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ILLIQUIDITY COMPONENT OF CREDIT RISK*

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We provide a theoretical decomposition of bank credit risk into insolvency risk and illiquidity risk, defining illiquidity risk to be the counterfactual probability of failure due to a run when the bank would have survived in the absence of a run. We show that illiquidity risk is (i) decreasing in the "liquidity ratio"—the ratio of realizable cash on the balance sheet to short-term liabilities; (ii) decreasing in the excess return of debt; and (iii) increasing in the solvency uncertainty—a measure of the variance of the asset portfolio.

1. INTRODUCTION

Credit risk refers to the risk of default by borrowers. In the simplest case, where the term of the loan is identical to the term of the borrower's cash flow, credit risk arises from the uncertainty over the cash flow from the borrower's project. However, the turmoil in credit markets in the financial crisis that erupted in 2007 once again highlighted the limitations of focusing just on the value of the asset side of banks' balance sheets. The problem can be posed most starkly for institutions such as Bear Stearns or Lehman Brothers that financed themselves through a combination of short-term and long-term debt, but where the heavy use of short-term debt made the institution vulnerable to a run by the short-term creditors.

The issue is highlighted in an open letter written by Christopher Cox, the (then) chairman of the U.S. Securities and Exchange Commission (SEC) explaining the background and circumstances of the run on Bear Stearns in March 2008.[2]

> [T]he fate of Bear Stearns was the result of a lack of confidence, not a lack of capital. When the tumult began last week, and at all times until its agreement to be acquired by JP Morgan Chase during the weekend, the firm had a capital cushion well above what is required to meet supervisory standards calculated using the Basel II standard.

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[2] Letter to the Chairman of the Basel Committee on Banking Supervision, dated March 20, 2008, posted on the SEC website on: http://www.sec.gov/news/press/2008/2008-48.htm.

> Specifically, even at the time of its sale on Sunday, Bear Stearns' capital, and its broker–dealers' capital, exceeded supervisory standards. Counterparty withdrawals and credit denials, resulting in a loss of liquidity—not inadequate capital—caused Bear's demise.

Thus, in spite of Bear Stearns meeting the letter of its regulatory capital requirements, it got into trouble because its lenders stopped lending. The implication is that the run was liquidity-based instead of solvency-based.

The idea that self-fulfilling bank runs are possible is well established in the banking literature (see Bryant, 1980; Diamond and Dybvig, 1983).[3] But the sharp distinction between solvency and liquidity in the SEC Chairman's letter may not be so easy to draw in practice, even ex post. Our current understanding of the relation between insolvency risk and illiquidity risk is not well developed. Existing models tend to focus on one or the other and not on the interaction between the two. We regard this division of attention as untenable. Runs do not happen out of the blue; they tend to occur when there are also concerns about the quality of the assets, as in the case of Bear Stearns in 2008 and as documented by Gorton (1988) for U.S. bank runs during the 1863–1914 National Banking Era. It is sometimes difficult to tell (even ex post) whether the run merely hastened the failure of a fundamentally insolvent bank or whether the run scuppered an otherwise sound institution. Nevertheless, the distinction between insolvency and illiquidity is meaningful as a counterfactual proposition asking what *would* have happened in unrealized states of the world. The distinction is also important for the policy choices, since the policy response will depend on whether the bank is fundamentally solvent or not. A solvent but illiquid bank could be given emergency funding to tide it over the crisis, but an insolvent bank is best dealt through least cost resolution. In order to address counterfactual "what if" questions, a theoretical framework is required.

For the ex ante pricing of total credit risk, it is important to take account of the probability of a run. This is both because the occurrence of a run will undermine the debt value and because a run will tend to destroy recovery values through disorderly liquidation under distressed circumstances. Merely focusing on the credit risk associated with the fundamentals of the assets will underestimate the total credit risk faced by a long-term creditor. In what follows, we describe a framework that can be used to address these questions. A leveraged financial institution funds its assets using short- and long-term debt as well as its own equity. We show how global game methods (introduced by Carlsson and van Damme, 1993, and used in Morris and Shin, 1998, 2003) can solve for the unique equilibrium in the rollover game among short-term creditors. In particular, we provide an accounting framework to decompose total credit risk into its components. First, the eventual asset value realization may be too low to pay off all debt; this is the insolvency component of credit risk, or "insolvency risk." Second, a run by the short-term creditors may precipitate the failure of the institution even though, in the absence of the run, the asset realizations would have been high enough to pay all creditors; this is the illiquidity component of credit risk, or "illiquidity risk." We illustrate how total credit risk can be decomposed into insolvency risk and illiquidity risk and how the two are jointly determined as a function of the underlying balance sheet.

Our analysis can be used to provide comparative statics and thus may inform policy analysis.[4] Illiquidity risk is decreasing in the "liquidity ratio"—the ratio of realizable cash on the balance sheet to short-term liabilities. This ratio is analogous to the *liquidity coverage ratio* (LCR) in the Basel III bank capital rules[5] and provides further theoretical rationale for liquidity regulation. In general, a bank with low holdings of liquid assets but that is funded by large runnable short-term liabilities will have higher credit risk due to the run risk.

[3] See Gorton (2010) for a modern variation on the classic bank run scenario with an account of the crisis of 2007 as a banking panic with a run on repos instead of deposits.

[4] Because we are taking the balance sheet as given, policy interpretations of comparative statics should be seen describing one channel, ignoring the impact of policy changes on the choice of balance sheet.

[5] http://www.bis.org/publ/bcbs238.pdf.

One advantage of a microfounded model such as ours is that we can gain additional insights into the properties of the equilibrium. For instance, a less obvious comparative statics result is that illiquidity risk is more sensitive to changes in the liquidity ratio when the level of liquidity is low. In addition, illiquidity risk is increasing (and in fact linear) in fundamental uncertainty, as measured by the standard deviation of ex post returns. The conclusion that there is no illiquidity risk without solvency uncertainty is an intuitive one, but not present in many models and much discussion of illiquidity risk. Illiquidity risk is also decreasing in the excess return offered to short-term creditors over their outside options.

Our benchmark analysis examines the pure case of illiquidity risk, where bank failure is due to the inability to meet the withdrawals by creditors. We refer to this case of pure illiquidity risk as "run risk." The risk emanates on the liabilities side of the balance sheet.

