# FROM HYPERINFLATION TO STABLE PRICES: ARGENTINA'S EVIDENCE ON MENU COST MODELS\*

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In this article, we analyze how inflation affects firms' price-setting behavior. For a class of menu cost models, we derive several predictions about how pricesetting changes with inflation at very high and at near-zero inflation rates. Then, we present evidence supporting these predictions using product-level data underlying Argentina's consumer price index from 1988 to 1997—a unique experience where monthly inflation ranged from almost 200% to less than zero. For low inflation rates, we find that (i) the frequency and absolute size of price changes as well as the dispersion of relative prices do not change with inflation, (ii) the frequency and size of price increases and decreases are symmetric around zero inflation, and (iii) aggregate inflation changes are mostly driven by changes in the frequency of price increases and decreases, as opposed to the size of price changes. For high inflation rates, we find that (iv) the elasticity of the frequency of price changes with respect to inflation is close to two-thirds, (v) the frequency of price changes across different products becomes similar, and (vi) the elasticity of the dispersion of relative prices with respect to inflation is one-third. Our findings confirm and extend available evidence for countries that experienced either very high or nearzero inflation. We conclude by showing that a hyperinflation of 500% a year is associated with a cost of approximately 8.5% of aggregate output a year as a result of inefficient price dispersion alone. JEL Codes: E31, E50.

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#### I. Introduction

Infrequent nominal price adjustments are at the center of much of the literature studying positive and normative implications of inflation. In this article, we study how price-setting behavior changes with inflation, both theoretically and empirically. We consider menu cost models with idiosyncratic firm-level shocks, that is, models where monopolistic firms set prices subject to a fixed cost of adjustment and are hit by shocks to their real marginal cost. We derive sharp predictions concerning how changes in steady-state inflation affect price-setting behavior at near-zero and very high inflation rates. Then we confront these predictions with their empirical counterparts using product-level data underlying Argentina's consumer price index from 1988 to 1997. Argentina's experience provides a unique opportunity to analyze price-setting behavior through the lens of menu cost models because it encompasses several years of price stability (and even deflation) as well as years of sustained very high inflation.

We find evidence supporting many predictions of menu cost models. Our empirical findings involve confronting a number of variables (summarizing price-setting behavior) across time periods with different inflation rates. Our theoretical results involve comparative statics about how these same variables change across steady states with different inflation rates. However, we show that when idiosyncratic shocks are persistent and discount rates are low, these comparative statics depend on the ratio of inflation to the variance of firm-level idiosyncratic shocks. Therefore, they are comparable to our empirical findings under the assumption that the variance of idiosyncratic shocks remains approximately constant across time periods and that current inflation approximates expected future inflation. The first set of results concerns lowinflation economies. Theoretically and empirically, we show that in a neighborhood of zero inflation (i) the frequency of price changes is unresponsive to inflation; (ii) the dispersion of relative prices is unresponsive to inflation; (iii) the frequency of price increases is equal to the frequency of price decreases; (iv) conditional on a price change, the size of price increases is equal to the size of price decreases; and (v) inflation changes mostly (to be precise, 90%) due to the changes in the difference between the frequency

<sup>1.</sup> Later we discuss how we empirically implement measures of expected future inflation.

of price increases and of price decreases—as opposed to changes in the size of price increases and decreases. The second set of results concerns high-inflation economies. We find that (vi) the frequency of price adjustment becomes the same for all products/firms, (vii) the elasticity of the average frequency of price changes with respect to inflation converges to two-thirds, (viii) the elasticity of the dispersion of relative prices converges to one-third, (ix) the elasticity of the average size of price changes with respect to inflation converges to one-third, and (x) the frequency of price decreases converges to zero.

We believe these results are interesting because they underlie the welfare costs of inflation in menu cost models (as well as other models of price stickiness) and because they test this class of models. First, the menu cost paid when changing prices is a direct welfare cost of inflation because these resources are wasted. Second, the "extra" price dispersion created by nominal variation in prices is another avenue for inefficiency in menu cost models as well as in other models with sticky prices. For example, in models with an exogenous frequency of price adjustment, as described in Woodford (2003, ch. 6). We find that the direct cost of more frequent price changes is unlikely to be significant for inflation rates below 5% a year (see Figure V later). Moreover, the costs of inflation arising from inefficient price dispersion is also likely to be negligible for inflation rates below 10% a year (Figure X later). Therefore, findings (i) and (ii) imply that the welfare cost of increasing the rate of inflation is negligible around zero inflation. For higher rates of inflation, the welfare costs of inflation increase as the frequency of price changes and the standard deviation of relative prices increase. We estimate that the welfare costs of the additional relative price dispersion caused by inflation are approximately 1.5% of GDP for an inflation rate of 100% a year and approximately 8.5% of GDP for inflation rates of 500% a year.

We begin by presenting a model of monopolistic firms that are hit by idiosyncratic shocks to their real marginal cost and face

<sup>2.</sup> See Bénabou (1992) and Burstein and Hellwig (2008) for earlier and recent examples of analysis that takes both effects of inflation into account, the former using heterogeneous consumers that search for products and homogeneous firms, and the latter using differentiated products on the demand side and heterogeneity in the firm's cost. Recently, and most related to our article, Nakamura et al. (2018) reexamine these issues for the United States using new micro data including the higher inflation period for the United States as well as calibrated menu cost models.

a fixed cost of changing prices. Similar models have been introduced by Barro (1972), Bertola and Caballero (1990), Danziger (1999), Golosov and Lucas (2007), and Gertler and Leahy (2008). Then, in Sections II.B and II.C, we derive our comparative static results about how inflation affects price-setting behavior and illustrate them with a numerical example based on the model in Golosov and Lucas (2007), so that one can evaluate how the local theoretical results apply to a large range of quantitatively relevant parameter values. The results for low inflation are new: in particular, the prediction that most of the changes in inflation (90%) are accounted for by changes in the difference between the frequency of price increases and decreases.<sup>3</sup> Furthermore, a new insight of this work is that under some simplifying assumptions, price-setting behavior depends only on the ratio between inflation and the variance of the idiosyncratic shocks. In this sense, high-inflation economies are equivalent to economies in which firms do not face idiosyncratic shocks for nominal price adjustment decisions. Hence, we can apply benchmark results on the effect of inflation on price-setting behavior in the deterministic case (Sheshinski and Weiss 1977; and Bénabou and Konieczny 1994, which consider models without idiosyncratic shocks) to the case in which firms face persistent idiosyncratic shocks and there is high inflation.

We continue with our main empirical findings in Section IV. Hereinafter, we discuss the most notable ones as well as their relationship to previous literature. We first describe our data set in Section IV.A. We use the micro-data underlying the construction of the Argentine consumer price index from 1988 to 1997 for 506 goods covering 84% of expenditures. The unique feature of these data is the range of inflation during this time period. The inflation rate was almost 5,000% during 1989 and 1,500% during 1990. After the stabilization plan of 1991, there was a quick disinflation episode and after 1992 there was virtual price stability with some deflationary periods.

Using these data, we estimate the frequency of price changes, the average size of price changes, and the dispersion of relative prices for different time periods. We find that the frequency of price changes as a function of the rate of inflation is flat at low

<sup>3.</sup> Alvarez et al. (2011) and Alvarez and Lippi (2014) show that the frequency of price changes and the dispersion of relative prices are insensitive to inflation at low inflation rates under a more restrictive set of assumptions.

inflation levels and has a constant elasticity for high inflation levels (a novel empirical finding). We estimate this elasticity to be between one-half and two-thirds, which is close to our theoretical prediction of two-thirds.<sup>4</sup> There is a large literature that estimates the frequency of price changes. Various papers do this for different countries, different time periods, different rates of inflation, and different sets of goods.<sup>5</sup> A feature of our data set (and a success of the theory) is that our estimates of the frequency of price changes for each level of inflation in the Argentine data are similar to the estimates of the other studies with the same rate of inflation. This is illustrated in Figure VI later. Further details on the samples and inflation ranges considered in other studies in the literature are provided in Online Appendix G.

Then, we find that even though the frequency of price changes is unresponsive to inflation when inflation is low, the difference between the frequency of price increases and the frequency of price decreases is an increasing function of inflation. This is consistent with previous evidence (see, for example, Nakamura and Steinsson 2008 for the United States; Gagnon 2009 for Mexico; Berardi, Gautier, and Le Bihan 2015 for France; and Cavallo 2015 for cross-country evidence). Our contribution is to provide a new theoretical interpretation of this fact through the lens of menu cost models. We show that under some assumptions, when inflation is low, this fact is consistent with 90% of the changes in inflation resulting from the extensive margin of price adjustments, that is, changes in the difference between the frequency of price increases and decreases. Furthermore, we document that the cross-good dispersion of the frequency of price changes falls with inflation when inflation is high. As is well known in the literature on low-inflation economies, there is large variation in the frequency of price changes across firms selling different goods.

- 4. To accommodate the range of estimates in the data, our simple model could be extended, allowing firms to freely change prices at random times. Nakamura and Steinsson (2010) consider a version of this model (also see Dotsey and King 2005; Caballero and Engel 2007; and Alvarez, Le Bihan, and Lippi 2016).
- 5. For example, for low-inflation economies Álvarez et al. (2006) for the Euro area; Bils and Klenow (2004), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008) for the United States. For moderate levels of inflation, Gagnon (2009) for Mexico, Barros et al. (2009) for Brazil, and Wulfsberg (2016) for Norway. Baharad and Eden (2004) and Lach and Tsiddon (1992) for Israel, as well as Konieczny and Skrzypacz (2005) for Poland, look at high-inflation economies, but not as high as our Argentine sample.

The new finding is that as inflation rises, the frequency of price changes becomes similar across different products, reflecting that the importance of idiosyncratic differences across products disappears as a motive for changing prices when inflation is high.

Next we are concerned with the dispersion of relative prices and size of price changes. Despite being of theoretical and practical importance because of its welfare implications, this is the first article to look at the relationship between inflation and the dispersion of relative prices across stores, selling the same good, for a wide range of inflation rates. We document that the dispersion of relative prices across stores is insensitive to inflation for low inflation rates and it becomes an increasing function of inflation for high inflation rates. In fact, for very high inflation rates, the empirical elasticity of the dispersion of relative prices with respect to inflation is close to the theoretical upper bound of one-third. The average size of price changes, conditional on a price change taking place, exhibits a similar pattern. It is insensitive to inflation at low inflation rates and then increases with inflation as inflation becomes higher.

We conclude by showing that the welfare costs of inflation due to inefficient price dispersion are only relevant for very high inflation rates. In particular, we find that the cost of the additional price dispersion resulting from a hyperinflation of 500% a year is approximately 8.5% of aggregate output a year.

Some of our results for the frequency and size of price changes have recently been documented for the United States as well by Nakamura et al. (2018). In particular, using newly found post 1977 BLS micro-data on U.S. consumer prices, they find that in calibrated models the frequency of price changes increases with inflation and that the size of price changes is

6. Many papers look at the dispersion of inflation rates across goods, which is a different concept. Reinsdorf (1994), Sheremirov (2015), and Perez and Drenik (2015) are some exceptions. However, due to the low variation of inflation in their samples, it is hard to make inferences about the elasticity of interest in those studies. In Reinsdorf (1994), inflation varies between 3.7% and 10.4% in the United States in the early 1980s and has conflicting results depending on the measure of inflation. Nakamura et al. (2018) have a better sample, and their results are consistent with ours. Sheremirov (2015) looks at U.S. data for 2001–2011 when the variation in the rate of inflation is small. Perez and Drenik (2015) find a positive correlation between inflation and the dispersion of relative prices for inflation rates ranging from 10% to 20% a year, which they attribute to other confounding factors.

insensitive to inflation for inflation rates under 14% a year. While they study the consequences of these findings for the welfare cost of inflation in calibrated New Keynesian models, our article differs in that we are interested in providing a more general theoretical characterization within menu cost models of these as well as other statistics for both very low and very high inflation. In addition, on the empirical side we are able to study very high and very low inflation during this period, since the first two years of the sample had annual inflation rates in the thousands, and the last few years essentially enjoyed price stability, with a disinflation period in between (see Figure IV and the figures in the background discussion in Online Appendix H).

Finally, several online appendixes provide supplementary material. Online Appendix A contains the proofs of the propositions in Section II.A. Some of these proofs are of independent interest because they fully characterize the solution of the menu cost model with a closed-form solution. Online Appendix BA contains an analytical characterization of our version of the Golosov and Lucas (2007) model. Online Appendix C describes some details of our data. Both in the main body and in Online Appendix D. we perform extensive robustness checks to evaluate the sensitivity of our estimates. Our findings are robust to different treatment of sales, product substitutions, and missing values in the estimation of the frequency of price changes and with respect to the level of aggregation of price changes. They are also robust to using contemporaneous inflation or an estimate of expected future inflation for the relevant time frame, and to excluding observations corresponding to periods with inflation above some threshold for which we have reasons to believe that discrete sampling might bias the estimates. Online Appendix H contains a short description of the history of economic policy and inflation in Argentina for the years before and during our sample.

#### II. COMPARATIVE STATIC PROPERTIES OF MENU COST MODELS

In this section, we study how inflation affects price-setting behavior in menu cost models. We show several theoretical predictions for a stylized menu cost model where competitive monopolistic firms face a fixed cost of changing their nominal price in the presence of idiosyncratic real marginal-cost shocks and common constant inflation.

