

Endogenous Institutions and Political Extremism*

Alexander Wolitzky

Stanford University

May 3, 2013

Abstract

The election of extreme political leaders is often associated with changes in political institutions. This paper studies these phenomena through a model in which the median voter elects a leader anticipating that he will impose institutional constraints—such as constitutional amendments, judicial appointments, or the implicit threat of a coup—that influence the behavior of future political challengers. It is typically optimal for the median voter to elect an extreme incumbent when democracy is less fully consolidated, when the costs of imposing institutional constraints are intermediate, and when the distribution of potential challengers is asymmetric. The median voter typically elects a more right-wing incumbent when the distribution of potential challengers shifts to the left. Implications of the model for the consolidation of democracy and institutional constraints are discussed, as are several related mechanisms through which politicians' ability to affect institutions may lead voters to optimally elect extremists.

JEL Codes: D72, D74

*This paper was previously circulated under the title, “A Model of Political Repression and Extremism.” I thank Daron Acemoglu for advice and thank the editor and two anonymous referees, as well as seminar participants at MIT and the IAE Concentration on Conflict, for helpful comments.

1 Introduction

According to North (1990, p. 3), institutions are “the humanly devised constraints that shape human interaction,” and are thus inherently endogenous. One important class of institutions consists of constraints that a political leader may impose on his successors. These constraints can take many forms. A leader can amend the constitution to ban certain policies. He can stack the courts or the bureaucracy with loyalists. He can curry favor with the military or secret policy, implicitly threatening a future coup. Two prominent examples of such behavior are the institutional constraints imposed by Atatürk and the Kemalist establishment in Turkey in the 1920s and 30s, and those imposed by Pinochet in Chile in the 1980s. In both cases, political leaders used a variety of tactics—including constitutional amendments, changes to the judiciary, and the implicit threat of future coup attempts—to ensure that policy choices made after they left office would not be too unfavorable to themselves.¹

This paper develops a model of endogenous institutional constraints of this kind, with an emphasis on the question of which politicians will be elected by voters who anticipate that they will impose such constraints on future leaders. A main finding is that anticipating politicians’ choice of institutional constraints can have a dramatic effect on voters’ behavior. In particular, it typically leads the (by definition centrist) median voter to elect *extremists*, because extremists have the greatest incentive to constraint future leaders from implementing policies that are even more extreme. Thus, this paper provides a new rationale for why voters may elect extreme political leaders, as well as developing a range of comparative statics on when elected leaders will be more extreme, when they will impose more stringent institutional constraints on their successors, and how the extent of democratic consolidation interacts with institutional constraints.

At the core of the model is the problem of an incumbent politician who has the ability to influence state institutions and who faces a challenger with different political preferences than his own. The more resources the incumbent devotes to influencing institutions—stacking

¹For a discussion of the Turkish case, see Yavuz (2009). For a discussion of the Chilean case, see Acemoglu and Robinson (2005).

the courts, courting the army, etc.—the more he can affect the challenger’s choice of policy (by imposing greater costs on him if he chooses unfavorable policies). Thus, the incumbent invests more in influencing institutions when the difference between his preferences and the challenger’s is greater. This simple observation is of course quite consistent with the Turkish and Chilean cases, where Atatürk and Pinochet invested heavily in institutional change because they anticipated future political support for dramatically different leaders (“Islamists” and comparative left-wingers, respectively).

The more subtle part of the model concerns the behavior of voters who anticipate that institutions will be influenced in this manner. If democracy is fully consolidated—in that the median voter can always freely choose the political leader—then institutional constraints are irrelevant as a means of controlling future challengers, and the median voter will always elects leaders who share her own political preferences (i.e., centrists). The conclusion that the median voter always elects a centrist also holds if influencing institutions is prohibitively difficult. Interestingly, it also holds if influencing institutions is very easy, as in this case even a centrist incumbent imposes strong institutional constraints that ensure that almost any challenger implements centrist policies. But, when democracy is not fully consolidated and the cost of influencing institutions is intermediate, it is typically optimal for the median voter to elect an extremist. A final comparative static is that the median voter tends to elect a more right-wing incumbent when the distribution of potential challengers shifts to the left. The intuition is that, since an incumbent imposes stronger institutional constraints when his preferences differ more from the challenger’s, a more right-wing incumbent does more to moderate the implemented policy of a left-wing challenger, and is therefore more appealing to the median voter when challengers are more left-wing.

The model also has implications about the value to the median voter of democratic consolidation (modelled as the probability that the median voter is able to choose the political leader) and institutional consolidation (modelled as the ease of influencing institutions). In particular, democratic consolidation and institutional consolidation are complements for the median voter when the incumbent is sufficiently moderate (which typically holds in the leading case where the incumbent was chosen optimally by the median voter), but are substitutes when the incumbent is sufficiently extreme. The idea is that if the incumbent is

moderate then institutional constraints are a moderating influence on average, so elections and institutional constraints are alternative ways of preventing extreme policies from being implemented in the future. But if the incumbent is extreme then institutional constraints are an extremizing influence on average, so elections are more important to the median voter when influencing institutions is easy. This result has the interesting implication that citizens in societies with easily influenced institutions may have little incentive to agitate for democratic consolidation, but that this strategy can backfire if an extreme leader comes to power.

I consider three extensions of the main model. First, I assume that an incumbent must decide whether or not to pay an upfront cost to influence institutions before the identity of the challenger is determined, perhaps corresponding to setting up a new branch of the bureaucracy or the armed forces. A consequence of this assumption is that only very extreme incumbents will pay the upfront cost. Thus, introducing an upfront cost of influencing institutions in effect forces the median voter to choose between giving up on institutional constraints by electing a centrist and electing a very extreme incumbent who is willing to pay the upfront cost (while in the baseline model it may be optimal for her to elect a “moderately extreme” incumbent).

Second, I assume that the median voter can only choose between reelecting the incumbent and electing a randomly chosen challenger (rather than being able to elect any challenger she desires). This leads to a commitment problem for the incumbent which can cause him to lose elections that he would have won without the ability to influence institutional constraints.² In addition, it can counterintuitively make it optimal for the median voter to elect a more extreme incumbent.

Finally, I assume that the incumbent can influence the extent of democratic consolidation as well as institutional constraints. While naively one might think that this would lead the median voter to elect a more moderate incumbent (as moderate incumbents have preferences closer to the median voter’s, and thus are better off when the median voter is decisive than are extreme incumbents), it turns out that this is not always the case.

²This effect also arises in one of the versions of the model of Ellman and Wantchekon (2000), discussed below.

There is a voluminous political economy literature on endogenous institutions (Acemoglu and Robinson, 2000; Aghion, Alesina, and Trebbi, 2004; Greif and Latin, 2004). Since the central feature of my model is that the median voter chooses an incumbent anticipating that the incumbent's actions (in particular, his choice of institutional constraints) will affect the distribution of future policies, one particularly related strand is the literature on political delegation and on "choosing how to choose" (Lagunoff, 2001; Aghion, Alesina, and Trebbi, 2004; Barbera and Jackson, 2004; Messner and Polborn, 2004; Acemoglu, Egorov, and Sonin, 2012b; Svulik, 2012). For example, the finding that the median voter may elect a right-wing incumbent when she is worried about left-wing challengers is reminiscent of, say, Rogoff's (1985) analysis of the optimality of delegating monetary policy to a conservative central banker, although the mechanisms involved are completely different.

This paper is also related to several others that ask why a median voter may elect extreme political leaders, especially Ellman and Wantchekon (2000), Padro i Miquel (2007), Acemoglu, Egorov, and Sonin (2012a), and the related contributions of Ortuño-Ortín (1997), Alesina and Rosenthal (2000), Ghosh (2002), and Faulí-Oller, Ok, and Ortuño-Ortín (2003). Ellman and Wantchekon (2000) show that the possibility that a strong party may incite political unrest can lead to policy divergence. Thus, their paper studies electoral competition in the presence of an exogenous threat of conflict, while my paper focusses on the electoral consequences of endogenous threats or constraints. Padro i Miquel (2007) develops a model in which voters in one group tolerate bad behavior on the part of their leader because removing him could cause a badly behaved leader from another group to take power, which would be even worse. In Acemoglu, Egorov, and Sonin (2012a), voters elect candidates with left-wing platforms because adopting a left-wing platform credibly signals that one is not a right-wing extremist, which voters are otherwise afraid of. My model shares with Acemoglu, Egorov, and Sonin's the property that fear of extremists from one side of the political spectrum can lead the median voter to elect extremists from the other side of the spectrum, although the two models are very different; for example, there is no incomplete information about incumbents in my model. Finally, in the models of Ortuño-Ortín (1997), Alesina and Rosenthal (2000), Ghosh (2002), and Faulí-Oller, Ok, and Ortuño-Ortín (2003), political parties have an incentive to choose extremist candidates or platforms because the

realized policy is assumed to result from a compromise between the chosen candidates. This mechanism (which takes different forms in each of these papers) has a flavor of the institutional constraints in my model, but in all of these papers it is specified exogenously and concerns interactions between participants in the same election rather than between an incumbent and a future political challenger.³

The paper proceeds as follows. Section 2 introduces the model. Sections 3 through 5 contain the main part of the analysis. Section 3 studies an incumbent’s optimal choice of institutional constraints. Sections 4 and 5 study the median voter’s optimal choice of incumbent, taking into account his subsequent choice of institutional constraints; Section 4 presents this analysis for a simplified version of the model, while Section 5 considers the general case. Section 6 presents the extensions of the main model, and Section 7 concludes.