An alternative notion of liquidity has to do with the asset side of the balance sheet and has to do with the market risk arising from loss of value on the assets held by the bank. We refer to this second risk as "fire sale risk." In this case, the bank may be able to meet the withdrawals of short-term creditors but—when it does so—it may have to dispose of assets at prices that are at a discount to prices that will rule in normal circumstances. If the fire sale is severe enough, the assets may be depleted sufficiently to lead to subsequent failure.

Bringing all strands together, we can therefore offer the following perspective. As an analytical device, we can decompose illiquidity risk into two components: *run risk* is the probability of failure through inability to meet withdrawals by short-term creditors, even though the bank would have been solvent in the absence of a run; *fire sale risk* is the probability of failure via balance sheet erosion, even when the bank would have been solvent had the assets been disposed without any price discount due to market conditions.

The modeling of run risk provides yet further perspective. As an analytical device, we can subsume run risk into fire sale risk for the calculation of default probabilities by supposing that the bank never fails due to short-term withdrawals per se, but that even if the bank survives the run, withdrawals above a certain level will wipe out the value of assets, inducing failure for sure at a later date.

Building on this special case, we can proceed more generally by incorporating less extreme versions of fire sale risk, thereby allowing more general decomposition of illiquidity risk. In particular, we consider a piecewise linear model of balance sheet degradation, due to Rochet and Vives (2004), where withdrawals above some critical level give rise to a linear cost to the balance sheet, which can be interpreted as the cost associated with selling at fire sale prices. An important distinction is then that fire sale risk (unlike run risk) can arise even as solvency uncertainty becomes negligible.

The distinction between illiquidity risk and insolvency risk—within the same model but with incomplete information—arises in a number of contexts. An early reference is Postlewaite and Vives (1987). Most relevant for us is Rochet and Vives (2004), together with a follow-up paper of Vives (2014). Rochet and Vives (2004) first applied global game methodology to study illiquidity risk for a bank with a stylized balance sheet like that considered here.[6] Our model is a stripped down version of their model, making even more simplifying assumptions to focus on key channels giving rise to the decomposition of credit risk. But there are a number of differences in substance and interpretation. Their focus is on what we call fire sale risk and the piecewise linear model of balance sheet degradation described above. In this setting, illiquidity risk survives even without solvency risk. We emphasize the tight connection between insolvency risk and illiquidity risk. This tight connection arises in our benchmark model with only "run risk," and we note how run risk can be given a fire sale risk interpretation. Given the importance of underlying insolvency risk in our model, the value of preventing runs is much higher when insolvency risk is low than when insolvency risk is high. Adding liquid assets when

[6] The focus of Rochet and Vives (2004) was on the role of lender of last resort policies. Vives (2014) focuses on comparative statics and policy questions similar to those in this article.

a bank is likely to fail anyway is not a good use of resources. It is this that implies decreasing returns to liquidity on the balance sheet.

Other early papers using global game methods to study illiquidity risk are Morris and Shin (2004) and Goldstein and Pauzner (2005). The former gives a reduced-form treatment of the ability of the bank to survive a run. The latter model allows partial repayment in a model of banks offering demand-deposits for the reasons highlighted in Diamond and Dybvig (1983). This article follows Morris and Shin (2004) and Rochet and Vives (2004) in removing partial repayment as a consideration in rollover decisions. Schilling (2016) analyzes the implications of modeling partial repayment in models like this one. Eisenbach et al. (2014) provide an analysis of illiquidity and insolvency risk with richer balance sheets but with a more reduced-form (sunspot) treatment of resolution of the coordination problem.

Our article is a methodological one and is devoted to analytical issues. However, the framework is sufficiently flexible for applications in bank liquidity regulations and for applications beyond the banking sector. Mutual funds and other collective investment vehicles are instances where investor stakes are in the form of equity, not debt-like stakes such as bank deposits. Nevertheless recent literature has highlighted how such funds may still exhibit procyclical tendencies through the variation of cash holdings in the face of redemptions by investors. Since one investor's withdrawal impinges on the interest of other investors, elements of run-like incentives can be present. We return to these issues in the concluding section.

2. DECOMPOSITION OF CREDIT RISK

Consider the stylized balance sheet of a leveraged financial institution, called a "bank" for convenience. On the asset side, the bank holds two assets: a safe asset and a risky asset. We assume that the safe asset M ("bonds") is liquid and safe. The risky asset Y ("loans") is risky and cannot be sold (or repoed).[7] On the liability side, the bank has short-term debt S and long-term debt L. Writing E for the value of equity (Assets minus Liabilities), we thus have balance sheet

Assets	Liabilities
Cash M	Equity E
Risky asset Y	Short debt S
	Long debt L

The interest rate on long and short debt and the return on cash and bonds are all normalized to be 0. At date 1, the gross return θ on the risky asset is uncertain. It is uniformly distributed on the interval $[\overline{\theta} - 1/2 \cdot \sigma, \overline{\theta} + 1/2 \cdot \sigma]$. Thus, $\overline{\theta}$ is the mean return of the asset at date 1 and σ is a measure of the variation. We will be interested in decomposing credit risk at date 1 as a function of $\overline{\theta}$ and later allow there to be uncertainty about $\overline{\theta}$ at a prior time 0. The return is realized at date 2. An important parameter in our analysis will be the liquidity ratio, specifying what proportion of creditors could be paid if all creditors ran: $\lambda = M/S$. A maintained assumption is that the liquidity ratio is less than 1, so that short-term debt exceeds the assets that can be sold to pay it. Without this assumption, there will not be illiquidity risk.

2.1. *Insolvency Risk.*

If nothing has happened before date 2, the equity of the bank is

$$M + \theta Y - S - L.$$

The bank is *solvent* at date 2 if this expression is positive, that is, if

[7] For simplicity, we focus on two assets, one liquid and safe and one illiquid and risky. The analysis can be extended to allow independent variation in illiquidity and riskiness: see Morris and Shin (2010) on this issue.