In Section II.A. we write down a simple setup and obtain the main analytical results. In Section II.B and Section II.C, we obtain the results for low and high inflation, respectively; explain the nature of the assumptions needed for the results; highlight which form of these results are already present in the literature; and discuss pros and cons of applying these comparative statics results to time series data. Section II.D decomposes changes in the rate of inflation into those arising from changes in the frequency of price changes and those arising from changes in the size of price changes. In Section III, we present Kehoe and Midrigan's (2015) version of Golosov and Lucas's (2007) model and illustrate the theoretical results in this section for both low and high inflation. This model relaxes some assumptions and we parameterize the model in an empirically reasonable fashion to show that the theoretical results of Section II.A are quantitatively applicable to typical economies.

In Online Appendix A, we prove the theoretical propositions in this section in a setup with a more general profit function and a less restrictive process for the shocks, but that retains certain symmetry properties.

#### II.A. The Basic Menu Cost Model

We study the problem of a monopolist adjusting the nominal price of its product in an environment with inflation, idiosyncratic real marginal-cost shocks, and a fixed cost (the menu cost) of changing nominal prices. We think of this problem as a simplified version of the problem in Golosov and Lucas (2007), and as an almost identical problem to the one considered in Barro (1972).

We assume that the instantaneous profit of the monopolist depends on its price relative to the economy- or industry-wide average price and on an idiosyncratic shock. We let  $F(p,z)=\zeta(z)-B(p-z)^2$  be the real value of the profit per period as a function of p, the log of the nominal price charged by the firm relative to the log of the economy-wide price, and z, an idiosyncratic shifter of the profit function. We let z be the static profit-maximizing real price. Thus, our quadratic specification can be interpreted as an approximation of a general function F around the static profit-maximizing price. In this case,  $\zeta(z)$  is the static maximized profit, and B>0 is half the second derivative of F around that price. We assume that the economy-wide price grows at a constant inflation rate  $\pi$ , so that when the firm does not change its nominal price its relative log-price decreases, that is,  $dp=-\pi dt$ . We also allow

the menu cost to depend on z. In this case, we write  $C_t = c \, \zeta(z_t)$ , where  $c \geqslant 0$  is a constant, so c = 0 represents the frictionless problem. We assume that  $\{z_t\}$  is a diffusion with law of motion  $dz = \sigma \, dW$ , where  $\{W(t)\}$  is a standard Brownian motion with  $W(t) - W(0) \sim N(0, t)$ . We use  $r \geqslant 0$  for the real discount rate of profits and adjustment costs. We let  $\{\tau_i\}$  be the stopping times at which prices are adjusted and  $\{\Delta p(\tau_i)\}$  the corresponding price changes, so that the problem of the firm can be written as

$$(1) V(p,z) = \max_{\substack{\left\{\tau_{i}, \Delta p_{i}\right\}_{i=0}^{\infty}}} \mathbb{E}\left[\int_{0}^{\infty} e^{-rt} F(p(t), z(t)) dt - \sum_{i=0}^{\infty} e^{-r\tau_{i}} c \, \zeta(z(t)) \, \Big| z(0) = z\right],$$

with  $p(t) = p(0) + \sum_{i=0}^{\tau_i < t} \Delta p(\tau_i) - \pi t$  and  $z(t) = \sigma W(t)$  for all  $t \ge 0$ , and the initial state is given by p(0) = p and z(0) = z.

Given the simple form of the period return function and the law of motion of the state, the optimal policy that solves this problem can be described by three numbers that control the difference between p and z, namely:

$$\Psi(\pi, \sigma^2) = \left[\underline{\psi}(\pi, \sigma^2), \bar{\psi}(\pi, \sigma^2), \hat{\psi}(\pi, \sigma^2)\right],$$

where we include the parameters  $\pi$  and  $\sigma^2$  as explicit arguments of the decision rules to conduct comparative statics. The numbers  $\underline{\psi}(\pi,\sigma^2)$  and  $\bar{\psi}(\pi,\sigma^2)$  define the inaction set as follows:

$$\mathcal{I}(\pi,\sigma^2) = \left\{ (p,z) \in \mathbb{R} \times Z : \underline{\psi}(\pi,\sigma^2) + z \leqslant p \leqslant \bar{\psi}(\pi,\sigma^2) + z \right\}.$$

If the firm's relative price is within the inaction set, that is, if  $(p,z)\in\mathcal{I}$ , then it is optimal not to change prices. Outside the interior of the inaction set the firm will adjust prices so that its relative price just after adjustment is given by  $p=\hat{\psi}(\pi,\sigma^2)+z$ . Since  $\{z(t)\}$  has continuous paths, all adjustments will occur at the boundary of the inaction set (given additional regularity conditions). For instance, a firm with a relative price p and an idiosyncratic shock z such that the relative price hits the lower boundary of the inaction set—that is, such that  $p=\psi(\pi,\sigma^2)+z$ — will raise its price by  $\Delta p=\hat{\psi}(\pi,\sigma^2)-\psi(\pi,\sigma^2)>0$ . Likewise, when hitting the upper boundary, it will change its price (decrease it) by  $\Delta p=\hat{\psi}(\pi,\sigma^2)-\bar{\psi}(\pi,\sigma^2)<0$ .

Using the optimal decision rules, we can compute the density of the invariant distribution of the state,  $g(p, z; \pi, \sigma^2)$ , as well as the expected time between adjustments  $\mathcal{T}(p, z; \pi, \sigma^2)$  starting from the state (p, z). Note that using  $g(\cdot)$  we can readily find the distribution of relative prices in the economy (or industry) and we can compute the expected time elapsed between consecutive adjustments under the invariant distribution, and its reciprocal, the expected number of adjustments per unit of time, which we denote by  $\lambda_a(\pi, \sigma^2)$ .

We denote by  $\lambda_a^+(\pi,\sigma^2)$  and  $\lambda_a^-(\pi,\sigma^2)$  the frequencies of price increases and decreases, respectively. Furthermore, we let  $\Delta_p^+(\pi,\sigma^2)$  be the expected size of price changes, conditional on having an increase, and  $\Delta_p^-(\pi,\sigma^2)$  the corresponding expected size of price changes, conditional on having a decrease. Formally  $\Delta_p^+(\pi,\sigma^2)=\int_Z \left[\hat{\psi}(z)-\underline{\psi}(z)\right] \frac{g(\underline{\psi}(z),z)}{\int_Z g(\underline{\psi}(z'),z')dz'}\,dz$  where we omit  $(\pi,\sigma^2)$  as arguments of  $g,\hat{\psi}$ , and  $\underline{\psi}$  to simplify notation.  $\Delta_p^-(\pi,\sigma^2)$  is defined analogously.

#### II.B. Comparative Statics with Low Inflation

In this section, we show that when inflation is zero and firms face idiosyncratic profit shocks, changes in the inflation rate do not have a first-order effect on the frequency of price changes or on the distribution of relative prices. The intuition for this result is that at zero inflation, price changes are triggered by idiosyncratic shocks and small variations in inflation have only a second-order effect. Moreover, we show that there is a type of symmetry in this case: the frequency of price increases and decreases and the size of price increases and decreases are the same.

We let  $h(\hat{p}; \pi, \sigma^2) = \int g(\hat{p}, z; \pi, \sigma^2) dz$  be the invariant distribution of log relative prices  $\hat{p} = p - \bar{p}$ , for an economy, or industry, with  $(\pi, \sigma^2)$ , when it exists. Using h we can compute several statistics of interest, such as  $\bar{\sigma}(\pi, \sigma^2)$ , the standard deviation of relative prices. As in the case of the frequency of price changes, we include  $(\pi, \sigma^2)$  explicitly as arguments of this statistic.

#### Proposition 1.

(i) If the frequency of price changes  $\lambda_a(\pi, \sigma^2)$  is differentiable at  $\pi = 0$ , then the frequency of price changes is insensitive to inflation,

$$\frac{\partial}{\partial \pi} \lambda_a(0, \sigma^2) = 0.$$

(ii) If the density of the invariant distribution  $h(\hat{p}; \pi, \sigma^2)$  is differentiable at  $\pi = 0$ , then the dispersion of relative prices under the invariant distribution is insensitive to inflation,

$$\frac{\partial}{\partial \pi} \bar{\sigma}(0, \sigma^2) = 0.$$

(iii) The frequencies of price changes and the size of price adjustment are symmetric at  $\pi = 0$  in the sense that

$$\lambda_a^+(0,\sigma^2) = \lambda_a^-(0,\sigma^2), \quad \frac{\partial \lambda_a^+(0,\sigma^2)}{\partial \pi} = -\frac{\partial \lambda_a^-(0,\sigma^2)}{\partial \pi} \text{ and }$$

$$\Delta_p^+(0,\sigma^2) = \Delta_p^-(0,\sigma^2), \quad \frac{\partial \Delta_p^+(0,\sigma^2)}{\partial \pi} = -\frac{\partial \Delta_p^-(0,\sigma^2)}{\partial \pi}$$

where  $\lambda_a^+$  is the frequency of price increases,  $\Delta_p^+$  is the average size of price increases, and  $\lambda_a^-$ ,  $\Delta_p^-$  are the analogous concepts for price decreases.

The proof, for a more general case, is in Online Appendix A. The main idea is to use the symmetry of the objective function F(p,z) with respect to (p,z) to show the results. Indeed, Online Appendix A extends the model to one with a general F as well as a law of motion for z given by  $dz=a(z)dt+b(z)\sigma dW$  where both  $F(\cdot,\cdot)$  and  $a(\cdot)$ ,  $b(\cdot)$  satisfy certain symmetry properties. The proof shows that the expected number of adjustments is symmetric around zero inflation, that is,  $\lambda_a(\pi,\sigma^2)=\lambda_a(-\pi,\sigma^2)$  for all  $\pi$ . Given the symmetry of the profit function we view this property as quite intuitive: a 1% inflation should give rise to as many price changes as a 1% deflation. Symmetry implies that if  $\lambda_a$  is differentiable then  $\frac{\partial}{\partial \pi}\lambda_a(\pi,\sigma^2)=-\frac{\partial}{\partial \pi}\lambda_a(-\pi,\sigma^2)$ , which establishes the first result.

Analogously, for the distribution of relative prices, the main idea is to show that the marginal distribution of relative prices is symmetric in the sense that  $h(\hat{p};\pi,\sigma^2)=h(-\hat{p};-\pi,\sigma^2)$  for all  $\hat{p},\pi$ , that is, the probability of high relative prices with positive inflation is the same as that of low relative prices with deflation. As these symmetric functions are locally unchanged with respect to  $\pi$  when  $\pi=0$ , inflation has no first-order effect on the second moment of the distribution of relative prices at  $\pi=0$ . Similarly, the symmetry of the frequency and of the average size of price

increases and decreases also follow from the symmetry assumptions.

The assumption of differentiability of  $\lambda_a$  and  $\bar{\sigma}$  with respect to  $\pi$  is not merely a technical condition. The function  $\lambda_a(\pi, \sigma^2)$  could have a local minimum at  $\pi=0$  without being smooth. This is indeed the case for  $\sigma^2=0$  to which we turn in the next subsection. Likewise, the differentiability of  $h(\hat{p};\pi,\sigma^2)$  at  $\pi=0$  requires  $\sigma>0$ . In Sheshinski and Weiss's (1977) model, that is, when  $\sigma=0$ , the distribution h is degenerate, uniform at  $\pi\neq 0$ , but nondifferentiable at  $\pi=0$ .

1. Remarks and Relation to the Literature. We find Proposition 1's theoretical predictions interesting because they extend an important result on the welfare cost of inflation from sticky-price models with exogenous price changes (e.g., the Calvo model) to menu cost models with endogenous frequency of price changes. The result is that in cashless economies with low inflation, there is no first-order welfare effect of inflation, that is, the welfare cost of inflation can be approximated by a "purely quadratic" function of inflation.

Inflation imposes welfare costs through two channels in cashless economies. First, the "extra" price dispersion created by inflation is an avenue for inefficiency in models with sticky prices, because it creates "wedges" between the marginal rates of substitution in consumption and the marginal rates of transformation in production. See, for example, Woodford (2003, ch. 6), and references therein for the analysis of this effect. Part (ii) of Proposition 1 extends this result to the menu cost model. Second, a higher endogenous frequency of price adjustments because of inflation is an obvious source of welfare losses when these adjustments are costly. Part (i) of Proposition 1 establishes that this second channel is also negligible for low inflation rates.

- 7. We conjecture, but have not proved at this level of generality, that as long as  $\sigma^2>0$ , the problem is regular enough to become smooth, that is, the idiosyncratic shocks will dominate the effect of inflation. For several examples one can either compute all the required functions or show that they are smooth, given the elliptical nature of the different ODEs involved. Based on this logic, as well as on computations of different models, we believe that the length of the interval for inflations around zero for which  $\lambda_a(\cdot,\sigma^2)$  is approximately flat is increasing in the value of  $\sigma^2$ .
- 8. There are other publications that take more than one effect into account. Burstein and Hellwig (2008) compute numerical examples in a model closer to

The results of Proposition 1 should apply to a wider class of models as long as one essential assumption is maintained: the symmetry of the profit function around the profit-maximizing price. For example, a version of Proposition 1 applies to models with both observations and menu costs (Alvarez et al. 2011), to models that have multiproduct firms (Alvarez and Lippi 2014), and to models that combine menu costs and Calvo-type adjustments (Nakamura and Steinsson 2010; Alvarez, Le Bihan, and Lippi 2016). In Section III, we solve numerically for Golosov and Lucas's (2007) version of the model (which does not satisfy the aforementioned symmetry properties). We show that for empirically reasonable parameter values, the functions  $\lambda_a$  and  $\bar{\sigma}$  are approximately flat for a wide value of inflation rates around zero.

