Online Appendices for

Optimal Monetary Policy with Informational Frictions

George-Marios Angeletos
MIT and NBER

Jennifer La'O
Columbia and NBER

December 28, 2018

Abstract

This document is comprised of two parts. The first part develops the extension with rational inattention, or endogenous information acquisition. The second part collects the proofs for all the results of the paper, except those already contained in the In-print Appendix.

Online Appendix A: Rational Inattention

In main text we have treated φ, the distribution from which a firm's signal is drawn conditional on the state of Nature, as an exogenous object. We now allow each firm to choose her φ optimally, subject to some cost. One can think of this either as costly acquisition of information or as the firm's decision of how much attention to pay to the available data (Sims, 2003, 2010). The key finding is that the policies that are optimal in our baseline framework remain optimal in the extended framework. This means that these policies implement not only the optimal allocation taking the stochastic process of the signals as given but also the socially optimal choice of this process itself.

A1. Set up

We augment our baseline framework with a generalized form of rational inattention, as in Angeletos and Sastry (2018). For any i, let $\varphi_i \equiv \{\varphi_i^t\}_{t=0}^{\infty}$, where φ_i^t denotes the distribution from which ω_i^t is drawn conditional on s^t. Note that φ_i represents a complete description of how the information or the cognitive state of firm i evolves over time and over the different realizations of the underlying state of Nature. So far, φ_i was restricted to be the same across all i and was exogenously fixed. We now let each each firm choose her own φ_i, at the beginning of time, from some set Φ, subject to a cost represented by a function $\kappa : \Phi \to \mathbb{R}_+$.

To simplify the exposition, we shut down capital[1] and assume that the aforementioned cost is in terms of utility or "cognitive effort."[2] As will become evident, the arguments we develop in this section do not hinge on these simplifications. We also bypass the technical issue of the existence of an equilibrium or existence of a Ramsey optimum by requiring that all maximization and fixed-point problems defined henceforth admit a solution. We finally impose that for every $\varphi \in \Phi$, the firm learns the realization of an extrinsic random variable that is independent of s^t for all t, is i.i.d. across firms, and is drawn from a uniform distribution over $[0, 1]$. This guarantees that it is without loss of generality to concentrate on equilibria and optima in which all firms end up choosing the same distribution and the same strategies.[3]

No restriction of economic substance is imposed on the set Φ nor on the function κ. For instance, there is no need to order the elements of Φ in terms of more or less information or to model κ in terms of relative (Shannon) entropy or Kullback-Leibler divergence. There is also no need to take a stand on whether firms can recall their past signals effortlessly or suffer from partial amnesia, nor specify whether the cost κ is separable across time or signals. We can thus nest, *inter alia*, the specifications considered in Sims (2003), Myatt and Wallace (2012), Paciello and Wiederholt (2014), and Pavan (2016).

A2. Equilibria, Implementability, and Optimality

We now proceed to define and characterize the equilibria and the Ramsey optimum of the economy with endogenous information (or endogenous cognition). To simplify, we concentrate on the case with flexible prices; the case with sticky prices is analogous.

Consider the problem faced by an arbitrary firm i. This problem can be split into two subproblems: the "outer" problem of choosing a φ_i; and the "inner" problem of choosing the optimal input and output strategies for given φ. Recall that any given triplet (ξ, ρ, θ) contains a unique collection $\{Y_t(\cdot), C_t(\cdot), W(\cdot), \theta_t(\cdot)\}_t^\infty$, that is, it is associated with a unique stochastic process for aggregate output, aggregate consumption, the wage rate, and taxes. With this in mind, we can represent the

[1] That is, we set $k_{it} = 1$ and $x_{it} = 0$ for all i, all t, and all realizations of uncertainty.

[2] This assumption guarantees that, whenever $\varphi_i = \varphi$ for all i and for some φ, the definition and the characterization of the sets of feasible, flexible-price, sticky-price, and optimal allocations *conditional* on φ remain exactly the same as in our baseline model. If, instead, we had specified the cost in terms of final good (or, say, labor), we would have to adjust appropriately all the earlier analysis: the cost would show up in firm profits and in the resource constraint.

[3] This is because any asymmetric equilibrium (or optimum) can be replicated by a symmetric one that let's each firm condition her production choices on the aforementioned extrinsic variable.

firm's inner problem as follows:

$$\Pi\left(\varphi;\xi,\rho,\theta\right) \;=\; \max_{y,\ell,h}\sum_t\sum_{\omega,s}\beta^t\mathcal{M}(s^t)\pi\left(\omega^t,s^t\right)\varphi^t\left(\omega^t\middle|\,s^t\right)\mu^t(s^t) \qquad (58)$$

$$s.t.\quad y\left(\omega^t,s\right)=A(s^t)F\left(h\left(\omega^t\right),\ell\left(\omega^t,s^t\right)\right),$$

where $\mathcal{M}(s^t)=\frac{U_c(C(s^t))}{1+\tau^c(s^t)}$ and

$$\pi\left(\omega^t,s^t\right)\equiv\left(1-\tau^r\left(s^t\right)\right)\left(\frac{y(\omega^t,s)}{Y(s)}\right)^{-\frac{1}{\rho}}y\left(\omega^t,s^t\right)-h\left(\omega^t\right)-W\left(s^t\right)\ell\left(\omega^t,s^t\right).$$

We can then represent the solution to the outer problem as follows:

$$\varphi\in\Gamma\left(\xi,\rho,\theta\right)\equiv\arg\max_{\phi}\left\{\Pi\left(\phi;\xi,\rho,\theta\right)-\kappa\left(\phi\right)\right\} \qquad (59)$$

To interpret these representations, note that the first problem takes φ as given but lets the firm optimize her input and output choices. The second problem then describes the optimal choice of φ.

The above determines the firm's optimal choice of φ for *any* triplet (ξ,ρ,θ). But not every such triplet is relevant: (ξ,ρ,θ) can be part of an equilibrium of the "overall game" in which firms choose both their information structures and their input/output strategies only if it is also an equilibrium of the "continuation game" that obtains once the firms' information structures have been fixed. We therefore define an equilibrium as follows.

Definition 3. *In the economy with endogenous information, a flexible-price equilibrium is a collection* $(\varphi,\xi,\rho,\theta)$ *such that: (i)* $(\xi,\rho,\theta)\in\mathcal{E}^{flex}(\varphi)$*; and (ii)* $\varphi\in\Gamma\left(\xi,\rho,\theta\right)$*.*

An equilibrium now contains not only the triplet (ξ,ρ,θ) that describes the allocation (or the firm strategies), the price system, and the government policy, but also the information structure φ. Part (i) requires that, taking φ as given, the triplet (ξ,ρ,θ) constitutes an equilibrium in the sense of Definition 2. Part (ii) on the other hand requires that φ is itself a solution to the optimal information/cognition problem that the typical firm faces when the rest of the economy is described by (ξ,ρ,θ). An equilibrium of the economy with endogenous information is therefore a fixed point between the mapping $\mathcal{E}$, which was studied earlier (see especially Proposition 1), and the mapping Γ, which is defined by condition (59) above.

Consider next the planner's problem. By manipulating the available policy instruments, the planner can now influence not only the equilibrium allocation in the "continuation game" that obtains once φ is fixed but also the optimal choice of φ in the first place. To understand how this modifies the planner's problem relative to the one studied earlier on, pick an arbitrary $\hat{\varphi}$ and let

$\hat{\xi}$ be the allocation that is optimal in the sense described in Section 5 (that is, when treating $\hat{\varphi}$ as exogenous). Relative to this benchmark, the planner's problem has been eased by the introduction of the option to choose a $\varphi \neq \hat{\varphi}$. However, the planner's problem has also been worsened by the introduction of an additional implementability constraint: namely the requirement that the pair (φ, ξ) must be consistent with the individually optimal information/cognition problem the firms.

To formalize this point, we first adapt the notion of implementability as follows.

Definition 4. *A pair (φ, ξ) of an information or cognition structure and an allocation is implementable (under flexible prices) if there exists a policy θ and a price system ρ such that the collection $(\varphi, \xi, \rho, \theta)$ is an equilibrium in the sense of Definition 3.*

We then state the following result, which can be proved following similar steps as in the proof of Proposition 1.

Proposition 10. *A pair (φ, ξ) is implementable if and only if the following properties hold.*

(i) The following constraint is satisfied at the aggregate level:

$$\sum_{t,s^t} \beta^t \mu\left(s^t\right)\left[U_c\left(s^t\right) C\left(s^t\right) + U_\ell\left(s^t\right) L\left(s^t\right)\right] = 0; \tag{60}$$

(ii) There exist wedges ψ=$(\psi^c, \psi^\ell \psi^r) : \mathcal{S}^{3t} \to \mathbb{R}$ such that the following conditions hold at the firm level:

$$\psi^r\left(s^t\right) \tfrac{\rho-1}{\rho} MP_\ell\left(\omega^t, s^t\right) - \psi^\ell\left(s^t\right) \quad = \quad 0 \quad \forall\ \omega_i^t, s^t \tag{61}$$

$$\sum_{s^t} \left\{\psi^r\left(s^t\right) \tfrac{\rho-1}{\rho} MP_h\left(\omega^t, s^t\right) - \psi^c\left(s^t\right)\right\} \varphi\left(s^t \middle| \omega^t\right) \quad = \quad 0 \quad \forall\ \omega_i^t \tag{62}$$

$$\varphi \ \in \ \arg\max_{\phi} \left\{\tilde{\Pi}(\phi; \xi, \psi) - \kappa(\varphi)\right\} \tag{63}$$

where

$$\tilde{\Pi}(\phi; \xi, \psi) \equiv \tag{64}$$

$$\max_{y,\ell,h} \sum_t \sum_{\omega,s} \beta^t \left\{\psi^r\left(s^t\right)\left(\tfrac{y\left(\omega^t,s\right)}{Y(s)}\right)^{-\frac{1}{\rho}} y\left(\omega_i^t, s^t\right) - \psi^c\left(s^t\right) h\left(\omega^t\right) + \psi^\ell(s^t)\ell\left(\omega^t, s^t\right)\right\} \phi\left(\omega^t \middle| s^t\right) \mu^t(s^t)$$

$$s.t. \quad y\left(\omega_i^t, s\right) = A(s^t) F\left(h\left(\omega_i^t\right), \ell\left(\omega_i^t, s^t\right)\right),$$

Comparing this result to Proposition 1 makes clear that the option to choose φ adds an extra degree of freedom to the planner's problem, whereas condition (63) adds an extra implementability constraint. This constraint reflects the lack of a certain class of policy instruments, namely instruments that would permit the planner to manipulate the equilibrium value of φ while holding ξ

constant. Think, in particular, of a direct Pigouvian tax or subsidy on the firm's acquisition of information or cognition effort. If the planner had access to such an instrument, condition (63) would drop out of Proposition 10, and the planner would be free to control the equilibrium allocation ξ without having to worry how this affects the firms' choice of φ. Now, by contrast, the planner must take into account the feedback effect from the equilibrium value of ξ to that of φ. In other words, the planner faces a potential trade off between influencing the *use* of information and influencing the *collection* of information.[4]

With slight abuse of notation, let $\mathcal{X}^{flex}$ denote the set of the pairs (φ, ξ) that are implementable in the sense of Definition 3. The planner's problem is to maximize welfare (defined as the ex ante utility of the representative agent, net of the cost κ) over the set $\mathcal{X}^{flex}$.

Definition 5. *The Ramsey optimum is given by a pair (φ, ξ) that maximizes welfare over $\mathcal{X}^{flex}$.*

We characterize the Ramsey optimum again by adapting the methods developed in Section 5 to the endogeneity of φ. In particular, we let $\mathcal{X}^{relax}$ denote the set of the pairs (φ, ξ) that satisfy *only* condition (60) and note that, trivially, $\mathcal{X}^{flex} \subset \mathcal{X}^{relax}$. We then consider the following object.

Definition 6. *The relaxed optimum is given by a pair (φ^*, ξ^*) that maximizes welfare over $\mathcal{X}^{relax}$.*

The next lemma provides two necessary conditions for a pair (φ^*, ξ^*) to a be relaxed optimum.

Lemma 4. *If (φ^*, ξ^*) is a relaxed optimum, the following two properties must hold.*

(i) taking φ^* as given, ξ^* is optimal in the sense of Section 5; and

(ii) taking ξ^* as given, φ^* satisfies

$$\varphi^* \in \arg\max_{\varphi} \left\{ \mathcal{Z}(\varphi; \xi^*) - \kappa(\varphi) \right\}, \tag{65}$$

where

$$\mathcal{Z}(\varphi; \xi) \equiv \max_{y,\ell,h} \sum_t \sum_{\omega^t, s^t} \beta^t \left[\tilde{U}_c(s^t) \left(\int_0^{y(\omega,s)} \left(\tfrac{z}{Y(s)} \right)^{-\frac{1}{\rho}} dz - h(\omega^t) \right) + \tilde{U}_\ell(s^t)\, \ell(\omega_i^t, s^t) \right] \varphi(\omega^t | s^t)\, \mu^t(s^t).$$

Part (i) states that ξ^* is optimal whether the planner takes into account the endogeneity of φ or treats φ as fixed at φ^*. This is trivially true because the relaxed problem has dropped the implementability constraint (63): the aforementioned trade off between the collection and the use of information has been removed by assumption. To understand part (ii), note that, because each firm is infinitesimal, the planner can vary *both* a firm's production choices *and* her information structure without affecting the aggregate outcomes. It follows that the contribution of any firm

[4] We have qualified the trade off as a *potential* one because it remains to be seen whether this trade off is relevant for understanding the solution to the planner's problem.

to social welfare is captured by $\mathcal{Z}(\varphi;\xi)$; this measures the social surplus generated by the optimal production choices of the firm, when her information structure is fixed at φ. By the same token, the socially optimal choice of φ maximizes the aforementioned surplus net of the information cost, which is what part (ii) states.

We next prove that any pair (φ^*, ξ^*) that satisfies the aforementioned two properties belongs to the set $\mathcal{X}^{flex}$. This guarantees that the solution to the relaxed problem coincides with the solution to the actual Ramsey problem, a property that mirrors the one encountered in Section 5. We furthermore prove that the same taxes that are optimal in the baseline economy in which φ is exogenously fixed at φ^* permit the planner to implement the pair (φ^*, ξ^*) as an equilibrium of the (extended) economy in which φ is endogenously chosen.

Proposition 11. *Let (φ^*, ξ^*) be a relaxed optimum. This can be implemented as part of a flexible-price equilibrium (in the sense of Definition 3) with the same taxes as in Theorem 1.*

This follows directly from Lemma 4 together with the following argument. Let θ^* be the taxes identified in Theorem 1 and let ρ^* be the associated price system. From our earlier analysis, we know that $(\xi^*, \theta^*, \rho^*)$ is an equilibrium of the (restricted) economy in which the φ is exogenously fixed at φ^*. What remains to show is that, when the firm faces $(\xi^*, \theta^*, \rho^*)$, she finds it individually optimal to pick $\varphi = \varphi^*$.