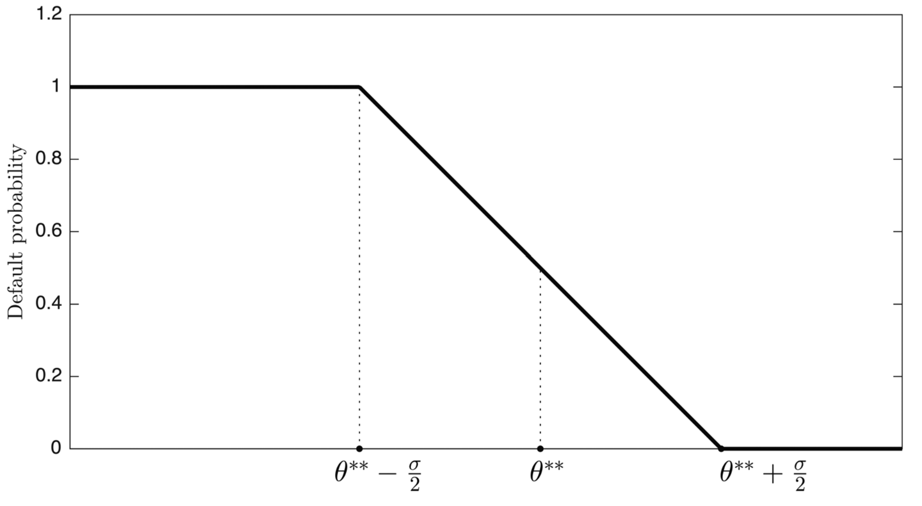

FIGURE 1

UNIFORM INSOLVENCY

$$\theta \geq \theta^{**} = \frac{S + L - M}{Y}. \tag{1}$$

We will refer to θ^{**} as *the solvency point*. We define *insolvency risk* at date 1 to be the probability that the bank fails under this scenario. Insolvency risk is then given by

$$\mathcal{S}\left(\overline{\theta}\right) = \Pr\left(\theta \leq \theta^{**}\right) = \begin{cases} 1, & \text{if} \quad \overline{\theta} \leq \theta^{**} - \dfrac{1}{2}\sigma \\ \dfrac{1}{2} + \dfrac{\overline{\theta} - \theta^{**}}{\sigma}, & \text{if} \quad \theta^{**} - \dfrac{1}{2}\sigma \leq \overline{\theta} \leq \theta^{**} + \dfrac{1}{2}\sigma \\ 0, & \text{if} \quad \theta^{**} + \dfrac{1}{2}\sigma \leq \overline{\theta}. \end{cases}$$

Insolvency risk is plotted in Figure 1.

2.2. *Illiquidity Risk.* Illiquidity risk is defined to be the risk that the bank fails due to a run at date 1 when it would have been solvent at date 2 without the run. Short-term creditors have the option to roll over their debt at date 1, or to run (i.e., not roll over). If they choose to run, they have an outside option α with $0 < \alpha < 1$. Another maintained assumption will be that $\alpha < \lambda$. If this assumption failed, we will see that creditors would always want to run in equilibrium. Put differently, this assumption ensures that there will be no runs in the absence of insolvency risk. If either the bank cannot cover short-term withdrawals at date 1 or if the bank is insolvent at date 2, then debt holders receive no repayment. If both short-term withdrawals are met at date 1 and the bank is solvent at date 2, then short-term debt holders get a payment of 1.

There is thus a coordination problem for creditors. If proportion π of creditors run, then the bank will survive if

$$\pi S \leq M. \tag{2}$$

In calculating illiquidity risk, we initially assume[8] that—if the bank survives the run—there is no deterioration of the bank's balance sheet, even though assets are sold to meet withdrawals. Thus if the bank survives the run, it is possible to buy the assets back at par.

[8] The assumption is relaxed in Section 3.

In characterizing illiquidity risk, assume for now that there is a critical θ_0^* below which all creditors run and above which all creditors will roll over and that, at the critical θ_0^*, each creditor is indifferent between running and rolling over if he has a uniform belief over the proportion of other creditors running. We will see in the next subsection that this feature holds at the equilibrium switching point.

Thus, assume that π is uniformly distributed on the interval [0, 1]. Then, the probability that condition 2 holds will be equal to the liquidity ratio $\lambda = M/S$. If the bank does survive, it may still be insolvent. So the expected return of short-term creditors will be the probability that bank survives a run at date 1 times the probability that the bank is not insolvent at date 2. This gives an expected return of

$$\lambda \left(1 - \mathcal{S}\left(\overline{\theta}\right)\right).$$

Short-run creditors will be indifferent between rolling over or running when this expression equals the outside option. This occurs when insolvency risk is between 0 and 1 and

$$\lambda \left(\frac{1}{2} + \frac{\theta^{**} - \overline{\theta}}{\sigma} \right) = \alpha. \tag{3}$$

Write θ_0^* for the critical value of $\overline{\theta}$ where this holds. This implies[9]

$$\theta_0^* = \theta^{**} - \sigma \left(\frac{\alpha}{\lambda} - \frac{1}{2} \right).$$

Illiquidity risk is the probability that the bank fails due to a run when it would have survived in the absence of a run. Illiquidity risk is therefore given by

$$\mathcal{I}\left(\overline{\theta}\right) = \begin{cases} 0, & \text{if} \quad \overline{\theta} \leq \theta^{**} - \dfrac{1}{2}\sigma \\ \dfrac{1}{2} - \dfrac{1}{\sigma}\left(\theta^{**} - \overline{\theta}\right), & \text{if} \quad \overline{\theta} \in \left[\theta^{**} - \frac{1}{2}\sigma, \theta^{*} + \sigma\left(\frac{\alpha S}{M} - \frac{1}{2}\right)\right] \\ 0, & \text{if} \quad \overline{\theta} > \theta^{**} + \sigma\left(\dfrac{\alpha S}{M} - \dfrac{1}{2}\right). \end{cases}$$

Insolvency risk and illiquidity risk are plotted together in Figure 2.

