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MARGINAL q

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ABSTRACT

The standard version of q theory, in which investment is positively related to marginal q, breaks down in the presence of fixed costs of adjustment. With fixed costs, investment is a non-monotonic function of q. Therefore its inverse, which is the traditional investment function, does not exist. Depending upon auxiliary assumptions, the correlation between investment and marginal q can be either positive or negative. Given certain homogeneity assumptions, a version of the theory based on *average* q still holds, although under the same assumptions profits and sales perform as well as average q. More generally, q is no longer a sufficient statistic.

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Fixed Costs: The Demise of Marginal q

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Abstract

The standard version of q theory, in which investment is positively related to marginal q, breaks down in the presence of fixed costs of adjustment. With fixed costs, investment is a non-monotonic function of q. Therefore its inverse, which is the traditional investment function, does not exist. Depending upon auxiliary assumptions, the correlation between investment and marginal q can be either positive or negative. Given certain homogeneity assumptions, a version of the theory based on *average* q still holds, although under the same assumptions profits and sales perform as well as average q. More generally, q is no longer a sufficient statistic.

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"In writing this note, I feel at one and the same time as if I were preaching in the wilderness and belaboring the obvious. For the major conclusions of this paper are important and widely neglected, yet they seem distressingly obvious."
Milton Friedman (1953)

1 Introduction

In the sixties, Tobin posited a relationship between a firm's investment and the ratio of its stock market value to the replacement cost of its capital, a ratio that has subsequently become known as Tobin's q or average q. It was a simple arbitrage argument. If the market

valuation of capital held by a firm exceeds the cost of capital on the open market, then the firm can increase its value by investing.

Subsequent formalizations of Tobin's insight have downplayed the importance of average q, and instead focused on marginal q, defined as the value of a marginal unit of capital installed in the firm relative to its price outside the firm. In the presence of convex adjustment costs, the theory goes, investment decisions are determined on the margin. The firm weighs the marginal benefit of investment, measured by q, against the marginal cost of investment. Since marginal cost increases with the size of investment, this consideration naturally leads to a positive relationship between investment and q. The higher is q, the greater is the firm's willingness to incur the cost of adjustment, and the higher is investment.

The message of this paper is that the elegance of modern marginal-q theory comes at the cost of a serious lack of robustness. While the use of Tobin's average q as *one* of the main explanatory variables of investment remains valid under a wide variety of scenarios, the validity of marginal q, as well as on the sufficiency of such a statistic, rests on very stringent assumptions concerning markets and adjustment costs.

The attempt to explain movements in investment by movements in marginal q suffers from a simultaneity problem: in general, marginal q depends not only on the expected future evolution of exogenous variables, but also on the firm's current and future investment decisions. Other things equal, a unit of installed capital is worth more if the firm does not plan to make large investments in the near future. It turns out that whether or not investment and q are positively related depends on whether the effect of investment on q is weak or strong. The standard positive relationship between investment and marginal q arises from one of two sets of assumptions. First, in partial equilibrium, if there are constant returns to scale and perfect competition, then investment has no effect on the marginal value of capital. In this case marginal q is independent of investment, and we are left only with the effect of q on investment. Second, in a world in which adjustment costs are convex, firms have an incentive to smooth investment over time; an expected need for capital tomorrow increases the desire to invest today. The negative effect of future investment on q is therefore weaker than the positive effect of q on current investment.[1]

In more realistic scenarios, where the firm's adjustment technology includes fixed costs

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[1] Empirically, researchers reduce the impact of this negative effect by using lagged (beginning of period) as opposed to current q on the right hand side of investment regressions.

and its profit function is concave with respect to capital, the standard formulation of q-theory breaks down.[2] With fixed costs, investment occurs infrequently and in a lumpy fashion.[3] In such cases, events that make investment more likely, such as positive demand or productivity shocks, may lower rather than increase the value of an installed unit of capital; very much as the threat of an avalanche of entry into a market may reduce the value of an incumbent firm. Positive shocks raise the current marginal profitability of capital, but by increasing the probability of large investments in the near future, may reduce the expected marginal profitability of capital. Since marginal q is the present value of the marginal profitability of capital, whether it rises or declines on the face of a positive shock is ambiguous. Below we argue that marginal q is no longer a monotonic function of investment, and therefore its inverse —investment as a function of q— *no longer exists*.