To establish that this is indeed true, consider the firm's market valuation, as given in condition (58). At $(\xi, \theta, \rho) = (\xi^*, \theta^*, \rho^*)$, this reduces to the following:

$$\Pi(\varphi;\xi^*,\theta^*,\rho^*) = \\ \max_{y,\ell,h} \sum_t \sum_{\omega^t,s^t} \beta^t \left[\tilde{U}_c(s^t)\left(\tfrac{\rho}{\rho-1} Y^*(s^t)^{\frac{1}{\rho}} y(\omega^t,s^t)^{1-\frac{1}{\rho}} - h(\omega^t)\right) + \tilde{U}_\ell(s^t)\,\ell(\omega^t,s^t)\right]\varphi(\omega^t,s^t).$$

Next, evaluating the innermost integral of (65), the social surplus generated by firm i can be expressed as follows:

$$\mathcal{Z}(\varphi;\xi^*) = \max_{y,\ell,h} \sum_t \sum_{\omega^t,s^t} \beta^t \left[\tilde{U}_c(s^t)\left(\tfrac{\rho}{\rho-1} Y^*(s^t)^{\frac{1}{\rho}} y(\omega^t,s^t)^{1-\frac{1}{\rho}} - h(\omega^t)\right) + \tilde{U}_\ell(s^t)\,\ell(\omega^t,s^t)\right]\varphi(\omega^t,s^t).$$

It follows that $\Pi(\varphi;\xi^*,\theta^*,\rho^*) = \mathcal{Z}(\varphi;\xi^*)$ for *every* φ. Combining this property with part (ii) of Lemma 4, we conclude that

$$\varphi^* \in \Gamma(\xi^*,\theta^*,\rho^*),$$

which verifies the claim that φ^* is optimal in the eyes of the typical firm and completes the proof of Proposition 11.

This result echoes the second welfare theorem of Angeletos and Sastry (2018), the key differences here being the presence of monopoly power and the absence of lump sum taxation. To understand this result, it is useful to build an analogy. Consider a neoclassical growth model in which a monopolist can choose her production technology (e.g., as in Romer, 1990) and ask the following question: can a uniform subsidy on firm sales induce both the efficient level of output for given technology and the efficient choice of technology? The answer to this question is positive as long as one maintains the usual Dixit-Stiglitz specification for intermediate good demand and abstract from any knowledge spillovers. These conditions suffice for the aforementioned subsidy to equate both the *marginal* revenue of the firm with the marginal utility of the consumer and the *total* profit made from any given technology with the corresponding social surplus.[5] Our result can thus be understood as a variant of this observation: the choice of an information structure in our context is the analogue of the choice of technology in the growth context, the Dixit-Stiglitz specification has been maintained, and spillovers are ruled out, i.e., the cost κ faced by each firm is independent of the choices of other firms.

It is straightforward to extend the above arguments to the more general case that allows for capital accumulation and for nominal rigidity (in the sense of Property 2). We conclude that the policy lessons provided in the earlier sections of our paper are robust to endogenous acquisition of information or rational inattention.

Theorem 3. *Theorems 1 and 2 continue to hold in the extended framework described in this section, despite the influence that the policy instruments can exert on the information acquisition, or the cognitive effort, of the firms and thereby on the severity of the considered friction.*

Online Appendix B: Proofs

In the first part of this appendix, we first state and solve for the household and firms' problems, and then prove two auxiliary lemmas which offer a complete characterization of the sets of sticky- and flexible-price equilibria. In the second part, we then proceed with the proofs for all the results of the paper, except those already included in the In-print Appendix. Throughout, we ease the notation by dropping the subscript t from the functions $C_t(.), L_t(.)$, etc, except for few occasions in which it is useful to make explicit the dependence on t.

B1. Preliminary Analysis

We start by characterizing the optimal behavior of the representative household and of the typical monopolist. Consider first the household. The statement of her problem is standard.

[5] Without the aforementioned conditions, the planner may need a *non-linear* subsidy, as in a two-part tariff, in order to hit both goals.

Household's Problem. *The household chooses* $\{C(.), L(.), K(.), B(.), D(.)\}$ *so as to maximize expected utility,*

$$\mathcal{W} = \sum_{t=0}^{\infty} \sum_{s^t} \beta^t \mu\left(s^t\right) \left[U\left(C\left(s^t\right), L\left(s^t\right), s^t\right)\right],$$

subject to her budget constraint,

$$\begin{aligned}\left(1+\tau^c\left(s^t\right)\right) C\left(s^t\right) + X(s^t) + \frac{1}{P\left(s^t\right)}\left\{B\left(s^t\right) + \sum_{s^{t+1}} Q(s^{t+1}) D(s^{t+1})\right\} = \left(1-\tau^\ell\left(s^t\right)\right) w\left(s^t\right) L\left(s^t\right) + \\ + \left(1-\tau^k\left(s^t\right)\right) r\left(s^t\right) K\left(s^{t-1}\right) + \frac{1}{P\left(s^t\right)}\left\{\left(1+R\left(s^{t-1}\right)\right) B\left(s^{t-1}\right) + D(s^t)\right\} \quad \forall t, s^t,\end{aligned}$$

and the law of motion for capital,

$$K\left(s^t\right) = (1-\delta) K\left(s^{t-1}\right) + X\left(s^t\right) \quad \forall t, s^t.$$

Consider next the typical monopolistic firm. Her (ex ante) valuation is given by

$$\mathcal{V} \equiv \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \mathcal{M}(s^t) \frac{\Pi(\omega_i^t, s^t)}{P(s^t)}\right] = \sum_{t=0}^{\infty} \sum_{\omega_i^t, s^t} \left\{\beta^t \mathcal{M}(s^t) \frac{\Pi(\omega_i^t, s^t)}{P(s^t)} \varphi\left(\omega_i^t, s^t\right)\right\},$$

where $\mathcal{M}(s^t) \equiv \frac{U_c(s^t)}{1+\tau^c(s^t)}$ is the "pricing kernel," $U_c\left(s^t\right)$ is a shortcut for $\frac{\partial}{\partial c} U\left(C\left(s^t\right), L\left(s^t\right), s^t\right)$, and

$$\Pi(\omega_i^t, s^t) \equiv \left(1-\tau^r\left(s^t\right)\right) \frac{p\left(\omega_i^t\right)}{P\left(s^t\right)} y\left(\omega_i^t, s^t\right) - h\left(\omega_i^t\right) - w\left(s^t\right) \ell\left(\omega_i^t, s^t\right) - r(s^t) k(\omega_i^{t-1})$$

is the firm's real profit net of the revenue tax. The demand faced by the monopolist is given by

$$y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho} Y\left(s^t\right). \tag{66}$$

We may thus express the monopolist's problem as follows.

Monopolist's Problem. *The typical monopolist chooses* $\{p, k, h, \ell, y\}$ *so as to maximize its valuation,*

$$\sum_t \sum_{\omega_i^t, s^t} \left\{\beta^t \mathcal{M}(s^t) \left[\left(1-\tau^r\left(s^t\right)\right) \frac{p(\omega_i^t)}{P(s^t)} y\left(\omega_i^t, s^t\right) - h\left(\omega_i^t\right) - w\left(s^t\right) \ell\left(\omega_i^t, s^t\right) - r(s^t) k(\omega_i^t)\right] \varphi\left(\omega_i^t, s^t\right)\right\},$$

subject to technology,

$$y\left(\omega_i^t, s^t\right) = A\left(s^t\right) F\left(k_i\left(\omega_i^t\right), h_i\left(\omega_i^t\right), \ell_i\left(\omega_i^t, s^t\right)\right) \quad \forall t, s^t, \omega_i^t,$$

and the demand for its product,

$$y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho} Y\left(s^t\right) \quad \forall t, s^t, \omega_i^t.$$

Finally, since the cross-sectional distribution of the signal in period t and state s^t is given by $\varphi(.|s^t)$, the following properties are self-evident: aggregate output is given by

$$Y\left(s^t\right) = \left[\sum_{\omega\in\Omega^t}\left(y\left(\omega, s^t\right)\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}} \quad \forall t, s^t; \tag{67}$$

the price level (the price of the final good) is given by

$$P(s^t) = \left[\sum_{\omega\in\Omega^t}\left(p\left(\omega\right)\right)^{\rho-1}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{\rho-1}} \quad \forall t, s^t; \tag{68}$$

the market for the final good clears if and only if

$$C\left(s^t\right) + X(s^t) + G\left(s^t\right) + \sum_{\omega\in\Omega^t} h\left(\omega\right)\varphi\left(\omega|s^t\right) = Y\left(s^t\right) \quad \forall t, s^t; \tag{69}$$

the market for labor clears if and only if

$$\sum_{\omega\in\Omega^t}\ell\left(\omega\right)\varphi\left(\omega|s^t\right) = L\left(s^t\right) \quad \forall t, s^t; \tag{70}$$

and the market for capital clears if and only if

$$\sum_{\omega\in\Omega^t}k\left(\omega\right)\varphi\left(\omega|s^t\right) = K\left(s^t\right) \quad \forall t, s^t. \tag{71}$$

Lemma 3 stated in the in-print appendix provides a complete characterization of the set flexible-price equilibria. We now state a similar auxiliary lemma which offers a complete characterization of the set sticky-price equilibria We follow this with the proofs of both lemmas.

Lemma 5. *An allocation ξ, a policy θ, and a price system ϱ are part of a sticky-price equilibrium if and only if the following four properties hold.*

(i) The following household optimality conditions are satisfied:

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)} = \beta\left[\left.\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}\left(1+R\left(s^t\right)\right)\right| s^t\right] \tag{72}$$

$$-U_\ell\left(s^t\right) = U_c\left(s^t\right)\frac{\left(1-\tau^\ell\left(s^t\right)\right)}{\left(1+\tau^c\left(s^t\right)\right)}w\left(s^t\right) \tag{73}$$

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)} = \beta\left[\left.\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)}\left(1+\tilde{r}\left(s^{t+1}\right)-\delta\right)\right| s^t\right] \tag{74}$$

$$Q\left(s^{t+1}\right) = \beta\frac{\mu\left(s^{t+1}\right)}{\mu\left(s^t\right)}\frac{U_c\left(s^{t+1}\right)}{U_c\left(s^t\right)}\frac{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)} \tag{75}$$

where

$$\tilde{r}\left(s^t\right) = \left(1-\tau^k\left(s^t\right)\right)r\left(s^t\right) \tag{76}$$

is the net-of-taxes return on savings.

(ii) The following firm optimality conditions are satisfied:

$$\begin{aligned}
\lambda\left(\omega_i^t, s^t\right) A(s^t) f_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) &= 0 \quad (77)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left(\lambda(\omega_i^t, s^t) A(s^t) f_h\left(\omega_i^t, s^t\right) - 1\right)\middle|\, \omega_i^t\right] &= 0 \quad (78)\\
\mathbb{E}\left[\mathcal{M}(s^t)\left(\lambda(\omega_i^t, s^t) A(s^t) f_k\left(\omega_i^t, s^t\right) - r\left(s^t\right)\right)\middle|\, \omega_i^t\right] &= 0 \quad (79)\\
\mathbb{E}\left[\mathcal{M}(s^t) y\left(\omega_i^t, s^t\right)\left\{\left(1-\tau^r(s^t)\right)\left(\tfrac{\rho-1}{\rho}\right)\frac{p(\omega_i^t)}{P(s^t)} - \lambda\left(\omega_i^t, s^t\right)\right\}\middle|\, \omega_i^t\right] &= 0 \quad (80)
\end{aligned}$$

with $\mathcal{M}(s^t) \equiv \frac{U_c(s^t)}{1+\tau^c(s^t)}$*, along with the intermediate-good demand condition, namely,*

$$y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho} Y\left(s^t\right). \quad (81)$$

.

(iii) The household's and the government's budget constraints are satisfied.

(iv) All markets clear, namely, conditions (69), (70), and (71) are satisfied.

Proof of Lemma 5. We first derive the household's optimality conditions. Following this we derive the firm's optimality conditions.

Household. Consider the Household's problem stated above. Let $\Lambda\left(s^t\right)$ be the Lagrange multiplier on the Household's budget constraint in history s^t. The Lagrangian for the household's problem is given by

$$\begin{aligned}
L &= \sum_{t=0}^{\infty}\sum_{s^t}\beta^t\mu\left(s^t\right)\left[U\left(C\left(s^t\right), L\left(s^t\right), s^t\right)\right]\\
&\quad -\sum_{t=0}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\begin{array}{c}\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)C\left(s^t\right) + B\left(s^t\right) + \sum_{s^{t+1}} Q(s^{t+1})D(s^{t+1})\\ +P\left(s^t\right)\left(K\left(s^t\right) - (1-\delta)K\left(s^{t-1}\right)\right)\end{array}\right]\\
&\quad +\sum_{t=0}^{\infty}\sum_{s^t}\Lambda\left(s^t\right)\left[\begin{array}{c}\left(1-\tau^\ell\left(s^t\right)\right)P\left(s^t\right)w\left(s^t\right)L\left(s^t\right) + \left(1-\tau^k\left(s^t\right)\right)P\left(s^t\right)r\left(s^t\right)K\left(s^{t-1}\right)\\ +\left(1+R\left(s^{t-1}\right)\right)B\left(s^{t-1}\right) + D(s^t)\end{array}\right]
\end{aligned}$$

The household's first order conditions for consumption, labor, bonds, and state-contingent securities are given by

$$\begin{aligned}
\beta^t\mu\left(s^t\right)U_c\left(s^t\right) - \Lambda\left(s^t\right)\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right) &= 0, \text{ for all } s^t \quad (82)\\
\beta^t\mu\left(s^t\right)U_\ell\left(s^t\right) + \Lambda\left(s^t\right)\left(1-\tau^\ell\left(s^t\right)\right)P\left(s^t\right)w\left(s^t\right) &= 0, \text{ for all } s^t \quad (83)\\
-\Lambda\left(s^t\right) + \sum_{s^{t+1}|s^t}\Lambda\left(s^{t+1}\right)\left(1+R\left(s^t\right)\right) &= 0, \text{ for all } s^t \quad (84)\\
-Q\left(s^{t+1}\right)\Lambda\left(s^t\right) + \Lambda\left(s^{t+1}\right) &= 0, \text{ for all } s^{t+1} \quad (85)
\end{aligned}$$

By combining (82) and (84) we derive the household's Euler equation,

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)} = \beta \sum_{s^{t+1}|s^t} \mu\left(s^{t+1}|s^t\right) \frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)} \left(1+R\left(s^t\right)\right). \tag{86}$$

And by combining (82) and (83) we derive the household's intratemporal condition,

$$-U_\ell\left(s^t\right) = U_c\left(s^t\right) \frac{\left(1-\tau^\ell\left(s^t\right)\right)}{\left(1+\tau^c\left(s^t\right)\right)} w\left(s^t\right)$$

Thus we obtain optimality conditions for the household stated in (72) and (73). From (85), we have that the state-contingent price satisfies:

$$Q\left(s^{t+1}\right) = \frac{\Lambda\left(s^{t+1}\right)}{\Lambda\left(s^t\right)} = \beta \frac{\mu\left(s^{t+1}\right)}{\mu\left(s^t\right)} \frac{U_c\left(s^{t+1}\right)}{U_c\left(s^t\right)} \frac{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}.$$

Next, the household's optimality condition for capital is given by

$$-\Lambda\left(s^t\right)P\left(s^t\right) + \sum_{s^{t+1}|s^t} \left[\Lambda\left(s^{t+1}\right)\left(1-\tau^k\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)r\left(s^{t+1}\right) + \Lambda\left(s^{t+1}\right)P\left(s^{t+1}\right)(1-\delta)\right] = 0$$

which may be rewritten as

$$\Lambda\left(s^t\right)P\left(s^t\right) = \sum_{s^{t+1}|s^t} \Lambda\left(s^{t+1}\right)P\left(s^{t+1}\right)\left[1+\left(1-\tau^k\left(s^{t+1}\right)\right)r\left(s^{t+1}\right)-\delta\right] \tag{87}$$

Using (82) to replace $\Lambda\left(s^t\right)P\left(s^t\right)$ and $\Lambda\left(s^{t+1}\right)P\left(s^{t+1}\right)$ in the above equation, we get

$$\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)} = \beta \sum_{s^{t+1}|s^t} \mu\left(s^{t+1}|s^t\right) \frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)} \left[1+\left(1-\tau^k\left(s^{t+1}\right)\right)r\left(s^{t+1}\right)-\delta\right].$$

Thus we obtain the household optimality condition stated in (74).