2.3. *Aside: Global Game Foundation for Creditor Beliefs.* It was assumed in the previous section that creditors had a uniform belief over the proportion of creditors running at the critical θ_0^*, and thus assigned probability λ to a run-induced bank failure. Suppose instead that creditors do not know $\overline{\theta}$ precisely and that there is variation in their expectations of $\overline{\theta}$. Assume that for sufficiently low $\overline{\theta}$ creditors would have a dominant strategy to run, and for sufficiently high $\overline{\theta}$ creditors would have a dominant strategy to roll over.[10] Say that a creditor

[9] Observe that, under out maintained assumptions that $0 < \alpha < 1$ and $\alpha \leq \lambda \leq 1$, the expression $\frac{\alpha}{\lambda} - \frac{1}{2}$ is in the interval $[-\frac{1}{2}, \frac{1}{2}]$ and thus insolvency risk is strictly between 0 and 1 at θ_0^*. As $\lambda \downarrow \alpha$, $\theta_0^* \to \theta^{**} + \frac{1}{2}\sigma$, and so runs always occur when insolvency risk is positive in that limit. As $\lambda \uparrow 1$, $\theta_0^* \to \theta^{**} + \sigma(\alpha - \frac{1}{2})$ and thus runs occur when the insolvency risk is greater than $1 - \alpha$.

[10] The former condition is implied by payoffs in our game: for sufficiently low $\overline{\theta}$, creditors are certain that the bank is insolvent and will surely run. The latter condition is not implied by the payoffs, since a run can always occur for any $\overline{\theta}$. But the latter condition is easily justified by a natural perturbation of the model. Suppose that the bank had an extra risky asset on the balance sheet that could be used as collateral to borrow an amount at date 1 that increases without bound in $\overline{\theta}$. An arbitrarily small amount of this extra asset would ensure that there was a dominant strategy to roll over if $\overline{\theta}$ was sufficiently high.

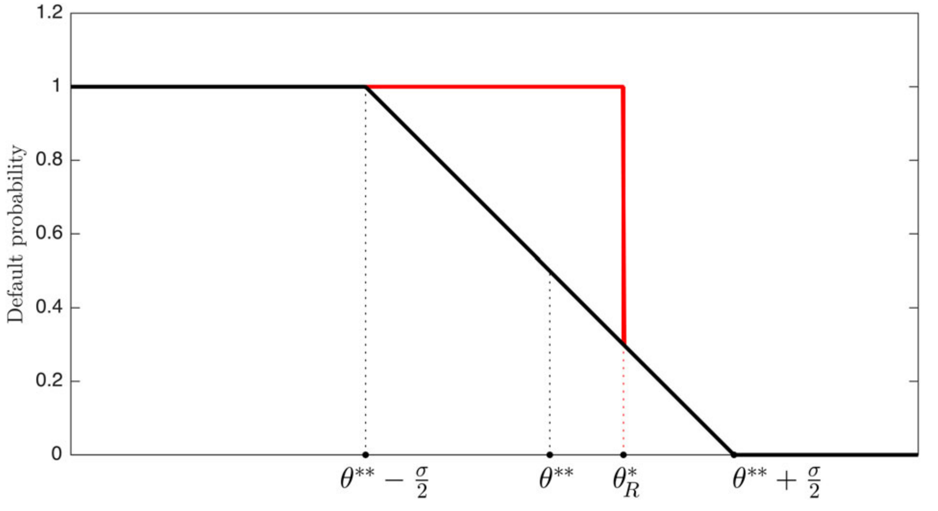

FIGURE 2

UNIFORM ILLIQUIDITY

has "rank belief" r if the proportion of creditors with a higher expectation of $\overline{\theta}$ is r. There is "common certainty of uniform rank beliefs" if each creditor always has a uniform belief about his rank independent of his expectation of $\overline{\theta}$. Morris et al. (2016) show if there is common certainty of uniform rank beliefs in games like that studied here, then there will be a unique strategy surviving iterated deletion of dominated strategies where a creditor withdraws if his signal is below his best response to uniform beliefs about his opponents' actions (i.e., θ_0^* in this example).[11] If agents' expectations of $\overline{\theta}$ are tightly distributed around the true $\overline{\theta}$, this argument rationalizes the assumption pinning down θ_0^* in the previous section.

Common certainty of rank beliefs is implied by assumptions made in the (symmetric) global games literature (Carlsson and van Damme, 1993; Morris and Shin, 1998). In particular, suppose that each creditor observed a noisy signal of $\overline{\theta}$, $x_i = \overline{\theta} + \tau\varepsilon$, where the noise terms ε were distributed in the population according to $\varepsilon \sim f(\cdot)$. Assume that the prior distribution of $\overline{\theta}$ is given by density $g(\cdot)$. With this information, what rank beliefs will creditors hold? If $g(\cdot)$ is uniform, or if τ is small, the creditor will have uniform beliefs on π independent of x_i (Morris and Shin, 2003). Thus there will be common certainty of rank beliefs.[12]

2.4. *Ex Ante Credit Risk.* It is possible to carry out comparative statics and policy analysis on the run points in the "interim" period 1. However, in analyzing comparative statics and policy, it is important to characterize illiquidity risk at an ex ante stage, where policymakers do not know exactly how much liquidity is needed prevent a run. Thus we consider a prior time 0, where $\overline{\theta}$—the time 1 expectation of θ—is distributed uniformly on

$$\left[\widehat{\theta} - \frac{1}{2}\xi, \widehat{\theta} + \frac{1}{2}\xi\right].$$

We assume that ξ is large enough so that all $\widehat{\theta} \in [\theta^{**} - 1/2 \cdot \sigma, \theta^{**} + 1/2 \cdot \sigma]$ are possible at date 0. Recall that

[11] This is true under weaker conditions: It is enough that there is common certainty of approximately uniform rank beliefs conditional on there not being dominant actions. Morris et al. (2016) show this for a class of games where payoffs are separable in others' actions and the state. This class does not include the "regime change" game discussed here, but the argument easily extends.

[12] More precisely, the weakening of common certainty of uniform rank beliefs described in footnote 11.

$$\theta_0^* = \theta^{**} - \sigma\left(\frac{\alpha}{\lambda} - \frac{1}{2}\right).$$

Now ex ante illiquidity risk will be $1/\xi$ times the area of the illiquidity risk triangle in Figure 2:

$$\frac{\sigma}{2\xi}\left(\frac{\alpha}{\lambda}\right)^2. \qquad \text{(ex ante liquidity risk)}$$

This expression allows the type of comparative statics analysis discussed in the Introduction. Note that this expression is decreasing in the liquidity ratio, λ; increasing (and in fact linear) in solvency uncertainty, σ; and decreasing in the excess return offered to short-term creditors over outside their outside options, $1/\alpha$. Also observe that the size of the derivative of liquidity risk with respect to its level, $\sigma\alpha^2/(\xi\lambda^3)$, is decreasing in λ. The most interesting and robust channel driving this result is that the run point θ_0^* is decreasing in the liquidity ratio and the reduction in illiquidity risk associated with any reduction of θ_0^* is equal to 1 minus the insolvency risk. And the insolvency risk is decreasing in $\overline{\theta}$. In short, there is a small benefit to preventing runs when the probability of failure due to insolvency is high.