2 Model

There is a continuum of individuals with preferences over a one-dimensional policy space: an individual with bliss point $\theta \in \mathbb{R}$ receives payoff

$$-l(x - \theta)$$

whenever policy x is implemented, where the loss function $l : \mathbb{R} \rightarrow \mathbb{R}$ is even (i.e., $l(x) = l(-x)$), twice-differentiable, increasing, and strictly convex, with $l(0) = l'(0) = 0$. The bliss point of the median voter is normalized to 0. There are two periods—so the policy is chosen twice—and no discounting. The timing is as follows, and everyone observes everything that has happened at all previous stages:

1. At the beginning of period 1, the median voter (voter 0) chooses the bliss point of the period 1 politician (i.e., incumbent) θ_1 .
2. The incumbent chooses the period 1 policy x_1 .

³An earlier version of this paper took a narrower view of institutional constraints as “repression” of the political challenger by the original incumbent. From that perspective, the paper offers a view of political repression that is complementary to the usual view of repression as voter disenfranchisement, as studied by Robinson and Torvik (2009) and Collier and Vicente (2012) among others.

3. With probability $p \in [0, 1]$, the median voter chooses the period 2 politician (i.e., challenger) θ_2 . With probability $1-p$, the bliss point of the period 2 politician is instead drawn from an exogenous cdf F satisfying the technical condition that $E[l(\theta)] < \infty$.⁴
4. The incumbent chooses how much to invest in institutional capacity (or influence) $k \geq 0$, at cost $\alpha c(k)$, and then chooses a function $\kappa : \mathbb{R} \rightarrow [0, k]$ that specifies the institutional resistance to each policy $x_2 \in \mathbb{R}$. The pair (k, κ) is referred to as the incumbent's choice of *institutional constraints*.
5. The challenger chooses the period 2 policy x_2 and incurs cost $\kappa(x_2)$ of facing institutional resistance.

Thus, an individual with bliss point θ receives payoff

$$-l(x_1 - \theta) - l(x_2 - \theta)$$

if she never invests in nor faces institutional constraints; receives payoff

$$-l(x_1 - \theta) - l(x_2 - \theta) - \alpha c(k)$$

if she invests in constraints; and receives payoff

$$-l(x_1 - \theta) - l(x_2 - \theta) - \kappa(x_2)$$

if she faces constraints. Assume that $\alpha > 0$ and that the function $c : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ is twice-differentiable, increasing, and weakly convex, with $c(0) = 0$.

The solution concept is subgame perfect equilibrium, henceforth *equilibrium*. The equilibria are characterized by backward induction: Section 3 studies the choice of institutional constraints of an arbitrary incumbent θ_1 facing an arbitrary challenger θ_2 and the challenger's response, and Sections 4 and 5 study the median voter's choice of incumbent, taking the subsequent choice of institutional constraints into account.

Before beginning the analysis, five remarks on the model are in order.

First, the timing of the model should be interpreted as follows: Sometime during the incumbent's tenure, the identity of his successor becomes apparent; the successor is elected

⁴Throughout, $E[\cdot]$ denotes expectation with respect to F .

by the median voter in the event that democracy holds up, and is otherwise drawn from an exogenous distribution that represents challengers that may take power through force or through the support of a newly empowered, ideological group of voters.⁵ Before leaving power, the incumbent has the opportunity to set up institutions that will partially constrain his successor's policy choice, and it is assumed that he can tailor the choice of institutional constraints to the challenger (implicitly, there is a lag between when the challenger's identity becomes known and when he actually takes power). The choice of institutional constraints consists of two parts: institutional capacity (k) and the affected policies (κ). Institutional capacity might be determined by, for example, the fraction of judges and government bureaucrats appointed by the incumbent, the amount of favor the incumbent retains with the army after the challenger takes power, or the size of any secret police or paramilitary force loyal to the incumbent. The affected policies might represent banned political parties or particular policies that have been declared unconstitutional. Alternatively, noting that the assumption that institutional resistance is non-negative is just a normalization, the affected policies could represent all of those parties or policies that are not actively favored by the judiciary, bureaucracy, or security apparatus. Finally, the challenger takes power and sets policy, but if he may face institutional resistance in the form of, for example, protests from the judiciary and government agencies, the threat of a coup from the military, or harassment from the secret police. A stylized example that fits the model well is the case of Turkey, where Atatürk and his followers established a military and judiciary strongly committed to secularism as a way of constraining future leaders from instituting Islamist policies.

Second, the assumption that the median voter acts as a dictator is a significant simplifying assumption. This is because it turns out that the preferences of a voter with bliss point θ over incumbents θ_1 (taking into account what will happen in subsequent stages) are not single-crossing in θ and θ_1 , so the optimal incumbent from the median voter's perspective may not be a Condorcet winner. However, it may be verified that preferences *are* single-

⁵Most of the qualitative conclusions of the analysis would be unaffected if there were also a possibility that the incumbent could remain in power in period 2, and would remain unaffected if the incumbent could take actions to affect the probability of this event in addition to affecting the incentives of the challenger in the event that he takes power. See Sections 6.2 and 6.3 for related extensions.

crossing in the leading special case where the cost function c is linear and the loss function l is quadratic, which I focus on in much of the analysis (including all of Section 4). For general cost and loss functions, where a Condorcet winner may not exist, assuming that the median voter is dictatorial remains a natural specification of collective decision making, though of course it might be interesting to consider alternative specifications as well.

Third, it is important that the cost of investing in institutional capacity is borne by the incumbent rather than the median voter. If costs were borne equally by all individuals, then the median voter would always elect an incumbent exactly like herself, and such an incumbent would then choose institutional constraints optimally from the median voter's perspective. The assumption that the cost is borne by the incumbent seems realistic, in that the time, money, and political capital that go into amending the constitution, stacking the courts, and influencing the army could otherwise be diverted into private consumption or into political projects that yield direct private benefits for the incumbent or his party. The conclusions of the model would also go through if the cost of investing in institutional capacity was shared partially but unequally between the incumbent and the median voter.

Fourth, another modelling choice is the assumption that once the incumbent acquires influence k , he can impose cost k on the challenger for choosing any number of policies (rather than, for example, having k total "units of cost" that he must spread across policies). This assumption seems natural given the motivating examples: it is as if the incumbent acquires k "hired guns" whom he can then direct as he sees fit. But of course alternative assumptions might also be worth examining.

Finally, it should be emphasized that the point of the model is to examine just one particular aspect of institutional choice: institutional constraints aimed at future political leaders. However, we do also interpret the parameters p and α as the extent of democratic and institutional consolidation (which are of course other kinds of "institutions"), as well as endogenizing p in Section 6.3.⁶

⁶The interpretation of p as the extent of democratic consolidation seems natural. For the interpretation of α , recall that α parametrizes the cost of investing in institutional influence. So the "consolidation" measured by α is in the sense of rigidity or resistance to influence.

3 The Incumbent's Choice of Institutional Constraints

This section studies the incumbent's choice of institutional constraints and the challenger's response. I first characterize the optimal choice of institutional constraints (k, κ) and resulting period 2 policy x_2 for an incumbent with bliss point θ_1 facing a challenger with bliss point θ_2 , and then present comparative statics.

I first consider the optimal resistance function κ and resulting period 2 policy taking institutional capacity k as given. For any resistance function κ , the challenger chooses a policy x_2 in the set

$$\arg \max_{x_2 \in \mathbb{R}} -l(x_2 - \theta_2) - \kappa(x_2).$$

Observe that an incumbent with institutional capacity k can induce the challenger to choose any policy $x_2 \in [\theta_2 - l^{-1}(k), \theta_2 + l^{-1}(k)]$ by choosing the resistance function κ given by $\kappa(x_2) = 0$ and $\kappa(x'_2) = k$ for all $x'_2 \neq x_2$ (throughout, $l^{-1}(k)$ is to be read as the unique *positive* number x such that $l(x) = k$). In addition, he cannot induce the challenger to choose any policy that is more than distance $l^{-1}(k)$ away from θ_2 , as for any function κ the challenger would rather choose policy $x_2 = \theta_2$ than choose a policy more than distance $l^{-1}(k)$ from θ_2 . Thus, when the incumbent has institutional capacity k , the resulting period 2 policy x_2 is given by

$$x_2 = \left\{ \begin{array}{l} \theta_1 \text{ if } \theta_2 \in [\theta_1 - l^{-1}(k), \theta_1 + l^{-1}(k)] \\ \theta_2 + l^{-1}(k) \text{ if } \theta_2 < \theta_1 - l^{-1}(k) \\ \theta_2 - l^{-1}(k) \text{ if } \theta_2 > \theta_1 + l^{-1}(k) \end{array} \right\}^7. \quad (1)$$

Hence, the incumbent uses the threat of institutional resistance to shift the resulting policy from the challenger's bliss point toward his own bliss point by up to $l^{-1}(k)$ units.

