined by the solution to the following game:

$$k_i = \mathbb{E}_i\left[(1-\delta_X)X + \delta_X K\right],$$

with $X \in \{\tau, Y\}$ depending on the form of forward guidance. Aggregating this and replacing $\bar{\mathbb{E}}[X] = q$, we get

$$K = (1-\delta_X)qX + \delta_X \bar{\mathbb{E}}[K],$$

which makes clear that aggregate behavior depends, not only on the average beliefs of $K$, but also on the average belief of $X$, which now moves less than to one-to-one with $X$ insofar as $q < 1$. That said, the following property still holds:

$$\bar{\mathbb{E}}[K] = \lambda K.$$

This makes clear that $\lambda$ alone pins down the perceived responsiveness of others relative the truth.

Theorem 1 readily extends modulo the following change in the slopes of the implementability constraints:

$$\mu_\tau = \left(1-\alpha + \frac{1-\alpha\gamma}{1-\alpha\gamma\lambda}\alpha q\right)^{-1} \qquad \text{and} \qquad \mu_Y = \frac{1-\alpha+\alpha(1-\gamma)\lambda - \alpha(1-\alpha\gamma)q}{(1-\alpha)\left(1-\alpha+\alpha(1-\gamma)\lambda\right)}. \tag{95}$$

Instrument communication necessarily produces attenuation, or $\mu_\tau > 1$, because both frictions ($q < 1$ and $\lambda < 1$) work in the same direction. By contrast, the case for target communication is ambiguous ($\mu_Y \lesseqgtr 1$), because the amplification induced by rigid higher-order beliefs ($\lambda < 1$) opposes the attenuation induced by inattention ($q < 1$). Which effect dominates depends on the belief parameters $(q, \lambda)$ *and* the GE feedback $\gamma$, because the last interacts with rigid higher-order beliefs as explained in our main analysis.[2]

Finally, note that $\mu_\tau = \mu_Y$ if and only if $q = \lambda$. In this knife-edge case, agents' perception of *all* variables, communicated directly or not, is uniformly dampened by a single parameter and, as a result, we recover irrelevance of the instrument-versus-target choice. We allude to this fact in our discussion of clarity and confidence in Section 7—a model with "plain" inattention does *not* capture our desired feature of relaxing the most essential property of (full information) rational expectations equilibrium, which is the interchangeability of different objects that appear in the equilibrium allocation.

### F.2 A signal extraction model

To provide a more specific structure for what $q$ and $\lambda$ mean as independent parameters, consider the following model of inattention with a behavioral twist. Let the announcement $X$ be Gaussian with mean 0 and known variance $\sigma_X^2$.[3] Each agent, because of their inattention, observes in effect a noisy signal

[2] Indeed, attenuation is obtained with target communication (i.e., $\mu_Y > 1$) if and only if $q < \tilde{q}(\lambda,\gamma) \equiv \frac{1-\alpha(1-(1-\gamma)\lambda)}{1-\alpha\gamma}$. The threshold $\tilde{q}$ is increasing in both $\lambda$ and $\gamma$, always exceeds $\lambda$, and reaches 1 when either $\lambda = 1$ or $\gamma = 1$.

[3] This property will be maintained in the policy problem we consider if the underlying shock $\theta$ is Gaussian with known variance, because $X$ is itself proportional to $\theta$ in equilibrium

$s_i = X + u_i$, where $u_i$ is idiosyncratic Gaussian noise with mean 0. Agent $i$ *perceives* $u_i$ to have variance $\omega^2$; the noise actually has variance $\xi^2$, where $\xi$ may or may not be the same as $\omega$ depending on whether the agent has the correct prior about his cognitive capacities. It follows, from simple signal-extraction math, that the agent's own expectation of $X$ is

$$\mathbb{E}_i[X] = \frac{\sigma_X^2}{\sigma_X^2 + \omega^2}(X + u_i).$$

This expression, averaged and then mapped to the reduced-form model for average expectations introduced in the previous subsection, gives $\bar{\mathbb{E}}[X] = qX$ with

$$q \equiv \frac{\sigma_X^2}{\sigma_X^2 + \omega^2}.$$

When agents perceive their internal representations to have more noise (i.e., $\omega^2$ is higher), $q$ becomes smaller and first-order beliefs are more attenuated. Note that there is no direct role for the actual noise variance $\xi^2$ in determining the mean belief, which is sufficient for characterizing implementable allocations. Nonetheless, we can also define a "rational" signal-to-noise ratio, or

$$q^* \equiv \frac{\sigma_X^2}{\sigma_X^2 + \xi^2},$$

which is a benchmark to which we can compare the actual outcome whenever subjective perceptions diverge from reality.When agents over-estimate their cognitive capacities or the precision of their information, $\omega^2 < \xi^2$ and $q > q^*$. When they make the opposite mistake, $\omega^2 > \xi^2$ and $q < q^*$.

The above completes the description of how agents think about *themselves*. Let us now turn to how they think about *others*. Agent $i$ perceives any other agent $j$ to receive a signal of the form $X + u_j$, where $u_j$ has mean 0 and variance $\tilde{\omega}^2$, which again may not equal the true variance $\xi^2$. Agent $i$ believes further that agent $j$ will associate variance $\tilde{\omega}^2$ with the signals of agents $k \neq j$, and so forth. It is simple to show that second-order beliefs thus satisfy

$$\mathbb{E}_i[\mathbb{E}_j[X]] = \mathbb{E}_i\left[\frac{\nu^2}{\nu^2 + \tilde{\omega}^2}(X + u_j)\right] = \left(\frac{\nu^2}{\nu^2 + \tilde{\omega}^2}\right)\mathbb{E}_i[X]$$

Averaging and iterating this argument, we get $\bar{\mathbb{E}}^h[X] = \lambda^{h-1}\bar{\mathbb{E}}[X]$ with

$$\lambda \equiv \frac{\nu^2}{\nu^2 + \tilde{\omega}^2}.$$

This scalar therefore depends exclusively on what each agent perceives to be the quality of *others'* information.

Note now that $q \in (0, 1)$ and $\lambda \in (0, 1)$ are guaranteed respectively by $\omega^2 > 0$ and $\tilde{\omega}^2 > 0$, or positive perceived variances. The case $q > \lambda$ is guaranteed by $\omega^2 > \tilde{\omega}^2$, or a given agent believing he is more informed and/or attentive than the average other agent. The opposite case, $q < \lambda$, is associated with the opposite, or a given agent's belief that others are more likely to be paying attention.