2.5. *General Payoffs.* In order to derive simple expressions and visualizations, it was convenient for us to focus on the case where θ had a uniform distribution. We will briefly note how the analysis extends to general distributions on θ. Beyond checking robustness and illustrating our results in a more general case, this extension will give a useful comparison with the analysis in the next section.

So now suppose that $\theta = \sigma\varepsilon$ where ε is drawn according to density $f(\cdot)$. Now the expression for the run point θ_0^*, analogous to (3), will be

$$\theta_0^* = \theta^{**} - \sigma F^{-1}\left(1 - \frac{\alpha}{\lambda}\right). \tag{4}$$

Insolvency and illiquidity risk are plotted in Figures 3 and 4, respectively, for the case where $\sigma = 0.25$.[13]

3. BALANCE SHEET DEGRADATION

So far, we have maintained the assumption that if the bank survives the run, there is no long-term degradation of its balance sheet. However, illiquidity problems often lead to bank failure not because the banks cannot pay off their short-term creditors at the time of withdrawal ("run risk"), but because the problems lead to degradation of the balance sheet ("fire sale risk") that leads to their later failure. In this section, we make the observation that run risk—such as that in our benchmark model—can always be reinterpreted as fire sale risk. But we also study the decomposition of illiquidity risk into run risk—corresponding to the illiquidity risk as modeled and interpreted in our benchmark model—and fire sale risk operating through balance sheet degradation.

3.1. *Interpretation of the Benchmark Model in Terms of FireSale Risk.* Suppose now that the bank can always come up with enough cash to pay off short-term creditors but the cost of doing so impairs the balance sheet. We can define an impairment function $\widetilde{\delta} : \mathbb{R}_+ \to \mathbb{R}_+$ where $\widetilde{\delta}(Z)$ is the cost to the balance sheet if mass Z of creditors withdraw. The assumption in our benchmark model was that the bank fails at the time of creditor withdrawals if those withdrawals exceed M. We can give an alternative interpretation to this outcome by means of the impairment function $\widetilde{\delta}(Z)$. Consider the assumption that the impairment of balance sheet is ∞ if withdrawals exceed

[13] All figures are plotted for the case where $M = 3$, $y = 1$, $S = 5$, $L = 2$, and $\alpha = \frac{1}{2}$.

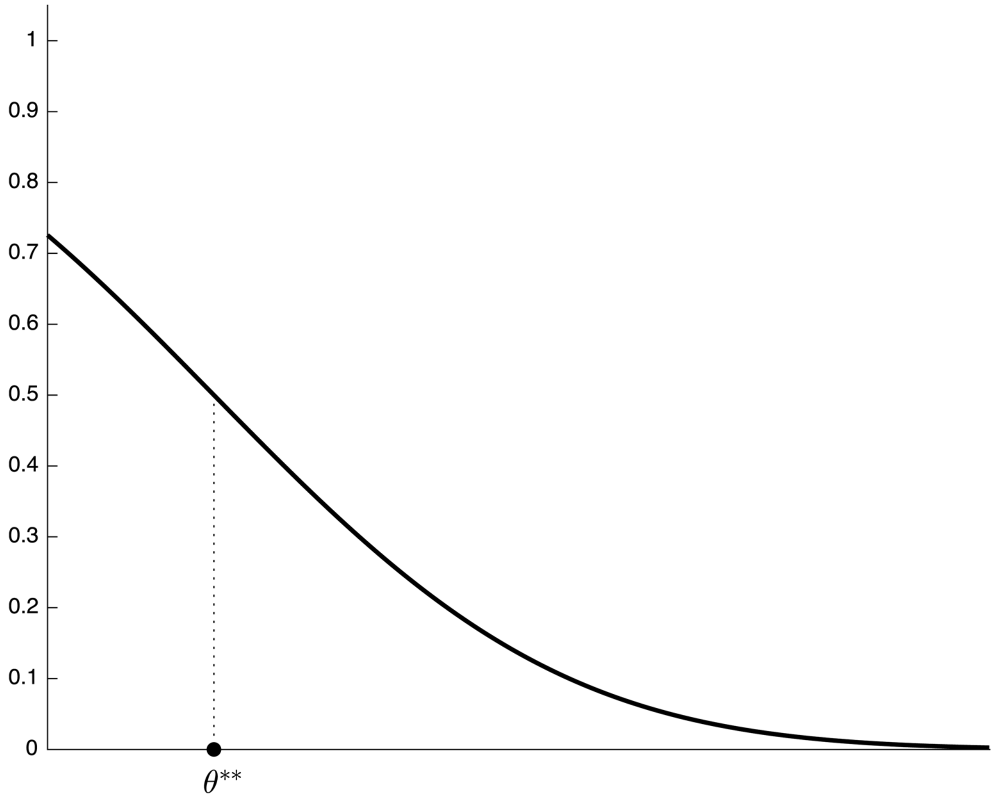

FIGURE 3

NORMAL INSOLVENCY

M, but is 0 otherwise. In this case, the bank is doomed to failure if withdrawals exceed M. For the purpose of calculating the ex ante probability of failure, adopting the impairment function yields results identical to the benchmark pure run-risk model. Formally, the benchmark model can be interpreted as one with only fire sale risk if

$$\widetilde{\delta}(Z) = \begin{cases} 0, & \text{if} \quad Z \leq M \\ \infty, & \text{if} \quad Z > M. \end{cases}$$