Given equation (1), if $\theta_1 \geq \theta_2$ then the incumbent chooses institutional capacity k to solve

$$\max_{k \in \mathbb{R}_+} -l(\theta_1 - \theta_2 - l^{-1}(k)) - \alpha c(k). \quad (2)$$

This problem yields first-order condition

$$\alpha c'(k) l'(l^{-1}(k)) = l'(\theta_1 - \theta_2 - l^{-1}(k)). \quad (3)$$

⁷There is some leeway in the choice of resistance function κ . But only the (unique) resulting policy x_2 matters for the subsequent analysis.

This condition is necessary and sufficient, because the incumbent's problem is strictly concave (by strict convexity of l and weak convexity of c) and the assumption that $l(0) = l'(0) = 0$ guarantees an interior solution. The incumbent's problem when $\theta_1 \leq \theta_2$ is given by switching θ_1 and θ_2 in (2), so in this case k is given by

$$\alpha c'(k) l'(l^{-1}(k)) = l'(\theta_2 - \theta_1 - l^{-1}(k)). \quad (4)$$

Equations (1), (3), and (4) completely characterize the optimal choice of institutional capacity and resulting period 2 policy for an incumbent with bliss point θ_1 facing a challenger with bliss point θ_2 , which I denote by $k^*(\theta_1, \theta_2)$ and $x_2^*(\theta_1, \theta_2)$ respectively. In addition, comparative statics on institutional capacity and period 2 policy are straightforward: when his bliss point is farther from the challenger's, an incumbent invests more in institutional capacity, but the resulting period 2 policy is still farther from his bliss point. This is because the incumbent balances his loss from the period 2 policy against the cost of investing in institutional capacity, and both the loss and cost functions are convex.

Proposition 1 *The incumbent's investment in institutional capacity k is increasing in the distance between his bliss point and the challenger's, $|\theta_1 - \theta_2|$. The distance between the induced period 2 policy x_2 and the incumbent's bliss point is also increasing in $|\theta_1 - \theta_2|$.*

Proof. Suppose that $\theta_1 \geq \theta_2$. Then it is immediate from (3) and convexity of l and c that k is increasing in $\theta_1 - \theta_2$. This in turn implies that the left-hand side of (3) is increasing in $\theta_1 - \theta_2$, and therefore the right-hand side of (3) must also be increasing in $\theta_1 - \theta_2$. Hence, $\theta_1 - \theta_2 - l^{-1}(k)$ (which is the distance between x_2 and θ_1) is increasing in $\theta_1 - \theta_2$. The $\theta_1 \leq \theta_2$ case is symmetric. ■

This simple observation that an incumbent invests more in institutional capacity when facing a challenger whose preferences differ more from his own will be the key consideration of the median voter when determining which incumbent to elect.

I conclude this section by introducing the *linear cost–quadratic loss case*, which is a focus of the analysis because of its tractability. Suppose that $c(k) = k$ and $l(x) = x^2$. Then (3) becomes $2\alpha\sqrt{k} = 2(\theta_1 - \theta_2 - \sqrt{k})$, or

$$\sqrt{k} = \frac{\theta_1 - \theta_2}{1 + \alpha},$$

and the resulting period 2 policy is

$$x_2 = \theta_2 + \sqrt{k} = \frac{\theta_1 + \alpha\theta_2}{1 + \alpha}. \quad (5)$$

Thus, in the linear-quadratic case the resulting period 2 policy is simply a weighted average of the incumbent's and challenger's bliss points, where the weight on the incumbent's bliss point (i.e., $\frac{1}{1+\alpha}$) is greater when institutions are easier to influence.

4 The Median Voter's Choice of Incumbent: Linear-Quadratic Case

I now analyze the median voter's choice of incumbent, anticipating his subsequent choice of institutions. This section considers the simple linear-quadratic case ($c(k) = k$; $l(x) = x^2$), while Section 5 considers the general case. Section 4.1 solves for the median voter's choice of incumbent in closed form (which is not possible in the general case). In particular, it shows that it is generally optimal for the median voter to elect an extremist (θ_1 significantly different from 0) in period 1. Section 4.2 provides comparative statics. Finally, Section 4.3 considers the value of democratic and institutional consolidation for the median voter.

4.1 The Median Voter's Problem

An important preliminary observation is that if the median voter is decisive in period 2 then she can always choose a challenger θ_2 such that the resulting period 2 policy equals her bliss point of 0, regardless of the bliss point of the incumbent θ_1 (both in the linear-quadratic case and in general). In other words, she can always choose θ_2 to “undo” whatever institutional constraints the incumbent will end up imposing. Formally, we have the following.

Proposition 2 *For any incumbent θ_1 , there exists a unique challenger θ_2 such that the resulting policy $x_2^*(\theta_1, \theta_2)$ equals 0. The median voter elects this challenger θ_2 whenever she is decisive in period 2.*

Proof. Suppose that $\theta_1 \geq 0$. Then, for any $\theta_2 \leq 0$, the resulting policy is $x_2^*(\theta_1, \theta_2) = \theta_2 + l^{-1}(k)$, where k is given by equation (3). Since $l^{-1}(k) \geq 0$, it follows that $x_2^*(\theta_1, 0) \geq 0$.

In addition, equation (3) implies that $\lim_{\theta_2 \rightarrow -\infty} k^*(\theta_1, \theta_2) = \infty$. Hence, as $\theta_2 \rightarrow -\infty$, the left-hand side of (3) goes to ∞ , which implies that the right-hand side of (3) must also go to ∞ , and thus $x_2^*(\theta_1, \theta_2) = \theta_2 + l^{-1}(k) \rightarrow -\infty$. Furthermore, it follows easily from Proposition 1 that $x_2^*(\theta_1, \theta_2)$ is increasing and continuous in θ_2 . Therefore, by the Intermediate Value Theorem, there exists a unique value of θ_2 such that $x_2^*(\theta_1, \theta_2) = 0$. Finally, electing this challenger θ_2 yields period 2 payoff 0 for the median voter, while electing any other challenger yields a strictly negative payoff. So the median voter elects this challenger whenever she is decisive in period 2. The $\theta_1 \leq 0$ case is symmetric. ■

In particular, the median voter receives period 2 payoff 0 whenever she is decisive in period 2, regardless of the bliss point of the incumbent θ_1 . Therefore, the median voter's period 1 problem is

$$\max_{\theta_1} -l(\theta_1) - (1-p) \int l(x_2^*(\theta_1, \theta_2)) dF(\theta_2) - p(0). \quad (6)$$

By (5), in the linear-quadratic case this simplifies to

$$\max_{\theta_1} -\theta_1^2 - (1-p) \int \left(\frac{\theta_1 + \alpha\theta_2}{1 + \alpha} \right)^2 dF(\theta_2).$$

This is a strictly concave problem with first-order condition

$$\theta_1 = - (1-p) \int \frac{\theta_1 + \alpha\theta_2}{(1 + \alpha)^2} dF(\theta_2) = - (1-p) \left(\frac{\theta_1 + \alpha E[\theta]}{(1 + \alpha)^2} \right),$$

or equivalently

$$\theta_1 = - \frac{\alpha(1-p)}{(1 + \alpha)^2 + 1 - p} E[\theta]. \quad (7)$$

Thus, the bliss point of the *optimal incumbent* (from the perspective of the median voter) is a negative constant times the average bliss point of the challenger. The intuition is that, since an incumbent invests more in institutional capacity when facing a challenger whose preferences differ more from his own, an incumbent with bliss point on the opposite side of 0 from the average challenger bliss point tends to shift challengers' policies toward 0 by more than a centrist incumbent does, and this effect becomes more important the more extreme is the average challenger bliss point.⁸

⁸An interesting observation here is that in the linear-quadratic case it is generally optimal for the median voter to elect an extremist in period 1, and then elect an extremist from the opposite side of the political spectrum in period 2 (if she is decisive in period 2). Thus, the ex ante possibility that the median voter may not be decisive in period 2 only leads her to elect extremists in *both* periods.

4.2 Comparative Statics

The simple solution to the median voter’s problem in the linear-quadratic case (given by (7)) yields sharp and intuitive comparative statics on the effects of the extent of democratic and institutional consolidation (p and α , respectively) and the distribution of challengers. In particular, the following result—which is one of the main findings of the paper—identifies unconsolidated democracy, intermediate cost of influencing institutions, and asymmetry of the distribution of political challengers as the key factors that lead the median voter to elect an extremist incumbent.

