Idiosyncratic Sentiments and Coordination Failures

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Motivation

- how rational investors can have **differing degrees of optimism** regarding the prospects of economy
- even if they share the same information regarding all economic fundamentals

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- even if they share the same information regarding all economic fundamentals
- key insight: **idiosyncratic extrinsic uncertainty**

This Paper

- model 1: simple real investment game
- model 2: a financial market

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- independence $\Rightarrow$ unique equilibrium, identical choices
- complementarity $\Rightarrow$ **endogenous heterogeneity**, despite strong incentive to coordinate

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- modeling instrument: **private sunspots**
- payoff-irrelevant (like public sunspots), but imperfect (Aumann)
- examples: “how bright is the sun?”, “what did the leader say?”

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- examples: "how bright is the sun?", "what did the leader say?"
- devices that permit the construction of equilibria with **self-fulfilling heterogeneity in beliefs**

Novel Positive and Normative Properties

- capture strategic uncertainty
- rationalize idiosyncratic investor sentiment
- source of heterogeneity in investment/portfolio choices

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- higher welfare
- render apparent coordination failures evidence of efficiency

Model 1: Real Investment Game

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- continuum of investors, each choosing $k = 0$ or $k = 1$
- return to investment increasing in K :

$$A(K) \equiv \begin{cases} 1 & \text{if } K \geq \hat{\kappa} \\ 0 & \text{if } K < \hat{\kappa} \end{cases}$$

for some $\hat{\kappa} \in (0, 1)$

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- best response:

$$k_i = BR(K) \equiv \begin{cases} 1 & \text{if } K \geq \hat{\kappa} \\ 0 & \text{if } K < \hat{\kappa} \end{cases}$$

- no or only public sunspots$\Longrightarrow$ two equilbrium outcomes, $K = 0$ or $K = 1$

Private Sunspots

- nature draws an unobserved payoff-irrelevant random variable s, with support $\mathbb{S} \subseteq \mathbb{R}$ and c.d.f. $F : \mathbb{S} \to [0, 1]$
- each investor observes a private signal m regarding s
- conditional on s, m is i.i.d. across investors, with support $\mathbb{M} \subseteq \mathbb{R}$ and c.d.f. $\Psi : \mathbb{M} \times \mathbb{S} \to [0, 1]$
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Definition

An equilibrium with private sunspots consists of a sunspot structure $(\mathbb{S}, F, \mathbb{M}, \Psi)$ and a strategy $k : \mathbb{M} \to \{0, 1\}$ such that

$$k(m) \in \arg \max_{k \in \{0,1\}} \int_{\mathbb{S}} U(k, K(s)) dP(s|m) \;\; \forall m \in \mathbb{M},$$

with $K(s) = \int_{\mathbb{M}} k(m) d\Psi(m|s) \; \forall s \in \mathbb{S}$, and with $P(s|m)$ being the c.d.f. of the posterior about s conditional on m (as implied by Bayes' rule.

Gaussian Private Sunspots

- $s \sim N(\mu_s, \sigma_s^2)$
- $m_i = s + \varepsilon_i$, $\varepsilon_i \sim N(0, \sigma_\varepsilon^2)$

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Proposition

For any $(\mu_s, \sigma_s, \sigma_\varepsilon)$, there exists an equilibrium in which the following are true:

- *An investor invests when $m > m^*$ and not when $m < m^*$, for some $m^* \in \mathbb{R}$.*
- *The aggregate level of investment is stochastic, with full support on $(0, 1)$.*
- *The cross-sectional distribution of expectations regarding the aggregate level of investment, $\mathbb{E}[K|m]$, has full support on $(0, 1)$.*

Gaussian Private Sunspots

Proof. Given the proposed strategy,

$$K(s) = \Pr(m \geq m^* | s) = \Phi\left(\frac{s - m^*}{\sigma_\varepsilon}\right)$$

$$K(s) \geq \hat{\kappa} \text{ iff } s \geq s^*, \text{ where } s^* = m^* + \sigma_\varepsilon \Phi^{-1}(\hat{\kappa})$$

Since the posterior about s conditional on m is Normal,

$$\mathbb{E}\left[A(K(s))|m\right] = \Pr(s \geq s^* | m) - c = \Phi(...) - c$$

Proposed strategy is an equilibrium iff m^* satisfies $\mathbb{E}\left[A|m^*\right] = 0$. Equivalently,

$$m^* = \mu_s - \sigma_s \left\{ \frac{\sigma_s^2 + \sigma_\varepsilon^2}{\sigma_s \sigma_\varepsilon} \Phi^{-1}(\hat{\kappa}) + \sqrt{1 + \left(\frac{\sigma_s}{\sigma_\varepsilon}\right)^2} \Phi^{-1}(c) \right\}$$

QED

Extension: Dynamics and Learning

- $s_t = \rho s_{t-1} + v_t$
- $m_{it} = s_t + \varepsilon_{it}$
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- stationary equil where an agent invests at t iff $\hat{m}_t > \hat{m}^*$

$$K_t(s_t) = \Phi\left(\frac{s_t - \hat{m}^*}{\hat{\sigma}}\right)$$

- up to a monotone transformation, K_t follows a smooth $AR(1)$ process

Extension: Dynamics and Learning

- s constant over time, but learning through new signals
- non-stationary equil where an agent invests at t iff $\hat{m}_t > \hat{m}_t{}^*$

$$K_t(s) = \Phi\left(\frac{s - \hat{m}_t^*}{\hat{\sigma}_t}\right)$$

- more and more coordination over time:

$$\lim_{t \to \infty} K_t(s) \in \{0, 1\}$$

Model 2: Financial Market

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- re-interpret $k$as investment in an asset traded in a financial market
- dividend of the asset $A(K)$
- price of the asset p
- payoff of an investor

$$\pi = \Pi(k, K, p) \equiv [A(K) - p]\, k$$

- exogenous supply of the asset:

$$Q = Q(p, u)$$

where u is an unobserved supply shock

Private Susnpots: Correlated Eq meets REE

- sunspot structure $(\mathbb{S}, F, \mathbb{M}, \Psi)$ as before
- but now equilibrium price partially reveals s
- Aumann meets Grossman-Stiglitz!