The canonical noisy rational expectations case is nested for $\tilde{\omega}^2 = \omega^2 = \xi^2$, or $\lambda = q = q^*$. But $\omega^2 = \tilde{\omega}^2$, or $\lambda = q$, alone is necessary for a model that, in terms of the equivalence between instrument and target communication irrelevance outcomes, is isomorphic to the noisy rational-expectations model.

Going back to the analysis of the previous subsection, recall that the implementability constraints depend only on $q$ and $\lambda$, not on $q^*$. This is because, in a linear model such as ours, the actual level, or the value of $q^*$, does not matter at all for the *positive* properties of aggregate behavior; what matters is only people's subjective view of the world. But as we explain below, $q^*$ matters for judging the *normative* implications of any given behavior.

### F.3 Rational inattention or "one-distortion case"

Our baseline analysis and the loss function (3) compared all allocations to the full-information, rational-expectations allocation. This may not be appropriate in an environment with *rational* inattention, as in Sims (2003) and large follow up literature. Angeletos and Sastry (2019) show that the introduction of such inattention *alone* does not upset the Welfare Theorems: there is no policy that can improve upon market outcomes. .

The basic intuition is that there is no good reason for the policymaker to try to correct people's behavior if any "friction" in it is merely the product of the agents' optimal use of limited information or limited cognitive capacity. To capture this idea in reduced form, we now consider an altered policy problem that is "re-centered" around the rational expectations equilibrium (i.e., the one with *correct* perceptions of the noise variance).

Let us first consider the simplest such case, in which mis-perception of others' precision of information is the only behavioral distortion. This means $q = q^*$, or agents correctly perceive their own precision, but $\lambda \neq q = q^*$, or agents mis-perceive others' precision. Let $\mu_{\text{in}}$ be the slope of the implementability constraint in a counterfactual world in which $\lambda = q = q^*$. Re-centering the policymaker's objective around this reference point amounts to the following modification of the loss function:

$$L(\tau, Y, \theta) \equiv (1-\chi)(\tau-\theta)^2 + \chi(Y - \theta/\mu_{\text{in}})^2. \tag{96}$$

This problem features only one distortion, relative over or under confidence, and thus resembles our baseline policy problem with a new "center point." Given this adjustment in the relevant benchmark for optimality, we can prove that our main result is once again generic for $\lambda \neq q$:

**Proposition 1.** *Assume the combination of first and higher-order uncertainty described above and a policy objective that treats the noisy rational expectations equilibrium as the first-best. For $q \leq 1$ and $\lambda \neq q$, there exists some critical threshold $\hat{\gamma} \in [0,1)$ such that target communication is strictly preferred for $\gamma > \hat{\gamma}$.*

This result, and all others in this section, are proved in a final subsection of this Online Appendix. The result intuitively "re-isolates" our main friction of interest as the *only* source of distortion.

### F.4 Irrational inattention or "two-distortion case"

Let us now consider a situation in which there is a second *competing* distortion induced by irrational inattention or some other "wedge" in first-order beliefs.

A first path forward for evaluating optimal policy is to treat inattention and the behavioral bias as joint sources of inefficiency. This is tantamount to evaluating $\mu_{\text{in}}$ in (96) with $q^* = 1$, or continuing to use the original objective (3). Provided $\lambda < q$, the paternalistic planner again uses a threshold strategy:

**Proposition 2.** *Let* $c(\gamma) \equiv \mathbb{I}\{\mathscr{A}^* = \mathscr{A}_Y\}$ *be a 0 or 1 indicator for using target communication. For* $\lambda < q \leq q^* = 1$, $c(\gamma)$ *weakly increases on the domain* $[0,1]$.

In this case, target communication may be preferred on the entire domain. This has the opposite intuition from the previous result: some over-reaction in GE reasoning helps offset the attenuation from incomplete information.

Note that the previous two propositions do not cover the case of $q < \lambda < q^* = 1$, with agents believing they are "worse than average." In such a case, the considerations of canceling out the friction in higher-order reasoning and "fighting" the wedge in first-order beliefs do not stack with one another. Instead, there is now room for the familiar second-best logic of using one distortion to fight another.

Next, consider the case of $q^* < 1$ but $q > q^*$. There is optimally some inattention, but agents over-perceive the precision of their own signals. As discussed later, the empirical evidence in Kohlhas and Broer (2018) and Bordalo et al. (2020) supports such a case in the data. We now consider a policy problem with the objective (96), but with $\mu_{\text{in}}$ evaluated at $q = q^*$ and $\lambda = q^*$. In such a case, we can show that if $\lambda < q$ our result extends in the following sense:

**Proposition 3.** *Let* $c(\gamma) \equiv \mathbb{I}\{\mathscr{A}^* = \mathscr{A}_Y\}$ *be a 0 or 1 indicator for using target communication. For* $\lambda < q \leq 1$ *and* $q^* < q$, $c(\gamma)$ *weakly increases on the domain* $[0,1]$.

**Empirical (and psychological) context.** The combination of the evidence provided in Bordalo et al. (2020), Coibion and Gorodnichenko (2012, 2015), Coibion et al. (2018), Kohlhas and Broer (2018), and Kohlhas and Walther (2018) from various surveys of macroeconomic forecasts rejects the representative-agent, rational-expectations benchmark. Much of this evidence concentrates on professional forecasters, but some of it covers firms and consumers as well. Notwithstanding the difficulty of extrapolating from such broad-scope evidence to the specific counterfactual studied in our paper, we now explain why this evidence points towards the following combination of parameters, which (per Proposition 3) suffices for our main result to survive even when inattention is irrational:

- $q < 1$, meaning that people are inattentive or imperfectly informed;
- $q^* < q$, meaning that people over-estimate the precision of their information relative to the truth; and
- $\lambda < q$, meaning that people under-estimate the precision of others' information relative to their own.

The first property is documented in Coibion and Gorodnichenko (2012, 2015) by showing that average forecasts under-react to news. These papers also offer a structural interpretation of this fact in terms of models with dispersed noisy information and rational expectations, along the lines of Morris and Shin (2002) and Woodford (2003). But they do not contain any evidence that would support this hypothesis against the richer alternative. That is, they *presume* $q = \lambda = q^* < 1$, but the provided evidence actually only proves $q < 1$ and $\lambda < 1$, leaving the $q - \lambda$ gap and the value of $q^*$ free. Accordingly, Gabaix (2020) interprets the same fact as evidence of a certain form of *irrational* inattention, or in terms of a model where $q < 1$ but $q^* = 1$.