Proposition 3 *In the linear-quadratic case, the optimal incumbent is more extreme when democracy is less consolidated (i.e., p is lower) and when the expected challenger is more extreme (i.e., $|E[\theta]|$ is higher), and the extremism of the optimal incumbent is inverse U-shaped in the extent of institutional consolidation α .*

Proof. The comparative statics on p and $|E[\theta]|$ are immediate from (7). For the comparative static on α , it is easy to check that the derivative of $\frac{\alpha(1-p)}{(1+\alpha)^2+1-p}$ with respect to α is positive if $\alpha < \sqrt{2-p}$ and negative if $\alpha > \sqrt{2-p}$. ■

The intuition for the comparative static with respect to $|E[\theta]|$ is given in the previous subsection. For the comparative static with respect to p , the intuition is that the benefit of electing an extremist is decreasing in p , as when p is larger institutional constraints are less likely to affect the distribution of period 2 policies (as they affect this distribution only in the event that the median voter is not decisive in period 2, by Proposition 2), while the cost of electing an extremist is that extremists choose worse period 1 policies, which is independent of p . Finally, for the comparative static with respect to α , the intuition is as follows: If α is very small, then the median voter can obtain an expected payoff close to 0 (her best possible payoff) by electing a centrist incumbent (θ_1 close to 0), because a centrist incumbent will use institutional constraints to concentrate period 2 policy at 0. If α is very large, then institutional constraints have little influence on period 2 policy, so it is essentially optimal for the median voter to choose the incumbent to maximize her period 1 payoff only, which again entails electing a centrist. Hence, it is only for intermediate values of α that the median voter finds it optimal to elect an extreme incumbent.

4.3 Democratic and Institutional Consolidation

A last set of results concerns the relationship between the extent of democratic and institutional consolidation (as measured by p and α). The first observation is that, from the perspective of the median voter in period 1, democratic and institutional consolidation are complements, intuitively because institutional flexibility is useful as a means of controlling challengers precisely when democracy breaks down (i.e., when the median voter is not decisive). However, for an arbitrary, exogenously given incumbent, democratic and institutional consolidation are complements if the incumbent is sufficiently moderate but may be substitutes if the incumbent is sufficiently extreme, because in the latter case greater institutional flexibility leads to more extreme policies if the median voter is not decisive, and this makes being decisive more important for the median voter. These results imply that voters in a society with less consolidated (i.e., easier to influence) institutions have little incentive to devote resources to consolidating democracy, but that failing to do so can backfire dramatically if a sufficiently extreme politician comes to power.

It is straightforward to show that democratic and institutional consolidation are complements in the linear-quadratic case.

Proposition 4 *In the linear-quadratic case, democratic and institutional consolidation are complements from the perspective of the median voter.*

Proof. Recall that the period 2 policy given incumbent θ_1 and challenger θ_2 is $x_2 = \frac{\theta_1 + \alpha\theta_2}{1 + \alpha}$. Therefore, the cross-partial of the median voter's problem with respect to p and α equals

$$\begin{aligned} \int 2 \left(\frac{\theta_1 + \alpha\theta_2}{1 + \alpha} \right) \left(\frac{\theta_2 - \theta_1}{(1 + \alpha)^2} \right) dF(\theta_2) &= \int 2 \left(\frac{\alpha\theta_2^2 + (1 - \alpha)\theta_1\theta_2 - \theta_1^2}{(1 + \alpha)^3} \right) dF(\theta_2) \\ &\geq 2 \left(\frac{\alpha E[\theta^2] + (1 - \alpha)\theta_1 E[\theta] - \theta_1^2}{(1 + \alpha)^3} \right). \end{aligned} \quad (8)$$

Now recall that the optimal incumbent is $\theta_1 = -\frac{\alpha(1-p)}{(1+\alpha)^2 + 1 - p} E[\theta]$. Making this substitution,

the sign of (8) equals the sign of

$$\begin{aligned}
& \alpha E[\theta^2] - (1 - \alpha) \left(\frac{\alpha(1-p)}{(1+\alpha)^2 + 1 - p} \right) E[\theta]^2 - \left(\frac{\alpha(1-p)}{(1+\alpha)^2 + 1 - p} \right)^2 E[\theta]^2 \\
\geq & \alpha E[\theta]^2 \left[1 - \left(\frac{\alpha(1-p)}{(1+\alpha)^2 + 1 - p} \right) \left(1 - \alpha + \frac{\alpha(1-p)}{(1+\alpha)^2 + 1 - p} \right) \right] \\
\geq & \alpha E[\theta]^2 \left[1 - \left(\frac{\alpha}{(1+\alpha)^2 + 1} \right) \left(1 - \alpha + \frac{\alpha}{(1+\alpha)^2 + 1} \right) \right] \geq 0,
\end{aligned}$$

where the first inequality follows because $E[\theta^2] \geq E[\theta]^2$ and the second inequality follows because both terms in parentheses are less than 1. Hence, p and α are complements. ■

It is also straightforward to examine the relationship between democratic and institutional consolidation in the linear-quadratic case where the incumbent θ_1 is taken as exogenous. Most interestingly, p and α are substitutes conditional on the incumbent's being sufficiently extreme, in contrast to Proposition 4 (and the case where the exogenous incumbent is moderate). The intuition is that a sufficiently extreme incumbent primarily uses institutional constraints to shift challengers toward implementing more extreme policies. Hence, making it easier to acquire institutional capacity hurts the median voter if she is not decisive in period 2 and thus makes being decisive in period 2 more valuable for the median voter.

Proposition 5 *Suppose that $E[\theta] \neq 0$. Then, in the linear-quadratic case with an exogenous incumbent θ_1 , democratic and institutional consolidation are complements from the perspective of the median voter if $|\theta_1|$ is sufficiently small and are substitutes from the perspective of the median voter if $|\theta_1|$ is sufficiently large.*

Proof. Recall from the proof of Proposition 4 that the cross-partial of the median voter's problem with respect to p and α equals

$$\int 2 \left(\frac{\alpha\theta_2^2 + (1-\alpha)\theta_1\theta_2 - \theta_1^2}{(1+\alpha)^3} \right) dF(\theta_2).$$

Since $E[\theta] \neq 0$, this is positive whenever $|\theta_1|$ is sufficiently small, and it is clearly negative whenever $|\theta_1|$ is sufficiently large. Thus, p and α are complements if $|\theta_1|$ is sufficiently small and substitutes if $|\theta_1|$ is sufficiently large. ■

5 The Median Voter's Choice of Incumbent: General Case

This section considers the median voter's problem (given by (6)) without restricting to the linear-quadratic case. While this problem cannot be solved in closed form in general, several results can still be derived. Section 5.1 provides necessary conditions for the median voter to elect an extreme incumbent. Section 5.2 contains comparative statics. Section 5.3 studies the relationship between democratic and institutional consolidation. In general, many of the insights from the linear-quadratic case carry over to the general case, but several caveats as well as additional possibilities arise.

5.1 When Does the Median Voter Elect an Extremist?

Recall that the basic tradeoff faced by the median voter in choosing the incumbent is that a centrist incumbent (θ_1 close to 0) implements her bliss point in period 1, but an extremist incumbent might use institutional constraints to favorably influence the distribution of period 2 policies in the event that the median voter is not decisive in period 2. In this subsection, I show that electing a centrist is optimal if democracy is nearly fully consolidated (so that the median voter is almost always decisive in period 2), if influencing institutional constraints is either very difficult (so that the incumbent has little influence on period 2 policy) or very easy (so that electing a centrist incumbent leads period 2 policy to be concentrated at 0), or if F is symmetric, but that otherwise electing an extremist may be optimal. Thus, as in the linear-quadratic case, the median voter elects an extremist incumbent only if democracy is unconsolidated, the cost of influencing institutions is intermediate, and the distribution of political challengers is asymmetric.

The following result formalizes the above conditions under which the median voter elects a centrist.⁹

⁹For the symmetric F case, the result states only that $\theta_1 = 0$ satisfies the first-order condition of the median voter's problem, not that $\theta_1 = 0$ is the unique optimal incumbent. This distinction matters because in general the median voter's problem may not be concave (though it is concave in the linear-quadratic case).

Proposition 6 *For all $\varepsilon > 0$, there exists $\delta > 0$ such that if $p > 1 - \delta$ (democracy is sufficiently consolidated), if $\alpha < \delta$ (influencing institutions is sufficiently easy), or if $\alpha > 1/\delta$ (influencing institutions is sufficiently difficult), then every optimal incumbent satisfies $|\theta_1| < \varepsilon$. In addition, if F is symmetric then $\theta_1 = 0$ satisfies the first-order condition of the median voter's problem (given by (6)).*

The intuition for the result on α is as in Proposition 3. The intuition for the result on p is that if p is sufficiently high then the median voter can obtain expected payoff close to 0 by electing a centrist incumbent and then electing a centrist challenger whenever she is decisive, so she never takes the certain loss in terms of period 1 policy that would result from electing an extremist. Finally, if F is symmetric then any gain from electing an incumbent with bliss point slightly different from 0 in terms of the period 2 policy that results when the challenger is on one side of 0 is exactly offset by a loss that results when the challenger is on the other side.

Proof of Proposition 6. Note that the resulting period 2 policy given incumbent $\theta_1 = 0$ and challenger θ_2 satisfies $|x_2| \leq |\theta_2|$. Therefore, choosing incumbent $\theta_1 = 0$ yields payoff at least $-(1-p)E[l(\theta_2)]$ for the median voter. On the other hand, choosing any incumbent with $|\theta_1| \geq \varepsilon$ yields payoff at most $-l(\varepsilon)$. Thus, if $p > 1 - \frac{l(\varepsilon)}{E[l(\theta_2)]}$ then choosing incumbent $\theta_1 = 0$ is strictly better than choosing any incumbent with $|\theta_1| \geq \varepsilon$, which implies that every optimal incumbent satisfies $|\theta_1| < \varepsilon$ whenever $p > 1 - \frac{l(\varepsilon)}{E[l(\theta_2)]}$.