While the theoretical literature on investment, armed with the convex-adjustment-cost model, has emphasized marginal q, the empirical literature has generally employed average q. The primary reason for this divergence between theory and practice is convenience: average q is easily derived from market data, whereas marginal q is more difficult to measure.[4] It turns out that with fixed costs of adjustment, this may not be such a bad practice. Average q may, in fact, be a better explanatory variable in an investment regression than marginal q. The reason for this is quite simple and in line with Tobin's original intuition: events that make firms valuable (thus raise average q) also tend to make them good places to invest. We show that under certain homogeneity assumptions, average q is a sufficient statistic for investment in spite of the presence of concave profits and fixed costs.

The superiority of average q, however, should not be exaggerated. While there are

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[2]Concavity results when the firm faces either a less than fully elastic demand for its goods or supply of its factors and inputs, or experiences decreasing returns to its flexible and quasi-flexible factors. Real firms are likely to suffer from each of these features to some degree. Doms and Dunne (1994), Cooper, Haltiwanger and Power (1995), and Caballero, Engel, and Haltiwanger (1995) have documented the importance of lumpy-investment for U.S. manufacturing plants.

[3]The fixed costs we consider here are —as is standard in the literature— *stock* fixed costs. This contrasts with Abel and Eberly's (1994) *flow* fixed costs. Intuitively, stock fixed costs are the cost of turning on a tap independent of how much water flows through it or how long the water flows, whereas flow fixed costs are the costs of running the tap per unit of time water flows and is independent of how much water flows. Flow fixed costs do not behave very differently from convex adjustment costs, investments are still infinitesimal, thus q-theory still works. Abel and Eberly (1995), on the other hand, work in discrete time, where the distinction between flow and stock costs is less meaningful; however, by assuming perfect competition and constant returns to scale, they eliminate (in partial equilibrium, or with respect to idiosyncratic shocks) the feedback from investment onto q, which is our concern here. The reader should be careful, therefore, when comparing the apparently contradictory conclusions of these papers with ours.

[4]See Abel and Blanchard (1986) for an important —although not successful— exception to this practice, and Hayashi (1982) for conditions under which average and marginal q coincide.

conditions under which average q is a sufficient statistic for investment, these conditions are quite strong. Moreover, under the same circumstances other variables, such as cash flows and sales, are also sufficient statistics for investment.

In section 2 we present a model of investment with fixed costs and discuss our main results concerning marginal q, average q, and fixed costs. Section 3 concludes and is followed by an appendix.

2   The Model

Our purpose in this section is to provide a model of a firm's investment decision in the presence of fixed costs of adjustment, and to discuss the implications of these costs for q-theory. Except for isolated comments, we focus on fixed costs and abstract from other types of frictions that are popular in the investment literature such as differences in the purchase and sale price of capital, irreversibilities, and adjustment costs that are convex in investment. We assume the firm away from the unlikely combination of perfect competition in goods and factor markets, as well as constant returns to scale. The marginal revenue product of capital is therefore decreasing in the capital stock. We also make a number of homogeneity assumptions that reduce the model to a single state variable and, therefore, simplify the analysis. None of the conclusions regarding marginal q depend on these homogeneity assumptions, but they play an important role in our discussion of average q later on.

We now present the model in detail. We consider a firm that uses capital, K_t, to produce output. Time is discrete. The firm's per period revenue function is $\Pi(\theta_t, K_t)$. This revenue function incorporates the optimal choice of flexible factors, as well as the level of fixed factors. θ is an index of profitability capturing both demand and productivity conditions, as well as the cost of factors of production other than capital. We place standard restrictions on the revenue function to ensure that the firm's problem is well behaved: revenue is zero when the capital stock, and hence output, is zero; revenue is strictly increasing in profitability and strictly increasing and strictly concave in capital; and, for each θ, $\lim_{K \to \infty} \Pi_K(\theta, K) = 0$.