This ambiguity is resolved by the combination of Bordalo et al. (2020) and Kohlhas and Broer (2018). These papers provide evidence that, whereas forecast errors are positively related to past forecast revisions at the *aggregate* level (as originally shown in Coibion and Gorodnichenko, 2015), they are negatively related at the *individual* level.

The second fact, by itself, rejects rational expectations: with rational expectations, an individual's forecast error cannot be forecastable by his *own* past information. Furthermore, the sign of the documented bias points towards individual over-reaction to own information. Kohlhas and Broer (2018) attribute such over-reaction to the tendency of an individual to think that his information is more precisely than it actually is ("absolute over-confidence"). In the language of the simple model presented above, this means $q > q^*$. Bordalo et al. (2020) propose a variant explanation, based on "representativeness bias," which though works in essentially the same way and, for our purposes, can also be captured by $q > q^*$.

To match the first fact, or the under-reaction of the average forecasts, it is then necessary to have dispersed noisy information. To understand why, recall that this fact alone could be explained either by dispersed noisy information, as originally shown by Coibion and Gorodnichenko (2012, 2015) themselves, or by a bias that causes individual beliefs to under-react, as suggested by Gabaix (2020). But we just argued that the bias in individual beliefs, as evidenced in the second fact, is of the opposite kind. The two facts *together* therefore point towards the combination of over-confidence and dispersed noisy information, which in the language of the model presented above means $q^* < q < 1$.

Both Bordalo et al. (2020) and Kohlhas and Broer (2018) reach basically the same conclusion. Angeletos and Huo (2020) further clarify why information has to be not only noisy but also dispersed: the aforementioned facts together imply one agent's forecast error is predictable by the *another* agent's information. Angeletos and Huo (2020) also develop the precise mapping between these facts and a model that has a similar formal structure as our framework—and that adds various extra features that are needed for quantitative purposes, including richer micro-foundations, long horizons and learning dynamics, but are of course beyond the scope of our paper.

More importantly for the present purposes, Kohlhas and Broer (2018) provide a *third* fact, which points towards $\lambda < q$: individual forecasts over-react to consensus forecasts. This is consistent with the hypothesis that the typical individual under-estimates the information of others and is thus relatively over-confident in their own assessment. As mentioned in the main text, such a perception in being "better than average" is documented by psychologists in various contexts (see, for instance, Alicke and Govorun, 2005). In our context, it translates into a lack of confidence in other agents' attentiveness, or $\lambda < q$.

Of course, the literature reviewed here may not be the final word on what the best structural interpretation of the available evidence on expectations is. Also, this evidence need not be directly importable to the context of interest. In particular, Garcıa-Schmidt and Woodford (2019) and Farhi and Werning (2019) argue that, because this was the first time the United States had hit the ZLB context and nobody could draw from past data to infer the GE effects of the various unconventional policies the Fed had to experiment with, people may have naturally resorted to introspection and deductive (iterative) reasoning, of the kind seen in experiments. If this argument is valid, it offers offers a more direct justification for our baseline analysis. Still, the evidence discussed above is complementary: not only it rejects the representative-agent, rational-expectations benchmark but also favors, within the extension presented in this Online Appendix,the particular scenario of $\lambda < q$ and $q^* < q$, which in turn suffices for our main policy prescription to continue to hold (Proposition 3) despite the presence of confounding distortions.

### F.5 Proofs

**Proof of Proposition 1**

Note first the following properties of $(\mu_{\text{in}}, \mu_\tau, \mu_Y)$, which can be verified by direct calculation:

1. $\mu_\tau = \mu_{\text{in}}$ when $\gamma = 0$, and $\mu_\tau > \mu_{\text{in}}$ when $\gamma \in (0,1]$.
2. $\mu_Y = \mu_{\text{in}}$ when $\gamma = 1$, and $\mu_Y < \mu_{\text{in}}$ when $\gamma \in [0,1)$

Note finally that $\mu_Y > 0$ if and only if $\lambda > 1 + q - 1/\alpha$, which by the same argument provided in Lemma D.2 is always true if we have specified $|\delta_Y| < 1$ for all $\gamma \in [0,1]$. As with the main result, we will focus on such a case in the proof.

Assume that the policymaker's objective function is given by (96), where $\mu_{\text{in}}$ defines the slope of the implementability constraint in the noisy rational expectations case of a given model (i.e., in which $\lambda$ is set equal to $q$).

The objective in terms of the message slope $r$ and the implementability slope $\mu$ is

$$(1-\chi)(r-1)^2 + \chi(r/\mu - 1/\mu_{\text{in}})^2$$

The optimal $r$ in closed-form, as a function of other parameters, is

$$r(\mu) = \frac{(1-\chi)\mu^2 + \chi\frac{\mu}{\mu_{\text{in}}}}{(1-\chi)\mu^2 + \chi}$$

and the new objective function, in terms of $(\mu, \mu_{\text{in}})$, is a function $\ell_0$:

$$\mathscr{L} = \ell(\mu, \mu_{\text{in}}) \equiv \chi(1-\chi)\frac{(\mu - \mu_{\text{in}})^2}{\mu_{\text{in}}^2(\mu^2(1-\chi)+\chi)} \tag{97}$$

Note that the derivative of the loss function f with respect to $\gamma$ comes through two components, and is

$$\frac{\partial \ell}{\partial \gamma} = \frac{\partial \ell}{\partial \mu}\frac{\partial \mu}{\partial \gamma} + \frac{\partial \ell}{\partial \mu_{\text{in}}}\frac{\partial \mu_{\text{in}}}{\partial \gamma} \tag{98}$$

The two partial derivatives of $\ell$ are

$$\frac{\partial \ell}{\partial \mu} = 2(1-\chi)(\chi)\frac{(\mu-\mu_{\text{in}})(\mu\mu_{\text{in}}(1-\chi)+\chi)}{\mu^2(\mu^2(1-\chi)+\chi)^2}$$

which is positive if and only if $\mu > \mu_{\text{in}}$, and

$$\frac{\partial \ell}{\partial \mu_{\text{in}}} = -2(1-\chi)(\chi)\frac{\mu}{\mu_{\text{in}}}\cdot\frac{(\mu-\mu_{\text{in}})}{\mu_{\text{in}}^2(\mu^2(1-\chi)+\chi)}$$

which is positive if $\mu < \mu_{\text{in}}$.