Next, note that (3) implies that, for any θ_1 and θ_2 , $|\theta_1 - x_2| \rightarrow 0$ as $\alpha \rightarrow 0$. Also, by Proposition 1, $|\theta_1 - x_2|$ is increasing in $|\theta_1 - \theta_2|$. Hence, for every distance $d > 0$, there exists $\delta > 0$ such that if $\alpha < \delta$ then $|\theta_1 - x_2| < \frac{\varepsilon}{2}$ for all θ_1 and θ_2 such that $|\theta_1 - \theta_2| < d$. Therefore, if $\alpha < \delta$ then choosing incumbent $\theta_1 = 0$ yields payoff at least $-(1-p)[\Pr(|\theta_2| < d)l(\frac{\varepsilon}{2}) + \Pr(|\theta_2| \geq d)E[l(\theta_2) | |\theta_2| \geq d]]$ for the median voter, which converges to $-(1-p)l(\frac{\varepsilon}{2})$ as $d \rightarrow \infty$, by the assumption that $E[l(\theta_2)] < \infty$. But choosing any incumbent with $|\theta_1| \geq \varepsilon$ yields payoff at most $-l(\varepsilon)$. So every optimal incumbent satisfies $|\theta_1| < \varepsilon$ whenever α is sufficiently small.

Similarly, (3) implies that, for any θ_1 and θ_2 , $k \rightarrow 0$ as $\alpha \rightarrow \infty$, and Proposition 1 states that k is increasing in $|\theta_1 - \theta_2|$. Hence, for every constant $\eta > 0$ and distance $d > 0$ there exists $\delta > 0$ such that if $\alpha > 1/\delta$ then $k < \eta$ for all θ_1 and θ_2 such that $|\theta_1 - \theta_2| < d$.

Therefore, choosing any incumbent with $|\theta_1| \in [\varepsilon, E[l(\theta_2)]]$ yields payoff at most

$$\begin{aligned} & -l(\theta_1) - (1-p) \Pr(|\theta_1 - \theta_2| < d) (E[l(\theta_2)] - \eta l'(|\theta_1| + d)) \\ \leq & -l(\varepsilon) - (1-p) \Pr(|\theta_1 - \theta_2| < d) (E[l(\theta_2)] - \eta l'(E[l(\theta_2)] + d)), \end{aligned}$$

which converges to $-l(\varepsilon) - (1-p) E[l(\theta_2)]$ as $\eta \rightarrow 0$ and then $d \rightarrow \infty$, uniformly over $|\theta_1| \in [\varepsilon, E[l(\theta_2)]]$. In addition, choosing any incumbent with $|\theta_1| > \max\{\varepsilon, E[l(\theta_2)]\}$ also yields payoff strictly less than $-(1-p) E[l(\theta_2)]$. But choosing incumbent $\theta_1 = 0$ yields payoff at least $-(1-p) E[l(\theta_2)]$. So every optimal incumbent satisfies $|\theta_1| < \varepsilon$ whenever α is sufficiently large.

Finally, for the F symmetric case, simplify notation by letting $y^*(\theta_1, \theta_2)$ be the value of $l^{-1}(k)$ that satisfies (3) (if $\theta_1 \geq \theta_2$) or (4) (if $\theta_1 \leq \theta_2$). Note that the function y^* is differentiable in θ_1 and θ_2 by the Implicit Function Theorem. Then the derivative with respect to θ_1 of the median voter's problem (6), evaluated at $\theta_1 = 0$, exists and equals

$$\begin{aligned} & -l'(0) - (1-p) \int_{-\infty}^0 l'(\theta_2 + y^*(0, \theta_2)) \left(\frac{\partial y^*(\theta_1, \theta_2)}{\partial \theta_1} \Big|_{\theta_1=0} \right) dF(\theta_2) \\ & - (1-p) \int_0^{\infty} l'(\theta_2 - y^*(0, \theta_2)) \left(\frac{\partial y^*(\theta_1, \theta_2)}{\partial \theta_1} \Big|_{\theta_1=0} \right) dF(\theta_2). \end{aligned} \quad (9)$$

Comparing (3) and (4), it follows that $y^*(0, \theta_2) = y^*(0, -\theta_2)$ and that $\frac{\partial y^*(\theta_1, \theta_2)}{\partial \theta_1} \Big|_{\theta_1=0} = -\frac{\partial y^*(\theta_1, -\theta_2)}{\partial \theta_1} \Big|_{\theta_1=0}$ for all θ_2 . In addition, the fact that l is strictly convex and even implies that $l'(0) = 0$ and $l'(x) = -l'(-x)$ for all x . Combining these observations implies that (9) equals zero whenever F is symmetric, which is to say that $\theta_1 = 0$ satisfies the first-order condition of the median voter's problem whenever F is symmetric. ■

5.2 Comparative Statics

Recall that in the linear-quadratic case all of the conditions in Proposition 6 have corresponding comparative statics results: not only does the median voter elect a centrist for p close to 1, for $E[\theta]$ close to 0, or for α close to 0 or ∞ , but the optimal incumbent always becomes more extreme as p decreases, as $|E(\theta)|$ increases, or as α approaches $\sqrt{2-p}$. However, it turns out that not all of these comparative statics continue to hold in the general case. The

main difficulty is that the median voter's problem is neither concave in θ_1 nor supermodular in θ_1 and F (with, for example, the first-order stochastic dominance order on distributions). However, one can establish that the comparative static on p holds very generally, so that the optimal incumbent is always more extreme when democracy is more weakly consolidated. I also present an example that shows that a first-order stochastic dominance shift of F to the left can sometimes shift the optimal incumbent to the left, which shows that not all of the comparative statics from the linear-quadratic case generalize.

The general comparative static with respect to the extent of democratic consolidation is the following.

Proposition 7 *Fix $p' > p$ and let*

$$\bar{\theta}_1(p') \in \underset{\theta_1 \in \arg\max -l(\theta_1) - (1-p')E[l(x_2)|\theta_1]}{\operatorname{argmax}} |\theta_1|$$

and

$$\underline{\theta}_1(p) \in \underset{\theta_1 \in \arg\max -l(\theta_1) - (1-p)E[l(x_2)|\theta_1]}{\operatorname{argmin}} |\theta_1|,$$

so that $\bar{\theta}_1(p')$ is one of the most extreme incumbents that is an optimal choice of the median voter given parameter p' , and $\underline{\theta}_1(p)$ is one of the least extreme incumbents that is an optimal choice of the median voter given parameter p . Then $|\bar{\theta}_1(p')| \leq |\underline{\theta}_1(p)|$; that is, $\bar{\theta}_1(p')$ is less extreme than $\underline{\theta}_1(p)$.

Proof. To simplify notation, let $u(\theta_1, p) \equiv -l(\theta_1) - (1-p) \int l(x_2^*(\theta_1, \theta_2)) dF(\theta_2)$, the median voter's payoff from electing incumbent θ_1 given parameter p .

Note that $u(\bar{\theta}_1(p'), p') \geq u(\theta_1, p')$ for all θ_1 , and that in addition $\int l(x_2^*(\bar{\theta}_1(p'), \theta_2)) dF(\theta_2) < \int l(x_2^*(\theta_1, \theta_2)) dF(\theta_2)$ for all θ_1 such that $|\theta_1| < \bar{\theta}_1(p')$ (as otherwise $\bar{\theta}_1(p')$ could not be an optimal incumbent). Therefore, for all θ_1 such that $|\theta_1| < \bar{\theta}_1(p')$, it follows that

$$\begin{aligned} u(\bar{\theta}_1(p'), p) &= u(\bar{\theta}_1(p'), p') - (p' - p) \int l(x_2^*(\bar{\theta}_1(p'), \theta_2)) dF(\theta_2) \\ &> u(\theta_1, p') - (p' - p) \int l(x_2^*(\theta_1, \theta_2)) dF(\theta_2) \\ &= u(\theta_1, p). \end{aligned}$$

Hence, no incumbent θ_1 such that $|\theta_1| < \bar{\theta}_1(p')$ can be an optimal choice of the median voter given parameter p . This implies that $|\underline{\theta}_1(p)| \geq |\bar{\theta}_1(p')|$. ■

Finally, I present a simple example that demonstrates that a first-order stochastic dominance shift to the left in F may not shift the set of optimal incumbents to the right (unlike in the linear-quadratic case).