A less extreme assumption would that balance sheet impairment is increasing in Z with derivative at most 1 and convex when $\widetilde{\delta}'(Z) < 1$. This case now has a natural interpretation: $\widetilde{\delta}'(Z)$ is the price discount associated with the Zth unit of the asset sold, with assets with the smallest price discount sold first. A simple example of this impairment function would be a piecewise linear one with

$$\widetilde{\delta}(Z) = \begin{cases} 0, & \text{if} \quad Z \leq M_0 \\ \delta(Z - M_0), & \text{if} \quad M_0 \leq Z \leq M \\ \infty, & \text{if} \quad M < Z. \end{cases}$$

3.2. *Another Risk Decomposition.* Instead of reinterpreting the illiquidity risk in our benchmark model, we can also explicitly model a natural decomposition of illiquidity risk into run risk and fire sale risk. We now suppose that the bank will fail in period 1 if withdrawals exceed M; the bank will survive with an unimpaired balance sheet if withdrawals are less than M_0; but if

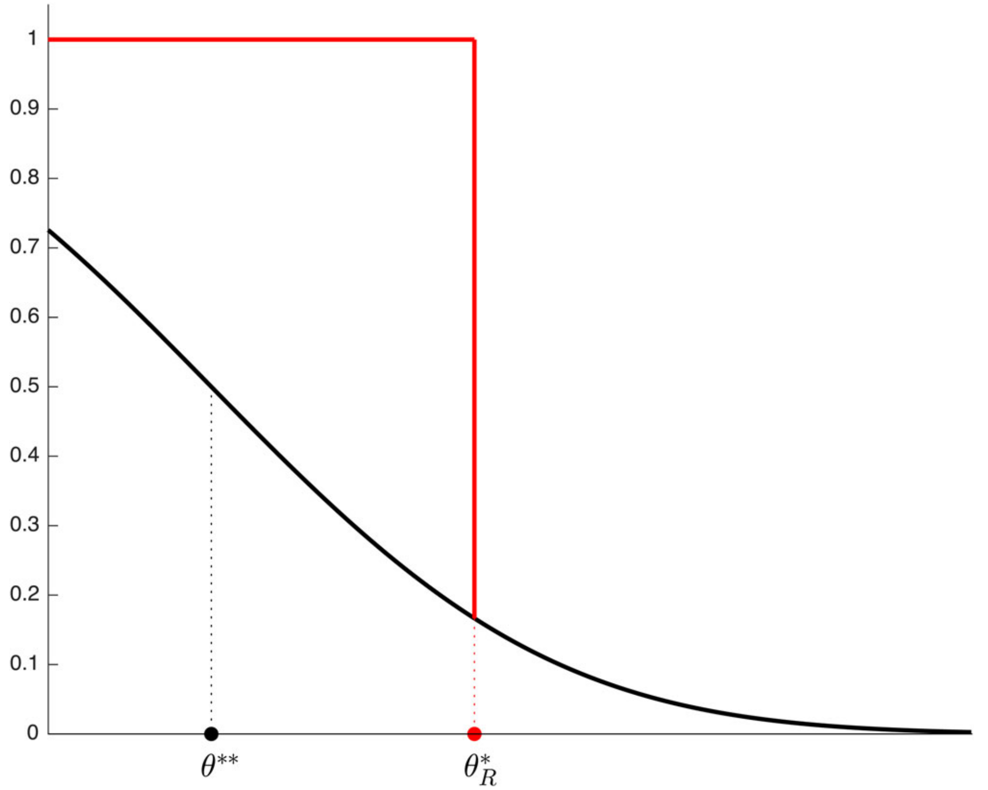

FIGURE 4

NORMAL ILLIQUIDITY

withdrawals Z are greater than M_0 and less than M, the bank will survive but the balance sheet will take a loss of $\delta(\pi S - M_0)$.

With this interpretation, the piecewise linear impairment function is that studied by Rochet and Vives (2004) and Vives (2014). Now if we write π for the proportion of creditors who run, we have that if $M_0 \leq \pi S \leq M$, there is an adjusted solvency point of

$$\begin{aligned}\theta^{**}_{\delta}(\pi) &= \frac{S + L + \delta(\pi S - M_0) - M}{Y} \\ &= \theta^{**} + \frac{\delta(\pi S - M_0)}{Y}.\end{aligned}$$

Now the run point θ^*_δ, analogous to (3) and (4), will be implicitly defined by

$$\frac{M_0}{S} F\left(\frac{\theta^{**} - \theta^*_\delta}{\sigma}\right) + \int_{\pi=\frac{M_0}{S}}^{\frac{M}{S}} F\left(\frac{\theta^{**} - \theta^*_\delta + \frac{\delta}{Y}(\pi S - M_0)}{\sigma}\right) d\pi = \frac{M}{S} - \alpha.$$

We cannot solve this in closed form. But we can compute θ^*_δ and identify fire sale risk with one minus the insolvency risk when $\theta \in [\theta^*_0, \theta^*_\delta]$. Fire sale risk is added into Figure 5.

Now if solvency uncertainty is reduced (to $\sigma = 0.05$), we have Figure 6.

The figures illustrate a striking property: as $\sigma \to 0$, "run risk" disappears, but fire sale risk does not disappear. In fact, we can give a closed-form expression for the fire sale point when $\sigma \to 0$. If $\overline{\theta}$ is just above θ^{**}, balance sheet degradation will imply eventual failure even when