Plugging the previous expressions into (98), we have that $\partial\ell/\partial\gamma > 0$ is positive if $\mu > \mu_{\text{in}}$ and

$$\frac{\partial \mu}{\partial \gamma} > \frac{\mu}{\mu_{\text{in}}}\frac{\mu^2(1-\chi)+\chi}{\mu\mu_{\text{in}}(1-\chi)+\chi}\frac{\partial \mu_{\text{in}}}{\partial \gamma} \tag{99}$$

or if $\mu < \mu_{\text{in}}$ and

$$\frac{\partial \mu}{\partial \gamma} < \frac{\mu}{\mu_{\text{in}}}\frac{\mu^2(1-\chi)+\chi}{\mu\mu_{\text{in}}(1-\chi)+\chi}\frac{\partial \mu_{\text{in}}}{\partial \gamma} \tag{100}$$

Finally, note that the partial derivative of $\mu_{\text{in}}$ with respect to $\gamma$ is

$$\frac{\partial \mu_{\text{in}}}{\partial \gamma} = \frac{\alpha^2(1-q)\lambda}{(1-\alpha+\alpha q(1-q))^2} > 0$$

**Monotonicity of loss with instrument communication.** Note that the derivative of $\mu_\tau$ in $\gamma$ is given by

$$\frac{\partial \mu_\tau}{\partial \gamma} = \frac{q\alpha^2(1-\lambda)}{(1-\alpha+q\alpha(1-\alpha\gamma)-\alpha\gamma\lambda(1-\alpha))^2} > 0$$

Consider first this case $q > \lambda$ which entails $\mu_\tau > \mu_{\text{in}}$. A looser version of (99) is

$$\frac{\partial \mu_\tau}{\partial \gamma} > \left(\frac{\mu_\tau}{\mu_{\text{in}}}\right)^2\frac{\partial \mu_{\text{in}}}{\partial \gamma}$$

and this can be verified by "brute force": the previous expression is

$$\frac{(1-\lambda)}{(1-q)} > \frac{(1-\lambda\alpha\gamma)^2}{(1-q\alpha\gamma)^2} \tag{101}$$

Note that an upper bound for the right-hand-side is given for $\gamma = 1$, or

$$\frac{(1-\lambda)}{(1-q)} > \frac{(1-\lambda\alpha)^2}{(1-q\alpha)^2}$$

But this is guaranteed if we impose $\alpha < 1/2$, which was consistent with $|\delta_Y| > -1$ on the entire domain of study.

Now consider $q < \lambda$. The loose version of (100) is

$$\frac{\partial \mu_\tau}{\partial \gamma} < \left(\frac{\mu_\tau}{\mu_{\text{in}}}\right)^2\frac{\partial \mu_{\text{in}}}{\partial \gamma}$$

because for $\mu_\tau < \mu_{\text{in}}$ the right-hand-side is a lower bound. From the exact same math of (101), the key condition is now

$$\frac{(1-\lambda)}{(1-q)} < \frac{(1-\lambda\alpha\gamma)^2}{(1-q\alpha\gamma)^2} \tag{102}$$

which is satisfied for the exact same reason.

Together, these arguments suffice to show that in any case, $\ell(\mu_\tau, \mu_{\text{in}})$ increases in $\gamma$. Note finally that this loss function is 0 at $\gamma = 0$, where $\mu_\tau = \mu_{\text{in}}$, and strictly positive at $\gamma = 1$, where $\mu_\tau \neq \mu_{\text{in}}$.

**Monotonicity of loss with target communication.** The derivative of $\mu_Y$ in $\gamma$ is given by

$$\frac{\partial \mu_Y}{\partial \gamma} = \frac{q\alpha^2(1-\lambda)}{(1-\alpha+\alpha\lambda(1-\gamma))^2} > 0$$

First consider $q > \lambda$, which entails $\mu_Y < \mu_{\text{in}}$. It is simple to show that (100) is never satisfied because $\frac{\partial \mu_Y}{\partial \gamma} > \frac{\partial \mu_{\text{in}}}{\partial \gamma}$, since

$$\frac{\partial \mu_Y}{\partial \gamma} = \frac{(1-\lambda)}{(1-q)} \frac{(1-\alpha+\alpha q(1-\gamma))^2}{(1-\alpha+\alpha\lambda(1-\gamma))^2} \frac{\partial \mu_{\text{in}}}{\partial \gamma} > \frac{\partial \mu_{\text{in}}}{\partial \gamma}$$

Next consider the case $q < \lambda$, which entails $\mu_Y > \mu_{\text{in}}$. Note that condition (99) is violated because

$$\frac{\partial \mu_Y}{\partial \gamma} = \frac{(1-\lambda)}{(1-q)} \frac{(1-\alpha+\alpha q(1-\gamma))^2}{(1-\alpha+\alpha\lambda(1-\gamma))^2} \frac{\partial \mu_{\text{in}}}{\partial \gamma} < \frac{\partial \mu_{\text{in}}}{\partial \gamma} < \left(\frac{\mu_Y}{\mu_{\text{in}}}\right)^2 \frac{\partial \mu_{\text{in}}}{\partial \gamma}$$

Together, these arguments suffice to show that $\ell(\mu_Y, \mu_{\text{in}})$ decreases in $\gamma$. Note finally that this loss function is 0 at $\gamma = 1$, where $\mu_Y = \mu_{\text{in}}$, and strictly positive at $\gamma = 0$, where $\mu_Y \neq \mu_{\text{in}}$.

**Proving the threshold strategy.** Given the monotonicities established above, proving the sought-after result—that target communication is optimal if and only if $\gamma > \hat{\gamma}$, for some $\hat{\gamma} \in (0,1)$—requires only using continuity arguments like in the proof of Theorem 2.

**Proof of Proposition 2**

First, we note the monotonicity of $(\mu_\tau, \mu_Y)$ in $\gamma$. The derivative of $\mu_\tau$ with respect to $\gamma$ is

$$\frac{\partial \mu_\tau}{\partial \gamma} = \frac{1}{\mu_\tau^2} \frac{\alpha q(1-\lambda)}{(1-\lambda\gamma)^2} > 0$$

and the derivative of $\mu_Y$ is

$$\frac{\partial \mu_Y}{\partial \gamma} = \frac{1}{\mu_Y^2} \frac{\alpha q(1-\alpha)(1-\lambda)}{(\alpha q(\delta_Y - 1) - \lambda\delta_Y + 1)^2} > 0$$

Next, we want to show that $\mu_\tau > \mu_Y$. The correct condition in terms of parameters is

$$\frac{1 + \frac{\lambda\alpha(1-\gamma)}{1-\alpha} - \alpha q \frac{1-\alpha\gamma}{1-\alpha}}{1-\alpha+\lambda\alpha(1-\gamma)} \leq \frac{1-\lambda\alpha\gamma}{(1-\alpha)(1-\lambda\alpha\gamma)+\alpha q(1-\alpha\gamma)}$$