We do not know of other theoretical results analyzing the sensitivity of  $\lambda_a(\pi,\sigma^2)$  and  $\bar{\sigma}(\pi,\sigma^2)$  to inflation around  $\pi=0$  in this setup. However, there is a closely related model that contains a complete analytical characterization by Danziger (1999). In fact, we can show that for a small cost of changing prices, Proposition 1 holds in Danziger's characterization.

#### II.C. Comparative Statics with High Inflation

Now we turn to the analysis of price-setting behavior for large values of inflation. In highly inflationary environments, the main reason for firms to change nominal prices is to keep their relative price in a target zone as the aggregate price level grows. Idiosyncratic shocks in the high-inflation case become less important and therefore the analysis of the deterministic case is instructive. This leads us to proceed in two steps. First we derive comparative statics results in the deterministic case—that is, when  $\sigma^2=0$ . This is a version of the problem studied by Sheshinski and Weiss (1977). Then we study the conditions under which these comparative statics are the same as for the case of  $\sigma^2>0$  and very large  $\pi$ .

Sheshinski and Weiss (1977) study a menu cost model similar to the deterministic case in our basic setup. The firm's problem is to decide when to change prices and by how much when aggregate prices grow at the rate  $\pi$ . In Sheshinski and Weiss's (1977) model, the time elapsed between adjustments is simply a constant, which we denote by  $\mathcal{T}(\pi)$ . Sheshinski and Weiss (1977) find sufficient

conditions so that the time between adjustments decreases with the inflation rate (see their Proposition 2), and several authors have further refined the characterization by concentrating on the case where the fixed cost c is small. Let  $p^* = \arg \max_{p} F(p, 0)$  be the log price of the static monopolist maximization profit, where z=0 is a normalization of the shifter parameter, which stays constant. In the deterministic setup the optimal policy for  $\pi > 0$ is to let the log price reach a value s, at which time it adjusts to S, where  $s < p^* < S$ . The time between adjustments is then  $\mathcal{T}(\pi) =$  $\frac{S-s}{\pi}$ . Furthermore, we highlight another implication obtained in Sheshinski and Weiss's (1977) model: the setup with  $\sigma^2 = 0$ . The distribution of the log relative price is uniform in the interval [s, S]. Thus, the standard deviation of the log relative prices in this economy, denoted by  $\bar{\sigma}$ , is given by  $\bar{\sigma} = \sqrt{\frac{1}{12}}(S-s)$ . As established in Proposition 1 in Sheshinski and Weiss (1977), the range of prices S-s is increasing in the inflation rate  $\pi$ . Obviously the elasticities of  $\lambda_a$  and of  $\bar{\sigma}$  with respect to  $\pi$  are related because  $S - s = \pi T$  and  $\lambda_a = \frac{1}{\tau}$ .

LEMMA 1. Assume that  $\sigma^2=0$  and  $\pi>0$ . Then it follows immediately that  $\lambda_a^-(\pi,\sigma^2)=0$  and that  $\Delta_p^+(\pi,\sigma^2)=S-s$ . Furthermore assume that  $F(\cdot,0)$  is three times differentiable, then

(2a) 
$$\lim_{c \to 0} \frac{\partial \lambda_a}{\partial \pi} \frac{\pi}{\lambda_a} = \frac{2}{3} \text{ and }$$

(2b) 
$$\lim_{c \to 0} \frac{\partial \bar{\sigma}}{\partial \pi} \frac{\pi}{\bar{\sigma}} = \frac{1}{3}$$

Proof. See Online Appendix AB.

The lemma establishes that in the deterministic case when menu costs c are small, there are no price decreases, and the magnitude of price increases, S-s, increases with inflation at a rate of one-third. Also, as inflation increases, the time between consecutive price changes shrinks and the frequency of price adjustment increases with an elasticity of two-thirds.

Next, Lemma 2 analyzes the conditions under which the limiting values of the elasticities in Lemma 1 for the Sheshinski and Weiss (1977) model are the same as for the case with idiosyncratic costs,  $\sigma > 0$ , and very large  $\pi$ . Lemma 2 establishes that when the idiosyncratic shocks z are persistent and interest rates and menu

costs are small, the frequency of price adjustment is homogeneous of degree one in  $(\pi, \sigma^2)$  so that it can be written as a function of the ratio  $\frac{\sigma^2}{\pi}$ .

For the next results we write the frequency of price adjustment as a function of the rate of inflation,  $\pi$ , the variance of the idiosyncratic shock,  $\sigma^2$ , the discount factor, r, and the inverse of the menu cost,  $\frac{1}{c}$ , that is,  $\lambda_a(\pi,\sigma^2,r,\frac{1}{c})$ . We also write the policy rules as functions of the parameters for each z;  $\Psi(\pi,\sigma^2,r,\frac{1}{c};z) = \left[\underline{\psi}(z;\pi,\sigma^2,r,\frac{1}{c}),\bar{\psi}(z;\pi,\sigma^2,r,\frac{1}{c}),\hat{\psi}(z;\pi,\sigma^2,r,\frac{1}{c})\right]$  and the expected price change functions as  $\Delta_p^+(\pi,\sigma^2,r,\frac{1}{c})$  and  $\Delta_p^-(\pi,\sigma^2,r,\frac{1}{c})$ .

Lemma 2. The function  $\lambda_a(\pi, \sigma^2, r, \frac{1}{c})$  is homogeneous of degree one and the policy functions  $\Psi(\pi, \sigma^2, r, \frac{1}{c})$  are homogeneous of degree zero in all the parameters. Therefore,

$$(3a) \qquad \lim_{\pi \to \infty} \left[ \lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \right]_{\sigma > 0}$$

$$= \lim_{\sigma \to 0} \left[ \lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma^2, r, \frac{1}{c})}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma^2, r, \frac{1}{c})} \right]_{\tau > 0}.$$

Also,

$$(3b) \quad \frac{\lim_{\pi \to \infty} \left[ \lim_{c \downarrow 0, r \downarrow 0} \Psi(\pi, \sigma^2, r, \frac{1}{c}; z) \right]_{\sigma > 0}}{\lim_{\sigma \to 0} \left[ \lim_{c \downarrow 0, r \downarrow 0} \Psi(\pi, \sigma^2, r, \frac{1}{c}; z) \right]_{\pi > 0}} = \mathbb{1} \quad \text{ for all } z,$$

and

$$(3c) \qquad \frac{\lim_{\pi \to \infty} \left[ \lim_{c \downarrow 0, r \downarrow 0} \Delta_p^+(\pi, \sigma^2, r, \frac{1}{c}) \right]_{\sigma > 0}}{\lim_{\sigma \to 0} \left[ \lim_{c \downarrow 0, r \downarrow 0} \Delta_p^+(\pi, \sigma^2, r, \frac{1}{c}) \right]_{\pi > 0}} = 1.$$

*Proof.* See Online Appendix AC.  $\Box$ 

The intuition underlying Lemma 2's proof is that multiplying r,  $\pi$ ,  $\sigma^2$ , and the profit function  $F(\cdot)$  by a constant k>0 is akin to changing the units in which we measure time. Moreover, the objective function on the right-hand side of equation (1) is homogeneous of degree one in  $F(\cdot)$  and c and hence the policy function is the same whether we multiply  $F(\cdot)$  by k or divide c by it. Thus  $\lambda_a$  is homogeneous of degree one in  $(\pi, \sigma^2, r, \frac{1}{c})$ . Likewise,  $\lambda_a(\pi, \sigma^2)$  is homogeneous of degree one in  $(\pi, \sigma^2)$  when menu costs are

small,  $c\downarrow 0$ , and the interest rate is very low,  $r\downarrow 0$ . The interpretation of r going to zero is that instead of maximizing the expected discounted profit, the firm is maximizing the expected average profit, a case frequently analyzed in stopping-time problems (see, for example, Maurice 1981; Andrew and Zervos 2006), which we study in detail in Online Appendix AD.

Lemma 2 extends the result on the elasticity of the frequency of price adjustment with respect to inflation of equation (2a) in Lemma 1 to the case with  $\sigma>0$  and with an arbitrarily large  $\pi$ . This lemma requires that the shifter z has only permanent shocks. In a model with permanent shocks, there is no invariant distribution of z, and hence no invariant distribution of relative prices. However, in the version of Golosov and Lucas's (2007) model considered in Section III, the invariant distribution is well defined. Indeed, the elasticity of the standard deviation of relative prices conditional on z with respect to inflation converges to one-third, as in Sheshinski-Weiss. Furthermore, as discussed in Section III, for large inflation rates, this implies that the elasticity of the unconditional standard deviation of relative prices with respect to inflation has an upper bound of one-third.

Using Lemma 1 and Lemma 2, we obtain the following result for the high-inflation case:

PROPOSITION 2. Assume that F is three times differentiable. Consider two firms with  $\sigma_1$ ,  $\sigma_2 > 0$ . Then,

$$\lim_{\pi \to \infty} \left\lceil \frac{\lim_{c \downarrow 0, r \downarrow 0} \lambda_a(\pi, \sigma_1^2)}{\lim_{c \downarrow 0, r \downarrow 0} \lambda_a(\pi, \sigma_2^2)} \right\rceil = 1$$

$$(4b) \qquad \lim_{\pi \to \infty} \left[ \lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \lambda_a(\pi, \sigma_i^2)}{\partial \pi} \frac{\pi}{\lambda_a(\pi, \sigma_i^2)} \right] = \frac{2}{3} \ for \ i = 1, 2$$

$$(4c) \quad \lim_{\pi \to \infty} \left[ \lim_{c \downarrow 0, r \downarrow 0} \frac{\partial \Delta_p^+(\pi, \sigma_i^2)}{\partial \pi} \frac{\pi}{\Delta_p^+(\pi, \sigma_i^2)} \right] = \frac{1}{3} \ for \ i = 1, 2.$$

Proposition 2 contains strong predictions about the limiting behavior of the frequency of price adjustment as inflation becomes large. The first part is a direct consequence of Lemma 2. It implies

that if we think that different industries have systematically different idiosyncratic shocks, we would expect the variance of these shocks to differ across industries and, hence, the frequency of price adjustment to be different across industries when inflation is low. Equation (4a) implies that differences in the frequency of price adjustment observed with low inflation should wash away as inflation becomes large. This is illustrated in the numerical example in the next section (see Figure II) and verified in the data (see Section IV.D). The intuition is that when inflation is low, the main driver of idiosyncratic nominal price changes are idiosyncratic shocks, and when inflation is high, the main driver of price changes is the growth of aggregate prices. The second part of Proposition 2 is a sharp prediction about the rate at which firms change the frequency of price adjustment when inflation grows. It states that this elasticity should be two-thirds in the limit when  $\pi \to \infty$ . Finally, equation (4c) states that the elasticity of price increases with respect to inflation converges to one-third as  $\pi \to 0$  $\infty$ . It follows from the fact that  $\Delta_p^+$  is  $S-s=\frac{\pi}{\lambda_q}$  when  $\sigma=0$ . The results of Proposition 2 apply to a wider set of models

such as those mentioned in the comments to Proposition 1.

### II.D. Decomposition of Changes in the Rate of Inflation

This section shows how steady-state changes in the rate of inflation can be decomposed into changes in the extensive and in the intensive margins of price adjustment, that is, changes in inflation accounted for by the frequency of price changes and changes in inflation accounted for by the size of price changes conditional on a price change taking place. Our main theoretical result is that for low inflation, the extensive margin accounts for 90% of changes in inflation while for high inflation it accounts for two-thirds of inflation changes.

As a matter of accounting, we can decompose the inflation rate as the difference between the product of the frequency of price increases times the average size of price increases and the product of the frequency of price decreases times the average size of price decreases. Formally:

$$\pi = \lambda_a^+ \Delta_p^+ - \lambda_a^- \Delta_p^-.$$

Totally differentiating the previous expression with respect to the inflation rate, and using the optimal decision rules, we can decompose the changes in the inflation rate into those due to changes in the frequency, denoted by  $\delta$ , and those due to changes in the average size, denoted by  $1 - \delta$ :

$$1 = \underbrace{\frac{\partial \lambda_a^+}{\partial \pi} \, \Delta_p^+ - \frac{\partial \lambda_a^-}{\partial \pi} \, \Delta_p^-}_{\text{Extensive Margin}} + \underbrace{\frac{\partial \Delta_p^+}{\partial \pi} \, \lambda_a^+ - \frac{\partial \Delta_p^-}{\partial \pi} \, \lambda_a^-}_{\text{Intensive Margin}}.$$

We derive the decomposition of inflation for  $\pi=0$  and for  $\pi\to\infty$  with a quadratic profit function,<sup>9</sup> no discounting, and where z represents the (log of the) product cost and follows a driftless continuous time random walk.

PROPOSITION 3. Assume that  $\sigma > 0$  and  $F(p - \bar{p}, z) = -B(p - z)^2$ , where B > 0 is a positive constant. Then,

$$\lim_{r\downarrow 0} \delta(0;r) = \frac{9}{10} \ and \ \lim_{\pi\to\infty} \lim_{r\downarrow 0} \delta(\pi;r) = \frac{2}{3} \,.$$

*Proof.* See Online Appendix AD.  $\Box$ 

Notably, while Proposition 3 states that 90% of changes in inflation around zero inflation are accounted for by the extensive margin of price adjustment, Proposition 1 states that the frequency of price changes is insensitive to inflation. To gain insights into the interplay between the two propositions observe that for zero inflation the frequency of price increases and decreases are the same, that is  $\lambda_a^+ = \lambda_a^-$ , and also that the sizes are the same, that is  $\Delta_p^+ = \Delta_p^-$ . Proposition 1 implies that  $\frac{\partial \lambda_a^+}{\partial \pi} = -\frac{\partial \lambda_a^-}{\partial \pi}$ so the extensive margin at zero is  $\delta(0) = 2\Delta_p \frac{\partial \lambda_a^+}{\partial \pi}$ . Since inflation introduces a negative trend in relative prices, it induces them to hit more often the lower limit of the inaction set, prompting more price increases and fewer price decreases. The characterization of optimal policies in the proof of Proposition 3 shows that these changes in the frequency of price increases and decreases account for 90% of changes in the rate of inflation at  $\pi = 0$ . A similar argument holds for the decomposition of the change of inflation in a mild deflation.