Private Susnpots: Correlated Eq meets REE

Definition

A REE with private sunspots consists of a sunspot structure $(\mathbb{S}, F, \mathbb{M}, \Psi)$, a price function $P : \mathbb{S} \times \mathbb{R} \to \mathbb{R}$, an individual demand function $k : \mathbb{M} \times \mathbb{R} \to \left[\underline{k}, \bar{k}\right]$, and a belief $\mu : \mathbb{S} \times \mathbb{R} \times \mathbb{M} \times \mathbb{R} \to [0, 1]$, such that:

- μ consistent with Bayes rule, given P
- given μ and P, the demand function satisfies individual rationality:

$$k(m, p) \in \arg \max_{k \in \{0,1\}} \int_{\mathbb{S} \times \mathbb{U}} \Pi(k, K(s, P(s, u)), P(s, u)) d\mu(s, u|m, p)$$

  where $K(s, p) \equiv \int_{\mathbb{M}} k(m, p) d\Psi(m|s) \; \forall s \in \mathbb{S}$.
- given the demand function, the price function satisfies market-clearing:

$$K(s, P(s, u)) = Q(s, u) \quad \forall (s, u).$$

Gaussian example

- Normality: $u \sim N\left(0, \sigma_u^2\right)$, $s \sim N\left(\mu_s, \sigma_s^2\right)$, $m_i = s + \varepsilon_i$, $\varepsilon_i \sim N\left(0, \sigma_\varepsilon^2\right)$
- Functional forms:

$$A(K) = \begin{cases} 1 & \text{if } K \geq 1/2 \\ 0 & \text{otherwise} \end{cases} \quad \text{and} \quad Q(p, u) = \Phi\left(u + \lambda \Phi^{-1}(p)\right)$$

Gaussian example

Proposition

For any (σ_u, λ), there exists a REE with private sunspots in which:

- *The equilibrium price is $p = P(s, u)$, where P is a continuously increasing function of s and a continuously decreasing function of u.*
- *An investor's equilibrium demand is*

$$k(m, p) = \begin{cases} 1 & \text{if } m \geq m^*(p) \\ 0 & \text{otherwise} \end{cases}$$

  where $m^*(p)$ is a continuous decreasing function of p.
- *The aggregate demand for the asset, $K(s, p)$, is continuously increasing in s and continuously decreasing in p.*

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- **key point to take:** too high K in best sunspot-less equilibrium

Private Sunspots and Efficiency

- variant of model 1:

$$A(K) = \begin{cases} 1 - c - hK & \text{if } K \geq \hat{\kappa} \\ -c - hK & \text{if } K < \hat{\kappa} \end{cases}$$

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Proposition

Suppose $0 < 1 - c - h < h$.

- *There exist only two sunspot-less equilibria:* $K = 1$ *and* $K = 0$.
- *The equilibrium in which* $K = 1$ *achieves higher welfare than the equilibrium in which* $K = 0$, *as well as than any equilibrium with public sunspots.*
- *The first-best level of aggregate investment is* $K^* \in [\hat{\kappa}, 1)$.

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- low investment ($K = 0$) evidence of coordination failure, symptom of inefficiency
- **neither of the above true once we allow for private sunspots**

Best Equilibrium with Private Sunspots

Proposition

Suppose $0 < 1 - c - h < h(1-h)$, *allow for private sunspots, and consider the set of equilibria that maximize welfare. There exists a unique pair* (q^*, p^*), *with* $K^* < q^* < 1$ *and* $0 < p^* < 1$, *such that all these equilibria are characterized by the following properties:*

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- $K(s) = q^*$ *with probability* p^* *and* $K(s) = 0$ *with probability* $1 - p^*$; *that is, the economy fluctuates between "normal times", events during which aggregate investment is positive, and "crashes", events during which investment collapses to zero.*

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- q^* *and* p^* *decrease with* c *or* h; *that is, the probability of a crash increases, and the level of investment in normal times decreases, as fundamentals get worse.*

Best Equilibrium with Private Sunspots

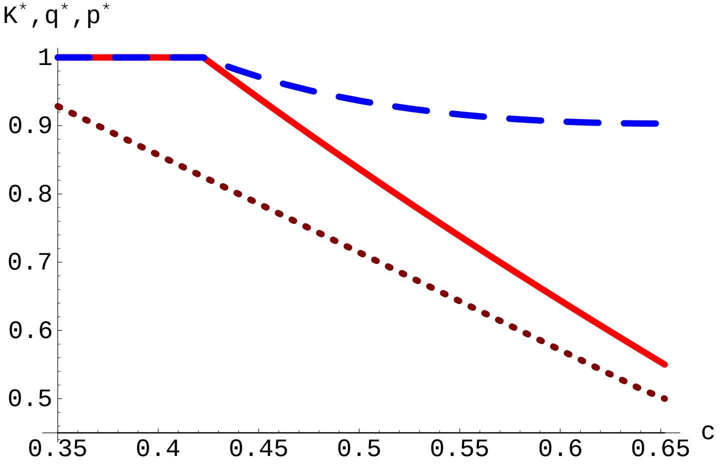

Figure: Comparative statics of best private-sunspot equilibrium.

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- apparent coordination failures become evidence of efficiency
- policies that fight such coordination failures may reduce efficiency