Given that $\mu_Y > 0$, which is guaranteed like just as in Lemma D.2, the left denominator is positive. The other three terms are necessarily positive. Thus an equivalent statement, after cross-multiplying, is the following:

$$(1-\lambda\alpha\gamma)(1-\alpha+\lambda\alpha(1-\gamma)) \geq \left((1-\lambda\alpha\gamma) + \frac{\alpha q(1-\alpha\gamma)}{1-\alpha}\right)(1-\alpha+\lambda\alpha(1-\gamma)-\alpha q(1-\alpha\gamma)$$

Subtracting like terms from each side, and dividing by $\alpha > 0$, yields the following condition:

$$(q-\lambda)(1-\alpha\gamma) \geq 0$$

Hence $q > \lambda$ and $\alpha\gamma < 1$ are a sufficient condition for $\mu_\tau > \mu_Y$, and either $q = \lambda$ or $\alpha\gamma = 1$ are a sufficient condition for $\mu_\tau = \mu_Y$.

Finally, let us return to the proof of optimality. It is straightforward to solve the expression for $\mu_Y$ for some $\tilde{\gamma}_Y(\alpha,\lambda,q) \in (0,1]$ such that $\mu_Y|_{\gamma=\tilde{\gamma}_Y} = 1$. One can apply the argument in the proof of Theorem 2 to the loss functions $\mathscr{L}_\tau(\gamma)$ and $\mathscr{L}_Y(\gamma)$ on the domain $[0,\tilde{\gamma}_Y]$. There is some $\hat{\gamma} \in [0,\tilde{\gamma}_Y)$ where the functions cross.

For $\gamma \in (\tilde{\gamma}_Y, 1)$, we know that (i) $\mu_Y$ and $\mu_\tau$ both increase in $\gamma$ and (ii) $\mu_\tau > \mu_Y$. It is straightforward to deduce that $\mu_\tau > \mu_y > 1$ for $\gamma > \tilde{\gamma}_Y$ (and hence $\mathscr{L}_Y < \mathscr{L}_\tau$), which shows the optimality of target communication and completes the proof.

**Proof of Proposition 3**

We proceed with the same parameter restriction assumed in the proof of Proposition 1. Note also that the same expressions for the loss functions, the partial derivatives thereof, and sufficient conditions for monotonicity of the loss function in $\gamma$ still apply.

Applying arguments from the proof of Proposition 1, it is simple also to show that $\mu_\tau > \mu_Y$ and $\mu_{\text{in}} > \mu_Y$ on this domain.

**Case 1: $q^* \leq \lambda < q$.** In this case, $\mu_\tau \leq \mu_{\text{in}}$ with equality only for $q^* = \lambda$ and $\gamma = 1$, verified by the direct calculation

$$\frac{1-\alpha\lambda\gamma}{(1-\alpha)(1-\alpha\lambda\gamma)-q\alpha(1-\alpha\gamma)} \leq \frac{1-\alpha q^*\gamma}{1-\alpha-q^*\alpha(1-\gamma)} \tag{103}$$

In this case, we have $\mu_Y < \mu_\tau \leq \mu_{\text{in}}$, and all three increasing in $\gamma$. It follows that instrument communication always produces less loss and is preferred on the entire domain $\gamma \in [0,1]$. To see this, note that for $\mu < \mu_{\text{in}}$, the loss function is decreasing in $\mu$.

**Case 2: $\lambda < q^* \leq \frac{q}{1+\alpha(q-\lambda)} < q$.** Re-arrangement of (103), with this condition, again shows $\mu_\tau \leq \mu_{\text{in}}$ with equality only at $\gamma = 1$ and $q^* = \frac{q}{1+\alpha(q-\lambda)}$. Again, instrument communication is preferred on the entire domain.

**Case 3: $\frac{q}{1+\alpha(q-\lambda)} < q^* < q$ and $\lambda < q^*$.** In this final case, there exists a $\check{\gamma}$ such that $\mu_\tau > \mu_{\text{in}}$ for $\gamma > \check{\gamma}$ and $\mu_\tau \leq \mu_{\text{in}}$ for $\gamma \leq \check{\gamma}$. The previous argument applies to show the optimality for instrument communication for $\gamma \leq \check{\gamma}$. For $\gamma > \check{\gamma}$, we want to show that the loss for instrument communication strictly increases and the loss from target communication strictly decreases.

From the proof of Proposition (1), a sufficient condition for the first is that

$$\frac{\partial\mu_\tau}{\partial\gamma} > \left(\frac{\mu_\tau}{\mu_{\text{in}}}\right)^2 \frac{\partial\mu_{\text{in}}}{\partial\gamma}$$

This condition simplifies to

$$\frac{q}{q^*}\frac{(1-\lambda)}{(1-q^*)} > \frac{(1-\lambda\alpha\gamma)^2}{(1-q^*\alpha\gamma)^2}$$

Taking a lower bound on the left (with $q/q^* \geq 1$) and an upper bound on the right (evaluating at $\gamma = 1$) gives

$$\frac{(1-\lambda)}{(1-q^*)} > \frac{(1-\lambda\alpha)^2}{(1-q^*\alpha)^2}$$

which, as used in the proof of Proposition 1, will always hold for $\lambda < q^*$ and $\alpha < 1/2$. Thus we have shown that $\mathscr{L}_\tau(\gamma)$, the loss function associated with instrument communication, strictly increases for $\gamma > \check{\gamma}$.

Next, a sufficient condition for $\mathscr{L}_Y(\gamma)$, the loss function from target communication, to decrease for $\gamma > \check{\gamma}$ is $\frac{\partial \mu_Y}{\partial \gamma} > \frac{\partial \mu_{\text{in}}}{\partial \gamma}$. By direct calculation,

$$\frac{\partial \mu_Y}{\partial \gamma} = \frac{q}{q^*}\frac{(1-\lambda)}{(1-q^*)}\frac{(1-\alpha+\alpha q^*(1-\gamma))^2}{(1-\alpha+\alpha\lambda(1-\gamma))^2}\frac{\partial \mu_{\text{in}}}{\partial \gamma} > \frac{\partial \mu_{\text{in}}}{\partial \gamma}$$

so this is always true.

We have thus established that the difference in loss between target and instrument communication, or $\Delta \equiv \mathscr{L}_Y(\tau) - \mathscr{L}_\tau(\tau)$, decreases in $\gamma$ for $\gamma > \check{\gamma}$.