Firms. Turning attention now to the firms, we first consider the final-good retail sector. Its optimal input choices satisfy

$$y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{-\rho} Y\left(s^t\right). \tag{88}$$

This gives the demand function faced by the typical intermediate-good monopolistic firm.

Consider now the monopolist's problem stated above. The demand function (88) implies that we may write monopolistic firm's real revenue as

$$\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)} y\left(\omega_i^t, s^t\right) = \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{1-\rho} Y\left(s^t\right).$$

We can thus state the monopolistic firm's pricing and production problem as follows:

$$\max \mathbb{E}\left[\sum_{t=0}^{\infty} \beta^t \mathcal{M}(s^t) \left\{ \left(1-\tau^r\left(s^t\right)\right) \left(\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)}\right)^{1-\rho} Y\left(s^t\right) - h\left(\omega_i^t\right) - w\left(s^t\right)\ell\left(\omega_i^t, s^t\right) - r(s^t)k(\omega_i^{t-1}) \right\} \middle| \omega_i^t \right]$$

subject to

$$A\left(s^{t}\right)F\left(k_{i}\left(\omega_{i}^{t}\right),h_{i}\left(\omega_{i}^{t}\right),\ell_{i}\left(\omega_{i}^{t},s^{t}\right)\right)=\left(\frac{p\left(\omega_{i}^{t}\right)}{P\left(s^{t}\right)}\right)^{-\rho}Y(s^{t})\quad\forall\omega_{i}^{t},s^{t}$$

The first constraint is simply the law of motion for capital. The second constraint, which follows from combining condition (88) with the production function, dictates how labor adjusts so as to meet the realized demand, whatever that might be.

Let $\beta^{t}\mathcal{M}(s^{t})\lambda(\omega_{i}^{t},s^{t})$ be the Lagrange multiplier on the second constraint. The first order conditions with respect to labor, intermediate inputs, and investment are given by the following:

$$\lambda\left(\omega_{i}^{t},s^{t}\right)A(s^{t})f_{\ell}\left(\omega_{i}^{t},s^{t}\right)-w\left(s^{t}\right)=0 \tag{89}$$

$$\mathbb{E}\left[\mathcal{M}(s^{t})\left(\lambda(\omega_{i}^{t},s^{t})A(s^{t})f_{h}\left(\omega_{i}^{t},s^{t}\right)-1\right)\middle|\,\omega_{i}^{t}\right]=0 \tag{90}$$

$$\mathbb{E}\left[\mathcal{M}(s^{t})\left(\lambda(\omega_{i}^{t},s^{t})A(s^{t})f_{k}\left(\omega_{i}^{t},s^{t}\right)-r\left(s^{t}\right)\right)\middle|\,\omega_{i}^{t}\right]=0 \tag{91}$$

The first-order condition with respect to the price $p\left(\omega_{i}^{t}\right)$, on the other hand, can be stated as follows:

$$\mathbb{E}\left[\mathcal{M}(s^{t})y\left(\omega_{i}^{t},s^{t}\right)\left\{\left(1-\tau^{r}(s^{t})\right)\left(\tfrac{\rho-1}{\rho}\right)\frac{p(\omega_{i}^{t})}{P(s^{t})}-\lambda\left(\omega_{i}^{t},s^{t}\right)\right\}\middle|\,\omega_{i}^{t}\right]=0 \tag{92}$$

Thus we obtain optimality conditions for the firm stated in (77)-(79) and (80). **QED.**

Proof of Lemma 3. The household's problem is the same as in the sticky-price equilibrium, and hence follows the proof of Lemma 5. On the firm's side, the demand for intermediate goods from the final-good retail sector continues to satisfy (81) but with state-contingent prices $p\left(\omega_{i}^{t},s^{t}\right)$ instead of $p\left(\omega_{i}^{t}\right)$.

Thus, the only difference between the sticky-price and flexible-price equilibria are the intermediate good firms' problem. We may state the monopolistic firm's production problem as follows:

$$\max\mathbb{E}\left[\sum_{t=0}^{\infty}\beta^{t}\mathcal{M}(s^{t})\left\{\left(1-\tau^{r}\left(s^{t}\right)\right)y\left(\omega_{i}^{t},s^{t}\right)^{\frac{\rho-1}{\rho}}Y\left(s^{t}\right)^{\frac{1}{\rho}}-h\left(\omega_{i}^{t}\right)-w\left(s^{t}\right)\ell\left(\omega_{i}^{t},s^{t}\right)-r(s^{t})k(\omega_{i}^{t-1})\right\}\middle|\,\omega_{i}^{t}\right]$$

subject to the production function

$$y\left(\omega_{i}^{t},s^{t}\right)=A\left(s^{t}\right)F\left(k_{i}\left(\omega_{i}^{t}\right),h_{i}\left(\omega_{i}^{t}\right),\ell_{i}\left(\omega_{i}^{t},s^{t}\right)\right)$$

The FOCs of this problem are given by

$$\left(1-\tau^{r}(s^{t})\right)\frac{\rho-1}{\rho}\left(\frac{y\left(\omega_{i}^{t},s^{t}\right)}{Y\left(s^{t}\right)}\right)^{-\frac{1}{\rho}}A(s^{t})f_{\ell}\left(\omega_{i}^{t},s^{t}\right)-w\left(s^{t}\right) = 0 \quad (93)$$

$$\mathbb{E}\left[\mathcal{M}(s^{t})\left(\left(1-\tau^{r}(s^{t})\right)\frac{\rho-1}{\rho}\left(\frac{y\left(\omega_{i}^{t},s^{t}\right)}{Y\left(s^{t}\right)}\right)^{-\frac{1}{\rho}}A(s^{t})f_{h}\left(\omega_{i}^{t},s^{t}\right)-1\right)\middle|\omega_{i}^{t}\right] = 0 \quad (94)$$

$$\mathbb{E}\left[\mathcal{M}(s^{t})\left(\left(1-\tau^{r}(s^{t})\right)\frac{\rho-1}{\rho}\left(\frac{y\left(\omega_{i}^{t},s^{t}\right)}{Y\left(s^{t}\right)}\right)^{-\frac{1}{\rho}}A(s^{t})f_{k}\left(\omega_{i}^{t},s^{t}\right)-r\left(s^{t}\right)\right)\middle|\omega_{i}^{t}\right] = 0 \quad (95)$$

Combining these with the intermediate good demand in (81) yields equations (44)-(46). **QED.**

B2. Proofs of Results in Main Text

Equipped with the previous auxiliary results, we now offer the proofs for all results that appear in the main text, except for those already included in the in-print appendix.

Proof of Proposition 1. *Necessity.* We first prove necessity. First, take equation (93). This may be rewritten as

$$\left(1-\tau^{r}\left(s^{t}\right)\right)\frac{\rho-1}{\rho}MP_{\ell}\left(\omega_{i}^{t},s^{t}\right)-w(s^{t})=0 \quad \forall t,\omega_{i}^{t},s^{t}$$

Combining this with the household's intratemporal condition (73), we obtain

$$\frac{U_{c}\left(s^{t}\right)}{\left(1+\tau^{c}\left(s^{t}\right)\right)}\left(1-\tau^{r}(s^{t})\right)\frac{\rho-1}{\rho}MP_{\ell}\left(\omega_{i}^{t},s^{t}\right)-\frac{-U_{\ell}\left(s^{t}\right)}{\left(1-\tau^{\ell}\left(s^{t}\right)\right)}=0$$

thereby proving necessity of (7) with

$$\psi^{r}\left(s^{t}\right)\equiv\frac{U_{c}(s^{t})(1-\tau^{r}(s^{t}))}{1+\tau^{c}(s^{t})},\ \chi^{*}\equiv\frac{\rho-1}{\rho}\quad\text{and}\quad\psi^{\ell}\left(s^{t}\right)\equiv\frac{-U_{\ell}(s^{t})}{1-\tau^{\ell}(s^{t})} \quad (96)$$

Next, take equation (94). This may be rewritten as

$$\mathbb{E}\left[\frac{U_{c}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)}\left\{\left(1-\tau^{r}\left(s^{t}\right)\right)\frac{\rho-1}{\rho}MP_{h}\left(\omega_{i}^{t},s^{t}\right)-1\right\}\middle|\omega_{i}^{t}\right]=0 \quad \forall t,\omega_{i}^{t}$$

We thereby prove necessity of (8) with

$$\psi^{c}\left(s^{t}\right)\equiv\frac{U_{c}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)} \quad (97)$$

Next, take equation (95). This may be rewritten as follows

$$\mathbb{E}\left[\frac{U_{c}\left(s^{t}\right)}{1+\tau^{c}\left(s^{t}\right)}\left(\left(1-\tau^{r}\left(s^{t}\right)\right)\frac{\rho-1}{\rho}MP_{k}\left(\omega_{i}^{t},s^{t}\right)-r(s^{t})\right)\middle|\omega_{i}^{t}\right]=0 \quad \forall t,\omega_{i}^{t}$$

We thereby prove necessity of (9) with

$$\psi^k\left(s^t\right) = \frac{U_c(s^t)}{1+\tau^c(s^t)} r\left(s^t\right) = \frac{U_c(s^t)}{1+\tau^c(s^t)} \frac{\tilde{r}\left(s^t\right)}{1-\tau^k\left(s^t\right)}. \tag{98}$$

So far we have established the necessity of conditions (7)-(9). The necessity of the resource constraint follows from the combination of budgets and market clearing. What remains is to prove the necessity of the implementability condition (6).

To obtain this condition, we multiply the household's budget constraint at s^t by $\Lambda\left(s^t\right)$ and then sum over s^t and t. This gives us the following

$$\begin{aligned} &\sum_{t,s^t} \Lambda\left(s^t\right) \left[\begin{array}{c} \left(1+\tau^c\left(s^t\right)\right) C\left(s^t\right) + B\left(s^t\right) + \sum_{s^{t+1}} Q(s^{t+1}) D(s^{t+1}) \\ + \left(K\left(s^t\right) - (1-\delta) K\left(s^{t-1}\right)\right) \end{array} \right] \\ = \;& \sum_{t,s^t} \Lambda\left(s^t\right) \left[\begin{array}{c} \left(1-\tau^\ell\left(s^t\right)\right) w\left(s^t\right) L\left(s^t\right) + \left(1-\tau^k\left(s^t\right)\right) r\left(s^t\right) K\left(s^{t-1}\right) \\ + \left(1+R\left(s^{t-1}\right)\right) B\left(s^{t-1}\right) + D(s^t) \end{array} \right] \end{aligned}$$

Substituting in the FOCs for debt (84) and state contingent bonds (85) we get that

$$\begin{aligned} \sum_{t,s^t} \Lambda\left(s^t\right) \left[\left(1+\tau^c\left(s^t\right)\right) C\left(s^t\right) + K\left(s^t\right)\right] \;=\; & \sum_{t,s^t} \Lambda\left(s^t\right) \left[\left(1-\tau^\ell\left(s^t\right)\right) w\left(s^t\right) L\left(s^t\right)\right] \\ & + \sum_{t,s^t} \Lambda\left(s^t\right) \left(1 + \left(1-\tau^k\left(s^t\right)\right) r\left(s^t\right) - \delta\right) K\left(s^{t-1}\right) \end{aligned}$$

where we have used $B_0 = D_0 = 0$. Next, substituting in the FOC for capital (87), we get

$$\sum_{t,s^t} \Lambda\left(s^t\right) \left(1+\tau^c\left(s^t\right)\right) C\left(s^t\right) = \sum_{t,s^t} \Lambda\left(s^t\right) \left(1-\tau^\ell\left(s^t\right)\right) w\left(s^t\right) L\left(s^t\right)$$

Now, using the household's FOCs for consumption and employment, (82) and (83), to substitute out all prices, we obtain

$$\sum_{t,s^t} \beta^t \mu\left(s^t\right) U_c\left(s^t\right) C\left(s^t\right) = -\sum_{t,s^t} \beta^t \mu\left(s^t\right) U_\ell\left(s^t\right) L\left(s^t\right)$$

which we may re-write as follows

$$\sum_{t,s^t} \beta^t \mu\left(s^t\right) \left[U_c\left(s^t\right) C\left(s^t\right) + U_\ell\left(s^t\right) L\left(s^t\right)\right] = 0$$

We thus obtain condition in (6) and complete the proof of necessity.

Sufficiency. Consider now sufficiency. Take any allocation ξ_t that satisfies (6)-(9). We now prove that there exists a set of tax rates

$$\left\{\tau^c\left(s^t\right), \tau^\ell\left(s^t\right), \tau^k\left(s^t\right), \tau^r(s^t)\right\},$$

a real wage $w\left(s^t\right)$, relative prices $\left(p\left(\omega_i^t, s^t\right)\right)_{i\in I}$, a real rental rate $r\left(s^t\right)$, an interest rate function $R\left(s^t\right)$ and a path for nominal debt holdings $B\left(s^t\right)$ that implement this allocation as an equilibrium. We construct the equilibrium prices and policies as follows.

First, relative prices satisfy

$$\frac{p(\omega_i^t, s^t)}{P(s^t)} = p(\omega_i^t, s^t) = \left(\frac{y(\omega_i^t, s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}$$

where we normalize the aggregate price level to one: $P(s^t) = 1$. With these prices we satisfy the equilibrium conditions (81) for intermediate good demand.