<sup>9.</sup> Equivalently, we can write the result for a small fixed  $\cos c$ , so that prices are close to the profit-maximizing value, and thus a second-order expansion of the profit function is accurate.

The key technical insight in the proof of Proposition 3 is that  $\lim_{r\to 0} rV(x,r)$  is finite and independent of x, where V(x,r) is the value function and where  $x\equiv p-z$  is the markup. This allows us to obtain an analytical solution of the value function and to characterize optimal policies.

Finally, the following corollary to Proposition 3 presents a sharp prediction about how changes in inflation affect the frequency of price increases and decreases when inflation is low.

COROLLARY 1. Around  $\pi = 0$ , the difference between the frequency of price increases and decreases rises with inflation. Formally,

$$\frac{\partial(\lambda_a^+ - \lambda_a^-)}{\partial \pi}|_{\pi=0} = \frac{\delta(0)}{\Delta_p^+|_{\pi=0}} = \frac{\frac{9}{10}}{\Delta_p^+|_{\pi=0}} > 0.$$

Taken together, the results at low inflation in Proposition 3 and its corollary imply that inflation rises when inflation is low mostly because the frequency of price increases rises and that of price decreases falls (the extensive margin), as opposed to the size of price increases rising and that of price decreases falling (the intensive margin).

1. General Remarks. We conclude this section with a few remarks on the applicability of these comparative static results to the time series variation in our data set. The propositions in this section were obtained under the assumption that inflation is to remain constant at the rate  $\pi$ , and that the frequency of price changes is computed under the invariant distribution. Thus, strictly speaking, our propositions are not predictions for time series variation but comparative static results.

We give three comments in this regard. First, this should be less of a concern for very high inflation, since the model becomes close to static, that is, firms change prices very often and thus the adjustment to the invariant distribution happens very fast. Second, when we analyze the Argentinean data, we correlate the current frequency of price changes with an average of the current and future inflation rates. We experiment with different definitions of these averages and find that the estimates of the elasticities in the first two propositions of this section are not sensitive to this choice. Moreover, with Argentina's experience in mind, Beraja (2013) studies the transitional dynamics in a menu cost

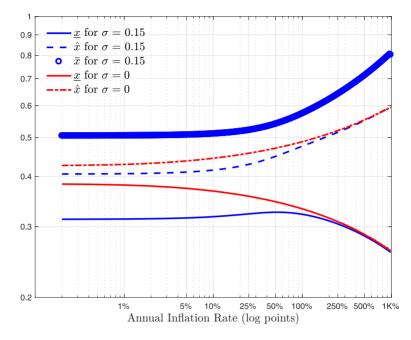
model where agents anticipate a disinflation in the future and performs the same comparative statics with artificial data generated from such model. He finds that the theoretical results in this section are robust to conducting the analysis in a nonstationary economy during a disinflation process calibrated to the Argentine economy. Third, in Section III we numerically solve a standard version of the menu cost model for reasonable parameter values for menu cost c and discount rate r, which are positive but small, and for finite but large inflation rates  $\pi$ , of the order that are observed in Argentina. We find that the propositions in this section (which use limit values for c, r, and  $\pi$ ) accurately predict the behavior of the statistics of interest computed in the calibrated model.

# III. ILLUSTRATING THE THEORY WITH GOLOSOV AND LUCAS'S (2007) MODEL

In this section, we specify a version of the firm's problem studied in Section II.A to illustrate the theory. We characterize the solution of the model analytically and numerically and show how changes in the rate of inflation affect optimal pricing rules, the frequency of price changes, and the size of price adjustments. The example also verifies the robustness of the analytical predictions obtained so far. In Section II.A, we obtained sharp analytical results under a variety of simplifying assumptions such as limit values of parameters (e.g., vanishing menu cost c and or discount rate r), or the shape of profit functions r. Also, our analytical results were obtained at two extreme values of inflation. In this section, we check the robustness of the simplifying assumptions by computing a version of the model away from the limit cases, considering values of inflation in the range observed in Argentina.

The example is a version of the Golosov and Lucas (2007) model, identical to the one in Kehoe and Midrigan (2015). Decifically, we assume a constant elasticity of demand, a constant returns to scale production technology, idiosyncratic shocks to marginal cost that are permanent, an exponentially distributed product life, and a cost of changing prices that is proportional to current profits (but independent of the size of the price change).

<sup>10.</sup> We zero out the transitory shock that gives rise to sales in Kehoe and Midrigan (2015) and write the model in continuous time.



 $\label{eq:Figure I} Figure \ I$  Optimal Thresholds for Different Inflation Rates

Online Appendix B presents a more detailed description of the setup.

Furthermore, in Online Appendix BA, we present several propositions with an analytical characterization of the solution of this model. A novel contribution of this article is to derive a system of three equations in three unknowns for optimal pricing rules, as well as the explicit solution to the value function. We also derive an explicit solution for the expected number of adjustments per unit of time  $\lambda_a$ , and we characterize the density g of the invariant distribution of (p, z). We believe these derivations could be useful for researchers interested in menu cost models more generally.

The remainder of this section contains several figures that describe numerically how changes in the rate of inflation affect the optimal pricing rules, the frequency of price changes, and the size of price adjustments. Again, see Online Appendix B for details on the calibration of the model underlying these figures.

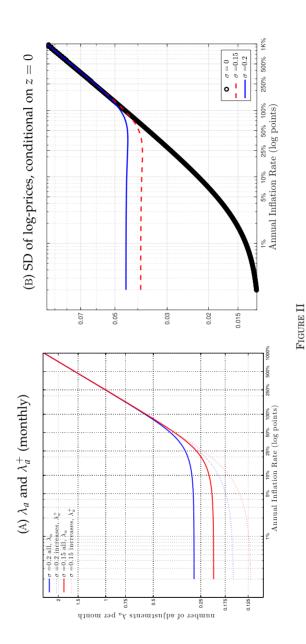
Figure I illustrates how the optimal pricing policies vary with inflation for two cases,  $\sigma=0.15$  and  $\sigma=0$ . The dashed center

lines are the optimal return markup and the outer lines are the boundaries of the inaction set in each case. With no inflation and  $\sigma>0$ , the markup will drift away from the starting optimal markup, driven by the idiosyncratic shock. The firm will keep its nominal price fixed as long as it does not hit the boundaries. Once the markup hits either boundary,  $\underline{x}$  or  $\bar{x}$ , the firm resets the price and the markup returns to  $\hat{x}$ . The light gray (in color version, online, red) lines depict the optimal thresholds for Sheshinski and Weiss's (1977) case with  $\sigma=0$ . Markups always fall when there are no idiosyncratic shocks after the firm resets its nominal price. Hence, the upper limit of the inaction set becomes irrelevant. The firm resets its nominal price to  $\hat{x}+z$  when the markup hits the lower bound x and waits for it to fall again.

Figure I shows several properties of the menu cost models we study. At very low inflation rates, and when  $\sigma > 0$ , the thresholds are symmetric, that is, the distance between x and  $\hat{x}$  is the same as the distance between  $\bar{x}$  and  $\hat{x}$ . This symmetry implies that the size of price increases is equal to the size of price decreases,  $\Delta_n^+(0,\sigma) = \Delta_n^-(0,\sigma)$ , and that the frequency of price increases is equal to the frequency of price decreases,  $\lambda_a^+(0,\sigma) = \lambda_a^-(0,\sigma)$ . These illustrate the results obtained in part (iii) of Proposition 1. At very high inflation rates, the models with  $\sigma > 0$  and with  $\sigma = 0$  are equivalent in the sense that the critical values x and  $\hat{x}$  in Golosov and Lucas's (2007) model converge to the Ss bands in Sheshinski and Weiss's (1977) model as established in equation (3b) in Lemma 2. As a result, the magnitude of price changes in the two models is the same as in equation (3c) in Lemma 2,  $\Delta_n^+(\pi,0) = S - s = \hat{x} - \underline{x}$ . For rates of inflation above 250% a year, Figure I also shows that the elasticity of  $\Delta_n^+(\pi,0)$  with respect to inflation is close to one-third—equation (4c) in Proposition 2.

Figure II, Panel A (color version online) displays the frequency of price increases  $\lambda_a^+$ , together with the frequency of all adjustments  $\lambda_a$ , for two values of the cost volatility  $\sigma$ . There are several interesting observations about this figure. First, the frequency  $\lambda_a$  is insensitive to inflation in the neighborhood of zero inflation as established in part (i) of Proposition 1. Second, the length of the inflation interval around  $\pi=0$  where  $\lambda_a$  is

<sup>11.</sup> This example does not exactly satisfy all the assumptions of the model in Section II.A since the profit function F derived from a constant elasticity demand is not symmetric. Yet for small cost c, the terms in the quadratic expansion, which are symmetric by construction, should provide an accurate approximation.



Frequency of Price Changes and Standard Deviation of log-prices

approximately constant increases with  $\sigma$ —(see the discussion in note 7). Third, the last part of Proposition 1 predicts that the frequency of price increases and price decreases is the same when  $\pi=0$ . The figure shows that for low inflation the frequency of price increases is about half of the frequency of price changes, indicating that half the price changes are increases and half are decreases. Fourth, for values of inflation above 250% a year, the frequency of price changes  $\lambda_a$  for different values of  $\sigma$  are approximately the same, consistent with the limiting results in equation (4a) of Proposition 2. Fifth, since the graph is in log scale, it is clear that the common slope is approximately constant for high inflation, and close to two-thirds as established in equation (4b) in Proposition 2. Finally, as inflation becomes large all price adjustments are price increases—as can be seen from the fact that  $\lambda_a^+$  converges to  $\lambda_a$  for each value of  $\sigma$ .

Figure II, Panel B displays the standard deviation of log prices, conditional on z=0. It shows that the elasticity of the standard deviation of relative prices conditional on z with respect to inflation is approximately zero for  $\pi=0$  and  $\sigma>0$  (as in Proposition 1) and it is approximately one-third for large  $\pi$  (as in Lemma 1). Moreover, for the case when  $\sigma>0$ , Panel B shows that the standard deviation of relative prices converges to the case with  $\sigma=0$  and large enough  $\pi$ . Thus, the elasticity with respect to inflation of the standard deviation of relative prices, conditional on z=0, converges to one-third as in Sheshinski and Weiss. Even though the figure only shows this property conditional on z=0, this also holds for all z. This can be seen by using the characterization of the invariant distribution of relative prices in Proposition 6 in the Online Appendix and taking limits as  $\pi\to\infty$ .

Next we analyze the unconditional dispersion of relative prices. The standard deviation of relative prices conditional on z mostly captures the price dispersion coming from asynchronous price adjustments to inflation. The only other remaining source of dispersion in log prices is due to firm idiosyncratic shocks z. To see this, it is helpful to decompose the unconditional variance of relative prices  $\bar{\sigma}^2(p;\pi,\cdot)$  for a given inflation rate  $\pi$  as follows:

(5) 
$$\bar{\sigma}^{2}(p;\pi,\cdot) = \mathbb{E}\left[\mathbb{V}ar\left(p|z;\pi,\cdot\right)\right] + \mathbb{V}ar\left[\mathbb{E}\left(p|z;\pi,\cdot\right)\right].$$

Here, the first term,  $\mathbb{E}\left[\mathbb{V}ar\left(p|z;\pi,\cdot\right)\right]$  corresponds to Figure II, Panel B because, as we show in Online Appendix BA, the log-price distribution  $g(\cdot,z)$  has the same shape for any value

of z. As for the term  $\mathbb{V}ar\left[\mathbb{E}\left(p|z;\pi,\cdot\right)\right]$ , this source of dispersion is mostly exogenous because the variance of the average price is equal to the cross-sectional dispersion of z for all values of  $\pi$  when menu costs are zero.

Taken together, the foregoing discussion implies that relative price dispersion is insensitive to inflation when inflation is low because it reflects idiosyncratic firm shocks. However, for large enough inflation rates, relative price dispersion has an elasticity with respect to inflation that has an upper bound of one-third because the conditional variance for a given z has an elasticity of exactly one-third and the second term in equation (5) is insensitive to inflation. To illustrate this, Figure XII in Online Appendix BB plots the unconditional standard deviation of log-prices for different values of volatility of the idiosyncratic shocks and average product life. We observe that the elasticity increases when the second term in equation (5) becomes smaller (for example, when the average product life decreases). Yet it takes much higher inflation rates than the ones observed in the peak months in Argentina for the elasticity to reach the upper bound of one-third for high inflation rates.