Example 1 *Suppose that $\frac{1}{5}$ of the population has each of the five bliss points $-5, -1, 0, 1,$ and 5 . Let $p = 0$, $l(x) = x^2$, and $\alpha = 1$, and let c be given by*

$$c(k) = \left\{ \begin{array}{l} 0 \text{ if } k \in [0, 1] \\ 9 \text{ if } k \in (1, 4] \\ 100 \text{ if } k > 4 \end{array} \right\}.$$

In particular, it is costless for an incumbent to move the challenger's policy 1 unit from his bliss point, it costs 9 for an incumbent to move the challenger's policy 2 units from his bliss point (because $l^{-1}(4) = 2$), and it costs 100 for an incumbent to move the challenger's policy more than 2 units from his bliss point (which makes doing so prohibitively costly). Thus, in equilibrium an incumbent will move the challenger's policy 2 units away from his bliss point if and only if $|\theta_1 - \theta_2| \geq 6$ (as if $|\theta_1 - \theta_2| = 6$ then moving the challenger's policy 2 units yields payoff $-(4^2) - 9 = -25$, and moving the policy only 1 unit yields payoff $-(5^2) = -25$), and otherwise will move the challenger's policy 0 or 1 units away from his bliss point. Given this observation, it can be verified that electing any incumbent $\theta_1 \in \{-1, 0, 1\}$ is optimal for the median voter, as electing incumbent $\theta_1 = \pm 1$ yields payoff

$$-(1^2) - \frac{1}{5}(4^2) - \frac{2}{5}(1^2) - \frac{1}{5}(0^2) - \frac{1}{5}(3^2) = -\frac{32}{5},$$

and electing incumbent $\theta_1 = 0$ yields payoff

$$-(0^2) - \frac{2}{5}(4^2) - \frac{3}{5}(0) = -\frac{32}{5},$$

while electing any other incumbent is strictly worse (as only incumbents with $|\theta_1| \geq 1$ shift the policy of either challenger with bliss point ± 5 by 2 units, and the incumbents with bliss points ± 1 are clearly optimal among those incumbents with $|\theta_1| \geq 1$).

Next, suppose that the population mass with $\theta = 5$ shifts to $\theta = 0$. This is a first-order stochastic shift to the left in F . But now the unique optimal incumbent is $\theta_1 = 0$, as electing incumbent $\theta_1 = 0$ yields payoff

$$-(0^2) - \frac{1}{5}(4^2) - \frac{4}{5}(0) = -\frac{16}{5},$$

while electing incumbent $\theta_1 = 1$, for example, yields payoff

$$-(1^2) - \frac{1}{5}(3^2) - \frac{3}{5}(1^2) - \frac{1}{5}(0^2) = -\frac{17}{5}.$$

Thus, $\theta_1 = 1$ was an optimal incumbent before the shift in F , but after the shift the unique optimal incumbent is $\theta_1 = 0$. This shows that shifting the distribution of challengers to the left does not always shift the set of optimal incumbents to the right.

5.3 Democratic and Institutional Consolidation

The intuition for why democratic and institutional consolidation are complements is simply that elections and institutional constraints are alternative ways of preventing extreme policies from being enacted. More precisely, it may be verified that p and α are complements if the median voter's expected period 2 payoff when her optimal incumbent is in power is decreasing in α . This is always the case if the optimal incumbent is a centrist (see below). It is thus intuitive that it should also be true when the optimal incumbent is an extremist, as the only reason the median voter ever elects an extremist is that an extremist imposes institutional constraints in a way that is more favorable for her, so one might expect that making imposing these constraints easier for such an incumbent would benefit the median voter. However, this intuition does not seem to easily turn into a proof (outside of the linear-quadratic case), and therefore I present the formal result only for the special case where the optimal incumbent is a centrist.

When the optimal incumbent is a centrist, democratic and institutional consolidation can be seen to be complementary because the incumbent always uses institutional constraints to shift period 2 policy toward the center (regardless of the identity of the challenger). Therefore, making it easier to acquire institutional capacity always improves period 2 policy from the median voter's perspective in the event that she is not decisive in period 2, and thus makes being decisive in period 2 less valuable for the median voter.

Proposition 8 *Suppose that the optimal incumbent is a centrist ($\theta_1 = 0$) for an interval of parameters p and α . Then democratic and institutional consolidation are complements from the perspective of the median voter over that interval.*

Proof. Let $y^*(\theta_1, \theta_2; \alpha)$ be as in the proof of Proposition 6, where I have made the dependence on α explicit, and note that the function y^* is differentiable in α by the Implicit Function Theorem. When the optimal incumbent is $\theta_1 = 0$, the cross-partial of the median voter’s problem (6) with respect to p and α equals

$$\int_{-\infty}^0 l'(\theta_2 + y^*(0, \theta_2; \alpha)) \left(\frac{\partial y^*(\theta_1, \theta_2; \alpha)}{\partial \alpha} \Big|_{\theta_1=0} \right) dF(\theta_2) \\ - \int_0^{\infty} l'(\theta_2 - y^*(0, \theta_2; \alpha)) \left(\frac{\partial y^*(\theta_1, \theta_2; \alpha)}{\partial \alpha} \Big|_{\theta_1=0} \right) dF(\theta_2).$$

By (3) and (4), $\theta_2 + y^*(0, \theta_2; \alpha) \leq 0$ if $\theta_2 \leq 0$, $\theta_2 - y^*(0, \theta_2; \alpha) \geq 0$ if $\theta_2 \geq 0$, and $\frac{\partial y^*(\theta_1, \theta_2; \alpha)}{\partial \alpha} \Big|_{\theta_1=0} \leq 0$ for all θ_2 . Since l is increasing and convex, $l'(x) \leq 0$ if $x \leq 0$ and $l'(x) \geq 0$ if $x \geq 0$. Combining these observations implies that the cross-partial is non-negative. Hence, p and α are complements. ■

6 Extensions

This section studies three extensions of the main model. Section 5.1 introduces fixed costs of developing institutional influence. Section 5.2 examines the possibility that the median voter may only be able to choose between the incumbent and a random challenger in period 2. Finally, Section 5.3 allows the incumbent to influence the extent of democratic consolidation as well as institutional constraints. A common theme among the extensions is that all of them lead to new reasons why it may be optimal for the median voter to elect an extreme incumbent.

6.1 Fixed Cost of Influencing Institutions

This subsection considers an alternative model of the cost of imposing institutional constraints which provides a reason why the median voter may elect either a centrist or a very extreme incumbent (rather than a “moderately extreme” incumbent). In particular, I assume that the incumbent must decide whether or not to pay a fixed cost $\chi > 0$ to acquire the the ability to invest in institutional capacity *before the challenger is determined* (in addition to paying $\alpha c(k)$ to develop institutional capacity k after the challenger is determined).

This fixed cost reflects the fact that an incumbent may not be able to wait until he learns the identity of his successor before making any investments in institutional capacity. For example, he may have to spend χ to establish a new branch of the government or armed forces, which takes a long time and thus must be done before he is sure what challenger will emerge, but if he establishes the branch then he can quickly ramp up its effectiveness after the challenger emerges. The key implication of having a fixed cost of institutional capacity is that only sufficient extreme incumbents will pay it (at least in the linear-quadratic case). Hence, while in the baseline model without fixed costs the median voter may find it optimal to elect a “moderately extreme” incumbent, in the model with fixed costs the median voter will always either elect a centrist (if he gives up on electing an incumbent who will influence institutional constraints) or a more extreme incumbent.

The following result shows that, in the linear-quadratic case, when the fixed cost of institutional capacity increases, the optimal incumbent either becomes more extreme or becomes a centrist ($\theta_1 = 0$). Since the baseline model is equivalent to $\chi = 0$, this implies that introducing fixed costs of institutional capacity causes the median voter to elect either a more extreme incumbent or a centrist. The key step in the proof is showing that more extreme incumbents benefit more from being able to influence institutional constraints in the linear-quadratic case. This implies that only sufficiently extreme incumbents pay the fixed cost, which leads to the result.

Proposition 9 *In the linear-quadratic case, if θ_1 is an optimal incumbent when the fixed cost of institutional capacity is χ and θ'_1 is an optimal incumbent when the fixed cost of institutional capacity is $\chi' > \chi$, then either $|\theta'_1| \geq |\theta_1|$ or $\theta'_1 = 0$. That is, increasing the fixed cost of institutional capacity causes the optimal incumbent either to become more extreme or to become a centrist.*

Proof. It is easy to see that the sign of any optimal incumbent is always opposite to the sign of $E[\theta]$, regardless of χ . So suppose that $E[\theta] \leq 0$ and that $\theta_1, \theta'_1 \geq 0$ (the $E[\theta] \geq 0$ case is symmetric).

If an incumbent θ_1 does not pay the fixed cost, his expected payoff is $-\int (\theta_1 - \theta_2)^2 dF(\theta_2)$. If he pays the fixed cost, then when challenger θ_2 is drawn he spends $\alpha \left(\frac{\theta_1 - \theta_2}{1 + \alpha}\right)^2$ on institu-

tional capacity and the resulting policy is $\frac{\theta_1 + \alpha\theta_2}{1 + \alpha}$. Hence, his expected payoff is

$$\begin{aligned} & - \int \left[\alpha \left(\frac{\theta_1 - \theta_2}{1 + \alpha} \right)^2 + \left(\theta_1 - \frac{\theta_1 + \alpha\theta_2}{1 + \alpha} \right)^2 \right] dF(\theta_2) - \chi \\ & = - \left(\frac{\alpha}{1 + \alpha} \right) \int (\theta_1 - \theta_2)^2 dF(\theta_2) - \chi. \end{aligned}$$

Therefore, an incumbent pays the fixed cost if and only if $\chi \leq \frac{1}{1 + \alpha} \int (\theta_1 - \theta_2)^2 dF(\theta_2)$. Now $\frac{1}{1 + \alpha} \int (\theta_1 - \theta_2)^2 dF(\theta_2)$ is increasing in θ_1 whenever $\theta_1 \geq 0$ (as $E[\theta] \leq 0$). Hence, there exists a cutoff $\bar{\theta}_1 > 0$ such that the incumbent pays the fixed cost if and only if $\theta_1 \geq \bar{\theta}_1$, and $\bar{\theta}_1$ is increasing in χ .