Let the choice of target communication be a 0 or 1 indicator variable, $c \equiv \mathbb{I}\{\mathscr{A}^* = \mathscr{A}_Y\} = \mathbb{I}\{\Delta < 0\}$. $c$ weakly decreases in $\Delta$, so the choice of target communication weakly increases in $\gamma$ for $\gamma \in (\check{\gamma}, 1]$. Because $c = 0$ for any $\gamma \in [0, \check{\gamma}]$, this completes the proof that $c$ weakly increases in $\gamma$ in $[0, 1]$.

# G Sophisticated Forward Guidance and Policy Rules

## G.1 Set-up

Assume that, after observing $\theta$, the policymaker can commit to and communicate a flexible *relation* between the instrument $\tau$ and the outcome $Y$, given by

$$\tau = T(Y;\theta),$$

for some function $T : \mathbb{R}^2 \to \mathbb{R}$.[4] Without serious loss of generality, we restrict attention to linear reaction functions of the form

$$T(Y;\theta) = a + bY, \qquad \text{with} \qquad a = A(\theta) \quad \text{and} \quad b = B(\theta), \tag{104}$$

for arbitrary $A(\cdot) : \mathbb{R} \to \mathbb{R}$ and $B(\cdot) : \mathbb{R} \to (\underline{b}, 1)$, where $\underline{b} \equiv \frac{1+\alpha\gamma}{1-2\alpha+\alpha\gamma} < -1$. The bounds on $b$ are necessary and sufficient for "reasoning to converge," or for infinite-order beliefs not to have undue influence on behavior.[5] The simpler strategies considered in our baseline analysis are nested with $b = 0$ and $a = \hat{\tau}$ for instrument communication, and $b \to -\infty$ and $-a/b \to \hat{Y}$ for target communication. With the flexibility added

[4] Clearly, the outcomes implemented with such a policy rule coincide with those implemented with a rule of the form $\tau = T(K;\theta)$, since $Y$ is a (fixed) function of $\tau$ and $K$.

[5] See the proof of Proposition 5 for the details. In the New Keynesian framework, the analogue of $b < 1$ is the Taylor principle, and the analogue of $b > \underline{b}$ is the additional bound on the slope of the Taylor rule identified by Guesnerie (2008) as necessary and sufficient for the unique linear REE of that model to be also the unique rationalizable outcome.

here, forward guidance amounts to announcing, conditional on $\theta$, a pair of numbers $(a,b) = (A(\theta), B(\theta))$, or an intercept and a slope for the reaction function, instead of a single number $\hat{\tau}$ or $\hat{Y}$.[6]

All assumptions about depth of knowledge and rationality now relate to agents' understanding of the function $T$, or the pair $(a,b)$. In particular, Assumption 4 is adapted as follows: agents believe that only a fraction $\lambda \in [0,1]$ of the others are both rational and aware of the actual $(a,b)$, like themselves; the rest are expected to play the "default" action $k=0$, either because of inattention or because of irrationality.[7]

### G.2 Optimal policy

In our main analysis, we contrasted how the choice between instrument and target communication was irrelevant in the rational-expectations benchmark ($\lambda = 1$) to how it became crucial in managing expectations once we accommodated bounded rationality ($\lambda < 1$). The next result generalizes this insight to the richer policy strategy space allowed here.

**Proposition 4.** *Consider a reaction function $T$ and let $Y(\theta)$ and $\tau(\theta)$ be, respectively, the induced equilibrium values of the outcomes and the supporting policies (together, "allocations"). Next, consider any other reaction function $T'$ such that $T'(Y(\theta),\theta) = T(Y(\theta),\theta)$ for all $\theta$.*

*(i) When $\lambda = 1$, $T'$ induces the* same *equilibrium outcomes and policies as $T$.*

*(ii) When instead $\lambda < 1$, $T'$ induces* different *equilibrium outcomes and policies than $T$.*

Part (i) is familiar from the existing literature on Ramsey problems, in which there is often a large family of policy rules that implement the same equilibrium allocations and policies. The analogue of this property in the 3-equation New Keynesian model is also well known: there are multiple combinations of a state-contingent intercept and a slope for the Taylor rule that implement the same equilibrium paths for output, inflation, and interest rates.[8]

Part (ii) shows that this kind of irrelevance breaks once we bound agents' depth of knowledge and rationality. Fix $T$ and let $\theta \mapsto (\tau^*(\theta), Y^*(\theta))$ be the equilibrium mapping from states to allocations implemented by $T$. Next take any other $T'$ that satisfies $T'(Y^*(\theta),\theta) = \tau^*(\theta)$. This property guarantees that agents find it optimal to play the same action under $T'$ as under $T$ insofar as long as they conjecture that $T'$ continues to induce the same allocations as $T$. When $\lambda = 1$, one can close the loop to prove this conjecture is self-fulfilling and hence that $T'$ induces the same behavior as $T$. But once $\lambda < 1$, agents doubt that *others* make

[6]This interpretation is under the maintained timing, which has the policymaker choose and communicate the scalars $(a,b)$ *after* observing $\theta$. But the same outcomes obtain also with an alternative timing that has the policymaker choose and communicate the entire mappings $(A(\cdot), B(\cdot))$ *prior* to observing $\theta$. The first perspective seems more natural in the context of forward guidance and under the interpretation of $\theta$ as the policymaker's current assessment of the best thing to do. The second perspective is more appropriate for connecting to the macroeconomic literature on policy rules and for re-interpreting $\theta$ as a *future* shock. Finally, note that for now we are allowing *both* the intercept and the slope of the policy rule to vary with $\theta$, but below we will show that optimality requires that *only* the intercept varies with $\theta$.

[7]This specification imposes $\lambda \leq 1$. But the results stated below readily extend to $\lambda > 1$, or a situation where agents over-estimate the responses of others and the GE effects of policy, along the lines of Section 5.3.

[8]The most applied segment of the New Keynesian literature (e.g., that on estimated DSGE models) often removes the state-contingency of the intercept of the Taylor rule. We return to this issue at the end of this section.

the same conjecture. This causes them to form different expectations about $K$ under $T'$ than under under $T$, which in turn leads them to follow different behavior under $T'$ than under $T$.