# IV. ARGENTINA'S EVIDENCE ON MENU COST MODELS OF PRICE DYNAMICS

In Section II we uncovered several properties of menu cost models that can be contrasted with data. The presentation of the empirical results in this section is organized around those predictions. As a reminder, these are:

- (i) The elasticity of the frequency of price changes  $\lambda$  with respect to changes in the rate of inflation is zero at low inflation rates, and it approximates two-thirds as inflation becomes very large. <sup>12</sup>
- (ii) The dispersion of the frequency of price changes across goods decreases with inflation. It is zero when inflation goes to infinity and the model converges to the Sheshinski and Weiss (1977) model with no idiosyncratic shocks.
- 12. This result is robust to extending the menu cost model to one where the menu cost is zero at some random times, so that they combine menu costs and Calvo-type price adjustments. See, for example Nakamura and Steinsson (2010) and Alvarez, Le Bihan, and Lippi (2016). In such random-menu-cost models, as  $\pi \to \infty$ , the frequency also converges to two-thirds.

- (iii) Intensive and extensive margins of price increases and decreases.
  - (a) The frequency of price increases and of price decreases are similar at low inflation rates.
  - (b) The size of price increases and price decreases are similar at low inflation rates.
  - (c) At low inflation rates, as inflation grows, the frequency of price changes remains constant while the frequency of price increases rises and the frequency of price decreases falls.
  - (d) For high inflation rates, the frequency of price increases converges to  $\lambda$  and the frequency of price decreases converges to zero.
  - (e) The size of price changes is an increasing function of the inflation rate.
- (iv) The elasticity of the dispersion of prices across stores with respect to inflation is zero for low inflation rates and it is bounded by one-third when inflation becomes very large.

Next, we look at each of these predictions in the Argentinean data.

## IV.A. Description of the Data Set

Our data set contains 8,618,345 price quotes underlying the consumer price index for the Buenos Aires metropolitan area in the period December 1988 to September 1997. Each price quote represents an item, that is, a good or service of a determined brand sold in a specific outlet in a specific period of time. Goods<sup>13</sup> and outlets are chosen to be representative of consumer expenditure in the 1986 consumer expenditure survey. Price quotes are for 506 goods that account for about 84% of household expenditures.

Goods are divided into two groups: homogeneous and differentiated goods. Differentiated goods represent 50.5% of the expenditure in our sample while homogeneous goods account for the remaining 49.5%. <sup>15</sup> Prices are collected every two weeks for all homogeneous goods and for those differentiated goods sold in

<sup>13.</sup> To simplify the exposition, when it is clear, we use goods to refer to either goods or services.

<sup>14.</sup> Encuesta Nacional de Gasto de los Hogares.

<sup>15.</sup> Examples of homogeneous goods are barley bread, chicken, and lettuce. Examples of differentiated goods are moccasin shoes, utilities, tourism, and professional services.

supermarket chains. They are gathered every month for the rest of the differentiated goods. The data set contains 233 prices collected every two weeks and 302 prices collected every month. Twentynine of each of these goods are gathered both monthly and twice a month. <sup>16</sup>

An important feature of the data set is the rich cross section of outlets where prices are recorded at each point in time. Over the whole sample there are 11,659 outlets, roughly around 3,200 outlets per month for homogeneous goods and about the same number for differentiated goods. On average, across the nine years, there are 166 outlets per good (81 outlets per product collected monthly and 265 per good collected bimonthly). Online Appendix C contains further information on data collection and on the classification of goods.

We exclude from the sample price quotes for baskets of goods, rents, and fuel prices. Baskets correspond to around 9.91% of total expenditure and are excluded because their prices are gathered for any good in a basket, that is, if one good is not available, it is substituted by another in the basket. Examples are medicines and cigarettes. Rents are sampled monthly for a fixed set of representative properties. Reported prices represent the average of the sampled properties and include what is paid on that month, as opposed to what is paid for a new contract. Rents represent 2.33% of household expenditure. Fuel prices account for 4% of total expenditure, and we exclude them because they were gathered in a separate database that we do not have access to.

The data set has some missing observations and flags for stock-outs, price substitutions, and sales. We treat stock-outs (10.5% of observations) and price quotes with no recorded information (2.25% of observations) as missing observations. The statistical agency substitutes the price quote of an item for a similar item, typically when the good is either discontinued by the producer or not sold any longer by an outlet. Using this definition, across the nine years of our data set, we have an average of 2.39% of price quotes that have been substituted. The data set contains an indicator of whether an item was on sale. Around 5% of items have a sale flag. This is small compared with the 11% frequency

<sup>16.</sup> The outlets are divided into 20 waves, corresponding to the 20 working days of the month. Each outlet is visited roughly on the same working day every 10 working days in the case of homogeneous goods and differentiated goods gathered at supermarkets. The data set includes the particular day when each price is gathered.

of sales reported by Klenow and Kryvtsov (2008) for the United States. Seventy percent of the sales correspond to homogeneous items (this is similar to Klenow and Kryvtsov 2008. They report that sales are more frequent for food items). The time series data for the number of outlets per good and for the frequencies of missing observations, substitutions, and sales are depicted in Figure III (color version online).

#### IV.B. Estimating the Frequency of Price Changes

We extend the methodology of Klenow and Kryvtsov (2008) for estimating the frequency of price changes to the case of time-varying frequencies of price changes.<sup>17</sup> We assume a constant probability of a price change per unit of time (a month for differentiated goods and two weeks for homogeneous goods) so that the arrival rate of a price change follows a Poisson process. In this case, the maximum likelihood estimator of the frequency of price changes is

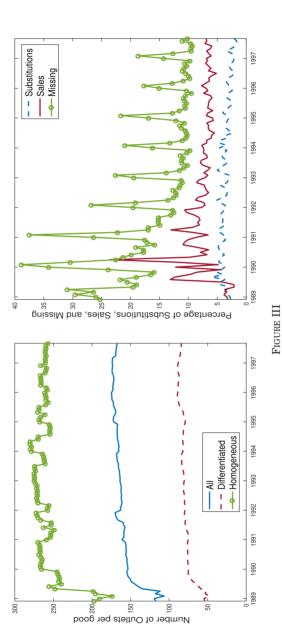
$$\lambda_t = -\ln(1 - \text{fraction of outlets that changed price})$$
 between  $t$  and  $t-1$ ).

The fraction of outlets that changed their price between periods can be calculated for individual goods or for the aggregate by pooling the data for all outlets and all goods together. In this computation, we drop observations with missing price quotes. This simple estimator just counts the fraction of price changes in a period of time, and transforms it into a per unit of time rate,  $\lambda$ . We refer to  $\lambda$  as the "instantaneous" frequency of price changes.

Later, we perform robustness checks by using different methods of aggregation across goods, by considering different treatments for sales, substitutions, and missing observations, and by dropping the assumption that price changes follow a Poisson process.

Figure IV plots the monthly time series of the simple pooled estimator of  $\lambda$  and the expected inflation rate. It assumes that all homogeneous and all differentiated goods have the same frequency of price changes and estimates this aggregate frequency by using the simple pooled estimator for the homogeneous and for the

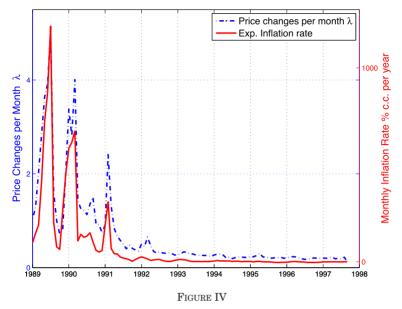
<sup>17.</sup> Using the same methodology makes our study comparable to most of the papers in the literature.



Number of Outlets per Good, and Frequencies of Substitution and Sales

For the homogeneous goods during a month, we count a sale or substitution if there was one such event

in any of the two 15-day subperiods. Missing includes stock-outs.



Estimated Frequency of Price Changes  $\lambda$  and Expected Inflation

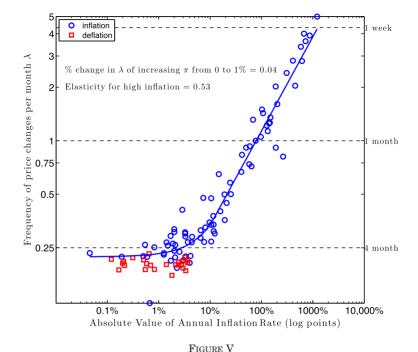
The  $\lambda$  shown is the expenditure-share-weighted average of the homogeneous and differentiated goods' simple estimator  $\hat{\lambda} = -\log(1-f_t)$ , where  $f_t$  is the fraction of outlets that changed price in period t. Inflation is the average of the log-difference of monthly prices multiplied by 1,200 and weighted by expenditure shares. Expected inflation is the average inflation rate  $\frac{1}{\lambda}$  periods ahead.

differentiated goods. The biweekly estimates of the homogeneous goods are aggregated to a monthly frequency, <sup>18</sup> and the plot shows the weighted average of these two estimators, using the share of household expenditures as weights. Finally, the expected inflation is computed as the average inflation rate  $\frac{1}{\lambda_i}$  periods ahead. We observe that the two variables are correlated. For instance, during the mid-1989 hyperinflation, the implied expected duration of a price spell is close to one week; after 1993, the implied expected duration is close to half a year.

## IV.C. The Frequency of Price Changes and Inflation

In this section, we report how the estimated frequency of price changes varies with inflation. We find that, as predicted by

<sup>18.</sup> The monthly frequency is the sum of the biweekly frequencies of each month.



The Frequency of Price Changes  $(\lambda)$  and Expected Inflation

The  $\lambda$  shown is the expenditure-share-weighted average of the homogeneous and differentiated goods' simple estimator  $\hat{\lambda} = -\log(1-f_t)$ , where  $f_t$  is the fraction of outlets that changed price in period t. Inflation is the average of the log difference of monthly prices weighted by expenditure shares. Expected inflation is the average inflation rate  $\frac{1}{\lambda t}$  periods ahead. The fitted line is  $\log \lambda = a + \epsilon \min \left\{ \pi - \pi^c, \, 0 \right\} + \nu (\min \left\{ \pi - \pi^c, \, 0 \right\})^2 + \gamma \max \left\{ \log \pi - \log \pi^c, \, 0 \right\}$ . The squares represent negative expected inflation rates and the circles positive ones.

the menu cost model, the frequency of price changes is insensitive to inflation when inflation is low. Moreover, for high inflation rates, we find that the elasticity of the frequency of price changes with respect to inflation is between one-half and the theoretical two-thirds.

Figure V plots the frequency of price changes against the rate of inflation using a log scale for both variables. <sup>19</sup> On the right axis we indicate the implied instantaneous duration, that is,  $\frac{1}{\lambda}$ . In interpreting this figure, as well as the other estimates presented

<sup>19.</sup> See Section II for caveats on these results and for the interpretation of contemporaneous correlations.

below, it is worth noting that  $\frac{1}{\lambda_t}$  is the expected duration of prices at time t if  $\lambda_t$  would remain constant in the future and provided that the probability of a price change is the same within the smallest period of observation (one month for differentiated goods and two weeks for homogeneous goods).

Motivated by the theoretical considerations in Section II, as well as the patterns evidenced in Figure V, we fit (estimated via nonlinear least squares) the following statistical model to the data:

(7) 
$$\log \lambda = \alpha + \epsilon \min \{ \pi - \pi^c, 0 \} + \nu (\min \{ \pi - \pi^c, 0 \})^2 + \gamma \max \{ \log \pi - \log \pi^c, 0 \}.$$

This model assumes that  $\log \lambda$  is a quadratic function of inflation for inflation rates below the critical value,  $\pi^c$  and that  $\log \lambda$  is a linear function of  $\log \pi$  for inflation rates above  $\pi^c$ . In Figure V we observe that  $\lambda$  is insensitive to inflation at low inflation rates. Increasing inflation from 0% to 1% a year increases the frequency of price changes by only 0.04%. Moreover, the behavior of  $\lambda$  is symmetric around zero. The frequency of price changes starts to rise for inflation rates under 5% a year. For high inflation rates, the elasticity of  $\lambda$  with respect to inflation is captured by the parameter  $\gamma$ . We estimate  $\gamma$  to be at least one-half but smaller than the theoretical limit of two-thirds. Also, as predicted by the menu cost model, as inflation rises this elasticity becomes constant—that is, the linear fit for  $\log \lambda$  as a function of  $\log \pi$  works well for high inflation rates. In this estimation, the critical value  $\pi^c$  in the statistical model, which has no theoretical interpretation, is of 14% a year.<sup>20</sup> The expected duration of price spells for zero inflation is 4.5 months, which is consistent with international evidence, as the next section shows.

The strong results in Figure V are surprising as the model applies to steady states in which the inflation rate has been at the same level for a long time. Section IV.C.2 presents two sets of robustness checks along these lines. First, we replicated Figure V using current inflation as well as different measures of expected

20. The comparative static of the models discussed in Section II does not imply a kink like the one in equation (7), we merely use this specification because it is a low-dimensional representation of interesting patterns in the data that provides a good fit and has properties at the extreme values that are consistent with our interpretation of the theory.

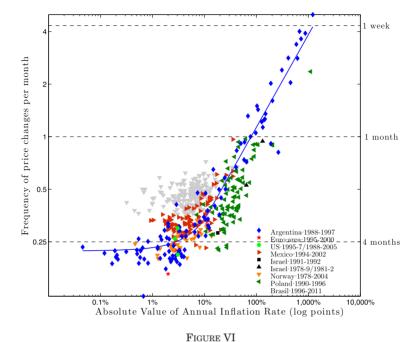
inflation instead of inflation. The results are similar to those in Figure V but somewhat weaker. Second, we redo the analysis on data simulated from our model in Section III, but in a nonstationary economy where agents anticipate a disinflation like the one that occurred in Argentina following the exchange rate peg. Again, our results are similar to conducting the analysis in a stationary economy with a fixed inflation rate.