The firm is risk neutral and a price taker in the capital goods market. We fix the real interest rate so that the firm discounts future profits at a constant rate β. We assume the firm rents the capital it employs in production at a constant cost r. This latter assumption is formally equivalent to assuming that the purchase and sale price of capital are fixed, but

more tractable algebraically.[5] In addition to the rental cost, the firm incurs a fixed cost whenever it chooses to invest.[6]

We make two homogeneity assumptions that ensure the form of the firm's problem does not change with the firm's size. First, we assume that the fixed cost of adjustment is proportional to the firm's stock of capital, $c \cdot K_t$. This ensures that the firm cannot grow out of the fixed cost. Second, we assume that Π is homogeneous of degree one in θ and K.[7]

It remains to describe the dynamics of capital and the profitability index. The capital accumulation equation is the standard one (to save in notation, we assume no depreciation):

$$K_{t+1} = K_t + I_t,$$

where I_t denotes investment in period t that becomes productive in period $t + 1$. The profitability index follows a random walk in logs. Let,

$$\eta_t \equiv \frac{\theta_t}{\theta_{t-1}}.$$

The realization of the profitability shock is characterized by the density $\phi(\eta)$.

2.1 A detour: a two period model

Before continuing with the full model, it is helpful to pause and consider a simpler version that illustrates some of the intuition behind our main results. Consider a firm that exists for two periods. It begins the first period endowed with a capital stock K_0 and profitability index θ and can increase its capital stock in either period.[8] We slightly modify the timing assumptions by allowing investment to add to the capital stock immediately instead of with a one period delay. All other aspects of the model are as before, including the fixed cost of adjustment and the shock to profitability η that occurs between periods.

How well does marginal q reflect the firm's incentive to invest in this setting? Or, more to the point, since it is apparent that given K_0 the incentive to invest is increasing with respect to θ, is it the case that q increases with θ as well? The answer is no. To see this, we start by describing the firm's actions in the second period, given the stock of capital $(K_1 = K_0 + I_1)$ inherited from period 1. We then step back and show how marginal q in

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[5]See Harrison, Sellke, and Taylor (1983).

[6]Where investment corresponds to a change in the stock of capital under rental agreement.

[7]Among other things, this condition implicitly selects θ's normalization.

[8]Considering only positive investment simplifies the exposition, but has no substantive consequences.

the first period depends on the firm's expected second period action.

The value of the firm (net of rental cost of capital) in the second period, V^2, is the maximum over net revenue with and without adjustment:

$$V^2(\eta\theta, K_1) = \max \left\{ \max_{I_2} \Pi(\eta\theta, K_1 + I_2) - r \cdot (K_1 + I_2) - c \cdot K_1, \ \Pi(\eta\theta, K_1) - r \cdot K_1 \right\}.$$

Since the size of the adjustment cost is independent of the size of investment, it is apparent that if the firm adjusts, it chooses I_2 so that:

$$V_K^2 = 0.$$

Whether the firm adjusts or not, depends on whether the benefit from adjusting is larger or smaller than the fixed cost. Since the benefit of adjustment rises with the value of the profitability index in period 2, it is apparent that there is a level of $\eta\theta$, which we shall denote by U, above which the firm pays the fixed cost and invests. We can now go back to the first period and ask the question, what happens to q as we vary the first period's profitability index, θ? The economic forces are better illustrated if we focus on a range of values of θ for which there is no adjustment in the first period (so $K_1 = K_0$).

We define marginal q as the value of an extra unit of installed capital in period 1:

$$q = \pi_K(\theta, K_0) - r + \ \mathrm{E}\left[V_K^2(\eta\theta, K_0)\right].$$

Note that because we have included the rental cost of capital in V, q is "centered" around zero. Because $V_K^2 = 0$ if the firm adjusts in the second period, we may rewrite q as:

$$q = \pi_K - r + p\left(\eta < \frac{U}{\theta}\right) \mathrm{E}\left[V_K(\eta\theta, K_0)|\eta < \frac{U}{\theta}\right], \tag{1}$$

where $p(\cdot)$ denotes the probability that the firm does not adjust in period 2. π_K increases with θ, but the last term is ambiguous. As θ rises, the probability of *not* adjusting in the second period declines, while V_K conditional on not adjusting generally rises. We will show below that the effect that dominates depends on how near the firm is to adjustment. When θ is low and adjustment is distant, increases in θ raise q. Eventually, as θ rises the probability of adjustment in the second period rises sufficiently that q falls. Moreover, if θ rises so far as to prompt investment in the first period, then the first order condition for first period investment dictates that q must be equal to zero.