Let us propose the following tax rates τ^ℓ, τ^c, and τ^r:

$$1+\tau^c\left(s^t\right) = \frac{U_c(s^t)}{\psi^c(s^t)},\quad 1-\tau^\ell\left(s^t\right) = \frac{-U_\ell(s^t)}{\psi^\ell\left(s^t\right)},\quad \text{and} \quad 1-\tau^r(s^t) = \frac{\psi^r\left(s^t\right)}{\psi^c(s^t)} \tag{99}$$

We then satisfy the household's necessary optimality condition for labor (73) with the following real wage:

$$w\left(s^t\right) = \frac{\psi^\ell\left(s^t\right)}{\psi^c(s^t)} = \frac{-U_\ell\left(s^t\right)}{U_c\left(s^t\right)\frac{\left(1-\tau^\ell(s^t)\right)}{\left(1+\tau^c(s^t)\right)}} \tag{100}$$

Next, take condition (7). We may replace this with the wage from (100) and obtain

$$\chi^*\psi^r\left(s^t\right)MP_\ell\left(\omega_i^t, s^t\right) - \psi^c(s^t)w\left(s^t\right) = 0$$

Substituting in for ψ^r and ψ^c from (99) gives us:

$$\left(1-\tau^r(s^t)\right)\tfrac{\rho-1}{\rho}MP_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) = 0$$

This satisfies the firm's optimality condition for labor in (93).

Next, take implementability condition (8). Again substituting in for ψ^r and ψ^c from (99) gives us the following:

$$\mathbb{E}\left[\frac{U_c(s^t)}{1+\tau^c(s^t)}\left((1-\tau^r(s^t))\tfrac{\rho-1}{\rho}MP_h\left(\omega_i^t, s^t\right) - 1\right)\middle|\,\omega_i^t\right] = 0$$

This satisfies the firm's optimality condition for the intermediate good (94).

Next take implementability condition (9). Again substituting in for ψ^r from (99) gives us:

$$\mathbb{E}\left[\frac{U_c(s^t)(1-\tau^r(s^t))}{1+\tau^c(s^t)}\tfrac{\rho-1}{\rho}MP_k\left(\omega_i^t, s^t\right) - \psi^k\left(s^t\right)\middle|\,\omega_i^t\right] = 0$$

This satisfies the firm's optimality condition for capital (95) as long as we set the real rental rate on capital be equal to

$$r\left(s^t\right) = \psi^k\left(s^t\right)\left(\frac{U_c(s^t)}{1+\tau^c(s^t)}\right)^{-1} \tag{101}$$

This implies that we may satisfy the household's Euler condition (74) with the following capital-income tax rate

$$1-\tau^{k}\left(s^{t}\right)=\frac{\tilde{r}\left(s^{t}\right)}{r\left(s^{t}\right)} \tag{102}$$

with $r\left(s^{t}\right)$ given by (101). Moreover, given the allocation, the following interest rate function

$$1+R\left(s^{t}\right)=\frac{U_c\left(s^{t}\right)}{\left(1+\tau^{c}\left(s^{t}\right)\right)}\left\{\beta\mathbb{E}\left[\left.\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^{c}\left(s^{t+1}\right)\right)}\right| s^{t}\right]\right\}^{-1}$$

ensures that condition (72) holds.

Finally we construct bond holdings such that the household's Euler equation (72) holds. We multiply the budget by $\Lambda\left(s^{t}\right)$ and sum over all periods and states following s^{r}:

$$\begin{aligned}&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^{t}\right)\left[\begin{array}{c}\left(1+\tau^{c}\left(s^{t}\right)\right)C\left(s^{t}\right)+B\left(s^{t}\right)+\sum_{s^{t+1}}Q(s^{t+1})D(s^{t+1})\\ +\left(K\left(s^{t}\right)-\left(1-\delta\right)K\left(s^{t-1}\right)\right)\end{array}\right]\\ =&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^{t}\right)\left[\begin{array}{c}\left(1-\tau^{\ell}\left(s^{t}\right)\right)w\left(s^{t}\right)L\left(s^{t}\right)+\left(1-\tau^{k}\left(s^{t}\right)\right)r\left(s^{t}\right)K\left(s^{t-1}\right)\\ +\left(1+R\left(s^{t-1}\right)\right)B\left(s^{t-1}\right)+D(s^{t})\end{array}\right]\end{aligned}$$

Substituting in the FOCs for debt (84) and state contingent bonds (85) we get that

$$\begin{aligned}&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^{t}\right)\left[\left(1+\tau^{c}\left(s^{t}\right)\right)C\left(s^{t}\right)+K\left(s^{t}\right)\right]\\ =&\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^{t}\right)\left[\left(1-\tau^{\ell}\left(s^{t}\right)\right)w\left(s^{t}\right)L\left(s^{t}\right)+\left(1+\left(1-\tau^{k}\left(s^{t}\right)\right)r\left(s^{t}\right)-\delta\right)K\left(s^{t-1}\right)\right]\\ &+\sum_{s^{r+1}|s^r}\Lambda\left(s^{r+1}\right)\left(1+R\left(s^{r}\right)\right)B\left(s^{r}\right)\end{aligned}$$

Next, substituting in the FOC for capital (87), we get

$$\Lambda\left(s^{r}\right)B\left(s^{r}\right)=\sum_{t=r+1}^{\infty}\sum_{s^t}\Lambda\left(s^{t}\right)\left[\left(1+\tau^{c}\left(s^{t}\right)\right)C\left(s^{t}\right)-\left(1-\tau^{\ell}\left(s^{t}\right)\right)w\left(s^{t}\right)L\left(s^{t}\right)\right]$$

Next, using the household's focs for consumption and labor (82) and (83) gives us

$$\frac{\beta^{r}\mu\left(s^{r}\right)U_c\left(s^{r}\right)}{\left(1+\tau^{c}\left(s^{r}\right)\right)}B\left(s^{r}\right)=\sum_{t=r+1}^{\infty}\sum_{s^t}\beta^{t}\mu\left(s^{t}\right)\left[U_c\left(s^{t}\right)C\left(s^{t}\right)+U_{\ell}\left(s^{t}\right)L\left(s^{t}\right)\right]$$

which we may rewrite as follows

$$\frac{U_c\left(s^{r}\right)}{\left(1+\tau^{c}\left(s^{r}\right)\right)}B\left(s^{r}\right)=\sum_{t=r+1}^{\infty}\sum_{s^t}\beta^{t-r}\mu\left(s^{t}|s^{r}\right)\left[U_c\left(s^{t}\right)C\left(s^{t}\right)+U_{\ell}\left(s^{t}\right)L\left(s^{t}\right)\right]$$

Therefore real bond holdings are given by

$$B\left(s^{r}\right)=\left(\frac{U_c\left(s^{r}\right)}{1+\tau^{c}\left(s^{r}\right)}\right)^{-1}\sum_{t=r+1}^{\infty}\sum_{s^t}\beta^{t-r}\mu\left(s^{t}|s^{r}\right)\left[U_c\left(s^{t}\right)C\left(s^{t}\right)+U_{\ell}\left(s^{t}\right)L\left(s^{t}\right)\right]$$

for any period r, state s^{r}. **QED.**

Proof of Proposition 2. *Necessity.* We first prove necessity. Feasibility follows from the combination of budgets and market clearing.

Next, using the intermediate demand equation in (88), we may rewrite (80) as

$$\mathbb{E}\left[\frac{U_c(s^t)}{(1+\tau^c(s^t))}y\left(\omega_i^t,s^t\right)\left\{\left(1-\tau^r(s^t)\right)\left(\tfrac{\rho-1}{\rho}\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}-\lambda\left(\omega_i^t,s^t\right)\right\}\middle|\,\omega_i^t\right]=0$$

We re-write this condition as

$$\mathbb{E}\left[\frac{U_c(s^t)}{(1+\tau^c(s^t))}y\left(\omega_i^t,s^t\right)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\left\{\chi(\omega_i^t,s^t)-\chi^*\right\}\middle|\,\omega_i^t\right]=0$$

with $\chi^*=\frac{\rho-1}{\rho}$ and

$$\chi(\omega_i^t,s^t)\equiv\frac{\lambda\left(\omega_i^t,s^t\right)}{\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}}\tag{103}$$

Using the definition of $\psi^r\left(s^t\right)$ in (96) we obtain

$$\mathbb{E}\left[\psi^r\left(s^t\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}y\left(\omega_i^t,s^t\right)\left\{\chi(\omega_i^t,s^t)-\chi^*\right\}\middle|\,\omega_i^t\right]=0$$

thereby proving necessity of (14).

Next, we combine the intratemporal optimality conditions of the household and of the firm for labor. Substituting (73) into the firm's condition (77) to replace the real wage, we obtain:

$$\lambda\left(\omega_i^t,s^t\right)\frac{U_c\left(s^t\right)}{(1+\tau^c(s^t))}A(s^t)f_\ell\left(\omega_i^t,s^t\right)-\left(\frac{-U_\ell\left(s^t\right)}{1-\tau^\ell\left(s^t\right)}\right)=0.\tag{104}$$

From our definition of $\chi(\omega_i^t,s^t)$ in (103), we have that

$$\lambda\left(\omega_i^t,s^t\right)=\chi(\omega_i^t,s^t)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}$$

Substituting this into (104) we obtain

$$\chi(\omega_i^t,s^t)\left(1-\tau^r(s^t)\right)\frac{U_c\left(s^t\right)}{(1+\tau^c(s^t))}\left(\frac{y\left(\omega_i^t,s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}A(s^t)f_\ell\left(\omega_i^t,s^t\right)-\left(\frac{-U_\ell\left(s^t\right)}{1-\tau^\ell\left(s^t\right)}\right)=0.$$

We may write this as

$$\chi(\omega_i^t,s^t)\psi^r\left(s^t\right)MP_\ell\left(\omega_i^t,s^t\right)-\psi^\ell\left(s^t\right)=0.$$

where $\psi^r\left(s^t\right)$ and $\psi^\ell\left(s^t\right)$ are given by (96), thereby proving necessity of (11).

Next, we have the firm's optimality condition for intermediate goods given by (78). We again substitute for $\lambda\left(\omega_i^t, s^t\right)$ from (103) into (78) and obtain

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)}\left(\chi(\omega_i^t, s^t)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_h\left(\omega_i^t, s^t\right)-1\right)\middle|\,\omega_i^t\right] \quad = 0$$

We may write this as

$$\mathbb{E}\left[\chi(\omega_i^t, s^t)\psi^r\left(s^t\right) MP_h\left(\omega_i^t, s^t\right)-\psi^c\left(s^t\right)\middle|\,\omega_i^t\right] \quad = 0$$

where $\psi^r\left(s^t\right)$ and $\psi^c\left(s^t\right)$ are given by (96) and (97), thereby proving necessity of (12).

Similarly we have the firm's optimality condition for capital investment given by (79). We again substitute for $\lambda\left(\omega_i^t, s^t\right)$ from (103) into (79) and obtain

$$\mathbb{E}\left[\frac{U_c(s^t)}{\left(1+\tau^c\left(s^t\right)\right)}\left(\chi(\omega_i^t, s^t)\left(1-\tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} A(s^t) f_k\left(\omega_i^t, s^t\right)-r\left(s^t\right)\right)\middle|\,\omega_i^t\right] \quad = 0$$

We may write this as

$$\mathbb{E}\left[\chi(\omega_i^t, s^t)\psi^r\left(s^t\right) MP_k\left(\omega_i^t, s^t\right)-\psi^k\left(s^t\right)\middle|\,\omega_i^t\right] \quad = 0$$

where $\psi^r\left(s^t\right)$ and $\psi^k\left(s^t\right)$ are given by (96) and (98), thereby proving necessity of (13).

We now prove part (iii) of Proposition 2. In any sticky-price equilibrium, prices must satisfy the intermediate good demand equation (81). Consider then the relative prices between two firms. Fix a period t and a state s^t, and take an arbitrary pair of firms (i, j), with $j \neq i$. From the consumer demand equation (81), the *relative* price of the two firms is pinned down by their relative output:

$$\frac{p\left(\omega_i^t\right)}{p\left(\omega_j^t\right)} = \left[\frac{y\left(\omega_i^t, s^t\right)}{y\left(\omega_j^t, s^t\right)}\right]^{-1/\rho}$$

Clearly, the above condition can hold for all realizations of ω_i^t, ω_j^t and s^t only if the right-hand side of this condition is independent of s^t conditional on the pair (ω_i^t, ω_j^t). This can be true if and only if there exist positive-valued functions Ψ^ω and Ψ^s such that the output of a firm can be expressed as $y\left(\omega_i^t, s^t\right) = \Psi^\omega(\omega_i^t)\Psi^s(s^t)$.

Next, we may write the Cobb-Douglas production function more generally as iso-elastic in labor:

$$F\left(k, h, \ell\right) = \ell^\alpha F\left(k, h, 1\right) = \ell^\alpha g\left(k, h\right) \tag{105}$$

for all (k, h, ℓ) and some $\alpha \in (0, 1)$. Output may thereby be written as

$$y_i\left(\omega_i^t, s^t\right) = A\left(s^t\right)\ell\left(\omega_i^t, s^t\right)^\alpha g\left(k\left(\omega_i^t\right), h\left(\omega_i^t\right)\right) = A\left(s^t\right)\ell\left(\omega_i^t, s^t\right)^\alpha g\left(\omega_i^t\right). \tag{106}$$

where, with some abuse of notation, $g\left(\omega_i^t\right) = g\left(k\left(\omega_i^t\right), h\left(\omega_i^t\right)\right)$. Thus, log-separability of output $y\left(\omega_i^t, s^t\right)$ along with iso-elastic production imply log-separability of labor $\ell\left(\omega_i^t, s^t\right)$.

In any sticky-price equilibrium $\ell\left(\omega_i^t, s^t\right)$ is pinned down by condition (11). Given technology (106), condition (11) may be expressed as

$$\chi(\omega_i^t, s^t)\frac{\psi^r\left(s^t\right)}{\psi^\ell\left(s^t\right)}\left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\alpha\frac{y\left(\omega_i^t, s^t\right)}{\ell\left(\omega_i^t, s^t\right)} = 1 \tag{107}$$

Thus, condition (107) along with log-separability of $y\left(\omega_i^t, s^t\right)$ and $\ell\left(\omega_i^t, s^t\right)$ imply log-separabilty of $\chi(\omega_i^t, s^t)$.

What remains is the implementability condition (6) in part (i) of Proposition 2. To obtain this necessary condition, we follow the exact same steps used to obtain this condition in the proof of Proposition 1.

Sufficiency. Consider now sufficiency. Take any allocation ξ_t that satisfies (6), (11)-(14), and is log-separable in the sense of (15). We now prove that there exists a set of tax rates

$$\left\{\tau^c\left(s^t\right), \tau^\ell\left(s^t\right), \tau^k\left(s^t\right), \tau^r(s^t)\right\},$$

a real wage $w\left(s^t\right)$, nominal prices $\left(p\left(\omega_i^t\right)\right)_{i\in I}, P\left(s^t\right)$, a real rental rate $r\left(s^t\right)$, a nominal interest rate function $R\left(s^t\right)$, and a path for nominal debt holdings $B\left(s^t\right)$ that implement this allocation as an equilibrium. We construct the equilibrium prices and policies as follows.