Now suppose, towards a contradiction, that $\chi' > \chi$ but $\theta_1 > \theta'_1 > 0$, where θ_1 and θ'_1 are as in the statement of the proposition. Then it must be that incumbent θ'_1 invests in institutional capacity when the fixed cost is χ' , as otherwise the median voter would rather elect a centrist when the fixed cost is χ' . Hence, incumbent θ_1 also invests in institutional capacity when the fixed cost is χ' , because $\theta_1 > \theta'_1$ (and since θ'_1 invests in institutional capacity it must be that $\theta'_1 \geq \bar{\theta}_1$). Therefore, incumbents θ_1 and θ'_1 must also invest in institutional capacity when the fixed cost is only χ . It follows that the median voter's payoff from electing incumbent θ_1 or θ'_1 is the same whether the fixed cost is χ' or χ , and that in particular both θ_1 and θ'_1 are optimal incumbents when the fixed cost is either χ or χ' . But the median voter's payoff is strictly concave in θ_1 over the range of θ_1 that pay the fixed cost (and $\theta_1 = 0$ is the unique optimal incumbent among those who do not pay the fixed cost, if any incumbents do not pay the fixed cost), so this is impossible. ■

6.2 Limited Choice of Challenger

The observation that a decisive median voter can always choose a challenger so as to “undo” whatever institutional constraints the incumbent will subsequently impose—which plays an important role in the analysis—seems somewhat unrealistic, as voters typically do not have the option of choosing a challenger from any point on the political spectrum. To partially explore the implications of relaxing this assumption, this subsection considers the alternative model where the challenger is instead drawn from F in the event that the median voter is decisive and the median voter then chooses between electing the challenger and reelecting

the incumbent himself.

Formally, suppose the challenger is drawn from F when the median voter is decisive, the median voter then either elects the challenger or reelects the incumbent, and if she elects the challenger then he faces institutional constraints that the incumbent imposes before leaving office. There are two interesting observations to be made about this model. First, the fact that the incumbent has the ability to influence institutional constraints poses a commitment problem for him that may lead him to lose the period 2 election in situations where he would have won had he not had this ability, making him worse off.¹⁰ Consider, for example, the case where $\alpha c(k) = k$, $l(x) = x^2$, $\theta_1 = 1$, and the challenger turns out to be $\theta_2 = -2$. If it were impossible to influence institutional constraints, the median voter would reelect the incumbent, resulting in policy $x_2 = 1$ and a period 2 payoff of 0 for the incumbent. However, the possibility of influencing institutional constraints causes the median voter to instead elect the challenger, anticipating that the incumbent will use institutional constraints to shift the resulting policy to $x_2 = \frac{\theta_1 + \theta_2}{2} = -\frac{1}{2}$ (so the incumbent is worse off both because the resulting policy is worse for him and because he spends resources investing in institutional influence). Thus, in this model having access to institutional influence can actually cause an incumbent to lose power in cases where he could have retained power were it impossible to influence institutional constraints.

Second, the median voter's optimal choice of incumbent may be different in the model with a limited choice of challenger than in the baseline model. One's first thought might be that limiting the median voter's choice of challenger should make her choose a more moderate incumbent, as she may not have the option of electing a moderate challenger in period 2. However, it turns out that whether limiting the median voter's choice of challenger makes her elect a more or less extreme incumbent is ambiguous. Intuitively, it may be that the median voter's optimal strategy involves reelecting the incumbent with high probability in the event that she is decisive, in which case she elects a more moderate incumbent than in the baseline model. It is easy to construct examples where this is the case; for example, it

¹⁰This idea is closely related to Proposition 1 of Ellman and Wantchekon (2000), which shows that a strong party with the ability to launch a coup can lose to a weak party without this ability, because voters anticipate that the weak party will moderate its policy to avoid the coup.

can be verified that if in the linear-quadratic case fraction $\frac{1}{3}$ of the population has bliss point -1 , fraction $\frac{2}{3}$ of the population has bliss point 0 , and $p = \frac{1}{2}$, then the optimal incumbent in the baseline model is $\theta_1 = \frac{1}{27}$ while the optimal incumbent in the model with a randomly drawn challenger is $\theta_1 = \frac{1}{33}$. But it is also possible that the median voter’s optimal strategy may involve electing an extreme challenger with high probability and relying on an extreme incumbent on the opposite side of the political spectrum to use institutional constraints to moderate the resulting period 2 policy. This possibility is illustrated by the following example.

Example 2 *Suppose that fraction $\frac{1}{1000}$ of the population has bliss point -3000 , fraction $\frac{499}{1000}$ has bliss point -1 , and fraction $\frac{1}{2}$ has bliss point 0 (technically, consider a perturbation of this example where $\frac{1}{2} + \varepsilon$ of the population has bliss point 0 , so that the median voter is actually 0). Consider the linear-quadratic case with $p = \frac{1}{2}$ and $\alpha = 1$. Recall that in the baseline model the optimal incumbent is $\theta_1 = -\frac{1-p}{5-p}E[\theta] \approx .389$. But in the model with a randomly drawn challenger, it can be verified that the optimal incumbent is $\theta_1 \approx .400$. The intuition is that the median voter will reelect the optimal incumbent only if the realized challenger is $\theta_2 = -3000$; in particular, the median voter would rather elect a challenger with $\theta_2 = -1$ than reelect an incumbent with $\theta_1 \approx .4$, as doing so results in policy $x_2 \approx \frac{-1+.4}{2} = -.3$ (which is better for the median voter than $x_2 \approx .4$). Since the event that $\theta_2 = -3000$ is very rare, the main consequence of moving from the baseline model to the model with a randomly drawn challenger is that institutional constraints are imposed in period 2 with probability close to 1 rather than with probability $\frac{1}{2}$, and this makes it optimal to elect a more extreme incumbent.*

6.3 Endogenizing Democratic Consolidation

Throughout the paper, I have studied the incumbent’s optimal choice of institutional constraints, taking the extent of democratic and institutional consolidation—the other kinds of “institutions” considered in the paper—as exogenous. As a final extension, I make the natural assumption that the extent of democratic consolidation (p) is also endogenous (one could easily also let α be endogenous, but this seems less interesting).

Formally, modify the baseline model by assuming that before the challenger is determined

the incumbent chooses p at cost $C(p)$, where C is convex and reaches its minimum of 0 at $p = p^*$ (so that $p < p^*$ corresponds to encouraging democratic consolidation, while $p > p^*$ corresponds to discouraging it). Also, restrict attention to the linear-quadratic case, for tractability. To build some intuition for this model, note that since the median voter always elects a challenger who implements $x_2 = 0$ when she is decisive (by Proposition 2), more extreme incumbents are worse-off when the median voter is decisive. Thus, one might naively think that extreme incumbents suppress democracy (i.e., reduce p) more than moderate incumbents, and that therefore the median voter chooses a more moderate incumbent in the model with endogenous p than in baseline model. However, note that at the optimum in the baseline model the median voter elects an incumbent that is more moderate than the expected challenger. Hence, when the median voter is decisive incumbent θ_1 faces challenger $-\frac{\theta_1}{\alpha}$, which may be closer to θ_1 than $E[\theta]$. So the optimal incumbent (from the perspective of the baseline model) prefers the median voter to be decisive than to face a random challenger. In addition, since loss functions are convex, this preference is stronger for a slightly more extreme incumbent. Hence, on the margin there is an effect pushing for the median voter to elect a more extreme incumbent than in the baseline model. Finally, there is also a countervailing marginal effect, which is that electing a more extreme incumbent makes the median voter elect a more extreme challenger (from the other side of the political spectrum) in period 2, and this makes the median voter's being decisive less appealing to more extreme incumbents. The following result shows that the overall effect is ambiguous, so that endogenizing democratic consolidation can lead the median voter to elect either a more moderate or a more extreme incumbent than in the baseline model.