In short, the above result generalizes our earlier insights about the role of policy in regulating the error in the public's reasoning and its footprint on actual behavior. The upshot for optimality is given below:

**Proposition 5.** *(i) When $\lambda = 1$, the optimal rule is indeterminate and its slope can be anything: the first best is implemented if and only if the intercept satisfies $a = (1-b)\theta$, for an arbitrary (possibly $\theta$-contingent) slope $b$.*

*(ii) When instead $\lambda \neq 1$, the optimal rule is unique and its slope is inversely related to the GE feedback: the first best is implemented if and only if*

$$b = -\frac{\gamma}{1-\gamma} \qquad \text{and} \qquad a = \frac{1}{1-\gamma}\theta \qquad \forall\theta. \tag{105}$$

With rational expectations, optimality requires that $\tau = Y = \theta$, but there is a continuum of policy rules that induce this as an equilibrium. The analogue in the New Keynesian model (without a binding ZLB and markup shocks) is that the first best can be implemented with a continuum of Taylor rules, whose state-contingent intercept tracks the natural rate of interest and whose non-contingent slope with respect to inflation or the output gap is indeterminate.

With bounded rationality, this indeterminacy disappears. The slope of the optimal rule is now inversely tied to the strength of the GE feedback, in a way that smooths out our baseline main result (Theorem 2): as $\gamma$ increases, the policymaker gives more emphasis on anchoring the public's expectations of $Y$ rather than their expectations of $\tau$.

To see this more clearly, let us first re-express the optimal rule as follows:

$$\tau - \theta = -\frac{\gamma}{1-\gamma}(Y-\theta).$$

From this perspective, the optimal forward guidance consists of two components: the policymaker's assessment of the "fundamentals" and of the corresponding "rational" outcome (e.g., the central bank's forecast about the natural rate of output) in the form of $\theta$; and a commitment about how much she will tolerate a gap in terms of $\tau$ versus a gap in terms of $Y$. Building on our discussion of categorizing policy communication in Online Appendix XX, the former piece might be reflected in the policymaker's overall outlook (e.g., as reflected in the Summary of Economic Projections and dot plots), whereas the latter requires an explicit discussion of policy's contingency on different outcomes. The latter piece, our analysis shows, is *required* to achieve the first best—forecasts, by themselves, cannot do the job.

The next result expands on how policy optimally manages the expectations of interest rates and aggregate employment when both of them are distorted due to bounded rationality:

**Proposition 6.** *Let $f_\tau(\gamma) \equiv |\tau - \bar{\mathbb{E}}[\tau]|$ and $f_Y(\gamma) \equiv |Y - \bar{\mathbb{E}}[Y]|$ denote the aggregate errors in the expectations of, respectively, the instrument and the outcome, evaluated at the optimal policy, as functions of $\gamma$. Then, for all $\gamma \in (0,1)$: (i) $f_\tau(\gamma) > 0$ and $f_Y(\gamma) > 0$; (ii) $f'_\tau(\cdot) > 0$, and $f'_Y(\cdot) < 0$.*

The first property shows that, away from the extreme values of $\gamma$, the optimal policy is never completely clear: it does not eliminate the mistakes in either kind of expectations. This might be surprising given the previous discussion of "forecasts plus commitments"—in this interpretation, if the Fed had provided forecasts and a dot plot that described $\tau = Y = \theta$, its optimal forward guidance would induce the public to think something else.

The second property shows that policy shifts clarity from $\tau$ to $Y$ as the GE feedback increases. This makes even clearer how the optimal policy rule "smooths out" the main insight of Theorem 2 about switching the spotlight from instruments to targets, or from interest rates to unemployment.

Although the optimal rule does not eliminate the mistakes in people's reasoning about equilibrium, under the assumptions made thus far it insulates their actual behavior from such reasoning and recovers the policymaker's first best. The intuition is similar to the one developed for the extremes $\gamma = 0$ and $\gamma = 1$ in our baseline analysis, except that it now extends to interior $\gamma$: the optimal rule zeros out equilibrium reasoning about others' reactions.[9]

One should not take the present result too literally. First, there may be costs (left outside our analysis) for communicating sophisticated strategies. Second, if the policymaker is uncertain about the precise value of $\gamma$, the policymaker implements a "second-best approximation" of the policy described in Proposition 5: the first best is not attainable any more, but the optimal $b$ increases (in the sense of first-order stochastic dominance) in the policymaker's beliefs about $\gamma$. We expect a related result to hold in a multi-decision extension with limited policy instruments: the policymaker would try to eliminate the distortion in all decisions, but might succeed in doing so for only some.

But the basic logic is always the same. The optimal policy aims at minimizing the public's need to reason about the economy. And this is achieved by shifting emphasis from anchoring the public's expectations of $\tau$ ("interest rates") to anchoring the public's expectations of $Y$ ("unemployment") as the GE feedback increases.

### G.3 Optimal policy without commitment

We now expand on the role played by commitment. In the absence of commitment, the policymaker chooses $\tau$ in stage 2 so as to minimize $L$ subject to condition (1), taking $K$ as given. This gives the following optimality condition, which trades off the marginal effect of the policy on the two "gaps:"

$$(1-\chi)(\tau-\theta)+\chi(Y-\theta)(1-\alpha)=0;$$

Rearranging gives the follow ex post optimal reaction function:

$$\tau = \tfrac{1-\alpha\chi}{1-\chi}\theta - \tfrac{\chi(1-\alpha)}{1-\chi}Y. \tag{106}$$

The coefficients of this reaction function do not depend on the parameter $\gamma$, which determined how expectations of $\tau$ and $Y$ mapped to $K$, because $K$ itself is already determined. By contrast, the optimal policy

[9] In the language of best-response condition (7), the optimal rule ensures a zero slope on $\mathbb{E}_i[K]$. This implements the first best without correctly anchoring beliefs about *either* the instrument or target, but instead by making sure that the distortions in those beliefs are exactly irrelevant for choices.

rule with commitment, given in (105), depends on $\gamma$ precisely because it internalizes the effect it has on public reasoning and thereby on $K$.

Except for the knife-edge case in which $\frac{\chi(1-\alpha)}{1-\chi} = \frac{\gamma}{1-\gamma}$, the two rules are different. Commitment is *necessary* for implementing the first best—but only insofar as there is a distortion in equilibrium reasoning ($\lambda \neq 1$). When instead $\lambda = 1$, the following class of rules implements the first best under full commitment and rational expectations:

$$\left\{\tau = (1+b) - bY, \text{for any } b\right\}$$

But this class now includes the policy rule in (106), which means that commitment is *not* needed under rational expectations.

When the policymaker follows (106) and agents have rational expectations, agents *correctly* expect that all others will play $K = \theta$ and that this together with (106) will induce $\tau = Y = \theta$. But once $\lambda \neq 1$, this rule causes agents to believe that $K \neq \theta$, which in turn distorts their behavior away from the first best. The only way to fix this distortion is to make sure that agents find it optimal to play the first best action *regardless* of their beliefs of $K$ (or regardless of their higher-order beliefs), which in turn is possible if and only if the policymaker commits to the rule described in (106).