First, because $\chi(\omega_i^t, s^t)$ is log-separable, then iso-elastic technology (106) and condition (11) jointly imply that $y\left(\omega_i^t, s^t\right)$ and $\ell\left(\omega_i^t, s^t\right)$ are log-separable. Thereby, we have that $y\left(\omega_i^t, s^t\right) = \Psi^\omega(\omega_i^t)\Psi^s(s^t)$ for some functions Ψ^ω and Ψ^s. Let us then propose the following nominal prices:

$$p(\omega_i^t) = \Psi^\omega\left(\omega_i^t\right)^{-\frac{1}{\rho}},$$

which are by construction measurable in ω_i^t. It follows that the price level satisfies

$$P(s^t) = \left[\sum_{\omega\in\Omega^t} p\left(\omega_i^t\right)^{1-\rho}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}} = \left[\sum_{\omega\in\Omega^t}\Psi^\omega\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}},$$

while aggregate output satisfies

$$Y\left(s^t\right) = \Psi^s\left(s^t\right)\left[\sum_{\omega\in\Omega^t}\Psi^\omega\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}},$$

and therefore relative prices satisfy

$$\frac{p(\omega_i^t)}{P(s^t)} = \frac{\Psi^\omega\left(\omega_i^t\right)^{-\frac{1}{\rho}}}{\left[\sum_{\omega\in\Omega^t}\Psi^\omega\left(\omega_i^t\right)^{\frac{\rho-1}{\rho}}\varphi\left(\omega|s^t\right)\right]^{\frac{1}{1-\rho}}} = \left(\frac{y(\omega_i^t, s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}$$

That is, we can find nominal prices that implement the right relative prices while being measurable in ω_i^t. These prices satisfy the equilibrium necessary condition (81) for intermediate good demand.

We propose tax rates τ^ℓ, τ^c, and τ^r as in (99). We then satisfy the household's necessary optimality condition for labor (73) with the real wage proposed in (100).

Next, take implementability condition (11). We may replace this with the wage from (100) and obtain

$$\chi(\omega, s^t)\psi^r\left(s^t\right) MP_\ell\left(\omega_i^t, s^t\right) - \psi^c(s^t) w\left(s^t\right) = 0.$$

Substituting in for ψ^r and ψ^c from (99) gives us:

$$\chi(\omega, s^t)\left(1 - \tau^r(s^t)\right) MP_\ell\left(\omega_i^t, s^t\right) - w\left(s^t\right) = 0$$

This satisfies the firm's optimality condition for labor (77) as long as we let

$$\lambda\left(\omega_i^t, s^t\right) \equiv \chi(\omega_i^t, s^t)\left(1 - \tau^r(s^t)\right)\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}. \tag{108}$$

Next, take implementability condition (12). Again substituting in for ψ^r and ψ^c from (99) gives us:

$$\mathbb{E}\left[\frac{U_c(s^t)}{1 + \tau^c(s^t)}\left(\chi(\omega, s^t)(1 - \tau^r(s^t)) MP_h\left(\omega_i^t, s^t\right) - 1\right)\middle|\, \omega_i^t\right] = 0$$

This satisfies the firm's optimality condition for the intermediate good (78) with $\lambda\left(\omega_i^t, s^t\right)$ given by (108).

Next take implementability condition (13). Substituting in for ψ^r from (99) gives us:

$$\mathbb{E}\left[\frac{U_c(s^t)(1 - \tau^r(s^t))}{1 + \tau^c(s^t)}\chi(\omega, s^t) MP_k\left(\omega_i^t, s^t\right) - \psi^k(s^t)\middle|\, \omega_i^t\right] = 0$$

This satisfies the firm's optimality condition for capital (79) with $\lambda\left(\omega_i^t, s^t\right)$ given by (108) and with a real rental rate on capital given by (101). This implies further that we may satisfy the household's Euler condition (74) with the a capital-income tax rate τ^k as in (102).

Next, take implementability condition (14). Substituting in for ψ^r from (99) gives us:

$$\mathbb{E}\left[Y\left(s^t\right)^{1/\rho} y\left(\omega_i^t, s^t\right)^{1-1/\rho}\frac{U_c(s^t)(1 - \tau^r(s^t))}{1 + \tau^c(s^t)}\left\{\chi(\omega_i^t, s^t) - \chi^*\right\}\middle|\ \omega_i^t\right] = 0$$

which we may rewrite as

$$\mathbb{E}\left[\frac{U_c(s^t)}{(1 + \tau^c(s^t))} y\left(\omega_i^t, s^t\right)(1 - \tau^r(s^t))\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\left\{\chi(\omega_i^t, s^t) - \chi^*\right\}\middle|\, \omega_i^t\right] = 0$$

Substituting $\chi(\omega, s^t)$ from (103) gives us

$$\mathbb{E}\left[\frac{U_c(s^t)}{(1 + \tau^c(s^t))} y\left(\omega_i^t, s^t\right)\left\{\lambda(\omega_i^t, s^t) - \tfrac{\rho-1}{\rho}(1 - \tau^r(s^t))\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\right\}\middle|\, \omega_i^t\right] = 0$$

Using the optimality for intermediate good demand (81) we may rewrite this as

$$\mathbb{E}\left[\frac{U_c(s^t)}{(1+\tau^c(s^t))}y\left(\omega_i^t,s^t\right)\left\{\lambda(\omega_i^t,s^t)-\tfrac{\rho-1}{\rho}(1-\tau^r(s^t))\frac{p(\omega_i^t)}{P(s^t)}\right\}\middle|\,\omega_i^t\right]=0$$

and therefore the firm's optimality condition for its nominal price (80) is satisfied.

Given the allocation and the path for the nominal price level, the following nominal interest rate ensures that condition (72) holds:

$$1+R\left(s^t\right)=\frac{U_c\left(s^t\right)}{\left(1+\tau^c\left(s^t\right)\right)P\left(s^t\right)}\left\{\beta\mathbb{E}\left[\frac{U_c\left(s^{t+1}\right)}{\left(1+\tau^c\left(s^{t+1}\right)\right)P\left(s^{t+1}\right)}\middle|\ s^t\right]\right\}^{-1}$$

Finally what remains is to construct bond holdings such that the household's Euler equation (72) holds. For this we follow the exact same steps used to obtain bond holdings in the sufficiency proof of Proposition 1. Following these steps, real bond holdings are given by

$$\frac{B\left(s^r\right)}{P\left(s^r\right)}=\left(\frac{U_c\left(s^r\right)}{1+\tau^c\left(s^r\right)}\right)^{-1}\sum_{t=r+1}^{\infty}\sum_{s^t}\beta^{t-r}\mu\left(s^t|s^r\right)\left[U_c\left(s^t\right)C\left(s^t\right)+U_\ell\left(s^t\right)L\left(s^t\right)\right]$$

for any period r, state s^r. **QED.**

Proof of Corollary 1. Follows from the main text.

Proof of Lemma 1. Suppose preferences are homothetic as follows:

$$U\left(C,L\right)=\frac{C^{1-\gamma}}{1-\gamma}-\eta\frac{L^{1+\epsilon}}{1+\epsilon}, \tag{109}$$

for $\gamma,\epsilon,\eta>0$. This implies

$$\tilde{U}\left(C,L\right)=\frac{C^{1-\gamma}}{1-\gamma}-\eta\frac{L^{1+\epsilon}}{1+\epsilon}+\Gamma\left[\left(1-\gamma\right)\frac{C^{1-\gamma}}{1-\gamma}-\left(1+\epsilon\right)\eta\frac{L^{1+\epsilon}}{1+\epsilon}\right]. \tag{110}$$

where $\Gamma\geq 0$ is a constant. Then in the optimal allocation ξ^*,

$$\frac{U_c\left(s^t\right)}{\tilde{U}_c\left(s^t\right)}=\frac{1}{1+\Gamma\left(1-\gamma\right)},\quad\text{and}\quad\frac{U_\ell\left(s^t\right)}{\tilde{U}_\ell\left(s^t\right)}=\frac{1}{1+\Gamma\left(1+\epsilon\right)}.$$

We now prove that the optimal allocation is implemented with a zero tax on capital ($\tau^k=0$), a zero tax on consumption ($\tau^c=0$), and a time- and state-invariant tax on labor. A zero tax rate on consumption implies that in order to obtain the optimal labor tax given in (21), it must satisfy

$$1-\tau^\ell=\frac{U_\ell\left(s^t\right)}{\tilde{U}_\ell\left(s^t\right)}\left(\frac{U_c\left(s^t\right)}{\tilde{U}_c\left(s^t\right)}\right)^{-1}=\frac{1+\Gamma\left(1-\gamma\right)}{1+\Gamma\left(1+\epsilon\right)}$$

The tax rate on capital follows directly from Theorem 1. **QED.**

Proof of Lemma 2. In any sticky-price equilibrium, prices must satisfy the intermediate good demand equation:

$$\frac{p\left(\omega_i^t\right)}{P\left(s^t\right)} = \left(\frac{y\left(\omega_i^t, s^t\right)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}}. \tag{111}$$

Consider then the relative prices between two firms. Fix a period t and a state s^t, and take an arbitrary pair of firms (i, j), with $j \neq i$. From the demand equation (111), the *relative* price of the two firms is pinned down by their relative output:

$$\frac{p\left(\omega_i^t\right)}{p\left(\omega_j^t\right)} = \left[\frac{y\left(\omega_i^t, s^t\right)}{y\left(\omega_j^t, s^t\right)}\right]^{-1/\rho}.$$

Clearly, the above condition can hold for all realizations of ω_i^t, ω_j^t and s^t only if the right-hand side of this condition is independent of s^t conditional on the pair (ω_i^t, ω_j^t). This can be true if and only if y is log-separable. **QED.**

Proof of Proposition 7. Take any flexible-price equilibrium. For any realization of $\left(\omega_i^t, s^t\right)$, the following two equations must hold:

$$1 = \chi^* \frac{\psi^r\left(s^t\right)}{\psi^\ell\left(s^t\right)} \left(\frac{y\left(\omega_i^t, s^t\right)}{Y\left(s^t\right)}\right)^{-\frac{1}{\rho}} \alpha \frac{y\left(\omega_i^t, s^t\right)}{\ell\left(\omega_i^t, s^t\right)} \tag{112}$$

$$y\left(\omega_i^t, s^t\right) = A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)} \ell\left(\omega_i^t, s^t\right)^{\alpha} \tag{113}$$

Note that the main difference between these two equations and those stated previously in conditions (106) and (54) is that firm specific productivity A is now measurable in ω_i^t. Following the proof for Lemma 5, we can solve (112) and (113) simultaneously for $y\left(\omega_i^t, s^t\right)$ and $\ell\left(\omega_i^t, s^t\right)$. We thus find that in any flexible-price equilibrium, output $y\left(\omega_i^t, s^t\right)$ and labor $\ell\left(\omega_i^t, s^t\right)$ are log-separable in ω_i^t and s^t and satisfy

$$y\left(\omega_{it}, s^t\right) = \Psi^\omega(\omega_i^t)\Psi^s(s^t) \tag{114}$$

$$\ell\left(\omega_i^t, s^t\right) = \Psi^\omega(\omega_i^t)^{\frac{\rho-1}{\rho}} \Psi^s(s^t)^{\frac{1}{\alpha}} \tag{115}$$

with

$$\Psi^\omega(\omega_i^t) = \left[A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{116}$$

$$\Psi^s(s^t) = \left[\alpha\chi^* \frac{\psi^r\left(s^t\right)}{\psi^\ell\left(s^t\right)} Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{117}$$

Next, consider the proposed tax policy. The revenue tax (and the associated wedges) takes the following form,

$$\log\left(1 - \tau^r\left(A_t, Y_t\right)\right) = \hat{\tau}_0 - \hat{\tau}_A \log A_t - \hat{\tau}_Y \log Y_t \tag{118}$$

so that it is log-normally distribued for some scalars $\hat{\tau}_0, \hat{\tau}_A, \hat{\tau}_Y \in \mathbb{R}$, and the remaining tax rates satisfy $\tau^k(s^t) = \tau^c(s^t) = 0$, and $1 + \tau^\ell(s^t) = 1/\left(1 - \tau^r(s^t)\right)$.

Combining this last condition with the tax expressions in (99) implies

$$\frac{\psi^r\left(s^t\right)}{\psi^c(s^t)} = \frac{\psi^\ell\left(s^t\right)}{-U_\ell(s^t)}.$$

Rearranging and combining it with the expression for $\psi^c(s^t)$ in (99) gives us

$$\frac{\psi^r\left(s^t\right)}{\psi^\ell\left(s^t\right)} = \frac{U_c(s^t)}{-U_\ell(s^t)}$$

Using the above expression to replace the wedges in (117) gives us

$$\Psi^\omega(\omega_i^t) = \left[A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{119}$$

$$\Psi^s(s^t) = \left[\frac{U_c(s^t)}{-U_\ell(s^t)} Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \tag{120}$$

where we abstract from the constant scalar $(\alpha\chi^*)^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$.

Aggregate output may be expressed as

$$Y\left(s^t\right) = \left[\sum_{\omega\in\Omega^t} y\left(\omega_{it}, s^t\right)^{\frac{\rho-1}{\rho}} \varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}} = \Psi^s(s^t)\mathcal{B}(s^t) \tag{121}$$

Similarly using (115), aggregate labor may be expressed as

$$L\left(s^t\right) = \sum_{\omega\in\Omega^t} \ell\left(\omega_i^t, s^t\right)\varphi\left(\omega|s^t\right) = \Psi^s(s^t)^{\frac{1}{\alpha}}\mathcal{B}(s^t)^{\frac{\rho-1}{\rho}}. \tag{122}$$

Finally, the assumed specification for $U(C, L)$ in (109) allows us to rewrite $\Psi^s(s^t)$ in (120) as

$$\Psi^s(s^t) = \left[\frac{C(s^t)^{-\gamma}}{L\left(s^t\right)^\epsilon} Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}. \tag{123}$$

Taking logs of equations (121), (122), and (123) produces the following three equations.

$$\log Y\left(s^t\right) = \log\Psi^s(s^t) + \log\mathcal{B}(s^t) \tag{124}$$

$$\log L\left(s^t\right) = \frac{1}{\alpha}\log\Psi^s(s^t) + \frac{\rho-1}{\rho}\log\mathcal{B}(s^t) \tag{125}$$

$$\log\Psi^s(s^t) = \alpha\zeta\left[\frac{1}{\rho}\log Y(s^t) - \gamma\log C(s^t) - \epsilon\log L\left(s^t\right)\right] \tag{126}$$

where $\zeta \equiv \frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}$.