The previous discussion reveals that q is a non-monotonic function of θ. Since the firm's desired capital stock rises monotonically with θ, it follows that q is a non-monotonic function of desired investment. Therefore, the inverse of this function *does not exist*.[9]

Whereas marginal q bares little relation to the firms incentive to invest, average q behaves somewhat better. Average q is equal to the value of the firm per unit of capital, $V(\theta, K_0,)/K_0$. Since V is monotonically increasing in θ, desired investment will be monotonically related to average q after controlling for K_0.[10]

2.2 Back to the infinite horizon model

We now continue with the development of the full intertemporal model and make our claims more precise. The firm makes its investment decision in order to maximize the present value of profits. The Bellman equation for the firm's problem is:

$$V(\theta, K) = \max_I \ \Pi(\theta, K) - rK - c \cdot K \cdot 1_{\{I \neq 0\}} + \beta \int V(\eta\theta, K + I) \, \phi(\eta) \, d\eta, \qquad (2)$$

where the indicator function $1_{\{A\}}$ is equal to one if the event A occurs and zero otherwise.

It is easy to see that V is homogeneous of degree one in K and θ. Let $z \equiv K_t/\theta_t$, $v(z) \equiv V(1, z)$, and $\pi(z) \equiv \Pi(1, z)$. Dividing (2) by θ, we obtain:

$$v(z) = \max_I \ \pi(z) - (r + c1_{\{I \neq 0\}})z + \beta \int v(z') \, \eta \, \phi(\eta) \, d\eta, \qquad (3)$$

where

$$z' \equiv \frac{K + I}{\theta\eta} = \frac{z}{\eta} \left(1 + \frac{I}{K} \right),$$

represents the future value of the state variable z.

Proposition 1 below shows that given the homogeneity assumptions there exists a unique $v(z)$ that satisfies (3). All propositions are proved in the appendix.

Proposition 1 *There exists a unique solution to the functional equation (3).*

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[9] By adding a probabilistic adjustment rule in which the probability of investment is smoothly increasing with respect to the desire to invest —as opposed to the sharp (S, s) boundaries we study here— one can transform our statements about the relation between desired investment and q into statements about the (smooth) relation between actual average investment and q. We avoid this complication here. See Caballero and Engel (1994) for a probabilistic fixed-costs model relating desired with actual (average) investment.

[10] To see that increases in θ increase V note that for any given investment plan, an increase in θ increases the value of the firm. Choosing the optimal investment plan can only make the firm even better off.

We can now study the implications of fixed costs for marginal q in this model.

2.3 Marginal q

Following Abel and Blanchard (1986), we define marginal q_t as the present value of a marginal unit of investment,

$$q_t = \beta \int V_K(\eta\theta_t, K_{t+1})\,\phi(\eta)\,d\eta,$$

which, in terms of z, becomes,

$$q_t = \beta \int v'(z_{t+1})\,\phi(\eta)\,d\eta.$$

Since investment becomes productive with a one period delay, marginal q is the expected value of an additional unit of capital in the next period. Current profits are sunk and do not enter into the calculation of marginal q.

Once the firm decides to invest and incurs the fixed cost, it faces no further adjustment costs. The firm therefore chooses the level of investment that maximizes the present value of profits. The first order condition for investment in period t is, therefore:

$$q_t = 0. \tag{4}$$

Hence $q = 0$ whenever the firm invests.[11] It follows immediately from (4) that q cannot be a sufficient statistic for investment. A value of q equal to zero provides no information about the size or sign of investment. Since q is also equal to zero if the firm happens to posses a level of capital that maximizes the present value of profits, q does not allow one to distinguish between a firm with a pressing need to invest and a one that fortuitously possesses an optimal capital stock. In this setting it is not possible to think of investment as a function of marginal q.

Adding a convex cost of adjustment on top of our fixed cost does not solve the problem. Suppose that the cost function were instead $c + k(I)$ with $k(0) = 0$ and $k''(\cdot) > 0$. Then

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[11]Recall that because we have included the current and future cost of capital in V, our measure of marginal q is centered around zero not one. That is, rather than paying the machines at the time of investment, we have assumed the firm commits to an infinite stream of flow payments, with present value equal to the value of the machines.

conditional on investment, we would get a more standard first order condition,

$$k'(I) = q\left(\frac{K+I}{\theta}\right),$$

that would allow us to distinguish between various levels of investment I. The problem remains, however, that since adjustment with fixed costs is discrete there will be cases in which a firm that does not invest has the same q as a firm that has just invested. It is still not possible to write investment as a function of q.