We summarize these lessons below.

**Proposition 7.** *In the absence of commitment, the unique optimal policy rule is given by*

$$\tau = \frac{1-\alpha\chi}{1-\chi}\theta - \frac{\chi(1-\alpha)}{1-\chi}Y. \tag{107}$$

*This implements the first best when $\lambda = 1$ but not when $\lambda \neq 1$ (except for the knife-edge case in which $\frac{\chi(1-\alpha)}{1-\chi} = \frac{\gamma}{1-\gamma}$ or $\theta = 0$).*

**Corollary 1.** *Commitment is valuable only when $\lambda \neq 1$ (away from rational expectations). In these circumstances, the rule described in (106) is ex ante optimal, even though ex post suboptimal, because and only because the commitment embedded in it helps regulate the distortion in equilibrium reasoning.*

Of course, the property that commitment is useless under rational expectations is special to our model. There is a large literature in macroeconomics studying time-inconsistency issues under rational expectations in the context of both flexible policy rules (Kydland and Prescott, 1977; Barro and Gordon, 1983) and simpler, instruments-versus-targets implementations (Atkeson, Chari and Kehoe, 2007; Halac and Yared, 2018). But by assuming away these familiar considerations, we have illustrated a *new* function that commitment can play away from rational expectations.

Circling back to our baseline analysis, this also makes clear the following point: what was crucial about the two kinds of forward guidance studied there is that they communicated a policy plan (i.e., a commitment to get something done) as opposed to information about fundamentals (e.g., the central bank's forecasts of future fundamentals).

### G.4 Proofs

**Proofs of Propositions 4 and 5**

For a policy rule to implement the first best, it is necessary that $T(\theta,\theta) = \theta$, which restricts $a$ and $b$ as follows:

$$a = (1-b)\theta.$$

We can henceforth focus on the class of policy rules that satisfy this restriction. This is a one-dimensional class indexed by $b$.

Solving (104) and (1) jointly for $\tau$ and $Y$, using $a = (1-b)\theta$, and substituting the solution into (2), we obtain the following game representation for the agents' behavior in stage 1:

$$k_i = (1-\delta)\theta + \delta \mathbb{E}_i[K] \tag{108}$$

where

$$\delta = \delta(b;\alpha,\gamma) \equiv \frac{\alpha(\gamma + b(1-\gamma))}{1-(1-\alpha)b}.$$

Note that $\delta \in (-1,+1)$ if and only if $b \in (\underline{b},+1)$, where $\underline{b} \equiv \frac{1+\alpha\gamma}{1-2\alpha+\alpha\gamma} < -1$. This explains the assumed bounds imposed on $b$: outside these bounds, "reasoning fails to converge" (this is the present analogue of the restriction $\delta_X \in (-1,+1)$, $X \in \{\tau, Y\}$, in the baseline analysis).

Consider now the case with rigid beliefs from higher-order doubts, as in Section 3.3. Iterating the best response (108) yields the unique equilibrium average action as

$$K = \sum_{h=1}^{\infty} \delta^{h-1}\lambda^{h-1}\zeta\theta = \frac{1-\delta}{1-\lambda\delta}\theta, \tag{109}$$

with $\delta = \delta(b;\alpha,\gamma)$ defined above. For this to coincide with the first best action, it is therefore necessary and sufficient that

$$\frac{1-\delta}{1-\lambda\delta} = 1$$

Clearly, this is automatically satisfied when $\lambda = 1$ (rational expectations), regardless the value of $\delta$, or equivalently of $b$. This verifies the indeterminacy of the optimal policy rule under rational expectations. When instead $\lambda < 1$, the above is satisfied if and only if $\delta = 0$, or equivalently $b = -\gamma/(1-\gamma)$. Along with $a = (1-b)\theta$, this completes the characterization of the unique policy rule that implements the first best once $\lambda < 1$.[10]

**Proof of Proposition 6**

Note that $\bar{\mathbb{E}}[K] = \mathbb{E}_i[K] = \lambda K$ by an argument essentially identical to the one supporting Lemma 2. Evaluated at the equilibrium under optimal policy, this is $\bar{\mathbb{E}}[K] = \lambda\theta$. The expected policy instrument and outcome, evaluated at the optimal rule, are

$$\bar{\mathbb{E}}[\tau] = \frac{\theta}{1-\gamma} - \frac{\gamma}{1-\gamma}\bar{\mathbb{E}}[Y]$$

$$\bar{\mathbb{E}}[Y] = (1-\alpha)\bar{\mathbb{E}}[\tau] + \alpha\lambda\theta$$

[10] Clearly, the argument extends to $\lambda > 1$, modulo the re-interpretation of the friction along the lines of Section 5.3.

Solving this system of equations gives

$$\bar{\mathbb{E}}[\tau] = \frac{1-\lambda\alpha\gamma}{1-\alpha\gamma}\cdot\theta$$
$$\bar{\mathbb{E}}[Y] = \frac{1-\alpha+\alpha\lambda(1-\gamma)}{1-\alpha\gamma}\cdot\theta$$

Recall also that the equilibrium satisfies $\tau = Y = \theta$.

The instrument forecast gap as a function of $\gamma$ is

$$f_\tau(\gamma) \equiv |\tau - \bar{\mathbb{E}}[\tau]| = \left|\frac{(1-\lambda)}{1-\alpha\gamma}\alpha\gamma\right|\cdot|\theta|$$

which satisfies $f_\tau(0) = 0$ and $f'_\tau(\gamma) > 0$ for $\gamma \in (0,1)$. This obviously implies $f_\tau(\gamma) > 0$ for $\gamma \in (0,1)$. Similarly the target forecast gap is

$$f_Y(\gamma) \equiv |Y - \bar{\mathbb{E}}[Y]| = \left|\frac{(1-\lambda)}{1-\alpha\gamma}\alpha(1-\gamma)\right|\cdot|\theta|$$

which satisfies $f_Y(1) = 0$ and $f'_Y(\gamma) < 0$ for $\gamma \in (0,1)$. This obviously implies $f_Y(\gamma) > 0$ for $\gamma \in (0,1)$.

Finally, going slightly beyond the original statement of the Proposition, it is simple to see that the instrument gap is always smaller for $\gamma < 0$: $f_\tau(\gamma) > f_Y(\gamma)$ since $|1-\gamma| > |\gamma|$. This generalizes our results for the optimality of instrument communication with negative GE effects.

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