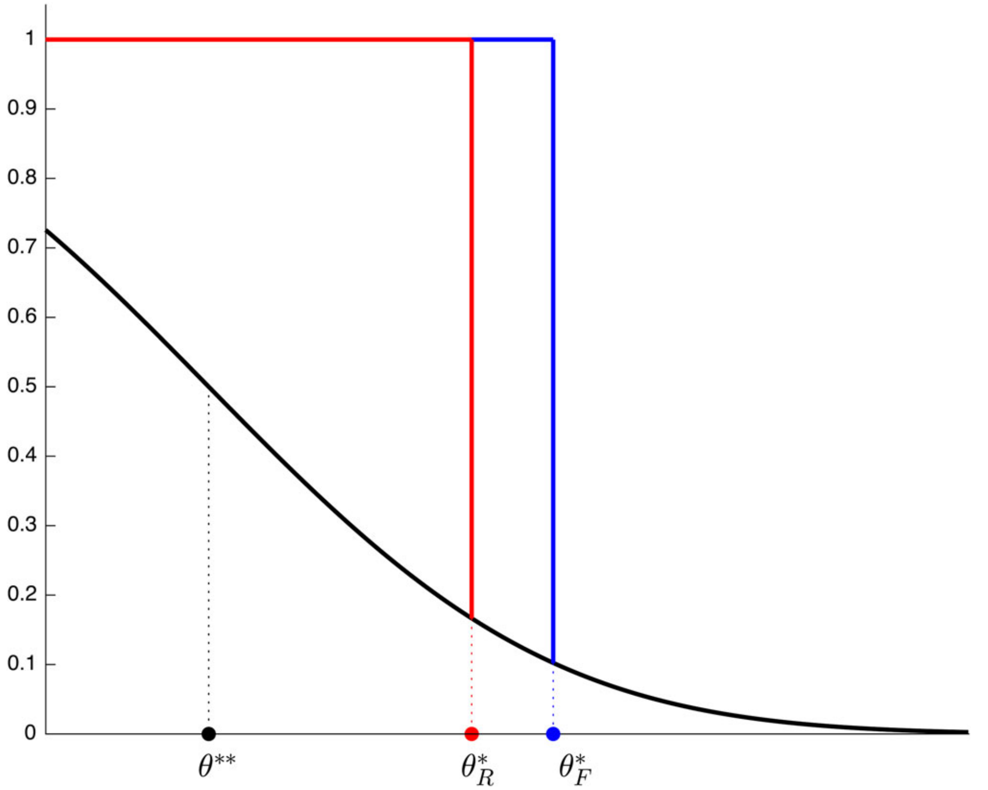

FIGURE 5

NORMAL FIRE SALE

solvency uncertainty is small. Suppose that $M_0 \leq \alpha S \leq M$. In this case, if the fire sale point were θ^*_δ, there will not be a run as $\sigma \to 0$, only if π satisfies

$$\theta^{**} + \frac{\delta\,(\pi S - M_0)}{Y} \geq \theta^*_\delta.$$

Rearranging gives the critical proportion of creditors at which a run will occur:

$$\pi^*_\delta = \frac{1}{S}\left(\frac{(\theta^*_\delta - \theta^{**})\,Y}{\delta} + M_0\right). \tag{5}$$

Assuming as before that there will be a uniform probability distribution over the proportion of creditors running (justified by the arguments in Subsection 2.3), we have that the probability of the bank surviving is π^*_δ. Equating this to the outside option, we have

$$\frac{1}{S}\left(\frac{(\theta^*_\delta - \theta^{**})\,Y}{\delta} + M_0\right) = \alpha. \tag{6}$$

But this in turn gives an expression for the fire sale point as $\sigma \to 0$:

$$\theta^*_\delta = \theta^{**} + \delta\left(\frac{\alpha S - M_0}{Y}\right). \tag{7}$$

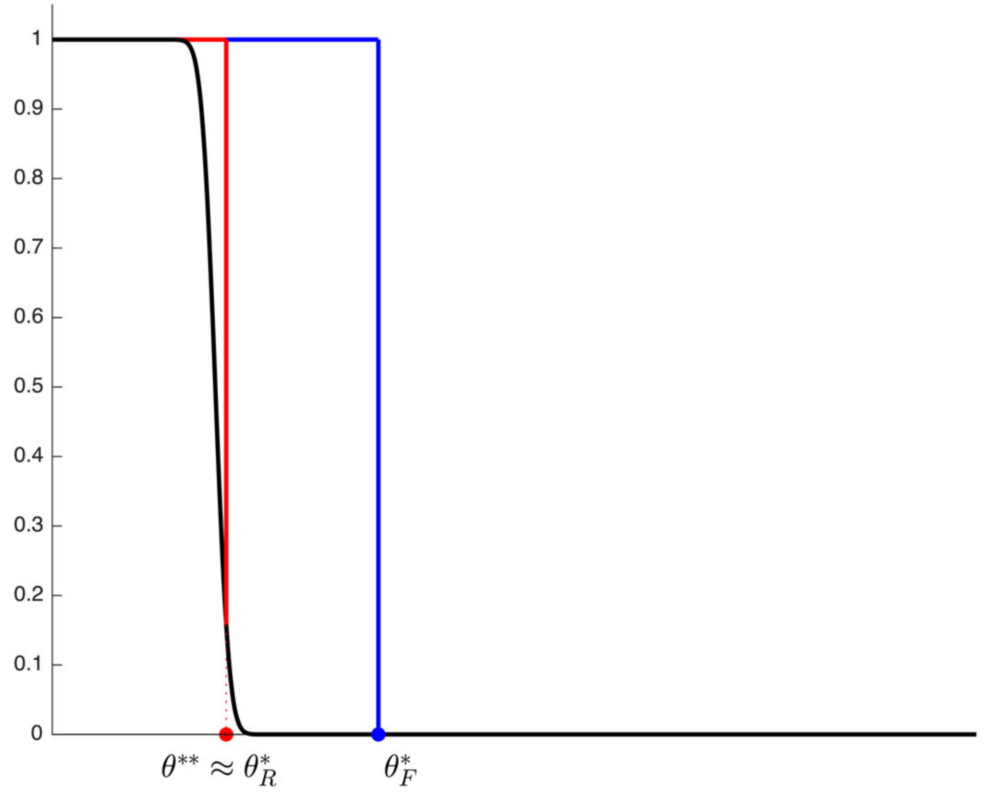

FIGURE 6

VERY LOW SOLVENCY UNCERTAINTY

Now the intuition for the existence of illiquidity risk surviving even as insolvency risk disappears is as follows: Suppose that $\overline{\theta}$ was just above θ^{**}. As solvency uncertainty disappears, all creditors must be certain that the bank would survive if no one runs. Without balance sheet degradation, no one would run under our maintained assumptions. But with balance sheet degradation and strategic uncertainty about how many creditors will run, creditors will have an incentive run when $\overline{\theta}$ is sufficiently close to θ^{**}.

To understand the exact formulas, observe that θ_δ^* corresponds to the shift in the solvency point if exactly proportion α of creditors ran: $\delta(\alpha S - M_0)$ if the loss of assets, and so $\delta(\alpha S - M_0)/Y$ is change in the return on risky assets sufficient to compensate for that loss. But if θ_δ^* took this value, the critical proportion who would need to run in order generate insolvency through balance sheet degradation would be α (this can be verified by substituting (7) into (5)). But now uniform strategic uncertainty implies that the expected return from rolling over equals the outside option.