Returning to the case of purely fixed costs, the fact that q is equal to zero when investment occurs, implies that either investment or q is zero at all times. Hence

$$E(I \cdot q) = 0$$

and

$$\text{Cov}(I, q) = E(I \cdot q) - E(q)E(I) = -E(q)E(I).$$

We shall see below that this implication of the theory makes it easy to construct examples in which the covariance between investment and q is negative. Furthermore, the fact that $q(z)$ is equal to zero more than once (in z-space), suggests that it is a non-monotonic function. Unfortunately, without placing more structure on the problem there is little that we can say about the shape of $q(z)$. We therefore turn to a convenient special case.

2.3.1 A special case

One particularly useful restriction is that $\ln \theta$ follows a simple random walk with drift:

$$\eta = \begin{cases} e^{\nu}, & \text{with probability } p; \\ e^{-\nu}, & \text{with probability } 1 - p. \end{cases}$$

where ν is a positive constant. We call this Assumption (*).

Assumption (*) has two appealing implications. First, if we set $\nu = \sqrt{\sigma^2 \Delta t + \mu^2 \Delta t^2}$ and $p = \frac{1}{2}[1 + \frac{\mu \Delta t}{\nu}]$, then θ approaches a geometric Brownian motion with drift μ and infinitesimal variance σ^2 as Δt approaches zero. Hence all of our results using this assumption will extend to this common class of continuous-time models. Second, with Assumption (*), the optimal investment policy and the value function take simple forms. We state these properties in Proposition 2.

Proposition 2 *Given Assumption (*), there exists a set of z, Z, with the following prop-*

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erties:

(a) $\mathcal{Z}$ *is ergodic.*

(b) $\mathcal{Z}$ *has a finite set of elements and is of the form:*

$$\mathcal{Z} = \{\underline{z}, \underline{z}e^{\nu}, \ldots z^* e^{-\nu}, z^*, z^* e^{\nu}, \ldots \bar{z}\},$$

where z^* is a maximum of $E_z v(z/\eta)\eta$.

(c) *For $z \in \mathcal{Z}$ the firms optimal policy is a two-sided (S,s) rule: if $z \leq \underline{z}$ or $z \geq \bar{z}$ the firm invests to set the value of z before the realization of η equal to z^*.*

(d) $v(z)$ *is quasiconcave on $\mathcal{Z}$.*

With Assumption (*) the optimal policy is an (S, s) rule. So long as z remains in the interval $(\underline{z}, \bar{z})$, the benefit of adjustment is not sufficient to justify the fixed cost and the firm does nothing; z rises and falls with the profitability index. As soon as z leaves the range of inaction, however, the firm incurs the fixed cost and adjusts its capital stock so that z reaches the level, z^*, that maximizes its expected discounted value.

We can use Proposition 2 to study the shape of q given Assumption (*). We know that q is equal to zero when $z = z^*$, $z \leq \underline{z}$, or $z \geq \bar{z}$. The quasiconcavity of $v(z)$ on $\mathcal{Z}$, implies that q is positive for $z \in (\underline{z}, z^*) \cap \mathcal{Z}$ and negative for $z \in (z^*, \bar{z}) \cap \mathcal{Z}$.

The non-zero q's in $\mathcal{Z}$ are related to each other by the Bellman equation. Differentiating equation (3) with respect to z_{t+1} and taking expectations as of period t yields,

$$q(z_t) = \beta E_t[\pi'(z_{t+1}) - r] + \beta E_t[q(z_{t+1})]. \tag{5}$$

Repeated substitutions, together with the fact that $q = 0$ at times of adjustment, yields,

$$q(z_t) = E_{z_t} \sum_{s=t+1}^{T} \beta^{s-t}[\pi'(z_s) - r], \tag{6}$$

where T denotes the first (random) time the state of the economy z reaches one of the investment triggers $\underline{z}$ or $\bar{z}$.

According to equation (6), q is the present value of marginal profitability up until the next time of investment. The equation illustrates the two forces that determine the shape of q. Given K, as θ rises (i.e., as z falls) the marginal profitability of capital rises. This

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effect argues that $q'(z) < 0$. Changes in z, however, also alter T, the time of the next adjustment. For the process in Assumption (*), and for many other standard processes, the time of the next investment is quasiconcave; it is lowest as z approaches the triggers $\underline{z}$ and $\bar{z}$ and highest when z is in the middle of the range of inaction. Reductions in T reduce the horizon over which q is calculated and push it closer to zero.