We combine these three equations as follows. Substituting (125) into (126) for $L\left(s^t\right)$ yields

$$\log\Psi^s(s^t) = \alpha\zeta\left[\frac{1}{\rho}\log Y(s^t) - \gamma\log C(s^t) - \epsilon\left(\frac{1}{\alpha}\log\Psi^s(s^t) + \frac{\rho-1}{\rho}\log\mathcal{B}(s^t)\right)\right].$$

We can solve this for $\Psi^s(s^t)$ and get

$$\log \Psi^s(s^t) = \frac{\alpha\zeta}{(1+\epsilon\zeta)}\left[\frac{1}{\rho}\log Y(s^t) - \gamma \log C(s^t) - \epsilon\frac{\rho-1}{\rho}\log \mathcal{B}(s^t)\right] \tag{127}$$

Combining this expression with equation (124) yields

$$\log Y\left(s^t\right) = \frac{\alpha\zeta}{(1+\epsilon\zeta)}\left[\frac{1}{\rho}\log Y(s^t) - \gamma \log C(s^t) - \epsilon\frac{\rho-1}{\rho}\log \mathcal{B}(s^t)\right] + \log \mathcal{B}(s^t)$$

Solving the above equation for $\mathcal{B}(s^t)$ gives us

$$\log \mathcal{B}(s^t) = \frac{1}{1-\epsilon\frac{\rho-1}{\rho}\frac{\alpha\zeta}{1+\epsilon\zeta}}\left[\left(1-\frac{\alpha\zeta}{1+\epsilon\zeta}\frac{1}{\rho}\right)\log Y\left(s^t\right) + \gamma\frac{\alpha\zeta}{1+\epsilon\zeta}\log C(s^t)\right]. \tag{128}$$

Finally, from the definitions of $\mathcal{B}(s^t)$ and $\Psi^\omega\left(\omega_i^t\right)$, we have the following equation.

$$\mathcal{B}(s^t) = \left[\sum_{\omega\in\Omega^t}\left[A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}\varphi\left(\omega|s^t\right)\right]^{\frac{\rho}{\rho-1}}$$

If we log-linearize our model about the complete information equilibrium, the previous equation becomes[6]

$$\log \mathcal{B}(s^t) = \zeta \log A\left(s^t\right) + \eta\left(1-\alpha\right)\zeta \log H\left(s^t\right). \tag{129}$$

In summary, thus far to describe the flexible price equilibrium we have a system of two equations, (128) and (129), in four unknowns: $Y\left(s^t\right), C\left(s^t\right), H\left(s^t\right)$, and $\mathcal{B}\left(s^t\right)$.

Complete Information Case. Our solution for the incomplete-information equilibrium will be a log-linear approximation around the complete-information Ramsey optimum. Without yet solving for the complete-information optimum, we characterize it below.

Lemma 6. *In the complete information optimum, aggregate intermediate good purchases and aggregate consumption are log-linear in aggregate productivity:*

$$\log H^{LS}\left(s^t\right) = \phi_A^{LS}\log A\left(s^t\right) + const \tag{130}$$
$$\log C^{LS}\left(s^t\right) = \gamma_A^{LS}\log A\left(s^t\right) + const \tag{131}$$

where ϕ_A^{LS} and γ_A^{LS} are scalar constants.

Thus, the complete information optimum is log-linear in the aggregate productivity shock, with a coefficient γ_A^{LS} on productivity for all aggregate variables. The scalar γ_A^{LS} is pinned down by

[6] Alternatively, we would obtain equation (129) in an exact version of our model if we assume that the information and shock structure are jointly log-Normal.

preference and technology parameters along with the level of government spending (equivalently, the tightness of the government budget). For now, we take this allocation as given. We will prove Lemma 6 later when we consider the Ramsey planner's problem in the proof of Proposition 8.

Incomplete Information Log-Linearization. We now return to characterizing the equilibrium under incomplete information. First, we log-linearize the resource constraint around the complete information equilibrium characterized in Lemma 6; this gives us

$$\log Y\left(s^t\right) = (1-\varsigma)\log C\left(s^t\right) + \varsigma \log H\left(s^t\right) \tag{132}$$

where $\varsigma = \eta(1-\alpha)$ is the proportion of output that goes to intermediate good use under complete information. Substituting (132) for $Y\left(s^t\right)$ into equation (128) produces the following expression for $\mathcal{B}(s^t)$:

$$\log \mathcal{B}(s^t) = \zeta\left(\Gamma_C \log C\left(s^t\right) + \Gamma_H \log H\left(s^t\right)\right) \tag{133}$$

where

$$\begin{aligned} \Gamma_H &\equiv \frac{1+\epsilon-\alpha}{1+\epsilon}\varsigma \in (0,1), \quad \text{and} \\ \Gamma_C &\equiv \frac{1+\epsilon-\alpha}{1+\epsilon}(1-\varsigma) + \frac{\alpha\gamma}{1+\epsilon} > 0. \end{aligned}$$

Note that the coefficients Γ_H and Γ_C depend only on the parameters $(\alpha, \gamma, \epsilon, \eta)$ and are both strictly positive. Next, we combine (129) with (133) to obtain

$$\Gamma_C \log C\left(s^t\right) = \log A\left(s^t\right) + (\varsigma - \Gamma_H)\log H\left(s^t\right) \tag{134}$$

We thus reach an expression for aggregate GDP (consumption) in terms of $\log A\left(s^t\right)$ and $\log H\left(s^t\right)$.

Derivation of Beauty Contest. What remains to be characterized is the equilibrium behavior of intermediate good purchases $H\left(s^t\right)$. We show that there exists a fixed point in $h\left(\omega_{i,t}\right)$ and $H\left(s^t\right)$ which pins down their joint solution. To do this, we use the optimality condition for intermediate good purchases given in (8). With our specification of preferences, technology, and the proposed tax scheme, this condition may be written as follows:

$$\mathbb{E}\left[U_c(s^t)\left(\left(1-\tau^r(s^t)\right)\frac{\rho-1}{\rho}\left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}}\eta(1-\alpha)\frac{y\left(\omega_i^t,s^t\right)}{h\left(\omega_i^t\right)} - 1\right)\middle|\,\omega_i^t\right] = 0$$

where $1-\tau^r(s^t)$ satisfies (118). Next, the log-separability of $y\left(\omega_{it}, s^t\right)$ implies that this condition may be further expressed as

$$\mathbb{E}\left[U_c(s^t)\left(\left(1-\tau^r(s^t)\right)\bar{\chi}Y(s^t)^{\frac{1}{\rho}}\Psi^\omega(\omega_i^t)^{\frac{\rho-1}{\rho}}\Psi^s(s^t)^{\frac{\rho-1}{\rho}} - h\left(\omega_i^t\right)\right)\middle|\,\omega_i^t\right] = 0$$

where $\bar{\chi} \equiv \left(\frac{\rho-1}{\rho}\right)\eta(1-\alpha)$. Next, substituting in for $\Psi^{\omega}(\omega_i^t)$ from (119) gives us

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\left(1-\tau^r(s^t)\right)\bar{\chi}Y(s^t)^{\frac{1}{\rho}}\Psi^s(s^t)^{\frac{\rho-1}{\rho}}\left[A\left(\omega_i^t\right)h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}-h\left(\omega_i^t\right)\right)\;\middle|\;\omega_i^t\right]=0$$

Solving the above equation for h, we obtain the following equation characterizing the firm's optimal choice of intermediate good purchases

$$h\left(\omega_i^t\right)^{1-\eta(1-\alpha)\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}=\bar{\chi}A\left(\omega_i^t\right)^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}\frac{\mathbb{E}\left[U_c\left(s^t\right)\left(1-\tau^r(s^t)\right)Y(s^t)^{\frac{1}{\rho}}\Psi^s(s^t)^{\frac{\rho-1}{\rho}}\;\middle|\;\omega_i^t\right]}{\mathbb{E}\left[U_c\left(s^t\right)\mid\omega_i^t\right]}$$

We may re-write this in logs as follows:

$$\log h\left(\omega_i^t\right)=\frac{1}{1-\eta(1-\alpha)\zeta\left(\frac{\rho-1}{\rho}\right)}\left\{\begin{array}{c}\zeta\frac{\rho-1}{\rho}\log A\left(\omega_i^t\right)+\frac{1}{\rho}\mathbb{E}_i\log Y(s^t)\\+\frac{\rho-1}{\rho}\mathbb{E}_i\log\Psi^s(s^t)+\mathbb{E}_i\log\left(1-\tau^r(s^t)\right)\end{array}\right\}$$

where we have abstracted from the constant scalar and used $\mathbb{E}_i$ as shorthand for the conditional expectation operator: $\mathbb{E}_i x=\mathbb{E}\left[x|\;\omega_i^t\right]$. Finally, we substitute in for the tax $1-\tau^r(s^t)$ from (118), giving us

$$\log h\left(\omega_i^t\right)=\frac{1}{1-\eta(1-\alpha)\zeta\left(\frac{\rho-1}{\rho}\right)}\left\{\begin{array}{c}\zeta\frac{\rho-1}{\rho}\log A\left(\omega_i^t\right)-\hat{\tau}_A\mathbb{E}_i\log A\left(s^t\right)\\+\left(\frac{1}{\rho}-\hat{\tau}_Y\right)\mathbb{E}_i\log Y(s^t)+\frac{\rho-1}{\rho}\mathbb{E}_i\log\Psi^s(s^t)\end{array}\right\}. \tag{135}$$

Next, using the fact that $\Psi^s(s^t)$ and $\mathcal{B}(s^t)$ simultaneously satisfy equations (127) and (133), we combine these to obtain

$$\log\Psi^s(s^t)=\frac{\alpha\zeta}{(1+\epsilon\zeta)}\left[\frac{1}{\rho}\log Y(s^t)-\gamma\log C(s^t)-\epsilon\frac{\rho-1}{\rho}\zeta\left(\Gamma_C\log C\left(s^t\right)+\Gamma_H\log H\left(s^t\right)\right)\right]. \tag{136}$$

Replacing $\Psi^s(s^t)$ in (135) with (136) gives us the following representation

$$\log h\left(\omega_i^t\right)=G_1\left(\log A\left(\omega_i^t\right),\;\mathbb{E}_i\log A(s^t),\;\mathbb{E}_i\log Y(s^t),\;\mathbb{E}_i\log C(s^t),\;\mathbb{E}_i\log H\left(s^t\right)\right) \tag{137}$$

where G_1 is a linear function of five variables. Next, using the log-linearized resource constraint (132) to replace $Y\left(s^t\right)$, equation (137) may be reduced to

$$\log h\left(\omega_i^t\right)=G_2\left(\log A\left(\omega_i^t\right),\;\mathbb{E}_i\log A(s^t),\;\mathbb{E}_i\log C(s^t),\;\mathbb{E}_i\log H\left(s^t\right)\right) \tag{138}$$

where G_2 is a linear function of four variables. Note that from (134) we may write aggregate consumption as follows:

$$\log C\left(s^t\right)=\Gamma_C^{-1}\log A\left(s^t\right)+\Gamma_C^{-1}\left(\varsigma-\Gamma_H\right)\log H\left(s^t\right). \tag{139}$$

Using this expression to replace $C(s^t)$ in (138) gives us the following result.

Lemma 7. *Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases satisfy the fixed point*

$$\log h\left(\omega_i^t\right) = m_\omega \log A\left(\omega_i^t\right) + m_A\left(\hat{\tau}\right)\mathbb{E}_i \log A(s^t) + m_H\left(\hat{\tau}\right)\mathbb{E}_i \log H(s^t) \tag{140}$$

with $H\left(s^t\right) = \sum h\left(\omega_i^t\right)\varphi\left(\omega|s^t\right)$, *where* m_ω *is a constant given by*

$$m_\omega = \frac{\frac{\rho-1}{\rho}}{1-\left(\alpha+\eta\left(1-\alpha\right)\right)\left(\frac{\rho-1}{\rho}\right)} > 0 \tag{141}$$

and $m_A\left(\hat{\tau}\right)$ *and* $m_H\left(\hat{\tau}\right)$ *are the following linear functions of the tax coefficients* $\hat{\tau} = \left(\hat{\tau}_A, \hat{\tau}_Y\right)$*:*

$$\begin{aligned} m_A\left(\hat{\tau}\right) &= \delta_A + \delta_{AA}\hat{\tau}_A + \delta_{AY}\hat{\tau}_Y, \\ m_H\left(\hat{\tau}\right) &= \delta_H + \delta_{HY}\hat{\tau}_Y. \end{aligned}$$

The coefficients δ_A, δ_H, δ_{AA}, δ_{AY}, *and* δ_{HY} *are scalars that are functions only of the primitive parameters* $(\alpha, \gamma, \epsilon, \eta, \rho)$.

$$\delta_A = \frac{\alpha^2\epsilon\eta\left(\rho-1\right)-\alpha\left(\gamma\left(1-\rho\right)+\epsilon+\eta-\epsilon\rho\left(1-\eta\right)\right)-\left(1+\epsilon\right)\left(1-\eta\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)} \tag{142}$$

$$\delta_H = \frac{\left(1-\alpha\right)\eta\left(\alpha^2\left(\gamma+\epsilon\eta\right)\left(\rho-1\right)-\alpha\left(\gamma+\epsilon+\eta-\epsilon\rho\left(1-\eta\right)\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)} \tag{143}$$

$$\delta_{AA} = \frac{-\left(\rho-\alpha\left(\rho-1\right)\right)}{\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)}$$

$$\delta_{AY} = \frac{\left(1+\epsilon\right)\left(1-\eta\left(1-\alpha\right)\right)\left(\rho-\alpha\left(\rho-1\right)\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)}$$

$$\delta_{HY} = \frac{\eta\left(1-\alpha\right)\left(\left(1+\epsilon\right)\left(1-\eta\right)+\alpha\left(\gamma+\left(1+\epsilon\right)\eta\right)\right)\left(\rho-\alpha\left(\rho-1\right)\right)}{\left(\alpha^2\eta+\alpha\left(1-\gamma-\left(2+\epsilon\right)\eta\right)-\left(1+\epsilon\right)\left(1-\eta\right)\right)\left(\eta+\left(1-\eta\right)\left(\rho-\alpha\left(\rho-1\right)\right)\right)}$$

The fixed-point representation in (140) pins down the flexible-price alllocation $h\left(\omega_i^t\right)$ and $H\left(s^t\right)$ for any Gaussian information structure. Given the linear structure of $m_A\left(\hat{\tau}\right)$ and $m_H\left(\hat{\tau}\right)$ the following corollary is immediate.

Corollary 3. *The tax elasticities* $\left(\hat{\tau}_A, \hat{\tau}_Y\right)$ *form a spanning set of* $\left(m_A\left(\hat{\tau}\right), m_H\left(\hat{\tau}\right)\right)$.

Morevoer, note that one may use $\hat{\tau}_Y$ to pin down any value for m_H, and given this, one may use $\hat{\tau}_A$ to pin down any value for m_A.