1. International Evidence on the Frequency of Price Changes and Inflation. The previous section shows that certain aspects of price-setting behavior in Argentina are consistent with the predictions of menu cost models. In particular, the elasticity of the frequency of price changes is close to zero at low inflation rates and close to two-thirds for high inflation rates. Here we show that Argentina's inflationary experience is of special interest because it both spans and extends previous findings in the literature.

There are several studies that estimate the frequency of price changes for countries experiencing different inflation rates. Figure VI provides a visual summary of these studies and compares them to ours by adding the international evidence to Figure V.

First, observe how the wide range of inflation rates covered by our sample makes this article unique: none of the other papers covers inflation rates ranging from a mild deflation to 7.2 million percent a year (annualized rate of inflation in July 1989). This is what allows us to estimate the elasticity of the frequency of price changes with respect to inflation both at low and high inflation rates. In the other samples it is hard to test these hypotheses because of their limited inflation range. Second, we note how the patterns of the data for each country are consistent with the two predictions of the menu cost model. Third, we note that in most cases, the level of the estimated frequency of price changes is similar to Argentina's. The similarity between our results and the existing literature is remarkable since the other studies involve different economies, different goods, and different time periods. It is a strong indicator that our results are of general interest, as the theory suggests, and are not a special feature of Argentina.<sup>21</sup>

21. Table XV in Online Appendix G provides a succinct comparison of the data sets used in these studies and of the inflationary environment in place in each case. The table shows that in addition to covering a wider range of inflation rates our data set is special due to its broad coverage that includes more than 500 goods representing 85% of Argentina's consumption expenditures.



The Frequency of Price Changes  $(\lambda)$  and Expected Inflation: International Evidence

A color version of this figure is available online. Price changes per month for Argentina are the simple pooled estimator of  $\lambda.$  For the other cases we plot  $-\log(1-f),$  where f is the reported frequency of price changes in each study. The  $(\lambda,\pi)$  pairs for Argentina, Mexico, and Brazil are estimated once a month and for the other countries once a year. Expected inflation is the average inflation  $\frac{1}{\lambda}$  months ahead. Data for the Euro area is from Álvarez et al. (2006), for the United States from Bils and Klenow (2004), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008), for Mexico from Gagnon (2009), for Israel from Baharad and Eden (2004) and Lach and Tsiddon (1992), for Poland from Konieczny and Skrzypacz (2005), for Brazil Barros et al. (2009), and for Norway from Wulfsberg (2016). Logarithmic scales for both axes.

The studies included in the figure are all the ones we could find covering a wide inflation range. For the low inflation range we included studies for the United States by Bils and Klenow (2004), Klenow and Kryvtsov (2008), and Nakamura and Steinsson (2008), and for the Euro area by Álvarez et al. (2006).<sup>22</sup> Our estimates of the frequency of price changes are consistent with all

<sup>22.</sup> Recently Nakamura et al. (2018) have extended the U.S. data to cover an earlier period, which includes inflation rates going up to 14% a year.

of them.<sup>23</sup> We have three data points for Israel corresponding to an inflation rate of 16% a year between 1991 and 1992 (Baharad and Eden 2004), 64% a year between 1978 and 1979, and 120% a year between 1981 and 1982 (Lach and Tsiddon 1992). The frequency of price changes for these three points is well aligned with the Argentine data. The same is true for the Norwegian data (Wulfsberg 2016) that ranges from 0.5% to 14% a year. For Poland, Mexico. and Brazil, we were able to obtain monthly data for a wide range of inflation rates. The Polish sample ranges from 18% to 249% a year (Konjeczny and Skrzypacz 2005) and the Mexican one ranges from 3.5% to 45% a year (Gagnon 2009). In both cases, the observations are aligned with the Argentina sample. The Brazilian data (Barros et al. 2009) yields an elasticity of the frequency of price changes at high inflation that is consistent with ours. However, it yields a higher level of the frequency of price changes than ours and other studies.

2. Robustness. Next we conduct a number of robustness exercises to evaluate the sensitivity of the main results regarding the frequency of price changes. The first set of exercises deals with recurrent issues when analyzing micro-price data sets, such as missing observations and price changes due to substitutions or sales, as well as issues of aggregation across products. Second, we address biases resulting from discrete sampling. Third, we present results using different measures of expected inflation. Finally, we address the possibility that the theoretical propositions that hold in the steady state are a poor description of the Argentine experience in the high-inflation period leading to the stabilization plan in 1991 where agents are likely to have anticipated the strong disinflation that followed.

The conclusions are twofold. First, at low inflation rates the empirical findings of this section go through intact. Second, at high inflation, we observe some quantitative but not qualitative differences. Most notably, depending on the estimator used to aggregate the data, the elasticity of the frequency of price changes can range from approximately one-half to the theoretical two-thirds.

i. Missing Data, Substitutions, Sales, and Aggregation. Table I reports the sensitivity of the estimates of the elasticity  $\gamma$ , the

<sup>23.</sup> There are other studies for low-inflation countries, especially for the Euro area, but since they mostly yield estimates similar to those of Álvarez et al. (2006) we do not report them (see Álvarez et al. 2006 and Klenow and Malin 2011 for references to these studies).

 ${\bf TABLE~I}$  The Frequency of Price Adjustment and Inflation: Robustness Checks

	Elasticity $\gamma$			Semi-elasticity at zero Δ%λ			Expected duration at $\pi = 0$ (months)		
Aggregation	Diff.	Hom.	Agg.	Diff.	Hom.	Agg.	Diff.	Hom.	Agg.
Panel A: Simple estimator	(no info	ormatio	n from	missing	price qu	otes)			
Pooled	0.51	0.5	0.53	0.08	-0.01	0.04	9.1	2.9	4.5
Weighted average	0.52	0.48	0.52	0.07	-0.01	0.04	8.2	2.8	4.4
Median	0.64	0.64	0.68	0.1	0.01	0.05	15	7	9.3
Weighted Median	0.65	0.64	0.68	0.09	0	0.04	12.2	5.5	7.8
Pooled (excluding sales)	0.5	0.47	0.52	0.08	0.01	0.05	10	3.8	5.7
Panel B: All price quotes									
Pooled	0.51	0.5	0.52	0.08	0	0.04	8.8	3	5.1
Weighted average	0.52	0.45	0.49	-0.05	-0.01	0.04	8.9	2.9	4.6
Median	0.62	0.58	0.65	0.09	-0.1	0.02	15.3	4.4	10.2
Weighted median	0.62	0.65	0.65	0.09	0.01	0.04	12.9	5.4	7.5
Panel C: Excluding substitu	ution q	uotes							
Pooled	0.55	0.5	0.52	0.09	-0.01	0.03	7	3.2	6.2
Weighted average	0.52	0.45	0.51	0.06	-0.01	0.03	10.7	3	6.7
Median	0.66	0.65	0.68	0.13	0.01	0.02	18.8	9	12.4
Weighted median	0.66	0.62	0.66	0.07	-0.03	0.02	16.5	6.1	10.4
Panel D: Excluding substit	ution s	pells							
Pooled	0.52	0.5	0.52	0.07	0	0.05	10.7	3	5.5
Weighted average	0.53	0.44	0.49	-0.1	-0.01	0.04	8.6	2.9	4.8
Median	0.62	0.64	0.64	0.09	0.02	-0.02	18.4	8.3	10.9
Weighted median	0.63	0.6	0.66	0.09	-0.07	-0.05	15.4	4.7	8.2
Panel E: Excluding substitu	ution a	nd sales	s quote	s					
Pooled	0.5	0.47	0.52	0.08	0.03	0.05	9.9	2.6	6.7

Notes. Diff. denotes differentiated goods, which are sampled once a month. Hom. denotes homogeneous goods, which are sampled twice a month. Agg. denotes the weighted average of the differentiated and homogeneous goods, with weights given by the expenditure shares and where the homogeneous goods have been aggregated to monthly frequencies. For each case we use NLLS to fit:  $\log \lambda_t = a + \epsilon \min{\{\pi_t - \pi^c, 0\}} + \nu (\min{\{\pi_t - \pi^c, 0\}})^2 + \nu \max{\{\log \pi_t - \log \pi^c, 0\}} + \omega_t$ . The semi-elasticity at zero  $\Delta\%$  is the percentage change in  $\lambda$  when inflation goes from 0% to 1%. Panel A estimates  $\lambda$  with the simple estimator in equation (6) discarding information from missing prices, Panel B is the full information maximum likelihood estimator described in Online Appendix DA, Panel C replaces price quotes with a product substitution by missing data, Panel D replaces price spells ending in a product substitution by missing data, and Panel E replaces sales quotes by the previous price and product substitutions by a missing quote.

semi-elasticity  $\Delta\%\lambda$  and the duration at low inflation obtained with the simple estimator to different treatments of missing data, sales, product substitution, and broad aggregation levels. In Online Appendix DA we describe in detail the methodologies and the definitions of all estimators in Table I.

We report estimates of the three parameters of equation (7) for the sample of differentiated goods (sampled monthly), for the sample of homogeneous goods (sampled twice a month) and for the aggregate. The latter is obtained by averaging the estimated  $\lambda$ s with their expenditure shares after converting the biweekly estimates to monthly ones.

The first and second columns show the elasticities at high and low inflation. The third block of columns shows the implied duration of price spells when inflation is low (below the threshold) under the assumption that the frequency of price adjustment is constant.

The first row in Table I corresponds to the pooled simple estimator reported in Figure V. The estimates of the elasticity of the frequency of price adjustment with respect to inflation,  $\nu$ , are very similar for the  $\lambda s$  in the two samples and for the aggregate λ. The estimates for the semi-elasticity and expected duration at low inflation are markedly higher for differentiated goods in comparison to homogeneous goods. The other lines in the table provide estimates of the three parameters of interest for different aggregation methods and for the different treatments of missing observations, product substitutions, and sales. The values for the elasticity  $\gamma$  across all these estimation techniques ranges from approximately one-half to two-thirds. The variation in  $\Delta\%\lambda$  estimates is much smaller across methodologies. Both differences in the estimators result from alternative aggregation methodologies and not from the treatment of sales, substitutions, and missing values. For instance, the elasticity at high inflation when using the simple estimator with pooled data climbs from 0.53 to 0.68 when using the median estimate across industries.

The treatment of sales and substitutions does seem to have an effect on the estimates of the expected duration of price spells when inflation is low, as in other papers in the literature (see Klenow and Malin 2011). For example, durations increase from 4.5 months to 5.7 months when sales price quotes are replaced by the price quote of the previous regular price. In Klenow and Kryvtsov (2008) durations go from 2.2 months to 2.8 after the sales treatment and in Nakamura and Steinsson (2008) they go from 4.2 to 3.2 months. Time series for frequency of substitution, sales, and missing values in the sample can be seen in Figure III.

What accounts for the differences in the estimated implied duration at low inflation between the sample of differentiated and homogeneous goods? Expected durations are much higher for differentiated goods than for homogeneous goods. In principle, we believe that this discrepancy can be attributed to two features: an intrinsic difference between the type of goods or due to the fact that the prices of homogeneous goods are sampled bimonthly and prices for differentiated goods once a month.

	Elasticity $\gamma$		Semi-elasticity at zero Δ%λ		Duration at $\pi = 0$	
	Diff.	Hom.	Diff.	Hom.	Diff.	Hom.
Mean	0.58	0.56	0.03	0.01	18.5	7.5
Median	0.58	0.55	0.03	0	15	6.4
Perc 75	0.7	0.6	0.1	0.02	25.6	9.3
Perc 25	0.48	0.5	-0.03	-0.02	6.9	4.7
Std. dev.	0.14	0.08	0.1	0.07	14.2	5.2

TABLE II
DISTRIBUTION OF FITTED COEFFICIENTS AT THE FIVE-DIGIT LEVEL

Notes. Diff. denotes differentiated goods. Hom. denotes homogeneous goods. For each five-digit industry we use NLLS to fit:  $\log \lambda_t = a + \epsilon \min \left\{ \pi_t - \pi^c, 0 \right\} + \nu (\min \left\{ \pi_t - \pi^c, 0 \right\})^2 + \nu \max \left\{ \log \pi_t - \log \pi^c, 0 \right\} + \omega_t$ . The semi-elasticity at zero  $\Delta \% \lambda$  is the percentage change in  $\lambda$  when inflation goes from 0 to 1%.  $\lambda$  is estimated with the simple estimator in equation (6).

Finally, we explore the robustness of the parameter estimates for the elasticity of the frequency of price changes with respect to inflation at high and low inflation rates by fitting equation (7) for each of the five-digit industries, using the simple estimator of  $\lambda$ .<sup>24</sup>

Table II presents statistics describing the distribution of the coefficient estimates derived from equation (7) across five-digit industries. The elasticity estimates confirm our previous findings: (i) the elasticity of the frequency of price changes at high inflation,  $\gamma$ , varies between one-half and two-thirds; and (ii) the semi-elasticity  $\Delta\%\lambda$  is approximately zero regardless of the industry. Consistent with the results in Table III in the next section, there is large variation in the expected duration at low inflation, particularly so for differentiated goods.

ii. Sampling Periodicity. So far we have been using the estimator of the theoretical frequency  $\lambda_a$  that has been proposed in the literature,  $\hat{\lambda}_t = -\ln{(1-f_t)}$ . If price changes follow a Poisson process, this is the maximum likelihood estimator of  $\lambda_a$ . However, since we only observe frequency of price changes  $f_t$  at discrete times, a well known bias may arise if prices change more than once within the time interval and these changes are not independent. In particular, we would expect the bias to become larger as inflation increases and prices change more frequently.