Figure 1 illustrates the "natural" shape of q as a function of z. It crosses zero at z^*. It is decreasing near the center of the interval $[\underline{z}, \bar{z}]$, reflecting the influence of the declining marginal profitability of capital. It bends back toward zero as it approaches the edge of the interval $[\underline{z}, \bar{z}]$, reflecting impending prospect of adjustment. It is zero outside the interval.

Figure 1 resembles the relationship between exchange rates and fundamentals in Flood and Garber's (1991) model of a target zone with discrete intervention.[12] The similarity is not a coincidence. Like Flood and Garber's exchange rate, our q is the price of an asset. It is a claim to a stream of marginal profits. There is therefore an arbitrage condition that links the value of q before and after investment. This condition is the source of the non-monotonicity in $q(z)$.

Not surprisingly the unconventional behavior of q may be attributed to the fixed costs. In models with convex adjustment costs, a positive profitability shock raises the marginal profitability of capital, and the incentive to invest. Since investment is "small" due to the convex costs, the future marginal profitability of capital rises as well. q, being the present value of marginal profits, therefore rises. With fixed costs, on the other hand, it is still true that a positive profitability shock raises the marginal profitability of capital and the incentive to invest. It is future marginal profits that do not necessarily rise. Because investment is lumpy, future marginal profits fall when investment is imminent, so that q falls as investment approaches.

With Assumption (*) it is easy to construct examples in which the covariance between investment and q is negative. Suppose, for example, that there are only positive profitability shocks ($p = 0$), so that z only falls in the absence of investment. In this case the firm always remains in the interval $[\underline{z}, z^*]$. Both investment and q are non-negative and sometimes positive, so $\text{Cov}(I, q) = -\text{E}[I]\text{E}[q] < 0$.[13]

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[12]It is also related to Dixit's (1993, p. 44) depiction of optimal regulation.

[13]*Aggregation.* The possibility of a negative correlation between investment and marginal q arises naturally from the fact that when investment takes place, q is always zero. While this is true for an individual firm, there is no reason to expect it to be true for aggregates. The fact that some firms invest does not imply that q is on average zero across firms. Aggregation, therefore, may alter the correlation between investment and q. To illustrate this possibility consider a market with two identical firms indexed by i and j. Let the

2.4 Average q

While marginal q is the appropriate concept in the standard convex-adjustment cost model, it is rarely used in empirical work. Instead, researchers often appeal to the "theoretically incorrect" but more measurable, average q.[14]

Paradoxically, it turns out that with fixed costs of adjustment, average q may do better than marginal q as a right hand side variable in an investment regression. Indeed, we showed above that marginal q does not work, while here we describe assumptions under which average q is a sufficient statistic for investment. Although we also argue that under these circumstances profits and sales perform as well as average q.

As is standard, we define average q, which we denote by Q, as the value of the firm per unit of capital:

$$Q(z) = \frac{V(\theta, K)}{K} = \frac{v(z)}{z}.$$

In the next Proposition, we show that given our homogeneity assumptions Q is strictly decreasing in z. It follows immediately that observers can recover z from Q and that Q is a sufficient statistic for investment.[15]

Proposition 3 *Average Q is strictly decreasing in z.*

It is easy to see why Q performs so well. A little algebra shows that:

$$\frac{dQ}{dz} \sim -V_\theta(\theta, K).$$

Therefore the monotonicity of Q is a reflection of the fact that a positive profitability shock increases the value of the firm for any given capital stock.

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aggregates $\bar{q}$ and I denote market-wide averages of marginal q and investment respectively. It follows that:

$$\mathrm{Cov}(\bar{q}, I) = \frac{1}{2}E\{q_i \cdot I_j\} + \mathrm{Cov}(q_i, I_i).$$

The second term is exactly as before. It is easy to construct examples in which the first term is positive.

[14] As noted earlier, Hayashi (1982) derived the conditions under which marginal and average q coincide. While illuminating, these assumptions, which include perfect competition in all markets as well as constant returns to scale, are unlikely to be met in practice, especially by the large firms that are often used in empirical studies. They are often invoked to theoretically justify what is truly a selection dictated by data availability rather than by a believe on the underlying assumptions.