4. CONCLUDING REMARKS

We conclude with a discussion of possible applications of our framework. We will mention two in particular.

A natural application of our framework is in the calibration of the Basel III LCR,[14] which is at the core of the Basel III bank capital and liquidity rules.

[14] http://www.bis.org/publ/bcbs238.pdf.

The LCR is defined as the ratio of High-Quality Liquid Assets (HQLA) to a weighted sum of the liabilities of the bank, where the weights reflect the "runability" of the liabilities in question. The weights vary from 3% for stable retail deposits to 100% in the case of some wholesale funding. The Basel III LCR rule stipulates that this ratio be maintained at 1 or higher. A bank can meet this requirement either by holding liquid assets that qualify for HQLA status or to fund itself with more stable liabilities that are less prone to run.

The solution for the illiquidity risk is possible in closed form in some cases—for instance for the "pure run" scenario—but in general numerical methods will be necessary to ascertain the actual magnitudes. Even so, from a conceptual viewpoint, our framework provides basic conceptual anchors that will allow more detailed modeling and numerical calibration, as necessary.

As well as tackling analytical issues in the banking sector, our model may also prove useful in the discussions concerning the financial stability risks associated with asset managers—especially with mutual funds and other similar collective investment vehicles where investors are able to redeem their stakes at short notice.

Although asset managers do not employ much leverage (if at all), episodes of concerted selling by investors may generate fire-sale externalities in the sense examined in our article. As a result, the asset manager who manages the mutual fund may respond by hoarding cash as a precautionary measure. In turn, however, the cash position of the mutual fund will feed into the redemption decision of the investors themselves. In this way, redemption risk (a type of run risk) and cash hoarding by the asset manager will depend on each other and will need to be solved jointly.

A recent BIS study (Shek et al., 2015) examined the issue of cash hoarding by emerging market bond mutual fund managers. Their approach was to distinguish investor-driven sales and discretionary sales by comparing changes in cash holdings with the inflows and outflows of investors' money. At its simplest, consider a hypothetical passive mutual fund that holds no cash and is fully invested in bonds at all times. Then, redemptions by investors result in sales of the same amount. In this case, we define all sales to be driven by investor flows, and there are no discretionary sales by the fund managers.

But now consider an alternative scenario with the same amount of investor redemptions. Suppose that the fund starts with no cash holding at the beginning of the period, but ends the period with a positive holding of cash, in spite of the investor redemptions. Then the positive cash holding at the end of the period can be regarded as the additional, discretionary sales undertaken by the fund, as the fund has ended up selling more than was strictly necessary to meet investor redemptions. This simple logic can be extended to funds that start the period with positive cash holdings. We can define discretionary sales so that the fund has undertaken discretionary sales by the amount of the increase in cash holdings during the period. This would be quite a conservative definition of discretionary sales that allows funds to hold some cash, but only deems sales to be discretionary if the cash holdings increase in spite of investor redemptions.

Shek et al. (2015) find that for every 100 dollars' worth of sales due to redemptions by ultimate investors, there is another 10 dollars' worth of discretionary bond sales by the manager that can be attributed to cash hoarding. In this way, an element of procyclicality is injected into the problem through the precautionary liquidity hoarding by the asset managers.

Our framework has been developed in the context of banking. However, the basic ideas would be applicable also in the context of collective investment vehicles such as mutual funds. The insights gained may be useful in examining the financial stability implications of fire sale risk associated with mutual funds.

REFERENCES

BRYANT, B., "A Model of Reserves, Bank Runs, and Deposit Insurance," *Journal of Banking and Finance* 4 (1980), 335–44.

CARLSSON, H., AND E. VAN DAMME, "Global Games and Equilibrium Selection," *Econometrica* 61 (1993), 989–1018.

DIAMOND, D., AND P. DYBVIG, "Bank Runs, Deposit Insurance, and Liquidity," *Journal of Political Economy* 91 (1983), 401–19.

EISENBACH, T., T. KEISTER, J. MCANDREWS, AND T. YORULMAZER, "Stability of Funding Models," *Federal Reserve Bank of New York Economic Policy Review* 20 (2014).

GOLDSTEIN, I., AND A. PAUZNER, "Demand Deposit Contracts and the Probability of Bank Runs," *Journal of Finance* 60 (2005), 1293–328.

GORTON, G., "Banking Panics and Business Cycles," *Oxford Economic Papers* 40 (1988), 751–81.

———, *Slapped by the Invisible Hand: The Panic of 2007* (Oxford: Oxford University Press, 2010).

MORRIS, S., AND H. S. SHIN, "Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks," *American Economic Review* 88 (1998), 587–97.

———, AND ———, "Global Games: Theory and Applications," in M. Dewatripont, L. Hansen, and S. Turnovsky, eds., *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society)* (Cambridge, England: Cambridge University Press, 2003), 56–114.

———, AND ———, "Coordination Risk and the Price of Debt," *European Economic Review* 48 (2004), 133–53.

———, AND ———, "Illiquidity Component of Credit Risk," available at http://www.princeton.edu/˜smorris/pdfs/Morris-IlliquidityComponentofCreditRisk.pdf (2010).

———, ———, AND M. YILDIZ, "Common Belief Foundations of Global Games," *Journal of Economic Theory* 163 (2016), 826–48.

POSTLEWAITE, A., AND X. VIVES, "Bank Runs as an Equilibrium Phenomenon," *Journal of Political Economy* 95 (1987), 485–91.

ROCHET, J.-C., AND X. VIVES, "Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?" *Journal of the European Economic Association* 2 (2004), 1116–47.

SCHILLING, L., "Capital Structure, Liquidity and Miscoordination on Runs," Social Science Research Network working paper, 2016. Available at http://ssrn.com/abstract=2673980 or http://dx.doi.org/10.2139/ssrn.2673980.

SHEK, J., I. SHIM, AND H. S. SHIN, "Investor Redemptions and Fund Manager Sales of Emerging Market Bonds: How Are They Related?" BIS Working Papers 509, 2015.

VIVES, X., "Strategic Complementarity, Fragility, and Regulation," *Review of Financial Studies* 27 (2014), 3547–92.