In this section, we consider an alternative estimator  $\hat{\lambda}_t^{SW} = f_t$  and compare it to  $\hat{\lambda}_t$ . In the Sheshinski-Weiss model with no

<sup>24.</sup> We performed the same exercise at a six-digit level obtaining qualitatively similar results. See Table V for disaggregation levels.

		λ.	
Annual inflation range (%)	Median duration	75-25 pct difference	90-10 pct difference
Homogeneous goods			
<10	6.3	6.4	14.0
[10, 100]	1.7	3.0	6.4
[100, 500]	0.6	0.7	1.9
<b>≽</b> 500	0.2	0.3	0.6
Differentiated goods			
[0, 10]	9.9	12.0	27.0
[10, 100]	2.9	3.8	6.9
[100, 500]	0.8	0.7	1.6
≥500	0.3	0.2	0.4

TABLE III Cross Industry Dispersion of Duration  $\frac{1}{i}$ 

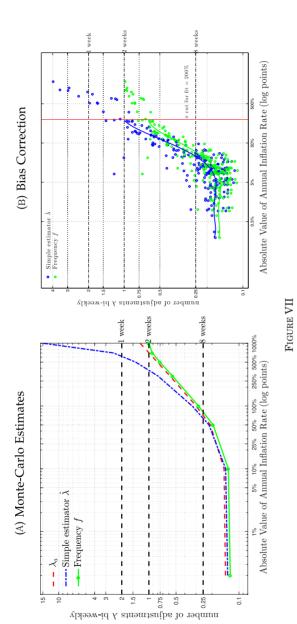
*Notes.* Duration is in months and calculated as  $\frac{1}{\lambda}$  for each five digit industry. The cross-industry statistics, for example, 75-25 pct, are computed by pooling all  $\lambda$ s corresponding to inflation rates in the interval.

idiosyncratic shocks (or in the limit as inflation becomes very large compared to the volatility of the shocks), this is a maximum likelihood estimator of  $\lambda_a$ .

In the left panel of Figure VII we present the results of Monte Carlo simulations using data generated by the model in Section II. We sample observations every two weeks and calculate both estimators of  $\lambda_a$  for different inflation rates. The true frequency of price adjustment,  $\lambda_a$ , is represented by the medium gray (red) line, the frequency, f, by the light gray (green) line and the simple estimator  $\hat{\lambda}_t$  by the dark gray (blue) line (color version online). The figure points to the existence of an upward bias in  $\hat{\lambda}_t$  for high inflation rates, and as such, in the elasticity of the frequency of price adjustments to inflation.

To reduce the incidence of such bias in our empirical estimates, we proceed by reestimating the elasticity of the frequency of price changes to inflation by excluding observations corresponding to inflation above some threshold. In the right panel of Figure VII, we show this for a threshold inflation of 200%. This threshold is where our Monte Carlo estimates show that the bias starts becoming more pronounced. The estimated elasticities are 0.63 and 0.48, respectively, much in line with our benchmark estimates.<sup>25</sup>

<sup>25.</sup> Excluding observations corresponding to inflation below 50% and above 200% or below 50% and above 100% results in estimated elasticities of 0.64 and 0.76 when using  $\hat{\lambda}$ ; when using f instead, these are 0.44 and 0.59.



Sensitivity to Sampling Periodicity.

A color version of this figure is available online. Panel A shows the theoretical  $\lambda_a$  and estimators  $\hat{\lambda}$ , f when we simulate the model in Section II and sample observations twice a month. Panel B shows both estimators for goods sampled twice a month in our data (i.e., homogeneous goods) and the fit of a polynomial of order 5 when we drop observations associated with inflation above 200%. iii. Expected Inflation. Next we check the robustness of our results to measuring expected inflation differently. So far, we have used the average realized inflation for the expected duration of the price set in period t,  $\frac{1}{\lambda_t}$ . We now consider the average of the actual inflation rate of the following  $k_t$  months, where  $k_t = \left[\frac{n}{\lambda_t}\right]$  and the operator  $[\cdot]$  refers to the integer part of a number. Formally, this is  $\pi_t^e = \frac{1}{k_t} \sum_{s=t}^{t+k_t} \pi_s$ . We refer to n as the forward-looking factor. Thus, as inflation falls (and implied durations rise) in our sample, agents put an increasing weight on future inflation. When n=0 expected and actual inflation are the same.

Table XIII in Online Appendix DG shows that the results presented here are not very sensitive to estimating equation (7) using different forward-looking factors in equation (59).

iv. Expected Disinflation. Motivated by Argentina's history in the years prior to the exchange rate peg of the 1990s, it seems reasonable to believe that forward-looking agents anticipating lower future money growth rates and inflation would have altered their pricing behavior before the exchange rate peg was actually in place. This could cast doubts on our interpretation of the evidence on menu cost models by studying the Argentinean economy during this exact period, since our theoretical results are derived for stationary economies with constant money growth rates.

Beraja (2013) studies this issue. He conducts the same comparative statics analysis from this section on data simulated from the model in Section III during a disinflation process calibrated to the Argentine economy. He finds that our theoretical results are robust to studying a nonstationary economy where agents anticipate a disinflation in the future.

# IV.D. Inflation and the Dispersion of the Frequency of Price Changes

This section reports how the dispersion of the frequency of price changes varies as inflation grows. Proposition 2 states that under certain conditions, the firm's pricing behavior when inflation is high is independent of the variance of the idiosyncratic shocks. This implies that as inflation becomes higher, it swamps the effect of idiosyncratic differences across firms that result in differences in the frequency with which they change prices. Figure II illustrates this point in the numerical example of our version of the Golosov and Lucas (2007) model in Section III. We

use this result assuming that firms in an industry have the same parameters, but that the parameters differ across industries.

In Table III we estimate  $\lambda$  for each narrowly defined industry (at a five-digit level of aggregation), <sup>26</sup> calculate the implied average duration  $\frac{1}{\lambda}$  and present two measures of the dispersion of  $\lambda$ 's across such industries: the 75-25 and 90-10 percentile differences. We observe a significant decline in dispersion as inflation rises both across homogeneous and differentiated good industries. For example, for homogeneous goods, the 90-10 percentile difference in  $\lambda$ s when inflation is above 500% a year is about 23 times smaller than the percentile difference at single-digit inflation.

# IV.E. Inflation and the Intensive and Extensive Margins of Price Adjustments

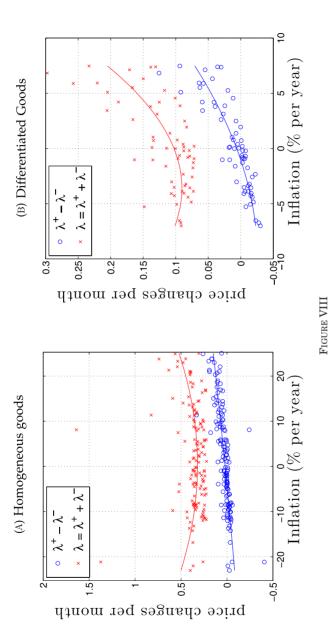
In this section, we confront theoretical predictions about the behavior of the intensive and extensive margins<sup>27</sup> of price changes with the data.

We first look at the predictions of the theory (Propositions 1 and 3 and Corollary 1) with respect to the frequency of price changes (the extensive margin of price adjustments) for near-zero inflation rates. According to theory, for near-zero inflation, the frequency of price increases and of price decreases is the same, the frequency of price changes is insensitive to inflation, and the difference between the frequency of price increases and decreases rises with inflation. Figure II illustrates some of these properties of the menu cost model in the numerical example in Section III.

Figure VIII (color version online) takes these predictions to the data for our two groups of goods. The red crosses plot the frequency of price changes  $\lambda$  against inflation and the blue circles represent the difference between the frequency of price increases and that of price decreases,  $\lambda^+ - \lambda^-$ . The range of inflation in the figure was chosen by picking the lowest, negative rate of inflation (excluding outliers) and its positive opposite. The quadratic function fitting the red crosses reflects both the insensitivity of the frequency of price changes to inflation as well as the symmetry between the frequency of price increases and price decreases—that is, quadratic functions have zero derivative and are symmetric around zero. This fact is particularly evident for homogeneous

<sup>26.</sup> Examples of five-digit aggregation are citrus fruits, soaps, and detergents. See Table V in Online Appendix C.

<sup>27.</sup> See Section II.D for a definition of these margins.



Decomposition of Inflation for Low Inflation Rates

A color version of this figure is available online.  $\lambda$  is the frequency of price changes per month,  $\lambda^+(\lambda^-)$  is the frequency of price increases (decreases) per month. Inflation is the annualized log difference of the average price between two consecutive periods. The inflation range is chosen by picking the one-percentile inflation (minimum inflation rate removing outliers) and its positive opposite. Lines are least squares second-degree polynomials. goods (Panel A), but somewhat less apparent for differentiated goods (Panel B).<sup>28</sup> Furthermore, the two panels in Figure VIII show that the prediction that the derivative of  $\lambda^+ - \lambda^-$  with respect to inflation is positive for low rates of inflation seems to be consistent with the data.

We conclude the analysis of the extensive margin of price changes at low inflation with a variance decomposition of inflation. Proposition 3 states that in near-zero inflation, most of the changes in inflation result from the extensive margin (90% to be precise). Then, in our data, we compute the extensive margin contribution as follows. Remember that inflation can be decomposed as:

$$\pi = (\lambda^+ - \lambda^-)\Delta_p^+ + (\Delta_p^+ - \Delta_p^-)\lambda^-,$$

so that its variance can be written as

$$\operatorname{var}(\pi) = \operatorname{cov}\left[(\lambda^+ - \lambda^-)\Delta_p^+, \pi\right] + \operatorname{cov}\left[(\Delta_p^+ - \Delta_p^-)\lambda^-, \pi\right].$$

Therefore, the extensive margin contribution is simply  $\frac{Cov[(\lambda^+-\lambda^-)\Delta_p^+,\pi]}{Var(\pi)}$ . Because at  $\pi=0$  the frequency of price increases and decreases are identical, this calculation approximates the theoretical extensive margin contribution in Proposition 3.<sup>29</sup> We find that for both homogeneous and heterogeneous goods, the contribution of the extensive margin to total inflation variance is between 80% and 90%, depending on whether we define "near-zero inflation" as inflation belonging to a large range (e.g., between 20% and -20%) or small (e.g., between 5% and -5%).

28. Figure IX, Panel A shows the same fact by plotting the frequency of price increases (green circles) and the frequency of price decreases (red squares) against the absolute value of inflation. The axes in Figure VIII are not on a log scale, unlike Figure V and Figure IX, thus ensuring that the insensitivity of the frequency of price adjustment to inflation is not an artifact of the scale.

29. This variance decomposition, guided by Proposition 3, differs from Klenow and Kryvtsov (2008), who do not distinguish between the frequency of price increases and decreases. Because the frequency of price changes is unresponsive to inflation for low inflation rates Klenow and Kryvtsov (2008), as well as Gagnon (2009), conclude that the variance of inflation at low inflation rates is mostly explained by the intensive margin, whereas we conclude that it is explained by the extensive margin. In our case, the latter is the change in the difference between the frequency of price increases and that of price decreases captured by  $\frac{Cov\left[(\lambda^+ - \lambda^-) \Delta_p^+, \pi\right]}{Var(\pi)}.$ 

Next, we look at extensive and intensive margin predictions for high inflation rates. We present results for the homogeneous goods alone. The results for differentiated goods are similar. As a reminder, our theoretical propositions showed that at high inflation rates, the frequency of price increases  $\lambda^+$  should converge to  $\lambda$  and the frequency of prices decreases  $\lambda^-$  should converge to zero (Lemma 1 and Lemma 2). Figure IX, Panel A shows that this is indeed the case in the Argentine data. Furthermore, Panel B shows the empirical behavior of the intensive margin. We find that for low inflation rates, the size of price changes is insensitive to inflation and that the size of price increases and decreases is the same (approximately 10%).<sup>30</sup> This is consistent with the last part of Proposition 1. As inflation rises, the size of price increases and decreases rises, with the magnitude of price increases becoming larger than that of price decreases. This is consistent with the properties of our numerical example, shown in Figure I and Figure IX.31

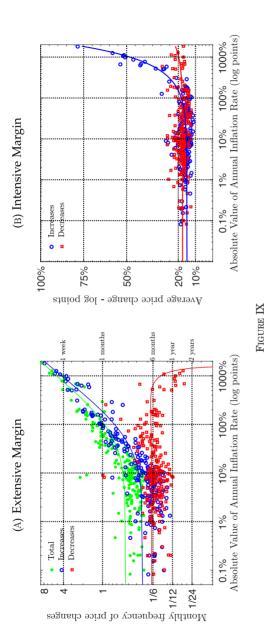
As with the results for the frequency of price changes in the previous section, one concern is that the average size of price increases in Figure IX, Panel B are biased at very high inflation rates because of the aforementioned issues with time aggregation. Thus, we repeat the analysis of Figure VII, Panel B. We calculate the size of price increases for different inflation rates using data generated by the model in Section III and sample observations every two weeks. Figure XIII in Online Appendix E compares this to the theoretical average size of price increases in the model. As opposed to what we found in Figure VII, there is almost no bias in the average size of price increases because of the sampling periodicity.