[15] Our homogeneity assumptions differ from those in Hayashi (1982). Hayashi requires that both the flow profit and the adjustment cost be homogeneous in capital and investment. Our model necessarily violates both of these. Optimal investment in our model would be infinite if the flow profit were homogeneous in capital, and a fixed cost is by definition independent of the level of investment. Instead, our model requires that the flow profit and the adjustment cost be homogeneous in capital and profitability.

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It is important to realize, however, that sufficiency in our case represents a substantially simpler and different result from that which makes the standard q-theory so appealing. In the latter, q (and possibly Q) is a powerful summary of all the information —possibly many state variables— about the future that is relevant for current investment. In our context, on the other hand, we have reduced the problem to the simple case where a single state variable, z, is all that is needed to plan investment. Any variable monotonically related to it will serve the role of a sufficient statistic. Q is one of them, but so are $\pi(z)$ and $\pi'(z)$. The statistic that performs better in a simple investment equation will depend on the non-linearities of the model and the nature of measurement error.

If either the homogeneity conditions fail or the shock does not follow a random walk, then Q generally will not be a sufficient statistic for investment. The reason is that in such cases there will be no single statistic that describes the state of the firm. With many state-variables Q, and cash flow will convey different information. Cash flow will mostly reflect immediate factors, while Q will reflect both present and future influences on investment. It is likely that both would prove to be significant predictors of investment. The significance of cash flows in this case would therefore not signal the presence of liquidity constraints or other capital market imperfections. Instead it would reflect the fact that in a world of many state variables a single variable like Q may not capture all available information.[16]

3 Conclusion

The standard formulation of q-theory, which posits a monotonic relationship between marginal q and investment, is elegant but of limited practical use. It depends critically on auxiliary assumptions, such as convex adjustment costs or perfect competition and constant returns to scale, that are not likely to hold in practice. We have shown that in a more realistic model in which these assumptions are violated, this standard formulation of q-theory breaks down and marginal q ceases to represent a firm's incentive to invest.

While marginal q performs poorly, we show that average q may still perform quite well. In a sense, this is not surprising; while marginal q is the construct of a very specific optimization problem, q-theory as originally conceived by Tobin is closer to a generic ar-

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[16]We are not arguing against liquidity constraints, which we believe affect many firms. Rather, we are arguing against the claim that by putting Q on the right hand side of investment regressions, the researcher has controlled for all information relevant for investment when there are no credit constraints. Gilchrist and Himmelberg (1994) argue similarly, although the source of the problems they highlight is measurement error in Q rather than misspecification and omitted variables, which is what is behind our argument.

bitrage condition, and is therefore more robust to changes in the specifics of the economic environment.

Our arguments imply that average q should be a useful right hand side variable in applied investment equations. It is important, however, not to take these arguments too far. For example, we have shown that average q is more naturally related to investment than is marginal q, but the reason for this also makes other variables, such as cash flows, good candidates for right hand side variables in investment equations.

4 Appendix: Proofs of the Propositions

Proof of Proposition 1

It is inconvenient to work with $v(z)$ since neither the current period payoff $\pi(z) - rz$ nor the cost of adjustment cz is bounded as z approaches infinity. Instead we analyze $\tilde{v}(z) = v(z) - \pi(z) + rz + cz$. Note that this does not affect the optimization problem since z is fixed at the time of the investment decision. $\tilde{v}(z)$ is bounded above by $\frac{\beta}{1-\beta} \max_z \pi(z) - rz < \infty$ which is the maximum possible payoff, and bounded below by $\frac{\beta}{1-\beta} (-c + \max_{\tilde{z}} E\pi(\tilde{z}/\eta)\eta) > -\infty$, which is the value of adjusting each period.

Let $\mathcal{V}$ be the space of bounded, continuous functions. Given a function $g(z)$, consider the mapping $g \to Mg$:

$$
(Mg)(z) = \max\left\{ cz + \beta \int [g(z/\eta) + \pi(z/\eta) - rz/\eta]\, \eta\, \phi(\eta) d\eta, \right. \tag{7}
$$
$$
\left. \max_{\tilde{z}} \beta \int \left[g(\tilde{z}/\eta) + \pi(\tilde{z}/\eta) - \frac{r\tilde{z}}{\eta} - \frac{c\tilde{z}}{\eta} \right] \eta\, \phi(\eta) d\eta \right\}.
$$

It is easy to see that if $g \in \mathcal{V}$ then $Mg \in \mathcal{V}$. $\tilde{v}(z)$ is a fixed point of M should one exist. Since M satisfies Blackwell's conditions for a contraction mapping, the existence and uniqueness of such a fixed point follows from the contraction mapping theorem. Therefore, $\tilde{v}(z)$ exists and is unique. It follows that $v(z)$ exists and is unique.