Fixed Point Solution to Beauty Contest. We now solve the fixed point described in Lemma 7. We take the beauty contest formulation given in (140) and transform it as follows. Let us define $\tilde{h}\left(\omega_i^t\right)$ as follows

$$\log \tilde{h}\left(\omega_i^t\right) \equiv \log h\left(\omega_i^t\right) - m_\omega \log A\left(\omega_i^t\right) \tag{144}$$

Then combining this with (140) implies

$$\log \tilde{h}\left(\omega_i^t\right) = m_A\left(\hat{\tau}\right)\mathbb{E}_i \log A(s^t) + m_H\left(\hat{\tau}\right)\mathbb{E}_i \log H(s^t) \tag{145}$$

Next, aggregating over (144) gives us

$$\log H\left(s^t\right) = \log \tilde{H}\left(s^t\right) + m_\omega \log A(s^t) \tag{146}$$

Finally, substituting the above expression into (145) we get

$$\log \tilde{h}\left(\omega_i^t\right) = \left(m_A\left(\hat{\tau}\right) + m_H\left(\hat{\tau}\right)m_\omega\right)\mathbb{E}_i \log A(s^t) + m_H\left(\hat{\tau}\right)\mathbb{E}_i \log \tilde{H}\left(s^t\right)$$

From this formulation the following result is immediate.

Lemma 8. *Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases satisfy the fixed point*

$$\log \tilde{h}\left(\omega_i^t\right) = \left(1-\tilde{\alpha}\right)\tilde{\chi}\mathbb{E}_i \log A(s^t) + \tilde{\alpha}\mathbb{E}_i \log \tilde{H}\left(s^t\right) \tag{147}$$

with $\tilde{H}\left(s^t\right) = \sum \tilde{h}\left(\omega_i^t\right)\varphi\left(\omega|s^t\right)$ *and*

$$\tilde{\alpha} = m_H\left(\hat{\tau}\right) \qquad \text{and} \qquad \tilde{\chi} \equiv \frac{m_A\left(\hat{\tau}\right) + m_H\left(\hat{\tau}\right)m_\omega}{1-m_H\left(\hat{\tau}\right)} \tag{148}$$

Morevoer, any pair $(\tilde{\alpha},\tilde{\chi}) \in \mathbb{R}^2$ *can be attained by an appropriate choice of the pair* $(\hat{\tau}_A,\hat{\tau}_Y)$

Proof of Lemma 8. Equation (147) follows from the above analysis. As for the last claim in Lemma 8, the proof is straightforward. For any $(\tilde{\alpha},\tilde{\chi}) \in \mathbb{R}^2$, choose $m_H = \tilde{\alpha}$ and $m_A = \tilde{\chi}\left(1-m_H\right) - m_H m_\omega$. This is the pair (m_A, m_H) that attains $(\tilde{\alpha},\tilde{\chi})$ given (148). Next recall that that for any pair $\left(m_A\left(\hat{\tau}\right), m_H\left(\hat{\tau}\right)\right) \in \mathbb{R}^2$ there exists a pair $(\hat{\tau}_A,\hat{\tau}_Y)$ that implements these coefficients. Therefore, there exists a pair $(\hat{\tau}_A,\hat{\tau}_Y)$ that attains $(\tilde{\alpha},\tilde{\chi})$. QED.

Although any value of $\tilde{\alpha} \in \mathbb{R}$ can be achieved with appropriate tax instruments, from now on we restrict attention to $\tilde{\alpha} \in (-\infty, 1)$ so as to ensure a unique equlibrium. Equivalently, $m_H\left(\hat{\tau}\right) < 1$. With this qualification, next we note that the game in (147) is the same as in Bergemann and Morris (2013) and hence can be spanned by a private and public signal. Thus suppose the agent gets two Gaussian signals, a private and public signal, call these (x,z) with mean zero and precisions (κ_x,κ_z). Then the solution to this system is given by

Lemma 9. *Suppose managers have Gaussian information about the aggregate state. Then the equilibrium level of intermediate good purchases are given by*

$$\log \tilde{h}\left(x,z\right) = \phi_0 + \phi_x x + \phi_z z \tag{149}$$

where

$$\phi_x = \frac{(1-\tilde{\alpha})\,\kappa_x}{\kappa_0 + (1-\tilde{\alpha})\,\kappa_x + \kappa_z}\tilde{\chi} \tag{150}$$

$$\phi_z = \frac{\kappa_z}{\kappa_0 + (1-\tilde{\alpha})\,\kappa_x + \kappa_z}\tilde{\chi} \tag{151}$$

Let $r_\phi \equiv \phi_z/\phi_x$ be the ratio of these coefficients, so that $\phi_z = r_\phi \phi_x$. Any pair $(\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+$ can be attained by an appropriate choice of the pair $(\hat{\tau}_A, \hat{\tau}_Y)$.

Proof of Lemma 9. Choose any pair $(\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+$. First, note that

$$r_\phi = \frac{\phi_z}{\phi_x} = \frac{1}{(1-\tilde{\alpha})}\frac{\kappa_z}{\kappa_x} \tag{152}$$

One may choose any $\tilde{\alpha}$ to satisfy (152). However, recall there is an upper bound on $\tilde{\alpha} \in (-\infty, 1)$. This imposes certain bounds on the ratio r_ϕ as follows.

$$\lim_{\alpha \to -\infty} r_\phi = 0 \quad \text{and} \quad \lim_{\alpha \to 1} r_\phi = \infty$$

Therefore the ratio r_ϕ must be weakly positive. Next given the $\tilde{\alpha}$ that satisfies (152), one need only choose the $\tilde{\chi}$ that implements ϕ_x in equation (150). Finally, recall that from Lemma 8 we know that any pair $(\tilde{\alpha}, \tilde{\chi})$ can be attained by an appropriate choice of the pair $(\hat{\tau}_A, \hat{\tau}_Y)$. This implies that for any pair $(\phi_x, r_\phi) \in \mathbb{R} \times \mathbb{R}_+$ can be attained by an appropriate choice of $(\hat{\tau}_A, \hat{\tau}_Y)$. QED.

This implies that the ratio between ϕ_x and ϕ_z must be weakly positive. This is intuitive: if actions are increasing in the fundamental under complete information, then also under incomplete information agents will put a positive weight on both the private and public signal; conversely if actions are decreasing in the fundamental under complete information, then under incomplete information agents will put a negative weight on both the private and public signal. Thus in either case the pair ϕ_x, ϕ_z are of the same sign.

Equilibrium Aggregate Intermediated Good Purchases and Consumption (GDP). Next we compute aggregate intermediate good purchases. Equation (149) in Lemma 9 implies that the aggregate intermediate good purchases satisfies

$$\log \tilde{H}\left(s^t\right) = \phi_0 + (\phi_x + \phi_z)\,a_t + \phi_z u_t$$

We may transform this back into the true $H\left(s^t\right)$ from (146) as follows

$$\begin{aligned} \log H\left(s^t\right) &= \log \tilde{H}\left(s^t\right) + m_\omega \log A(s^t) \\ &= \phi_0 + (\phi_x + \phi_z)\,a_t + \phi_z u_t + m_\omega a_t \end{aligned}$$

We thus obtain the following result.

$$\log H\left(s^t\right) = \left(\phi_x + \phi_z + m_\omega\right)\log A\left(s^t\right) + \phi_z u_t + const \tag{153}$$

This is the solution to the original beauty contest game in (140). Equation (153) characterizes the equilibrium behavior of intermediate good purchases $H\left(s^t\right)$ as a function of the aggregate productivity shock and the common noise u_t.

Equilibrium Aggregate Consumption (GDP). Finally, we compute aggregate consumption (GDP). Using the expression in (153) to replace $H\left(s^t\right)$ in equation (139) gives us

$$\log C\left(s^t\right) = \Gamma_C^{-1}\log A\left(s^t\right) + \Gamma_C^{-1}\varsigma\left(\frac{\alpha}{1+\epsilon}\right)\left(\left(\phi_x + \phi_z + m_\omega\right)\log A\left(s^t\right) + \phi_z u_t\right)$$

where we have used the fact that $\varsigma - \Gamma_H = \varsigma\left(\frac{\alpha}{1+\epsilon}\right)$. Therefore

$$\log C\left(s^t\right) = \Gamma_C^{-1}\left(1 + \varsigma\frac{\alpha}{1+\epsilon}\left(\phi_x + \phi_z + m_\omega\right)\right)\log A\left(s^t\right) + \Gamma_C^{-1}\varsigma\frac{\alpha}{1+\epsilon}\phi_z u_t + const$$

We thus obtain the following characterization of aggregate consumption:

$$\log C\left(s^t\right) = \log GDP\left(s^t\right) = \gamma_0 + \gamma_a\log A\left(s^t\right) + \gamma_u u_t$$

where $(\gamma_0, \gamma_A, \gamma_u)$ are constants. The coefficients (γ_a, γ_u) satisfy

$$\gamma_a = \hat{\gamma} + \upsilon\left(\phi_x + \phi_z\right), \text{ and} \tag{154}$$

$$\gamma_u = \upsilon\phi_z \tag{155}$$

where $(\hat{\gamma}, \upsilon)$ are strictly positive scalars given by

$$\hat{\gamma} = \Gamma_C^{-1}\left(1 + \varsigma\frac{\alpha}{1+\epsilon}m_\omega\right) > 0 \quad \text{and} \quad \upsilon = \Gamma_C^{-1}\varsigma\frac{\alpha}{1+\epsilon} > 0. \tag{156}$$

We have thus derived equation (28) in Proposition 7. What remains to be derived are the values of (γ_a, γ_u) that may be spanned with the appropriate tax instruments. To do so, we use $\phi_z = r_\phi\phi_x$ to rewrite (154) and (155) as follows

$$\gamma_a = \hat{\gamma} + \upsilon\left(1 + r_\phi\right)\phi_x \quad \text{and} \quad \gamma_u = \upsilon r_\phi\phi_x \tag{157}$$

This implies that for any γ_u, the following relation must hold

$$\phi_x = \frac{\gamma_u}{\upsilon r_\phi}$$

where $\upsilon r_\phi > 0$. This implies that γ_u, ϕ_x, ϕ_z must all have the same sign. Plugging this into (157) gives us

$$\gamma_a = \hat{\gamma} + \upsilon\left(1 + r_\phi\right)\frac{\gamma_u}{\upsilon r_\phi} = \hat{\gamma} + \left(1 + \frac{1}{r_\phi}\right)\gamma_u$$

Recall that r_ϕ can take any positive number. Therefore the pair (γ_A, γ_u) may take *any* value in the set Υ defined in (29). **QED.**

Proof of Proposition 8. For any realization of $\left(\omega_i^t, s^t\right)$, at the Ramsey Optimum the following two equations must hold:

$$\begin{aligned} \frac{-\tilde{U}_\ell(s^t)}{\tilde{U}_c(s^t)} &= \left(\frac{y(\omega_i^t,s^t)}{Y(s^t)}\right)^{-\frac{1}{\rho}} \alpha \frac{y\left(\omega_i^t, s^t\right)}{\ell\left(\omega_i^t, s^t\right)} && (158) \\ y\left(\omega_i^t, s^t\right) &= A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)} \ell\left(\omega_i^t, s^t\right)^\alpha && (159) \end{aligned}$$

The first is the labor-optimality condition of the Ramsey planner and the second is the production function. Note that the only main difference between (158) and the corresponding labor-optimality condition for the flexible price equilibrium, (112), is that (158) holds specifically at the Ramsey optimum. Thus in (158), $-\tilde{U}_\ell(s^t)/\tilde{U}_c\left(s^t\right)$ is the Ramsey planner's marginal rate of substitution between consumption and labor and there are no tax wedges.

However, recall that with homothetic preferences, the function $\tilde{U}\left(s^t\right)$ is given by (110). We thereby replace (158) with the following equation:

$$-\left(\frac{1+\Gamma\left(1+\epsilon\right)}{1+\Gamma\left(1-\gamma\right)}\right)\frac{U_\ell\left(s^t\right)}{U_c\left(s^t\right)} = \left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} \alpha \frac{y\left(\omega_i^t, s^t\right)}{\ell\left(\omega_i^t, s^t\right)} \tag{160}$$

Following the proof of Proposition 7, we can solve (159) and (160) simultaneously for $y\left(\omega_i^t, s^t\right)$ and $\ell\left(\omega_i^t, s^t\right)$. We find that output at the Ramsey optimum must satisfy

$$y\left(\omega_i^t, s^t\right) = \left[\frac{U_c\left(s^t\right)}{-U_\ell\left(s^t\right)} Y(s^t)^{\frac{1}{\rho}} \left(A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right)^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$$

where we have abstracted from the constant scalar $\left(\alpha\frac{1+\Gamma(1-\gamma)}{1+\Gamma(1+\epsilon)}\right)^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}}$. Thus, output $y\left(\omega_i^t, s^t\right)$ and labor $\ell\left(\omega_i^t, s^t\right)$ are log-separable in ω_i^t and s^t and satisfy

$$\begin{aligned} y\left(\omega_{it}, s^t\right) &= \Psi^\omega(\omega_i^t)\Psi^s(s^t) && (161) \\ \ell\left(\omega_i^t, s^t\right) &= \Psi^\omega(\omega_i^t)^{\frac{\rho-1}{\rho}}\Psi^s(s^t)^{\frac{1}{\alpha}} && (162) \end{aligned}$$

with

$$\begin{aligned} \Psi^\omega(\omega_i^t) &= \left[A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{1}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} && (163) \\ \Psi^s(s^t) &= \left[\frac{U_c\left(s^t\right)}{-U_\ell\left(s^t\right)} Y(s^t)^{\frac{1}{\rho}}\right]^{\frac{\alpha}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} && (164) \end{aligned}$$

Comparing (163) and (164) to the corresponding equations for Ψ^ω and Ψ^s in the flexible price allocation with the proposed tax scheme, (119) and (120), it is clear that these are identical up to a scalar multiple. This implies that we may write aggregate output as (121) and aggregate labor as in (122). Following the exact same steps as in the proof of Proposition 7, we may describe the Ramsey optimum with equations (132) for the resource constraint, (133) for aggregate sentiment, and (134) for aggregate consumption. We reproduce equation (139) here:

$$\log C\left(s^t\right) = \Gamma_C^{-1} \log A\left(s^t\right) + \Gamma_C^{-1}\left(\varsigma - \Gamma_H\right) \log H\left(s^t\right). \tag{165}$$

We thus reach the same expression for aggregate GDP (consumption) in terms of $\log A\left(s^t\right)$ and $\log H\left(s^t\right)$, abstracting from all constants.