# IV.F. Inflation and the Dispersion of Relative Prices

In this section, we document the empirical sensitivity of the cross-sectional price dispersion to the inflation rate at very low and very high values of inflation. In Section II and in the example in Section III, we analyzed how inflation affects the dispersion of relative prices in menu cost models. We showed that the

<sup>30.</sup> Gagnon (2009) finds similar patterns in Mexico. For low inflation, the frequency and the absolute size of price changes are unresponsive to inflation, while the share of price increases (decreases) rises (falls) with inflation

<sup>31.</sup> Note that Figure IX, Panel B is the analog to Figure XI in Online Appendix BB, which we computed with the numerical example described in Section III. The theoretical and the empirical figures are indeed qualitatively very similar.



Intensive and Extensive Margins of Price Adjustments for Homogeneous Goods

where fis the fraction of outlets increasing or decreasing price on a given date. In Panel B, the average price change is the log difference A color version of this figure is available online. In Panel A, the frequency of price increases and decreases is calculated as -log(1-f), in prices, conditional on a price change taking place, averaged with expenditure weights over all homogeneous and differentiated goods, on a given date. Both panels use data on homogeneous goods alone. Lines are least squares second-degree polynomials. dispersion of relative prices is insensitive to changes in inflation when inflation is low, whereas it increases with inflation when inflation is high. In the limit, as the rate of inflation relative to the variance of idiosyncratic shocks becomes infinite,  $\frac{\pi}{\sigma} \to \infty$ , the elasticity of the standard deviation of relative prices with respect to inflation is bounded by one-third. In this section, we contrast these predictions with our data set and find that the empirical elasticities are remarkably close to the ones predicted by the theory.

This aspect of the data is of independent interest because at the core of the welfare costs of inflation in sticky-price models is that higher inflation introduces relative price dispersion that decouples marginal rates of substitution from marginal rate of transformation. In our model, this is captured by the effect of inflation on the standard deviation of relative prices.

Our strategy is to estimate the cross-sectional dispersion of prices in each period and correlate it with inflation. The assumption behind it is that the cross-sectional dispersion is changing through time only because of the time series variation in inflation. We think that in our case this is a reasonable assumption due to the very large changes in inflation in relatively short periods of time. We measure the dispersion of relative prices through the residual variance in a regression of prices at each time, store, and good on a rich set of fixed effects.

To estimate the effect of inflation on the distribution of relative prices, ideally we need to compare identical goods or, at least, control for factors that affect individual price levels (for example, quality or store characteristics). We proxy for these factors by controlling for goods, stores, and nonsubstitution spells, which we define next.

The Argentine statistical agency (INDEC) fixes the exact characteristics of a good in each store during what we call a non-substitution spell (see Online Appendix C). In particular, when an INDEC enumerator first goes to a store, she fixes all the characteristics of a good and records them. For instance, suppose we are talking at the most disaggregated level of goods defined as "carbonated drink of top brand in a small bottle"—this will be i

<sup>32.</sup> As pointed out by Nakamura et al. (2018) this may be a more pressing issue for their U.S. data which has much smaller changes in inflation over a longer time period.

in the notation below.<sup>33</sup> The first time the enumerator goes to the store s, she fixes the brand and the exact package of the good i, based on the information given by the manager of store s on which brand and package is sold most often in that particular store. We refer to this particular brand-package as j(i, s, t). <sup>34</sup> From then on, when the enumerator visits store s she keeps measuring the price of that particular brand and package for the good i "carbonated drink of top brand in small bottle." The first time the particular brand and package is no longer available in that store, the enumerator records this as a substitution. Subsequently the enumerator fixes a new brand and package for that store, again using the information from the store manager. We refer to the times t between these two events as a nonsubstitution spell, that is, a period where the variable j(i, s, t) takes the same value. After a substitution takes place for the same (s, i) then j(i, s, t) increases by 1. Thus, for each good-store combination (i, s), the variable i(i, s)s, t) takes positive integer values. Although we do not have access to the agency record of the description of the good in each substitution spell, we have access to the indicator of the times at which substitutions have taken place, and hence we can compute j(i, s, j)t) for each time t, good i, and store s.

We consider five cases for the specification of fixed effects where we progressively include more dummies. In each case, we estimate a weighted regression for prices of goods at each store in each time period, using the CPI weights. Then we compute the residual variance for each time across the goods and stores and convert it into a standard deviation. We plot this standard deviation against the expected inflation for each date—which correspond to a two-week period. We use the homogeneous goods because for these goods there are more outlets per good and because of the higher frequency of these goods, which is required to have a large number of good × stores with more than one nonsubstitution spell. The data set has about 5.5 million price quotes (combination of times, goods, and stores with valid prices for homogeneous goods). In case 1 we have 222 time dummies, one for each two-week period. In case 2 we have about 5,000 separate dummies for times, goods, and stores. In case 3 we have about 75,000 dummies for time and for goods × store combinations. In

<sup>33.</sup> To be concrete, we have 233 different i's for homogeneous goods.

 $<sup>34. \, {\</sup>rm To}$  be concrete, this can be regular Coca-Cola in a particular type of plastic bottle.

0.38

0.28

0.37

0.30

REGRESSIONS USED TO COMPUTE RESIDUAL VARIANCE OF PRICE LEVELS									
Models <i>i</i> : indicate dummies	# of dummies	$\operatorname{Adj} R^2$	Elast at $\pi = 100\%$	Elast at $\pi = 500\%$	Elast at $\pi = 700\%$				
1: time	212	0.751	0.03	0.21	0.31				
2: time + good + store	4,978	0.982	0.06	0.26	0.34				
3: time + good $\times$ store	74,755	0.987	0.14	0.35	0.37				

TABLE IV

4: time + good  $\times$  store

× non-subs-spell

5: time  $\times$  store + time

 $\times$  good + good  $\times$  store  $\times$  non-subs-spell

Notes, 5,497,452 price observations are used in each regression for 233 goods with prices collected twice a month over 212 periods.

0.989

0.996

0.16

0.13

153.896

464,505

case 4 we have dummies for time and for each nonsubstitution spell in each of the store × good combinations, which requires us to estimate 155,000 parameters. For many good x store combinations there is only one nonsubstitution spell during the time spanned by our data set, while for some there are dozens. Finally, in case 5 we have dummies for combinations of time × store, dummies for combinations of time × good, and separate dummies for nonsubstitution spells of each store  $\times$  good combination. In this case we have about 470,000 dummies. Table IV summarizes this information. On conceptual grounds, our preferred specification is number 4: time dummies and nonsubstitution-spell dummies for each good and store. Yet we are mindful that we are borderline in terms of the degrees of freedom left to be able to accurately estimate the residual variance in each two-week period.

Figure X displays the standard deviation of the residuals for each case, plotted against the expected inflation at each point of time. For each case we also include a fitted polynomial regression. It is clear that for low inflation the cross-sectional standard deviation of relative prices varies very little, just as the theory predicts. Instead, at very high inflation, the elasticity of the crosssectional dispersion of relative price with respect to inflation is about one-third. In particular, the last three columns of Table IV display the elasticities of the fitted polynomial regressions evaluated at annual continually compounded inflation rates of 100%, 500%, and 700% for each case. We view the values of the elasticity of the fitted line at both low and high inflation as consistent with the theory. The elasticity is zero at low inflation rates and

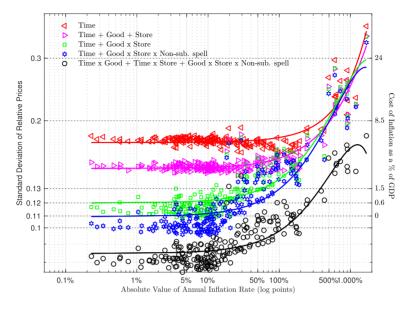


FIGURE X

Cross-Sectional Standard Deviation of Prices and Costs of Price Dispersion versus Inflation

A color version of this figure is available online. Each point is an inflation-standard deviation pair. The residual standard deviation is derived from each regression in Table IV for the sample of goods with two visits a month. The lines are OLS fitted values for a second-order polynomial in levels. The right axis shows the costs of inflation captured by equation (8) with  $\eta=6$  for  $\pi=0$  ( $\sigma=0.108$ ),  $\pi=50$  ( $\sigma=0.1175$ ),  $\pi=100$  ( $\sigma=0.129$ ), and  $\pi=500$  ( $\sigma=0.2$ ), measured as a % of GDP.

approaches the upper bound of one-third for sufficiently large inflation rates, as discussed in Section II.B and Section III. $^{35}$ 

Finally, we perform two types of exercises to evaluate the robustness of these results. As for the frequency and size of price changes, the first concern is that the standard deviation of relative prices is biased at very high inflation rates because of issues with

35. These results are sensitive to the method employed to fit the relation between the standard deviation of relative prices and expected inflation. In Figure X we fit a second-degree polynomial in levels. Fitting an equation similar to the one in Figure V, for example, the elasticity of the standard deviation with respect to inflation for low inflation is still zero, but for high inflation it is lower.

time aggregation.<sup>36</sup> Thus we repeat the Monte Carlo analysis of previous sections. Using data generated by the model in Section III, we sample observations every two weeks and calculate the standard deviation of log prices for different inflation rates. Figure XIV in Online Appendix E compares it to the theoretical standard deviation of log prices. We observe that for inflation rates higher than 500%, some bias exists. However, in Table IV, we found that the estimated elasticity remains close to the theoretical upper bound of one-third even at 500% inflation rates.

The second concern is that of sample selection. Because our sample is an unbalanced panel, it might be the case that the goods or stores for which we have price quotes at different rates of inflation have different variance of relative prices. To account for this we reproduced Figure X and Table IV for a sample of store-good pairs with 190 (out of 212) nonmissing observations. The sample is reduced by 4 million observations, so it has approximately 1.5 million observations. The results are very similar to those from the full sample and can be found in Online Appendix F.

1. The Cost of Inflation. Taken together, the foregoing results imply that the welfare cost of inflation due to the inefficient dispersion in relative prices—as emphasized in Woodford (2003, ch. 6)—is likely to be relevant only for high rates of inflation, as evidenced by the insensitivity of relative price dispersion to inflation for inflation rates below ten percent per year.

Using a second-order approximation to the expression for the decrease in output due to price dispersion, one obtains the following expression for the cost of inflation (expressed as a fraction of output lost per period):

(8) 
$$\operatorname{cost}(\pi) = \frac{\eta}{2} \left( \bar{\sigma}^2(\pi) - \bar{\sigma}^2(0) \right),$$

where  $\bar{\sigma}^2(\pi) - \bar{\sigma}^2(0)$  is the change in the variance of relative prices between an annual inflation rate of zero and  $\pi$ , and  $\eta$  is the

36. Prices of the same good are gathered from different stores over a rolling window of two weeks, as opposed to being measured simultaneously across all stores on the same day. Thus, if the prices gathered late in the two-week period are systematically higher than those gathered early on because almost all prices do change due to inflation, we obtain an upward-biased estimate of the cross-sectional standard deviation of prices at a point in time.

elasticity of substitution between the different goods. Note that this is a "typical" Harberger's triangle formula: proportional to half of the elasticity and to the (average) square of the tax wedge. Furthermore, it is worth emphasizing that this is only the part of the cost that corresponds to the distortion due to extra price dispersion: the total cost also includes the average menu cost spent per year at the inflation rate  $\pi$ .

Equation (8) and Figure X indicate that the cost of inefficient price dispersion due to inflation is highly nonlinear. The right axes of Figure X shows the costs of inefficient price dispersion for the benchmark case—regression 4 in Table IV—for different (log) annual inflation rates with  $\eta=6$ . We take the standard deviation of relative prices with no inflation to be the level of the fitted line close to zero. For an inflation rate of  $\pi=50\%$  a year the cost of inflation is approximately 0.6% of GDP, and for  $\pi=100\%$  it is approximately 1.5% of GDP. Thus, for inflation rates below 100% a year the costs of inflation arising from inefficient price dispersion are relatively moderate. For higher rates of inflation, these costs rise quickly. An inflation of  $\pi=500\%$  a year, for example, results in a cost of approximately 8.5% of aggregate output per year due to the additional price dispersion alone.

#### V. Conclusions

After deriving several predictions of menu cost models of nominal price setting at very high and near-zero inflation rates, we empirically analyzed how inflation affects price-setting behavior by using a novel micro-data set underlying Argentina's consumer price index. Argentina's experience is unique because it encompasses periods of very high and near-zero inflation, thus allowing us to test sharp predictions of menu cost models in these extreme scenarios.

We found that when inflation is low, the frequency of price changes, the dispersion of relative prices, and the absolute size of price changes are insensitive to inflation. Furthermore, we showed, both theoretically and empirically, that the difference between the frequency of price increases and decreases rises with inflation when inflation is low. These findings are consistent with predictions of menu cost models at low inflation, where idiosyncratic firm shocks swamp inflation as a motive for changing prices.

At high inflation, we found that inflation swamps idiosyncratic shocks as a driver of price changes. The frequency of price

changes across different products becomes similar, and the frequency of price changes, the dispersion of relative prices, and the average size of price changes all rise with inflation with elasticities that are quantitatively in line with Sheshinski and Weiss's (1977) menu cost model with no idiosyncratic shocks.

Furthermore, we confirmed and extended available evidence for the relationship between the frequency of price changes and inflation for countries that experienced either very high or low inflation. Despite large structural differences between these countries, we view these findings as reflecting common, robust economic mechanisms captured by menu cost models driving price changes and inflation.

Finally, we showed that the cost of inflation resulting from inefficient price dispersion is likely to be quantitatively large only for very high inflation rates.

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### SUPPLEMENTARY MATERIAL

An Online Appendix for this article can be found at *The Quarterly Journal of Economics* online. Data and code replicating tables and figures in this article can be found in Alvarez et al. (2018), in the Harvard Dataverse, doi:10.7910/DVN/C8ZOAS.

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