Proposition 10 *In the linear-quadratic case, the optimal incumbent in the model with endogenous democratic consolidation is more moderate than the optimal incumbent in the baseline model with $p = p^*$ if and only if $\alpha(1 + \alpha) < 1 - p$.*

Proof. Recall that $k^*(\theta_1, \theta_2) = \left(\frac{\theta_1 - \theta_2}{1 + \alpha}\right)^2$ and $x_2^*(\theta_1, \theta_2) = \frac{\theta_1 + \alpha\theta_2}{1 + \alpha}$, so incumbent θ_1 's payoff from facing challenger θ_2 is

$$-\alpha \left(\frac{\theta_1 - \theta_2}{1 + \alpha}\right)^2 - \left(\theta_1 - \frac{\theta_1 + \alpha\theta_2}{1 + \alpha}\right)^2 = -\frac{\alpha}{1 + \alpha} (\theta_1 - \theta_2)^2.$$

When the median voter is decisive in period 2, she elects the challenger θ_2 such that $x_2^*(\theta_1, \theta_2) = 0$, which is easily seen to be $\theta_2 = -\frac{\theta_1}{\alpha}$. In this case, incumbent θ_1 's payoff is

$$-\frac{\alpha}{1+\alpha} \left(\theta_1 + \frac{\theta_1}{\alpha} \right)^2 = -\frac{1+\alpha}{\alpha} \theta_1^2.$$

Hence, the incumbent's problem of choosing p is

$$\max_p -p \frac{1+\alpha}{\alpha} \theta_1^2 - (1-p) \frac{\alpha}{1+\alpha} \int (\theta_1 - \theta_2)^2 dF(\theta_2) - C(p).$$

This yields first-order condition

$$C'(p) = -\frac{1+\alpha}{\alpha} \theta_1^2 + \frac{\alpha}{1+\alpha} \int (\theta_1 - \theta_2)^2 dF(\theta_2). \quad (10)$$

Differentiating the right-hand side of (10) with respect to θ_1 yields

$$-2 \frac{1+2\alpha}{\alpha(1+\alpha)} \theta_1 - 2 \frac{\alpha}{1+\alpha} E[\theta].$$

Letting $p^*(\theta_1)$ denote incumbent θ_1 's optimal choice of p , we see that $p^*(\theta_1) > 0$ if and only if

$$\theta_1 < -\frac{\alpha^2}{1+2\alpha} E[\theta].$$

Now recall that the optimal choice of θ_1 in baseline model is $\theta_1^* \equiv -\frac{\alpha(1-p)}{(1+\alpha)^2+1-p} E[\theta]$.

Assume that $E[\theta] \geq 0$ (the $E[\theta] \leq 0$ case is symmetric). Then $p^*(\theta_1^*) > 0$ if and only if

$$\frac{\alpha(1-p)}{(1+\alpha)^2+1-p} > \frac{\alpha^2}{1+2\alpha},$$

or equivalently

$$\alpha(1+\alpha) < 1-p. \quad (11)$$

If (11) holds, then choosing $\theta_1 < \theta_1^*$ cannot be optimal in the model with endogenous p , as $p^*(\theta_1) > 0$ so slightly increasing θ_1 both increases $p^*(\theta_1)$ (which clearly benefits the median voter) and also increases the median voter's payoff holding p^* fixed (from the median voter's first-order condition in the baseline model). Similarly, if (11) fails then choosing $\theta_1 > \theta_1^*$ cannot be optimal in the model with endogenous p , as $p^*(\theta_1) < 0$ so slightly decreasing θ_1 increases the median voter's payoff. Hence, it is optimal for the median voter to choose more a moderate (i.e., greater) θ_1 if and only if (11) holds. ■

7 Conclusion

This paper has examined a model of endogenous institutional constraints and their implications for electoral behavior. The main argument is that a political leader will set up institutions that constrain his successor more severely when his preferences are more different from his successor's, and that therefore the median voter benefits from choosing a leader whose preferences are opposed to those of most potential challengers. Hence, the median voter tends to elect a more right-wing politician when the distribution of potential challengers shifts to the left. In addition, the median voter elects a more extreme politician when democracy is less fully consolidated, when the costs of developing institutional capacity are neither too large nor too small, and when the distribution of potential challengers is more asymmetric. The model also implies that democratic and institutional consolidation are complements from the perspective of the median voter. This suggests that citizens may face weak incentives to consolidate democracy when they live in societies where institutions are easily influenced, as in such societies they can rely on influential leaders to constrain future challengers from implementing extreme policies. Of course, this strategy can backfire if an extremist nonetheless manages to come to power.

A final comment on the interpretation of the model is that, while I have assumed throughout that the median voter is decisive in choosing the period 1 incumbent, all of the analysis in the paper can be replicated for any other period 1 decision-maker. An interesting consequence of the period 1 decision-maker's being, say, to the right of the median voter is that from her perspective the distribution of future challengers is skewed to the left, which can make it optimal for her to choose an incumbent even further to the right. Perhaps an example of this kind of effect is the United States Central Intelligence Agency's support of Pinochet in the 1970s: it seems plausible that one of the CIA's motivations in supporting Pinochet was the possibility that he would create institutions that would constraint future Chilean leaders from implementing left-wing policies, even if Pinochet himself did not remain in power.¹¹

¹¹As this example indicates, one contribution of this paper is characterizing which local politician an outside power would like to delegate the choice of institutional constraints to. From this perspective, this paper is related to Padro i Miquel and Yared (2012), who study how an outside power can incentivize a

The model presented in this paper seems amendable to several directions for future study. Technically, in this model the median voter’s problem of choosing which politician to elect may not be concave or supermodular, and it remains to be seen whether further comparative statics results can be established in this or related models. Another direction is introducing a more structured model of electoral competition: in the current model, the median voter freely chooses an incumbent who then sets up institutional constraints optimally, but this might be nicely complemented by an analysis of party competition or of different models of commitment to policy and institutions. Finally, it might also be worth considering more fully dynamic models, in which electing an extremist today can have consequences for elections and policy several periods in the future.

References

- [1] Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin (2012a), “A Political Theory of Populism,” *Quarterly Journal of Economics*, forthcoming.
- [2] Acemoglu, Daron, Georgy Egorov, and Konstantin Sonin (2012b), “Dynamics and Stability of Constitutions, Coalitions, and Clubs,” *American Economic Review*, 102, 1446–1476.
- [3] Acemoglu, Daron and James A. Robinson (2005), *Economic Origins of Dictatorship and Democracy*, Cambridge University Press: Cambridge, UK.
- [4] Acemoglu, Daron and James A. Robinson (2000), “Why Did the West Extend the Franchise? Democracy, Inequality and Growth in Historical Perspective,” *Quarterly Journal of Economics*, 115, 1167-1199.
- [5] Aghion, Philippe, Alberto Alesina, and Francesco Trebbi (2004), “Endogenous Political Institutions,” *Quarterly Journal of Economics*, 119, 565-611.
- [6] Alesina, Alberto and Howard Rosenthal (2000), “Polarized Platforms and Moderate Policies with Checks and Balances,” *Journal of Public Economics*, 75, 1-20.

given local politician to prevent political disturbances.

- [7] Barbera, Salvador and Matthew O. Jackson (2004), “Choosing How to Choose: Self-Stable Majority Rules and Constitutions,” *Quarterly Journal of Economics*, 119, 1011-1048
- [8] Collier, Paul and Pedro C. Vicente (2012), “Violence, Bribery, and Fraud: The Political Economy of Elections in Sub-Saharan Africa,” *Public Choice*, 153, 117-147.
- [9] Ellman, Matthew and Leonard Wantchekon (2000), “Electoral Competition under the Threat of Political Unrest,” *Quarterly Journal of Economics*, 115, 499-531.
- [10] Faulí-Oller, Ramon, Efe Ok, and Ignacio Ortuno-Ortín (2003), “Delegation and Polarization of Platforms in Political Competition,” *Economic Theory*, 22, 289-309.
- [11] Ghosh, Parikshit (2002), “Electoral Competition, Moderating Institutions and Political Extremism,” *mimeo*.
- [12] Greif, Avner and David D. Laitin (2004), “A Theory of Endogenous Institutional Change,” *American Political Science Review*, 98, 633-652.
- [13] Lagunoff, Roger (2001), “A Theory of Constitutional Standards and Civil Liberties,” *Review of Economic Studies*, 68, 109-132.
- [14] Messner, Matthias and Matthias K. Polborn (2004), “Voting on Majority Rules,” *Review of Economic Studies*, 71, 115-132.
- [15] Ortuno-Ortín, Ignacio (1997), “A Spatial Model of Political Competition and Proportional Representation,” *Social Choice and Welfare*, 14, 427-438.
- [16] Pardo i Miquel, Gerard (2007), “The Control of Politicians in Divided Societies: The Politics of Fear,” *Review of Economic Studies*, 74, 1259-1274.
- [17] Pardo i Miquel, Gerard and Pierre Yared (2012), “The Political Economy of Indirect Control,” *Quarterly Journal of Economics*, 127, 947-1015.
- [18] Robinson, James A. and Ragnar Torvik (2009), “The Real Swing Voter’s Curse,” *American Economic Review: Papers & Proceedings*, 99, 310-315.

- [19] Rogoff, Kenneth (1985), “The Optimal Degree of Commitment to an Intermediate Monetary Target,” *Quarterly Journal of Economics*, 100, 1169-1189.
- [20] Svobik, Milan W. (2012), “Learning to Love Democracy: Electoral Accountability and the Success of Democracy,” *American Journal of Political Science*, forthcoming.
- [21] Yavuz, H. Hakan (2009), *Secularism and Muslim Democracy in Turkey*, Cambridge University Press: Cambridge, UK.