Derivation of Planner's Beauty Contest. What remains to be characterized is the optimal behavior of intermediate good purchases $H\left(s^t\right)$. As in the proof for the flexible price allocaiton, we show that there exists a fixed point in $h\left(\omega_{i,t}\right)$ and $H\left(s^t\right)$ which pins down their joint solution for the Ramsey optimum. To do this, we use the optimality condition for intermediate good purchases given by (17). With our specification of preferences and technology, this optimality condition may be written as follows:

$$\mathbb{E}\left[\tilde{U}_c\left(s^t\right)\left(\left(\frac{y\left(\omega_i^t, s^t\right)}{Y(s^t)}\right)^{-\frac{1}{\rho}} \eta\left(1-\alpha\right) \frac{y\left(\omega_i^t, s^t\right)}{h\left(\omega_i^t\right)} - 1\right) \,\middle|\, \omega_i^t\right] = 0. \tag{166}$$

Recall that with homothetic prefences, the function $\tilde{U}\left(s^t\right)$ satisfies (110). We thereby rewrite equation (166) as follows:

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\eta\left(1-\alpha\right) Y(s^t)^{\frac{1}{\rho}} y\left(\omega_i^t, s^t\right)^{\frac{\rho-1}{\rho}} - h\left(\omega_i^t\right)\right) \,\middle|\, \omega_i^t\right] = 0.$$

The log-separability of $y\left(\omega_{it}, s^t\right)$ implies that this condition may be further expressed as

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\eta\left(1-\alpha\right) Y(s^t)^{\frac{1}{\rho}} \Psi^{\omega}(\omega_i^t)^{\frac{\rho-1}{\rho}} \Psi^{s}(s^t)^{\frac{\rho-1}{\rho}} - h\left(\omega_i^t\right)\right) \,\middle|\, \omega_i^t\right] = 0.$$

Next, plugging in the definition of $\Psi^{\omega}(\omega_i^t)$ from (163) gives us

$$\mathbb{E}\left[U_c\left(s^t\right)\left(\eta\left(1-\alpha\right) Y(s^t)^{\frac{1}{\rho}} \Psi^{s}(s^t)^{\frac{\rho-1}{\rho}} \left[A\left(\omega_i^t\right) h\left(\omega_i^t\right)^{\eta(1-\alpha)}\right]^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} - h\left(\omega_i^t\right)\right) \,\middle|\, \omega_i^t\right] = 0$$

Solving the above equation for h, we obtain the following equation characterizing the firm's optimal choice of intermediate good purchases

$$h\left(\omega_i^t\right)^{1-\eta(1-\alpha)\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} = \eta\left(1-\alpha\right) A\left(\omega_i^t\right)^{\frac{\frac{\rho-1}{\rho}}{1-\alpha\left(\frac{\rho-1}{\rho}\right)}} \frac{\mathbb{E}\left[U_c\left(s^t\right) Y(s^t)^{\frac{1}{\rho}} \Psi^{s}(s^t)^{\frac{\rho-1}{\rho}} \,\middle|\, \omega_i^t\right]}{\mathbb{E}\left[U_c\left(s^t\right) \mid \omega_i^t\right]} \tag{167}$$

We may re-write this in logs as follows:

$$\log h\left(\omega_i^t\right) = \frac{1}{1-\eta\left(1-\alpha\right)\zeta\left(\frac{\rho-1}{\rho}\right)}\left[\zeta\frac{\rho-1}{\rho}\log A\left(\omega_i^t\right) + \frac{1}{\rho}\mathbb{E}_i \log Y(s^t) + \frac{\rho-1}{\rho}\mathbb{E}_i \log \Psi^{s}(s^t)\right] \tag{168}$$

where we have abstracted from the constant scalar and again used $\mathbb{E}_i$ as shorthand for the conditional expectation operator: $\mathbb{E}_i x = \mathbb{E}\left[x | \ \omega_i^t \ \right]$.

We use (136) to replace $\Psi^s(s^t)$ in (168), as the former holds true also in the Ramsey optimal allocation (with different constants). This gives us the following representation

$$\log h\left(\omega_i^t\right) = G_1^*\left(\log A\left(\omega_i^t\right), \ \mathbb{E}_i \log Y(s^t), \ \mathbb{E}_i \log C(s^t), \ \mathbb{E}_i \log H\left(s^t\right)\right) \tag{169}$$

where G_1^* is a linear function of four variables. Next, using the log-linearized resource constraint (132) to replace $Y\left(s^t\right)$, equation (169) may be reduced to

$$\log h\left(\omega_i^t\right) = G_2^*\left(\log A\left(\omega_i^t\right), \ \mathbb{E}_i \log C(s^t), \ \mathbb{E}_i \log H\left(s^t\right)\right) \tag{170}$$

where G_2^* is a linear function of three variables. Finally, using (165) to replace $C(s^t)$ in (170) yields the following result.

Lemma 10. *The Ramsey optimal level of intermediate good purchases satisfy the fixed point*

$$\log h\left(\omega_i^t\right) = m_\omega \log A\left(\omega_i^t\right) + m_A^* \mathbb{E}_i \log A(s^t) + m_H^* \mathbb{E}_i \log H(s^t) \tag{171}$$

with $H\left(s^t\right) = \sum h\left(\omega_i^t\right) \varphi\left(\omega | s^t\right)$, *where* $m_\omega > 0$ *is as defined in (141) and the coefficients* (m_A^*, m_H^*) *are scalars given by*

$$m_A^* = m_A(0) = \delta_A, \text{ and } \ m_H^* = m_H(0) = \delta_H,$$

with δ_A, δ_H *as defined in (142) and (143).*

The fixed-point representation in (171) pins down the Ramsey optimal $h\left(\omega_i^t\right)$ and $H\left(s^t\right)$ for any information structure. Note that this is the same fixed-point representation as in (140) of Lemma 7, but with the tax instruments set at $\hat{\tau}_A = 0$ and $\hat{\tau}_Y = 0$.

Fixed Point Solution to Beauty Contest. We now solve the fixed point described in Lemma 10. Following the exact same steps as in the previous derivation of Lemma 8, we may take the beauty contest formulation given in (171) and transform it as in (144). We thus reach the following result

Lemma 11. *Suppose managers have Gaussian information about the aggregate state. Then the optimal level of intermediate good purchases satisfy the fixed point*

$$\log \tilde{h}\left(\omega_i^t\right) = (1 - \alpha^*)\, \tilde{\chi}^* \mathbb{E}_i \log A(s^t) + \alpha^* \mathbb{E}_i \log \tilde{H}\left(s^t\right) \tag{172}$$

with $\tilde{H}\left(s^t\right) = \sum \tilde{h}\left(\omega_i^t\right) \varphi\left(\omega | s^t\right)$ *and*

$$\alpha^* = m_H^* \qquad \text{and} \qquad \tilde{\chi}^* \equiv \frac{m_A^* + m_H^* m_\omega}{1 - m_H^*} \tag{173}$$

Morevoer, any pair $(\tilde{\alpha}, \tilde{\chi}) \in \mathbb{R}^2$ *can be attained by an appropriate choice of the pair* $(\hat{\tau}_A, \hat{\tau}_Y)$

Without serious loss of generality, we henceforth impose that $\tilde{\chi}^* > 0$, which simply means that the optimal Ht comoves positively with A_t in the frictionless benchmark. Given the above characterization and the previous analysis that followed Lemma 8, it is immediate that the solution to the fixed point described in Lemma 11 is given by

$$\log \tilde{h}^* (x, z) = \phi_0^* + \phi_x^* x + \phi_z^* z \tag{174}$$

where

$$\phi_x^* = \frac{(1-\alpha^*)\kappa_x}{\kappa_0 + (1-\alpha^*)\kappa_x + \kappa_z}\tilde{\chi}^* \tag{175}$$

$$\phi_z^* = \frac{\kappa_z}{\kappa_0 + (1-\alpha^*)\kappa_x + \kappa_z}\tilde{\chi}^* \tag{176}$$

Equation (174) thus gives the optimal level of intermediate good purchases. Furthermore, aggregating over (174) and again transforming back into the true $H(s^t)$ using (146), the optimal level of aggregate intermediate good purchases satisfies

$$\log H(s^t) = (\phi_x^* + \phi_z^* + m_\omega)\log A(s^t) + \phi_z^* u_t + const \tag{177}$$

Equation (177) characterizes the optimal behavior of intermediate good purchases $H(s^t)$ as a function of the aggregate productivity shock and the common noise u_t.

Finally, we compute optimal aggregate consumption (GDP). Using the expression in (177) to replace $H(s^t)$ in equation (165) gives us the following characterization for aggregate consumption:

$$\log C(s^t) = \log GDP(s^t) = \gamma_0^* + \gamma_a^* \log A(s^t) + \gamma_u^* u_t \tag{178}$$

where $(\gamma_0^*, \gamma_A^*, \gamma_u^*)$ are constants. The coefficients (γ_a^*, γ_u^*) satisfy

$$\gamma_a^* = \hat{\gamma} + \upsilon(\phi_x^* + \phi_z^*) \quad \text{and} \quad \gamma_u^* = \upsilon\phi_z^*$$

where $(\hat{\gamma}, \upsilon)$ are strictly positive scalars as defined in (156).

Finally, what remains to be shown is $0 < \gamma_A^* < \gamma_A^{LS}$ and $\gamma_u^* > 0$, where γ_A^{LS} is the coefficient on aggregate productivity in the complete-information Ramsey optimum. First note that

$$\gamma_a^* = \hat{\gamma} + \upsilon\left(\frac{(1-\alpha^*)\kappa_x + \kappa_z}{\kappa_0 + (1-\alpha^*)\kappa_x + \kappa_z}\right)\tilde{\chi}^* \tag{179}$$

with $\hat{\gamma}$, $\upsilon > 0$. Thus $\tilde{\chi}^* > 0$ is sufficient for $\gamma_a^* > 0$ and $\gamma_u^* > 0$. Now, to compare γ_A^* to γ_A^{LS} we finally solve for the complete information optimum and offer the proof of Lemma 6 as promised previously.

Proof of Lemma 6. The optimal allocation under complete information is the same allocation as in (177) and (178), except with $\kappa_x \to \infty$. In this limit,

$$\phi_x^* + \phi_z^* \to \tilde{\chi}^* \quad \text{and} \quad \phi_z^* \to 0$$

Therefore at the complete information optimum,

$$\begin{aligned} \log H^{LS}\left(s^t\right) &= \phi_A^{LS} \log A\left(s^t\right) + const \\ \log C^{LS}\left(s^t\right) &= \gamma_A^{LS} \log A\left(s^t\right) + const \end{aligned}$$

as in (130) and (131), where ϕ_A^{LS} and γ_A^{LS} are scalar parameters given by

$$\phi_A^{LS} = \tilde{\chi}^* + m_\omega \quad \text{and} \quad \gamma_A^{LS} = \hat{\gamma} + \upsilon\tilde{\chi}^* \tag{180}$$

QED.

We now take the difference between γ_A^{LS} and γ_A^*; using the expressions in (179) and (180), this difference is given by

$$\gamma_A^{LS} - \gamma_a^* = \upsilon\tilde{\chi}^* - \upsilon\left(\frac{(1-\alpha^*)\,\kappa_x + \kappa_z}{\kappa_0 + (1-\alpha^*)\,\kappa_x + \kappa_z}\right)\tilde{\chi}^*$$

which implies

$$\gamma_A^{LS} - \gamma_a^* = \upsilon\left[\frac{\kappa_0}{\kappa_0 + (1-\alpha^*)\,\kappa_x + \kappa_z}\right]\tilde{\chi}^*$$

Therefore $\tilde{\chi}^* > 0$ is sufficient for $\gamma_A^{LS} - \gamma_a^* > 0$, and as a result, $0 < \gamma_A^* < \gamma_A^{LS}$. **QED.**

Proof of Proposition 9. Following the proof of Theorem 2, for any arbitrary common-knowledge process z_t, the optimal aggregate price level is given by

$$P(s^t) = e^{z_t}\mathcal{B}(s^t)^{-\frac{1}{\rho}}$$

Taking logs and combining this with expression (133) for $\mathcal{B}(s^t)$, we may express the optimal aggregate price level as

$$\log P(s^t) = -\frac{1}{\rho}\log\mathcal{B}(s^t) = -\frac{1}{\rho}\zeta\left(\Gamma_C \log C\left(s^t\right) + \Gamma_H \log H\left(s^t\right)\right)$$

where we abstract from the common-knowledge process z_t. Next, by substitution of $H\left(s^t\right)$ and $C\left(s^t\right)$ from (177) and (178), we may express the aggregate price level as a log-linear function of A_t and u_t as follows

$$\log P(s^t) = -\frac{1}{\rho}\zeta\left\{\left(\Gamma_C\gamma_a^* + \Gamma_H\left(\phi_x^* + \phi_z^* + m_\omega\right)\right)\log A\left(s^t\right) + \left(\Gamma_C\gamma_u^* + \Gamma_H\phi_z^*\right)u_t\right\}$$

This yields the following expression for the aggregate price level at the Ramsey optimum:

$$\log P(s^t) = -\delta_A^* \log A\left(s^t\right) - \delta_u^* u_t + const$$

as in (32) where δ_A^* and δ_u^* are constants given by

$$\delta_A^* \equiv \frac{1}{\rho}\zeta\left(\Gamma_C \gamma_a^* + \Gamma_H\left(\phi_x^* + \phi_z^* + m_\omega\right)\right) \text{ and } \quad \delta_u^* \equiv \frac{1}{\rho}\zeta\left(\Gamma_C \gamma_u^* + \Gamma_H \phi_z^*\right).$$

Finally, note that

$$\frac{\delta_A^*}{\gamma_a^*} = \frac{1}{\rho}\zeta\left[\Gamma_C + \Gamma_H\left(\frac{\phi_x^* + \phi_z^* + m_\omega}{\gamma_a^*}\right)\right] = \frac{1}{\rho}\zeta\left[\Gamma_C + \Gamma_H\left(\frac{\phi_x^* + \phi_z^* + m_\omega}{\hat{\gamma} + \upsilon\left(\phi_x^* + \phi_z^*\right)}\right)\right] > 0$$

and

$$\frac{\delta_u^*}{\gamma_u^*} = \frac{1}{\rho}\zeta\left(\Gamma_C + \Gamma_H \frac{\phi_z^*}{\gamma_u^*}\right) = \frac{1}{\rho}\zeta\left(\Gamma_C + \Gamma_H \frac{1}{\upsilon}\right) > 0$$

Therefore, the ratios δ_A^*/γ_u^* and δ_u^*/γ_u^* are strictly positive. This, along with $\gamma_a^* > 0$ and $\gamma_u^* > 0$, imply that δ_A^* and δ_u^* are strictly positive. **QED.**

References

Angeletos, George-Marios and Karthik Sastry. 2018. "Inattentive Economies: General Equilibrium and Welfare Theorems." *MIT mimeo* .

Bergemann, Dirk and Stephen Morris. 2013. "Robust Predictions in Games with Incomplete Information." *Econometrica* 81 (4):1251–1308.

Myatt, David P and Chris Wallace. 2012. "Endogenous Information Acquisition in Coordination Games." *Review of Economic Studies* 79 (1):340–374.

Paciello, Luigi and Mirko Wiederholt. 2014. "Exogenous Information, Endogenous Information, and Optimal Monetary Policy." *Review of Economic Studies* 81 (1):356–388.

Pavan, Alessandro. 2016. "Attention, Coordination, and Bounded Recall." *Northwestern University mimeo* .

Romer, Paul M. 1990. "Endogenous Technological Change." *Journal of Political Economy* 98 (5, Part 2):S71–S102.

Sims, Christopher A. 2003. "Implications of Rational Inattention." *Journal of Monetary Economics* 50 (3):665–690.

———. 2010. "Rational Inattention and Monetary Economics." *Handbook of Monetary Economics* 3:155–181.