Proof of Proposition 2

Let z^* maximize $E_z v(z/\eta)\eta$. As the firm is indifferent between z^* and other potential adjustment targets, we assume that the firm adjusts to z^* each time it adjusts.

Since z follows a log random walk in the absence of adjustment, z will deviate arbitrarily far from z^* with probability one if the firm does not adjust. Therefore the firm will eventually adjust its z' to z^*. The form of the ergodic set and the optimal policy follow immediately.

We show that v is quasiconcave on $\mathcal{Z}$ by contradiction. Suppose otherwise. Then there exists a point $\bar{z} \in \mathcal{Z}$ that is a local minimum situated between to local maxima. Now consider three firms: one begins at $\bar{z}$ and others begin at $\bar{z}e^\nu$ and $\bar{z}e^{-\nu}$. Consider arbitrary sequences of η and compare the firms' payoffs. Note that in each period prior to adjustment the firms occupy adjacent states and that their period payoffs are concave. Note also that the firm that began at $\bar{z}$ reaches a local maximum before it adjusts, and that at this point the continuation payoffs are concave. Hence the initial values must have been concave. This

contradiction completes the proof.

Proof of Proposition 3

Using the definition of average q and the homogeneity of V:

$$Q = \frac{V(\theta, K)}{K} = V(1/z, 1).$$

Differentiating with respect to z:

$$\frac{dQ}{dz} = -\frac{1}{z^2} V_\theta.$$

Note $z > 0$. A simple argument shows that $V_\theta > 0$. Consider two firms. Suppose that one firm is following an optimal investment strategy given the initial state $z_0 = K_0/\theta_0$. Suppose that the other firm has the same capital stock K_0, but has a higher level of the profitability index $\hat{\theta}_0 > \theta_0$. Now if the second firm mimics the first firm, the second firm will incur the same cost of adjustment and will earn strictly higher flow profits. Since an optimal policy for the second firm must do better than this arbitrary policy, we have $V(\hat{\theta}_0, K_0) > V(\theta_0, K_0)$. Hence $V_\theta > 0$ and $dQ/dz < 0$. This completes the proof.

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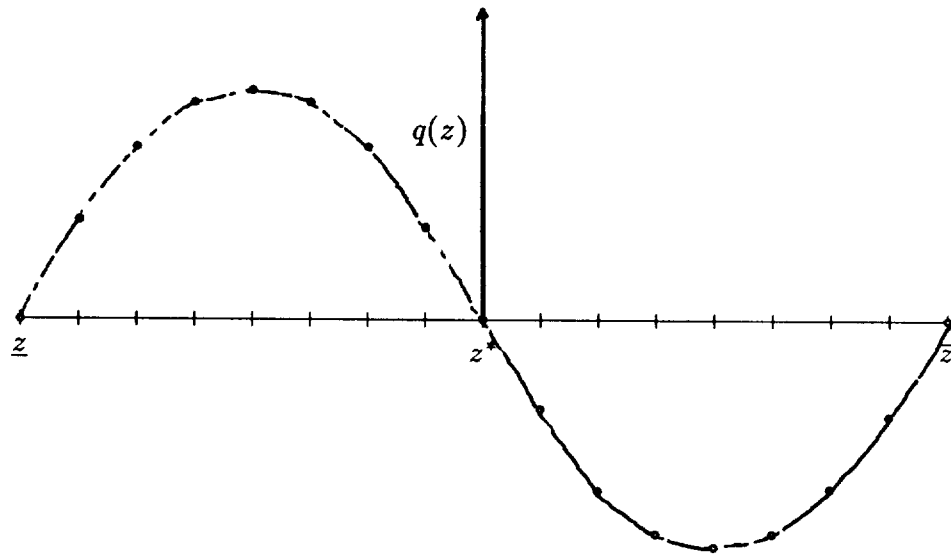
Figure 1. Marginal q **as a function of** z.